A Theory of Intelligence and Total Factor Productivity: Value Added Reflects the Fruits of Fluid Intelligence

Harashima, Taiji

Kanazawa Seiryo University

7 December 2012

A Theory of Intelligence and Total Factor Productivity: Value Added Reflects the Fruits of Fluid Intelligence

Taiji Harashima*

December 2012

Abstract

In this paper, a theory of total factor productivity (TFP) that incorporates a model of intelligence is formulated and described. In particular, the fluid intelligence of ordinary workers is emphasized as an important element in TFP because such workers have the intelligence to innovate, even though their innovations are minor. Nevertheless, these innovations are essential for production because they solve many small but unexpected problems that ordinary workers must address. The TFP model is based on item response theory, which is widely used in psychology and psychometrics. TFP is assumed to be an increasing function of ordinary workers' fluid intelligence, without which production is virtually impossible. Therefore, the model suggests that TFP is derived from the fruits of human intelligence.

JEL Classification code: D24, O15, O20, O31, O47
Keywords: Total factor productivity; Intelligence; Innovation; Item response theory; Experience curve effect

---

*Correspondence: Taiji HARASHIMA, Kanazawa Seiryo University, 10-1 Goshomachi-Ushi, Kanazawa-shi, Ishikawa, 920-8620, Japan.
Email: harashim@seiryo-u.ac.jp or t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

Estimates of total factor productivity (TFP) vary substantially among countries, particularly those of developed and developing countries. Neo-classical Ramsey growth models naturally predict that these currently diverse estimates of TFP will eventually converge. On the other hand, many endogenous growth models do not support the convergence hypothesis (e.g., Romer, 1986, 1987), because endogenized knowledge accumulation significantly influences growth trajectories and heterogeneous knowledge-acquisition processes (e.g., human capital accumulation) among economies do not lead to convergence of per capita GDP. However, Prescott (1998) has shown that arguments based on human capital are unconvincing. The conclusions of empirical studies are mixed and inconclusive (e.g., Abramovitz, 1986; Baumol, 1986; Barro, 1991; Mankiw et al., 1992; Bernard and Durlauf, 1995; Michelacci and Zaffaroni, 2000; Cheung and Garcia-Pascual, 2004). These mixed and inconclusive conclusions suggest that the question of why TFPs are diverse cannot be resolved without uncovering the mechanisms of TFP. Prescott (1998) has also concluded that a theory of TFP is needed to answer this question.

In this paper, a theory of TFP that incorporates a model of intelligence is formulated and presented. In particular, the intelligence of ordinary workers is emphasized as an important element in TFP. This idea is not a new. Arrow's (1962) theory of learning-by-doing argues that productivity is improved by workers regularly repeating the same type of action. The concept of learning-by-doing has been applied to many fields in economics (e.g., Sheshinski, 1967; Hall and Howell, 1985; Romer, 1986; Adler and Clark, 1991; Nemet, 2006). In addition, the importance of human capital has been argued since Mincer (1958) and Becker (1962, 1964). Nevertheless, theories of learning-by-doing and human capital focus almost exclusively on workers acquiring pre-existing knowledge. The idea that ordinary workers also create something new (i.e., innovate) has drawn little attention; in fact, it has been neglected in economics. Innovations have usually been attributed to researchers and other highly educated or trained employees, and this bounded nature of innovation has been explicitly or implicitly assumed in most economic analyses.

However, ordinary workers can also innovate, even if most of their innovations are minor because as human beings, they have the ability to create just as do researchers and other highly educated or trained employees. Although a robot or machine can deal with preprogrammed tasks quite well if nothing unexpected occurs, it may immediately stop working properly if an unexpected problem occurs, even if the problem is relatively minor. Moreover, not only will the machine stop working properly, it will also be unable to solve the unexpected problem by itself. Only human beings can solve unexpected problems by innovating and creating something new.

Most innovations of ordinary workers are so minor that they do not become part of the accumulated knowledge of humanity. Nevertheless, these innovations are indispensable for TFP because small unexpected problems occur frequently in the production process. These problems must be addressed by people who have the intelligence to innovate. In addition, because ordinary workers do possess intelligence, it is rational for firms to fully exploit the opportunities that their employees' creative activities offer. Rational firms will therefore offer incentives for their workers to innovate, and this rational behavior will have various impacts on economic activities. This paper shows, based on the experience curve theory (e.g., Wright, 1936; BCG, 1972), that the creation of minor innovations by ordinary workers is an essential element of TFP.

In psychology and psychometrics, the importance of the difference between fluid and crystalized intelligences is emphasized. The intelligence required to solve unexpected problems is fluid intelligence, which is the capacity to solve novel problems by thinking logically. In psychology and psychometrics, the item response function is widely used as a model that describes the relation between intelligence and outcomes (e.g., Lord and Novick, 1968; van der Linden and Hambleton, 1997). The TFP model presented in this paper uses this function; in addition, more broadly it draws upon studies in psychology and psychometrics. The model shows that TFP is an increasing function of ordinary workers' fluid intelligence. It also shows that without ordinary workers' fluid intelligence, production is virtually impossible. Therefore, the model implies that TFP is derived from the fruits of human intelligence.

The paper is organized as follows. In Section 2, the nature of workers' innovations is examined and ordinary workers are shown to be capable of minor innovations. In addition, a production function that incorporates workers' innovations by reflecting the experience curve effect is induced. In Section 3, the nature of the intelligence they need to solve unexpected problems is examined on the basis of theories in psychology and psychometrics. In Section 4, a model of TFP is constructed based on the production function shown in Section 2 and item response theory. In Section 5, the model is used to examine the role of fluid intelligence in TFP, and intelligence is shown to be the fundamental source of value added. Finally, concluding remarks are offered in Section 6.

2 WORKER'S INTELLIGENCE

To begin with, I examined how an unexpected problem is fixed through workers' innovations at dispersed production sites. Here, I use Harashima's (2009) TFP model that well describes the mechanism of innovation generation at dispersed production sites.

2.1 Innovations generated by ordinary workers

2.1.1 Non-accumulative innovation

2.1.1.1 Innovations need not be intrinsically accumulative

Innovations are usually considered to be intrinsically accumulative, and TFP reflects the total sum of innovations that have been created and accumulated in the long history of human beings. However, accumulativeness is not a necessary condition for innovation because, as discussed in the introduction, its core meaning is the act of introducing something new or the thing itself that has been newly introduced. Luecke and Katz (2003) argue that innovation is generally understood as the introduction of a new thing or method and the embodiment, combination, or synthesis of knowledge in original, relevant, valued new products, processes, or services. The essence of innovation is therefore not accumulativeness but newness.

Nevertheless, non-accumulative innovations have drawn little or no attention in economics because innovations that are not accumulated have been regarded as being without value from an economic point of view. Accumulated innovations are often thought of as knowledge or technology, and they are usually regarded as equivalent to TFP. An innovation that is not accumulated is not included as knowledge, technology, or TFP because these must be commonly accessible and non-accumulative innovations are not. From this perspective, non-accumulated innovations are considered to have no effect on production and therefore be meaningless. The neglect of non-accumulative innovation may also be partially attributed to the belief that innovations must be accumulated because they have the innate nature of spillover (i.e., transfer), which implies accumulation. If an innovation makes someone better off, rational people have incentive to obtain and utilize it; thus, the innovation spills over. To spill over, the innovation must be recorded and transferrable in advance, that is, accumulated as a common piece of knowledge or technology. Conversely, innovations must be accumulated if they are consistent with the incentives of rational people.

However, the above rationales do not necessarily hold, for the following reason. A non-accumulative innovation is without value to people who did not create it, and the above rationales are convincing if only those people are considered. There is, however, no *a priori* reason that a non-accumulative innovation is valueless to the person who created it because that person can utilize it personally for production even if others cannot. Therefore, even if an innovation is not accumulated and does not become common knowledge, it still can contribute to production. A non-accumulative innovation may even be an important production element for the person who created it. In addition, if the costs to acquire an innovation created by other persons are higher than its benefits, the innovation will not spill over. Therefore, the concept that some innovations do not spill over and are not accumulated is not inconsistent with rational people's incentives for using innovations. Clearly the accumulativeness of innovation is not a simple issue and requires more careful consideration.

2.1.1.2 Innovations that are not accumulated

Innovations will be used personally even if they are not recognized and recorded. In addition, some innovations may be deliberately kept personal. Hence, an innovation will not be accumulated if nobody is aware of the innovation's novelty, nobody records or reports the innovation, or the person who created the innovation keeps it secret. The above conditions will be satisfied in the following situations. An innovation will not be recognized or recorded if the innovation is minor or if the innovation can be applied only to an unrepeatable incident. In addition, an incentive to keep an innovation secret will be strong if the person who creates the innovation cannot gain enough benefits by making it public. Thus an innovation will not be recorded if the costs of making the innovation public are higher than its expected benefits.

2.1.1.2.1 Minor innovations

A person who creates an innovation may be unaware of having created it if its contribution to improving productivity is minor. The person may also notice the increased productivity but not seek to identify the reason for the improvement because such an investigation may seem too costly. Finally, even if the mechanism of the innovation is noticed and specified, the person who created it may not record it if it is deemed to be minor. It is therefore clearly possible that minor innovations are not noticed, identified, or recorded.

Even if an innovation is unnoticed or unrecorded, it still can be used for production by the person who created it, whether consciously or unconsciously, while the person continues doing that job. Unnoticed innovations will vanish when that person quits doing the job. If innovations are recognized but unrecorded, it is possible that at least some of them could be handed down to other workers. Because these are isolated and "personal" occurrences within a small closed group, they would not constitute a piece of accumulated knowledge common to all human beings.

2.1.1.2.2 Innovations for unrepeatable incidents

Even if an innovation is not minor, it will not be recorded if it can be applied only to an unrepeatable situation. For example, a negotiation between a seller and a buyer will be basically unrepeatable. Similar negotiations may occur, but an identical one will not. There are also incidents that occur, for example, only on a specific machine installed at a particular location; these incidents are never reproduced at other machines installed at other locations. This type of isolated and non-reproducible incident can be interpreted as unrepeatable in a broad sense. In addition to these spatially unrepeatable incidents, each machine has unique characteristics even if it was designed to be exactly the same as other machines. There will not be sufficient incentive to record or widely disseminate an innovation that can be applied only to an unrepeatable situation or to a machine with unique characteristics.

2.1.1.2.3 Costs of disseminating and acquiring information

There will be a strong incentive to keep an innovation secret if the innovation spills over freely without compensation to the innovator. However, even if a patent could be taken out to obtain appropriate compensation, the incentive to keep the innovation secret will still be strong if the cost of dissemination exceeds expected revenues. If an innovation was created for a minor incident, benefits gained from the innovation will usually be smaller than the cost of dissemination, and the incentive to keep the innovation personal will be strong. The costs for making an innovation public can be classified into two types: dissemination costs and acquisition costs. Dissemination costs are the costs paid to make an innovation public and to disseminate it, for example, patent application fees, advertising costs, marketing costs, and similar expenditures. Acquisition costs are the costs paid to acquire and utilize an innovation that some other person created, for example, search costs, transportation costs, and training costs. Patent royalties are included in acquisition costs only if the market value of the innovation exceeds the royalty plus other acquisition costs. Generally, dissemination costs are likely to be larger than acquisition costs, excluding patent royalties.

Let δ indicate dissemination costs, η indicate acquisition costs, and π indicate the market value of an innovation. As argued above, in general $\delta > \eta$ if $\delta > \pi$; therefore innovations are categorized into the following three ranges depending on the relative value of π compared with those of δ and η (see Figure 1):

Range I: $\pi \geq \eta \geq \delta$ or $\pi \geq \delta \geq \eta$; patented accumulative innovations
Range II: $\delta > \pi \geq \eta$; uncompensated spillovers of accumulative innovations
Range III: $\delta > \eta > \pi$; non-accumulative innovations

If the market value of an innovation exceeds its dissemination and acquisition costs, the patent of the innovation will be sold and disseminated widely (Range I). If the market value of an innovation does not exceed its dissemination costs but exceeds its acquisition costs, the innovation will disseminate widely without compensation (i.e., uncompensated spillover; Range II). If the market value of an innovation does not exceed either cost, the innovation will not be disseminated and will be kept personal (i.e., non-accumulative innovation; Range III). Because it is highly likely that the number of minor innovations is far larger than the number of innovations that have high market values, the shape of innovation distribution slopes downward and to the right (Figure 1), and the distribution will have a long tail. This shape can be approximated simply by an exponential or Pareto distribution, but it is not necessary to assume a specific functional form of distribution. The important point is not the specific functional form of the distribution but its properties—if $\delta > \eta > \pi$, then non-accumulative innovations exist and there will be far more of them than of accumulative innovations.

2.1.2 The origin of non-accumulative innovation

It seems clear that non-accumulative innovations exist, but who creates them? Researchers can certainly create them, but so can ordinary workers. Usually, workers are implicitly assumed to do only what they are ordered to do and nothing else. Workers in this sense can be substituted for capital. If the cost of using capital is lower than that of using workers, capital inputs will be chosen rather than labor inputs. Generally, such robot-like workers have been assumed as the labor input in typical production functions. Of course, workers are not robots. They are human beings that are fundamentally different from machines—only humans can fix unexpected problems by creating innovations.

2.1.2.1 Unexpected problems require innovation

Actions taken to deal with expected incidents are determined by calculating the solutions to optimization problems that are built based on models constructed in advance. These calculations can be implemented by machines given a specific objective function, structural equations, parameter values, and necessary environmental information. However, this is not true if actions taken to deal with unexpected problems are required, because the models constructed in advance are guaranteed to be useful only for expected incidents, and they are not necessarily guaranteed to be applicable to unexpected incidents. When an unexpected problem occurs, workers in charge of the production first have to grasp the situation and then prioritize their actions. During these actions, the workers conduct two types of important intellectual activities: (1) discover unknown mechanisms that prevail in the surrounding environment and (2) invent new ways to manage the environment. That the problem is unexpected indicates that correct mechanisms for this particular situation are not known and need to be discovered, and on the basis of the newly discovered mechanisms, the structural equations and parameters in the model used for the plan of action should be revised. The revised model may indicate that there is no solution to resume efficient production, and new ways of managing the environment should be invented. Discovery and invention commonly involve the creation of something new, that is, innovation.

Machines deal with programmed tasks quite well, often much better than human beings. Conversely, machines cannot deal with non-programmed tasks. The performance of machines declines and often they stop working if unexpected problems occur because the machines do not have a program to deal with unexpected problems. When encountering unexpected problems, machines will immediately reach a dead end. They cannot solve unexpected problems by simply applying their pre-programmed optimization algorithms, and they cannot rewrite these algorithms to make them applicable to unexpected incidents. The revision or creation of models in the face of unexpected incidents can be implemented only by human beings.

2.1.2.2 Workers' innovations to fix unexpected minor problems

Is it either necessary or expected to utilize workers' innovations for production? If workers are assumed to be robot-like beings, their abilities to solve unexpected problems will not be considered as part of production. However, it would be irrational for firms not to utilize workers' innovative abilities if the firms know that workers possess these abilities. An ordinary worker's ability to solve unexpected problems may be lower than that of educated and trained researchers, but the abilities of the former should be utilized fully for a firm to be rational. If anything, the workers' abilities to fix unexpected problems appear indispensable in production processes because many minor but unforeseeable incidents actually occur. It would be quite inefficient if a team of specialized highly educated and trained employees dealt with all unexpected incidents, no matter how minor, and workers had to wait for the team to arrive at the locations where a minor unexpected incident happened. If, however, an unexpected but minor problem is fixed by a worker at the location where the problem occurred, production can proceed more efficiently and smoothly. The well-known "Kaizen" method in Japanese manufacturing companies may be a way to more completely exploit such opportunities (e.g., Lee et al., 1999). Besides innovations by suppliers, "user innovation" by consumers and end users has drawn attention recently (e.g., Baldwin et al., 2006). It is quite reasonable and rational for firms to fully exploit any opportunity to improve productivity whether its source is an innovation created by a researcher, ordinary worker, or user.

Finally, a worker's ability to fix unexpected problems may seem to be part of the set of the worker's learned skills or techniques, but that ability is fundamentally different from learned skills or techniques because learning skills and techniques and creating skills and techniques are completely different activities.

2.1.3 Imperfections make workers' innovations indispensable

Although it is rational for employers to fully exploit workers' innovations, in this section, I explain why workers' innovations are truly an indispensable element in production.

2.1.3.1 Imperfect accumulated innovations

The current state of accumulated innovations is far from perfect, and, moreover, it always will be. Human beings will never know everything about the universe. Although we may be able to fully utilize known information, we still face many unexpected problems because the knowledge and technology we currently possess is imperfect. If accumulated innovations were perfect, machines that embody them would always work well in any situation. However, the accumulated innovations are not perfect, and thus machines malfunction occasionally or face other unexpected incidents. As stated previously, it is very efficient if workers' innovations are utilized to fix these minor but unexpected troubles. Imperfection of accumulated innovations therefore necessitates workers' innovations.

2.1.3.2 Incomplete information caused by the division of labor

Labor input has the property of decreasing marginal product, which is usually explained by congestion or redundancy. However, this explanation is not necessarily convincing. The inefficiency caused by congestion or redundancy can be removed by division of labor. If labor is sufficiently divided, there will be no congestion or redundancy, and the labor input will not exhibit decreasing marginal product. This suggests that division of labor cannot remove all inefficiencies with regard to labor input. With division of labor, each worker experiences only a fraction of the whole production process. These divided and isolated workers can access only a fraction of information on the whole production process. It is also difficult for a worker to know information that many other workers at different production sites accessed. Because all of the labor inputs are correlated owing to division of labor, this feature of fragmented information is especially problematic when workers engage in intellectual activities. Correlation of the entire labor input indicates that all pieces of information on the whole production process need to be completely known to each worker to enable correct decision making. However, only a portion of the information on the whole production process is available to each worker; that is, each individual worker has incomplete information. When an unexpected problem occurs, workers with fragmented and incomplete information will make different, usually worse, decisions than those with complete information. As a result, overall productivity decreases.

For example, a CEO of a large company may know the overall plan of production but not the local and minor individual incidents that happen at each production site each day. In contrast, each worker at each production site may know little of the overall plan but a great deal about local and minor individual incidents that occur for each specific task each worker engages in at each production site. To be most efficient, even if many unexpected incidents happen, all of the workers and the CEO need to know all of the information on the entire process because all of the labor inputs are correlated owing to division of labor. However, it is nearly impossible for each worker to access all of the experiences of every other worker. Division of labor therefore leads to information fragmentation and obstructs any person from knowing all the information about the entire production process.

Each worker therefore must use incomplete information when encountering unexpected problems. Conjecturing the full detailed structure of the whole production process is an intellectual activity to discover unknown mechanisms. If a worker can discover more correct mechanisms even in the absence of complete information, the inefficiency is mitigated. Because inefficiency is inevitably generated by incomplete information resulting from division of labor, workers' innovations are inevitably needed to mitigate inefficiency. However, completely mitigating the inefficiency will be impossible, and decisions based on less information will deviate from those made with full information. Sometimes actions that are relatively less urgent or important will be given priority, and efficiency will decline. As the division of labor increases, workers are less able to correctly estimate the full structure of the whole production process and less able to correctly prioritize actions to solve unexpected problems.

Division of labor cannot simultaneously solve inefficiency caused by congestion or redundancy and that caused by fragmented and incomplete information. Although a greater division of labor removes the former, it generates the latter. Inefficiency resulting from congestion and redundancy is probably much more serious than that caused by information fragmentation, and labor is divided almost completely despite the fact that information fragmentation harms productivity.

2.1.3.3 Indispensable and economically important workers' innovation

Even if workers can innovate to fix unexpected minor troubles, the question remains whether these innovations are important economically. In general, most non-accumulative innovations are minor, which suggests that they may not be economically important. However, as discussed in Section 2.1.1, there will be far more minor innovations than major innovations. There are also usually far more ordinary workers than researchers and other highly trained or educated employees. In addition, the distributions of innovations for researchers and other highly trained employees and for ordinary workers are certainly different. Ordinary workers are likely to have a limited contribution to accumulative innovations (i.e., Ranges I and II in Figure 1) as compared to that of researchers and other highly trained employees, but the former will have a much larger contribution to non-accumulative innovations (Range III). As previously discussed, non-accumulative innovations are indispensable for production at each production site because of imperfect accumulative innovations and fragmented and incomplete information. Without worker-created non-accumulative innovations, the efficiency of production will decline considerably. This indispensability indicates that workers' innovations are economically important. The economic importance of workers' innovations is further examined in Section 2.3.

2.2 The experience curve effect

2.2.1 The experience curve effect and workers' innovations

Workers' innovations are indispensable, but how are they created? The experience curve effect gives a clue to this mechanism.

2.2.1.1 The theory of the experience curve effect

The experience curve effect states that the more often a task is performed, the lower the cost of doing it. Workers who perform repetitive tasks exhibit an improvement in performance as the task is repeated a number of times. The primary idea of the experience curve effect (the "learning curve effect" in earlier literature) dates back to Wright (1936), Hirsch (1952), Alchian (1963), and Rapping (1965). The importance of the learning curve effect was emphasized by Boston Consulting Group (BCG) in the late 1960s and early 1970s (e.g., BCG, 1972). The experience (or learning) curve effect has been applied in many fields, including business management, strategy, and organization studies (e.g., on airplanes, Wright, 1936; Asher, 1956; Alchian, 1963; Womer and Patterson, 1983; in shipbuilding, Searle and Goody, 1945; on machine tools, Hirsch, 1952; in metal products, Dudley, 1972; in nuclear power plants, Zimmerman, 1982; Joskow and Rozanski, 1979; in chemical products, Lieberman, 1984; Argote et al., 1990; in food services, Reis, 1991). More recently, it has also been applied to technology and policy analysis, particularly energy technologies (e.g., Yelle 1979; Dutton and Thomas, 1984; Hall and Howell, 1985; Lieberman, 1987; Argote and Epple, 1990; Criqui et al., 2000; McDonald and Schrattenholzer, 2001; van der Zwaan and Rablc, 2003, 2004; Miketa and Schrattenholzer, 2004; Papineau, 2006). An empirical problem of the experience curve effect is to distinguish dynamic learning effects from static economies of scale. After surveying empirical studies, Lieberman (1984) concluded that, in general, static scale economies are statistically significant but small in magnitude relative to learning-based economies (see also Preston and Keachie, 1964; Stobaugh and Townsend, 1975; Sultan, 1976; Hollander, 2003).

The experience curve effect is usually expressed by the following functional form:

$$C_N = C_1 N^{-(1-\alpha)} \quad (1)$$

where C_1 is the cost of the first unit of output of a task, C_N is the cost of the nth unit of output, N is the cumulative amount of output and interpreted as experience of a worker engaging in the task, and α is a constant parameter ($0 < \alpha < 1$). $\frac{C_{2N}}{C_N}$ and $1 - \alpha$ are often called the progress ratio and learning rate, respectively. This log-linear functional form is most commonly used probably because of its simplicity and good fit to data. Empirical studies have shown that α is usually between 0.6 and 0.9. Studies by BCG in the 1970s showed that experience curve effects for various industries range from 10–25% cost reductions for every doubling of output (i.e., $0.58 \le \alpha \le 0.85$) (e.g., BCG, 1972). Dutton and Thomas (1984) present the distribution of progress ratios obtained from a sample of 108 manufacturing firms. The ratios mostly range from 0.7 to 0.9 (i.e., $0.48 \le \alpha \le 0.85$) and average 0.82 (i.e., $\alpha = 0.71$). OECD/IEA (2000) argues that industry-level progress ratios have a similar distribution as the firm-level ones shown in Dutton and Thomas (1984; see also, e.g., Hirsch, 1956; Womer and Patterson, 1983; Womer, 1984; Ayres and Martinas, 1992; Williams and Terzian, 1993).

The magnitude of α (or equivalently the progress ratio or learning rate) may be affected by various factors (e.g., Hirsch, 1956; Adler and Clark, 1991; Pisano et al., 2001; Argote et al., 2003; Sorenson, 2003; Wiersma, 2007). Nevertheless, the average α is usually observed to be almost 0.7 (i.e., a progress ratio of 0.8 and a learning rate of 0.3) as shown in BCG (1972), Dutton and Thomas (1984), and OECD/IEA (2000). It therefore seems reasonable to assume that α is 0.7 on average.

2.2.1.2 Information conveyed by experience

An important element that an experience conveys is information. By accumulating experiences of doing a task, a worker increases the amount of information known about the task and makes it more complete. In this sense, *N,* which indicates experience in equation (1), reflects the current amount of information a worker possesses about a task. Accumulated experiences will improve efficiency in implementing a task because the amount of information on the task increases. However, if other factors remain the same, the magnitude of improvement will diminish as N accumulates because the information on the task will approach saturation.

Let I be a set of the currently available maximum information on a task. Engaging in the task in a unit of period provides a subset of I to a worker. Engaging in more units of period (i.e., accumulating experience N) makes the information on the task the worker currently possesses ($\tilde{I}$) approach I (i.e., the difference between $\tilde{I}$ and I diminishes). A part of the subset of I the worker acquires in a unit of period will overlap the part of the subset of I the worker acquires in the next period. With more complete information, accordingly, efficiency will improve. Because $\tilde{I} \rightarrow I$ as $N \rightarrow \infty$, then the magnitude of improvement will asymptotically decrease as N increases. Nevertheless, this asymptotical decrease may not be a simple process. Some piece of information may be easily obtainable and some other piece may not be, and some portion of information may have a relatively large impact on efficiency and other portions have small effects. The functional form that describes the asymptotical decrease of the magnitude of improvement will depend on interaction between these effects. The log-linear functional form $C_N = C_1 N^{-(1-\alpha)}$ fits empirical data well and is simple, and thus it has been used mostly for the experience curve effect.

2.2.1.3 Extending the concept of the experience curve effect

Because the essence of experience is that it conveys information, the experience curve effect can be extended to a wide variety of tasks. The tasks need not be limited to a worker's repeated actions, that is, tasks whose experiences are divided by periods. For example, consider that a human activity can be divided into many experiences, each of which is obtained by different workers. Each experience conveys a subset of information, and a part of the subset overlaps with subsets regarding other experiences. The experience curve effect will be applicable to this kind of task by interpreting *N* as a subset all worker experiences, so a task in a period whose experiences are divided by workers will be also applicable to the experience curve effect in the same way that a task performed by a worker whose experiences are divided by periods is. Extending this logic suggests that tasks applied to the experience curve effect should not be limited to the ones whose experiences are divided only by periods or workers. As long as the task is a human intellectual activity and its experiences are divided by factors other than periods or workers, the task will also be applicable to the experience curve effect because it has the common nature that each divided experience conveys only a subset of all the information that affects the worker's intellectual activities. Nevertheless, the concept of the experience curve effect should not be expanded infinitely. It can be applied only to the tasks of workers, the performances of which differ depending on the amount of information the worker has.

2.2.2 The experience curve effect in the technology input

2.2.2.1 Dispersively embodied accumulative innovation in capital

To understand the mechanism for the creation of non-accumulative innovations, it is first necessary to examine how workers are in contact with capital inputs and the accumulative innovations embodied in them at each production site. Any single machine or tool cannot embody all the accumulated innovations in human history. Only a portion of accumulated innovations are embodied in each machine or capital input. Furthermore, different types of machines or tools embody different kinds of accumulative innovations. This relationship between accumulative innovation and capital suggests that accumulative innovations are varied, divisible, and dispersed among capital inputs. If there are negative effects of congestion and redundancy in the embodiment of accumulative innovation in capital, this division of accumulative innovation improves productivity. Embodying more types of accumulative innovations in a machine or tool may make it a more general purpose machine or tool. In implementing a specific task, however, a general purpose machine or tool will be less useful and efficient than a specialized one because congestion and redundancy of the accumulative innovations will occur and reduce efficiency.

Suppose that there is only one economy in the world and that all workers in the economy are identical. Let *Y*(*A*, *K*, *L*) be a production function where *Y* is production, *A* is technology (accumulated innovations), *K* is capital input, and *L* is labor input. *A* can be interpreted as indicating the total amount of technology and, at the same time, the total number of varieties of technology in the economy. Let also τA be the portion of *A* embodied on average in a unit of capital where τ is a positive parameter. To incorporate the idea that the division of *A* mitigates congestion and redundancy and improves efficiency for production, the following assumption is introduced:

$$\frac{\partial Y(\tau, A, K, L)}{\partial \tau} < 0 \quad , \tag{2}$$

which indicates that the smaller the value of τ (i.e., the smaller the magnitude of congestion and redundancy), the larger the production *Y*.

On the other hand, if τ is too small, there is the possibility that a piece of *A* is not embodied in any part of *K*. Without embodying any portion of *A*, *K* is no longer a machine or tool but merely a pile of useless materials. Avoiding this abnormal situation requires a condition that any *K* must embody at least some portion of *A*. If $\tau < K^{-1}$, then the total amount of *A* used in the economy is $\tau AK < A$, and thus some portion of *A* is not embodied in any *K*, which indicates that the condition $K^{-1} \leq \tau$ is necessary for avoiding the abnormal situation and that $\tau = K^{-1}$ is the threshold value. As the rationale for the condition $K^{-1} \leq \tau$ with the threshold value $\tau = K^{-1}$, it is assumed here that the total differential $dY(\tau, A, K, L)$ with respect to A and τ is positive such that

$$dY(\tau, A, K, L) = \frac{\partial Y(\tau, A, K, L)}{\partial A} dA + \frac{\partial Y(\tau, A, K, L)}{\partial \tau} d\tau > 0 \tag{3}$$

for $\tau < K^{-1}$, and thus

$$\frac{dY(\tau, A, K, L)}{d\tau} = \frac{\partial Y(\tau, A, K, L)}{\partial A} \frac{dA}{d\tau} + \frac{\partial Y(\tau, A, K, L)}{\partial \tau} > 0 \tag{4}$$

for $\tau < K^{-1}$, which means that if τ is smaller than the threshold value K^{-1}, then the reverse effect of the amount of A on production is much larger than the effect of the division of A on production. If $K^{-1} \leq \tau$, then any portion of A is embodied in some K, and thereby $\frac{dA}{d\tau} = 0$ and $\frac{dY(\tau, A, K, L)}{d\tau} = \frac{\partial Y(\tau, A, K, L)}{\partial \tau} < 0$.

Combining the characteristics of τ shown in inequalities (2) and (4) indicates that the optimal value of τ is K^{-1}. As a result of the rational behavior of firms, the optimal dispersion of accumulative innovation in capital is obtained when $\tau = K^{-1}$, and thus the portion of A embodied on average in a unit of capital is always

$$K^{-1}A$$

in the economy. A worker faces $K^{-1}A$ units of accumulative innovations at any time when the worker uses a unit of capital.[1] Because A indicates the total number of varieties of technology as well as the total amount of technology, dispersively embodied A in K indicates that a worker faces K^{-1} of varieties of A when the worker uses a unit of capital.

2.2.2.2 Specialized or generalized machines or tools

Suppose that the amount of A is fixed; that is, no new variety of innovation is added. If K increases and A remains fixed, the proportion of A embodied in a unit of K becomes smaller because the proportion of A embodied in a unit of K is kept equal to $K^{-1}A$. A smaller $K^{-1}A$ means that machines or tools become more specialized because the purpose of a machine or tool embodying less A will be more limited. The types of machines or tools used will change even if A does not increase. If K increases in this case, machines and tools will become more specialized and vice versa. The variety and type of machines or tools, that is, how specialized or generalized they are, depend not only on A but also on K.

Note, however, that generalized does not necessarily mean advanced. On the contrary, general purpose machines or tools are more primitive, and conversely, special purpose ones are more advanced. To be general purpose, machines or tools must rely more on basic or core technologies, and many specialized functions will be downgraded.

[1] In this paper, it is assumed that there is only one economy in the world. However, actually there are many smaller economies and a small economy may utilize only a small portion of A; i.e., the size of economy will matter to the optimal value of τ if there are many economies of various sizes. The problem of the size of economy as well as the problem of aggregation is discussed more in detail in Harashima (2009).

2.2.2.3 Effective technology input

As argued in Section 2.2.1, the experience curve effect can be applied to a task as long as the task is an intellectual creative activity and the experiences can be divided by some factor. The experience curve effect is applicable to the activity of creating non-accumulative innovations to supplement imperfect accumulative innovations because (1) the activity is an intellectual creative activity and (2) the experiences can be divided by varieties of A in K a worker encounters. A worker encounters a portion of the accumulated innovations ($K^{-1}A$) when the worker uses a unit of capital. The portion of accumulated innovations conveys a subset of all the information on accumulated innovations and a part of the subset overlaps with those conveyed in other portions of accumulated innovations that other workers encounter.

A worker encounters a unique combination of varieties of accumulative innovations ($K^{-1}A$) per unit capital. Let N_A be a worker's average encounter frequency (i.e., the worker's experience) with each variety of accumulative innovations per unit capital in a period. As $K^{-1}A$ increases, the number of varieties per unit capital increases; thus, N_A will decrease because the probability of encountering each of the varieties in $K^{-1}A$ in a period decreases. The amount of $K^{-1}A$ therefore will be inversely proportional to a worker's experience on a variety per capital N_A such that

$$N_A = \beta_A \left(\frac{A}{K} \right)^{-1}$$

where β_A is a positive constant. Standardizing the worker's average encounter frequency β_A equal to unity, then

$$N_A = \left(\frac{A}{K} \right)^{-1} . \tag{5}$$

Let C_{A,N_A} be the amount of inefficiency resulting from imperfect technology (which is equivalent to imperfect accumulative innovations) embodied in capital when a worker utilizes a variety of accumulative innovations in $K^{-1}A$ in a period. C_{A,N_A} does indicates not the inefficiency initially generated by imperfect technology but the one remaining after being mitigated by workers' innovations. Costs increase proportionally to increases in inefficiency; thus, C_{A,N_A} also indicates costs. Conversely, C_{A,N_A}^{-1} can be interpreted as a productivity in supplementing imperfect technology by creating non-accumulative innovations when a worker utilizes a variety of accumulative innovations in $K^{-1}A$ in a period. The creation of non-accumulative innovations will increase as the frequency of a worker encountering a variety of accumulative innovations in $K^{-1}A$ increases (i.e., the productivity in supplementing imperfect technology by creating non-accumulative innovations will increase as the number of experiences increases). Hence, the inefficiency C_{A,N_A} will decrease as the encounter frequency increases. The experience curve effect indicates that inefficiency C_{A,N_A} declines (i.e., productivity C_{A,N_A}^{-1} increases) as a worker's average encounter frequency on a variety per unit capital (N_A) increases (i.e., $K^{1}A$ becomes smaller) such that

$$C_{A,N_A} = C_{A,1} N_A^{-(1-\alpha)} \quad , \tag{6}$$

where $C_{A,1}$ is the inefficiency when $N_A = 1$. Note that α is the constant parameter ($0 < \alpha < 1$)

used in equation (1).

In addition, the amount of technology input per unit capital will increase as C_{A,N_A}^{-1} increases (i.e., C_{A,N_A} decreases) because the inefficiency is mitigated by an increased amount of workers' innovations. Thus, the amount of technology input per unit capital when a worker uses a variety of accumulative innovations in $K^{-1}A$ will be directly proportional to C_{A,N_A}^{-1} (i.e., inversely proportional to C_{A,N_A}) such that

$$W_A\left(\frac{A}{K}\right)^{-1} = \frac{\gamma_A}{C_{A,N_A}} \quad , \tag{7}$$

where W_A is the amount of technology input per unit capital when a worker utilizes a unique combination of varieties of accumulative innovations in $K^{-1}A$, and γ_A is a positive constant (i.e., γ_A indicates the amount of technology input per unit capital when a worker utilizes a unique combination of varieties of accumulative innovations $K^{-1}A$ in a period when $C_{A,N_A} = 1$). Substituting equations (5) and (6) into equation (7) gives

$$W_A = \frac{\gamma_A}{C_{A,N_A}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}N_A^{-(1-\alpha)}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}\left(\frac{A}{K}\right)^{1-\alpha}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}}\left(\frac{A}{K}\right)^{\alpha} . \tag{8}$$

As discussed in Section 2.2.2.1, the amount of technology embodied in a unit capital is $K^{-1}A$. Because technology is imperfect, however, that level of technology input cannot be effectively realized. At the same time, the inefficiency resulting from the imperfections is mitigated by non-accumulative innovations created by ordinary workers even though it is not completely removed. Equation (8) indicates that the magnitude of mitigation depends on $K^{-1}A$, and that, with the mitigation, technology input per unit capital is effectively not equal to $K^{-1}A$ but directly proportionate to $W_A = \frac{\gamma_A}{C_{A,1}}\left(\frac{A}{K}\right)^{\alpha}$. By equation (8), therefore, the effective technology input per unit capital ($\tilde{A}$) is

$$\tilde{A} = \upsilon_A W_A = \omega_A\left(\frac{A}{K}\right)^{\alpha} \tag{9}$$

where υ_A and ω_A are positive constant parameters and $\omega_A = \frac{\upsilon_A \gamma_A}{C_{A,1}}$.

2.2.3 The experience curve effect in the labor input

The task of mitigating the inefficiency resulting from fragmented and incomplete information caused by the division of labor satisfies the condition for applying the experience curve effect (Section 2.2.1). As shown in Section 2.1.3, workers' innovations reduce this inefficiency. In addition, production processes are divided by workers as part of the division of labor. Each worker encounters only a portion of the whole production process, a portion of the process conveys only a portion of information on the whole production process, and the information overlaps partially with that on other processes that other workers encounter. Hence, the experience curve effect can be applied to this task. Because labor is divided fully at the global level, inefficiency mitigation activities are correlated at the global level.

Let N_L be the production processes a worker encounters (i.e., the experience of a worker); it indicates the proportion of all production processes in the economy (N), which is here normalized such that $N = 1$. A proportion of the production process conveys a subset of all the information on the production process, and a part of the subset overlaps with subsets of information on processes that other workers encounter. Remember, in this discussion, I am assuming that there is only one economy in the world and that all workers are identical. Thus, because the experience of a worker (N_L) is inversely proportionate to the number of workers, then

$$N_L = L^{-1}\beta_L$$

where L is the number of workers in the economy and β_L is a constant. $\beta_L(= N_L L)$ indicates the total of all production processes in the economy such that $\beta_L = N$. Because $N = 1$, then

$$N_L = L^{-1} \quad . \tag{10}$$

Let C_{L,N_L} be the magnitude of inefficiency in a worker's labor input caused by fragmented and incomplete information when each worker's experience is N_L. C_{L,N_L} indicates not the inefficiency initially generated by fragmented and incomplete information but the inefficiency that remains after mitigation by a worker's innovations. Costs will increase proportionally with increases in inefficiency, and thus C_{L,N_L} also indicates costs. C_{L,N_L}^{-1} can be interpreted as a productivity in a worker's labor input, which increases as the amount of mitigation by the worker's innovations increases.

C_{L,N_L} increases as the amount of individually available information (i.e., experience) increases. The increased amount of information enables a worker to discover more correct mechanisms of the production processes, and this discovery reduces the inefficiency in a worker's labor input. As mentioned previously, the experience curve effect can be applied to this inefficiency mitigation mechanism. The experience curve effect indicates that C_{L,N_L} declines as the experience of a worker (N_L) increases (i.e., the number of workers deceases) such that

$$C_{L,N_L} = C_{L,1} N_L^{-(1-\alpha)} \quad , \tag{11}$$

where $C_{L,1}$ is the inefficiency when $N_L = 1$ (i.e., $N_L = N$ and $L = 1$). Note again that α is the constant parameter ($0 < \alpha < 1$) used in equation (1).

In addition, because the amount of a worker's provision of labor input increases as productivity (C_{L,N_L}^{-1}) increases (i.e., C_{L,N_L} decreases), then the amount of a worker's provision of labor input ($L^{-1}W_L$) is directly proportional to C_{L,N_L}^{-1} (i.e., inversely proportional to C_{L,N_L}) such that

$$\frac{W_L}{L} = \frac{\gamma_L}{C_{L,N_L}} \quad , \tag{12}$$

where W_L is the total amount of workers' provision of labor input that is supplemented by worker's innovations to mitigate the inefficiency resulting from fragmented and incomplete information, and γ_L is a constant (i.e., γ_L indicates the output per worker in a period when $C_{L,N_L} = 1$). Substituting equations (10) and (11) into equation (12) gives

$$W_L = \frac{\gamma_L}{C_{L,N_L}} L = \frac{\gamma_L}{C_{L,1} N_L^{-(1-\alpha)}} L = \frac{\gamma_L}{C_{L,1} L^{1-\alpha}} L = \frac{\gamma_L}{C_{L,1}} L^{\alpha} . \tag{13}$$

The inefficiency caused by fragmented and incomplete information constrains the labor provision by workers. As division of labor is widened (i.e., as L increases), the labor provision by workers is more constrained. The inefficiency, however, is mitigated by innovations created by workers, but it cannot be completely removed by workers' innovations. Hence, the labor input that is effectively provided by workers is not simply proportional to L. Equation (13) indicates that, instead of L, the labor input effectively provided by workers is directly proportional to $W_L = \frac{\gamma_L}{C_{L,1}} L^{\alpha}$; thus, the effective labor input $\tilde{L}$ is

$$\tilde{L} = \upsilon_L W_L = \omega_L L^{\alpha} \quad , \tag{14}$$

where υ_L and ω_L are positive constant parameters and $\omega_L = \frac{\upsilon_L \gamma_L}{C_{L,1}}$.

2.2.4 The experience curve effect and the capital input

As with $\tilde{A}$ and $\tilde{L}$, an inefficiency with regard to the capital input K may exist, and this inefficiency may be solved by intellectual activities of workers. If such inefficiency exists, the effective capital input would not be equal to K. However, I was unable to find a factor that significantly necessitates a worker's intellectual activities to lessen inefficiencies in utilizing capital, in particular inefficiencies that result from imperfectness or incompleteness of information on capital. Therefore, I have assumed that capital input does not necessitate workers' innovations. However, capital input is constrained by another element that is basically irrelevant to workers' intellectual activities. It is impossible for each worker to use all capital inputs existing in the economy; each worker can access only a fraction of the total amount. This accessibility constraint sets bounds to the use of capital. Nevertheless, the accessibility is basically irrelevant in terms of worker innovation because accessibilities of workers in the world are not correlated with each other at the global level and thus it is not difficult for a worker to find a correct way to access capital inputs when an unexpected incident occurs. Therefore, information on accessibility is not incomplete, and it is enough for a worker to know only local information with regard to accessibility to capital. Therefore, there is little differentiation among workers in finding correct ways to access capital inputs, and as a consequence, there is little differentiation in the workers' experiences.

Machines or tools are not necessarily in constant operation during production; they are idle during some periods. A worker often uses various machines or tools in turn in a period, or equivalently several workers often use the same machine or tool in turn in a period. Let σK be the portion of K used by a worker on average where $\sigma (0 < \sigma \leq 1)$ is a positive parameter. Because the total sum of K used in the economy must not be smaller than K, $K \leq \sigma K L$, $L^{-1} \leq \sigma$, and thereby $L^{-1} \leq \sigma \leq 1$ for $1 \leq L$. It is highly likely that production increases if more K is used per worker, in which case

$$\frac{\partial Y(\sigma, A, K, L)}{\partial \sigma} > 0 \quad . \tag{15}$$

Condition (15) and the constraint $L^{-1} \le \sigma \le 1$ lead to a unique steady state value of σ such that σ = 1, which indicates that each worker uses all K existing in the economy. Clearly, that is impossible—accessibility to capital is not limitless. Even if a worker wants to use K installed at a distant location, it is usually meaningless to do so because it is too costly. Thus, it is highly likely that there is a boundary of accessibility with regard to location. A worker can use only a small portion of K installed in the small area around the worker. That is, the value of the parameter σ has an upper bound such that

$$L^{-1} \le \sigma \le \bar{\sigma} \quad , \tag{16}$$

where $\bar{\sigma}$ $\left(0 < \bar{\sigma} < 1\right)$ is a positive constant. With the upper bound $\bar{\sigma}$, by conditions (15) and (16), the optimal portion of K used by a worker on average ($\tilde{K}$) for $1 \le L$ is

$$\tilde{K} = \bar{\sigma} K \ . \tag{17}$$

The parameter $\bar{\sigma}$ represents a worker's accessibility limit to capital with regard to location.[2] The average value of $\bar{\sigma}$ in the economy will depend on the availability of physical transportation facilities. Location constraints, however, are not limited to physical transportation facilities. For example, law enforcement, regulations, the financial system, and other factors will also influence accessibility. The value of $\bar{\sigma}$ reflects the combined effects of all of these factors. The values of $\bar{\sigma}$ with regard to workers who are obliged to work at a designated location using fixed machines in a factory (e.g., workers in manufacturing industries) may be nearly identical. However, values for workers in other jobs (e.g., in service industries) will be heterogeneous depending on conditions. Even in manufacturing industries, workers engage in a variety of activities (e.g., negotiating with financial institutions or marketing), so the values of $\bar{\sigma}$ will also be heterogeneous in manufacturing industries.

Suppose that the density of capital per unit area is identical in the industrial area in the economy with an upper bound of $\bar{\sigma}$.[3] An increase of the total sum of K indicates an increase of the density of K in the industrial area; thus, the portion of K used by a worker also increases at the same rate as K. On the other hand, an increase of the total sum of L does not indicate any change of the density of K in the industrial area, and the portion of K used by a worker does not change.

2.2.5 Related theories

2.2.5.1 Learning-by-doing

The theory of learning-by-doing originated in Arrow (1962), who argues that productivity is improved by workers' regularly repeating the same type of action through practice, self-perfection, and minor innovation. Arrow-type growth models assume that productivity is proportionate to accumulated investments in capital or production, which represent the accumulated effects of workers' learning-by-doing (e.g., Sheshinski, 1967; Romer, 1986). If accumulated experiences obtained through learning-by-doing are proportionate not to accumulated innovations (A) but to accumulated past investments in capital or production and are heterogeneous across economies, current significant income differences across economies,

---

[2] If there are many economies with various sizes, each economy's value of $\bar{\sigma}$ may be different. The effect of the size of economy on $\bar{\sigma}$ is discussed in Harashima (2009).

[3] An industrial area is considered here to be an area that is appropriate for economic activities and excludes deserts, deep forests, mountains, and other inaccessible areas. This concept is important when we consider the size of economy, which is examined in detail in Harashima (2009).

which are difficult to explain by attributing the fundamental cause to A because A is homogenous among economies, can be explained. Arrow (1962) argues that different economies have different production functions because of heterogeneous amounts of accumulated learning-by-doing.

The concept of learning-by-doing is similar to the concept of the effective technology and labor inputs $\tilde{A}$ and $\tilde{L}$ in some aspects. They both focus on activities of ordinary workers. Indeed, some researchers base the foundation of the experience curve effect on the theory of learning-by-doing (e.g., Hall and Howell, 1985; Adler and Clark, 1991; Nemet, 2006). However, the concepts are different in the following important aspects.

- Learning-by-doing mostly consists of activities to learn already-uncovered knowledge, technologies, or ideas, but the creation of non-accumulative innovations by workers consists only of activities to create something new.
- Experiences obtained through learning-by-doing in Arrow-type growth models accumulate in the economy, but non-accumulative innovations created by workers do not accumulate.
- The amount of accumulated learning-by-doing in Arrow-type growth models is proportionate to accumulated investments in physical capital and production. The amount of non-accumulative innovations to supplement imperfect accumulated innovations is proportionate to accumulated innovations (A) and inversely proportionate to the physical capital input (K). The amount of non-accumulative innovations to mitigate the inefficiency resulting from fragmented and incomplete information is proportionate to the labor input (L).

2.2.5.2 Human capital

Human capital usually refers to a worker's knowledge and skills that help increase productivity and performance at work and that are obtained by intentionally investing in education and training. The concept of human capital in the modern neoclassical economic literature dates back to Mincer (1958) and has been studied widely since Becker (1962, 1964). Human capital is similar to physical capital. Anyone can invest in it, and it is substitutable for physical capital and labor. Becker (1962) argues that investing in human capital means all activities that influence future real income through the embedding of resources in people. Investing in human capital takes the forms of formal schooling, on-the-job training, off-the-job training, medical treatment, and similar activities (e.g., Weisbroad, 1966; Lynch, 1991). Some researchers have argued that the currently observed international differences in investments and growth rates are closely related with human capital (e.g., Lucas, 1990; Barro, 1991; Benhabib and Spiegel, 1994).

The concept of human capital is similar to the concept of effective labor and technology inputs ($\tilde{A}$ and $\tilde{L}$) as well as learning-by-doing concepts in some aspects. These concepts commonly focus on the activities of ordinary workers. In Becker (1964), general and specific human capital inputs are distinguished because general human capital is useful not only with current workers but also with potential workers. Specific human capital in this sense is useful only with a current worker in a current job. Although researchers have argued that generating convincing examples of meaningful specific human capital is difficult (e.g., Lazear, 2003), specific human capital in the sense of Becker (1964) may consist partly of non-accumulative innovations. However, the concepts are different in the following fundamental aspects.

- A worker's human capital mostly consists of knowledge, technology, or ideas that have already been uncovered by other persons, but the creation of non-accumulative innovations by workers consists only of activities to create something new.
- Human capital obtained through education and training accumulates, but non-accumulative innovations do not.

- The amount of human capital is proportionate to variables that are unrelated to *A*, *K*, or *L* (e.g., periods of education or training). The amount of non-accumulative innovations to supplement imperfect accumulated innovations is proportionate to accumulated innovations (*A*) and inversely proportionate to physical capital input (*K*). The amount of non-accumulative innovations to mitigate the inefficiency resulting from fragmented and incomplete information is proportionate to the labor input (*L*).

These differences indicate that, as with learning-by-doing, the core concepts of human capital and effective technology and labor inputs are fundamentally different.

The concept of effective labor and technology inputs focuses more specifically on creativity and non-accumulative innovations. The concept of human capital appears infinitely elastic, and its broad but ambiguous nature may confuse arguments. Many studies of human capital have narrowed the scope to education or training to avoid this ambiguity, although the concept of education still appears too broad for analyses of economic growth (e.g., Krueger and Lindahl, 2001).

2.3 Production function

2.3.1 Effective production function

Suppose that production requires some strictly positive minimum amounts of *A*, *K*, and *L*. In addition, suppose that *A*, *K*, and *L* each do not exhibit increasing marginal product; that is, $\frac{\partial^2 f(A,K,L)}{\partial A^2} \leq 0$, $\frac{\partial^2 f(A,K,L)}{\partial K^2} \leq 0$, and $\frac{\partial^2 f(A,K,L)}{\partial L^2} \leq 0$. If $\lim_{A\to\infty} \frac{\partial^2 f(A,K,L)}{\partial A^2} = 0$, $\lim_{K\to\infty} \frac{\partial^2 f(A,K,L)}{\partial K^2} = 0$, and $\lim_{L\to\infty} \frac{\partial^2 f(A,K,L)}{\partial L^2} = 0$, then for sufficiently large *A*, *K*, and *L*, the production function is approximated by the production function in which any of *A*, *K*, and *L* exhibits constant marginal product such that

$$Y = \psi_1 (A + \psi_2)(K + \psi_3)(L + \psi_4) + \psi_5 \quad , \tag{18}$$

where ψ_i (i = 1, 2, 3, 4, 5) are constants. Here, by the assumption that production requires some strictly positive minimum amounts of *A*, *K*, and *L*, then $f(0,K,L) = 0$, $f(A,0,L) = 0$, and $f(A,K,0) = 0$. Among the approximated production functions (18), the production function that also satisfies this minimum requirement condition is

$$Y = \psi_1 AKL \quad .$$

If ψ_1 is standardized such that $\psi_1 = 1$, then

$$Y = AKL \quad . \tag{19}$$

Production function (19) appears intuitively understandable. Each of *L* workers uses *K* capital inputs per worker with *A* amount of technologies utilized in each *K*.[4] However, production function (19) cannot be realized as it is, because there are various constraints caused by various imperfections, as I argued in Section 2.2. The effective amounts of technology and labor inputs are not *A* and *L* but $\tilde{A}$ and $\tilde{L}$, and the portion of *K* usable for a worker on average is not *K* but $\tilde{K}$. Hence, the approximated production function is effectively

[4] Remember that all workers are assumed to be identical.

$$Y = \tilde{A}\tilde{K}\tilde{L} \quad . \tag{20}$$

Here, by equations (9), (14), and (17),

$$\tilde{A}\tilde{K}\tilde{L} = \omega_A \left(\frac{A}{K}\right)^{\alpha} \bar{\sigma} K \omega_L L^{\alpha} = \bar{\sigma}\omega_A \omega_L A^{\alpha} K^{1-\alpha} L^{\alpha} \quad . \tag{21}$$

Rational firms utilize inputs fully so as to maximize *Y*, and by equations (20) and (21), the approximate effective production function (AEPF) can be represented as

$$Y = \bar{\sigma}\omega_A \omega_L A^{\alpha} K^{1-\alpha} L^{\alpha} \quad . \tag{22}$$

2.3.2 The approximate effective production function

AEPF has the following properties, which have been widely assumed for production functions and are consistent with data across economies and time periods: a Cobb-Douglas functional form, a labor share of about 70%, and strict Harrod neutrality. The function therefore also has decreasing marginal products of labor, capital, and technology.

2.3.2.1 Cobb-Douglas functional form

The rationale and microfoundation of the Cobb-Douglas functional form have been long argued, but no consensus has been reached. For example, Jones (2005) argues that Cobb-Douglas production functions are induced if it is assumed that ideas are drawn from Pareto distributions. Growiec (2008), however, shows that Clayton-Pareto class of production functions that nest both the Cobb-Douglas functions and the CES are induced by assuming that each of the unit factor productivities is Pareto-distributed, dependence between these marginal distributions is captured by the Clayton copula, and that local production functions are CES. AEPF provides an alternative rationale and microfoundation of the Cobb-Douglas functional form. AEPF is the typical Cobb-Douglas production function, and the keys of its Cobb-Douglas functional form are workers' innovations and the experience curve effect.

2.3.2.2 A 70% labor share

The parameter α indicates the labor share in the distribution of income. Data in DME show that labor share is about 70% (see e.g., OECD.Stat Extracts[5]). No persuasive rationale has been presented on why the labor share is usually about 70%, but AEPF can offer one. In AEPF, the value of α is derived from the experience curve effect, and the average value of α has been shown to be about 70% in many empirical studies on the experience curve effect (e.g., Hirsch, 1956; Womer and Patterson, 1983; Dutton and Thomas, 1984; Womer, 1984; Ayres and Martinas, 1992: Williams and Terzian, 1993; OECD/IEA, 2000), which implies that workers' average rate of reducing inefficiencies is bounded. This boundary probably exists because newly added information decreases as the number of experiences increases and also because the marginal efficiency in a worker's analyzing, utilizing, and managing information (i.e., in creating innovations) decreases as the amount of information increases.

2.3.2.3 Strict Harrod neutrality and balanced growth

Because AEPF is a Cobb-Douglas production function, any of Harrod, Hicks, and Solow neutralities can be assumed as the type of technology change embodied in it. However, AEPF is

---

5 http://stats.oecd.org/Index.aspx

Harrod neutral in the strict sense such that a unit of A is neither $A^{-\frac{\alpha}{1-\alpha}}$ (Solow neutral) nor $A^{-\alpha}$ (Hicks neutral) but A^{-1} because a unit of A is defined before the functional form of AEPF is induced using the experience curve effect. This strict Harrod neutrality is a necessary condition for a balanced growth path. In the balanced growth equilibrium, the capital intensity of the economy $Y^{-1}K$ is kept constant, and $L^{-1}Y$, $L^{-1}K$, and A grow at the same rate. Because AEPF is strictly Harrod neutral, it is possible for a growth model based on AEPF to achieve a balanced growth path.

At first glance, the essential factor behind the strict Harrod neutrality in AEPF appears to be that both $\tilde{A}$ and $\tilde{L}$ are subject to workers' intellectual activities and the experience curve effect. However, this view is somewhat superficial. In a deeper sense, there is a more essential factor. For strict Harrod neutrality to be achieved, it is necessary that both AEPF with constant L and AEPF with constant A be homogeneous of degree 1 with regard to (A and K) and (K and L), respectively. These conditions are satisfied in AEPF because $\tilde{A}$ is $\omega_A \left(\frac{A}{K} \right)^{\alpha}$, and $\tilde{A}$ therefore is not proportionate simply to A but to $K^{-1}A$. That is, strict Harrod neutrality requires various types of accumulative innovations in A to be dispersed in K, which means that A and K are closely related (like two sides of the same coin). Production (Y) increases at the same rate as A and K; thus, the capital intensity ($Y^{-1}K$) is constant.

As shown in Section 2.2, the nature of dispersive accumulative innovations originates in the optimization of firms to minimize inefficiencies caused by congestion and redundancy of A (i.e., to maximize effects of the division of A). Because technology input is optimal when capital is as specialized as possible, then capital is actually as specialized as possible by the optimizing behaviors of firms, which implies that the very essence of the strict Harrod neutrality and the balanced growth path lies in the optimizing behaviors of rational firms.

3 THE NATURE OF INTELLIGENCE

3.1 Intelligence required by ordinary workers

In Section 2, ordinary workers' intelligence was shown to be necessary for production. The two productive inefficiencies ($C_{A,1}$ and $C_{L,1}$) can be reduced by ordinary workers' innovations. Here, innovations refer to newly uncovered "rules" that describe useful connections or relations among various factors. In this sense, $C_{A,1}$ and $C_{L,1}$ are regarded as consequences of a lack of knowledge on "rules" that are necessary for production.

$C_{A,1}$ indicates the degree to which ordinary workers are unaware of rules that are essential to the technologies used for production (e.g., certain scientific laws). Workers are not informed of these rules *ex ante* because they are so minor. Therefore, incorrect actions may be taken if problems related to these rules occur and thereby generate productive inefficiency $C_{A,1}$. However, if the workers can uncover the unknown rules by using their intelligence, $C_{A,1}$ can be reduced. Similarly, productive inefficiency $C_{L,1}$ indicates the extent to which ordinary workers are unaware of rules that are essential to a broad production strategy (e.g., details of various plans in related sections of a firm). Incorrect actions may be taken if problems related to these rules occur and thereby generate productive inefficiency $C_{L,1}$. However, $C_{L,1}$ can be reduced if the workers uncover the unknown rules by using their intelligence. Therefore, the intelligence required for ordinary workers to deal with unexpected problems is the ability to uncover unknown rules.

3.1.1 Innovation processes

The following three abilities are necessary to uncover unknown rules: perceiving signals from symbols, hypothesizing rules, and simulating hypothesized rules. The required intelligence is the combination of these three abilities. The recognition process (i.e., perceiving signals from symbols) can be understood using color as an example. Each color corresponds to a wavelength in the range of visible light. However, people perceive only a limited number of colors (i.e., signals) from all wavelengths in the range of visible light. Each color is perceived as one signal, e.g., red, yellow, green, blue or purple, but consists of an infinite number of wavelengths (i.e., symbols) within a range of wavelengths. Symbols are generated, used, or both generated and used in production, and a combination of symbols is described by a rule. Although workers must use the rules for production, they cannot perceive symbols directly but only indirectly through signals emitted from symbols.

Although there may be a one-to-one correspondence between a symbol and a signal, a number of symbols can correspond to a signal. The correspondences between symbols and signals differ among workers. Therefore, some workers may perceive the same signals from a given set of symbols, but others may perceive different signals from it. One worker may perceive only a single signal from many symbols, whereas another may perceive several signals from the same symbols. Therefore, the former can distinguish fewer symbols than can the latter; that is, the ability of the former to finely distinguish symbols is lower than that of the latter. For the former worker, uncovering unknown rules will be more difficult because a rule describes a unique combination of symbols.

3.1.2 Ability to perceive faithful signals

The first important ability for uncovering unknown rules is the degree of resolution in perceiving signals from symbols. This resolution can be measured by the probability of correctly distinguishing a symbol from the other symbols by signals. If the resolution is low, many symbols are perceived as being the same signal, and the signal provides a blurred picture of the symbols. The probability of uncovering the correct rule will be lower, and thus the worker may obtain a spurious rule. For example, if the probability that a worker can correctly distinguish a symbol (or set of symbols) is 80%, then the probability of normal production will be less than 80% because the worker may obtain only a spurious rule.

3.1.3 Ability to hypothesize

The second important ability is to be able to hypothesize rules. In the process of uncovering unknown rules, various hypotheses need to be formulated and examined. Given that rules describe combinations of symbols, hypotheses of a rule are also combinations of symbols. A problem here is that the number of hypotheses (i.e., the number of symbol combinations that may be true) will be extremely large if all combinations are treated equally. Therefore, the number of necessary tests of hypotheses will also be enormous, and completing them will take a very long time.

However, not all hypotheses are necessarily tested. Each is weighted in terms of importance by, for example, using knowledge obtained *a priori*, and tests are conducted in order of those with higher weights. If the weights are assigned properly, the number of hypotheses to test before uncovering the true rule will be greatly reduced.

At the same time, however, assigning weights by using knowledge obtained *a priori* risks delaying discovery of the rule. If the weights are improper, the true hypothesis may be left out. Preconceptions, biases, or prejudices in particular will seriously hinder the assignment of proper weights. Although the process of uncovering unknown rules can be made more efficient by assigning weights, the presence of preconceptions inevitably decreases efficiency. Therefore, the use of knowledge obtained *a priori* is a double-edged sword—a dilemma that may not easily be avoided. Nevertheless, younger people may be more able to avoid the influence of preconceptions, which are strengthened with accumulation of experience. Hence, innovations by younger people may be more likely than those by older people.

This dilemma also suggests that an accidental failure in the innovation process may unexpectedly lead to an important breakthrough. An accidental failure may provide the chance for a neglected hypothesis, assigned a very low weight and hence not expected to be tested early if at all, to be tested far earlier. This change in test order may fortuitously result in early discovery of the rule. Such innovations may be the result of luck because they would not be uncovered so quickly unless preconceptions had been discarded by previous accidental failures. Therefore, using knowledge obtained *a priori* does not provide a perfect solution for assigning proper weights to hypotheses, and chance will also play an important role in the innovation process.

3.1.4 Ability to compute

Tests of hypotheses are implemented by simulating the hypothesized rules. The simulation results are compared with observed data and the results of other simulated hypotheses. After comparisons, the validity of a hypothesis is determined. The test ends if the hypothesis is validated; otherwise, the hypothesis with the next largest weight is tested. This process is repeated until the true rule is found.

Hypothesis testing requires computational ability because extensive calculations by the brain are needed to simulate hypotheses. Therefore, if the calculation speed is higher, the true rule will be uncovered more rapidly. Hence, the speed of comparisons will directly influence the probability of uncovering the rule in a unit of time. In addition, if more complex calculations can be performed, the probability of not finding the true rule will decrease. To deal with complex calculations, the capacity of working memory should be sufficiently large. On the whole, the ability to compute will consist of calculation speed and working memory capacity.

3.2 Fluid intelligence

3.2.1 Fluid and crystallized intelligences

In psychology and psychometrics, many types of intelligence have been considered, including fluid intelligence, crystallized intelligence, short-term memory, long-term storage and retrieval, reading and writing ability, and visual processing. Among these, the importance of the difference between fluid intelligence and crystallized intelligence has been particularly emphasized. According to Cattell (1963, 1971), fluid intelligence is the capacity to solve novel problems by thinking logically, independent of acquired knowledge. This is the ability to deal with novel situations without relying on knowledge obtained from schooling or previous experience. With the help of fluid intelligence, people can flexibly adapt their thinking to new problems or situations. By contrast, crystallized intelligence is the capacity to acquire and use knowledge or experience. This is the ability to communicate one's knowledge and to reason by using previously learned experiences.

3.2.1 Raven’s Progressive Matrices

Raven’s Progressive Matrices test has been regarded as the best test to measure fluid intelligence (Raven, 1962; Snow et al., 1984; Raven et al., 1998). In this test, a subject (test-taker) is presented with a matrix of various images, one of which is missing, and asked to pick the answer that best completes the matrix from among a given set of possible answers. The images in the matrix are arranged by a rule; hence, selecting the correct answer is equivalent to correctly uncovering the rule.

Attempts have been made to solve Raven’s progressive matrices by computers since Carpenter et al. (1990) (see also e.g., Lovett et al., 2007). In these attempts, Raven’s test has had the following essential characteristics: the objective is to uncover unknown rules, and the required abilities to achieve the solution are the ability to hypothesize and the ability to compute.

Note, however, that these attempts are not necessarily regarded as sufficiently successful. Carpenter et al.'s (1990) model has the shortcoming that hypothesizing is heavily pre-processed by humans. Lovett et al.'s (2007) model is an attempt to overcome this shortcoming. Computers face another difficulty. For people, the ability to perceive faithful signals is unimportant in Raven's test because the symbols and signals are very simple and easily recognizable. However, the test is not an easy task for computers despite the simplicity of the symbols and signals. Hence, models that include the ability to perceive signals (e.g., by applying image processing techniques) are being developed (e.g., McGreggor et al., 2010).

3.2.3 Common intelligence: fluid intelligence

The characteristic features of solving unexpected problems (Section 3.1) are very similar to those of Raven's progressive matrices test (Section 3.2.1). The only difference is that Raven's test does not require the ability to perceive faithful signals, because the symbols and signals in the test are very simple and clearly distinguishable for people. Given that Raven's test is a good indicator of fluid intelligence (Raven, 1962; Snow et al., 1984; Raven et al., 1998), it is highly likely that the intelligence required by ordinary workers is almost the same as fluid intelligence. In addition, scores on Raven's test highly correlate with many different cognitive tests (see, e.g., Marshalek et al., 1983; Snow et al., 1984). Therefore, the abilities of perceiving faithful signals, hypothesizing, and computation are also likely to be highly correlated each other.

Note, however, that crystallized intelligence is not entirely unrelated to ordinary workers' ability to solve unexpected problems. For example, when hypothesizing, symbol combinations are weighted by their importance by using knowledge obtained *a priori*. Hence, crystallized intelligence plays an important role in this step. In other steps, the importance of crystallized intelligence will be far less than that of fluid intelligence. On the whole, although both types of intelligence are used, fluid intelligence is far more important for solving unexpected problems.

Note also that crystallized intelligence is closely related to the intelligence needed for "human capital" and "learning-by-doing." The concepts of human capital (e.g., Becker, 1962; Lucas, 1990; Barro, 1991) and learning-by-doing (e.g., Arrow, 1962; Sheshinski, 1967; Romer, 1986) are similar to the concept of ordinary worker's innovation in some aspects because they commonly focus on activities and contributions of ordinary workers to production. However, the concepts are fundamentally different because human capital and learning-by-doing consist of activities to learn already-uncovered knowledge, technologies, or ideas, whereas ordinary workers' innovation consists of activities to create something new. That is, human capital and learning-by-doing is related mostly to crystallized intelligence, whereas ordinary workers' innovation is related mostly to fluid intelligence.

4 WORKERS' FLUID INTELLIGENCE AND ITEM RESPONSE THEORY

4.1 Item response theory

The discussion above indicates that the ordinary workers' intelligence required for production amounts to fluid intelligence. Therefore, the former can be modeled on the basis of item response theory, which is widely used in psychometric studies (e.g., Lord and Novick, 1968; van der Linden and Hambleton, 1997). In particular, the item response function is used to describe the relationship between abilities and item responses (e.g., test scores or performances).

A typical item response function is

$$\tilde{p}\left(\tilde{\theta}\right) = \tilde{c} + \frac{1-\tilde{c}}{1+e^{-\tilde{a}\left(\tilde{\theta}-\tilde{b}\right)}} \quad ,$$

where $\tilde{p}$ is the probability of a correct response (e.g., answer) to an item (e.g., test or question), $\tilde{\theta}$ ($\infty > \tilde{\theta} > -\infty$) is a parameter that indicates an individual's ability, $\tilde{a}$ (> 0) is a parameter that characterizes the slope of the function, $\tilde{b}$ ($\infty \geq \tilde{b} \geq -\infty$) is a parameter that represents the difficulty of an item, and $\tilde{c}$ ($1 \geq \tilde{c} \geq 0$) is a parameter that indicates the probability that an item can be answered correctly by chance.

4.2 Item response model of ordinary workers' intelligence

On the basis of item response theory, the probability of ordinary workers solving unexpected problems in a unit of time, $p(\theta)$, can be modeled as

$$p(\theta) = c + \frac{1-c}{1+e^{-a(\theta-b)}} \quad ,$$

where θ ($\infty > \theta > -\infty$) indicates ordinary workers' average fluid intelligence, a (> 0) is a constant, b indicates the average difficulty of unexpected problems that ordinary workers are delegated to solve, and c ($1 \geq c \geq 0$) is the probability that unexpected problems are solved by chance. There is a lower boundary for b; $\underline{b}$ ($> -\infty$). As is evident from this function, the higher the ordinary workers' average intelligence (i.e., the higher the value of θ), the higher the probability of solving unexpected problems in a unit of time.

Because $\omega_A \omega_L$ indicates ordinary workers' ability to solve unexpected problems as shown in Section 2, $\omega_A \omega_L$ can be represented by $p(\theta)$. Given that $\omega_A = \frac{\upsilon_A \gamma_A}{C_{A,1}}$ and $\omega_L = \frac{\upsilon_L \gamma_L}{C_{L,1}}$ (Section 2), $\omega_A \omega_L$ is negatively and monotonically related to production inefficiencies $C_{A,1}$ and $C_{L,1}$, and is thus positively and monotonically related to $p(\theta)$. That is, $\omega_A \omega_L$ is determined by the degree of ordinary workers' average fluid intelligence θ. Hence, TFP can be modeled as

$$T(\theta, \bar{\sigma}, A) = \bar{\sigma} \omega_A \omega_L A^{\alpha} = \left[c + \frac{1-c}{1+e^{-a(\theta-b)}} \right] \omega \bar{\sigma} A^{\alpha} \quad ,$$

where ω is the unit of measurement and constant. Thus, the production function is

$$Y(\theta, \bar{\sigma}, A, K, L) = \left[c + \frac{1-c}{1+e^{-a(\theta-b)}} \right] \omega \bar{\sigma} A^{\alpha} K^{1-\alpha} L^{\alpha} \quad .$$

4.3. Parameter $\bar{\sigma}$ and fluid intelligence

Ordinary workers' fluid intelligence may influence not only $\omega_A \omega_L$ but also the parameter $\bar{\sigma}$ in TFP. $\bar{\sigma}$ indicates the accessibility of a worker to capital, where accessibility consists not only of physical transportation facilities but also of law enforcement, regulation, and financial systems as well as other institutional factors. Well-managed law enforcement systems, for example, require not only the intelligence of high-ranking officials but also that of many lower ranking personnel. Therefore, if the ordinary workers' fluid intelligence in one country is higher than that in another, systems such as those of law enforcement in the former country will be better. This conjecture indicates that $\bar{\sigma}$ is also a function of ordinary workers' fluid intelligence, and

$$\frac{\partial \bar{\sigma}(\theta)}{\partial \theta} > 0 \quad .$$

Hence, TFP is more precisely described as

$$T(\theta,\bar{\sigma},A) = \left[c + \frac{1-c}{1+e^{-a(\theta-b)}} \right] \omega \bar{\sigma}(\theta) A^{\alpha} \quad .$$

Thus, the production function is given as

$$Y(\theta,\bar{\sigma},A,K,L) = \left[c + \frac{1-c}{1+e^{-a(\theta-b)}} \right] \omega \bar{\sigma}(\theta) A^{\alpha} K^{1-\alpha} L^{\alpha} \quad .$$

Nevertheless, in the following sections, I focus on the effects of θ on $\omega_A \omega_L$. For simplicity, $\bar{\sigma}$ is assumed to be constant and unrelated to θ. Note, however, that the direction of effects of θ on $\omega_A \omega_L$ and $\bar{\sigma}$ are the same; hence, ignoring the effects on $\bar{\sigma}$ do not change the main conclusions of the following sections.

5 FLUID INTELLIGENCE AND TFP

5.1 Costs of dispatching high-ranking workers

The average fluid intelligence of ordinary workers (θ) is assumed to be given exogenously. In contrast, the average difficulty (b) is determined endogenously by firms on the basis of the given value of θ because firms have an option other than delegating the solution of unexpected problems to ordinary workers. Instead, they can dispatch experts or high-ranking employees to fix the problems by paying additional costs. A firm selects one of the two options by considering the maximization of its profits. That is, the value of b is determined through arbitrage between delegating the work to ordinary workers and dispatching experts by paying additional costs. If the ordinary workers have relatively high intelligence and can deal with relatively difficult problems, firms will refrain from dispatching experts because of the additional costs. Hence, b is a function of θ and the additional costs of dispatching experts or high-ranking employees.

Two types of workers are assumed: ordinary workers and high-ranking workers who are executives, managers, experts, specialists, or other highly educated and trained employees. The number of ordinary workers is far larger than that of high-ranking workers. When an unexpected problem occurs at one of its production sites, a firm has two options: (i) delegate ordinary workers at the site of concern to solve the problem or (ii) dispatch high-ranking workers at distant sites to deal with the problem by paying additional costs. The additional costs include transportation costs as well as opportunity costs due to the waiting time before the arrival of high-ranking workers.

5.2 Endogenous difficulty

5.2.1 Determination of *b*

If the problem is minor, option (i) will be favored because additional costs are unnecessary. If it is difficult, option (ii) will be selected despite the need to pay additional costs. Given the abilities of workers and the additional costs, the "optimal" b is determined through arbitrage between the two options.

Let θ_1 be the average fluid intelligence of ordinary workers and θ_2 ($> \theta_1$) be that of high-ranking workers. The values of θ_1 and θ_2 are given exogenously. Let also M be the number of unexpected problems that ordinary workers address. M increases as b increases, that is, as the ability of ordinary workers to deal with difficult problems increases. In addition, let q be the additional costs of dispatching high-ranking workers per unexpected problem. Dealing with a larger number of problems results in higher additional costs per unexpected problem because opportunity costs increase owing to the waiting time before the high-ranking workers arrive. A smaller value of b indicates that high-ranking workers must be dispatched to deal with a larger number of problems. Therefore, q is a monotonically decreasing function of b such that

$$\frac{dq(b)}{db} < 0 \quad .$$

It is assumed that

$$\lim_{b \to \underline{b}} q(b) = \infty$$

and

$$\lim_{b \to \infty} q(b) = 0 \quad ,$$

where $\underline{b}$ is the lower boundary of b. In addition, the unit price of a product is unity, and for simplicity, there is no difference in wages between ordinary and high-ranking workers. The number of firms is sufficiently large, and all firms are identical.

When unexpected problems are dealt with by high-ranking workers, production is larger than when they are addressed by ordinary workers, although additional costs $q(b)$ are required. Hence, the difference between firms' profits (R) when an unexpected problem is addressed by ordinary workers and that when it is addressed by high-ranking workers is described by the difference of products and the additional costs. Hence, the marginal R with respect to M is

$$\frac{dR}{dM} = \left[c + \frac{1-c}{1+e^{-a(\theta_1 - b)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - \left\{\left[c + \frac{1-c}{1+e^{-a(\theta_2 - b)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - q(b)\right\} \quad ,$$

where $\hat{\omega} = \dfrac{\omega}{\mu}$ and μ (> 1) is a constant (i.e., $\hat{\omega}$ indicates the value of ω when dealing with a unit of M).

Therefore,

$$\frac{dR}{db} = \frac{dM}{db}\frac{dR}{dM} = \frac{dM}{db}\left\{(1-c)\left[\frac{1}{1+e^{-a(\theta_1 - b)}} - \frac{1}{1+e^{-a(\theta_2 - b)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} + q(b)\right\} \quad . \quad (23)$$

By arbitrage, b is determined at the point $\dfrac{dR}{db} = 0$ where

$$\left(1-c\right)\left[\frac{1}{1+e^{-a(\theta_1-b)}}-\frac{1}{1+e^{-a(\theta_2-b)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha}+q(b)=0 \tag{24}$$

is satisfied by equation (23). Given the natures of functions $\frac{1}{1+e^{-a(\theta_2-b)}}-\frac{1}{1+e^{-a(\theta_1-b)}}$ and $q(b)$, there exists at least one point that satisfies equation (24). This means that even if θ_1 is very small (i.e., θ_1 has a large negative value), some unexpected problems are still delegated to ordinary workers.

5.2.2 Relation between θ_1 and b

Suppose that θ_2 is constant. By total differential of equation (24) with respect to θ_1 and b,

$$\frac{db}{d\theta_1}=\left\{1-\frac{\left[1+e^{-a(\theta_1-b)}\right]^2}{e^{-a(\theta_1-b)}}\frac{\frac{\partial q(b)}{\partial b}}{(1-c)a\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha}}-\frac{e^{-a(\theta_2-b)}}{e^{-a(\theta_1-b)}}\frac{\left[1+e^{-a(\theta_1-b)}\right]^2}{\left[1+e^{-a(\theta_2-b)}\right]^2}\right\}^{-1} .$$

Here, it is assumed that $b < \theta_2$ and $|\theta_2-b|$ is far larger than $|\theta_1-b|$; that is, $e^{-a(\theta_2-b)}$ is far smaller than unity whilst $e^{-a(\theta_1-b)}$ is not much larger than unity. Therefore, because $\theta_1 < \theta_2$ and thus $e^{-a(\theta_2-b)}<e^{-a(\theta_1-b)}$, then

$$\frac{e^{-a(\theta_2-b)}}{e^{-a(\theta_1-b)}}\frac{\left[1+e^{-a(\theta_1-b)}\right]^2}{\left[1+e^{-a(\theta_2-b)}\right]^2}<1 .$$

Hence, because $\frac{dq(b)}{db}<0$,

$$\frac{db}{d\theta_1}>0 .$$

If ordinary workers have higher θ_1, firms delegate more difficult unexpected problems to them because the firms' profits increase.

5.3 Nature of TFP and production

5.3.1 TFP and production

Let b_1 and b_2 (> b_1) be the average difficulties of problems that ordinary workers and high-ranking workers are respectively assigned to solve. The value of b_2 is exogenously given and constant, whereas b_1 is an endogenous variable. Suppose that one part of production in an economy is implemented by high-ranking workers and the other part by ordinary workers. If an unexpected problem of difficulty greater than b_1 occurs in the part of production assigned to ordinary workers, high-ranking workers are dispatched to solve the problem. Let w ($0 \leq w \leq 1$) be the share of ordinary workers' part in production, and assume that w is the share of the part of $K^{1-\alpha}L^{\alpha}$ that ordinary workers use among all inputs $K^{1-\alpha}L^{\alpha}$. The value of w is close to unity (i.e., $w \approx 1$) because the number of ordinary workers is far larger than that of high-ranking workers.

The production function is therefore,

$$Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) = w\left[c + \frac{1-c}{1+e^{-a(\theta_1-b_1)}}\right]\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} + (1-w)\left[c + \frac{1-c}{1+e^{-a(\theta_2-b_2)}}\right]\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha}$$
$$= \left\{c + (1-c)\left[\frac{w}{1+e^{-a(\theta_1-b_1)}} + \frac{1-w}{1+e^{-a(\theta_2-b_2)}}\right]\right\}\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} \quad . \tag{25}$$

Hence, TFP is

$$T(\theta_1,\theta_2,\bar{\sigma},A) = \left\{c + (1-c)\left[\frac{w}{1+e^{-a(\theta_1-b_1)}} + \frac{1-w}{1+e^{-a(\theta_2-b_2)}}\right]\right\}\omega\bar{\sigma}A^{\alpha} \quad .$$

5.3.2 TFP and θ_1

Given that

$$\left[c + \frac{1-c}{1+e^{-a(\theta_1-b_1)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} = \left[c + \frac{1-c}{1+e^{-a(\theta_2-b_1)}}\right]\hat{\omega}\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - q(b_1)$$

by equation (24) and $\hat{\omega} = \frac{\omega}{\mu}$, then by equation (25),

$$Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) = \left\{c + (1-c)\left[\frac{w}{1+e^{-a(\theta_2-b_1)}} + \frac{1-w}{1+e^{-a(\theta_2-b_2)}}\right]\right\}\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - w\mu q(b_1) \quad . \tag{26}$$

Hence, the sign of $\frac{dT(\theta_1,\theta_2,\bar{\sigma},A)}{d\theta_1}$ depends on the values of $\frac{d\left(1+e^{-a(\theta_2-b_1)}\right)^{-1}}{db_1}$ (< 0) and $\frac{dq(b_1)}{db_1}$ (< 0). If θ_1 increases (i.e., b_1 increases), then $w\mu q(b_1)$ decreases because $\frac{dq(b_1)}{db_1} < 0$. On the other hand, $\left(1+e^{-a(\theta_2-b_1)}\right)^{-1}$ decreases. Thus, the directions of changes for production and TFP are unclear. Nevertheless, if slight increases of θ_1 and b_1 substantially decrease q, then the sign of $\frac{dT(\theta_1,\theta_2,\bar{\sigma},A)}{d\theta_1}$ will be positive, and TFP will be an increasing function of θ_1.

On the other hand, if high-ranking workers are assumed to be able to solve almost all unexpected problems, then

$$\frac{1}{1+e^{-a(\theta_2-b_1)}} \approx 1$$

and

$$\frac{1}{1+e^{-a(\theta_2-b_2)}} \approx 1 \quad .$$

The assumption that high-ranking workers can solve almost all unexpected problems is natural because it is likely that they sufficiently know and understand the current level of technology A and the overall strategy of the firm. Therefore, by equation (26),

$$Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) \approx \omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - w\mu q(b_1) \quad .$$

Hence,

$$\frac{dT(\theta_1,\theta_2,\bar{\sigma},A)}{d\theta_1} > 0$$

will generally hold because $\frac{dq(b_1)}{db_1} < 0$ and $\frac{db_1}{d\theta_1} > 0$.

5.3.3 Range of TFP

Here,

$$Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) = \left\{c + (1-c)\left[\frac{w}{1+e^{-a(\theta_2-b_1)}} + \frac{1-w}{1+e^{-a(\theta_2-b_2)}}\right]\right\}\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} - \mu wq(b_1) \geq 0 \qquad (27)$$

always holds because production Y is never negative. The inequality (27) implies that even if θ_1 is very small, b_1 does not become so small so as to make $\mu wq(b_1)$ sufficiently large such that $Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) < 0$. That is, as mentioned in Section 5.2.1, even if θ_1 is very small, some intelligent tasks (b_1) are delegated to ordinary workers; otherwise, inequality (27) does not hold. Conversely, the role of ordinary workers' intelligence is always important.

By equation (25) and $\frac{1}{1+e^{-a(\theta_2-b_2)}} \approx 1$, a very small θ_1 also indicates that

$$\lim_{\theta_1\to-\infty} Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) = \lim_{\theta_1\to-\infty}\left\{c + (1-c)\left[\frac{w}{1+e^{-a(\theta_1-b_1)}} + \frac{1-w}{1+e^{-a(\theta_2-b_2)}}\right]\right\}\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha}$$
$$= \left[c + \frac{(1-w)(1-c)}{1+e^{-a(\theta_2-b_2)}}\right]\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha} \approx [1-w+wc]\omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha},$$

and thus

$$\lim_{\theta_1\to-\infty} T(\theta_1,\theta_2,\bar{\sigma},A) \approx [1-w+wc]\omega\bar{\sigma}A^{\alpha} \quad .$$

The value of w is close to unity ($w \approx 1$). Hence, if θ_1 is very small, TFP is very small such that

$$\lim_{\theta_1\to-\infty} T(\theta_1,\theta_2,\bar{\sigma},A) \approx c\omega\bar{\sigma}A^{\alpha} \quad . \qquad (28)$$

This is the lowest level of TFP.

On the other hand, if ordinary workers have the same level of intelligence as high-ranking workers (i.e., $b_1 = b_2$), then by equation (25), $\frac{1}{1+e^{-a(\theta_2-b_1)}} \approx 1$, and $\frac{1}{1+e^{-a(\theta_2-b_2)}} \approx 1$,

$$\lim_{\theta_1\to\theta_2} Y(\theta_1,\theta_2,\bar{\sigma},A,K,L) \approx \omega\bar{\sigma}A^{\alpha}K^{1-\alpha}L^{\alpha}$$

and

$$\lim_{\theta_1 \to \theta_2} T(\theta_1, \theta_2, \bar{\sigma}, A) \approx \omega \bar{\sigma} A^{\alpha} \quad .$$

This is the highest level of TFP.

Consequently, the TFP of an economy will fall somewhere between $c\omega\bar{\sigma}A^{\alpha}$ and $\omega\bar{\sigma}A^{\alpha}$. Note that even a slight decrease of θ_1 can make TFP significantly lower depending on the value of parameter *a*. This characteristic implies that even a small difference of intelligence can result in a substantial difference of TFP and per capita GDP.

5.3.4 Value of *c*

The lowest level of TFP shown by equation (28) depends on the value of *c*. However, *c* is highly likely to be near zero. The value of *c* indicates the probability that an unexpected problem is solved by chance. In Raven's test, the probability of a correct answer by chance is substantially higher than zero because a small number of possible answers for a question (e.g., 8 per question) are presented to the subject (test-taker) before the subject selects an answer from among them. Naturally, subjects can correctly answer some of the questions by chance (12.5% of questions in the above example). On the other hand, nobody gives possible answers *ex ante* to ordinary workers who face unexpected problems. Ordinary workers must uncover the correct rule from among a large number of hypotheses from scratch. Hence, the probability of uncovering the rule by chance will be far smaller than 12.5% and close to zero.

The solution of unexpected problems by chance includes cases in which the problems are unexpectedly and unconsciously solved by non-artificial accidental events. For example, an unintentional breath of air accidentally brushes off dust on a delicate part of a machine. Although such fortuitous incidents will occur, their probability of occurrence will be extremely low. Most unexpected incidents will instead have only negative effects.

On the whole, the value of *c* is highly likely to be almost zero. If *c* = 0, the production function is

$$Y(\theta_1, \theta_2, \bar{\sigma}, A, K, L) = \left[\frac{w}{1 + e^{-a(\theta_1 - b_1)}} + \frac{1 - w}{1 + e^{-a(\theta_2 - b_2)}} \right] \omega \bar{\sigma} A^{\alpha} K^{1-\alpha} L^{\alpha}$$

by equation (25), and TFP is

$$T(\theta_1, \theta_2, \bar{\sigma}, A) = \left[\frac{w}{1 + e^{-a(\theta_1 - b_1)}} + \frac{1 - w}{1 + e^{-a(\theta_2 - b_2)}} \right] \omega \bar{\sigma} A^{\alpha} \quad .$$

In addition, by equation (28), the lowest level of TFP (i.e., when both θ_1 and θ_2 are very small) is

$$\lim_{\theta_1 \to -\infty} T(\theta_1, \theta_2, \bar{\sigma}, A) \approx c\omega \bar{\sigma} A^{\alpha} \approx 0 \quad .$$

Hence, an economy's TFP will fall somewhere between 0 and $\omega\bar{\sigma}A^{\alpha}$.

5.4 Value added as the fruits of intelligence

The finding that the lowest level of TFP is almost zero indicates that if there is no fluid intelligence (neither θ_1 nor θ_2), production is almost impossible. Fluid intelligence is an indispensable factor of production. That is, value added comes from the fruits of fluid intelligence.

Note, however, that intelligence is not the only factor. Per capita production is constrained by not only intelligence but also A and K. Value added is thus the fruits of intelligence multiplied by A and K. Nevertheless, A and K indicate past intelligence and past fruits, respectively. In this sense, intelligence may be argued to be the fundamental source of value added.

The result that ordinary workers' fluid intelligence is a key factor in producing the value added indicates that improving this factor should be an essential element in strategies for economic development. Although the importance of accumulating physical and human capital for economic development has been emphasized, the results of this paper indicate that relying solely on accumulation of capital is insufficient. Measures to improve fluid intelligence are also needed. Studies in pedagogy, psychology, neuroscience, and other sciences have accumulated voluminous amounts of information suggesting that fluid intelligence can be improved. The fruits of these studies should be applied more intensely to economic development strategies.

6 CONCLUDING REMARKS

Estimates of TFP have varied widely among countries. Prescott (1998) concludes that a theory of TFP is needed to answer why TFPs are diverse. In this paper, a theory of TFP is proposed, and TFP is suggested to reflect the fruits of human intelligence. That the fluid intelligence of ordinary workers is an important element in TFP is particularly emphasized. The idea that ordinary workers can also create something new (i.e., innovate) has drawn little attention. However, they can do so because they have intelligence, just as do researchers and other highly educated or trained employees. Although most innovations of ordinary workers are so minor that they do not become part of the accumulated knowledge of humanity, they are indispensable for production because a large number of minor unexpected problems that ordinary workers must address occur in the process of production.

A model of TFP is formulated on the basis of the finding that ordinary workers' fluid intelligence is an essential element in TFP. TFP is modeled as a function of fluid intelligence, and item response theory, which is widely used in psychology and psychometrics, is used to specify the functional form. The model of TFP shows that TFP is an increasing function of ordinary workers' fluid intelligence. It also shows that TFP depends substantially on ordinary workers' fluid intelligence, and that production is almost impossible without it. Therefore, value added comes from the fruits of humans' fluid intelligence.

References

Abramovitz, Moses. (1986) “Catching Up, Forging Ahead, and Falling Behind,” *Journal of Economic History*, Vol. 46, No. 2, pp. 385-406.

Adler, Paul S. and Kim B. Clark. (1991) “Behind the Learning Curve: A Sketch of the Learning Process,” *Management Science*, Vol. 37, No. 3, pp. 267-281.

Alchian, Armen A., (1963) “Reliability of Progress Curves in Airframe Production,” *Econometrica*, Vol. 31, No. 4, pp. 679–693.

Argote, Linda, Sara L. Beckman and Dennis Epple. (1990): “The Persistence and Transfer of Learning in Industrial Settings,” *Management Science*, Vol. 36, No. 2, pp. 140-154.

Argote, Linda and Dennis Epple. (1990) “Learning Curves in Manufacturing,” *Science,* Vol. 247, No. 4945, pp. 920-924.

Argote, Linda, Bill McEvily and Ray Reagans. (2003) “Managing Knowledge in Organizations: An Integrative Framework and Review of Emerging Themes,” *Management Science*, Vol. 49, No. 4, pp. 571–582.

Arrow, Kenneth J. (1962) “The Economic Implications of Learning by Doing,” *Review of Economic Studies*, Vol. 29, No. 3, pp. 155–173.

Asher, Harold. (1956) *Cost-Quantity Relationships in the Airframe Industry*, Report 291, RAND Corporation, Santa Monica, Ca.

Ayres, Robert U. and Katalin Martinas. (1992), “Experience and Life Cycle: Some Analytical Implications”, *Technovation*, Vol. 12, No. 7, pp. 465-486.

Baldwin, Carliss, Christoph Hienerth and Eric von Hippel. (2006) “How User Innovations Become Commercial Products: A Theoretical Investigation and Case Study,” *Research Policy*, Vol. 35, No. 9, pp. 1291-1313.

Barro, Robert J. (1991) “Economic Growth in a Cross Section of Countries,” *Quarterly Journal of Economics*, Vol. 106, No. 2, pp. 407-43.

Baumol, William J. (1986) “Productivity Growth, Convergence, and Welfare: What the Long-run Data Show,” *American Economic Review*, Vol. 76, No. 5, pp. 1072-85.

Becker, Gary S. (1962) “Investment in Human Capital: A Theoretical Analysis,” *The Journal of Political Economy*, Vol. 70, No. 5, pp. 9-49.

Becker, Gary S. (1964) *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*, Columbia University Press, New York.

Benhabib, Jess and Mark M. Spiegel, (1994) “The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data,” *Journal of Monetary Economics*, Vol. 34, No. 2, pp. 143-173.

Bernard, Andrew B. and Steven N. Durlauf. (1995) “Convergence in International Output,” *Journal of Applied Econometrics*, Vol. 10, No. 2, pp. 97-108.

Boston Consulting Group (BCG) (1972) *Perspectives on Experience*, Boston, Mass.

Carpenter, Patricia A., Marcel Adam Just and Peter Shell. (1990) “What One Intelligence Test Measures: A Theoretical Account of the Processing in the Raven Progressive Matrices Test,” *Psychological Review*, Vol. 97, No. 3, pp. 404–431.

Cattell, Raymond Bernard. (1963) “Theory of Fluid and Crystallized Intelligence: A Critical Experiment,” *Journal of Educational Psychology*, Vo. 54, pp. 1-22.

Cattell, Raymond Bernard. (1971) *Abilities: Their structure, growth, and action*, Houghton Mifflin, Boston.

Cheung, Yin-Wong and Antonio Garcia-Pascual. (2004) “Testing for Output Convergence: A Re-examination,” *Oxford Economic Papers*, Vol. 56, No. 1, pp. 45-63.

Criqui, Patrick, Jean-Marie Martin, Leo Schrattenholzer, Tom Kram, Luc Soete and Adriaan Van Zon.. (2000) “Energy Technology Dynamics,” *International Journal of Global Energy Issues*, Vol. 14, No. 1-4, pp. 65-103.

Dudley, Leonard. (1972) “Learning and Productivity Changes in Metal Products,” *American*

Economic Review, Vol. 62, No. 4, pp. 662-69.
Dutton, John M. and Annie Thomas. (1984) "Treating Progress Functions as a Managerial Opportunity," *Academy of Management Review*, Vol. 9, No. 2, pp. 235-247.
Growiec, Jakub (2008) "A New Class of Production Functions and an Argument Against Purely Labor-Augmenting Technical Change," *International Journal of Economic Theory*, Vol. 4, No. 4, pp. 483-502.
Hall, Graham and Sydney Howell. (1985) "The Experience Curve from the Economist's Perspective," *Strategic Management Journal*, Vol. 6, No. 3, pp. 197-212.
Harashima, Taiji. (2009) "A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 15508.
Harashima, Taiji. (2011) "A Model of Total Factor Productivity Built on Hayek's View of Knowledge: What Really Went Wrong with Socialist Planned Economies?" *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 29107
Hirsch, Werner Z. (1952) "Manufacturing Progress Functions," *The Review of Economics and Statistics*, Vol. 34, No. 2, pp. 143-155.
Hirsch, Werner Z. (1956) "Firm Progress Ratios," *Econometrica*, Vol. 24, No. 2, pp. 136-143.
Hollander, Samuel. (2003) *The Sources of Increased Efficiency: A Study of DuPont Rayon Plants*, MIT Press, Cambridge.
Jones, Charles I. (2005) "The Shape of Production Functions and the Direction of Technical Change," *The Quarterly Journal of Economics*, Vol. 120, No. 2, pp. 517-549.
Joskow, Paul L. and George A. Rozanski. (1979) "The Effects of Learning by Doing on Nuclear Plant Operating Reliability," *Review of Economics & Statistics*, Vol. 61, No. 2, pp. 161-68.
Krueger, Alan B. and Mikael Lindahl. (2001) "Education for Growth: Why and for Whom?" *Journal of Economic Literature*, Vol. 39, No. 4, pp. 1101–1136.
Lazear, Edward P. (2003) "Firm-Specific Human Capital: A Skill-Weights Approach," *NBER Working Paper*, No. 9679.
Lee, Samson S., John C. Dugger and Joseph C. Chen. (1999) "Kaizen: An Essential Tool for Inclusion in. Industrial Technology Curricula," *Journal of Industrial Technology*, Vol. 16, No. 1.
Lieberman, Marvin B. (1984) "The Learning Curve and Pricing in the Chemical Processing Industries," *The RAND Journal of Economics*, Vol. 15, No. 2, pp. 213-228.
Lieberman, Marvin B. (1987) "The Learning Curve, Diffusion, and Competitive Strategy," *Strategic Management Journal*, Vol. 8, No. 5, pp. 441-452.
Lord, Frederic M. and Melvin R. Novick. (1968) *Statistical Theories of Mental Test Scores*, Addison-Wesley, Reading, MA.
Lovett, Andrew, Kenneth Forbus and Jeffery Usher. (2007) "Analogy with Qualitative Spatial Representations Can Simulate Solving Raven's Progressive Matrices," in *Proceedings from the 29th Annual Conference of the Cognitive Science Society*, pp. 449-454, Cognitive Science Society, Nashville, TN.
Lucas, Robert E. Jr. (1990) "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, Vol. 80, No.2, pp. 92-96.
Luecke, Richard and Ralph Katz (2003). *Managing Creativity and Innovation*, Harvard Business School Press, Boston, MA.
Lynch, Lisa. (1991) "The Role of Off-the-Job Vs. On-the-Job Training for the Mobility of Women Workers," *American Economic Review*, Vo. 81, No. 2, pp. 151-156.
Mankiw, N Gregory, David Romer and David N. Weil. (1992) "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, Vol. 107, No. 2, pp. 407-37
Marshalek, Brachia, David F. Lohman and Richard E. Snow. (1983) "The Complexity Continuum in the Radex and Hierarchical Models of Intelligence," *Intelligence*, Vol. 7, No. 2, pp. 107-127.

McDonald, Alan. and Leo Schrattenholzer. (2001) "Learning Rates for Energy Technologies," *Energy Policy*, Vol. 29, No. 4, pp. 255-261.
McGreggor, Keith, Maithillee, Kunda and Ashok Goel. (2010) "A Fractal Analogy Approach to the Raven's Test of Intelligence," in *AAAI workshops at the 24th AAAI conference on Artificial Intelligence*, pp. 69–75, Association for the Advancement of Artificial Intelligence, Atlanta.
Michelacci, Claudio and Paolo Zaffaroni. (2000) "(Fractional) Beta Convergence," *Journal of Monetary Economics*, Vol. 45, No. 1, pp. 129-153.
Miketa, Asami and Leo Schrattenholzer. (2004) "Experiments with a Methodology to Model the Role of R&D Expenditures in Energy Technology Learning Processes; First Results," *Energy Policy*, Vol. 32, pp. 1679–1692.
Mincer, Jacob. (1958) "Investment in Human Capital and Personal Income Distribution," *The Journal of Political Economy*, Vol. 6, No. 4, pp. 281-302.
Nemet, Gregory F. (2006) "Beyond the Learning Curve: Factors Influencing Cost Reductions in Photovoltaics," *Energy Policy*, Vol. 34, No.17, pp. 3218-3232.
OECD/IEA (2000) *Experience Curves for Energy Technology Policy*, The Organization for Economic Co-operation and Development (OECD), Paris, France.
Papineau, Maya. (2006) "An Economic Perspective on Experience Curves and Dynamic Economies in Renewable Energy Technologies," *Energy Policy*, Vol. 34, No. 4, pp. 422-432.
Pisano, Gary P., Richard M.J. Bohmer and Amy C. Edmondson. (2001) "Organizational Differences in Rates of Learning: Evidence from the Adoption of Minimal Invasive Cardiac Surgery," *Management Science*, Vol. 47, No. 6, pp. 752-768.
Prescott, Edward C. (1998) "Needed: A Theory of Total Factor Productivity," *International Economic Review*, Vol. 39, No. 3, pp. 525-51.
Preston, Lee E. and E. C. Keachie. (1964) "Cost Functions and Progress Functions: An Integration," *American Economic Review*, Vol. 54, No. 2, pp. 100-107.
Rapping, Leonard. (1965) "Learning and World War II Production Functions," *The Review of Economic Statistics*, Vol. 47, No. 1, pp. 81–86.
Raven, John Carlyle. (1962). *Advanced Progressive Matrices*: *Sets I and II,* H. K. Lewis, London.
Raven, J., John Carlyle Raven and J. H. Court, (1998) *Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 1: General overview*. Harcourt Assessment, San Antonio, TX.
Reis, Dayr A. (1991) "Learning Curves in Food Services," *The Journal of the Operational Research Society*, Vol. 42, No. 8, pp. 623-629.
Romer, Paul Michael. (1986) "Increasing Returns and Long-run Growth," *Journal of Political Economy*, Vol. 94, No. 5, pp. 1002-37.
Romer, Paul Michael. (1987) "Growth Based on Increasing Returns Due to Specialization," *American Economic Review*, Vol. 77, No. 2, pp. 56-62.
Searle, Allan D. and C. S. Goody. (1945) "Productivity Increases in Selected Wartime Shipbuilding Programs," *Monthly Labor Review*, Vol. 61, No. 6, pp. 1132-1147.
Sheshinski, Eytan. (1967) "Tests of the 'Learning by Doing' Hypothesis," *The Review of Economics and Statistics*, Vol. 49, No. 4. pp. 568-578.
Snow, Richard E., Patrick C. Kyllonen and Brachia Marshalek. (1984) "The Topography of Ability and Learning Correlations," in Sternberg, Robert J. ed. *Advances in the Psychology of Human Intelligence Vol. 2*, Erlbaum, Hillsdale, NJ:
Sorenson, Olav. (2003) "Interdependence and Adaptability: Organizational Learning and the Longterm Effect of Integration," *Management Science*, Vol. 49, No. 4, pp. 446-463.
Stobaugh, Robert B. and Phillip. L. Townsend. (1975) "Price Forecasting and Strategic Planning: The Case of Petrochemicals," *Journal of Marketing Research*, Vol. 12, pp. 19-29.

Sultan, Ralph G. (1976) *Pricing in the Electrical Oligopoly, Vol. 2: Business Strategy*, Division of Research, Graduate School of Business Administration, Harvard University.

van der Linden, Wim J. and Ronald K. Hambleton. (Eds.) (1997) *Handbook of Modern Item Response Theory*, Springer-Verlag, New York.

van der Zwaan, Bob and Ari Rablc. (2003) "Prospects for PV: A Learning Curve Analysis," *Solar Energy*, Vol. 74, No. 1, pp. 19–31.

van der Zwaan, Bob and Ari Rablc. (2004) "The Learning Potential of Photovoltaics: Implications for Energy Policy," *Energy Policy*, Vol. 32, No. 13, pp. 1545–1554.

Weisbroad, Burton A. (1966) "Investing in Human Capital," *The Journal of Human Resources*, Vol. 1, No. 1, pp. 5-21.

Wiersma, Eelke. (2007) "Conditions That Shape the Learning Curve: Factors That Increase the Ability and Opportunity to Learn," *Management Science*, Vol. 53, No. 12, pp. 1903-1915.

Williams, Robert H. and Terzian, G. (1993), "A Benefit/Cost Analysis of. Accelerated Development of Photovoltaic Technology," *PU/CEES Report No. 281*, Center for Energy and Environmental Studies, Princeton University, NJ.

Womer, Norman Keith. (1984) "Estimating Learning Curves from Aggregate Monthly Data," *Management Science*, Vol. 30, No. 8, pp. 982-992.

Womer, Norman Keith and J. Wayne Patterson. (1983) "Estimation and Testing of Learning Curves," *Journal of Business & Economic Statistics*, Vol. 1, No. 4, pp. 265-272.

Wright, Theodore Paul. (1936) "Factors Affecting the Cost of Airplanes," *Journal of the Aeronautical Sciences*, Vol. 3, No. 4, pp. 122-128.

Yelle, Louis E. (1979) "The Learning Curve: Historical Review and Comprehensive Survey," *Decision Sciences*, Vol. 10, No. 2, pp. 302-328.

Zimmerman, Martin B. (1982) "Learning Effects and the Commercialization of New Energy Technologies: The Case of Nuclear Power," *Bell Journal of Economics*, Vol. 13, No. 2, pp. 297-310.

Figure 1 ***The distribution of innovation***