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Abstract

In state-of-the-art macroeconomic and labor market models shocks are assumed to be homoscedastic. However, we show that this assumption is much too restrictive. We find significant evidence for strong time-varying volatility in all considered labor market time series.

First, we estimate the *unconditional* variance-covariance matrix and find significant evidence for time variability. Second, we estimate the *conditional* variance-covariance matrix and discuss the time-varying risk contained in labor market variables.

The implications are relevant for modelling purposes, welfare analysis, and the understanding of sources of fluctuations.

Keywords: Dynamic Correlation, Multivariate GARCH, Stochastic Volatility.

JEL codes: C30, E30, J60.

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1 Introduction

In macroeconomic models dynamic effects of shocks are simulated by imposing the ad hoc assumption of homoscedasticity of the underlying stochastic processes. However, recent research has shown that various macroceconomic time series exhibit a strong time-varying variance with clustering of periods with high and low volatility. Sims and Zha (2006) use a structural vector autoregression model allowing for Markov regime switching and show that the best fit is obtained with a model that features time-varying variances of structural disturbances. More recently, Fernandez-Villaverde and Rubio-Ramirez (2010) estimate the stochastic volatility present in aggregate time series and the estimation by Justiniano and Primiceri (2008) show a strong stochastic volatility of shocks in the United States that vary considerably across types of shocks.

A different, though related issue is the role played by uncertainty. Higher volatility of variables that influence the optimization problem of economic agents can have substantial effects on the optimal path of actions. This is especially important in the presence of non-convex adjustment costs, irreversibilities, or non-linearities in the production set. Bloom (2009) and Bloom et al. (2012) show that higher risk may lead a household to defer investments because of the fear to hit a liquidity or credit constraint that may impact the optimal consumption allocation in the future. Along this line, an increase in the volatility, for example, of the job separation rate may lead the household to increase savings out of precautionary motives or affect its effort and hence her productivity. Therefore, (increased) volatility in labor market variables can have substantial real effects.

While there is extensive evidence for macroeconomic time series, there is only very sparse evidence concerning labor market variables. Stock and Watson (2002) use time series for employment, unemployment, wages, and the help-wanted index and estimate an AR(4) process to address the change in the (time-varying) standard deviation. They find a significant time-varying component but they neither report the behavior of the unconditional mean of variance and covariance nor do they estimate the dynamic interdependencies between those variables.

In order to fully understand the source of fluctuations in the labor market, for welfare analysis, and for policy advice it is, however, important to consider all time series that are present in the workhorse labor market model, the Mortensen-Pissarides (1994) search and matching model. This paper aims at providing this missing empirical evidence.

The purpose of this paper is to discuss the stochastic volatility of labor market variables in the United States and hence is purely descriptive. We will leave a (micro)foundation of the observed stochastic volatility to future research. The contribution of this paper is twofold. First, we estimate the *unconditional* mean of the variance and covariance for the U.S. labor market to address the assumption of homoscedasticity in macroeconomic and, in particular, in labor market models. Here, we estimate a time-varying parameter VAR model as in Primiceri (2005). We find significant evidence for time variability in the unconditional mean and in the unconditional correlation.

Second, we estimate the *conditional* variance and covariance and therefore are able to analyze the behavior of uncertainty in the labor market. We do so by estimating a multivariate generalized ARCH model, namely the dynamic conditional correlation model proposed by Engle (1999). This model allows for stochastic variance and stochastic correlations and is therefore able to shed light on the behavior of the conditional second moments over time. We find that labor market variables contain large amount of time-varying risk.

ARCH models have predominantly been used in finance, while Hamilton (2008) discusses the scarse literature on ARCH in macroeconomics. However, there are various reasons why macroand labore economists should be interested in second moments as well. As discussed in detail in Hamilton (2008), misspecification of the variance-covariance matrix will make a hypothesis test on the mean invalid. Further, efficiency in estimating the first moments can be increased by including the observed heteroscedasticity. Moreover, time-varying first and second moments can be seen as evidence for non-linearities in the economy.

The paper is structured as follows. The next section discusses our data set. Section 3 will discuss the estimation of the unconditional variance-covariance, while section 4 will deal with the estimation of the conditional variance-covariance matrix. Section 5 concludes.

2 Data

In the subsequent chapters we will use different labor market time series, namely employment, vacancies, unemployment, wages, total factor productivity (TFP, for short), (job) separation rate, job finding rate, hours, and the gross domestic product (GDP, for short). All time series are seasonally adjusted and are on a quarterly basis and cover the period from 1955:Q1 to 2000:Q4, which gives us 184 data points.

For section 3 of this analysis, we use the unfiltered time series, while for section 4 we pass the time series through a Hodrick-Prescott filter with smoothing parameter 1600 in order to obtain the necessary zero-mean input series (see section 4 for more details).

Employment is measured by the number of nonsupervisory workers as constructed by the Bureau of Labor Statistics (BLS, for short). The series for vacancies is constructed from the help-wanted advertising index by the Conference Board. Unemployment is measured by the unemployment rate as published by the BLS in its Current Population Survey (CPS, for short). For wages we use the real hourly compensation series from the BLS.

The time series for TFP is taken from Basu et al. (2006) and is utilization-adjusted TFP for the U.S. Business Sector. For the separation rate and the job finding rate, we use the time series constructed by Shimer (2012) from employment, unemployment, and mean unemployment duration. The quarterly time series are averages of monthly rates.

Hours are constructed as the average weekly hours worked by all persons in nonfarm business multiplied by civilian employment and normalized by the labor force. All time series obtained by the BLS with the exception being average weekly hours being taken from the U.S. Department of Labor. Finally, the time series for GDP is taken from the NIPA tables.

3 Estimating the *Unconditional* Variance-Covariance Matrix

This section is concerned with the estimation of the unconditional mean of the variances and covariances of our time series. In order to do so, we will review the time-varying parameter VAR model developed in Primiceri (2005).

3.1 Model

We estimate a multivariate time series model that features not just time-varying parameters but also a time-varying variance-covariance matrix. The big advantage of this modelling approach is the fact that the model allows the time variation to be driven by changes in the propagation mechanism or by changes in the size of the shock.

Let $\{y_t\}$ denote a $N \times 1$ dimensional vector of random variables and assume that the $N \times 1$ dimensional vector c_t contains time-varying coefficients. Further, denote $N \times N$ dimensional matrices of time-varying coefficients by $\mathbf{B}_{i,t}, i = 1, \ldots, k$, where k determines the lag length.

Then, the model can be written as

$$y_t = c_t + \mathbf{B}_{1,t} y_{t-1} + \ldots + \mathbf{B}_{k,t} y_{t-k} + u_t, \quad t = 1, \ldots T, \tag{1}$$

where T is the sample size. Here, we assume that the exogenous disturbances, u_t, have a variance-covariance matrix $\mathbf{\Omega}_t$.

Using this variance-covariance matrix and applying a triangular factorization, we find

$$\mathbf{A}_t \mathbf{\Omega}_t \mathbf{A}_t' = \mathbf{\Xi}_t \mathbf{\Xi}_t', \tag{2}$$

where we denote the diagonal matrix with $\mathbf{\Xi}_t$

$$\mathbf{\Xi}_t = \begin{bmatrix} \sigma_{1,t} & 0 & \ldots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \sigma_{N,t} \end{bmatrix}, \tag{3}$$

with $\sigma_{i,t} > 0, \quad \forall \quad i = 1, \ldots, N$.

Further, $\mathbf{A}_t$ is a lower triangular matrix with ones a long the main diagonal and zeros above the main diagonal elements

$$\mathbf{A}_t = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \theta_{2,1,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \theta_{N,1,t} & \ldots & \theta_{N,N-1,t} & 1 \end{bmatrix}. \tag{4}$$

Then, we can write the model in eq. (1) as

$$y_t = c_t + \mathbf{B}_{1,t} y_{t-1} + \ldots + \mathbf{B}_{k,t} y_{t-k} + \mathbf{A}_t^{-1} \mathbf{\Xi}_t \varepsilon_t, \quad t = 1, \ldots T, \tag{5}$$

$$\mathbf{V}(\varepsilon_t) = \mathbf{I}_N, \tag{6}$$

where $\mathbf{V}(\varepsilon_t)$ is the variance-covariance matrix of the disturbances and $\mathbf{I}_N$ is a N dimensional identity matrix. Finally, we can write the model as

$$y_t = \mathbf{X}_t' G_t + \mathbf{A}_t^{-1} \mathbf{\Xi}_t \varepsilon_t, \tag{7}$$

$$\mathbf{X}_t' = \mathbf{I}_N \otimes \left[1, y_{t-1}', \ldots, y_{t-k}'\right], \tag{8}$$

where G_t is a vector that stacks the entire right-hand-side coefficients of (5) and $\otimes$ is the Kronecker product operator.

In the following, we need to impose some structure on the dynamic evolution of the time-varying parameters. We do so by assuming a random walk behavior, i.e.

$$G_t = G_{t-1} + \nu_t, \tag{9}$$

$$a_t = a_{t-1} + \zeta_t, \tag{10}$$

$$\log \sigma_t = \log \sigma_{t-1} + \eta_t, \tag{11}$$

where vector a_t stacks the non-zero, non-one elements of matrix $\mathbf{A}_t$ and vector σ_t stacks the diagonal elements of $\mathbf{\Xi}_t$. All disturbances are assumed to be Gaussian with the following variance-covariance matrix

$$\mathbf{V} = Var \begin{bmatrix} \varepsilon_t \\ \nu_t \\ \zeta_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} \mathbf{I}_N & 0 & 0 & 0 \\ 0 & \mathbf{Q} & 0 & 0 \\ 0 & 0 & \mathbf{S} & 0 \\ 0 & 0 & 0 & \mathbf{W} \end{bmatrix}, \tag{12}$$

where $\mathbf{Q}, \mathbf{S}, \mathbf{W}$ are all positive definite matrices.

Finally, it is worth noticing that the ordering of variables does, to some extend, affect the results. This caveat follows from the factorization in eq. (2) and the resulting lower triangular structure of $\mathbf{A}_t$. From the literature on search and matching models, we assume the following ordering of variables in our VAR

$$y_t = \begin{bmatrix} \text{Total Factor Productivity} \\ \text{Separation Rate} \\ \text{Unemployment} \\ \text{Job Finding Rate} \\ \text{Vacancies} \\ \text{Wages} \end{bmatrix}, \tag{13}$$

hence - by lower triangularity - we assume that total factor productivity drives all other variables and can be considered to be the most exogenous variable. On the contrary, we assume that wages is the most endogenous variable, as all variables ordered before will have an effect on it. In addition, this is not just an ordering issue but also an identification issue. With this structure, we are able to isolate the productivity shocks, as they are considered to be the main driving force in the canonical search and matching model.

3.2 Estimation Results

As in Primiceri (2005), we use Bayesian methods to estimate the parameters of interest. To be precise, those are the unobservable states $\mathbf{B}^T, \mathbf{A}^T, \mathbf{\Xi}^T$ and the variance-covariance matrix $\mathbf{V}$. The MCMC algorithm uses Gibbs sampling drawing a sample from $\left(\mathbf{B}^T, \mathbf{A}^T, \mathbf{\Xi}^T, \mathbf{V}\right)$ conditional on the data and the remaining parameters. Given this sample, a reduced form VAR of the type

$$y_t = \mathbf{X}_t'\mathbf{B}_t + \mathbf{\Upsilon}_t\varepsilon_t, \tag{14}$$

can be estimated, that results in posteriors for $\mathbf{B}$ and $\mathbf{\Omega}$ at any given point in time.[1] Then, the posterior values for $\mathbf{\Upsilon}_t$ are obtained by solving the system of equations for every draw of $\mathbf{\Omega}_t$

$$\mathbf{\Upsilon}_t\mathbf{\Upsilon}_t' = \mathbf{\Omega}_t, \forall t. \tag{15}$$

The solution is given by

$$\mathbf{\Upsilon}_t = \mathbf{A}_t^{-1}\mathbf{\Xi}_t, \tag{16}$$

because of the triangular identification approach.

We estimate the TVP-VAR with 70.000 draws of which 20.000 are used as burn-in-draws. Further, we use the first 60 observations to calibrate the prior distributions. Our results are presented in Figure 1 to 3.

Figure 1 presents the unconditional variance point estimates for the six shocks considered. We observe a significant and considerable amount of time variation in the time series at hand. The

[1] Identification is ensured by letting $\mathbf{\Upsilon}_t$ contain at least $\frac{n(n-1)}{2}$ restrictions.

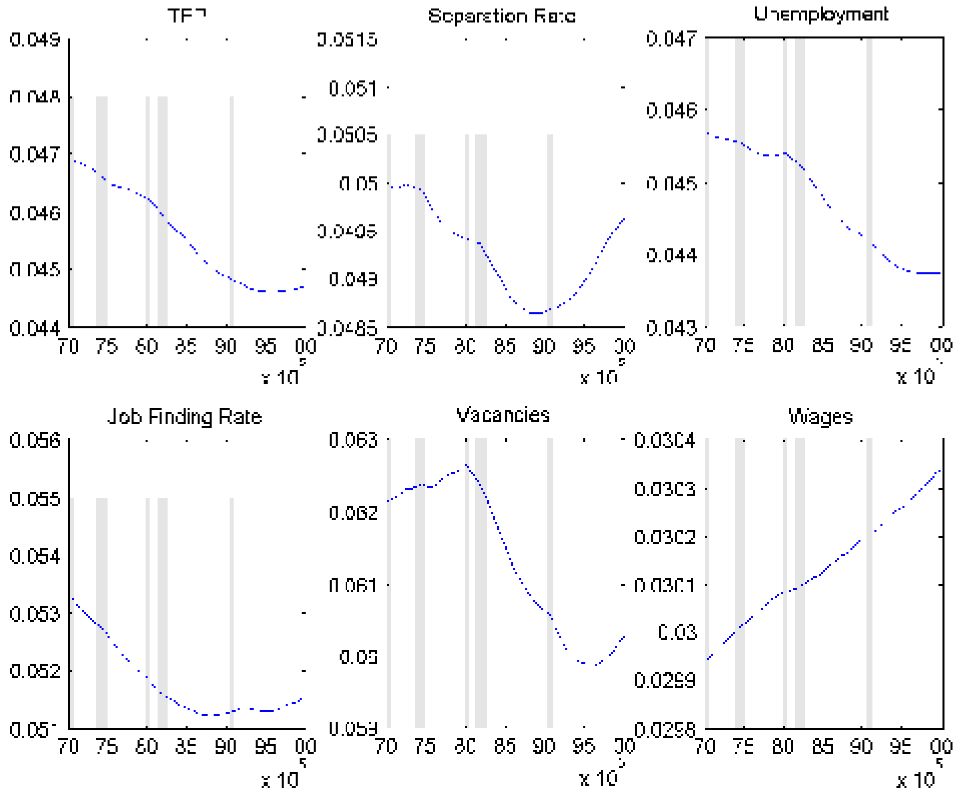

Figure 1: Estimated unconditional standard deviations. Shaded areas indicate NBER recession dates.

volatility in TFP declines nearly linearly from 0.047 at the beginning of our sample to a value close to 0.045. We observe a similar pattern for unemployment decreasing from a value close to 0.046 to a value between 0.0435 and 0.044. Further, a similar pattern is visible for the job finding rate. We observe a drop from 0.0535 to 0.0515 and a rough stabilization for the last 50 quarters on this level. On the contrary, we find that the volatility of wages has increased over the sample. Starting on a value of 0.0299 our estimation shows a final value of roughly 0.0304. Most interesting though is the evolution of the variances of the separation rate and vacancies. For the separation rate, we observe a sharp decline within the first half of the sample period from a value of roughly 0.05 to a value of 0.0486. Afterwards, in the second half of the sample, we find a sharp pick up of the variance reaching a value of 0.0496 at the end of the sample. Similarly, we find this pattern for vacancies, although less pronounced. The volatility of vacancies increases for almost 50 periods and then sharply declines for the next 50 quarters. At the end of the sample we again observe a pick up of the volatility.

Having discussed the conditional variance of our time series, we want to turn to the conditional correlations between the time series. For this purpose, Figures 2 and 3 show the fifteen correlations that can be estimated in our sample. Correlations of TFP with the separation rate and wages remain relatively stable over the sample period, while TFP and the job finding rate become significantly less countercyclical. A similar pattern is visible for the correlation between TFP and vacancies. Here, we observe that both variables are initially countercyclical, become acyclical and then again become countercyclical at the end of the sample. Along this finding, very interesting is the evolution

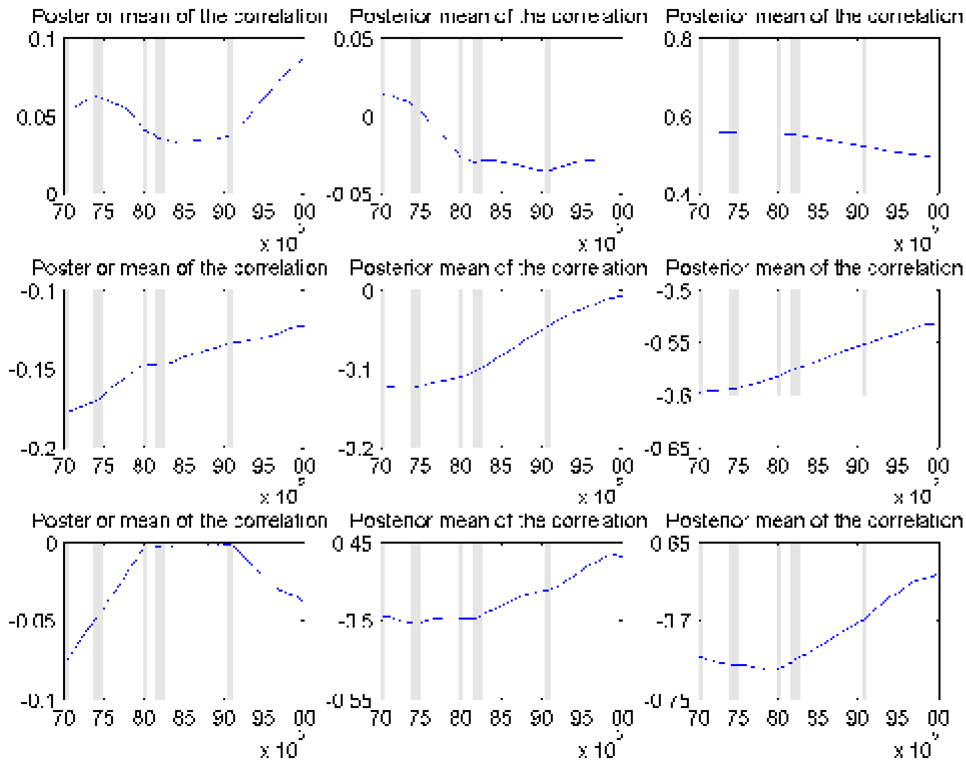

Figure 2: Estimated unconditional correlations. Shaded areas indicate NBER recession dates.

of the correlation structure between TFP and unemployment. While this correlation was positive (0.02) at the beginning of the sample, we observe that the cyclicality is reversed and after roughly 25 periods, the correlation is significantly negative, e.g. we find a final value of roughly -0.03.

Moreover, we observe a stable, positive relationship between the separation rate and unemployment. Moreover, we find a relatively stable relationship between the separation rate and the job finding rate, vacancies, and wages. Although the correlation tends to become smaller.

The correlation between the job finding rate and vacancies stays roughly constant over the sample period, though we observe a slight decline. The correlation between the job finding rate and wages varies considerably over the sample. Initially, it increases for the first 50 periods, then declines for the next 50 quarters, increases again, but starts to decline at the end of the sample.

Finally, we find that the correlation of unemployment with the job finding rate, vacancies, and wages all start to become less negative after some time. For vacancies and wages it takes roughly 50 quarters after which the correlation starts to become smaller, while it almost immediately starts to increase for the job finding rate.

4 Estimating the *Conditional* Variance-Covariance Matrix

In this section we are concerned with estimating the conditional variance-covariance matrix, therefore estimating the uncertainty contained in the time series. For this purpose, we use a multivariate GARCH model established by Engle (1999) and Engle and Sheppard (2001), namely the dynamic conditional correlation model (DCC, for short).

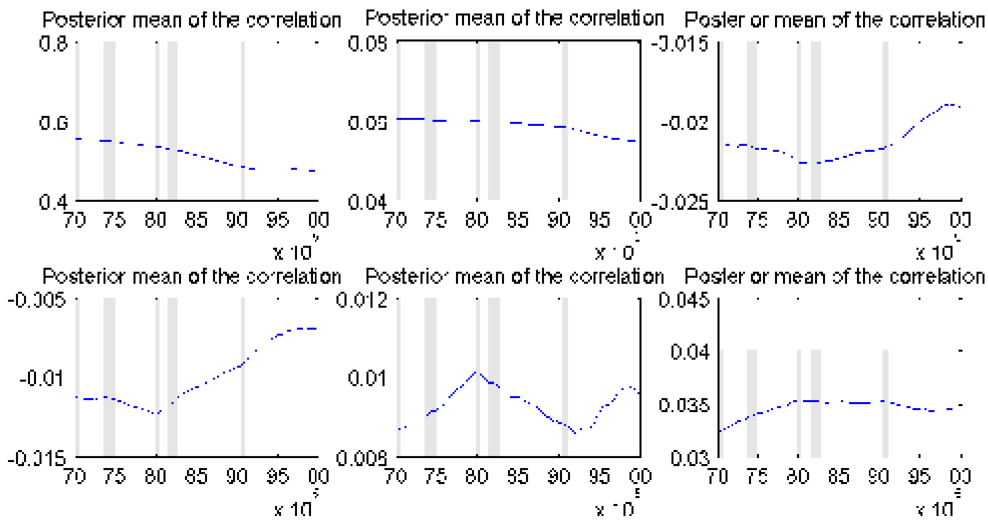

Figure 3: Estimated unconditional correlations. Shaded areas indicate NBER recession dates.

4.1 Model

Generally, let $\{y_t\}$ denote a $N \times 1$ dimensional vector of random variables with $\mathbb{E}[y_t] = 0$. Further, let $\mathcal{I}_t$ be the smallest σ-field generated by the history of values of y_t, i.e. $\mathcal{I}_t = \sigma(y_r, r \leq t-1)$. Moreover, assume that the time series in our vector $\{y_t\}$ are conditionally multivariate Gaussian with zero mean and covariance matrix $\mathbf{H}_t$. By imposing measurability of $\mathbf{H}_t$ with respect to $\mathcal{I}_t$ we can write the multivariate GARCH as

$$y_t|\mathcal{I}_t \sim \mathcal{N}(0, \mathbf{H}_t), \tag{17}$$

and, formally, $y_t = \sqrt{\mathbf{H}_t}\eta_t$, where η_t is a $N \times 1$ dimensional vector of i.i.d. errors. The next step is to put some structure on the $N \times N$ dimensional, positive semi-definite, symmetric conditional covariance matrix $\mathbf{H}_t$ and this is done by modelling the conditional variances and correlations, i.e.

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t. \tag{18}$$

Here, $\mathbf{D}_t$ is a $N \times N$ diagonal matrix of time-varying conditional standard deviations and $\mathbf{R}_t$ is a $N \times N$ dimensional time-varying conditional correlation matrix.

The former matrix is given by

$$\mathbf{D}_t = \begin{bmatrix} \sqrt{h_{1,t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{h_{N,t}} \end{bmatrix}, \tag{19}$$

where $\sqrt{h_{i,t}} \quad \forall \quad i \in \{1, ..., N\}$ is estimated from a univariate GARCH(q,p) model.

This GARCH(q,p) model can be written the usual way as

$$h_{i,t} = \alpha_{i,0} + \sum_{j=1}^{q} \alpha_{i,j} y_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j} h_{i,t-j}, \tag{20}$$

with $\alpha_{i,0} > 0, \alpha_{i,j} \geq 0$, and $\beta_{i,j} \geq 0 \quad \forall \quad i, j$. Further, we denote the standardized residuals as $\varepsilon_t = \mathbf{D}_t^{-1} y_t \sim \mathcal{N}(0, \mathbf{R}_t)$, where we standardize by the conditional standard deviation.

It can be shown that a necessary and sufficient condition for stationarity of those univariate GARCH processes is $\sum_{j=1}^{q} \alpha_{i,j} + \sum_{j=1}^{p} \beta_{i,j} < 1 \quad \forall \quad i$.

The symmetric correlation matrix $\mathbf{R}_t$ is given by

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{1,2,t} & \cdots & \rho_{1,N,t} \\ \rho_{1,2,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{N-1,N,t} \\ \rho_{1,N,t} & \cdots & \rho_{N-1,N,t} & 1 \end{bmatrix}, \tag{21}$$

where $\rho_{i,j,t}$ is the correlation estimator. We would like to ensure that $\mathbf{H}_t$ is positive definite, since it is a covariance matrix, and all elements of the correlation matrix, $\mathbf{R}_t$, are less or equal to one in absolute terms.

For this purpose, it is assumed that the conditional correlation matrix follows

$$\mathbf{R}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \tag{22}$$

$$\mathbf{Q}_t = \left(1 - \sum_{m=1}^{M} \alpha_m - \sum_{k=1}^{K} \beta_k\right) \bar{\mathbf{Q}} + \sum_{m=1}^{M} \alpha_m \varepsilon_{t-m} \varepsilon_{t-m}' + \sum_{k=1}^{K} \beta_k \mathbf{Q}_{t-k}, \tag{23}$$

where $\bar{\mathbf{Q}} = \mathbb{E}[\varepsilon_t \varepsilon_t']$ is estimated by the sample mean. Then, $\mathbf{Q}_t^* = diag\left(\sqrt{q_{11,t}}, \cdots, \sqrt{q_{NN,t}}\right)$ rescales the elements of $\mathbf{Q}_t$ to ensure that the elements of the correlation matrix are less or equal to one in absolute terms, i.e. $\left|\rho_{i,j,t}\right| = \left|\frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}}\right| \leq 1$. Positive definiteness of $\mathbf{H}_t$ with probability one is ensured by $\alpha_{i,j} \geq 0$, $\beta_{i,j} \geq 0 \quad \forall \quad i, j$, positive definite $\mathbf{Q}_0$, and the stationarity condition $\sum_{j=1}^{q} \alpha_{i,j} + \sum_{j=1}^{p} \beta_{i,j} < 1 \quad \forall \quad i$ as in Engle and Sheppard (2001).

After discussing the primitives of the model, we want to briefly describe the estimation strategy of the DCC model. Because we assumed our errors, η_t, to be multivariate Gaussian, we can derive the joint distribution of z_t as

$$f(\eta_t) = \prod_{t=1}^{T} (2\pi)^{-\frac{N}{2}} e^{-\frac{1}{2}\eta_t' \eta_t}, \tag{24}$$

where we used the properties of the i.i.d. shock $\mathbb{E}[\eta_t] = 0$ and $\mathbb{E}[\eta_t \eta_t'] = \mathbf{I}_N$. Then, by using our original definition, $y_t = \sqrt{\mathbf{H}_t} \eta_t$, we can write the likelihood function as

$$L(\phi) = \prod_{t=1}^{T} (2\pi)^{-\frac{N}{2}} (|\mathbf{H}_t|)^{-\frac{1}{2}} e^{-\frac{1}{2} y_t' \mathbf{H}_t^{-1} y_t}, \tag{25}$$

where $|\mathbf{H}_t|$ is the determinant of $\mathbf{H}_t$. Further, ϕ contains the parameters of the univariate GARCH model for the i^{th} time series, i.e. $(\phi,\psi) = (\phi_1, \ldots, \phi_N, \psi)$ and $\phi_i = \left(\alpha_{i,0}, \alpha_{i,1}, \ldots, \alpha_{i,p}, \beta_{i,1}, \ldots, \beta_{i,q}\right) \quad \forall \quad i$ and $\psi = (\alpha_1, \ldots, \alpha_M, \beta_1, \ldots, \beta_K)$ contains the parameters of the correlation structure.

After some algebra, the log-likelihood estimator is given by

$$\ln(L(\phi)) = -\frac{1}{2} \sum_{t=1}^{T} \left[N \ln(2\pi) + 2\ln(|\mathbf{D}_t|) + \ln(|\mathbf{R}_t|) + \eta_t' \mathbf{R}_t^{-1} \eta_t\right].$$

Estimating the DCC model involves two steps. First, one estimates univariate GARCH models for each residual time series. Here, one replaces the correlation matrix, $\mathbf{R}_t$, by the identity matrix, $\mathbf{I}_N$.

Then, the quasi maximum log-likelihood estimator is

$$\ln\left(QL_1\left(\phi|y_t\right)\right) = -\frac{1}{2}\sum_{n=1}^{N}\left[T\ln\left(2\pi\right) + \sum_{t=1}^{T}\left[\log\left(h_{i,t}\right) + \frac{y_{i,t}^2}{h_{i,t}}\right]\right]. \tag{26}$$

The first stage estimator intuitively is the summation of individual log-likelihood values of univariate GARCH models for each time series in y_t. The only remaining parameters to be estimated after the first step are the ones contained in ψ.

Consecutively, step two uses the correctly specified likelihood

$$\ln\left(QL_2\left(\psi|\hat{\phi}, y_t\right)\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[N\ln\left(2\pi\right) + 2\ln\left(|\mathbf{D}_t|\right) + \ln\left(|\mathbf{R}_t|\right) + \eta_t'\mathbf{R}_t^{-1}\eta_t\right], \tag{27}$$

here, we condition on the parameters estimated in the first step. Due to conditioning on the estimated parameters, we could exclude the constant parameters from the maximization and only consider the latter two terms in eq. (27).

Consistency and asymptotic normality of the two step QMLE estimator has been shown by White (1996) and Engle and Sheppard (2001).

4.2 Test on Constant Correlation

Engle and Sheppard (2001) suggest a test on constant correlation that standardizes the residuals of the estimate of univariate GARCH processes. Consecutively, the correlation of the standardized residuals is estimated, jointly standardized by the symmetric square root decomposition of the correlation matrix ($\bar{\mathbf{R}}^{-\frac{1}{2}}\mathbf{D}_t^{-1}y_t$). If correlation would truly be constant, then the residuals would be i.i.d. with an identity matrix as variance-covariance matrix.

Therefore, the hypothesis is

$$H_0 \quad : \quad \mathbf{R}_t = \bar{\mathbf{R}}, \tag{28}$$

$$vs.$$

$$H_a \quad : \quad vech^u\left(\mathbf{R}_t\right) = vech^u\left(\bar{\mathbf{R}}\right) + \beta_1 vech^u\left(\mathbf{R}_{t-1}\right) + \ldots + \beta_p vech^u\left(\mathbf{R}_{t-p}\right), \tag{29}$$

where $vech^u$ is a $vech$-operator that only uses elements above the diagonal. Then, the vector autoregression is

$$Y_t = \alpha + \beta_1 Y_{t-1} + \ldots \beta_s Y_{t-s} + \zeta_t, \tag{30}$$

where $Y_t = vech^u\left[\left(\bar{\mathbf{R}}^{-\frac{1}{2}}\mathbf{D}_t^{-1}y_t\right)\left(\bar{\mathbf{R}}^{-\frac{1}{2}}\mathbf{D}_t^{-1}y_t\right)' - \mathbf{I}_N\right]$. Again, if the null would be true, all regression coefficients would be zero. Finally, the test statistic is $\frac{\hat{\delta}X'X\hat{\delta}}{\sigma^2}$, where X contains the regressors and $\hat{\delta}$ contains the estimated regression coefficients. Then, the test statistic will be asymptotically $\chi^2\left(s+1\right)$.

If we apply this test to our data, we find that the value of the test statistic is 99.02 with a p-value of 0, using five lags. Put differently, the probability that the correlation is constant is effectively 0. Therefore, we reject the null hypothesis that the correlation in our data set is constant.

4.3 Model Selection

Before we discuss the results of our estimation, we want to justify our choice of the DCC model. For this purpose, we estimate five different, frequently used models (see e.g. the survey paper by

Silvennoinen and Teräsvirta (2007)). In order to judge the goodness of fit of our model, we employ the Akaike Information Criterion (AIC, for short) and search for the minimum value across models. The basic idea of the AIC is to estimate the Kullback-Leibler divergence that describes how close two probability distributions are to each other. Further, in large samples the Kullback-Leibler divergence can be estimated by the likelihood score, as the likelihood score is as an asymptotically unbiased estimator of the cross-entropy risk.

The AIC is defined as

$$AIC = -2\ln\left(L\left(\phi, \psi\right)\right) + 2p, \tag{31}$$

where p denotes the number of free parameters.

The five models considered are the Bollerslev (1990) constant conditional correlation model (CCC, for short), the dynamic conditional correlation model described in section 2, the integrated dynamic conditional correlation model (IDCC, for short)[2], the full BEKK model by Engle and Kroner (1995), and the full BEKK model with t-distributed error terms (TBEKK, for short). As

Table 1: Akaike Information Criterion for the five different models.

	CCC	DCC	IDCC	BEKK	TBEKK
AIC	-853.82	-863.77	-853.34	-854.67	-856.33

we can infer from Table 1, the DCC model gives the lowest AIC score and hence, the next chapter discusses the results from estimating the DCC model on our data set.

4.4 Estimation Results

We begin by discussing the results of the conditional standard deviation estimation following from the DCC model. The behavior of the standard deviations is shown in Figure 4. From this figure, we can draw several interesting conclusions.

- We find evidence for a significant decline of volatility in almost all time series which has been named the "Great moderation" and started around the mid 1980's. However, there are two counterexamples, namely wages and the job finding rate. For those two variables there is almost no difference in their behavior before or after the starting point of the Great moderation (1984 is accepted by most researchers). In fact, it appears that wages are more volatile after 1984 than before, as we observe four large peaks and see only very small peaks from 1955 to the early 70's. The job finding rate is quite volatile in the first half of the 1990's but seems to become much less volatile after this period of high volatility. Further, we find that the during the Great moderation the recession at the early 90's had only very limited impact on the volatility of almost all variables, which is significantly different to earlier recessions. Here, we observe a decoupling of recessions and peaks of volatility. Finally, the we find evidence that the length of the recession is uncorrelated with the size of the peak in volatility.

- We can cluster variables into four groups according to the size of their fluctuations, i.e. their riskiness. Employment, wages, and hours show the smallest values of conditional volatility. On the flipside, vacancies, the separation rate, and the job finding rate have the largest values.

[2] Integratedness here refers to the fact that the stationarity condition now holds with equality, i.e. $\sum_{j=1}^{q} \alpha_{i,j} + \sum_{j=1}^{p} \beta_{i,j} = 1 \forall i$.

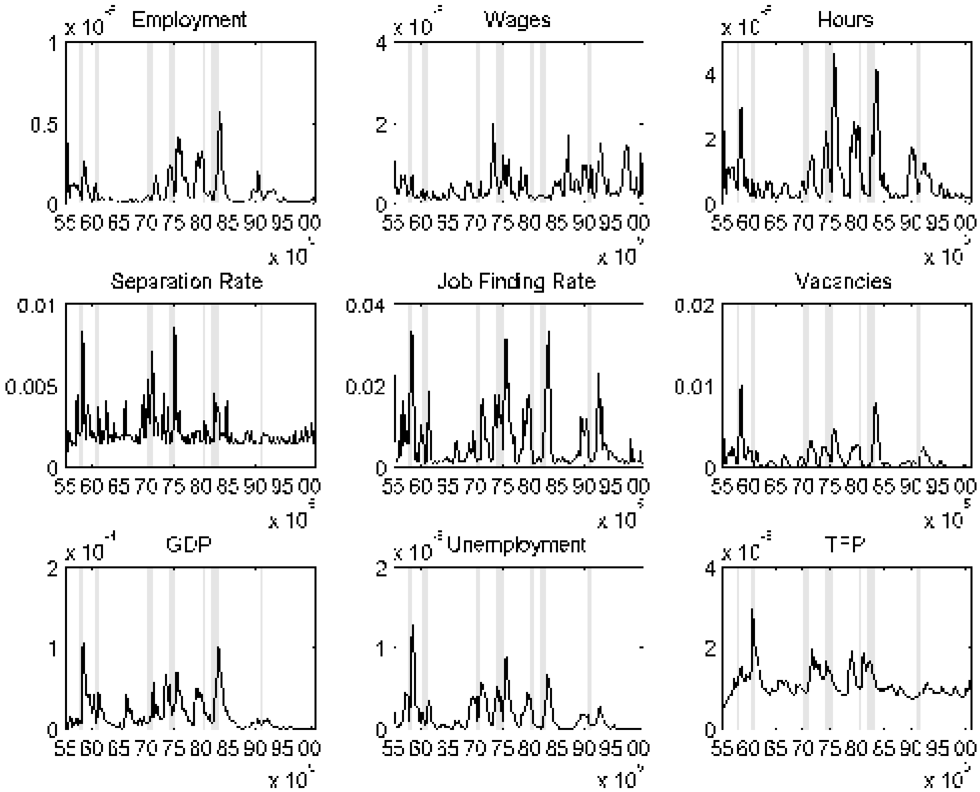

Figure 4: Estimated conditional standard deviations. Shaded areas indicate NBER recession dates.

Unemployment and TFP are more volatile as GDP but lie somewhere in between the two extremes. Further, while the volatility of all variables varies around zero, we find that the volatility of TFP varies around 0.001, while the separation rate varies around a level of 0.0015.

- In all figures NBER recession dates are shaded in grey in order to stress the synchronicity of recessions and peaks of volatility. We observe that recessions tend to lead peaks of volatility for most variables. However, this does not hold for the separation rate, wages, and TFP. For TFP we find that peaks of volatility occur during recessions. Similarly, we find this type of behavior for the separation rate. However, there seems to be no systematic pattern between recession dates and peaks of volatility.

  In general, there is an isomorphic mapping between recession dates and peaks, put differently every large peak is associated with a recession and every recession is associated with a peak of volatility. However, we should be careful in mistaking correlation with causation. Along this line, it would be interesting to think about the transmission of risk of an isolated type of shock (or policy reform) onto other variables. For example, consider the peak of the volatility from the job finding rate in the first half of the 90's. A fairly large peak is visible which is also visible in other time series but not in the separation rate. This leads to the conclusion that the underlying event occurred on the entry site of the labor market, or had larger effects on the (volatility of the) entry side.

Having discussed the conditional standard deviation we now turn to the dynamic conditional covariances between the labor market variables. Figures 5 to 8 present the results from our DCC

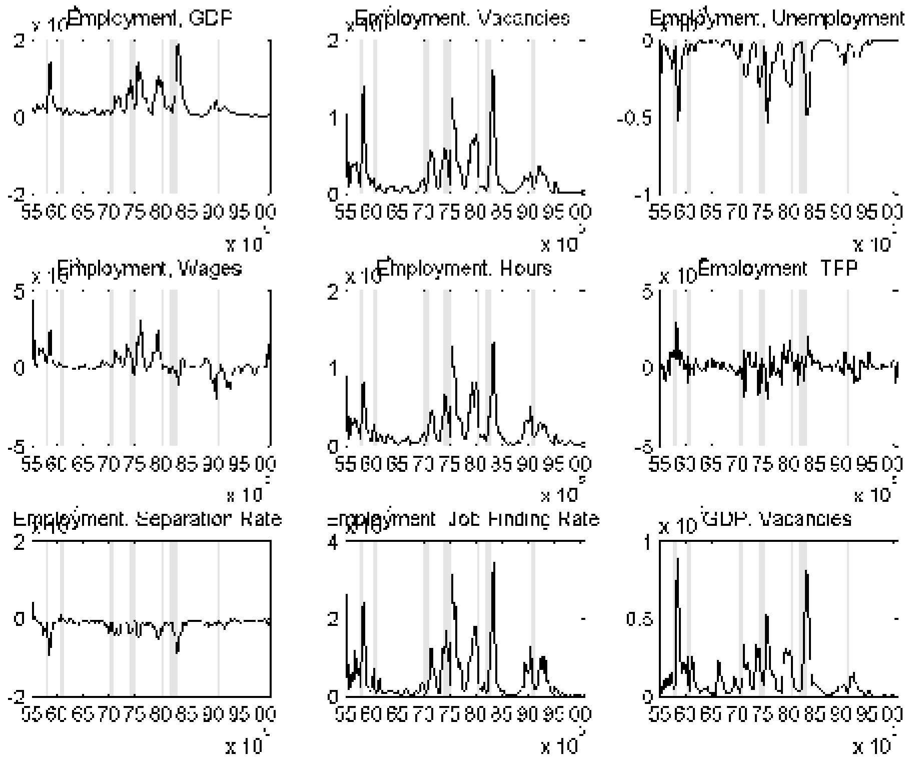

Figure 5: Estimated conditional covariances. Shaded areas indicate NBER recession dates.

estimation.

- As we have seen for the standard deviations, we observe the dampening effects of the Great moderation and a decoupling of recessions and covariance peaks. Further, we again observe that recessions lead peaks of correlations.
- However, we find that employment and wages are positively correlated which changes after 1980, where we observe small but negative peaks. This observation also holds for the covariance of wages with vacancies and hours.
- The covariance of TFP with all variables are highly volatile and switches from procyclical to countercyclical.

At the end of our discussion, we want to briefly discuss the resulting residuals from our DCC estimation. Table 2 present kurtosis and skewness values. We find that employment, vacancies, unemployment, and wages are platykurtotic, i.e. their distribution shows thinner tails. The remaining variables are all leptokurtotic. Further, we find that employment, GDP, vacancies, hours, and the job finding rate show left-skewed errors, while the remaining variables are right-skewed.

Finally, we want to answer the question whether the resulting residuals (or errors) from our DCC estimation are normally distributed. This is of interest for modelling purposes, where the canonical assumption is that errors are drawn from a normal distribution. For this purpose, we estimate kernel densities using the Epanechnikov–kernel from the estimated time series and plot them in black. Further, we estimate the mean and variance from the errors and plot the resulting normal distribution in red. Figure 9 compares those two estimates for our nine variables.

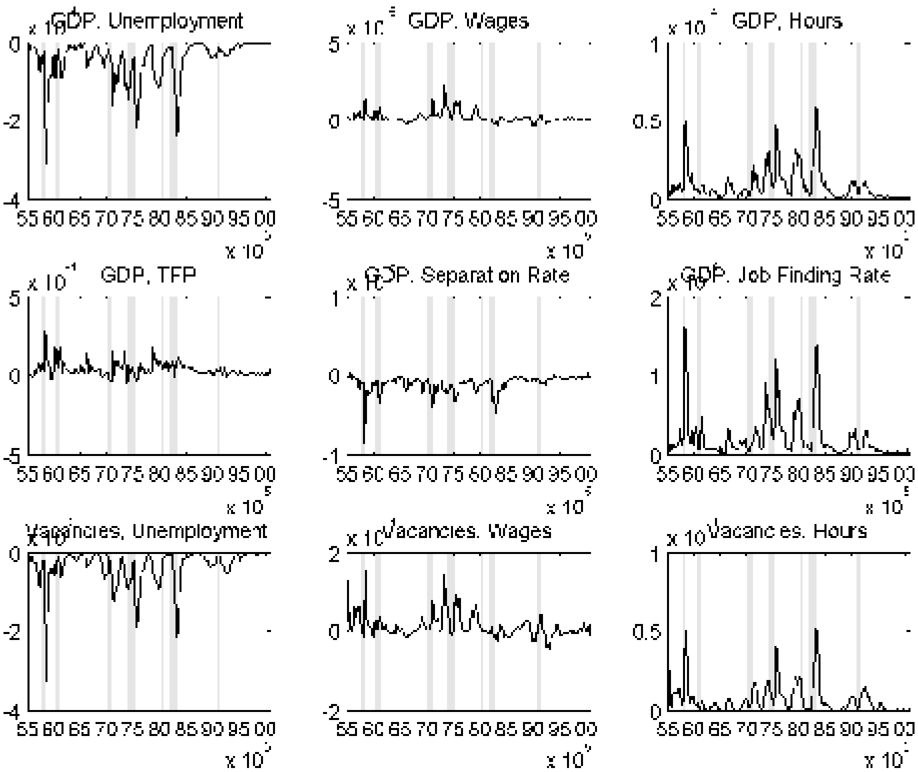

Figure 6: Estimated conditional covariances. Shaded areas indicate NBER recession dates.

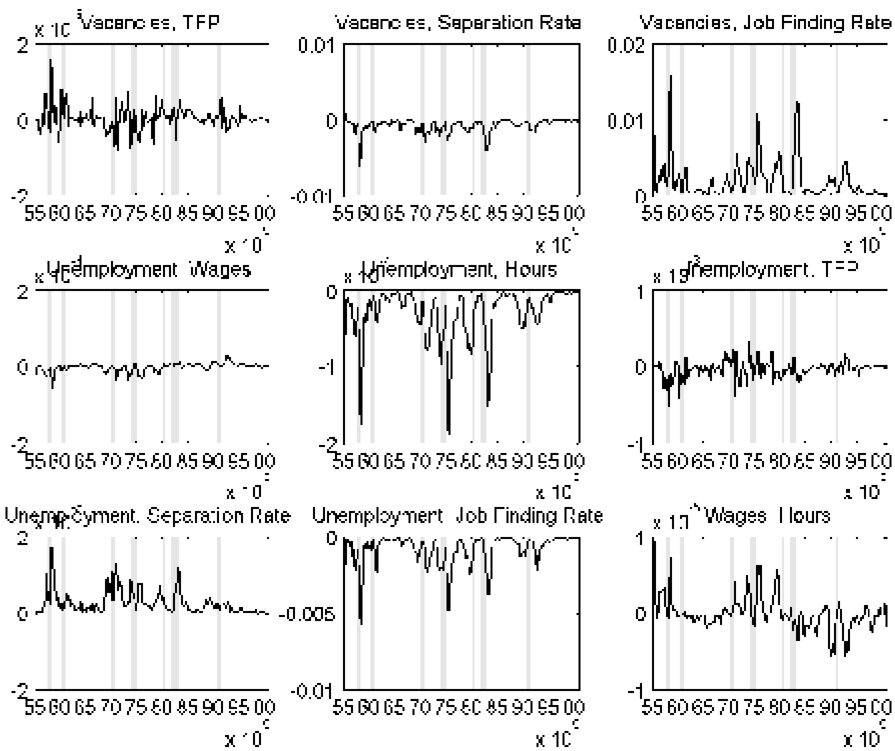

Figure 7: Estimated conditional covariances. Shaded areas indicate NBER recession dates.

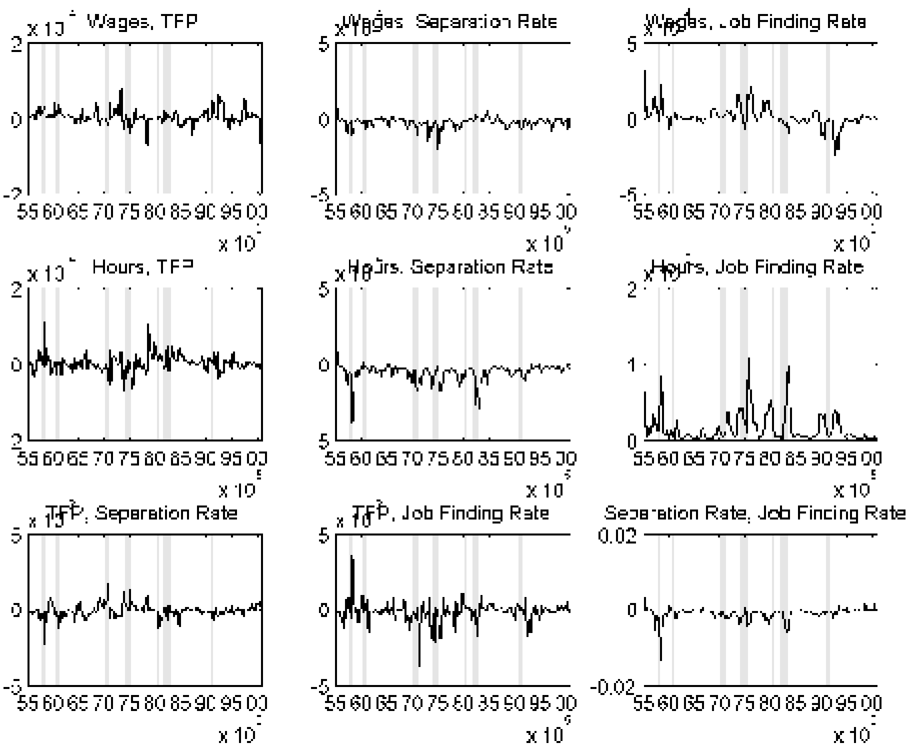

Figure 8: Estimated conditional covariances. Shaded areas indicate NBER recession dates.

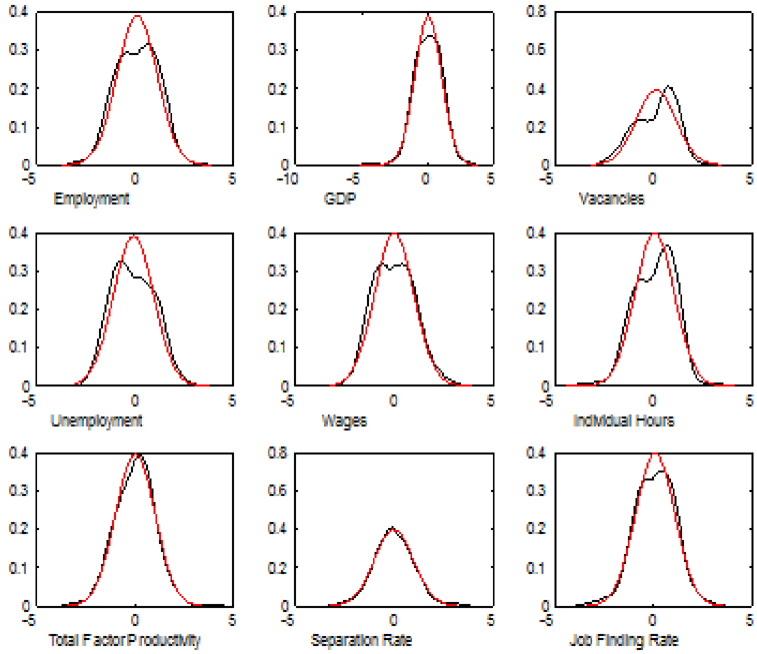

Figure 9: Kernel density estimation (black line) vs. normal estimation (red line) of residuals - DCC.

In order to assess whether the errors are in fact normally distributed, we use the Jarque-Bera test (JB, for short) on whether skewness and kurtosis of the distribution fits a normal distribution with unknown moments and the Lilliefors test (LF, for short) on whether the data is from a normal distribution with unknown moments. We find that the JB test rejects the null of normally distributed errors for GDP, vacancies, and unemployment, while the LF test rejects the null for employment, vacancies, unemployment, wages, hours, and the job finding rate. Both tests are performed on a 5 percent significance level.

We can conclude that the errors from unemployment and vacancies are in fact non-normally distributed, while for employment, wages, hours, and the job finding rate there is no clear evidence.

Table 2: Kurtosis and Skewness of DCC errors.

	Kurtosis	Skewness
Employment	2.49	-0.11
GDP	4.08	-0.33
Vacancies	2.32	-0.41
Unemployment	2.20	0.23
Wages	2.57	0.25
Hours	3.16	-0.34
TFP	3.83	0.09
Separation Rate	3.72	0.19
JFR	3.38	-0.22

5 Conclusion

In state-of-the-art macroeconomic and labor market models the dynamic effects of shocks are simulated by imposing the ad hoc assumption of homoscedasticity of the underlying stochastic processes. However, this assumptions seems to be arbitrary given the recent research agenda on time-varying variance in aggregate time series. Along this line, changes in volatility, or in the riskiness, in labor market variables can have substantial real effects. This paper provides missing evidence on stochastic volatility in the U.S. labor market. Using a number of labor market time series, covering the period from 1955 to 2000, we present evidence of strong time variability. First, we estimate the *unconditional* mean of the variance and covariance to address the assumption of homoscedasticity in macroeconomic and, in particular, in labor market models. Here, we estimate a time-varying parameter VAR model as in Primiceri (2005) and find clear evidence for time variability.

Second, we estimate the *conditional* variance and covariance matrix by using the DCC model proposed by Engle (1999). This model allows for stochastic variance and stochastic correlations and is therefore able to shed light on the behavior of the conditional second moments over time. We find a strong time varying risk component in the labor market. We observe the effects of the Great moderation in our sample and show an isomorphic relationship between recessions and peaks of risk. In particular, we show that wages and the job finding rate are *sui generis*, in the sense that they seem to be not affected by the Great moderation.

There are several implications for future research. First, the issue of the transmission of risk of an uncorrelated shock (or policy reform) onto other variables is of interest for policy makers and researchers. One example might be the recession at the early 1990's. Second, the role played by stochastic volatility for the estimation of search and matching models, as well as the role played for welfare results in those models needs to be addressed. Finally, while we have identified the types of stochastic volatility present in the labor market, we still lack an explanation. Here, one might think of underlying non-linearities in the model's primitives.

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