Indirect Taxation and Privatization in a Model of Government's Preference

Choi, Kangsik

30 November 2012

Indirect Taxation and Privatization in a Model of Government's Preference

Kangsik Choi[†]

Abstract

By introducing the government's preference for tax revenues into unionized mixed duopolies, this paper investigates how the preference can change the government's choice of tax regimes between ad valorem and specific taxes. Main results are as follows. Given that one of the tax regimes is predetermined, privatization never improves welfare and privatization is preferable for the government when it emphasizes its tax revenue. However, when the tax regime is *endogenously* determined by the government, privatization is preferable from the viewpoint of social welfare if the government heavily emphasizes its tax revenue. Thus, there are conflicts of interest between the public firm and the government: If it heavily emphasizes its tax revenue, then the government always has the incentive to levy specific tax, while the public firm has the incentive to be levied by ad valorem. However, there are no conflicts of interest between the public firm and the government when the government levies the specific tax if the government less emphasizes its tax revenue. Interestingly, the government never has the incentive for privatization if the government considers either tax as an option.

JEL Classification: C79, D43, H22, H44, J51, L13, L33.
Keywords: Ad Valorem, Specific Tax, Government's Payoff, Social Welfare, Privatization.

1 Introduction

It is well known that ad valorem taxes dominate specific taxes from the welfare perspective, as evidenced by analysis using the Cournot-type model of Seade (1985) and Delipalla and Keen (1992), among others (see also Keen's (1998) comprehensive survey paper). Surprisingly, in the literature on the mixed oligopoly, there has been few analyses of socially optimal taxation weigh the advantages of ad valorem and specific taxes. Mujumdar and Pal (1998) exceptionally showed that privatization could increase both social welfare and tax revenues, where an increase in tax does not change the total output but increases the output of the public firm and the tax revenues.

As stated above, concerning private firms, canonical arguments for comparing ad valorem and specific taxes implicitly assume that the government's (or social planner's) objective is to maximize social welfare. Given that with such government's objective, setting specific tax is adjusted ensures that both specific and ad valorem taxes lead to the same level of an industry output, there exists an ad valorem tax that yields the same social welfare with a higher tax revenue for any given specific tax. On the other hand, the public firm's welfare-maximizing behavior is assumed to be in support of the objective of benevolent government. Thus, it is generally understood that the public firm, as well as the government, traditionally maximizes the sum of the tax revenues or subsidies and the consumer and producer surpluses. However, in the real world, some conflicts of interest exist between the public firm in an industry and the government. For example, "Ministers were left to free to adopt their own definition of the public interest$\cdots$, and there have been frequent political interventions to influence operational decision

[†]Graduate School of International Studies, Pusan National University, Busandaehak-ro 63 beon-gil 2, Geumjeong-gu, Busan 609-735, Korea, Republic of. Tel: +82-51-510-2532. E-mail: choipnu@pusan.ac.kr

making" (Vickers and Yarrow, 1988, p. 128). Therefore, we investigate how the preference can change the government's choice of tax regimes between ad valorem and specific taxes when the public firm and the government have different objective functions in a model of mixed duopoly.

Consequently, the main motivation to the literature is that in this paper, the government does not simply maximize social welfare. That is, we assume that the public firm maximizes social welfare as usual definition, and the government attaches weight to both social welfare and its preference for tax revenues (i.e., a policy defined as government's "tax-inclusive social welfare," hereinafter called government's payoff. See also Section 2.). While the criterion for relative tax efficiency in the literature is a higher tax revenue for a *given total output* under fairly mild conditions, the government in this paper needs to indirectly control the level of industry output to balance social welfare and its tax revenue policy. It does so by choosing between the specific and ad valorem taxes as the optimal type of taxes[1].

In fact, the present study differs from the existing literature in at least two important ways. First, the existing studies on mixed oligopolies consider a monolithic entity that seeks to maximize social welfare at both the public firm and the government level. Second, prior studies on unionized mixed oligopolies mainly focus on Cournot competition without tax effects, whereas our study investigates tax effects, and compares social welfare with the government's payoff. Indeed, it also corresponds to the empirical results that in Europe, Japan and US, the government is heavily involved in the setting of public sector wages (Bordogna, 2003; Rose, 2004; Du Caju *et al.*, 2008). To investigate the optimal privatization policy, incorporating unions' behavior into the different objectives of the government and the public firm can serve to explain the government's use of taxes as a commitment device to control the unions' wage demands. Generally, a higher tax forces down both public and private firms' wages since wages are strategic complements between the unions[2]. This is why unions' behavior is considered in this paper.

The present study shows several results: Given that one of the tax regimes is predetermined, privatization never improves welfare and privatization is preferable for the government when it emphasizes its tax revenue. However, when the tax regime is *endogenously* determined by the government, privatization is preferable from the viewpoint of social welfare if the government heavily emphasizes its tax revenue of ad valorem tax. Thus, there are conflicts of interest between the public firm and the government. That is, if the government heavily emphasizes its tax revenue, then the government always has the incentive to levy specific tax, while the public

[1] If the setting specific tax is adjusted ensures that both specific and ad valorem taxes lead to the same level of industry output, there exists an ad valorem tax that yields the same social welfare with *higher government's tax revenue* as any given specific tax. Under this setting as in the literature, even if the government attaches weight to both social welfare and its preference for tax revenues, there are no conflicts of interest between the public firm and the government. This irrelevant case is excluded in the present paper.

[2] From real world examples related to the objective of the government and labor union, the present strength of either German or U.K. trade unions is major impediment to any privatization (Bos, 1991, pp. 3-6). From the recent empirical study, Bordogna (2003, pp. 62-63) pointed out that "even where bargaining has been decentralized, governments have often maintained strong, centralized, financial controls in order to contain public expenditures and avoid inflationary consequences of the decentralization process." See also Brock and Lipsy (2003) and references therein.

firm has the incentive to be levied by ad valorem. However, there are no conflicts of interest between the public firm and the government when the government levies the specific tax if the government's preference for tax revenues is sufficiently small. Interestingly, the government never has the incentive for privatization if the government considers a choice of tax regimes between ad valorem and specific taxes.

2 Relationship to the Literature

In this section, we discuss the relationship between our paper and some other previous theoretical or empirical papers on government's preference for tax revenues and closely follow Choi (2011).

We present some rationale for discussing objective functions from the perspective of government objectives. First, it has been argued in the literature that there is another way for fiscal centralization to limit the discretionary power of the government when a Leviathan government exists. Therefore, the literature contains a number of puzzles for fiscal centralization and the size of the public sector (Oates, 1989). For example, Brennan and Buchanan (1980) formulated the hypothesis of a Leviathan government attempting to maximize revenue for its own private agenda[3]. A similar idea lies behind Niskanen's (1971) model of a budget-maximizing bureau, although the bureau interacts with the government rather than with voters. This political power may fit with an analysis of the government[4].

Second, the objective of the public firm is not the same as that of the government because the government may find it hard to control some public firms. The advantage of the public firm is that it allows the government to distance itself from public sector activities that create discontent. This reducing of political interference is widely perceived as improving social efficiency. According to Vickers and Yarrow (1988), in the postwar period in the UK, a principal objective of the legislation that established public corporations was to create an "arm's length" relationship between government and management. Unsurprisingly, managers (of the public firms) and politicians took full advantage of the discretion allowed them (Vickers and Yarrow, 1988, pp. 127-133). As argued above, we can justify the viewpoint that the objective of the public firm is to promote social welfare, while that of the government is to secure both the social welfare and its attainment of tax revenues.

A recent paper that comes closest to differentiating the government objective function is that by Kato (2008), who only focuses on the specific tax. Kato (2008) showed that the government's privatization of the public firm would depend on its preference for tax revenues if the following two assumptions hold true. First, that the public firm gives full weight to social welfare net

[3] In theoretical studies of the Leviathan government, Edwards and Keen (1996) and Rauscher (2000) used formalized tax-competition models to address the issue and showed that the results of tax competition are ambiguous. For more detailed treatment of the Leviathan government, recent theoretical and empirical studies include Oates (1989), Keen and Kotsogiannis (2002) and Brülhart and Jametti (2006).

[4] According to Persson and Tabellini (2002, p. 7), the government raises taxes and spends the resulting revenue in three ways: (1) on general public goods; (2) on narrowly targeted redistribution to well-defined groups; (3) on rents for politicians.

of tax revenues (which is defined as the sum of the consumer and producer surpluses), and second, that the government attaches weight to both social welfare net of tax revenues and its preference for tax revenues. Instead of following Kato (2008) in ascribing to the government different objectives from those of the public firm, Choi (2011) demonstrated that regardless of the government's preference for tax revenues and the number of private firms, the government and the public firm do not always have an incentive to privatize the public firm *without* choice of tax regimes. In the sense of the specific tax as involving a transfer within the economy, Choi (2011) differs from Kato's (2008) assumption of the public firm.

3 The Model

Consider a mixed-duopoly situation for a homogeneous good that is supplied by a public firm and a private firm. Firm 1 is a profit-maximizing private firm and firm 0 is a public firm that maximizes the social welfare. Assume that the inverse demand is characterized by $p = 1 - x_0 - x_1$, where p is the price of the good, x_0 is the output level of the public firm and x_1 is the output level of the private firm.

On the demand side of the market, the representative consumer's utility is a quadratic function given by

$$V = x_0 + x_1 - \frac{1}{2}(x_0 + x_1)^2.$$

We assume that the public and private firms are unionized and that the firms are homogeneous with respect to productivity. Given that $L_i (i = 0, 1)$ is the number of ith firm workers, each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good. The price of labor (i.e., wage) that firm i has to pay is denoted by $w_i, i = 0, 1$. Let $\overline{w}$ denote the reservation wage. Taking $\overline{w}$ as given, the union's optimal wage-setting strategy, w_i regarding firm $i = 0, 1$, is defined as:

$$\max_{w_i} \ U_i = (w_i - \overline{w})^{\theta} L_i; i = 0, 1,$$

where θ is the weight that the union attaches to the wage level. Following Ishida and Matsushima (2009) in the literature on the unionized mixed duopoly, we assume that $\theta = 1$ and $\overline{w} = 0$ to demonstrate our results simply[5]. That is, the utility function of the union at the firm is its wage bill: $U_i(w_i; L_i) = w_i L_i = w_i x_i$. Thus, we consider the monopoly union model, which assumes that the unions set the wage while the firms choose the employment level once the wage is set by unions (see also Booth, 1995).

[5]As Ishida and Matsushima (2009), Barcena-Ruiz and Garzon (2009), Horn and Wolinsky (1988) and Haucap and Wey (2004) have suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit, as a special case of Nash bargaining solution, the monopoly union model (Oswald, 1982) is frequently adopted. Even if the present model of union's utility looses generality, the wage bill maximization enables us to gain insights otherwise very complicated when comparing a specific tax with an ad valorem tax under either privatization or mixed duopoly.

In what follows, we assume that either an ad valorem tax or a specific tax rate is imposed on the public and private firms. To distinguish notations, the superscript "s" (respectively, "v") is defined when the specific (respectively, ad valorem) tax rate is imposed on the public and private firms. In the case of an ad valorem tax t^v, the producer price, p_f for good obtains as $p_f = p(1 - t^v)$ where p the consumer price for good. If a specific tax is imposed, the producer price is defined by $p_f = p - t^s$. Thus, introducing ad valorem taxation at the (tax inclusive) rate t^v and a specific tax of t^s, each firm's profit follows the function

$$\pi_i = (p - w_i)x_i - t^s x_i, \quad \pi_i = (1 - t^v)px_i - w_i x_i, \quad i = 0, 1,$$

respectively. The profit expression is the same when taxes are set such that $w_i + t^s = w_i/(1 - t^v) \Leftrightarrow t^s = t^v w_i/(1-t^v)$, if the total output is fixed to be the same under the two taxes. However, as stated in Introduction, we do not consider this case since the government has preference for tax revenues to balance social welfare and the government's tax revenues by choosing between the specific and ad valorem taxes as the optimal type of taxes.

As usual, social welfare, $W^j, j = s, v$ can be defined as the sum of consumer surplus CS, producer surplus PS, the utilities of unions U, and total tax revenue $T^j, j = s, v$ collected by the government. Thus, the public firm aims to maximize social welfare, which is defined as

$$W^j = CS + PS + U + T^j, \quad j = s, v, \tag{1}$$

where $PS = \pi_1 + \pi_2$, $CS = V - \sum_{i=0}^{1} px_i$, $U = U_1 + U_0$ and $T^s = t^s(x_0 + x_1)$ (respectively, $T^v = t^v p(x_0 + x_1)$) denotes the tax revenues when the specific (respectively, ad valorem) tax is imposed on both firms. As tax revenues collected under each type of tax, T^j is a transfer within the economy.

Furthermore, we also assume that the government's payoff, G^j, is given by

$$G^j = CS + PS + U + (1 + \alpha)T^j - (t^j)^2 = V + \alpha T^j - (t^j)^2, \quad j = s, v, \tag{2}$$

where $\alpha(> 0)$ is the parameter that represents the weight of the government's preference for tax revenues, $(t^j)^2$ captures the cost of raising tax[6], and $\alpha T^j - (t^j)^2$ is defined as net tax revenues. Here, the government values the tax revenues, T^j, more than social welfare, W^j when $\alpha > 1$. Otherwise, the government values the tax revenues less than social welfare when $1 > \alpha > 0$. For the setup of different objectives, we introduce strict convexity of the cost function as an

[6] If this cost is not allowed, the government's payoff can indefinitely raise its ad valorem tax rate because the optimal ad valorem tax level of the government is independent of the government's preference for tax revenues. More clear extensions and government's cost analysis of depending on total output are left to future research to develop the analysis more generally. Another possible way to solve this problem is the taxes collected by the government is βT, $1 > \beta$. Parameter β represents the percentage of the taxes paid by firms that goes to the government; the remaining $1 - \beta$ percent represents the social cost of collecting the taxes (e.g., bureaucracy). Taking into account this assumption, social welfare is $W = CS + PS + U + \beta T$ and the government's payoff can be defined as: $G = V + (\alpha + \beta)T$. Generally, incorporating β into a transfer within the economy affects social welfare and government's payoff in complicated ways. As a start, this paper analyzes how the measures of W as in (1) and $G = V + \alpha T - t^2$ are affected by the government.

inefficiency of the government when it prefers tax revenues, to endogenize that inefficiency in collecting and comparing ad valorem and specific taxes[7].

Finally, the timing of the game is as follows. In the first stage, the government chooses whether or not to privatize the public firm, and simultaneously determines either the specific tax or ad valorem tax rate on the public and private firms. In the second stage, union i chooses its wage, w_i, after being made aware of each type of tax rate. In the third stage, firm i chooses its output x_i simultaneously to maximize its respective objective, knowing each type of tax of the government and the wage levels.

4 Results

Before comparisons of indirect taxation and market type with the government's payoff and social welfare, two cases are distinguished between the unionized mixed and privatized duopolies[8]: (i) the case where the tax is fixed by a specific tax rate; and (ii) the case where the tax is fixed by an ad valorem tax rate. Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

4.1 Unionized Mixed Duopoly

First, we consider specific tax under unionized mixed duopoly. In this case, the public firm's objective is to maximize social welfare, which is defined as the sum of the consumer surplus, individual firms' profits, unions' utilities and the specific tax revenues. Thus, given t^s and w_i for each firm i $(i = 0, 1)$, the public firm's maximization problem is as:

$$\max_{x_0} \quad W^s = V \quad \text{s.t.} \quad (p - w_0 - t^s)x_0 \geq 0.$$

As in Ishida and Matsushima (2009), the constraint implies there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint[9].

If the multiplier of the budget constraint is denoted as λ^s, the Lagrangian equation can be written as

$$L(x_0, \lambda^s) = V + \lambda^s(x_0 - x_0^2 - x_1x_0 - w_0x_0 - t^s x_0). \tag{3}$$

Given the specific tax rate, t^s, and the wage-levels, w_i, by solving the first-order conditions (3), we obtain

$$\frac{\partial L}{\partial x_0} = 1 - x_1 - x_0 + \lambda^s(1 - 2x_0 - x_1 - w_0 - t^s) = 0, \tag{4}$$

$$\frac{\partial L}{\partial \lambda^s} = 1 - x_1 - x_0 - w_0 - t^s = 0. \tag{5}$$

[7] This view of the government can be a focus for rent-seeking, in which the government is seen arguably as bureaucrats' or politicians' interests (e.g. Besley, 2006, chapter 1).

[8] We will discuss indirect taxation models as if the government chooses either mixed duopoly or privatization. Later, we will mention this why we use the fixed decision.

[9] In this model, if the public firm does not face the budget constraint, the public firm's union can indefinitely raise its wage because the optimal output level of the public firm is independent of the wage.

On the other hand, the optimal output for a private firm is given by

$$x_1 = \frac{1}{2}(1 - x_0 - w_1 - t^s). \tag{6}$$

Given these results, we now obtain the output level for each firm. By solving the first-order conditions, (4), (5) and (6), we obtain,

$$x_0 = 1 - t^s - 2w_0 + w_1, \quad x_1 = w_0 - w_1, \tag{7}$$

$$\lambda^s = \frac{x_1 + x_0 - 1}{1 - 2x_0 - x_1 - w_0 - t^s}. \tag{8}$$

For solving the first-order conditions of the Lagrangian equation, the budget constraint is momentarily treated as binding. We check ex-post whether this omitted constraint is binding[10].

In the second stage, the equilibrium wages, denoted as w_i^s is obtained by maximizing $U_i = x_i w_i$. In addition, the substitution of each optimal wage into (7) yields the respective equilibrium outputs, x_i^s. The equilibrium wages and outputs, w_i^s and x_i^s, respectively, can be obtained as:

$$w_0^s = \frac{2(1-t^s)}{7}, \quad w_1^s = \frac{1-t^s}{7}, \quad x_0^s = \frac{4(1-t^s)}{7}, \quad x_1^s = \frac{1-t^s}{7}. \tag{9}$$

We now move to the first stage of the game. From (9), the government's payoff, G^s, in the mixed duopoly can be rewritten as:

$$\max_{t^s} \; G^s = \frac{5(1-t^s)[14(1+\alpha t^s) - 5(1-t^s)] - 98(t^s)^2}{98}.$$

Straightforward computation yields the optimal tax rate as:

$$t^s = \frac{35\alpha - 10}{123 + 70\alpha}. \tag{10}$$

If the weight of the government's preference for tax revenues is sufficiently large as in the case of $\alpha > \frac{2}{7}$, the optimal specific tax rate becomes positive. Conversely, when it is small as in the case of $0 < \alpha < \frac{2}{7}$, the optimal specific tax rate becomes negative, and in the case of $\alpha = \frac{2}{7}$, the optimal specific tax rate is zero. We find that the greater the weight of the government's preference for tax revenues, the higher will be the specific tax rate that the government imposes. Thus, by using (10), we obtain the following result.

Lemma 1: *Suppose that the specific tax rate is imposed on the public and private firms. Then, the equilibrium wages, output, union's utilities, government's payoff, social welfare, consumer surplus and private firm's profit levels under a unionized mixed duopoly are given by*

[10]Some readers may argue that the budget constraint may be non-binding off equilibrium paths. Kuhn-Tucker conditions off equilibrium paths for any values of (x_i, w_i, t^s) are available from author upon request.

$$
\begin{aligned}
w_0^s &= \frac{38+10\alpha}{123+70\alpha}, \quad x_0^s = \frac{76+20\alpha}{123+70\alpha}, \quad U_0^s = \frac{(38+10\alpha)(76+20\alpha)}{(123+70\alpha)^2};\\
w_1^s &= \frac{19+5\alpha}{123+70\alpha}, \quad x_1^s = \frac{19+5\alpha}{123+70\alpha}, \quad U_1^s = \frac{(19+5\alpha)^2}{(123+70\alpha)^2};\\
G^s &= \frac{14145+14200\alpha+6575\alpha^2+1750\alpha^3}{2(123+70\alpha)^2}, \quad W^s = \frac{14345+14700\alpha+2875\alpha^2}{2(123+70\alpha)^2};\\
\pi_1^s &= \frac{(19+5\alpha)^2}{(123+70\alpha)^2}, \quad CS^s = \frac{9025+6650\alpha+1125\alpha^2}{2(123+70\alpha)^2}.
\end{aligned}
$$

From Lemma 1, it should be noted that the public firm would like to maximize W^s by setting the output at its zero profit level ($p = w_0^s + t^s$). Ordinarily, in the absence of unions' wage-setting power and also $\alpha = 0$, the government would give firms to a subsidy to derive price down to the marginal cost. However, for $\alpha > 0$, the price would remain above the marginal cost, and a subsidy would encourage the unions to set higher wages. Therefore, a tax is to be used to control the unions' wage demands. By substituting Lemma 1 into (8), we obtain

$$\lambda^s = \frac{-(28+45\alpha)}{-(51+48\alpha)} > 0,$$

which shows that the budget constraint is binding.

Second, we consider ad valorem under unionized mixed duopoly. Given the ad valorem tax, t^v and w_i for each firm $i = 0, 1$ in the third stage, the public firm's maximization problem is as:

$$\max_{x_0} \quad W^v = V \quad \text{s.t.} \quad (1-t^v)px_0 - w_0 x_0 \geq 0.$$

Denoting the multiplier of the budget constraint λ^v and repeating the same process as in previous case yields the first-order conditions of the Lagrangian equation with respect to x_0 and λ^v with the optimal output for a private firm[11]:

$$x_0 = \frac{1-t^v-2w_0+w_1}{1-t^v}, \quad x_1 = \frac{w_0-w_1}{1-t^v}, \tag{11}$$

$$\lambda^v = \frac{x_1+x_0-1}{(1-t^v)(1-2x_0-x_1)-w_0}. \tag{12}$$

In the second stage, given the output as a function of wage, each union at each firm sets the wage, w_i^v, that maximizes union rent, U_i^v. Straightforward computation yields the equilibrium wage and output. Repeating the same process as in previous cases yields the first-order conditions of the government's payoff, $G^v = [45 + 20\alpha t^v - 98(t^v)^2]/98$. That is, the optimal tax rate is given by $t^v = 5\alpha/49$. Thus, by using t^v, we have the following result.

Lemma 2: *Suppose that the ad valorem tax rate is imposed on the public and private firms. Then, the equilibrium wages, outputs, union's utilities, government's payoff, social welfare, consumer surplus and private firm's profit under a unionized mixed duopoly are given by*

[11] To solve for the Lagrangian equation, suppose that the budget constraint is momentarily binding. We check ex post that this constraint is binding.

$$w_0^v = \frac{98 - 10\alpha}{343}, \quad w_1^v = \frac{49 - 5\alpha}{343}, \quad x_0^v = \frac{4}{7}, \quad x_1^v = \frac{1}{7}, \quad U_0^v = \frac{392 - 40\alpha}{2401};$$
$$U_1^v = \frac{49 - 5\alpha}{2401}, \quad G^v = \frac{2205 + 50\alpha^2}{4802}, \quad W^v = \frac{45}{98}, \quad CS^v = \frac{25}{98}, \quad \pi_1^v = \frac{49 - 5\alpha}{2401}.$$

Lemma 2 suggests that as α becomes large, each firm's wage level decreases, and each union's utility thereby decreases since the ad valorem tax rate affects only the wage levels. Hence, the private firm's profit has a negative value if the government's preference for tax revenue is sufficiently high (i.e., $\alpha > 9.8$). By substituting Lemma 2 into (12), we obtain

$$\lambda^v = \frac{-33614}{-(67728 - 51450\alpha + 2450\alpha^2)} > 0,$$

which shows that the budget constraint is binding.

4.2 Unionized Privatized Duopoly

This subsection investigates the equilibrium of a unionized privatized duopoly in the case of indirect taxes. As discussed in the basic model, consider the situation of a unionized privatized duopoly for a homogeneous good that is supplied by a profit-maximizing private firm ($k = 1, 2$).

First, we consider specific tax under unionized privatized duopoly. In the third stage, given w_k and t^s, the firm k's profit-maximization problem is to maximize $\pi_k = (p - w_k - t^s)x_k$ where $p = 1 - x_1 - x_2$. Hence, the symmetry across private firms implies that each output level is given by

$$x_k = \frac{1 - t^s - 2w_k + w_l}{3}, \quad k \neq l; k, l = 1, 2. \tag{13}$$

In the second stage of this case, each wage is set to maximize its firm's union utility: $U_k = x_k w_k$. Repeating same process of previous subsection, straightforward computations and symmetry across private firms yield each firm's wage and output:

$$w_k = \frac{1 - t^s}{3}, \quad x_k = \frac{2(1 - t^s)}{9}. \tag{14}$$

Turning to the first stage and using the equilibrium outputs and wages, the government's payoff, G^s, in a unionized privatized duopoly can be rewritten as:

$$\max_{t^s} \ G^s = \frac{4(1 - t^s)[9(1 + \alpha t^s) - 2(1 - t^s)] - 81(t^s)^2}{81}.$$

Straightforward computation yields the optimal tax rate in the unionized privatized duopoly as:

$$\hat{t}^s = \frac{18\alpha - 10}{89 + 36\alpha}. \tag{15}$$

If the weight of the government preference for the tax revenues is sufficiently large (as in the case when $\alpha > \frac{5}{9}$), the optimal tax rate becomes positive. Conversely, when it is small (as in the case when $\alpha < \frac{5}{9}$), the optimal tax rate becomes negative. Further in the case when $\alpha = \frac{5}{9}$, the optimal tax rate is zero. Similar to the previous subsection, we have the following result.

Lemma 3: *Suppose that the specific tax rate is imposed on the private firms. Then, the equilibrium wages, outputs, union's utilities, government's payoff, social welfare, consumer surplus and private firm's profit levels under a unionized privatized duopoly are given by*

$$\hat{w}_k^s = \frac{33+6\alpha}{89+36\alpha}, \quad \hat{x}_k^s = \frac{22+4\alpha}{89+36\alpha}, \quad \hat{U}_k^s = \frac{(33+6\alpha)(22+4\alpha)}{(89+36\alpha)^2}, \quad \hat{W}^s = \frac{5896+3888\alpha+512\alpha^2}{2(89+36\alpha)^2};$$
$$\hat{G}^s = \frac{5696+3728\alpha+1288\alpha^2+288\alpha^3}{2(89+36\alpha)^2}, \quad \hat{CS}^s = \frac{968+352\alpha+32\alpha^2}{(89+36\alpha)^2}, \quad \hat{\pi}_k^s = \frac{(22+4\alpha)^2}{(89+36\alpha)^2}.$$

Similar to the previous case, we now analyze the case of the ad valorem tax in a unionized privatized duopoly. In the third stage, given w_k and t^v, the firm k's profit-maximization problem is to maximize $\pi_k = ((1-t^v)p - w_k)x_k$ where $p = 1 - x_1 - x_2$. Hence, the symmetry across private firms implies that each output level is given by

$$x_k = \frac{1-t^v-2w_k+w_l}{3(1-t^v)}, \quad k \neq l; k, l = 1, 2. \tag{16}$$

In the second stage, given the output as a function of wage, each union at each firm sets the wage, w_k^v, that maximizes union rent, U_k. Straightforward computation yields the equilibrium wage and output. Repeating the same process as in previous cases yields the first-order conditions of the government's payoff, $G^v = [28 + 20\alpha t^v - 81(t^v)^2]/81$. That is, the optimal tax rate is given by $\hat{t}^v = 10\alpha/81$. Thus, by using $\hat{t}^v$, we can compute each equilibrium value as follows:

Lemma 4: *Suppose that the ad valorem tax rate is imposed on the private firms. Then, the equilibrium wages, outputs, union's utilities, government's payoff, social welfare, consumer surplus and private firm's profit under a unionized privatized duopoly are given by*

$$\hat{w}_k^v = \frac{81-10\alpha}{243}, \quad \hat{x}_k^v = \frac{2}{9}, \quad \hat{U}_k^v = \frac{162-20\alpha}{2187};$$
$$\hat{G}^v = \frac{2268+100\alpha^2}{6561}, \quad \hat{W}^v = \frac{28}{81}, \quad \hat{CS}^v = \frac{8}{81}, \quad \hat{\pi}_k^v = \frac{4(81-10\alpha)}{6561}.$$

5 Comparisons of Indirect Taxation and Market Type

Having derived the equilibrium for two types of tax regimes in the previous section, we will find either *exogenously* or *endogenously* the subgame perfect Nash equilibrium in the first stage.

Moreover, for the sake of convenience, we shall choose a parameterization for the weight of the government's preference for ad valorem tax revenues from Lemma 2 (i.e., w_0^v, U_i^v and π_1^v) and 4 (i.e., $\hat{w}_k^v, \hat{U}_k^v$ and $\hat{\pi}_k^v$). In what follows, we shall make use of the higher-bound restriction on the government's preference for tax revenues:

Assumption: $\alpha \in (0, 8.1)$.

Since we shall appeal to assumption $\alpha \in (0, 8.1)$ in much of what follows, we observe that the assumption is satisfied for the optimal wages, private firm's profit, and utilities of unions (see Lemma 2 and 4). This assumption implies that if α is larger than $\alpha = 9.8$ from Lemma 2, the appropriate incentive is not provided by workers and unions, and the firm's maximization problem is not applicable when the ad valorem tax rate is imposed on the public and private firms[12].

5.1 Exogenous Comparisons of Indirect Taxation and Market Type

If the type of taxes is fixed and the Assumption holds, we can state the following results from easy comparisons of calculations that we can omit:

Proposition 1: *Suppose that the type of taxes is fixed under either mixed or privatized duopoly, then, in the first stage*
(i) $W^j > \hat{W}^j, j = s, v$. *Moreover,* $G^s > \hat{G}^s$.
(ii) $\hat{G}^v > G^v$ *if* $\alpha^v \approxeq 4.85 < \alpha < 8.1$; *otherwise,* $\hat{G}^v \leq G^v$ *if* $0 < \alpha \leq \alpha^v \approxeq 4.85$.

The intuition for Proposition 1(i) is as follows. The price under a unionized mixed duopoly is always smaller than under a unionized privatized duopoly (i.e., $X^j \equiv x_0^j + x_1^j > \hat{x}_1^j + \hat{x}_2^j \equiv \hat{X}^j, j = s, v$). This leads to more output under a unionized mixed duopoly than under a unionized privatized duopoly. On the other hand, privatization under specific taxes leads to a reduction in total output and an increase in market price. This is because $t^s > \hat{t}^s$ and $X^s > \hat{X}^s$, which lead to a lower price and increased tax revenue under a unionized mixed duopoly.

However, Proposition 1(ii) states that if the government levies ad valorem tax at the first stage, comparisons of the government's payoff in the first stage can vary with α. Due to $t^v < \hat{t}^v$ and $p^v < \hat{p}^v$, an increase in the producer's price requires a greater increase in the consumer price under privatized duopoly than under unionized mixed duopoly. This leads to increased ad valorem tax revenue if the government's preference for tax revenue is sufficiently large. On the other hand, a decrease in consumer price reduces the net price received by each firm by less than the decrease in consumer price. This part of the cost is borne by the government, and will be smaller under a mixed duopoly. As a result, when the government levies the ad valorem tax, conflicts of interest with respect to privatization will always arise between the public firm and the government if its preference for tax revenues is sufficiently large[13].

Next, we can state the following results with which the type of markets is fixed.

[12] If α is very large, the government has an incentive to maintain the public firm as a monopoly. This assumption is analytically more convenient for handling optimal solutions, which greatly simplifies the analysis.

[13] Proposition 1 differs from the results of Mujumdar and Pal (1998), which demonstrated that privatization can increase both social welfare and tax revenues. In contrast, our paper shows that when the government and the public firm do not have the same objectives, the privatization is not always desirable in terms of social welfare from the viewpoint of the public firm.

Proposition 2: *Suppose that the type of markets is fixed under either ad valorem or specific tax, then, in the first stage*
(i) $W^s > W^v$ *if* $\alpha \in (0, 0.29)$ *; otherwise,* $W^s \leq W^v$ *if* $\alpha \in [0.29, 8.1)$.
(ii) $\hat{W}^s > \hat{W}^v$ *if* $\alpha \in (0, 0.55)$*; otherwise,* $\hat{W}^s \leq \hat{W}^v$ $\alpha \in [0.55, 8.1)$.
(iii) $G^s < G^v$ *if* $\alpha \in [0.22, 0.44)$*; otherwise,* $G^s > G^v$ *if* $\alpha \in (0, 0.21]$ *or* $\alpha \in [0.45, 8.1)$.
(iv) $\hat{G}^s > \hat{G}^v$ *if* $\alpha \in (0, 0.73]$*; otherwise,* $\hat{G}^s \leq \hat{G}^v$ *if* $\alpha \in [0.74, 8.1)$.

Proof: See the appendix for the part of (i) and (ii). When comparing government's payoff, it can omit by simple calculations. Q.E.D.

The intuition for Proposition 2 (i) is as follows. Consider the condition that $\alpha > \alpha^* \approxeq 0.29$. In this case, the price under the imposition of the specific tax rate is higher than that under the imposition of the ad valorem tax rate. This condition implies that the total output level under ad valorem tax is greater than that under specific taxes (i.e., $X^v \equiv x_0^v + x_1^v > x_0^s + x_1^s \equiv X^s$ if $\alpha > \alpha^* \approxeq 0.29$). As a result, it turns out that social welfare under ad valorem is higher than that under specific tax if $\alpha > \alpha^* \approxeq 0.29$. However, if $\alpha \leq \alpha^* \approxeq 0.29$, those effects are reversed. When comparing $\hat{W}^s$ with $\hat{W}^v$, similar explanations are adopted by using the critical value of $\alpha^{**} \approxeq 0.55$[14]. Note that we will mention the intuitions of Proposition 2 (iii) and (iv) later since endogenous comparisons can endogenously exclude off the equilibrium paths.

5.2 Endogenous Comparisons of Indirect Taxation and Market Type

Given that the Assumption holds with the results of subsection 5.1, this subsection investigates endogenously the subgame perfect Nash equilibrium in the first stage. Since the government determines the choice variables (the type of taxes and markets) in the first stage, the government chooses the best choice from the four options of Section 4. However, we do not need to compare $\hat{G}^v$ with $\hat{G}^s$ and $\hat{W}^v$ with $\hat{W}^s$ since $G^s > \hat{G}^s$ in Proposition 1 (i). This implies that we can exclude the case of the unionized privatized duopoly under specific tax.

At the stage of choosing the type of markets and taxes, note that $\hat{W}^s$ is excluded by choosing G^s when comparing social welfare. Moreover, considering social welfare of the results of subsection 5.1, we see the impact of the government's payoff at the stage of choosing the type of taxes when we recall that "*net*" tax revenue under ad valorem tax is $\alpha p t^v (x_0^v + x_1^v) - (t^v)^2$ and that collected under specific tax is $\alpha t^s (x_0^s + x_1^s) - (t^s)^2$. Therefore, we can state the following results[15].

Proposition 3: *Suppose that the government prefers either specific or ad valorem tax revenue*

[14]By comparing $\hat{X}^s \equiv \hat{x}_1^v + \hat{x}_2^v$ with $\hat{X}^v \equiv \hat{x}_1^v + \hat{x}_2^v$, we get $\hat{X}^v < \hat{X}^s$ if $\alpha < \alpha^{**} \approxeq 0.55$; otherwise, $\hat{X}^v \geq \hat{X}^s$ if $\alpha \geq \alpha^{**}$.

[15]Since comparing government's payoff and net tax revenue obtains easily from direct calculations (i.e., Proposition 3 (i)-(v)), they are available from author upon request. Moreover, when comparing G^s with $\hat{G}^v$, the parameter of the government's preference for tax revenues needs to start from $\alpha > 4.85$ because of Proposition 1(ii). Otherwise, the government will choose G^s rather than $\hat{G}^s$. Also, if $\hat{G}^v \leq G^v$ in the range of $0 < \alpha \leq \alpha^v \approxeq 4.85$ from Proposition 1(ii), it should compare G^v with G^s.

when it considers such type of markets as an option, and the Assumption holds. Then, in the first stage,
(i) $G^s < G^v$ *if* $\alpha \in [0.22, 0.44)$*; otherwise,* $G^s > G^v$ *if* $\alpha \in (0, 0.21]$ *or* $\alpha \in [0.45, 8.1)$.
(ii) $G^s > \hat{G}^v$ *if* $\alpha \in (4.85, 8.1)$.
(iii) when comparing G^s *with* G^v*, net tax revenue under a mixed duopoly with ad valorem is greater than that under a mixed duopoly with specific tax if* $\alpha \in (0, 0.24]$*, and vice versa if* $\alpha \in [0.25, 8.1)$.
(iv) when comparing G^s *with* $\hat{G}^v$*, the net tax revenue under privatization with ad valorem is always smaller than that under a mixed duopoly with specific tax.*
(v) $W^s > W^v$ *if* $\alpha \in (0, 0.29)$*; otherwise,* $W^s \leq W^v$ *if* $\alpha \in [0.29, 8.1)$.
(vi) $W^s > \hat{W}^v$ *if* $\alpha \in (4.85, 6.6)$*; otherwise,* $W^s \leq \hat{W}^v$ *if* $\alpha \in [6.6, 8.1)$.

The intuition for Proposition 3 (i) is as follows[16]. Consider the first case where the government's preference for tax revenues is sufficiently small (i.e., $\alpha \in (0, 0.21]$ in Proposition 3 (i)) under a mixed duopoly. In this case, due to the fact that $t^v > t^s$ and $p^v > p^s$, there is higher net tax revenue under ad valorem tax. Besides this direct effect, social welfare under specific tax is higher than that under ad valorem tax. We call the former the "*net tax effect*" and the latter "*welfare effect.*" With Propositions 3 (i) and 4 (iii), the government's payoff improvement is possible when welfare effect under specific tax dominates net tax effect under ad valorem tax, once its preference for tax revenues is sufficiently small[17]. On the other hand, it is important to notice that in a mixed duopoly, the ad valorem tax rate is higher than the specific tax rate if the government's preference for tax revenues is sufficiently large (i.e., $\alpha \in [0.45, 8.1)$). Due to the fact that $p^s > p^v$ as lowering output under specific tax, the implication is that $W^v > W^s$ as in Proposition 3 (i). However, welfare effect under ad valorem tax is dominated by net tax effect under specific tax as Proposition 3 (iii) states. In addition, if the government's preference for tax revenues falls in the middle range, $\alpha \in [0.22, 0.44)$ under a mixed duopoly, the government's payoff under ad valorem tax is larger than that under specific tax since both welfare and net tax effects under ad valorem are always greater than under specific tax.

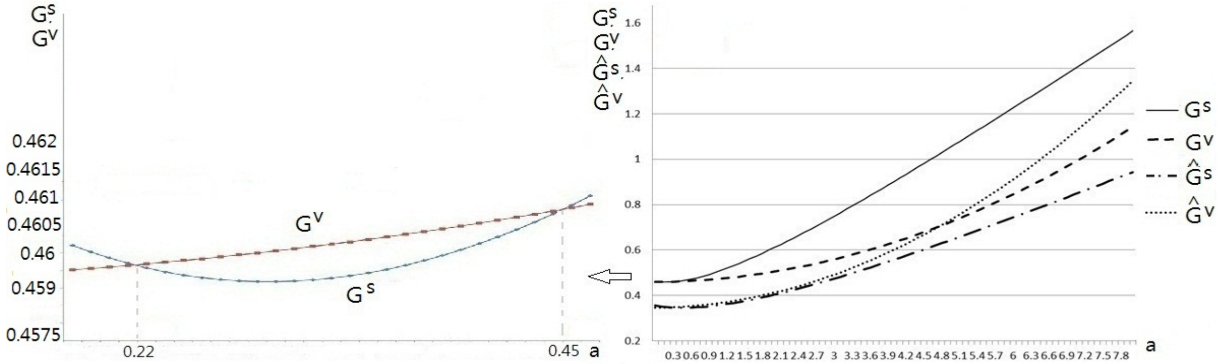
Finally, the intuition for Proposition 3 (ii) is as follows. The price under specific tax is higher than that under ad valorem tax since total output under specific tax is lower if α is sufficiently large. In this case, due to the fact that $p^s > \hat{p}^v$, lowering output under specific tax of the mixed duopoly, it turns out that $\hat{W}^v > W^s$ as in Proposition 3 (vi). However, welfare effect under ad valorem tax is dominated by net tax effect under specific tax as stated in Proposition 3 (iv). Therefore, we obtain Proposition 3 (ii).

Hence, noting either critical value, $\alpha = 0.22$ or $\alpha = 0.45$, when comparing government's

[16]The intuition for Proposition 3 (v) and (vi) is already explained in Proposition 2. Hence, as stated the intuition for Proposition 2 (i), when comparing W^s with $\hat{W}^v$, we get $\hat{X}^v < X^s$ if $\alpha < \alpha^\dagger = 6.6$; otherwise, $\hat{X}^v \geq X^s$ if $\alpha \geq \alpha^\dagger$.

[17]Strictly speaking, in the case of $\alpha \in [0.22, 0.24)$, where $G^s < G^v$, the government's payoff under ad valorem tax is larger than that under specific tax since both welfare and net tax effects under ad valorem are always greater than those under specific tax.

Figure 1: Comparisons



payoff under both market competitions with both tax regimes, the graphs of comparisons of G are shown in Figure 1.

In sum, Proposition 3 suggests that there are no conflicts of interest between the public firm and government when the government levies the specific tax under unionized mixed duopoly if its preference for tax revenues is sufficiently small. This is because the government always prefers specific tax to ad valorem under unionized mixed duopoly, and $W^s > W^v$. However, if the government's preference for tax revenues is sufficiently large, while the public firm has the incentive to be levied by ad valorem tax, the government always has an incentive to levy specific tax: $G^s > G^v$ and $W^s < W^v$. Interestingly, in line with Proposition 3, the government never has an incentive for privatization. Hence, Propositions 3 shows that depending on the government's preference for tax revenues, the conflict between these two views of objective functions typically induces a conflict with regard to imposing either tax.

6 Concluding Remarks

This study has investigated changes in social welfare and the government's payoff on the basis of the different objective functions of the government and the public firm; it compares the efficiency of ad valorem with that of specific taxes in both a unionized mixed duopoly and privatization.

We have found that the comparative effect on social welfare and government's payoff is endogenously determined by welfare and net tax effects as follows: There are no conflicts of interest between the public firm and government when the government levies the specific tax under unionized mixed duopoly if its preference for tax revenues is sufficiently small. However, if the government's preference for tax revenues is sufficiently large, while the public firm has the incentive to be levied by ad valorem, the government always has an incentive to levy specific tax. Interestingly, the government never has an incentive for privatization.

However, there remains much to be done in exploring the empirical impact of tax structure and asymmetric Bertrand competition with unionized mixed oligopoly. Moreover, there could be important economic implications if the analysis is expanded to include varying motives for bargaining among firms in the existing framework of mixed oligopolistic markets. The extension of our model in these directions remains an agendum for future research.

References

[1] Barcena-Ruiz, J. C. and M. B. Garzon, 2009, "Relocation and Public Ownership of Firms," *Journal of the Japanese and International Economies*, 23, 71-85.

[2] Besley, T., 2006, *Principled Agents? The Political Economy of Good Government*, Oxford University Press.

[3] Booth, A., 1995, *The Economics of the Trade Union,* Cambridge: Cambridge University Press.

[4] Bordogna, L., 2003, "The Reform of Public Firm Employment Relations in Industrialized Democracy," In Brock, J. and Lipsy, D. B. (eds), *Going Public: The Role of Labor-Management Relations in Delivering Quality Government Services,* Industrial Relations Research Association Series, pp. 23-67.

[5] Bos, D., 1991, *Privatization: A Theoretical Treatment*, Oxford University Press.

[6] Brennan, G. and J. Buchanan, 1980, *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge University Press.

[7] Brock, J. and D. B. Lipsy, 2003, *Going Public: The Role of Labor-Management Relations in Delivering Quality Government Services,* Industrial Relations Research Association Series.

[8] Brülhart, M. and M. Jametti, 2006, "Vertical versus Horizontal Tax Externalities: An Empirical Test," *Journal of Public Economics,* 90, 2027-2062.

[9] Choi, K., 2011, "Unions, Government's Preference, and Privatization," *Economic Modelling,* 28, 2502-2508.

[10] Delipalla, S. and M. Keen, 1992, "The Comparison Between Ad Valorem and Specific Taxation under Imperfect Competition" *Journal of Public Economics*, 49, 351-367.

[11] Du Caju, P., E. Gautier, D. Momferatou and M. Ward-Warmedinger, 2008, "Institutional Features of Wage Bargaining in 23 European Countries, the US and Japan," Working Paper, NERE 228, Banque de France.

[12] Edwards, J. and M. Keen, 1996, "Tax Competition and Leviathan," *European Economic Review,* 40, 113-140.

[13] Haucap, J. and C. Wey, 2004,"Unionisation Structures and Innovation Incentives," *Economic Journal*, 114, C149-C165.

[14] Horn, H. and A. Wolinsky, 1988, "Bilateral Monopolies and Incentives for Merger," *Rand Journal of Economics*, 19, 408-419.

[15] Ishida, J. and N. Matsushima, 2009, "Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach," *Journal of Public Economics*, 93, 634-646.

[16] Kato, H., 2008, "Privatization and Government Preference," *Economics Bulletin*, 12, 1-7.

[17] Keen, M., 1998, "The Balance between Specific and Ad Valorem Taxation," *Fiscal Studies*, 19, 1-37.

[18] Keen, M. and C. Kotsogiannis, 2002,"Does Federalism Lead to Excessively High Taxes?" *American Economic Review*, 92, 363-370.

[19] Mujumdar, S. and D. Pal, 1998, "Effects of Indirect Taxation in a Mixed Oligopoly," *Economics Letters*, 58, 199-204.

[20] Niskanen, W. A., 1971, *Bureaucracy and Representative Government,* New York: Aldine-Ahterton.

[21] Oates, W. E., 1989, "Searching for Leviathan: A Reply and Some Further Reflection," *American Economic Review*, 79, 578-583.

[22] Oswald, A. J., 1982, "The Microeconomic Theory of Trade Unions," *Economic Journal*, 367, 576-595.

[23] Persson, T. and G. Tabellini, 2002, *Political Economics: Explaining Economic Policy*, MIT Press.

[24] Rauscher, M., 2000, "Interjurisdictional Competition and Public-sector Prodigality: The Triumph of the Market over the State?" *Finanzarchiv,* 57, 89-105.

[25] Rose, J. B., 2004, "Public Sector Bargaining: From Retrenchment to Consolidation," *Relations Industrielles/Industrial Relations,* 59, 271-294.

[26] Seade, J. 1985, "Profitable Cost Increases and the Shifting of Taxation: Equilibrium Responses of Markets in Oligopoly," Working Paper, No. 260, University of Warwick.

[27] Vickers, J. and G. Yarrow, 1988, *Privatization: An Economic Analysis,* MIT Press, Cambridge.

Appendix: Proof of Proposition 2

Since $G^s > \hat{G}^s$ in Proposition 1, $\hat{W}^s$ is excluded by choosing G^s when comparing social welfare.
(i): Comparing W^s with W^v, and $\hat{W}^s$ with $\hat{W}^v$ yields $W^s - W^v$ and $\hat{W}^s - \hat{W}^v \Leftrightarrow 884 - 2184\alpha - 3185\alpha^2$ and $2125 - 2745\alpha - 1944\alpha^2$. By applying to a discriminant and ignoring the nonpositive roots for $\alpha > 0$ through the assumption, we have the root $\alpha^* \approxeq 0.29$ and $\alpha^{**} \approxeq 0.55$. Since the maximum value is attained from $-3185\alpha^2 < 0$ (respectively, $-1944\alpha^2 < 0$), $W^s \geq W^v$ (respectively, $\hat{W}^s \geq \hat{W}^v$) if $\alpha \in (0, 0.29]$ (respectively, $\alpha \in (0, 0.55]$); otherwise, $W^s < W^v$ (respectively, $\hat{W}^s < \hat{W}^v$)) if $\alpha \in (0.29, 8.1)$ (respectively, $\alpha \in [0.56, 8.1)$).
(ii): Comparing W^s with $\hat{W}^v$ yields $W^s - \hat{W}^v \Leftrightarrow 314721 + 226780\alpha - 41525\alpha^2$. Repeating the same process as in previous proof above yields the root $\alpha^\dagger \approxeq 6.6$. Thus, $W^s \geq \hat{W}^v$ if $\alpha \in (0, 6.6]$; otherwise, $W^s < \hat{W}^v$ if $\alpha \in (6.6, 8.1)$.

Supplement Material (Not for Publication)
Proof of Proposition 1

(i): Comparing W^v with $\hat{W}^v$ yields $W^v - \hat{W}^v = 712 > 0$, and comparing W^s with $\hat{W}^s$ yields $W^s - \hat{W}^s = 24426161 + 48068460\alpha + 31985451\alpha^2 + 9606360\alpha^3 + 1217200\alpha^4 > 0$. Moreover, comparing G^s with $\hat{G}^s$ yields

$$G^s - \hat{G}^s = 25867761 + 48633328\alpha + 49813383\alpha^2 + 29593838\alpha^3 + 5464640\alpha^4 + 856800\alpha^4 > 0.$$

(ii): Comparing G^v with $\hat{G}^v$ yields

$$\hat{G}^v - G^v \Leftrightarrow 152150\alpha^2 - 3576069 \Leftrightarrow \hat{G}^v \geq G^v \text{ if } \alpha \geq \alpha^v \approxeq \sqrt{23.50} \approxeq 4.85; \text{otherwise } \hat{G}^v < G^v.$$

Proof of Proposition 3

Since $G^s > \hat{G}^s$ in Proposition 1, $\hat{G}^s$ is excluded by choosing G^s when comparing government's payoff. Moreover, when comparing G^s with $\hat{G}^v$, the parameter of the government's preference for tax revenues needs to start from $\alpha > 4.85$ because of Proposition 1(ii). Otherwise, the government will choose G^s rather than $\hat{G}^s$. Also, if $\hat{G}^v \leq G^v$ in the range of $0 < \alpha \leq \alpha^v \approxeq 4.85$ from Proposition 1(ii), it should compare G^v with G^s.
(i) and (ii): Comparing G^s with G^v and G^s with $\hat{G}^v$ yield

$$G^s - G^v = 602700 - 3875900\alpha + 4225625\alpha^2 + 3340750\alpha^3 - 245000\alpha^4 < 0 \quad \text{if} \quad \alpha \in [0.22, 0.44];$$

$$\text{otherwise}, \quad G^s > G^v \quad \text{if} \quad \alpha \in (0, 0.21] \quad \text{or} \quad \alpha \in [0.45, 8.1).$$

$$G^s - \hat{G}^v = 24180201 + 15056280\alpha + 17886375\alpha^2 + 8037750\alpha^3 - 980000\alpha^4 > 0 \quad \text{if} \quad \alpha \in (4.85, 8.1].$$

Another ways of straightforward calculations of Proposition 3 are in the Table A-1. Using numerical examples to illustrate the impact of α, straightforward computations yield as follows.

Table A-1: Numerical Examples

α	$R^s - R^v$	$R^s - \hat{R}^v$	$G^s - G^v$	$G^s - \hat{G}^v$
0.01	-5025.785034	-13735.53956	564366.9008	24332560.47
⋮	⋮	⋮	⋮	⋮
0.21	-2069.399654	-6750.413541	5573.2698	28203340.63
0.22	-1520.103624	-5368.039896	-10479.3712	28441573.4
0.23	-929.776304	-3880.775191	-25260.1427	28684387.5
0.24	-298.043264	-2287.760256	-38750.3232	28931825.85
0.25	375.46875	-588.140625	-50931.25	29183931.16
0.26	1091.131816	1218.933464	-61784.3192	29440745.88
⋮	⋮	⋮	⋮	
0.43	20061.36076	49180.35355	-25381.9897	34567145.28
0.44	21598.02746	53061.55462	-9219.3872	34915722.76
0.45	23183.69125	57065.46188	8613.375	35269771.78
⋮	⋮	⋮	⋮	⋮
4.84	8571788.422	17081518.8	325159169.1	889587425.1
4.85	8612053.926	17146045.78	326767928.1	892673244.5
⋮	⋮	⋮	⋮	⋮
8.09	24823052.44	28408394.62	965203654.3	1374618766

(iii) and (iv): Comparing $R^s \equiv \alpha t^s X^s - (t^s)^2$ with $R^v \equiv \alpha t^v p^v X^v - (t^v)^2$ and R^s with $\hat{R}^v \equiv \alpha \hat{t}^v \hat{p}^v \hat{X}^v - (\hat{t}^v)^2$ yield

$$
\begin{aligned}
&R^s - R^v = 4802 - 24010\alpha + 16248\alpha^2 + 66815\alpha^3 - 4900\alpha^4 > 0 \quad \text{if} \quad \alpha \in (0, 0.24]; \\
&\text{otherwise} \quad R^s - R^v < 0 \quad \text{if} \quad \alpha \in [0.25, 8.1) \\
&R^s - \hat{R}^v = 13122 - 65610\alpha + 423998\alpha^2 + 160835\alpha^3 - 19600\alpha^4 < 0 \quad \text{if} \quad \alpha \in (0, 0.25]; \\
&\text{otherwise} \quad R^s - \hat{R}^v > 0 \quad \text{if} \quad \alpha \in [0.26, 8.1)
\end{aligned}
$$

respectively. However, when comparing the government's payoff under a mixed duopoly of specific tax under the privatization of ad valorem tax, the parameter of the government's preference for tax revenues needs to start from $\alpha > 4.85$ because of Proposition 1(ii). Otherwise, the government will choose G^s. Thus, when $\alpha \in (4.85, 8.1)$, the net tax revenue under the privatization of ad valorem tax is always smaller than that under a mixed duopoly of specific tax.