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Abstract

We consider information aggregation in national and local elections when voters are mobile and might sort themselves into local districts. Using a standard model of private information for voters in elections in combination with a New Economic Geography model, agglomeration occurs for economic reasons whereas voter stratification occurs due to political preferences. We compare a national election, where full information equivalence is attained, with local elections in a three-district model. We show that full information equivalence holds at a stable equilibrium in only one of the three districts when transportation cost is low. The important comparative static is that full information equivalence is a casualty of free trade. When trade is more costly, people tend to agglomerate for economic reasons, resulting in full information

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equivalence in the political sector. Under free trade, people sort themselves into districts, most of which are polarized, resulting in no full information equivalence in these districts. We examine the implications of the model using data on corruption in the legislature of the state of Alabama and in the Japanese Diet.

Keywords and Phrases: information aggregation in elections, informative voting, new economic geography, local politics

JEL Classification Numbers: D72, D82, R12

1 Introduction

We seek to address questions at the boundary of politics and geography: How much information is revealed in local as opposed to national elections? Does the mobility of voters help or hinder information aggregation in local elections? Of course, the electorate is generally smaller in local as opposed to national elections, but does voter migration for economic reasons result in polarization of local elections? *Under what circumstances do localities, such as cities, become politically polarized?*

Our theoretical model addressing these questions will be applicable to many contexts where mobile voters have both economic and political concerns, receiving a signal concerning an issue or candidate, as we shall detail below. For an empirical viewpoint, we examine the specific context of officials that are elected and later found (or found not) to have received outside money that might compromise their votes. The idea is that voters receive a signal about whether a particular candidate for office is corrupt. In polarized districts, our model predicts that candidates with an extreme view that coincides with the electorate in their district will be elected whether they are corrupt or not, whereas in unpolarized districts fewer corrupt candidates are elected, as people vote using their signals as if they were pivotal. A complicating factor, that we shall address specifically in our model, is the mobility of consumers and their endogenous selection of district.

Consider the following data, collected by Couch et al. (1992), on whether Alabama state-elected officials received income from serving on boards of local state-funded universities in 1987-1988. House districts are evidently smaller.

Table 1: 2×2 Contingency Table for Alabama's Legislature

Alabama	No Outside Income	Outside Income
Senate	31 (88.6%)	4 (11.4%)
House	77 (73.3%)	28 (26.6%)

Sources: Couch et al. (1992), http://www.legislature.state.al.us/

Note that House districts are not necessarily subsets of Senate districts.

$\chi^2 = 3.46$

Degrees of Freedom = 1

Probability = 0.063

From this table we can see that the likelihood that House and Senate members differ in their receipt of outside income is large but not definitive. Could it be that some elections for the House imply more information aggregation than others?

Next consider members of the Diet in Japan from July 2000 to March 2003. The Diet is bicameral, the House of Councilors having fewer members than the House of Representatives.

Table 2: 2×2 Contingency Table for Japan's Diet[1]

Japan	No Allegations	Resigned Under Duress or Convicted
House of Councilors	145 (99.3%)	1 (0.7%)
House of Representatives	289 (96.3%)	11 (3.7%)

Source: http://www.notnet.jp/data04index.htm

Note that House of Representatives districts are subsets of House of Councilors districts.

$\chi^2 = 3.33$

Degrees of Freedom = 1

Probability = 0.068

Again, there appears to be more corruption in elections involving smaller districts, but this is not definitive.

[1] In the House of Councilors of Japan's Diet, 146 of the 242 seats are elected in single-seat districts and 96 by proportional representation. In the House of Representatives, 300 of the 480 seats are elected in single-seat districts and 180 by proportional representation.

To address the theoretical questions we have posed as well as to explain the data, we formulate a model of politics and information aggregation in elections where voters are also economic agents and mobile. Geography and politics interact and feed back in interesting ways: On the one hand, economic factors might cause agglomeration of agents, thus affecting the polarization of districts, the aggregation of information in local elections, and the outcomes of local elections. On the other hand, the outcomes of elections in localities might affect the agglomeration of agents into these localities. This interplay leads us to the introduction of geography into models of politics, in particular those associated with the Condorcet jury theorem such as Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). It also leads us to introduce politics into models of stratification or agglomeration, such as Krugman (1991). In this respect, we could have used a model of local public goods for this purpose, but find the New Economic Geography model from urban economics to be both more tractable and less biased toward stratification. For example, in the US context, local education and quality of schools, along with property taxes, are the most important criteria used by consumers/voters for determining location of residence. Tiebout sorting models will generally lead directly to stratification by type of consumer in equilibrium, implying a failure of full information equivalence (defined in the next paragraph) in the various districts. In summary, we could use a model of equilibrium in a local public goods economy in place of the New Economic Geography part of our model, but we conjecture that results would be similar. In general, New Economic Geography models lead to agglomeration, but not directly to stratification. We shall elaborate on this in the conclusions.

Our main findings are summarized as follows. We compare a national election, where the same outcome is attained whether voters know everyone's private information or not (called *full information equivalence*[2] in the political science literature), with local elections in a three district model. We show that full information equivalence holds in only one of the three districts when transportation cost of goods is low. The important comparative static is that *full information equivalence is a casualty of free trade*. When trading goods is more costly, people tend to agglomerate for economic reasons, resulting in full information equivalence in the political sector. Under

[2]Equivalently, it can be said that full information aggregation occurs in the election.

free trade, people sort themselves into three districts, two of which are polarized, resulting in no full information equivalence in these districts. The remaining district still satisfies full information equivalence. Thus, if the signals voters receive concern the conflict of interest or corruption of candidates in their district, it is expected that elections in districts with smaller populations (local elections) will result in a higher proportion of compromised elected officials. This might even happen if the electorate is large, as in our model. But some of these districts will still satisfy full information equivalence, so the correlation between size of electorate and information aggregation in elections is imperfect. Nevertheless, our model endogenously generates politically polarized districts.

The literature on information aggregation in elections has a focus on an electorate that is exogenously given and thus is immobile. Austen-Smith and Banks (1996) presented the seminal work on the Condorcet jury theorem, showing in a game-theoretic context that for some states of nature, not all the information of voters is revealed in Nash equilibrium even if they all have the same objective functions and priors. Feddersen and Pesendorfer (1997) find sufficient conditions for which full information equivalence holds at Nash equilibrium, and that is the framework we employ below.

The literature on economic geography has almost no focus on voting, particularly when there is asymmetric information about candidates or ballot measures.

We wish to emphasize that one interpretation of the model, specifically taking the uncertainty to be about political corruption, is useful primarily because there are empirical implications that can be taken to data. Other interpretations of the alternatives over which there is uncertainty, for example the effects of policies regarding a local public good such as schooling, or candidate productivity or valence, are equally valid and possibly more interesting, but are harder to take to data. This will be discussed further in the conclusions.

The outline for the balance of the paper is as follows. Section 2 gives the model and definitions of equilibrium and stability. Section 3 contains a characterization of equilibrium and the comparative statics of the model with a focus on local politics. Section 4 discusses the general implications of the model, returning to our discussion of the data. Finally, Section 5 gives our conclusions.

2 The model

The spatial structure of the model consists of three districts indexed by $i = 1, 2, 3$, located at each vertex of a regular triangle. These can be cities, regions or jurisdictions within a city. There is an exogenously given mass $L > 0$ of consumers, each of whom supplies one unit of labor inelastically. Let the population of district i be denoted by L_i.

The model has a political as well as an economic sector. Overall utility is given by the sum of subutilities from the two sectors. The utility from the economic sector for a resident of district i is given by u_i, whereas the utility from the political sector is given by v. The total utility is given by

$$U_i \equiv u_i + v.$$

We will describe these subutility functions, including their domains, in detail. We begin by describing the economic sector.

2.1 The economic sector

Preferences are defined over a continuum of varieties of a horizontally differentiated good. The preferences of a typical resident of district i are represented by the following CES utility:

$$u_i = \left[\sum_{j=1}^{3} \left(\int_{\Omega_j} d_{ji}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \, d\omega \right) \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $d_{ji}(\omega)$ is the consumption in district i of variety ω produced in district j, and Ω_j is the set of varieties produced in district j with $j = 1, 2, 3$. The parameter $\varepsilon > 1$ measures both the constant own-price elasticity of demand for any variety, and the elasticity of substitution between any two varieties. Unlike standard models in the tradition of New Economic Geography, there is no freely traded homogeneous good. The freely traded homogeneous good is unrealistic and its presence might not be innocuous (Davis, 1998).

To explain how the economic sector works, first fix the locations of consumers.

Production of any variety of the differentiated good takes place under increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined in equilibrium by free entry and exit. In what follows, we denote by n_i the mass of firms located in district i. Production of each variety requires both a fixed and a constant marginal labor input requirement, denoted by $\bar{c}$ and c respectively. As for transportation costs, inter-district shipments of any variety are subject to iceberg transportation costs: $\tau_{ij} \geq 1$ units have to be shipped from district i to district j for one unit to reach its destination.

Given our assumptions, in equilibrium firms differ only by the district in which they are located. Accordingly, to simplify notation, we drop the variety label ω from now on. Let p_{ji} be the delivered price of a variety from district j to district i. Then, the maximization of (1) subject to the budget constraint

$$\sum_{j=1}^{3} n_j p_{ji} d_{ji} = w_i \tag{2}$$

yields the following individual demand in district i for a variety produced in district j:

$$d_{ji} = \frac{p_{ji}^{-\varepsilon}}{P_i^{1-\varepsilon}} w_i, \tag{3}$$

where w_i is the wage in district i and P_i is the CES price index in district i defined by:

$$P_i \equiv \left(\sum_{k=1}^{3} n_k p_{ki}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{4}$$

Due to the iceberg transportation cost assumption, a typical firm established in district i has to produce $q_{ij} = \tau_{ij} d_{ij} L_j$ units to satisfy final demand d_{ij} in district j, where L_j is the number of consumers in district j. The firm takes (3) into account when maximizing its profit given by:

$$\Pi_i = \left(\sum_{j=1}^{3} p_{ij} d_{ij} L_j \right) - w_i \left(c \sum_{j=1}^{3} q_{ij} + \bar{c} \right). \tag{5}$$

Profit maximization with respect to p_{ij}, taking the price index P_j as given because of the continuum of varieties, then implies that the price per unit delivered is:

$$p_{ij} = \frac{\varepsilon c}{\varepsilon - 1} \tau_{ij} w_i = \tau_{ij} p_{ii}. \tag{6}$$

Due to free entry and exit, profits must be non-positive in equilibrium. Then (5) and (6) imply that firms' equilibrium scale of operation in district i must satisfy

$\Pi_i = 0$, which is rewritten as:

$$(p_{ii} - cw_i) \sum_{j=1}^{3} \tau_{ij} d_{ij} L_j = w_i \bar{c}. \tag{7}$$

Because the labor input is given by $c \sum_{j=1}^{3} q_{ij} + \bar{c}$ in (5), the labor market clearing conditions are given by:

$$n_i \left(c \sum_{j=1}^{3} \tau_{ij} d_{ij} L_j + \bar{c} \right) = L_i. \tag{8}$$

Eliminating p_{ij} and $\sum_{j=1}^{3} \tau_{ij} d_{ij} L_j$ from (6), (7) and (8), we get:

$$n_i^* = \frac{L_i}{\varepsilon \bar{c}}. \tag{9}$$

That is, the number of firms in a district is proportional to the number of workers in that district at equilibrium.

Substituting (4), (6) and (9) into the zero profit condition (7), we have:

$$\sum_{j=1}^{3} \frac{\phi_{ij} w_j L_j}{\sum_k \phi_{kj} w_k^{1-\varepsilon} n_k^*} = w_i^{\varepsilon}. \tag{10}$$

Due to the geographically symmetric location of the districts, we set

$$\phi_{ij} \equiv \tau_{ij}^{1-\varepsilon} = \begin{cases} \phi \in [0,1] & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases},$$

which is a measure of how free trade is, since $\varepsilon > 1$. Its value is one when trade is free and zero when trade is prohibitively costly. There are three equilibrium conditions (10) and three unknowns: w_1, w_2 and w_3. However, one of the three equations in (10) is redundant by Walras' law. We set $w_1 = 1$ by choosing the wage in district 1 as the numéraire. As is standard in the New Economic Geography literature, it can be shown that there is a unique solution, namely $(w_1, w_2, w_3) = (1, w_2^*, w_3^*)$.

The indirect equilibrium utility (with a fixed distribution of consumers) is given by:

$$u_i^* = \frac{w_i^*}{P_i^*} = \frac{w_i^*}{\left[\frac{1}{\varepsilon \bar{c}} \sum_{j=1}^{3} \phi_{ji} \left(w_j^* \right)^{1-\varepsilon} L_j \right]^{\frac{1}{1-\varepsilon}}}. \tag{11}$$

It can also be shown that if $L_i > L_j$, then $w_i^* > w_j^*$ and $u_i^* > u_j^*$. This is called the market size effect: the nominal and real wages are higher in the larger district.

2.2 The political sector

There are two types of elections, namely national elections and local elections. For national elections, every consumer votes. For local elections, the alternatives are chosen in each district independently. Only the residents of a district vote in the local election for that district. We formulate two models, one with only a national election, and one with only local elections. We adopt the framework of Feddersen and Pesendorfer (1997) for the political sector. There are two alternatives in any election, A and Q. Let $\gamma \in \{A, Q\}$. A preference parameter for a voter is given by $x \in [-1, 1]$, whereas the state is given by $s \in [0, 1]$. The set of voter types is denoted by $X = [-1, 1]$.[3] The probability distribution over consumer types is given by F (if it has a density, call it f), whereas the common prior over states is given by G (if it has a density, call it g). Define the utility from the political sector of type x from alternative γ in state s to be $v(\gamma, s, x)$. We assume that $\Delta v(s, x) \equiv v(A, s, x) - v(Q, s, x)$ is continuous and increasing in s and x.

The total utility of a consumer in district i of type x is abbreviated as

$$U_i^x \equiv u_i + v.$$

Each voter receives a signal $\sigma \in \{1, ..., M\}$ at the beginning of the political stage, before voting, but after the economic stage. Denote by $p(\sigma \mid s)$ the probability that a consumer receives signal σ in state s.

2.3 Timing of the game

All players have perfect foresight. The timing of the game is as follows. First, the firms and consumers locate themselves in the three districts, knowing what lies ahead. The agents cannot relocate after this step. Then economic equilibrium in the districts is achieved. Next, each consumer receives a signal about the alternatives in the political sector. Then they simultaneously vote over the two alternatives, the winner determined by majority rule. For national elections, the outcome is independent of the district of residence. For local elections, the outcome is specific to each of the three districts. This is equilibrium in the political sector. Finally, all

[3] In the terminology of Feddersen and Pesendorfer (1997), there is only one information service.

players receive their utility payoffs. We seek the subgame perfect Nash equilibria in weakly undominated strategies of this game.

Notice that for national elections, only the economic sector matters in the choice of location, so the game reduces to a standard New Economic Geography model. Hence, we focus on local elections.

2.4 Equilibrium

Definition 1 *A* ***strategy profile*** *is a measurable map* $\pi = (\pi_1, \pi_2)$ *where* $\pi_1 : X \longrightarrow \{1, 2, 3\}$ *and* $\pi_2 : X \times \{1, 2, 3\} \times \{1, ..., M\} \longrightarrow [0, 1]$. *Here,* π_1 *denotes the strategy at stage 1, the economic stage, whereas* π_2 *denotes the strategy at stage 2, the political stage. In general, the range of* π_2 *denotes a mixed strategy where* 0 *is a pure strategy vote for* A *whereas* 1 *is a vote for* Q.

In stage 1, each consumer (of any type) chooses a location. In stage 2, they vote.

We face a technical issue here that is faced by most working on information aggregation in elections. In general, models with a finite number of voters are used due to division by zero in applying Bayes' rule when there is a continuum of voters. In other words, the event that a person is pivotal when there is a continuum of voters often has probability zero, so conditioning on this event is not possible. One option to address this problem is to use regular conditional probabilities, but that is not possible in our context. The alternative that we (and the literature) use is specified as follows.

The first stage of the game proceeds as an economy and game with a continuum of players. This yields a population distribution in each of the three districts. For national elections, votes from all districts are counted. For local elections, only votes from a district are counted for the election in that district. When there are local elections, there is an outcome for each district.

Fix population distributions F_1, F_2, and F_3 in districts 1, 2, and 3, respectively (if there is a density f for F, then F_i has density f_i for $i = 1, 2, 3$). For local elections (national elections follow in an obvious way) we draw randomly and independently N voters from each district using the appropriate district-wide distribution, where N is exogenous. Focus on a district i and a symmetric strategy profile for the district $\pi_2(\cdot, i, \cdot)$. Following Feddersen and Pesendorfer (1997, p. 1034), for each state $s \in [0, 1]$ we can calculate the updated posterior for the state, conditional on a voter being pivotal, on the signal they receive, and on others' strategies. Using this posterior, we can compute expected utility from the two alternatives, namely $E[v(A, s, x) \mid \pi_2, \sigma]$ and $E[v(Q, s, x) \mid \pi_2, \sigma]$. A voter can choose Q or A. Mixed strategies are defined in the obvious way. If the proportion of voters who choose Q is larger than 1/2, then Q is the outcome. Otherwise, A is the outcome. Expectations over the randomization are counted for mixed strategies.

A **second stage N-equilibrium** is a symmetric Bayesian Nash equilibrium in this second stage of the game, where no consumer/voter uses a weakly dominated strategy. Proposition 1 (actually the proof in the appendix) of Feddersen and Pesendorfer (1997) shows that such an equilibrium exists under their Assumption 1.

A **second stage equilibrium** is any limit point of second stage N-equilibria where N tends to infinity. Such exists if second stage N-equilibrium exists for each N due to the following argument. Let π_2^N be a second stage N-equilibrium with N voters drawn from F_i. If necessary, draw a converging subsequence so that for $i = 1, 2, 3$: $\int_X \pi_2^N(x, i, \sigma) dF_i$ converges for each σ.[4] This yields the expected number of votes for Q given σ at equilibrium. Then apply Fatou's lemma in several dimensions (see Hildenbrand, 1974, p. 69) to obtain a limit. The law of large numbers implies that if this number exceeds $\frac{1}{2}$ in district i, then given σ, the winner is Q. Otherwise, it is A.[5] Notice that the limit is not necessarily an equilibrium of the limiting game, due to problems with division by zero mentioned above. Rather, it is the limit of a sequence of equilibria for games with finitely many players, where the number of players tends to infinity. In the limit, there is a continuum of players in each district unless that district is empty. However, it is really the relative measure of players in the districts that matters for agglomeration. Moreover, each district electorate is large, so it is not the fact that local elections have smaller electorates than national elections that is driving the equilibrium information aggregation properties of voting.

[4]Notice that for each N this is just a list of real numbers of fixed, finite length, so such a converging subsequence exists provided that the sequence is bounded. Here, each element of the vector and sequence is in the unit interval.

[5]Of course, there is a continuity issue when the vote share converges to $\frac{1}{2}$ from above as $N \to \infty$ for some set of signals. But then a pivotal voter is indifferent between the two possible outcomes of the election.

Rather, it is the presence or absence of stratification.

Notice that even if the second stage N-equilibrium is unique, there might be multiple limit points, so the second stage equilibrium might not be unique. However, for our analysis we shall focus on an example where the second stage N-equilibrium is unique and independent of population, so the second stage equilibrium is unique.

Fix a strategy profile $\pi = (\pi_1, \pi_2)$. Fix a district i. At stage 2, the posterior over states conditional on being pivotal in that district and observing signal σ is denoted by $\beta_i(s \mid piv, \pi_2, \sigma)$. Then the explicit derivation of β_i can be found in Feddersen and Pesendorfer (1997, p. 1034) and below in the Appendix. The objective of a voter of type $x \in X$ is given by

$$\max_{\gamma \in \{A,Q\}} \int_0^1 v(\gamma, s, x) \cdot \beta_i(s \mid piv, \pi_2, \sigma) ds.$$

An **equilibrium** is the limit point of a sequence of subgame perfect, symmetric Bayesian Nash equilibria in this two stage game, where (almost) no consumer/voter in the sequence of games uses a weakly dominated strategy.

Informally, an equilibrium is said to satisfy **full information equivalence in district *i*** if the alternative that wins the election in that district is almost surely the one that would have been chosen if the electorate in that district were fully informed about the state s. The formal definition of full information equivalence is technical because it relies on statements about the asymptotic properties of large but finite elections, and can be found in Feddersen and Pesendorfer (1997, p. 1042).

2.5 Stability

To ease notation, we define $\lambda_i = \frac{L_i}{L}$. Take an equilibrium population distribution $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ with indirect economic utility u_i^* and with indirect political expected utility $Ev_i^*(x)$ for $i = 1, 2, 3$. Let f_i^* be the equilibrium density of types in district i, and let S_i^* be its support. We say that the equilibrium is *stable* if

$$\frac{d\left(U_j^* - U_i^*\right)}{d\lambda_j} = \frac{d\left(Ev_j^* - Ev_i^*\right)}{dx} \bullet \frac{dx}{d\lambda_j} + \frac{d\left(u_j^* - u_i^*\right)}{d\lambda_j}\Bigg|_{x \in S_i^* \cap S_j^*} < 0 \qquad \text{for } i, j = 1, 2, 3;\ i \neq j. \tag{12}$$

Here we are assuming that the economic equilibrium *does* change at the margin. However, the marginal change in the distribution of voters in the districts *does not* change the political equilibrium in either their origin or destination, as the first term only reflects the change in the expected utility evaluation of the difference in fixed equilibrium political outcomes with respect to the type or bias of a new district boundary consumer. The reason for this asymmetry between sectors is as follows. On the one hand, in the economic sector, even though no single consumer can affect prices, the consumers who are moved to a new district can observe that equilibrium prices, and thus their indirect utility, actually change. We take the limit of the change in utility divided by the measure of consumers moved as the measure of consumers goes to zero, resulting in the derivative of indirect utilities with respect to population. On the other hand, for the political sector, we are taking a different kind of limit, namely the limit of voting equilibria when there are random draws from the electorate as the size of the draw becomes large. When the distribution F has a density f, the probability that any particular person is even chosen as a member of a finite draw is zero. Thus, each individual agent does not think that their move to another district will affect the political outcome in either their origin or destination. (One can move a set of positive measure between districts and take limits as both the size of the draw and the measure of the set moved tend to zero. In that case, the order of limits is important. Since the limit of the equilibria as the size of the draw tends to infinity is not necessarily an equilibrium of the limiting game, we must focus on a fixed, finite size of draw and take the limit as the measure of agents moved tends to zero first, then focus on the limit of such equilibria as the sample size tends to infinity. In essence, we are testing for stability of the equilibria of the games with finite random draws of the electorate from the distribution rather than stability of the limit game. The latter has ill-defined conditional probabilities of being pivotal.)

If the supports don't overlap on an open set, $\frac{dx}{d\lambda_j} = \frac{1}{f_j^*}$. If they overlap on an open set, then on that open set, $\frac{dx}{d\lambda_j} = 0$.

3 Characterization of stable equilibrium and comparative statics

In order to study the comparative statics of equilibrium, we must be much more specific about the political sector. There are several reasons for this. First, since we want to be able to say something specific about the equilibrium distribution of population, we must know more about the equilibrium in the political sector for each given distribution of population, as agents can anticipate (at least in expectation) what will happen politically in each individual district, given the population distribution. The abstract framework of Feddersen and Pesendorfer (1997) tells us that equilibrium in *mixed* strategies exists and it has the form of a cutpoint equilibrium. But for our application, it is very useful to have an equilibrium in pure strategies. So we use one of their examples that does not fit their general framework, namely their Example 2, where for any distribution of population, equilibrium in pure strategies exists, is unique, and (under some further conditions) satisfies full information equivalence. The drawback of using this example is that since it does not satisfy their assumptions, we cannot claim the same generality in our results as they do in their paper.

A related issue pertaining to the modeling strategy concerns the fact that we have made functional form assumptions for the New Economic Geography sector of the model, for reasons detailed in that literature. This allows us to find equilibrium in that sector explicitly. If we were to use the general functional form we have specified for the political sector, then although we could know about existence of equilibrium and perhaps its general properties, we would not be exploiting the specific functional form assumptions made in the economic sector, and thus we could not use this to find equilibrium explicitly. In other words, we waste the additional information provided by functional form assumptions in the economic sector. With functional form assumptions in the political sector as well, we have balanced the assumptions in the two sectors so that we can exploit all of the functional form assumptions we use to find equilibrium explicitly, and thus find comparative statics explicitly.

Finally, there is the issue of existence of equilibrium in pure strategies when the second stage equilibrium is not unique. In that case, the classical problem that the equilibrium correspondence for the second stage equilibrium is not convex-valued arises, so existence of equilibrium in pure strategies is not assured.

For national elections, existence of a unique equilibrium satisfying full information equivalence follows directly from Feddersen and Pesendorfer (1997, Example 2).

Local election equilibrium for our example has the following features. Districts 1 and 3 are polarized, in that the outcome is always Q in district 1 and A in district 3, independent of signals received by the inhabitants. District 2 is the "swing district," in that it satisfies full information equivalence. Please refer to Figure 1 for an illustration of expected utilities for the political sector for each of the 3 districts as a function of x. In equilibrium, some districts can be empty, depending on the exogenous parameters. As in the literature on voting over local public goods, a person at a boundary between the swing district and a polarized district is just indifferent between always choosing one alternative for sure and the state-dependent, full information equivalence outcome. For each of these two districts, for example, such a person trades off ideological preferences with the probability that a candidate is corrupt. For the polarized districts, where full information equivalence does not hold, the assumption of Feddersen and Pesendorfer (1997) that is violated is: the distribution of preferences within a polarized district has full support (with density above a fixed, positive value). It does not have full support due to stratification.

To see that the second stage N-equilibrium (or the voting equilibrium) is unique in each district, we use the fact that "equilibrium strategies are not weakly dominated" in a strong way, as in Feddersen and Pesendorfer (1997). For example, one might think that in a national election, if everyone always votes for the same outcome independent of state, then nobody is ever pivotal and this "equilibrium" does not satisfy full information equivalence. The problem with this idea is that there are non-equilibrium strategies that will make a non-extreme voter pivotal, and the strategy that tells this voter to always vote for the same outcome is weakly dominated by a state-dependent one.

Similarly, for the polarized districts, if one has a suggested "equilibrium" where some person does not always vote for the same outcome, then there are non-equilibrium strategies that will render this person pivotal in the district, and the strategy they are using is weakly dominated by the strategy that tells them to always vote for the same outcome.

Assume that the political utility v is given by

$$v(\gamma, s, x) = K - \frac{1}{2}\left(x_\gamma - x\right)^2 - \left(x_\gamma + \frac{1}{2} - s\right)^2,$$

where $x_A = 1$ and $x_Q = -1$. Then,

$$\Delta v(s,x) \equiv v(A,s,x) - v(Q,s,x) = 2\left(-1 + x + 2s\right),$$

which is similar to the examples in section 5 of Feddersen and Pesendorfer (1997). Also assume that $f(x)$ is uniform over $[-1,1]$, $g(s) = 1$ for $s \in [0,1]$, there are only two signals so $M = 2$, and

$$p(1 \mid s) = \begin{cases} 1-\alpha & \text{if } s < 1/2 \\ \alpha & \text{if } s > 1/2 \end{cases},$$

where $\alpha < 1/2$.

Given our symmetric setting, we shall examine only equilibria with an *axisymmetric population distribution*:

$$(L_1, L_2, L_3) = (\lambda, 1 - 2\lambda, \lambda) \cdot L \tag{13}$$

Please refer to Figure 2 for a graphical illustration of Theorem 2. The vertical axis represents the population distribution, whereas the horizontal axis represents the exogenous parameter ϕ, trade freedom. The solid lines in the diagram illustrate the stable, axisymmetric equilibria for various values of ϕ; the thresholds ϕ_A and ϕ_B are defined in the Appendix.

Theorem 2 *For the national election model, there is a unique equilibrium, and it satisfies full information equivalence. For the local election model, there exists a stable equilibrium that has an axisymmetric population distribution. For every set of (exogenous) parameters aside from $\bar{c}$ and ϕ, there are $\bar{c}'$ and ϕ_A such that for all $\bar{c} < \bar{c}'$ and $\phi \leq \phi_A$, full agglomeration, namely $\lambda = 0$, is a population distribution for a stable equilibrium. For every set of parameters aside from ε and ϕ, there are ε' and ϕ_B such that for all $\varepsilon \geq \varepsilon'$ and for all $\phi \leq \phi_B$, stratified equilibrium with district 2 empty, namely $\lambda = \frac{1}{2}$, is a population distribution for a stable equilibrium. For every set of parameters aside from ϕ, for all $\phi \geq \phi_A$, partial agglomeration, namely an axisymmetric population distribution for some $\lambda \in (0, \frac{1}{2})$, is a population distribution for a stable equilibrium.*

The proof of Theorem 2 can be found in the Appendix.

Figure 2 illustrates the equilibrium distribution $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ as a function of trade freedom ϕ given $\varepsilon = 5$ and $L/\bar{c} = 100$.[6] Observe that there are multiple equilibria for small ϕ ($< \phi_B$).

The conclusion that should be drawn from this analysis is that for high and low freedom of trade, stable equilibria where not everyone is in the same district occur. Higher freedom of trade means location is less important for economic welfare, and hence the equilibrium location of consumers is driven by the political sector. With low trade freedom, either everyone is agglomerated in the same district, or the electorate is polarized in two separate districts. For moderate trade freedom, everyone is agglomerated in the same district, and the political outcome is state dependent. For high trade freedom, all three districts are occupied in equilibrium. Two of the districts are polarized, always voting for the same candidate or outcome independent of state, whereas the occupants of the larger moderate district vote according to their information.

4 Information aggregation in local elections

Using Feddersen and Pesendorfer (1997, Example 2), full information equivalence always holds in the *national elections* for this model, where every agent votes in the same election. Thus, elections aggregate information effectively, and we expect to see relatively few corrupt politicians elected.

On the other hand, *local elections* have different properties in this model with migration, where only the residents of a district have the opportunity to vote in that district's election. In this model with 3 districts and, for example, high trade freedom, only one of the 3 features full information equivalence at equilibrium. This is the largest district. The other two will always elect the same candidate, independent of individual signals and information. In other words, these two districts are polarized. The conclusion is that elections in larger geographical districts, called national elections in our terminology, will lead to the election of less corrupt candidates in those districts, whereas elections in smaller geographical districts, called local elec-

[6] Note that if $L/\bar{c}$ is sufficiently large, the "symmetric" equilibrium is unstable for small ϕ. This condition is somewhat similar to the black-hole condition that is standard in the New Economic Geography.

tions in our terminology, will lead to less information aggregation, and thus will lead to the election of more corrupt candidates as representatives of those districts. This matches the empirical evidence used as motivation for our work in the introduction. Notice that the theory does not predict that corrupt officials will be elected in every local district in every state of the world, but rather only for certain states of the world in the more polarized districts. Thus, one cannot expect a low p-value for this test.[7]

Ideally, we would want to use data from the US Congress to test this theory. The reason is that Senate districts are quite large and contain the House districts as subsets. However, there are data issues with this idea. Criminal convictions of members of the US Congress for corruption, for example by the Public Integrity Section of the Criminal Division of the US Department of Justice, are few. Although they are made public in their annual reports, most of the convictions are of officials in other branches of the federal government or of local officials. One could weaken the standards and look only at ethics investigations by congressional committees, but information about this is primarily confidential or leaked. Actual data, for example from the group Citizens for Responsibility and Ethics in Washington, is consistent with our hypotheses, but rather imprecise.[8]

Notice that the implication of the model that we apply to data has little to do with trade freedom, our comparative static parameter. Instead, it is based on the characterization of equilibrium, and relies on the idea that mobile voters can polarize the electorate endogenously in local as opposed to national elections, and polarization implies that corrupt politicians are more likely to be elected.

[7] We note that our data could be generated using a simple selection process for politicians moving from the lower to upper chamber: only those politicians who do not display corrupt tendencies are promoted. If we had panel data, we could test this theory. But we also note that in Japan, the lower house has more power than the upper house of the diet, so promotion might not be desirable.

[8] It would be interesting to examine data from lesser developed countries for consistency with the predictions of our model. A major issue with this idea is that many such countries use proportional representation systems rather than majority rule. We are not aware of a Condorcet jury theorem in this context.

5 Conclusions and extensions

We have constructed a model where politically polarized districts emerge endogenously. One consequence is that full information equivalence holds for only one of three districts in the local elections model, whereas it always holds in the national elections model. We have interpreted the model for empirical purposes as a model of politician corruption, and verified the informational implication of the theory in data. This particular interpretation of the model was used so that we could apply it easily to data. However, other interpretations of the uncertainty, such as the productivity or valence of the alternatives or candidates, are equally valid and perhaps more interesting, but less amenable to empirical applications. For example, the two alternatives in the model could represent different levels (high and low) of a local public good such as schooling, including appropriate taxes. The effectiveness of the policies might be unknown to voters, but they receive a signal about it. A big issue in trying to take this interpretation of the model to data is that many variables are changing as we observe the swing district's outcome changing, for example rents, wages, and populations of the districts.

Full information equivalence is a casualty of free trade. The reason is that under free trade, people sort themselves into districts, most of which are polarized. When trade is more costly, people tend to agglomerate for economic reasons, resulting in full information equivalence in the political sector.

It is interesting to discuss welfare in the context of this model. Originally, the New Economic Geography, representing the economic side of our model, was designed to answer the positive question: Why are there cities? The early literature shied away from normative questions, though more recent literature has examined efficiency. Similarly, the literature on information aggregation in elections also tends to focus on positive questions. There are reasons this has happened.

In the context of the Feddersen and Pesendorfer (1997) model, under assumptions that ensure full information equivalence, their model reduces to a standard political model where all policies, specifically A and Q, are Pareto optimal. As is standard in many political economy models, this represents a purely redistributive game, and thus welfare evaluation reduces to interpersonal utility comparisons. This is not desirable. Since our model is an adaptation of the Feddersen-Pesendorfer model, something similar happens here. An implication is that we cannot say that the swing district features a higher level of efficiency in provision of local public goods than the other districts. Beyond interpersonal utility comparisons, when discussing allocations that Pareto dominate equilibrium allocations but might not be equilibrium allocations themselves, it is unclear what information structure to use for evaluation of the political sector, for instance full information or a structure less informative to agents.[9]

Finally, it is clear that welfare evaluations in our model will hinge on the relative weight given to the economic and political sectors in the utility functions.

For all of these reasons, we eschew explicit welfare comparisons using our model.

If we were to use a model of local public goods in place of our New Economic Geography model for the economic sector, it is likely that stratification would always occur in equilibrium. Thus, it is likely that full information equivalence would never hold in local elections.[10] But there are also models of local public goods in the literature, such as Epple and Platt (1998), that do not imply complete one-dimensional stratification in equilibrium. Let us consider this model in a bit more detail. There are two dimensions of consumer heterogeneity in this framework, income and preferences. We wish to make 3 remarks. First, there is no theorem on existence of equilibrium in this model. Second, as illustrated in Figure 3 of that paper, if one considers income as our parameter x, there is some of every income type in every jurisdiction due to preference heterogeneity, leading to no polarization and full information equivalence. Finally, it makes less sense to us to look at a comparative static on the variance of idiosyncratic preference heterogeneity than on trade freedom, something that is observable.

Related to this, another variation of the model is of interest: combine a local public good model with an NEG model. Actually, this represents another interpretation of our model, provided there is some uncertainty. In essence, local elections allow sorting and let mobile consumers obtain their desired bundle of local public goods, in contrast with national elections. Thus, in regional economies such as ours, consumers face a tradeoff between a local policy match with their preferences, causing dispersion, and agglomeration for standard NEG purposes. One can conclude that the benefits

[9] These ideas will not be novel to those who work in this literature, as they are part of the folklore.

[10] Such a model would predict very low p-values in the data presented in the introduction.

of Tiebout sorting are a casualty of trade barriers, though this has no implication for overall welfare from both sectors. The equilibria of this model would be second best or worse. It would be possible to formulate a simpler combination Tiebout and NEG model with *no uncertainty* to make this precise, but that is beyond the scope of this paper.

With only 2 instead of 3 districts, the comparative statics reduce to the left hand half of Figure 2. That is, when trade costs are high, there is an equilibrium with full agglomeration of agents in one district, and an equilibrium with half the population in each district, sorted by voter type. For lower trade cost, only the stratified equilibria survive. Thus, our main conclusion still holds. With more than 3 districts, it is difficult to calculate the second stage (political) equilibria in the districts.

Many extensions of our work come to mind. It would be interesting to allow the alternatives or candidates A and Q move with the composition of the district, migrating toward the median or at least toward the endpoints of the support of the population distribution in a district. We conjecture that this would be possible if we force the candidates to move first and commit, before populations are determined. However, if we allow candidates to move after migration occurs, we run into the problem that this model has not been solved, to our knowledge, even in the national elections (or exogenous population) case. Castanheira (2003) makes some progress in a related model.

In the spirit of Maug and Yilmaz (2002), for the local elections model we could assume that each district elects a delegate corresponding to A or Q who would then vote in a legislature according to the wishes of the district, with the winner determined by the majority but applying to all districts. Thus, the outcome would not be district-specific. In our 3 district model with relatively free trade, there would be 3 delegates. The pivotal delegate would be determined by district 2, satisfying full information equivalence, so the outcome in the legislature would also satisfy full information equivalence. In contrast with Maug and Yilmaz (2002), we do not require that the same alternative win a majority in all 3 districts.

Our utility function has equal weights on the utility derived from the economic and political sectors, and is additive across the sectors. It would be interesting to consider more general utility functions, in particular asymmetric weights on the sectors, to see if it produces districts that might be asymmetric in various senses.

An interesting conjecture is that higher transportation cost into and out of a district leads to isolation, lower population, and polarized electoral outcomes. To analyze this conjecture, the New Economic Geography part of the model would have to be extended to allow asymmetries in either transportation cost or distance. This is not easy; see Ago et al. (2006) and Bosker et al. (2010).

Finally, it would be interesting to allow politicians to choose the transportation infrastructure. This involves the same complications as making transportation cost asymmetric, and more.

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Appendix: Proof of Theorem 2

Proof. Preliminaries:

The probability that a randomly selected voter votes for Q in state s is

$$\begin{aligned} t(s,\overline{\pi}) &= \sum_{\sigma=1}^{2} p(\sigma \mid s) \int_X \overline{\pi}(x,\sigma) f(x) dx \\ &= \begin{cases} (1-\alpha)\, F(x_1) + \alpha F(x_2) & \text{if } s < 1/2 \\ \alpha F(x_1) + (1-\alpha)\, F(x_2) & \text{if } s > 1/2 \end{cases} \end{aligned} \tag{14}$$

from the definition of $p(\sigma \mid s)$. The probability that a vote is pivotal in state s is given by

$$\Pr(piv \mid s, \overline{\pi}) = \binom{n}{n/2} t(s,\overline{\pi})^{n/2} \left[1 - t(s,\overline{\pi})\right]^{n/2},$$

where $t(s,\overline{\pi})$ is given by (14).

Analogous to Feddersen and Pesendorfer (1997), let x_1 and x_2 be cutpoints, $x_1 > x_2$, namely for $x < x_2$ the voter always votes for Q, for $x > x_1$ the voter always votes for A, and for $x_2 \leq x \leq x_1$ the voter uses a state-dependent strategy. Because of the symmetric setting relative to $x = 0$, it must be that the cutpoints are symmetric: $x_1 + x_2 = 0$, implying that $\Pr(piv \mid s, \overline{\pi})$ as calculated above is constant for all s. Then, the probability distribution over states conditional on being pivotal, $\beta(s \mid piv, \overline{\pi})$, is also constant, and hence, the probability distribution over states conditional on being pivotal and observing signal σ is reduced to

$$\begin{aligned}\beta(s \mid piv, \overline{\pi}, \sigma) &= \frac{\beta(s \mid piv, \overline{\pi})\, p(\sigma \mid s)}{\int_0^1 \beta(w \mid piv, \overline{\pi})\, p(\sigma \mid w) dw} \\ &= \frac{p(\sigma \mid s)}{\int_0^1 p(\sigma \mid w) dw}.\end{aligned}$$

Because

$$\begin{aligned}\beta(s \mid piv, \overline{\pi}, 1) &= \begin{cases} 1-\alpha & \text{if } s < 1/2 \\ \alpha & \text{if } s > 1/2 \end{cases} \\ \beta(s \mid piv, \overline{\pi}, 2) &= \begin{cases} \alpha & \text{if } s < 1/2 \\ 1-\alpha & \text{if } s > 1/2 \end{cases},\end{aligned}$$

we have

$$E[s \mid piv, \overline{\pi}, \sigma] = \frac{\int_0^1 \beta(s \mid piv, \overline{\pi}, \sigma)\, sds}{\int_0^1 sds} = \begin{cases} \frac{1}{4}(1+2\alpha) & \text{if } \sigma = 1 \\ \frac{1}{4}(3-2\alpha) & \text{if } \sigma = 2 \end{cases}.$$

Solving

$$\begin{aligned}E[v(x_1, s) \mid piv, \overline{\pi}, 1] &= -1 + 2x_1 + 2E[s \mid piv, \overline{\pi}, 1] = 0 \\ E[v(x_2, s) \mid piv, \overline{\pi}, 2] &= -1 + 2x_2 + 2E[s \mid piv, \overline{\pi}, 2] = 0\end{aligned}$$

respectively, we obtain the two cutpoints:

$$x_1 = \frac{1}{2} - \alpha \qquad \text{and} \qquad x_2 = \alpha - \frac{1}{2}.$$

Plugging them into (14) yields

$$\left| t(s, \overline{\pi}^n) - \frac{1}{2} \right| = \frac{1}{4}(1-2\alpha)^2.$$

As t is constant, conditioning on being pivotal provides no information at the equilibrium.

Hence, the political expected utilities are computed as

$$\begin{aligned}E[v(Q, s, x)] &= \int_0^1 v(Q, s, x) ds = K - \frac{1}{12}\left(6x^2 + 12x + 19\right) \\ E[v(A, s, x)] &= \int_0^1 v(A, s, x) ds = K - \frac{1}{12}\left(6x^2 - 12x + 19\right).\end{aligned}$$

In the case of full information equivalence,

$$E\left[v(\gamma(s),s,x)\right]=\int_0^{1/2}v(Q,s,x)ds+\int_{1/2}^{1}v(A,s,x)ds=K-\frac{1}{12}\left(6x^2+13\right).$$

See Figure 1 for these political expected utilities.

Existence of stable equilibrium:

The part of the Theorem concerning national elections follows from Feddersen and Pesendorfer (1997) and the argument that a second stage equilibrium exists if a second stage N-equilibrium exists for each N. The part of the proof concerning local elections proceeds as follows. First, we find a candidate symmetric allocation. Then we prove that it is an equilibrium. Finally, we prove that it is stable.

We will *guess* that district 1 always votes unanimously for Q, district 3 always votes unanimously for A, and district 2 satisfies full information equivalence, so the state-dependent outcome in district 2 is the same as the outcome with no uncertainty. For notational purposes, define that outcome to be $\gamma^*(s)$.

$$\begin{aligned}U_1^x &= u_1+E\left[v(Q,s,x)\right]\\ U_2^x &= u_2+E\left[v(\gamma^*(s),s,x)\right]\\ U_3^x &= u_3+E\left[v(A,s,x)\right].\end{aligned}$$

Given an axisymmetric distribution (13), we define the potential equilibrium value $\lambda^*\in[0,\frac{1}{2}]$ to be the minimal value of λ such that the marginal consumer is indifferent between districts 1 and 2:[11]

$$u_1^*(\lambda)+\int_0^1 v(Q,s,F^{-1}(\lambda))dG(s)=u_2^*(\lambda)+\int_0^1 v(\gamma^*(s),s,F^{-1}(\lambda))dG(s).$$

Next, we show that this is in fact an equilibrium. To accomplish this, we must simply consider the decision of one individual at this allocation. No individual can unilaterally affect the economic allocation in any district, no matter their action.

To show that this is a second stage equilibrium in the political sector, we must show that it is a second stage N-equilibrium for N tending to infinity, where we draw agents randomly in their respective districts. For the polarized districts, this is easy, since the strategy profile that has everyone voting for example for Q in district 1

[11] This condition is familiar from models of local public goods.

means that nobody is ever pivotal. As argued in section 3, this Nash equilibrium strategy profile is the only one that is weakly undominated. The harder case is district 2, the middle district. Since nothing is learned from conditioning on being pivotal, players rely only on their private signals, as in Feddersen and Pesendorfer (1997, Example 2). Hence a second stage N-equilibrium for district 2 exists and satisfies full information equivalence. So a second stage equilibrium exists.

So now it is simply a matter of showing that the agents we have assigned to each district are at least as happy with the outcomes in that district as they would be in any other. By symmetry, if the argument works for one side of the distribution, it works for the other. So we focus on the left side. Notice that since $\Delta v(s,x)$ is increasing in s and x, $v(\gamma^*(s),s,x) - v(Q,s,x)$ is non-decreasing in x for each s.

$$\begin{aligned} & u_1^*(\lambda) - u_2^*(\lambda) \\ = & \int_0^1 [v(\gamma^*(s), s, F^{-1}(\lambda^*)) - v(Q, s, F^{-1}(\lambda^*))] dG(s). \end{aligned}$$

So for $x \geq F^{-1}(\lambda^*)$,

$$u_1^*(\lambda) - u_2^*(\lambda) \leq \int_0^1 [v(\gamma^*(s), s, x) - v(Q, s, x)] dG(s)$$

and thus $U_1^x \leq U_2^x$. A similar argument works for $x \leq F^{-1}(\lambda^*)$, implying $U_2^x \leq U_1^x$. A symmetric argument holds for the boundary between districts 2 and 3. Notice that this argument also holds if $\lambda^* = \frac{1}{2}$, noting that $\gamma^*(s)$ is replaced by A in the expressions above.

Finally, we show that this equilibrium is stable. We must evaluate equation (12) for movement between the districts. There are two ways to attempt this. We could evaluate it directly for this system. Alternatively, we could apply the result in Tabuchi and Zeng (2004), reducing our work load. Although the latter approach is easier, and is the one we will use, the issue is that their framework is set up for homogeneous consumers. Obviously, we have heterogeneous consumers/voters. So in order to apply the result, we formulate an artificial model that gives all consumers in a district the utility of the consumer at the margin or boundary for that district. Then this artificial model fits into the framework of Tabuchi and Zeng (2004).

Define

$$\begin{aligned}
\overline{U}_1(\lambda_1) &= u_1^*(\lambda_1) + \int_0^1 v(Q, s, F^{-1}(\lambda_1))dG(s) \\
\overline{U}_2(\lambda_2) &= u_2^*(\lambda_2) + \int_0^1 v(\gamma(s), s, F^{-1}(\frac{1}{2} - \frac{\lambda_2}{2}))dG(s) \\
&= u_2^*(\lambda_2) + \int_0^1 v(\gamma(s), s, F^{-1}(\frac{1}{2} + \frac{\lambda_2}{2}))dG(s) \\
\overline{U}_3(\lambda_3) &= u_3^*(\lambda_3) + \int_0^1 v(A, s, F^{-1}(1 - \lambda_3))dG(s),
\end{aligned} \tag{15}$$

where

$$\lambda_1 + \lambda_2 + \lambda_3 = 1. \tag{16}$$

The remainder of the proof consists of 3 steps. First, apply Tabuchi and Zeng (2004, Theorem 2) to the system defined by (15) and (16) to obtain generic existence of a stable equilibrium for this system. Actually, we must delve into the proof of Lemma C1 of that paper, p. 659, as it is not immediately clear that Theorem 2 of that paper applies to a standard New Economic Geography model, such as our economic sector. (Recall that only the economic sector matters for stability, due to our definition of stability.) As it turns out, the Theorem does apply. From the discussion just below equation (11), it follows that the indirect utility of a nonempty region is higher than that of an empty region. Moreover, from standard calculations for the New Economic Geography model, we know that the derivative of indirect utility as a function of λ cannot be zero. Thus, conditions (i) and (ii) of Tabuchi and Zeng (2004, pp. 658-659) are *always* met (not just generically), so a stable equilibrium exists.

Second, we claim that there is no asymmetric equilibrium of the system (15) and (16), so the stable equilibrium must be symmetric, namely $\lambda_1 = \lambda_3$. This holds because if the equilibrium is not symmetric, then there is a discontinuity in equilibrium utility between some pair of districts, implying that it is not an equilibrium, a contradiction.

Third, we claim that any stable equilibrium of the system (15) is also a stable equilibrium for the original system in the sense of equation (12). It is an equilibrium of the original system because the utilities of the consumers at the boundaries between districts are equated. Stability holds because the derivatives of the two systems, evaluated at equilibrium populations, are the same. This is easily verified for each part of the left hand side of inequality (12), where each part is evaluated at the boundary (in consumers/voters) between districts.

Characterization of Stable Equilibria:

Due to symmetry, the necessary condition for interior equilibrium is given by

$$\Delta U(\lambda) \equiv U_2^x - U_1^x|_{x=1-\lambda} = 0.$$

(i) Full agglomeration in district 2 ($\lambda = 0$)

Suppose all individuals are agglomerated at district 2. Plugging $\lambda = 0$ into (10), we have $w^* = \phi^{-1/\varepsilon}$, and thus,

$$\Delta U(0) = \left(\frac{L}{\varepsilon \overline{c}}\right)^{\frac{1}{\varepsilon-1}} \left(1 - \phi^{\frac{2\varepsilon-1}{\varepsilon(\varepsilon-1)}}\right) - \frac{1}{2}.$$

Full agglomeration at district 2 is a stable equilibrium if and only if $\Delta U(0) \geq 0$. Solving $\Delta U(0) = 0$, we get the agglomeration sustain point:

$$\phi_A \equiv \left[1 - \frac{1}{2}\left(\frac{\varepsilon \overline{c}}{L}\right)^{\frac{1}{\varepsilon-1}}\right]^{\frac{\varepsilon(\varepsilon-1)}{2\varepsilon-1}} \in (0,1) \quad \text{if } 1 > \frac{1}{2}\left(\frac{\varepsilon \overline{c}}{L}\right)^{\frac{1}{\varepsilon-1}}.$$

Hence, full agglomeration emerges if the fixed labor requirement is sufficiently small relative to the mass of workers ($\overline{c} < \overline{c}' \equiv 2^{\varepsilon-1}\frac{L}{\varepsilon}$) and the transportation cost is large enough ($\phi \leq \phi_A$).

(ii) Stratified equilibrium with district 2 empty ($\lambda = 1/2$)

This is an equilibrium with a symmetric population distribution in districts 1 and 3. Substituting $\lambda = 1/2$ into (10) yields $w^* = \left(\frac{2\phi}{1+\phi}\right)^{\frac{1}{\varepsilon}}$. Stratified equilibrium with district 2 empty exists if

$$\Delta U(1/2) = \left(\frac{L}{\varepsilon \overline{c}}\right)^{\frac{1}{\varepsilon-1}} \left[\phi^{\frac{2\varepsilon-1}{\varepsilon(\varepsilon-1)}} \left(\frac{2}{1+\phi}\right)^{\frac{1}{\varepsilon}} - \left(\frac{1+\phi}{2}\right)^{\frac{1}{\varepsilon-1}}\right] + \frac{1}{2} \leq 0.$$

Solving for $\frac{L}{\overline{c}}$ yields

$$\frac{L}{\overline{c}} \geq h_1(\phi, \varepsilon), \tag{17}$$

where

$$h_1(\phi, \varepsilon) = \frac{\varepsilon}{2^{\varepsilon-1}\left[\left(\frac{1+\phi}{2}\right)^{\frac{1}{\varepsilon-1}} - \phi^{\frac{2\varepsilon-1}{\varepsilon(\varepsilon-1)}}\left(\frac{2}{1+\phi}\right)^{\frac{1}{\varepsilon}}\right]^{\varepsilon-1}}.$$

This equilibrium is stable if

$$\frac{d}{d\lambda}\left(U_3^\lambda - U_1^\lambda\right)\bigg|_{\lambda=1/2} < 0.$$

Solving for $\frac{L}{\bar{c}}$ by using (11) for this specific symmetric distribution yields

$$\frac{L}{\bar{c}} < h_2(\phi, \varepsilon), \tag{18}$$

where

$$h_2(\phi, \varepsilon) = \frac{2\varepsilon}{1+\phi}\left[\frac{(\varepsilon-1)(2\varepsilon+\phi-1)}{(2\varepsilon-1)(1-\phi)}\right]^{\varepsilon-1}.$$

Therefore, there exists a stable stratified equilibrium with district 2 empty if

$$h_1(\phi, \varepsilon) \leq \frac{L}{\bar{c}} < h_2(\phi, \varepsilon). \tag{19}$$

It can be shown that

$$\lim_{\varepsilon\to\infty,\ \phi\to 0} h_1(\phi, \varepsilon) < 0 \qquad \text{and} \qquad \lim_{\varepsilon\to\infty,\ \phi\to 0} h_2(\phi, \varepsilon) = \infty.$$

By continuity, (19) holds for any given L and $\bar{c}$ if ε is sufficiently large and ϕ is sufficiently small. Hence, the stratified equilibrium emerges and is stable if the goods are complementary ($\varepsilon \geq \varepsilon'$) and the transportation cost is large enough ($\phi \leq \phi_B$).

(iii) Partial agglomeration ($\lambda \in (0, 1/2)$)

When ϕ is sufficiently large, full agglomeration in district 2 is not an equilibrium because $\lim_{\phi\to 1}\Delta U(0) = -1/2 < 0$. Similarly, full agglomeration in district 1 or 3 is not an equilibrium.

When ϕ is sufficiently large, the stratified equilibrium with district 2 empty does not exist because $\lim_{\phi\to 1}\Delta U(1/2) = 1/2 > 0$. Similarly, there is no equilibrium with district 1 or 3 empty.

Thus, any corner solution is not an equilibrium. However, there exists at least one stable equilibrium, which should be interior $\lambda^* \in (0, 1/2)$ for sufficiently large ϕ. ■

Figure 1: Political expected utilities with K=10

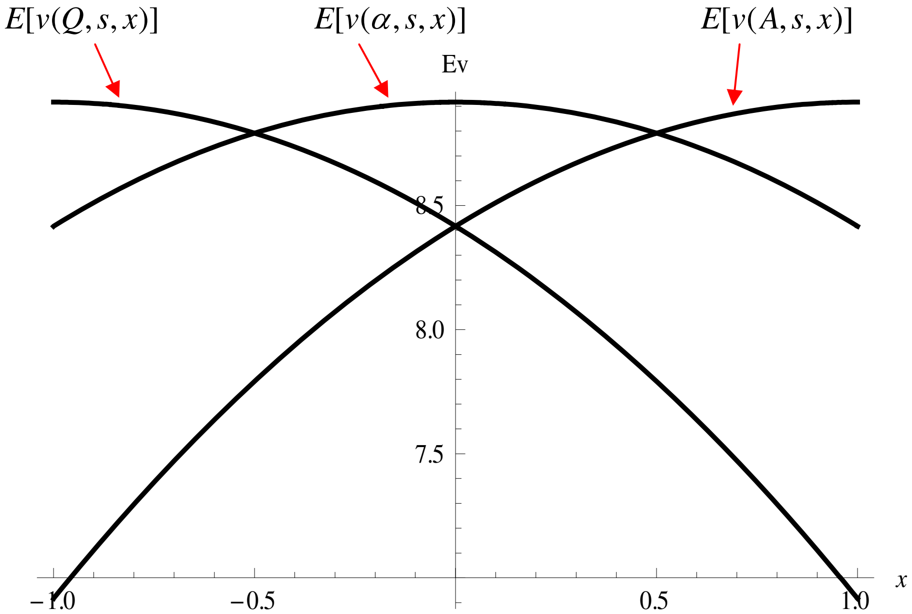

Figure 2: Equilibrium distributions when $\varepsilon = 5$ and $L/\bar{c} = 100$

Dotted arrow is L_1, solid arrow is L_2 and dashed arrow is L_3