Representing the effects of oligopolistic competition on risk-neutral prices in power markets

Vazquez, Miguel and Barquín, Julián

Universidad Pontificia Comillas (IIT)

June 2009

Representing the effects of oligopolistic competition on risk-neutral prices in power markets

Miguel Vazquez[a] and Julian Barquin[b]

[a]*Florence School of Regulation, RSCAS, EUI.* Villa Malafrasca, Via Boccaccio 151, I - 50133 Firenze (FI), ITALY, e-mail: miguel.vazquez@eui.eu, Tel: +39.055.4685.731

[b]*Universidad Pontificia Comillas, Escuela Técnica Superior de Ingeniería, Instituto de Investigación Tecnológica.* Alberto Aguilera 23, 28015 Madrid, España, email: julian.barquin@iit.upcomilals.es. Tel: +34 91 542-2800

Abstract

Forward transactions are becoming increasingly important in most of electricity markets. In this view, this paper develops a methodology able to capture the complexities of power markets and incorporate them into the framework of risk-neutral probabilities. This is done by the statement of a model that split up the power price dynamics into two different components: on the one hand, a component aimed at representing costs and market power, which will be based on a static, non-cooperative game; on the other, a component representing short-term deviations from the static model.

Key words: Fundamental pricing models, oligopoly, electricity, forward-looking information, stochastic discount factor.

1. INTRODUCTION

Since the 1980s, many electricity systems have chosen to open power production to competition, which in turn has paved the way for a significant amount of forward trading. For instance, NordPool, BETA, PJM, among many others have a relatively large amount of forward trading. In this context, one of the most important problems to be addressed when dealing with financial trading is the contract pricing, which seeks to obtain the price of financial products considering the uncertainty of the spot market. One could think that this could be reduced to calculating the future income stream associated to each of the forward products for each of a number of future spot prices scenarios, as wide as needed. However, risk affects the value that real players provide to forward products and this has to be included in the valuation problem.

One of the most used tools for the description of financial markets is based on the specification of the risk-neutral probability, which modifies the real probability to incorporate the effects of players' preferences (risk aversion, etc.), see for instance Duffie (2001) or Magill and Quinzii (2002). Therefore, the valuation of financial products can be obtained by obtaining the uncertain income associated to the financial contract and calculating their expected value under the risk-neutral probability. One of the ways of calculating these risk-neutral probabilities is based on the idea that the prices observed in the financial market represent its equilibrium and, thus, can be used to estimate from them a risk-neutral probability that represents the aggregated perceptions of market players. In practice, this is often done by selecting a certain class of stochastic processes, which is characterized by as many parameters as it is required to represent the shape of power price distributions. These parameters are then calibrated to match actual prices quoted in the market.

Nonetheless, several particular characteristics motivate the need for a careful study of forward trading in electricity markets. In particular, the complex structure of production costs and the impossibility of economic storage make the power price distribution considerably difficult to represent. As a means to cope with the problem, financial econometrics literature has proposed the use of auxiliary variables to describe electricity prices. The logic for the approach, which can be traced back to Box and Cox (1964), is based on the assumption that a complex behavior can be described by the transformation of several random variables, each of them distributed according to some simpler distribution. The central idea behind this approach is to describe, instead of power prices, the evolution of fundamental drivers, by means of the definition of the function transforming fundamental drivers into electricity prices.

Moreover, the representation of the information used to price contracts is difficult in power markets. As it is a non-storable commodity, the usual assumption that spot prices generate a filtration containing all the available information is dubious, as pointed out in Benth and Meyer-Brandis (2010). Forward-looking information is central in pricing electricity forwards, but it is not contained in any filtration generated by spot prices.

Besides the role of forward-looking information in the definition of production costs, this transformation is further complicated by the frequent existence of horizontal concentration. Hence, the transformation function must represent not only complex cost structures (including forward-looking information about production costs), but also the possible exercise of market power (including forward-looking information about strategies). In this view, it is useful to highlight that game theory models, and especially the ones based on static games, have been extensively used to represent power spot markets. Although these models reduce the need for statistical estimation, compared to econometrics approaches, their results depend on the model's assumptions regarding strategic interaction, and hence they may not describe the price dynamics in a robust quantitative manner.

The key objective of the paper is to develop a methodology able to capture the complexities of power markets and incorporate them into the framework of risk-neutral pricing, so it can be used for the valuation of electricity forward products. Broadly speaking, the class of models introduced in the paper builds on describing power prices by means of their fundamental drivers. To do so, this paper takes account of the fact that the representation of such relationship is one of the main objectives of the spot market models. That is, as these models are based on the description of the power price formation mechanism, they can be used to define the transformation of fundamental drivers into power prices. Therefore, it is possible to split up the power price model into two different components. On the one hand, the component aimed at representing costs and market power, which will be based on the statement of a static, non-cooperative game; on the other, the component representing short-term deviations from the static model. By doing so, the first component allows to represent forward-looking information about both costs and strategies, while the second one is aimed to represent any deviation from the dynamics represented in the first component.

The organization of the paper is as follows. After a literature review, section 3introduces a general overview of the pricing methodology. The idea is to state the model in the context of the stochastic discount factor, and to describe the general framework in detail, paying special attention to the requisite characteristics of the pricing model. Section 4proposes a particular model, describing the calibration process. Section 5applies the methodology to the analysis of prices in the Spanish power market, and section 6 collects our conclusions.

2. LITERATURE REVIEW AND MOTIVATION

2.1. Models that rely only on power markets representation

A first approach to analyze financial power markets builds on the calibration, using past prices or actual forward data, of a parameterized stochastic process (eg a Gaussian process), representing the spot price evolution. An important limitation of this approach, when applied to electricity markets, is the fact that power prices have several specific features (mainly related to complex production costs and lack of storage, eg price spikes). This typically forces the requisite model to have a large number of parameters, as the stochastic process should describe appropriately the shape of the price distribution, as this distribution is the tool used to identify the risks to be priced. Actually, power price models often use stochastic volatility, spikes and regime switches (see for instance Hamilton (1994) or Harvey (1989) for general surveys, or Pilipovic (1997), Eydeland and Wolyniec (2003) or Weron (2008) for a specialization on energy prices). However, the amount of available data is limited, and hence the model is usually hard to calibrate. Moreover, when dealing with power markets, this class of model faces the need for a definition of the relationship between spot and forward prices. In particular, the convenience yield –the risk premium associated with the value of owning the underlying asset– is a non-observable quantity, and hence it is hard to represent. An alternative approach is to use a process to model the entire forward curve, instead of just the spot price dynamics. The main advantage of the approach is that the convenience yield need not be estimated, because the forward prices imply this information. This kind of model is based on extending the methodology proposed in Heath, et al. (1992) to describe the evolution of interest rates, to the case of energy markets. The general idea behind the model is to describe the forward curve as a function made up of the forward prices, which evolves over time. These techniques are applied in Koekkerbaker and Ollmar (2001), where the forward curve of the NordPool is studied, or in Clewlow and Strickland (1999),where the forward curves in the NYMEX gas and oil markets are analyzed. More recently, Borovkova and Geman (2006) and Benth, et al. (2007) propose models to take account of the seasonality of electricity forward curves. Borak and Weron (2008) propose the use of a dynamic semiparametric factor model to avoid the need for an *a priori* definition of the seasonal behavior. However, these models still require a large number of parameters, and hence a considerable amount of data.

On the other hand, there is growing literature on the use of nonparametric methods in financial applications. One of the most active research areas in the financial literature has been the analysis of volatility. In this context, it is possible to consider the diffusion coefficient as a deterministic function of the available information, such as the underlying price or the time to maturity. This is the approach followed in Franke, et al. (2003)or Comte (2004), where a kernel estimator of the regression function is used to calculate the volatility function in a time-varying volatility model. Aït-Sahalia (1996) follows a different line, based on the estimation of the marginal density of the process by means of kernel density estimation. In addition, several models aim to obtain the state-price density directly from market data. A first approach consists in the definition of implied binomial trees, and it is proposed in Rubinstein (1994). It is based on proposing a candidate distribution to describe the risk-neutral density. Then, the risk-neutral density implied in the binomial tree is calculated, minimizing its distance to the candidate distribution, subject to the constraint that it prices correctly the available contracts. Aït-Sahalia and Lo (1998) propose another approach to estimate the risk-neutral density. This work is built on a result obtained in Breeden and Litzenberger (1978),which obtained that the risk-neutral density is the second derivative of the option price with respect to the strike. Alternatively, Hutchinson, et al. (1994)use neural network techniques to describe option valuation formulas, and show that it is possible to approximate the Black-Scholes formula using this approach.

Summing up, previous models are characterized by the increasing complexity of the distribution to be captured. The price to pay is the increasing number of parameters to be defined, which in turn creates the need for a large amount of data to estimate and calibrate the model. In addition, non-parametric methods have been shown to be aimed at relaxing the assumptions on distribution shapes, at the cost of additional data requirements. In fact, they need a considerable amount of liquidly traded contracts at each cross-section, which is almost never available in electricity markets.

2.2. Fundamental models

An alternative approach consists in the use of auxiliary variables to describe electricity prices, by defining the transformation of the fundamental drivers into power prices. This class of model is often called fundamental models. The rationale behind them is to express explicitly the dependence between the parameters of the model and a certain random variable underlying the price process. For instance, price spikes in electricity markets are highly correlated with demand values. Thus, one can state this dependence by means of a certain response function, such that the jump intensity is a function of the system demand. In addition to the price equation, the model considers the stochastic process corresponding to the system demand, and the relationship between demand values and power prices, statistically defined. Compared to the direct modeling, the fundamental model represents a model with a larger number of parameters, ie the former model has the great advantage of being much more parsimonious. However, the estimation of its parameters relies only on power price data. The advantage of using the fundamental model is that it expands the available data set. In particular, once the demand process is estimated, assuming that the response function is known, the power price process depends on less parameters than the direct model, because the jump-intensity factor is determined using the demand process. Therefore, although the latter model uses a larger amount of parameters, it can benefit from a larger data set.

This was the modeling approach pursued by Eydeland and Geman (1999), were the power price was defined as a function of a deterministic supply function and the system demand. Similar approaches can be found in Skantze, et al. (2000), which modeled supply and demand using Principal Component Analysis, or Barlow (2002), which proposed the use of supply and demand as fundamental variables, determining the relationship by a Box-Cox transformation of such variables.Burger, et al. (2004) consider a nonparametric approach to model such transformation. In addition, defining the transformation function by means of fundamentals may allow the introduction of forward-looking information. For instance, future reserve margins as fundamental drivers have been used by Mount, et al. (2006) andAnderson and Davison (2008).

Nonetheless, the transformation of power price fundamentals is extremely difficult to define. The characteristics of electricity production result in complex transformation functions, so that their statistical definition depends again on a large number of parameters, which require a large amount of historical data. Thus, this paper builds on an alternative description of the response function. The central idea behind the approach is to take advantage of the knowledge regarding the market structure to simplify the estimation of the response function. In particular, the description of the price formation mechanism will be the key of the approach proposed in this paper. In this case, the natural candidate to represent the fundamental structure of the power market is the aggregate supply curve, which can be intuitively related to the generation costs of each unit in the system, assuming that the cost of generating one megawatt can be represented as the fuel cost times the heat rate[1]. Thus, if one considers that the only generation costs of a certain plant is proportional to its fuel costs, the aggregate bid curve would be a step-wise, increasing function whose steps are defined by the variable cost of the plants.

This is the idea behind the methodology introduced in Eydeland and Wolyniec (2003). Fleten and Lemming (2003) uses a unit-commitment model to fit forward curves using bids and asks. Tipping, et al. (2004) uses a unit-commitment model, combined with a time series model, to represent the influence of water resources in New Zealand spot prices. Since this work, several authors have analyzed the problem of considering additional explanatory variables. For instance, Cartea and Villaplana (2008) introduced an additional process, representing the available capacity, to adjust the bid curve. Aïd, et al. (2009) consider the risk neutral process of the cost-based transformation. Coulon and Howison (2009) use both fuel prices and capacity processes.

Nonetheless, those proposals assume no strategic interaction in the spot market, which is an important feature to be considered in the spot price model (this will be shown in section 3.2). To put in another way,

[1] The heat rate is a measure of power plants ability to transform fuel energy content into electricity, ie the heat rate is a parameter that transforms fuel prices into variable costs.

we will extend the previous methodology, based on defining the transformation function by means of the system costs and the assumption of perfect competition, by including also a more detailed representation of system costs and the strategic interaction among market players. To do so, it is useful to consider that literature on spot markets has proposed the use of agent-based models (see for instance Bower and Bunn (2000) for an application to power markets). From this paper's point of view, agent-based approaches require the ex-ante definition of market players' behavior. Although they are rather useful to analyze the impact of different strategies in several market scenarios, they are of limited use when it comes to price description. Alternatively, self-dispatch models (see for instance Baíllo, et al. (2006) for a review) are often used to analyze short-term behavior of power markets. However, they do not represent market participants' reactions to different strategies, which usually play a central role when analyzing medium- to long-term price distributions.

Thus, we will make use of an alternative family of models proposed to describe the strategic interaction in electricity spot markets, game theory models (see for instance Rasmusen (1994), or Tirole (1988) for applications to industrial organization problems). Specifically, as capacity constraints are central in the description of power systems, the most likely candidates to represent the market equilibrium are quantity, supply-function or conjectured-supply-function games, rather than price games.

3. THE PRICING FRAMEWORK

3.1. General overview

The pricing model proposed in this paper is based on representingthe power price evolutionby means of two separate components: the dynamicsrelated to the dynamics of underlying factors, and electricity-specific dynamics. The logic for this is the same as the one for other fundamental models: to capture as much information as possible from the markets for the fundamental drivers. To show the reasoning of the model proposed in the paper, it is possible to use the stochastic discount factor framework, proposed in Hansen and Richard (1987) for two-period models.Let us denote the payoff of any financial contract written on electricity and expiring at $t+1$ by $v_{t+1}^{contract}(p_{t+1}^{power})$, where p_{t+1}^{power} is the power spot price. Thus, its price at time t, $p_t^{contract}(v_t^{contract})$, can be calculated as:

$$p_t^{contract}(v_t^{contract}) = E_t\left[\bar{\varepsilon}_{t+1}^{power} v_{t+1}^{contract}\right] \tag{1}$$

where$\bar{\varepsilon}_{t+1}^{power}$ is the stochastic discount factor used to price the contracts at time t, and $E_t[\cdot]$denotes the conditional expectation at time t. To consider more than one future period, it is possible to rely on the extension of the stochasticdiscountfactor methodology to the multi-period setting developed in Garcia, et al. (2003). Hence, we will denote the corresponding discount factor by:

$$\varepsilon_{t+i}^{power} = \left(\bar{\varepsilon}_{t+1}^{power}\right)\left(\bar{\varepsilon}_{t+2}^{power}\right)\cdots\left(\bar{\varepsilon}_{t+i}^{power}\right) \quad (2)$$

where$i$ is a natural number representing a certain future period, and $\bar{\varepsilon}_{t+1}^{power}$ is the discount factor defined for the two-period setting. For the sake of notational simplicity, let us consider the price of a forward contracts written on electricity$p_{t,t+T}^{F}$, where T denotes the time to the expiration of the contract:

$$p_{t,t+T}^{F} = E_t\left[\varepsilon_{t+T}^{power} p_{t+T}^{power}\right] \quad (3)$$

In this context, we will be usingthe reasoning of fundamental models, where one defines the transformation $F(\cdot)$ of fundamental drivers' prices into power prices. In addition, we will explicitly model that the transformation function may not contain all the relevant elements of the price dynamics. This is important in our approach, as the parameterization of $F(\cdot)$ is done a priori, based on fundamental considerations. Thus, the previous expression (3)can be rewritten as

$$p_{t,t+T}^{F} = E_t\left[\varepsilon_{t+T}^{power}\left\{F\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right) + y_{t+T}^{power}\right\}\right] \quad (4)$$

wherex_{t+T}^{j}represents each of the underlying factors (fuel prices, demand...),ε_{t+T}^{j} the corresponding discount factor, and y_{t+T}^{power} a certain stochastic process representing deviations from the behavior described by the fundamental transformation. Note that the above expression assumes that no state variables are considered in the model for fundamental drivers.

In addition, we are considering the stochastic discount factor corresponding to each market for fundamental drivers. In this regard, it is possible that, for a certain power system, some underlying factors are not liquidly traded, so that the pricing model is of limited use. In such a case, the power price model will consider the corresponding spot values, and the stochastic discount factor will be equal to one. For instance, when there are no derivatives on system demand available in the market, the model considers the demand values at the power spot market, being the discount factor equal to one.

On the other hand, the transformation function $F(\cdot)$will be defined in this paper by means of a model of the spot market behavior. It is aimed at representing in detail both the production costs and the strategic interaction among spot market players. To do so, such model will be calibrated to represent risk-neutral power prices, so the transformation function may be interpreted as a static transformation from risk-neutral fundamental drivers to risk-neutral power prices.

Put it another way, the discount factor $\varepsilon_{t+T}^{power}$ in equation (4) can be interpreted as representing electricity-specific risks. Thus, the pricing model can be expressed in the following way:

$$p_{t,t+T}^{F} = E_t\left[\bar{F}\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right) + \varepsilon_{t+T}^{power} y_{t+T}^{power}\right] \quad (5)$$

where $\bar{F}(\cdot)$ is the risk-neutral transformation function$\varepsilon_{t+T}^{power} F\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right)$. Furthermore, note that defining $\bar{F}(\cdot)$ as a model for the spot market behavior implies the representation of the cross-commodity portfolio optimization. As the problem characterizes both production costs and strategic interaction, it gives the indifference price between markets for underlying factors and power markets.

In this view, the risk-neutral transformation function might not be the same as the transformation for spot prices. Nonetheless, we will show that,by means of the characterization of the transformationstructure, it is possible to extend the relationship between fundamental and electricity spot pricesto the representation of electricity forward prices. The next section is devoted to discuss on the details of the modeling of the transformation function.

3.2. Analysis of strategic interaction effects

One of the effects that the proposed pricing methodology is intended to capture is the effect of strategic interaction. This section is devoted to describe how these effects are represented in the methodology, and why are important for pricing purposes. It begins by stating the oligopoly model that we will be using in the paper, and then analyzes the distributional effects of the oligopolistic behavior in power prices. The methodology also assumes that all dynamic effects of power markets (as opposed to the dynamics in the fundamental drivers)are represented in the deviation component, which will be analyzed in section 3.3.

Equilibrium description

To introduce the spot market model used in the pricing methodology, we will consider that the spot market equilibrium is defined by the solution of a static, non-cooperative game (next section will discuss the solutions proposed by the pricing methodology to cope with this assumption). Formally, the game is defined by the interaction of firms, each of whom solves a profit-maximizing problem, taking into account that their decisions effectively can modify the market price. In addition, the market operator clears the market, and provides the power price. For the sake of simplicity, we will consider in this section the aggregate output of each firm, instead of separating the production of every single plant owned by the firms. Let us define:

- g_i is the total output of firm i
- $C_i(g_i)$ is the generation cost of firm i
- $g_i^{\max}$ is the maximum output of firm i
- $\mu_i^{\min}$ and $\mu_i^{\max}$ are the Lagrange multiplier corresponding to minimum and maximum output constraints, respectively

- π is the equilibrium price

Thus, each firm solves the following problem:

$$\begin{aligned} \max_{g_i} \quad & \pi g_i - C_i\left(g_i\right) \\ s.t. \quad & 0 \leq g_i \leq g_i^{\max} \quad : \mu_i^{\min}, \mu_i^{\max} \end{aligned} \tag{6}$$

The optimality conditions of the firm i's problem are the following:

- The optimality with respect to output decisions

$$\pi + \frac{\partial \pi}{\partial g_i} g_i - \frac{\partial C_i\left(g_i\right)}{\partial g_i} + \left(\mu_i^{\max} - \mu_i^{\min}\right) = 0 \tag{7}$$

- The maximum output constraint

$$0 \leq g_i \leq g_i^{\max} \tag{8}$$

- The complementarity conditions

$$\begin{aligned} \left(g_i - g_i^{\max}\right)\mu_i^{\max} &= 0 \\ \left(g_i^{\min} - g_i\right)\mu_i^{\min} &= 0 \end{aligned} \tag{9}$$

The equilibrium point, thus, has to fulfill the set of equations defined by the optimality conditions of every market participant. However, in order to solve the Nash game we need equations that explain the behavior of the market operator. In this case, we will consider that the operator clearing process is represented just by imposing that demand is equal to supply. This implies that we are considering an inelastic demand. Formally,

$$\sum_i g_i = D \tag{10}$$

Equation (10)is thus the optimality condition of the market operator's problem. Hence, the solution of the game is provided by the optimality conditions (7)-(9) for each of the firms i and the market-clearing condition(10).

It is worth to analyze the results implied by the equilibrium model in some detail, since they play a key role in the characteristics of the price process proposed in this paper. In particular, equation (7) represents the relationship between the firms' decisions and the market price. The expression $-\frac{\partial C_i\left(g_i\right)}{\partial g_i} + \left(\mu_i^{\max} - \mu_i^{\min}\right)$ is the marginal cost of the firm i. If the output of the firm is below its limits, then the maximum output constraint is not active, and its Lagrange multiplier is equal to zero

$\mu_i^{\max} = 0$. Therefore, the production of the plant is at the margin. If the maximum output constraint is binding, the Lagrange multiplier is not equal to zero, and the firm is below the margin. Consider first that there is no opportunity to manipulate the price, or equivalently, the market is perfectly competitive. Then, $\frac{\partial \pi}{\partial g_i} = 0$ and the equation becomes $\pi = \frac{\partial C_i\left(g_i\right)}{\partial g_i} + \left(\mu_i^{\max} - \mu_i^{\min}\right)$, the traditional "price is equal to marginal cost" result. The term $\frac{\partial \pi}{\partial g_i} g_i$ shows the incentives for price manipulation that arises in the market. This is a value that makes the price result higher than the marginal cost (note that $\frac{\partial \pi}{\partial g_i}$ is negative). $\frac{\partial \pi}{\partial g_i}$ can be interpreted as the ability of the firm to modify the prices, while g_i measures how much does the firm benefits from that increment. One of the main elements of this class of oligopoly model is to define the previous term representing the ability of firms to manipulate prices.

A first alternative, the Cournot model Cournot (1838), builds on the idea that market players choose their quantities in order to maximize their profits, considering that competitors do not react to output decisions. Therefore, price changes associated with the output decisions of a certain agent are related only to changes in the quantity demanded. Thus, the previous derivativeis defined by the elasticity of the demand, so the model cannot deal with inelastic demands. Borenstein, et al. (1995) is an example of the application of Cournot competition to describe power prices.

A refinement of this model is the supply function equilibrium. Originally, Klemperer and Meyer (1989) developed the concept of supply function equilibrium as a compromise between price and quantity competition, suggesting that in an uncertain environment firms would not want to commit with either of these strategies, but instead firms would specify supply functions, ie functions specifying the bid price corresponding to each possible output.Compared to the Cournot model, the supply function equilibrium implies that the ability to manipulate the price is no longer demand's slope. Instead, market players take into account rivals' reactions, so that the price sensitivity is the residual demand's slope (and thus allowing the analysis of inelastic demands). The additional difficulty of the model is that the residual demand's slope is part of the equilibrium definition. Nonetheless, it constitutes an interesting proposal for the analysis of power markets, which was first adopted by Green and Newbery (1992) and Bolle (1992).

Supply function equilibrium, although providing many important insights, is often difficult to solve. This is the motivation for another approach, the conjectured supply function equilibrium. The central idea behind this approach is to define a parameterized supply function for each producer, so that the number of available decisions is reduced. A typical example of this methodology is to set the functions to be linear functions with known slope.That is, the ability of firms to affect spot pricesis a constant and knownparameter. Day, et al. (2002) and García-Alcalde, et al. (2002) are the first works applying this approach to electricity systems (note that this model is essentially the same as the conjectural variations approach, which goes back to Bowley (1924). We motivate the approach from the supply function model to highlight the static nature of the game considered).

From the viewpoint of our pricing methodology, the approach based on conjectured supply functions are especially convenient, because the pricing methodology is aimed at recovering the oligopolistic behavior from actual market data by means of the calibration of the model and with this approach, we only need to calibrate one parameter.On the other hand, the fact that this model fails to represent multi-period effectsis not, from our viewpoint, an important limitation of the model, as the strategic component is supposed to be, in our methodology,a static one.

Distributional effects of strategic interaction

Therefore, representing the market-clearing price as a certain calibration of the generation cost of the marginal plant, as in Eydeland and Wolyniec (2003) or Aïd, et al. (2009)(and consequently simplifying the game between producers considering no market power) may be not approximate enough. The strategic term implies that the bid price of the marginal plant depends not only on its own cost, but also on the production of the rest of the generation portfolio.

To see this effect, it is worth to consider the results of considering the supply curve based only on production costs. That is, the price is obtained using the aggregate supply curve, so that

$$\pi = \frac{\partial C_i\left(g_i\right)}{\partial g_i}; \quad \sum_i g_i = D \tag{11}$$

In this case, consider an increase in the generation cost of the marginal plant, possibly caused by an increased fuel price. According to(11), the increase of the market price is proportional to the increase in the generation cost. However, the impact of the change in the fuel price is measured by a change in the equilibrium of the game, which is in general different of the proportional change represented in(11). In fact, the merit order may change so that there is no effect at all. For instance, a big firm owning a generation portfolio with a high maximum output will enjoy a high market power term, because the production below the margin will be high. By contrast, smaller firms will face a smaller incentive to raise the price, since their infra-marginal production is low. Consider two plants: the first belongs to the big firm, and the second to the small firm. In addition, the first plant has lower generation costs than the second. The problem introduced by the strategic interaction is that it may cause the bid price of the second plant to be lower than the bid price of the first, because the incentive of the big firm to exercise market power is higher than the incentive of the small firm. In other words, the merit order is not known*a priori*, but it is determined through the solution of the game.

Let us consider as an instance to show the distributional effects of the strategic term, that the conjectured variations of all firms are the same. The residual between an oligopoly model and a model based only on production costs can be expressed by

$$ST_t^i = \theta_t \sum_i g_t^i = \theta_t D_t \tag{12}$$

which is a linear function of the demand. Assuming further the system demand is approximately normal, this term would be an increasing function of a normal distribution, and thus, it would contribute to the fat-tail effect of power prices.

Therefore, the oligopolistic competition plays a key role in the evolution of power prices, and consequently the transformation function of fundamental drivers into power prices must reflect these effects. Moreover, the above reasoning shows that the strategic interaction has a critical impact not only in the power price distribution, but also in the correlation between fuel and electricity prices, as the fuel used to fire the marginal plant may change because of firms' strategies.

3.3. Modeling short-term deviations from the static equilibrium

The previous section analyzed the class of model required to capture not only the effects of the fundamentals in power prices, but also the effects of the strategic behavior of market players. This latter analysis was based on the statement of a static game. It is thus necessary to analyze what kind of effects it cannot capture, and to develop a methodology able to represent such effects. This will be done in this section.

First, it is not easy to define the generation cost of the plants in advance, as power producers use to purchase in advance a large part of the supplies required for the production of electricity. For example, it is common practice to purchase natural gas supplies through long-term contracts (say 10 or 20 years in advance), whose prices has more to do with the particular negotiation of the contract than with international indexes, such as Henry Hub or ICE prices. From this point of view, it is difficult to estimate the real cost of producing electricity. However, taking into account that producers will consider the opportunity cost of their fuel supplies, such costs can be approximated by international prices representing fuel spot markets.

Second, it is in general difficult to model the short-term operation of the system in detail. The representation of technical characteristics of power systems operation usually results in complex optimization problems, which are computationally expensive. For instance, unit-commitment decisions, as start-ups, may be important to describe generation costs and thus price distributions. These characteristics could be described as input data to the fundamental structure model, so that the uncertainty associated with them can be modeled. The price to pay, however, is the increased difficulty of the resulting model.

The third and most delicate issue concerns the fundamental model used to find the transformation of the underlying variables. The equilibrium framework studied above captures the effect of the strategic interaction between market players by means of a static equilibrium model. However, the quantity game is just an approximation of the real behavior of a firm. A more detailed description would take into account that the bidding process involves not only the quantity decision, but the determination of the bid price as well. This fact would lead us to the statement of more sophisticated games, which calculates the profit-maximization problem deciding in both variables, as in Klemperer and Meyer (1989).Furthermore, we model the oligopolistic competition by means of a static game. The real decision-making process involves a sequence of stages where the acquisition of information with respect to competitors' decisions plays a key role. In addition, dynamic games include, as a part of the equilibrium, off-equilibrium strategies that have an important role even if they are rarely or even never played(threats, commitments...). However, the static game disregards the effects of dynamic competition. Moreover, electricity markets are very often more subject to regulatory decisions than any other commodity market. From this point of view, the true game is not only a game between market players, but additionally it should contain the strategic interaction between market players and the regulator.

Therefore, the equilibrium model cannot capture all the relevant features of power price dynamics. There is some uncertainty about the equilibrium price of the system that is difficult to anticipate through the behavior of fundamental drivers. The perturbation component aims to capture the uncertainty around the equilibrium price. It is important to highlight that the perturbation factor, defined as above, is in general intended to capture both structural uncertainty (misspecifications of the model used to represent the transformation of fundamental drivers) and underlying factors uncertainty (effects of drivers that have not been represented or misspecifications of the underlying factors dynamics). Hence, one should expect several features derived from this definition.

In this regard, the impact of these random shocks in the electricity price should tend to disappear, ie the short-term component should revert to zero. Consider the effect of an unexpected high price, in the sense that it is not explainedbecause of a random shock in the fundamentals. It may be motivated by a punctual bidding strategy of one or several agents, or even by a mistake in the bidding process. It will probably result in higher prices the next periods[2]. However, it seems reasonable to expect that the price will tend to

[2] This might be interpreted, for instance, as a reaction of market players to punish the rivals' strategy, or a reaction motivated by the uncertainty in the nature of the movement.

the long-term equilibrium of the system as the unexpected high price is forgotten. Therefore, the price should revert to the long-term equilibrium.

In addition, the equilibrium component naturally explains the spiky behavior of power prices. The rationale behind the price spikes is a period of scarcity in the system capacity, because of eventual outages, network failures, etc., implying that the opportunity costs of generators are wide higher than the pure marginal costs of producing –see Schweppe, et al. (1988) for the theory of spot pricing. Consequently, this behavior is not part of the perturbation factor. Turning to higher order moments, particularly to third and fourth moments, the equilibrium component of the model also captures the asymmetry characteristics of power prices (skewness) and the measure of fat tails (kurtosis), and hence the perturbation factor will be assumed to have zero third and fourth moments. Thus, the perturbation component will be modeled as stochastic process that will exhibit equilibrium reversion, that is, it will revert to the equilibrium factor.

4. A PROPOSAL FOR MODELING ELECTRICITY PRICES

Let t be the time and p_t the market price. The proposed model is made up of two components:

$$p_t = \pi_t + y_t \tag{13}$$

The rationale behind this model has been developed in previous sections. This decomposition in two factors, the first representing a seasonal pattern and the second representing stochastic perturbations, is not new in the literature. However, the model proposed in this section builds on the idea that the component π_t should be represented by a model relating the underlying random factors to electricity prices, and representing the strategic interaction.

4.1. Fundamental drivers

The general scheme of the model is represented in Figure 1.The first step of the model definition is to describe the evolution of the primary drivers, represented in the left part ofFigure 1, which are the basic input for the model.Regarding the fuel price description, we will consider separate models for the evolution of coal, heating-oil and gas prices. The rest of prices –namely the corresponding to nuclear plants– are modeled as a known variable cost.

The model for the evolution fuel prices is the model of forward curves proposed in Clewlow and Strickland (1999). It can be represented by the following expression:

$$\frac{dF_{t,T}}{F_{t,T}} = \sum_{i=1}^{N} \phi^i (T-t) dW_t^i \tag{14}$$

where the changes in the forward curve are explained by means of N random shocks, T is the maturity of the contract and t is the quotation date. Each of these perturbations is specified as a Gaussian factor dW_t^i multiplied by a deterministic function of the time-to-maturity $\phi^i(T-t)$, which is defined through principal components analysis.

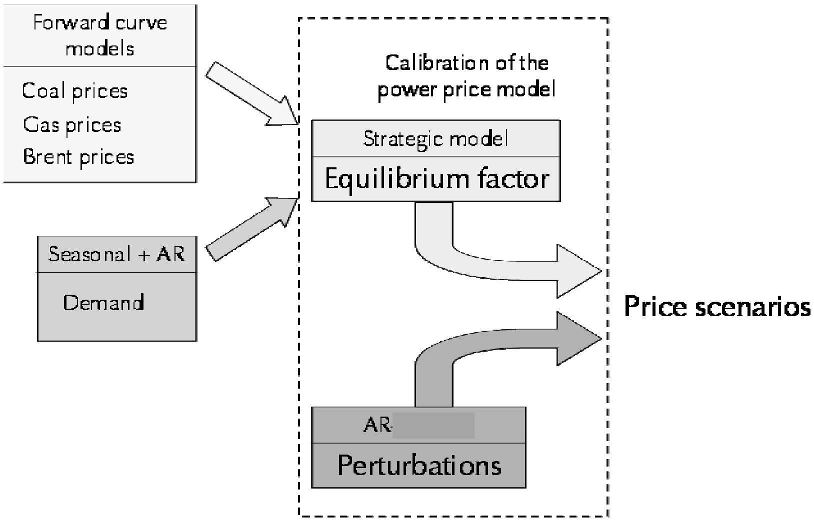

Figure 1.The model scheme.

We choose to model the power demand directly, instead of as a function of the temperatureor humidity. The main reason is that there is often little trading activity concerning the primary drivers, and then there is no market information available. Thus, the model for power demand is based on an autoregressive process, combined with a deterministic seasonal component:

$$D_t = M_t + \alpha_1 D_{t-1} + \ldots + \alpha_n D_{t-n} + \varepsilon_t \tag{15}$$

The model parameters $\{\alpha_1, \ldots \alpha_n\}$ are defined by maximizing the maximum likelihood using historical data, and the number of parameters *n* is chosen following the Akaike criterion, see for instance Lütkepohl (1993). The Linear Hinges Model, developed in Sánchez-Úbeda and Wehenkel (1998) and Sánchez-Úbeda (1999), estimates the seasonal component M_t.

In addition, hydro and wind production are modeled as known production in the system. Therefore, the demand values faced by equilibrium model should be thought of as the thermal demand of the system. That is, the system demand discounting the hydro production and the wind generation.

4.2. Long-term equilibrium

We will next describe a methodology to solve the equilibrium problem based on the statement of a quadratic optimization program. The approach follows the quadratic model developed in Hashimoto (1985) to study Cournot competition, which was extended to the conjectured-supply-function framework in Barquín, et al. (2004). One of the fundamental insights pointed out by this methodology is that the first order equilibrium conditions are equivalent to a quadratic program, which is easier to solve. Thus, we begin with the quantity game previously defined, that is, the solution of the game is characterized by the first order optimality conditions of the profit-maximization programs provided by equations (7)-(9), and the market clearing condition defined in equation (10). In addition, following the conjectured supply function approach, the strategic term $\frac{\partial \pi}{\partial g_i}$ is modeled as a known parameter: specifically, we will use $\theta_i = -\frac{\partial \pi}{\partial g_i}$. It is easy to check that equilibrium conditions are the same as the first order optimality conditions of the following quadratic program, see Barquín, et al. (2004) for details:

$$
\begin{aligned}
\min \quad & \sum_{i,t} \frac{1}{2}\theta_{i,t} g_{i,t}^2 + C_t\left(g_{i,t}\right) & \\
s.t. \quad & 0 \le g_{i,t} \le g_i^{\max} & : \mu_{i,t}^{\min}, \mu_{i,t}^{\max} \\
& \sum_i g_{i,t} = D_{i,t} & : \pi_t
\end{aligned}
\tag{16}
$$

where the subscript *t* denotes the time period. Since the program (16) is based on a quadratic optimization and its objective is convex, it can take advantage of a very well-developed theoretical corpus as well as very powerful algorithms and software. Therefore, this model can deal with real-size systems with little computational effort. In this regard, we think that the computational efficiency of the model is an important advantage with respect to complementarity-based approaches, such as Day, et al. (2002). A comparison between different approaches can be found Neuhoff, et al. (2005).

4.3. Short-term stochastic deviations

The model for the short-term component, which is denoted by y_t, will be a discrete-time autoregressive process, possibly with weekly seasonality. This model allows capturing the equilibrium-reverting behavior of power prices. The model is given by the following expression:

$$H\left(q^{-1}\right) y_t = u_t \tag{17}$$

where $u_i = N(0,\sigma)$ are random shocks, q^{-1} is the lag operator and $H\left(q^{-1}\right)$ is an autoregressive polynomial, or possibly the product of several polynomials. The logic for the use of several polynomials is the fact that the equilibrium perturbation component y_t should capture any seasonal behavior left by the long-term component. For instance, power prices typically present a weekly periodical component that is mainly related to different demands on working days and weekends. The internalization process of such effectsmay result in different patterns for dynamic strategies, and thus for equilibrium deviations, which are not part of the long-term component. Therefore, this behavior is described in the model through the product of two autoregressive polynomials. That is,

$$H\left(q^{-1}\right) = H_d\left(q^{-1}\right) H_w\left(q^{-7}\right) \tag{18}$$

where the base lag of $H_d\left(q^{-1}\right)$ is a day, and the base lag of $H_w\left(q^{-7}\right)$ is a week. This model is fit using the Akaike criterion, see for instance Lütkepohl (1993).

5. CASE STUDY

The main motivation of the pricing model developed in the previous sections is likely the need for capturing considerably complex distributions and correlation structures. Consequently, it is important to show, in a realistic case study, whether the model success at recovering power price characteristics. Hence, this case study analyzesits behavior in the context of the Spanish power market. The study uses daily prices in the Spanish market corresponding to the first eight months of 2008.

System definition

The Spanish system will be modeled using eighty-five power plants. They are classified under four different categories: coal, gas, fuel units and "other" units (the last term refers mainly to nuclear plants). Figure 2represents the thermal plants considered.

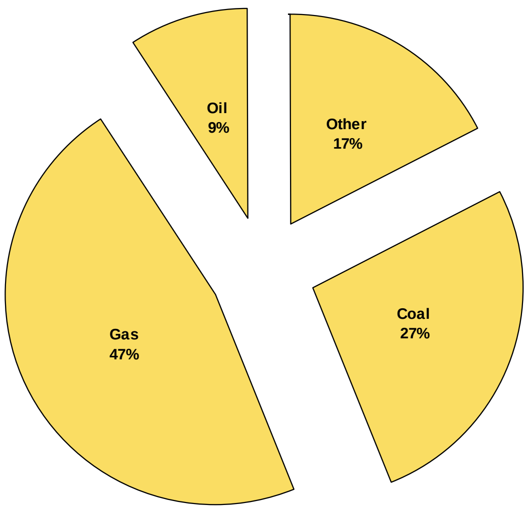

Figure 2. Thermal plants with respect to the fuel used to fire them.

In addition, it is necessary to transform the fuel prices into variable costs. We model such transformation, in the study, as the price of just one forward contract of the curve, multiplied by the efficiency of the plant. In particular, the variable cost will be the forward price of the contract expiring in three months, multiplied by the efficiency. The rationale behind this is that power producers need at least three months to get additional fuel, and their variable cost is the cost of refueling.

Furthermore, seven firms will be considered: Endesa (EN), Iberdrola (IB), Unión Fenosa (UF), Hidrocantábrico (HC), Viesgo (VI) and Gas Natural (GN). They own the thermal portfolio represented in Figure 3.

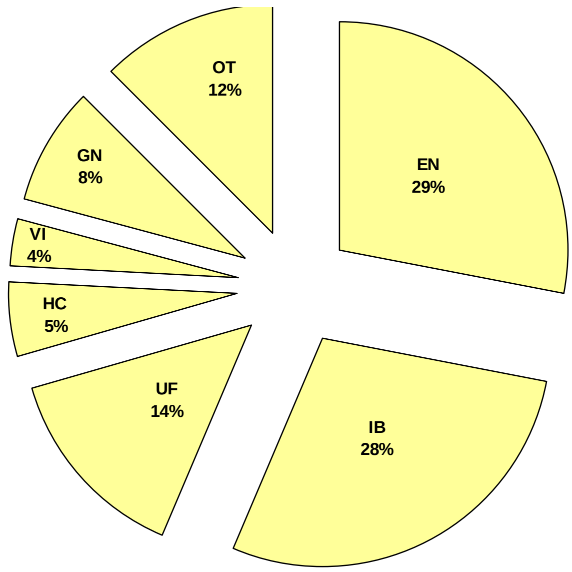

Figure 3. Thermal portfolio owned by the firms in the system.

Simulation

The simulation consists of 100 scenarios of the underlying factors, and 100 scenarios for the short-term factor. Both sets of scenarios are combined so that every short-term perturbation is added to each long-term scenario, resulting in a set of 10000 price scenarios.

Calibration of the equilibrium model

The main product traded in the Spanish market, during the period studied in this case study, is the forward contract, usually through over-the-counter agreements. The procedure to calibrate the model in this study is ultimately defined by market data available in the Spanish market. The only forward-looking contract traded in the market is the monthly forward. Thus, to use only the market approach to calibrate the model one should assume a unique, monthly strategic parameter. However, I will use a hybrid approach. First, I use historical prices to find out the pattern of the strategic parameter, so that the strategic parameter is fitted to describe historical prices. To do so, I assume the same parameter for every peak hour in a certain month, and the same parameter for the off-peak hours in a month (the off-peak period is defined as the first twelve hours in a day). This provides the variation pattern of a monthly strategic parameter, from peak to off-peak periods. Then, I use the forward contracts with expiration in 2008 to estimate a monthly parameter. Finally, peak and off-peak parameters are defined by applying the historical pattern to the monthly parameters.

Validation of the pricing model

The first step to validate the model is showing that it can recover appropriately the empirical characteristics of the electricity prices. To do so, we will show two types of validation results. On the one hand, we will test the power price distribution assuming that the evolution of primary drivers is perfectly known, so instead of simulating fuel prices and demand values, we will take actual realizations of the variables. Thus, the model obtains the set of price scenarios using real values for the input variables. On the other hand, using the set of scenarios for fuel prices and demand, we will test the correlation structure of the fundamental model.

Test of the transformation of underlying factors into power prices

Figure 4 shows the power price in the test time scope, compared to real prices.

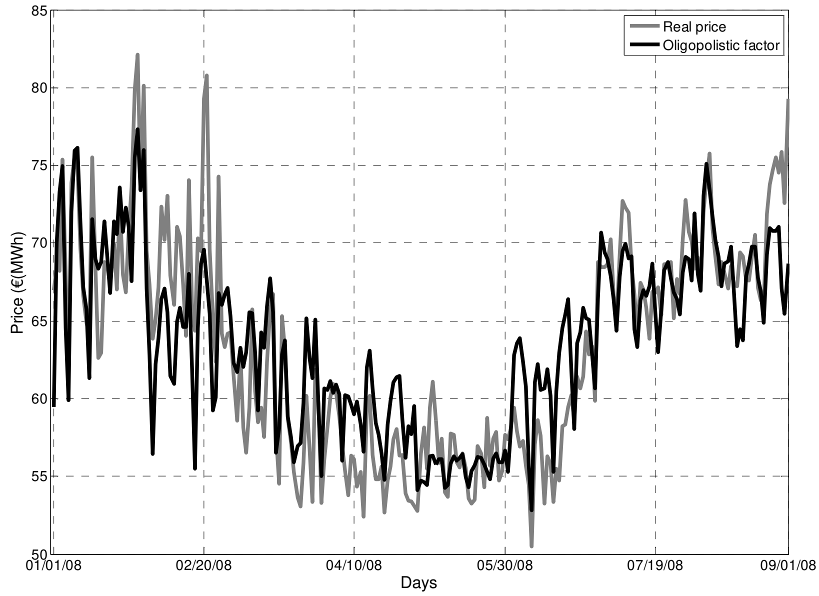

Figure 4.Model vs. real prices.

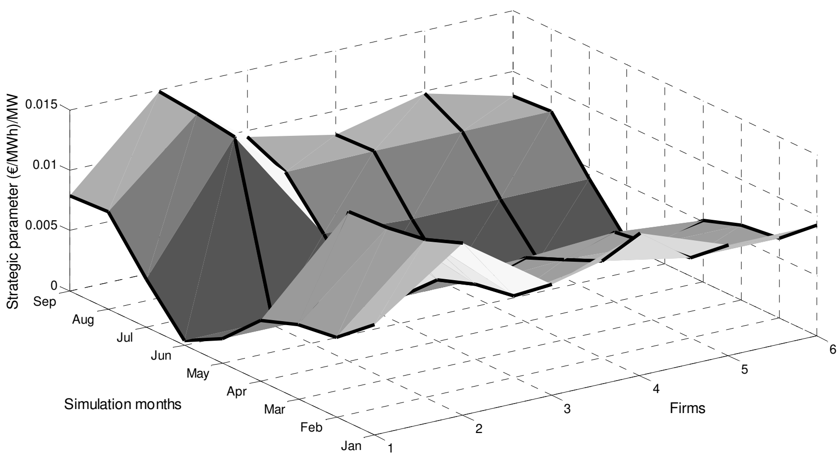

Figure 5.Calibrated strategic parameters for each of the firms in the system.

Figure 5 shows the resulting strategic parameters for the six largest firms in the system at each month of the simulation scope. Note that we have only calibrated monthly prices, but the daily pattern is very similar to the actual pattern of real daily prices. This supports the soundness of the fundamental structure model and provides some validation to it.

Regarding price volatility, Figure 6 compares the annualized volatility of the prices obtained with the model and the actual volatility. It is important to note that the parametric decomposition of this paper, describing the price as made up of long- and short-term factors, describes the volatility of real prices accurately. Again, as the only data used in the calibration process are forward prices, capturing the volatility pattern gives validation to the modeling approach.

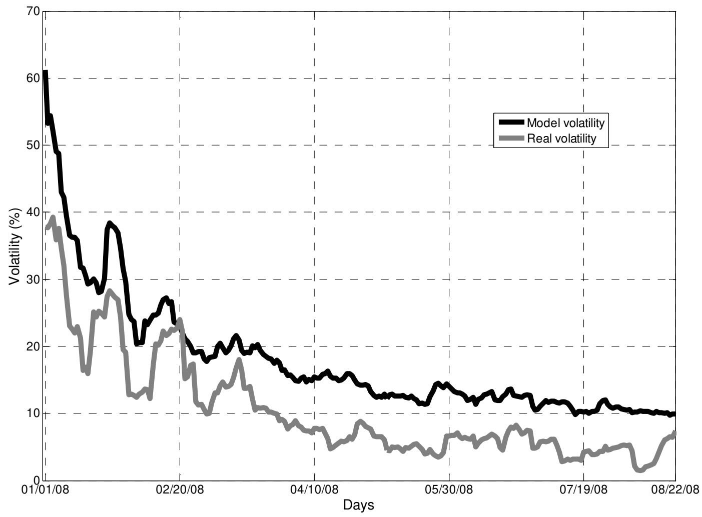

Figure 6.Model volatility.

Turning to higher order moments, Table 1 compares the model and empirical values of the third and fourth normalized moments for different periods. Q1 represents the first quarter of 2008, Q1+Q2 the first and second quarters, and Sept. is the time scope of the simulation, from January to August of 2008 (the different periods are chosen to show that the accuracy of the model results is relatively independent from the level of aggregation).

	Skewness		**Kurtosis**	
	Empirical	Strategic	Empirical	Strategic
Q1 (2008)	0.1675	0.1793	-0.3663	-0.4976
Q1+Q2 (2008)	0.8834	0.8926	2.3922	2.3426
Sept. (2008)	0.2768	0.2531	-0.8308	-1.0032

Table 1.Model and empirical third and fourth moments of the price distribution.

The numbers in the table shows the agreement of the higher normalized moments. This is particularly important because we did not use historical values to match price distributions, but market characteristics are captured by the fundamental structure of the model. Matching higher order moments, then, is an additional justification of the modeling approach.

Test of the correlation structure obtained with the fundamental structure model

In the following, we will test the correlation structure given by the fundamental structure model, using the set scenarios generated by the set of scenarios of the fundamental drivers (instead of using their real values in the simulation scope, as in the previous tests). One critical out-of-sample test of the agreement between model and real distributions is related to the representation of the volatility structure. Specifically, the volatility of power prices depends on the system demand. That is, when the system demand is high, the volatility of prices is higher than when system demand is lower. This is a typical characteristic of electricity prices. For high levels of demand, there is a great variability in the number of plants actually producing. When a peaking plant is producing, the infra-marginal production of the owner is typically high and thus the market power incentive. Therefore, each peaking plant that starts producing forces a steep variation in the price. Figure 7 shows this dependence of the volatility of weekly log-prices on the system demand.

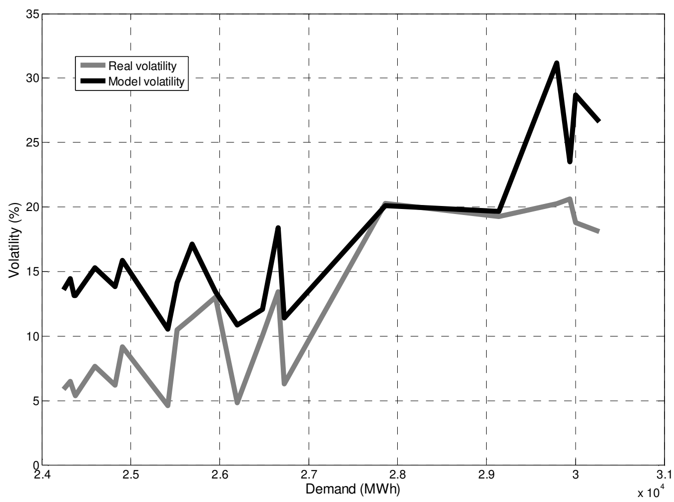

Figure 7. Volatility of weekly log-prices increases with the demand level.

Another important effect, strongly related to the previous dependence of volatility on demand levels, is the dependence of the volatility on the implied heat rate of the system. As shown in Figure 8, volatility is low for small values of the implied heat rate, and it increases as the implied heat rate increases. This effect can be explained by the dependence on the system demand. High levels of demand, in the Spanish system, correspond to high values of the implied heat rate. When the implied heat rate is low, the volatility is low, because in the Spanish system, this corresponds to a case where the marginal plant is typically a coal-fired unit, and there is little incentive to exercise market power. In a system with more nuclear production, so that sometimes the marginal plant is a nuclear unit, and sometimes is coal-fired, the volatility of the short end of the curve would be higher.

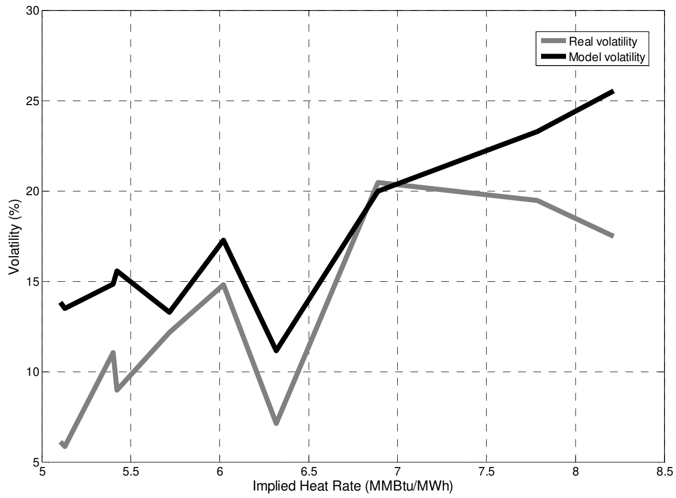

Figure 8. Volatility of weekly log-prices increases with the implied heat rate.

The middle part of the curve, where there is a reduction of the volatility, would correspond to the zone where the large amount of combined cycle units (corresponding to the Spanish system) is producing.

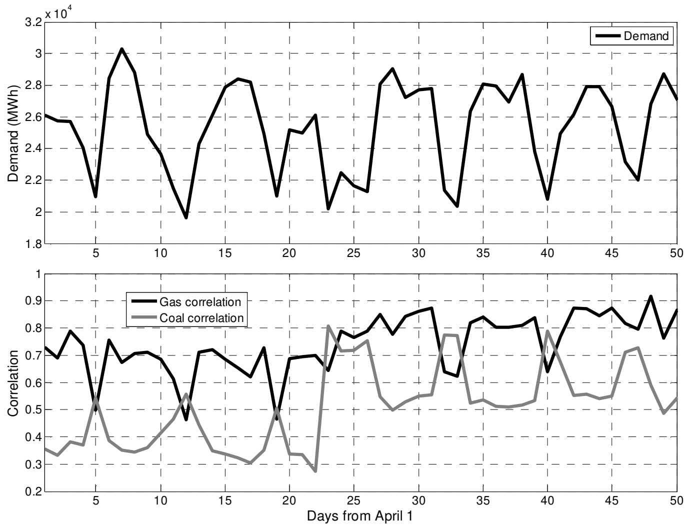

Figure 9.Evolution of system demand and fuel correlations.

The correlation structure of power prices is another important feature that must be captured by the price model. A power plant could be compared to a spread option between power and fuel prices, and consequently the correlation is central for hedging and pricing purposes. The correlation structure has not been calibrated, so the agreement between empirical and model results can be considered as a powerful out-of-sample test. Figure 9 shows the correlation between power prices, and gas and coal prices, in the lower panel, and the system demand in the upper panel. Both panels consider a time scope from April 1st to May 20th, to show in detail the dependence of correlation on the demand pattern. It can be observed that when the system demand is low, the correlation with gas prices is lower, and the correlation with coal prices is higher. Moreover, it is possible to explain this correlation structure from a fundamental viewpoint. When the system demand is low, the marginal plant will be probably a coal-fired unit, and on the contrary, when the system demand is high, the marginal plant will be a gas-fired unit.

6. CONCLUSION

The paper has proposed a new pricing model based on a fundamental representation of the structure of power prices. That is, instead of relying on the statistical estimation of the transformation function, the proposed model makes use of the fundamental knowledge about the price formation mechanism in electricity markets. Besides considering fuel prices and demand as underlying factors of power prices, the pricing model takes account of the fact that the strategic behavior is indeed a central characteristic of the power price formation, and thus it affects the risk-neutral distribution. On the other hand, electricity markets are a sequence of spot markets, which might result in extremely complex dynamics of strategic behavior. Moreover, from the viewpoint of cost representation, generation portfolios have a number of costs subject to strong variability (outages, network failures...). In this regard, a detailed representation of this kind of cost is often infeasible.

Therefore, the model developed is based on the decomposition of power prices into two factors: the first one represents a long-term equilibrium and the second term represents short-term, random deviation from the long-term component. These deviations, in addition, revert to the long-term equilibrium. Furthermore, in order to represent risk-neutral probabilities, an algorithm designed to calibrate the fundamental structure model was developed.

Therefore, the paper has developed a methodology to cope with the many special features of power markets. A central characteristic of the model is that it is defined by a relatively small number of parameters, and yet is capable to capture considerably complex dynamics of market prices. Moreover, as it is based on a fundamental description of the price formation mechanism, it can deal with changes in market conditions in a quantitatively robust manner.

The methodology developed in the paper has been designed to serve as a practical tool to analyze power prices. Thus, the pricing model has been validated in the context of the Spanish system. The case study shows the ability of the model to capture the observed characteristics of power prices.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge discussions and comments provided by Carlos Vázquez, Pablo Rodilla and Florian Leuthold. The authors also benefited from discussions provided by seminar participants at the Comillas University, the 2008 CLAIO conference, and the 2008 Madrid meeting of the Young Energy Engineers and Economists Seminar.

REFERENCES

Aïd, R., Campi, L., Huu, A. N. and Touzi, N. (2009). *A structural risk-neutral model of electricity prices*. International Journal of Theoretical and Applied Finance, 12 (7): pp. 925-47.

Aït-Sahalia, Y. (1996). *Nonparametric pricing of interest rate derivative securities*. Econometrica, 64: pp. 527-60.

Aït-Sahalia, Y. and Lo, A. W. (1998). *Nonparametric estimation of state-price densities implicit in financial asset prices*. Journal of Finance, 53 (2).

Anderson, C. L. and Davison, M. (2008). *A hybrid system-econometric model for electricity spot prices: considering spike sensitivity to forced outage distributions*. IEEE Transactions on Power Systems, 23 (3): pp. 927-37.

Baíllo, Á., Cerisola, S., Fernández-López, J. M. and Bellido, R. (2006). *Strategic bidding in electricity spot markets under uncertainty: a roadmap*. Power Engineering Society General Meeting, IEEE.

Barlow, M. T. (2002). *A diffusion model for electricity prices*. Mathematical finance, 12: pp. 287-98.

Barquín, J., Centeno, E. and Reneses, J. (2004). *Medium term generation programming in competitive environments: a new optimization approach for market equilibrium computing*. IEE Proceedings - Generation, Transmission and Distribution, 151 (1): pp. 119-26.

Benth, F. E., Koekebakker, S. and Ollmar, F. (2007). *Extracting and applying smooth forward curves from average-based commodity contracts with seasonal variation*. Journal of Derivatives, Fall: pp. 52-66.

Benth, F. E. and Meyer-Brandis, T. (2010). *The information premium for non-storable commodities*. Journal of Energy Markets, 2 (3).

Bolle, F. (1992). *Supply function equilibria and the danger of tacit collusion. The case of pot markets for electricity*. Energy Economics, 14: pp. 94-102.

Borak, S. and Weron, R. (2008). *A semiparametric factor model for electricity forward curve dynamics*. MPRA Working Paper 10421.

Borenstein, S., Bushnell, J., Kahn, E. and Stoft, S. (1995). *Market power in California electricity markets*. Utilities Policy, 5 (219-236).

Borovkova, S. and Geman, H. (2006). *Analysis and modelling of electricity futures prices*. Studies in Nonlinear Dynamics and Econometrics, 10 (3).

Bower, J. and Bunn, D. W. (2000). *Model-based comparisons of pool and bilateral markets for electricity*. The Energy Journal, 21 (3): pp. 1-29.

Bowley, A. (1924). *The mathematical groundwork of Economics*. Oxford, Oxford University Press.

Box, G. E. and Cox, R. (1964). *An analysis of transformations*. Journal of the Royal Statistical Society, Series B 26: pp. 211-52.

Breeden, D. T. and Litzenberger, R. H. (1978). *Prices of state-contingent claims implicit in option prices*. Journal of Business, 51 (4).

Burger, M., Klar, B., Muller, A. and Schindlmayr, G. (2004). *A spot market model for pricing derivatives in electricity markets*. Quantitative Finance, 4: pp. 109-22.

Cartea, A. and Villaplana, P. (2008). *Spot price modeling and the valuation of electricity forward contracts: the role of demand and capacity*. Journal of Banking and Finance, 32 (12): pp. 2502-19.

Clewlow, L. and Strickland, C. (1999). *A multifactor model for energy derivatives*. University of Technology, Sydney. QFRG Research Paper Series 28.

Comte, F. (2004). *Kernel deconvolution of stochastic volatility models*. Journal of Time Series Analysis, 25 (4): pp. 563-82.

Coulon, M. and Howison, S. (2009). *Stochastic behavior of the electricity bid stack: from fundamental drivers to power prices*. The Journal of Energy Markets, 2 (1).

Cournot, A. (1838). *Recherches sur les principies mathématiques de la théorie des richesses*. Paris, Hachette. English Translation: *Researches into the mathematical principles of the theory of wealth* (1897).

Day, C. J., Hobbs, B. F. and Pang, J.-S. (2002). *Oligopolistic competition in power networks: a conjectured supply function approach*. IEEE Transactions on Power Systems, 17 (3): pp. 597-607.

Duffie, D. (2001). *Dynamic asset pricing theory* (3rd edition). Princeton, Princeton University Press.

Eydeland, A. and Geman, H. (1999). *Fundamentals of electricity derivatives*. In: Energy Modelling & the Management of Uncertainty. Risk Books.

Eydeland, A. and Wolyniec, K. (2003). *Energy and power risk management: New developments in modeling, pricing, and hedging*. John Wiley & Sons.

Fleten, S.-E. and Lemming, J. (2003). *Constructing forward price curves in electricity markets*. Energy Economics, 25: pp. 409-24.

Franke, J., Härdle, W. and Kreiss, J. P. (2003). *Nonparametric estimation in a stochastic volatility model*. In: Recent Advances and Trends in Nonparametric Statistics. M. Arritas and D. N. Politis (eds.). Amsterdam, Elsevier.

García-Alcalde, A., Ventosa, M., Rivier, M., Ramos, A. and Relaño, G. (2002). *Fitting electricity market models. A conjectural variations approach*. 14th PSCC, Sevilla, Spain.

Garcia, R., Ghysels, E. and Renault, E. (2003). *The econometrics of option pricing*. In: Handbook of Financial Econometrics. Y. Aït-Sahalia and L. P. Hansen (eds.). Amsterdam, Elsevier-North Holland.

Green, R. J. and Newbery, D. M. (1992). *Competition in the British electric spot market*. Journal of Political Economy, 100: pp. 929-53.

Hamilton, J. D. (1994). *Time series analysis*. Princeton University Press.

Hansen, L. P. and Richard, S. (1987). *The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models*. Econometrica, 55: pp. 587-613.

Harvey, A. C. (1989). *Forecasting, structural time series and the Kalman filter*. Cambridge University Press.

Hashimoto, H. (1985). *A spatial Nash equilibrium model*. In: Spatial Price Equilibrium: Advances in Theory, Computation and Application. P. Harker (ed.). Berlin, Springer-Verlag.

Hastie, T. J. and Tibshirani, R. J. (1990). *Generalized additive models*. London, Chapman and Hall.

Heath, D., Jarrow, R. and Morton, A. (1992). *Bond pricing and the term structure of interest rates: A new methodology for contingent claim valuation*. Econometrica, 60: pp. 77-105.

Hutchinson, J. M., Lo, A. W. and Poggio, T. (1994). *A nonparametric approach to the pricing and hedging of derivative securities via learning networks*. Journal of Finance, 49: pp. 851-89.

Klemperer, P. D. and Meyer, M. (1989). *Supply function equilibria in oligopoly under uncertainty*. Econometrica, 57 (6): pp. 1243-77.

Koekkerbaker, S. and Ollmar, F. (2001). *Forward curve dynamics in the Nordic electricity market*. Norwegian Scool of Economics and Business Administration. Working Paper.

Lütkepohl, H. (1993). *Introduction to multiple time series analysis*. Springer-Verlag.

Magill, M. and Quinzii, M. (2002). *Theory of incomplete markets*. MIT Press.

Mount, T. D., Ning, Y. and Cai, X. (2006). *Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters*. Energy Economics, 28: pp. 62-80.

Neuhoff, K., Barquín, J., Boots, M. G., Ehrenmann, A., Hobbs, B. F., Rijkers, F. A. M. and Vázquez, M. (2005). *Network-constrained Cournot models of liberalized electricity markets: The devil is in the details*. Energy Economics, 27: pp. 495-525.

Pilipovic, D. (1997). *Energy risk: Valuing and managing energy derivatives*. McGraw-Hill.

Rasmusen, E. (1994). *Games and information: an introduction to game theory* (2nd edition). Blackwell Publishers.

Rubinstein, M. (1994). *Implied binomial trees*. Journal of finance, 49: pp. 771-818.

Sánchez-Úbeda, E. F. (1999). *Models for data Analysis: Contributions to automatic learning*. PhD. Thesis. Universidad Pontificia Comillas.

Sánchez-Úbeda, E. F. and Wehenkel, L. (1998). *The Hinges Model: A one-dimensional countinuous piecewise polynomial model*. Information Processing and Management of Uncertainty in Knowledge-based Systems, Paris.

Schweppe, F. C., Caramanis, M. C., Tabors, R. E. and Bohn, R. E. (1988). *Spot pricing of electricity*. Kluwer Academic Publishers.

Skantze, P. L., Gubina, A. and Illic, M. D. (2000). *Bid-based stochastic model for electricity prices: the impact of fundamental drivers on market dynamics*. Energy Laboratory, MIT. Working Paper MIT EL 00-004.

Tipping, J., McNickkle, D. and Read, G. (2004). *Incorporating storage levels into a model for New Zeland spot prices*. IAEE European Conference, Zurich.

Tirole, J. (1988). *The theory of industrial organization*. Cambridge, MIT Press.

Weron, R. (2008). *Wholesale electricity prices: a review of time series models*. In: Financial Markets: Principles of Modelling, Forecasting, and Decision-Making. W. Milo and P. Wdowinski (eds.). Lodz. FindEcon Monograph Series.

APPENDIX: CALIBRATION OF THE PRICING MODEL

The appendix will tackle the problem of specifying the parameters required to define the model (13). In this regard, the long-term equilibrium is defined, in addition to the underlying factors, by the parameter representing the strategic behavior of market agents. This parameter is not obtained by any underlying model, as in the case of the fundamental drivers, but it has to be estimated from market information or historical prices. By definition, the short-term component represents deviations from the long-term equilibrium, so it will be also estimated from market data or historical information.

Since the model is made up of the sum of two components, $p_t = \pi_t + y_t$, and without any additional criterion, an estimation only based on minimizing the squared residuals provides no way of disentangling the equilibrium behavior effects of π_t from the deviation factor y_t. For example, the strategic parameter may be zero, $\theta_{i,t} = 0$, and the short-term factor may explain any deviation from the cost-minimization problem defined by $\theta_{i,t} = 0$.

The solution proposed in this section is considering the calibration of the equilibrium price to drive the calibration process. That is, the equilibrium price π_t will match market prices for every available contract. For instance, consider a market where the only quoted contract is the monthly forward, ie a contract for delivery each hour of a determined month. The equilibrium price would be calibrated to represent the prices of these contracts, while the short-term component would only account for deviations of these prices within the month. Analogously, if monthly options were quoted, they would determine the monthly volatility, while the short-term component would capture deviations of the monthly volatility within the month. The rationale behind this strategy is capturing as many effects as possible by means of the equilibrium price, as it describes the relations between underlying factors and power prices.

From this viewpoint, the natural methodology for model calibration is to perform two consecutive steps: first, the strategic parameter $\theta_{i,t}$ is calibrated and hence the equilibrium price; second, the equilibrium deviation factor is determined by $y_t = p_t - \pi_t$. Note that the problem is related to the estimation of additive models, see for instance Hastie and Tibshirani (1990), as they also face the problem that the estimation through direct minimization of squared residuals results in any set of functions. However, the additional criterion proposed in that context is different: additive models introduce smoothness in the estimates of each factor, motivating the backfitting algorithm, while the criterion used in this section is based on fundamental considerations.

Strategic parameter calibration

We begin with the statement of the equilibrium problem defined by a certain scenario e of fuel prices and system demand. Thus, the equilibrium model is defined by $V^e\left(\theta_{i,t}\right)$, where

$$V^e\left(\theta_{i,t}\right)=\begin{cases}\min & \sum_{i,j,t}\frac{1}{2}\theta_{i,t}\left(\sum_j g_{i,j,t}^e\right)^2+C_t\left(g_{i,j,t}^e\right) & \\ s.t. & 0\le g_{i,j,t}^e\le g_{i,j}^{\max} & :\mu_{i,j,t}^{\min,e},\mu_{i,j,t}^{\max,e} \\ & \sum_{i,j} g_{i,j,t}^e=D_{i,t}^e & :\pi_t^e\end{cases}\tag{19}$$

The superscript e denotes the variables in the scenario considered. Thus, we can obtain the equilibrium factor of the market price as the solution of $V\left(\theta_{i,t}\right)$, and the equilibrium model is defined by the strategic parameters $\theta_{i,t}$. If there is no additional constraint on the values of the strategic parameter, it can take, each point in time, the value needed to match the actual market price. From a statistical point of view, since θ_t^i represents one parameter for each market player and each point in time, the equilibrium price can match any set of price values, ie the over-fitting problem.

The approach suggested in this section is relying again on fundamental considerations: the strategic behavior of market players should not change each point in time. Consider, for instance, a set $\{t_1,\ldots,t_s\}$ representing the period when the strategic behavior does not change. We impose an additional constraint so that $\theta_{t_1}^i=\ldots=\theta_{t_s}^i$, which can be understood as a version of the parsimonious principle, or alternatively as an assumption concerning the behavior of market players.

In practice, the above constraint is equivalent to stating an aggregated version of the equilibrium model. The subscript t in the problem (19)is substituted by τ, so that we have an aggregate production $G_\tau^i$of the firm i in the set τ, and the aggregate price in τ, π_τ. In addition, let us denote by m the set firms owning a marginal plant at τ. From the optimality conditions of the problem, we have that, for the marginal firm(s) *m*, it must be fulfilled the following equation:

$$\pi_\tau-\theta_\tau^m G_\tau^m-\frac{\partial C\left(G_\tau^m\right)}{\partial G_\tau^m}=0\tag{20}$$

From (20) we can obtain the strategic parameter of the marginal firm(s) as a function of model price:

$$\theta_\tau^m=\frac{1}{G_\tau^m}\left(\pi_\tau-\frac{\partial C\left(G_\tau^m\right)}{\partial G_\tau^m}\right)\tag{21}$$

Therefore,we state an iterative procedure that works as follows:

- First, we assume that every agent bids competitively, or in other words, no market power opportunities are taken into account $\theta_\tau^{m,0} = 0$. With this assumption, we obtain a collection of productions $G_\tau^{m,0}$ and an equilibrium price π_τ^0. The second superscript denotes the iteration 0

- The new strategic parameters $\theta_\tau^{m,k}$ are obtained using (21), but substituting π_τ by the mean price observed in the market in the set τ, π_τ^{real}. The superscript k denotes the iteration number. Note that the new parameters correspond only to the marginal firms. The rest of parameters remain zero

- The new parameters $\theta_\tau^{m,k}$ changes the output decisions of the power producers, so that we obtain a new set of productions $G_\tau^{m,k}$ and a new price π_τ^k

- The algorithm stops when the difference between equilibrium prices of two consecutive iterations is less than a tolerance ε, $\pi_\tau^{k+1} - \pi_\tau^k \leq \varepsilon$

Furthermore, if the marginal costs $\dfrac{\partial c\left(G_\tau^m\right)}{\partial G_\tau^m}$ are always less than the mean price π_τ^{real}, or in other words, if the parameters $\theta_\tau^{m,k}$ are always greater than zero, the algorithm converges to the real prices π_τ^{real}. Note in addition, that this is a condition for the problem (19) to have a solution. When this is not the case, it would be required an additional parameter. In fact, a negative value of $\theta_\tau^{m,k}$ makes the model meaningless, because it would represent that the firm m can raise the market price by increasing its output, and this is an unbounded problem. Negative values of the strategic parameter would require an additional parameter affecting the generation costs, so that the marginal cost is reduced to prevent the strategic parameter to take negative values.

Nonetheless, this situation has a difficult interpretation, and it will probably have more to do with a misspecification of generation costs than with the strategic parameter. For instance, model (19) does not take into account start-up costs. However, it may happen that the output of the model is such that some power plants are producing one day, the next day are shut down, and producing again the day after. Thus, the power plant would have to incur the costs of starting up twice. When the start-up costs are considered, it is likely that the power plant would choose to produce at its minimum output instead of shutting down. This latter option would imply to lower its bid price below the marginal cost of the plant. Consequently, following the reasoning of the paper, it is preferable to complicate the cost specification instead of adding parameters that ultimately will have to be statistically estimated.

Complete calibration scheme

Ideally, for valuation and hedging purposes, the model should be able to recover the price of the instruments available in the market, since they contain the information used by the market agents to price contracts. Let us assume that the market price obtained from the model, for the scenario e, is given by $p_t^e = \pi_t^e + y_t^e$. The market approach to adjusting the model consists in finding the parameters that match actual prices of liquidly traded instruments. Then, there is a set of prices, which will be denoted by $\hat{c}_q$, $q = \{1,\dots,Q\}$, assuming that Q products can be observed. On the other hand, with the simulation of the model, it is possible to obtain values for the observed contracts using the prices given by the model. Let me denote them as c_q. Therefore, we can state the following least squares problem:

$$\min_{\theta_{i,m},h_t} \quad \sum_{q=1}^{N} \left(c_q - \hat{c}_q\right)^2 \tag{22}$$

The key condition for the procedure stated above is that the process for power prices can calculate the value of the contracts. For example, the value of a forward contract $F_{t,T}$, quoted at time t with delivery at T, since the model gives a risk-adjusted process, is defined by $F_{t,T} = \frac{1}{E}\sum_{e=1}^{E} p_T^e$, and consequently allows the computation of the model parameters that match market prices. Analogously, the value of an option can be used to obtain the volatility implied by the market price of the option $\sigma_T^{implied}$, see for instance Eydeland and Wolyniec (2003). Then, the implied volatility can be used to introduce an additional constraint on model prices $\left(\sigma_T^{implied}\right)^2 = \frac{1}{E}\sum_{e=1}^{E}\left(p_T^e - mean\left[p_T^e\right]\right)^2$.

There are, however, several limitations of the previous approach. First, the problem (22) is usually ill-conditioned. In addition, many power markets do not have a large amount of liquidly traded instruments. Moreover, actively traded products may not contain all the relevant information for pricing any other contract. For example, monthly options account only for monthly volatility. If the contract studied is a daily option, for example, the volatility involved in the calculation is the daily volatility. Using only monthly contracts for the calibration of the price model will not capture the appropriate dynamics of daily volatility of market prices. Therefore, the calibration of the model just based on actual market data is not possible in most of the power markets.

This is closely related to the problem of defining which part of the market information is contained in the equilibrium component, and which part is contained in the deviation factor. The methodology defined by (22) does not help in the task of disentangling the equilibrium factor from the short-term process. Again, the suggested solution builds on fundamental arguments: the equilibrium component should match as much market information as possible. In fact, since the correlation structure between power prices and fundamental drivers is explained by the equilibrium price, matching market data will imply matching market information about this correlation structure. Consequently, the impact in power prices of future changes in the fundamentals will be better described.

From this viewpoint, consider that the only contract available in the market is a monthly forward. Thus, the strategic parameter may be calibrated the prices of these contracts. Note that, when doing so, one is making explicit assumptions about the strategic behavior of the firms: as the strategic parameter will not change during each month, the model will be assuming constant market power opportunities within the month. This may not be convenient when it implies long periods: for instance, calibrating the strategic parameter using only annual forwards would imply to assume that firms' strategic behavior is constant during the year, which would be likely a rough approximation. Hence, the calibration process should use in addition spot price information to obtain certain degree of variability in the strategic parameter. The extreme case of this situation is when there is no financial contract traded in the market. Then, one can only rely on spot price information, that is, on historical prices, to calibrate the model. When this is the case, the time intervals to fit the strategic component $\{t_1,\ldots,t_s\}$ must be defined using a priori knowledge of agents' behavior. In other words, the definition of the strategic parameter variability cannot be data driven, but must be set exogenously.

In any case, it is worth to highlight that the set $\{t_1,\ldots,t_s\}$ does not necessarily denotes a sequence of consecutive points in time. It just denotes the indexes used to aggregate the problem variables. For example, power markets usually trade peak-load forwards, which are forward contracts for delivery in some predefined hours of the day –supposedly the hours with highest demand. In addition, they do not have to be defined on a daily basis, but can be weekly or even monthly contracts. When dealing with such forwards, the set $\{t_1,\ldots,t_s\}$ will represent the peak hours of the contract period, (a week, a month...).If there were only spot price data, the calibration process would work as follows:

- Find the strategic parameters required for the definition of the equilibrium price. This step requires the definition of time intervals $\{t_1,\ldots,t_s\}$ for the aggregation of the problem

- Calculate the errors between real prices and equilibrium prices, $y_t^e = p_t^e - \pi_t^e$. Note that the model obtains the mean of the real price within a certain period, the set $\{t_1, \ldots, t_s\}$,but it does not match the price at each point in time

- Estimate the model (16) to describe historical errors

Adding market information to the above procedure is rather straightforward, although it can be computationally consuming. The first step required to compute mean values of the spot price in the set $\{t_1, \ldots, t_s\}$. Using contract prices thus will only consist in introducing additional constraints on equilibrium prices, defined by the contract values.