Developing Composite Indicators for Fiscal Decentralization: Which Is The Best Measure For Whom?

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Developing Composite Indicators for Fiscal Decentralization:
Which Is The Best Measure For Whom?

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November 28, 2012

Abstract

The right way to measure the degree and extent of the different aspects of fiscal decentralization has been a long-debated, yet underdeveloped issue. There has been little consensus on the right approach to developing a single indicator which is sometimes needed to show a general trend in fiscal decentralization and reveal relationship to other variables in empirical studies. In particular, several composite indicators of fiscal decentralization have been proposed, but there are very few attempts to evaluate and compare these measures in terms of implicit biases and different weights between revenue and expenditure decentralization. Critically reviewing and comparing various types of fiscal-relation indicators in a systematic way, this paper proposes two criteria to classify similar-looking composite indicators for fiscal decentralization while it also presents two new composite measures. The new fiscal decentralization indicators are symmetric in terms of the relative effects of revenue and expenditure decentralization on the value of the composite indicators at the same time that they are weighted for/against fiscal gaps and imbalances. It is argued that different composite indicators reflect different perspectives on which aspect of fiscal decentralization is more important and whether a growing fiscal gap means less fiscal decentralization or not.

Keywords: fiscal federalism; fiscal decentralization; decentralization measurement

JEL Codes: H77, H11, D63

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I. Introduction

A vast literature and national accounts data show that there is a general trend throughout OECD countries regardless of whether the country is federal or unitary to increasingly devolve the responsibility for key public sector functions from the central government to subnational tiers of government. Still, the “right” way to measure the degree and extent of the different aspects of the fiscal decentralization has been a long-debated, yet underdeveloped issue in theoretical research and more recently, in empirical works which try to investigate the effects of fiscal decentralization on economic growth, fiscal performance, and other important economic outcomes. Unfortunately, the sacrifice of rigour for tractability has seriously harmed the quality of public policy debate on the issue (Sharma, 2011). Furthermore, there has been little consensus on the right approach to developing a single indicator which is sometimes needed to show a general trend in fiscal decentralization and reveal relationship to other variables in empirical studies. In particular, several composite indicators of fiscal decentralization have been proposed, but there are very few attempts to evaluate and compare these measures in terms of implicit biases and different weights between revenue and expenditure decentralization. The reason being is that we do not have a clear criterion by which these composite measures can be judged in a consistent way. Critically reviewing and comparing various types of fiscal-relation indicators in a systematic way, this paper proposes two criteria to classify similar-looking composite indicators for fiscal decentralization while it also presents two new composite measures. The new indicators are symmetric in terms of the relative effects of revenue and expenditure decentralization on the value of the composite indicators at the same time that they are weighted for/against fiscal gaps and imbalances.

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2 Dziobek et al. (2011) provides a good example of the challenge in measuring decentralization.
The paper is organized as follows: Section II examines typical measures for fiscal relations involving decentralization and vertical fiscal imbalance on the same base before we focus on the composite indicators for fiscal decentralization to expose their implicit biases and limitations. Section III develops two new composite measures which are designed to overcome the shortcomings of the previous composite indicators. Section IV presents a summary and some conclusions for theoretical and empirical work concerning fiscal decentralization.

II. Decentralization Indicators and their Limitations

Fiscal decentralization is referred to as the multi-faceted multi-dimensional process of central government’s (CG) transferring decision-making powers concerning public finance to subnational government (SNG), covering both expenditure and revenue sides of decentralization. For practice purposes, however, there has been diversity of approaches to measuring fiscal decentralization. To facilitate the comparison and discussion of the relative advantages and disadvantages of various decentralization indicators, we utilize a two-dimensional representation, that is, the composition of revenue and expenditure in the general government (GG) level because decentralization may involve both expenditure and revenues as SNG’s are assigned the increasing amount of own revenue such as SNG own taxes and fees within its boundaries and (part of) shared taxes in order to match part of growing expenditure responsibilities. Figure 1 shows their relationship in a graphical representation.

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3 There has been a lack of consistency among researchers and articles concerning how to deal with shared taxes in terms of their inclusion in SNG own revenue, which means that it is difficult to have standardized data (Anderson, 2010). As late as in 1997, the Working Party on Tax Policy Analysis and Tax Statistics of the OECD Committee on Fiscal Affairs divided tax autonomy into five categories from full discretion over tax rates and
The horizontal dimension essentially break the total public revenue (GG revenue) into SNG own revenue (r), transfer from CG to SNG (R-r), and CG revenue (1-R).

The vertical dimension consists of SNG or subnationally-administered expenditure (e), centrally-administered local expenditure (E-e), and CG own expenditure (1-E). The distinction between subnationally-administered and centrally administered expenditure is not concerned with the funding source for the expenditure. In other words, whether the expenditure is funded by SNG own revenue or grants is irrelevant for this categorization. In case of a co-financed project, we look to who finally pay for it. The portion of the total project costs paid directly from the central budget to project contractors or recipients is considered as centrally-administered local expenditure, whereas the share paid by the subnational government is seen as SNG or subnationally-administered expenditure regardless of whether it is funded by its own revenue or transferred grants from the central government.

All the variables (r, R, e, E) are expressed as their shares in the GG revenue and expenditure, respectively. In other words, they are revenue ratio and expenditure ratio in terms of the GG total revenue and expenditure. It means that it suffices to know only two out of three components of the GG total revenue in order to have a complete picture of the revenue sources for SNG and CG; similarly, any two of three expenditure ratios tell us exactly the other one. SNG own revenue (r) is widely utilized to represent the degree of revenue decentralization while SNG expenditure (e) is the conventional indicator of

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bases to no discretion at all over any tax variable. One of the five categories is tax sharing arrangements, which are categorized to its four different subgroups in terms of the level of taxing power (OECD 1999). Stegarescu (2005) defines a more detailed notion of revenue decentralization which excludes part of shared tax revenue in his calculation of SNG own revenue, while many others utilize the conventional indicator of revenue decentralization where all the shared tax revenues are included in SNG own revenue regardless of their degree of tax autonomy or taxing power.

According to OECD (1999), the definition of SNG own revenue is the sum of SNG own tax revenue, shared tax revenue and non-tax and capital revenue. Each government’s revenue is defined as its total revenue minus the intergovernmental transfer revenue of that government level.

In order to focus only on governments’ expenditure responsibilities and corresponding financial means of financing that could reasonably be decentralized according to the theory of fiscal federalism, social security has been removed from the general government sector.
expenditure decentralization.\footnote{Feld and Schnellenbach (2008) show that the expenditure share of subnational governments or closely related measures is used as the fiscal decentralization variable in about 35% of models, the revenue share is used in about 10% of models, and the weighted average of expenditure and revenue decentralization is used in about 3% of models in their survey of 26 empirical studies on the effects of fiscal decentralization on economic growth. Others are the divergence between central and subnational government spending or revenue (about 12%), and the tax autonomy of subnational governments (about 25%).} True, it is well known that the expenditure shares derived from budgetary data is imperfect indicator of expenditure decentralization because it fails to reflect SNG’s discretion over its spending. For instance, expenditure mandated by the central government or spent on behalf of the central government is regarded as SNG or subnationally-administered expenditure in our framework. However, given that we differentiate between revenue and expenditure sides of fiscal decentralization, the power of administration can be measured as the own expenditures of a given government net of grants provided to other governments. The responsibility for service delivery is one thing while the composition of specific financial means matching the responsibility is another thing.
It can be easily verified that any measure of fiscal relations including fiscal decentralization can be expressed in terms of these break-down of public finances. For example, one of the most popular measures of fiscal decentralization measure is SNG own revenue and expenditure shares in GG revenue and expenditure, that is, $r$ and $e$; one of the definitions for vertical fiscal imbalance (VFI) is the share of transfer to SNG in SNG total revenue, which translates to $(R-r)/R$; one of revenue autonomy indicators is the share of SNG own revenue in SNG total revenue, which is simply $r/R$ in this space. Reviewing the relevant literature, we transform the typical definition of various fiscal relation measures into a functional form in our space, which means they are decomposed into the four elements, that is $r$, $e$, $R$, and/or $E$. Table 1 provide the representation of various fiscal relation measures in a functional form on the two-dimensional space. It should be noted that they are categorized into fiscal decentralization and vertical fiscal imbalance measure since fiscal relations are concerned mainly with intergovernmental fiscal interaction revolving around taxing power, transfer and spending responsibilities, which means that tax autonomy is not covered by this representation. In addition, each group is divided to the uncombined and composite measures according to whether a measure reflects both sides of revenue and expenditure.

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7 The degree of tax autonomy of subnational governments accounting for autonomous own taxes is derived either by using a qualitative scale for the different tax categories which are presented by OECD (1999) reflecting the deceasing extent of discretion, or simply by summing the different taxes that only accrue to and controlled by subnational governments as the corresponding shares in their total revenues.
Focusing on fiscal decentralization composite measures, we need an objective criterion in order to compare and evaluate the advantages and disadvantages of these several measures.

Table 1. Representation of Various Fiscal Relation Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Definition</th>
<th>Functional Form</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal Decentralization Measure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncombined Indicator</td>
<td>Revenue Decentralization Indicator (RDI)</td>
<td>$\frac{\text{SNG own revenue}}{\text{GG revenue}}$</td>
<td>$D'(r) = r$</td>
</tr>
<tr>
<td></td>
<td>Expenditure Decentralization Indicator (EDI)</td>
<td>$\frac{\text{SNG expenditure}}{\text{GG expenditure}}$</td>
<td>$D'(e) = e$</td>
</tr>
<tr>
<td></td>
<td>Revenue Autonomy I (RAI)</td>
<td>$\frac{\text{SNG own revenue}}{\text{SNG revenue}}$</td>
<td>$A^I(r, R) = \frac{r}{R}$</td>
</tr>
<tr>
<td></td>
<td>Revenue Autonomy II (RAII)</td>
<td>$\frac{\text{SNG own revenue}}{\text{SNG expenditure}}$</td>
<td>$A^{II}(r, e; \overline{D}) = \frac{r}{e \overline{D}}$</td>
</tr>
<tr>
<td></td>
<td>Production-Revenue Indicator (PRI)</td>
<td>Geometric mean of fiscal autonomy and fiscal importance</td>
<td>$V(r, e; \overline{D}) = \left( \frac{r}{e} \times \frac{1}{\overline{D}} \right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
<td>Fiscal Decentralization Index (FDI)</td>
<td>Composite Indicator</td>
<td>$\frac{\text{RDI}}{1 - \text{EDI}}$</td>
</tr>
<tr>
<td>Vertical Fiscal Imbalance (VFI) or Transfer Dependency (TD) Measure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncombined Indicator</td>
<td>VFI\textsuperscript{I} = TD\textsuperscript{I}</td>
<td>$\frac{\text{Transfer}}{\text{GG revenue}}$</td>
<td>$T^I(r, R) = R - r$</td>
</tr>
<tr>
<td></td>
<td>VFI\textsuperscript{II} = TD\textsuperscript{II}</td>
<td>$\frac{\text{Transfer}}{\text{GG expenditure}}$</td>
<td>$T^{II}(r, R; \overline{D}) = (R - r) \frac{1}{\overline{D}}$</td>
</tr>
<tr>
<td></td>
<td>VFI\textsuperscript{III} = TD\textsuperscript{III}</td>
<td>$\frac{\text{Transfer}}{\text{GG revenue}}$</td>
<td>$T^{III}(r, R) = \frac{R - r}{1 - r}$</td>
</tr>
<tr>
<td></td>
<td>VFI\textsuperscript{IV} = TD\textsuperscript{IV}</td>
<td>$\frac{\text{Transfer}}{\text{SNG revenue}}$</td>
<td>$T^{IV}(r, R) = \frac{R - r}{e \overline{D}}$</td>
</tr>
<tr>
<td></td>
<td>VFI\textsuperscript{V} = TD\textsuperscript{V}</td>
<td>$\frac{\text{Transfer}}{\text{SNG expenditure}}$</td>
<td>$T^{V}(r, e; \overline{D}) = \frac{R - r \overline{D}}{e \overline{D} - r}$</td>
</tr>
<tr>
<td>Compositive</td>
<td>VFI\textsuperscript{VI} = TD\textsuperscript{VI}</td>
<td>$\frac{(\text{SNG expenditure} - \text{SNG own revenue})/\text{GG revenue}}{\text{GG expenditure}}$</td>
<td>$T^{VI}(r, e; \overline{D}) = \frac{R - r}{e \overline{D} - r}$</td>
</tr>
</tbody>
</table>

Note: The asterisk (*) indicates the original source of its fiscal decentralization index. $\overline{D}$ is GG fiscal deficit defined as GG expenditure divided by GG revenue.

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indicators. We propose a pair of criteria as shown below in Definition 1 and 2: impact-symmetry and VFI-weightedness.

**Definition 1:** Suppose that a composite indicator for fiscal decentralization $F(r,e)$ is increasing, continuous, and differentiable in both arguments. Given the composite indicator for fiscal decentralization $F(r,e)$, we define the following concepts:

(i) $F(r,e)$ is **impact symmetric** if $F_r(r,e)$ and $F_e(r,e)$ are functionally symmetric.

(ii) $F(r,e)$ is **impact asymmetric** if $F_r(r,e)$ and $F_e(r,e)$ are not functionally symmetric.

where $F_r(r,e)$ is the partial derivative w.r.t. $r$ and $F_e(r,e)$ is the partial derivative w.r.t. $e$.

Definition 1 delineates the condition that must be met for a composite measure to weigh equally between the changes in revenue and expenditure decentralization. When a measure is functionally symmetric in its partial derivative function, incremental expenditure decentralization raises the composite measure at the same speed or to the same degree as incremental revenue decentralization. The impact-symmetry allows us to evaluate the previous composite indicators for fiscal decentralization in terms of this relative importance.

**Proposition 1:** $RA^II$, FDI, and CR are an impact asymmetric measure of fiscal decentralization, while PRI is an impact symmetric one.

Proof: See Appendix I.

Proposition 1 shows that three of the four previous composite indicators are weighted in favor of or against revenue decentralization. It is not surprising that PRI has equal weight on $r$ and $e$ because the functional form is the arithmetic mean of the two decentralization measures. $RA^II$ is decreasing in expenditure decentralization ($e$), but it can sometimes increase when $e$ is
rising. Martinez-Vazquez and Timofeev (2009) delineates the condition where more expenditure decentralization translates to higher RA\textsuperscript{II}. It is an increasing function of expenditure decentralization only when the elasticity of revenue decentralization with respect to expenditure decentralization is greater than 1. In other words, when revenue decentralization comes hand in hand with expenditure decentralization, it is possible to raise the level of RA\textsuperscript{II}. It is an obvious statement since the measure r/e can increase only when the numerator grows faster than the denominator.

While Vo (2008) presents a composite measure called FDI, which can be decomposed to the revenue decentralization (denoted by fiscal autonomy) and the SNG share of the public expenditure (denoted by fiscal importance), it turns out that a cancellation of the expenditure component results in a measure essentially equivalent to the revenue decentralization indicator (r) times GG fiscal balance. Thus, the increase in expenditure decentralization has no impact on the value of the composite index.

More recently, Martinez-Vazquez and Timofeev (2009) propose a new composite measure called Composite Ratio, which combines the information captured by both the revenue and expenditure decentralization. However, it weighs more heavily the former than the latter in some case, and vice versa in the other case, which is stated by following Lemma 1.

\textbf{Lemma 1:} The effect of revenue decentralization on CR measure is greater than that of expenditure decentralization only when the sum of the two decentralization measures, that is, r+e, is less than 1.

\textit{Proof:} See Appendix I.
It means that the increase in revenue decentralization tends to have much greater impact on the composite measure when comprehensive fiscal decentralization is at an early stage while expenditure decentralization becomes the more important factor in determining the value of CR measure at its mature stage in a relative term.

The second criterion which the paper proposes is to see how a composite measure adjusts for the gap between revenue and expenditure decentralization.

**Definition 2:** Given the above composite indicator for fiscal decentralization $F(r,e)$, we define the following concepts:

(i) $F(r,e)$ is *VFI-favorable* if its second-order cross partial derivative is negative.

(ii) $F(r,e)$ is *VFI-neutral* if its second-order cross partial derivative is zero.

(iii) $F(r,e)$ is *VFI-unfavorable* if its second-order cross partial derivative is positive.

where the second-order cross partial derivative of the function $F(r,e)$ is $\frac{\partial^2 F(r,e)}{\partial r \partial e}$.

The concept of VFI-favorableness is analogous with that of substitutability, while the meaning of VFI-unfavorableness is similar to that of complementarity. Note that in a cardinal von Neumann-Morgenstern context it is used to classify goods into substitutes and complements. In the first case of a negative cross-partial, one can think of revenue and expenditure decentralization as more than substitutes. A composite indicator could increase even more by decreasing the correlation between the two indicators. In the latter case of a positive cross-partial derivative, indicators are complements, and therefore a composite indicator could increase by increasing the correlation between them. In other words, the sign on the cross partial derivative of a composite indicator determines whether expenditure
decentralization is a complement or substitute for revenue decentralization in determining the level of the composite indicator.

With the help of this criterion, we can evaluate and compare the composite indicators in the previous literature in terms of their weights on VFI.

**Proposition 2:** RA\textsuperscript{II} is VFI-favorable; CR is VFI-unfavorable; and FDI and PRI is VFI-neutral, respectively.

Proof: See Appendix I.

Proposition 2 shows that a composite measure reflects the relative level of revenue and expenditure decentralization as well as their absolute degrees in one or another way. The larger a gap between revenue and expenditure decentralization, the higher RA\textsuperscript{II} is, whereas exactly the opposite holds for the case of CR. In other words, RA\textsuperscript{II} considers any increase in either revenue or expenditure decentralization as improvement for fiscal decentralization no matter which side of fiscal decentralization is increasing, while CR is designed to impose a kind of penalty against the fiscal imbalance. FDI and PRI take the level of the vertical fiscal imbalance into no account, which means that these indicators regard revenue and expenditure decentralization as perfect substitutes. In short, the bigger the gap between revenue decentralization \((r)\) and expenditure decentralization \((e)\) is, the greater the impact of incremental expenditure decentralization on a VFI-favorable measure, and vice versa for a VFI-unfavorable measure.
III. Impact-Symmetric and VFI-weighted Decentralization Indicators

Given that there is no single indicator that is both impact-symmetric and VFI-weighted, Section III attempts to develop a new composite indicator which has the two properties at the same time.

**Definition 3:** This paper develops two new composite indicators defined as follows:

(i) \( SF = S^f(r, e) = \frac{(r^2 + e^2)^{\frac{1}{2}}}{\sqrt{2}} \)

(ii) \( SU = S^u(r, e) = 1 - \frac{((1-r)^2 + (1-e)^2)^{\frac{1}{2}}}{\sqrt{2}} \)

The two new measures can be depicted on the \((r,e)\) space as in Figure 2. SF and SU are built on the distance from origin and (1,1), respectively. The way these new indicators form is based on a simple logic that the farther a pair \((r^*, e^*)\) is located from the origin, the higher value they should translate into in the single numerical expression of fiscal decentralization.

**Figure 2: Graphical representation of the new composite indicators in contour**
According to Definition 1, we evaluate the new measures in order to see whether they are impact symmetric or not.

**Proposition 3:** Both of the new composite indicators, i.e., SF and SU are a impact symmetric measure.

Proof: See Appendix I.

The functional form of the partial derivatives with respect to r and e is exactly the same in both measures. Both indicators are balanced in terms of relative importance between revenue and expenditure decentralization.

As with the previous indicators, we also examine how the composite indicators reflect the widening vertical fiscal imbalance according to Definition 2.

**Proposition 4:** SF is VFI-favorable whereas SU is a VFI-unfavorable measure.

Proof: See Appendix I.

Each of the new indicators reflects the opposite point of view on vertical fiscal imbalance. SF appreciates the growing gap between revenue and expenditure decentralization. Not surprisingly, we find that the different indicators have markedly different results from the gap. Suppose a country where r is 0.40 and e is 0.40. Its VFI-neutral PRI indicator is the arithmetic mean of r and e, that is, 0.40; and SF and SU are also 0.40. Now suppose that the country has devolved as more responsibility for public expenditure to the lower level government so that r is still 0.40 but e is raised all the way to 0.80. Its PRI increases to 0.60; SF jumps to 0.63; but SU goes up only to 0.55.
Table 2 presents the taxonomy for comparing the previous and new indicators. We
categorize the composite indicators according to our two criteria in the table. Given that in
the previous literature we have no indicator which is both impact-symmetric and VFI-
weighted, the two composite indicators that this paper proposes can fill the blank. It is easily
known that each composite indicator occupies a distinct position in the table, which means
that it has already made a value judgment about which side of public finance is the more
important and whether a widening VFI is seen as the process of fiscal decentralization or not.

<table>
<thead>
<tr>
<th>Relative Contribution of Revenue and Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric Impact</td>
</tr>
<tr>
<td>Asymmetric Impact</td>
</tr>
<tr>
<td>Symmetric Impact</td>
</tr>
<tr>
<td>Asymmetric Impact</td>
</tr>
</tbody>
</table>

Table 2. Taxonomy for Composite Indicators of Fiscal Decentralization

In addition, Appendix II presents the 3D graph and contour line of each composite
indicator predicated on \( r \) and \( e \) in order to help figure out what makes the difference between
the composite measures. A follow-up research project is underway to explore patterns and
changes over time of the historical trends of the composite indicators for cross sections of
countries.
IV. Conclusion

So far, this paper proposes the two criteria of impact-symmetry and VFI-favorableness by which varying composite indicators for fiscal decentralization can be judged and compared, and it also comes up with two new composite indicators in order to overcome the shortcomings of the previous measures. Now that we come to conclusion that a composite indicator always involves a certain viewpoint of the relative importance of revenue and expenditure decentralization and a distinct evaluation of vertical fiscal imbalance in an implicit or explicit way, a fundamental question could be raised: which is the best single indicator? It is not easy to answer, but there may be little rational reason for choosing an impact asymmetric indicator. Given that the rigorous meaning of fiscal decentralization consists of the equally-important two sub-concepts, the impact symmetric measures are recommended to reflect both sides of public finance.

Concerning VFI-favorableness criterion, VFI is a result of inappropriate assignments of tax-raising authority, which is seen as a structural problem. However, the revenue-expenditure imbalance between the two levels of a government does not always clearly indicate the extent to which the revenue-raising powers should be matched to spending responsibilities and the extent to which a restructuring of intergovernmental transfers is required (Sharma, 2011). Intergovernmental transfers turn out to be indispensable policy instruments for stabilizing macroeconomic variables, harmonizing taxes across the nation, internalizing fiscal externalities, supplying national minimum level of public goods through redistribution, utilizing inter-regional insurance, and exploiting the economies of scale for public service delivery. In the real world, some degree of revenue-expenditure imbalance along with a well-designed transfer system is inevitable. Thus, the optimal VFI is typically
positive; the question to be addressed is how much positive VFI is optimal. Theoretically, if we know the optimal level of transfers, then it is recommended that one rely on VFI-
favorable measures when the current level of revenue-expenditure imbalance is lower than the optimal one, whereas she should depend on VFI-unfavorable indicators when the current level is higher than the optimal level of VFI. In practice, the task of finding the optimal distribution of taxing and spending authority is a political one. Thus the best answer to the fundamental question concerning VFI-favorableness might be that it depends on to whom you talk. Even so, there has been vast literature insisting that SNG should finance their spending with their revenues due to a number of incentive problems with the transfer system (Eyraud and Lusinyan, 2011). Having said that, we suggest that one should rely on VFI-
unfavorable measures unless there are reasonable grounds for substituting a VFI-favorable indicator for them.
Reference


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fiscal performance?, IMF working paper 11/226
Feld and Schnellenbach (2008) Fiscal federalism, decentralization and economic growth: A meta-analysis, working paper
Appendix I

Proof of Proposition 1: Differentiate each composite indicator w.r.t. \( r \) and \( e \) and compare the partial derivatives, respectively.

1) \[ \frac{\partial A_{III}}{\partial r} = \frac{r}{e} \neq \frac{1}{e} \frac{1}{D} \quad \text{thus RA_{II} is an impact asymmetric indicator.} \]

2) \[ \frac{\partial V}{\partial r} = \frac{1}{2} \left( \frac{r}{D} \right)^{-\frac{1}{2}} \neq \frac{\partial V}{\partial e} = 0 \quad \text{thus FDI is an impact asymmetric indicator.} \]

3) \[ \frac{\partial C}{\partial r} = \frac{1}{1-e} \neq \frac{\partial C}{\partial e} = \frac{r}{(1-e)^2} \quad \text{thus CR is an impact asymmetric indicator.} \]

4) Since \[ \frac{\partial N}{\partial r} = \frac{1}{2} \quad \text{and} \quad \frac{\partial N}{\partial e} = \frac{1}{2} \] is identical, PRI is an impact symmetric indicator. The impact-symmetry is defined by Definition 1.

Proof of Lemma 1: Consider the first partial derivative of CR measure as in the proof of Proposition 1. The ratio between the two partial derivatives, \( \frac{C_R_1}{C_R_2} = \frac{1-e}{r} \) determines the relative effect on CR of the two decentralization indicators. The impact of revenue decentralization on CR is greater than that of expenditure decentralization only when \( 1-e \) is greater than \( r \). In other words, \( C_{R_1} > C_{R_2} \) only when \( r + e < 1 \).

Proof of Proposition 2: Differentiate each measure twice w.r.t. \( r \) and \( e \), respectively, to get the second-order cross partial derivative.

1) \[ \frac{\partial^2 A_{III}}{\partial r \, \partial e} = -\frac{r}{e^2} \frac{1}{D} < 0 \quad \text{thus RA_{II} is a VFI-favorable measure by Definition 2.} \]

2) \[ \frac{\partial^2 V}{\partial r \, \partial e} = 0 \quad \text{thus FDI is a VFI-neutral measure by Definition 2.} \]
3) \[ \frac{\partial^2 C}{\partial r \partial e} = \frac{1}{(1-e)^2} > 0 \] thus CR is a VFI-unfavorable measure by Definition 2.

4) \[ \frac{\partial^2 N}{\partial r \partial e} = 0 \] thus PRI is a VFI-neutral measure by Definition 2. ■

**Proof of Proposition 3:** Differentiate each composite indicator w.r.t. \( r \) and \( e \) and compare the partial derivatives, respectively.

1) Since \( \frac{\partial S_f}{\partial r} = \frac{r(r^2+e^2)^{-\frac{1}{2}}}{\sqrt{2}} \) and \( \frac{\partial S_f}{\partial e} = \frac{e(r^2+e^2)^{-\frac{1}{2}}}{\sqrt{2}} \) are functionally symmetric, SF is an impact symmetric indicator by Definition 1.

2) Since \( \frac{\partial S_u}{\partial r} = \frac{1}{\sqrt{2}} \left[ \frac{1-r}{(1-r)^2+(1-e)^2} \right] \) and \( \frac{\partial S_u}{\partial e} = \frac{1}{\sqrt{2}} \left[ \frac{1-e}{(1-r)^2+(1-e)^2} \right] \) are functionally symmetric, SU is an impact symmetric indicator by Definition 1. Therefore, all the new composite measures are impact symmetric. ■

**Proof of Proposition 4:** Differentiate the two new composite measures twice w.r.t. \( r \) and \( e \), respectively, to get the second-order cross partial derivative.

1) \[ \frac{\partial^2 S_f}{\partial r \partial e} = -\frac{r e(r^2+e^2)^{-\frac{3}{2}}}{\sqrt{2}} < 0 \] thus SF is a VFI-favorable measure by Definition 2.

2) \[ \frac{\partial^2 S_u}{\partial r \partial e} = \frac{1}{\sqrt{2}} (1-r)(1-e)[(1-r)^2+(1-e)^2]^{-\frac{3}{2}} > 0 \] thus SU is a VFI-unfavorable measure by Definition 2. ■
Appendix II

We draw the 3D graph and contour line of each composite indicator predicated on \( r \) and \( e \) in order to help figure out what makes the difference between the composite measures. The left figure represents three-dimensional representation of composite indicators, while the right one shows their contour lines.

Figure A.1. Revenue Autonomy II (RA\(^{II}\))

Figure A.2. Production-Revenue Indicator (PRI)
Figure A.3. Fiscal Decentralization Index (FDI)

Figure A.4. Composite Ratio (CR)

Figure A.5. Symmetric VFI-favorable (SF)
Figure A.6. Symmetric VFI-unfavorable (SU)