Forecasting Macedonian GDP: Evaluation of different models for short-term forecasting

Branimir, Jovanovic and Magdalena, Petrovska

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Branimir Jovanovic, MSc
Research Department
National Bank of the Republic of Macedonia

Magdalena Petrovska, MSc
Research Department
National Bank of the Republic of Macedonia

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Abstract

We evaluate the forecasting performance of six different models for short-term forecasting of Macedonian GDP: 1) ARIMA model; 2) AR model estimated by the Kalman filter; 3) model that explains Macedonian GDP as a function of the foreign demand; 4) small structural model that links GDP components to a small set of explanatory variables; 5) static factor model that links GDP to the *current* values of several principal components obtained from a set of high-frequency indicators; 6) FAVAR model that explains GDP through its own lags and lags of the principal components. The comparison is done on the grounds of the Root Mean Squared Error and the Mean Absolute Error of the one-quarter-ahead forecasts. Results indicate that the static factor model outperforms the other models, providing evidence that information from large dataset can indeed improve the forecasts and suggesting that future efforts should be directed towards developing a state-of-the-art dynamic factor model. The simple model that links domestic GDP to foreign demand comes second, showing that simplicity must not be dismissed. The small structural model that explains every GDP component as a function of economic determinants comes third, "reviving" the interest in these old-school models, at least for the case of Macedonia.

Keywords: GDP; forecasting; structural model; principal component; FAVAR; static factor model; Macedonia

JEL classification: C53; E27; E37

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1. Introduction

Forecasting future economic outcomes is crucial component of the decision-making process in central banks. Monetary policy decisions affect the economy with a lag, so, monetary policy authorities must be forward looking, i.e. must know what is likely to happen in the future. Furthermore, official data on most economic variables are available only with a lag: the first estimates of the GDP are usually available around two months after the end of the reference quarter. Finally, in the case of Macedonia, having an accurate forecast for the GDP on a horizon of one or two quarters is a necessary ingredient for the inflation forecasting model, which is used at the Macedonian central bank for inflation forecasting and policy analysis purposes. For these reasons, in this paper we evaluate the performances of several different models for short-term forecasting of the Macedonian GDP.

We evaluate six different models. The first one is a simple Autoregressive Integrated Moving Average model of the GDP series, developed following the Box-Jenkins methodology. The second one is an Autoregressive model of the GDP, estimated by the Kalman filter. The third one is a model that explains Macedonian GDP as a function of the foreign GDP, i.e. weighted average of the GDP of the biggest trading partners. The fourth model is a small structural model that links each of the expenditure components of the GDP to a small set of explanatory variables. The fifth and the sixth model are based on a principal components analysis, i.e. they extract a few principal components from a medium-size dataset of indicative variables, and then use these principal components to forecast the GDP. The fifth model is a simple Ordinary Least Squares (OLS) regression that links the GDP to the current values of the principal components, while the sixth model is a Vector Autoregression (VAR) model that includes the GDP and the principal components.

The models are compared on the grounds of two standard measures of forecasting performance - the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) of the one-quarter-ahead forecasts. We focus on the one-quarter-ahead forecasts, and not on a longer horizon, since the models will be primarily used for forecasting the next-quarter GDP. We also employ the Diebold-Mariano test, to see if the difference in the accuracy of the forecasts obtained from different models is statistically significant.

The paper is organized as follows. In the next section we give a brief overview of the forecasting methods that are most often found in the literature. In the third section we explain the models that we use in greater detail. Section 4 explains the data, while section 5 presents the design of the forecasting exercise. Section 6 gives the results and the final section concludes and points out to some areas for future research.

2. Overview of forecasting techniques

Recent decades have seen a proliferation of different methods for economic forecasting, following the computational advances and the development in econometric methods. Broadly speaking, models for economic forecasting can be classified into two groups - time series models and structural models. ***Time series models*** are mainly statistical, based on historical developments, traditionally with just a few variables and very little, if any, economic content. In ***structural models***, on the other hand, economic theory is used to specify the relationships between the variables, which can be done either by estimation or by calibration.

Earliest time series models were based on a methodology that was first developed in Box and Jenkins (1976), known as ***ARIMA (Auto-Regressive-Integrated-Moving-Average) methodology***. This approach was based on the Wold representation theorem, which states that every stationary time series has an *infinite* moving average (MA) representation, which actually means that its evolution can be expressed as a function of its past developments. This infinite MA representation can be approximated as a *finite* order autoregressive-moving average (ARMA) process. Thus, according to this methodology, a series is first differenced as many times as needed to achieve stationarity (therefore, "integrated"), after what a tentative ARMA model is fit to it. After a satisfying approximate representation is found, it can be used for forecasting. Nowadays, ARIMA models are usually found in studies as benchmark models against which other models are evaluated.

A multivariate extension (with more than one variable) to the univariate ARIMA models are ***Vector Autoregressions (VARs).*** A VAR specifies a group of economic series as a function of each series' past values. For instance, a VAR including GDP and inflation explains both GDP and inflation as depending on past values of the GDP and the inflation. Differently from the ARIMA models, VARs are not necessarily purely time series models. They can also incorporate theoretical considerations to some extent, and stand somewhere in between the purely time series models and the structural models. Since Sims (1980), they have received great attention in the economic literature. Although the question of how well the VAR toolkit has been successful in meeting its promises is debatable, VARs are considered very successful for forecasting purposes (see Stock and Watson 2001).

However, it is not the small scale VAR models, with just a few variables, that have proved to be good at forecasting, but ***larger, Bayesian VARs***, like those in Litterman (1986), Sims (1993) and Sims and Zha (1996). Small-scale VARs, with just a few variables and a small number of lags, usually have not-so-good forecasting properties. On the other hand, in large-scale VARs the number of parameters to be estimated can be very high, often impossible to estimate using traditional methods. For these reasons, Bayesian estimation methods have been employed. Bayesian VARs impose some restrictions on the model coefficients, reducing the dimensionality problem of VARs, resulting in more accurate forecasts. In other words, Bayesian methods impose some restrictions on the data, but also let the data speak for itself (Carriero et al 2007).

The Bayesian approach views the model parameters as distribution functions, where model coefficients, i.e. *the posterior distribution function* is equal to *the prior distribution function*, times *the sample likelihood*. Therefore, to obtain the model coefficients, one has to set the prior. The mean of the prior reflects one's best guess of the value of the parameter, while the variance of the prior reflects how strong one believes in their best guess. One way of setting the prior, present in Doan et al (1984) and Litterman (1986) is the so called Minnesota prior. This approach takes advantage of the fact that very often macroeconomic series are best described as random walks - that the best guess for the outcome tomorrow is - the outcome today. Thus, the prior distribution for the model parameters is specified as a normal distribution with mean equal to last period's value. The variance of the prior distribution is specified as a function of some hyperparameters, which determine how much the VAR coefficients can deviate from the prior means (Felix and Nunes, 2003).

Another branch of time series models are the ***unobserved components (UC) models***. According to these models, an observable economic series can be expressed as consisting of unobservable components. The observable series is linked to the unobservable components via the ***measurement equation.*** The unobservable component's dynamics is explained by the ***transition equation***, by some other variables, or by its past developments. For example, the GDP series can be expressed as a sum of the trend, i.e. potential GDP, and the cycle, i.e. output gap, which are both unobservable, by the ***measurement equation***. In the ***transition equation*** the trend and the cycle can be then expressed as some time series models (for instance, random walk with drift for the trend, and autoregressive process for the cycle). This way of expressing a time series is called ***state space representation***. UC models, written in state space form, can be estimated by the ***Kalman filter,*** which is an iterative algorithm that can be used for many purposes, including estimation. For more on UC models, see Harvey, 2006. UC models can be both univariate and multivariate. In the univariate UC model, a series depends only on its past values. The multivariate UC models, on the other hand, can incorporate economic theory, as well, and in these models the dynamics of a series is not completely explained by its past developments, but by other variables, as well.

Most recently, the focus of the literature in the field of economic forecasting has been moved towards extracting information from ***large datasets*** (e.g. more than 100 series). The methods developed in this area can be generally classified into two subgroups - forecast combination (with its extensions - Bayesian model averaging and empirical Bayes methods) and factor models. ***Forecast combination*** methods try to combine more than one forecast from different models into a single forecast, while ***factor models*** try to summarize large dataset of variables into a few common factors (for a thorough overview on methods for forecasting with a large number of variables, as well as for more on forecast combination, see Stock and Watson 2006).

Factor model methods view every series coming from a large dataset as consisting of two components, independent of each other - a *common component*, which is strongly correlated with the other series from the dataset, and an *idiosyncratic component*, which is specific to every series. Strict factor models consider the idiosyncratic components of all series independent of each other, while approximate factor models relax this assumption. The common component of the series is driven by a small number of factors. Factor models thus focus on extracting these common factors and on using them for a variety of purposes, including forecasting.

Barhoumi et al (2008) classify the factor models into three groups - static principal component as in Stock and Watson (2002), dynamic principal components estimated in the *time* domain, as in Doz et al (2006 and 2007) and dynamic principal components in the *frequency* domain, as in Forni et al (2000, 2004 and 2005). ***The Stock and Watson approach*** consists of deriving the static principal components in the conventional manner, as a weighted average of all the series, and then using them to forecast the economic series of interest, through OLS regressions. ***The Doz et al approach*** is slightly more involved and uses the Kalman filter to extract the common factors. ***The Forni et al approach***, also known as generalized dynamic factor model, estimates the dynamic principal components on the grounds of the spectral density matrix of the data, i.e. the data are weighted according to their signal-to-noise ratio (Barhoumi et al 2008). While the latter two approaches are more sophisticated, studies have shown that they perform no better than the static principal component approach (see Barhoumi et al 2008). Also, studies have shown that smaller datasets with about 40 series outperform larger datasets with disaggregated data, with more than 100 series (Bai and Ng, 2002, Watson, 2003, Boivin and Ng, 2006, Barhoumi et al, 2008).

Up till now, our discussion was focused on models that can be roughly classified as time series models. On the opposite side of the spectrum are the ***structural economic models***, which are based on relationships stemming from economic theory. Earliest structural economic models were large scale models, also known as Cowles Commission type models. Some of the most famous models of this type are the Klein-Goldberger model, the MPS model, the Brookings Quarterly Econometric model and the Wharton model. These models were based on Keynesian theory, they consisted of estimated regressions between many economic variables (for instance, the Brookings model consisted of nearly 400 equations), and were developed by famous economists at the time. Cowles Commission type models were very popular and successful until the 1970s. However, they started performing poorly in the 1970s, and were largely abandoned after the "revolution" that macroeconomics experienced since (see Mankiw, 1991, Woodford, 1999, Mankiw, 2006, Goodfriend, 2007). They have been criticized for ad-hockery in specifying the relationships, for lack of micro foundations, but first and foremost for not being policy invariant (i.e. the outcome of the model depends on the policy that is proposed - the famous Lucas critique).

The failure of these models and the "revolution" that the field of macroeconomics experienced since, eventually led to the development of the ***Dynamic Stochastic General Equilibrium (DSGE) models***. DSGE models are based on microeconomic foundations, assume general equilibrium in the economy and are deemed policy invariant. However, there is a sound and ongoing debate in the economic literature about the merits of these models (for a good overview, see Tovar, 2008). For a long time, DSGE models have been considered especially weak at forecasting, although Smets and Wouters (2003) and Del Negro and Shorffheide (2004) show that forecasts obtained from DSGE models can be as good as forecasts from Bayesian VARs.

3. Models for short term forecasting of Macedonian GDP

3.1. The "ARIMA" model

The first model that we consider is based on the ARIMA framework. ARIMA models are purely time series models, they are agnostic of economic considerations, but have still proven to be relatively robust for forecasting, especially on short horizons. The ARIMA model outlined in this part provides only a benchmark against which other models are compared.

We follow the Box-Jenkins methodology for fitting an ARIMA model to the Macedonian GDP (for details on this methodology, see Box and Jenkins, 1976, or Hamilton, 1994). As the GDP series was non-stationary, we first differenced it. We then fit the corresponding ARMA model to this transformation. The finally chosen model is ARIMA (3,1,3), shown below.

$$y_t = 0.01 + 0.86 * y_{t-1} - 0.34 * y_{t-2} - 0.26 * y_{t-3} - 1.25 * \varepsilon_{t-1} + 1.12 * \varepsilon_{t-2} - 0.11 * \varepsilon_{t-3} \qquad (1)$$

where y_t stands for the first difference of the log of the GDP. The correlograms of the Autocorrelation and the Partial correlation functions of the difference of the logged GDP, of the residuals after the ARMA model was fit, as well as the results of this model are shown in the Appendix.

3.2. The "Kalman AR" model

The Kalman AR model is slightly more advanced univariate time series model, an AR model estimated by the Kalman filter. Every ARIMA model can be generally represented in the following state space form:

$$y_t = z_t' a_t + \varepsilon_t \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \quad (2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t \qquad \eta_t \sim NID(0, Q_t) \quad (3)$$

for $t = 1, \ldots, n$. Equation (2) is called the *observation* or *measurement* equation, equation (3) is called the *transition* or *state* equation.

The terms y_t and ε_t are still scalars (i.e. of order 1 × 1). However, the remaining terms in (2) and (3) denote vectors and matrices. Specifically, z_t is an $m \times 1$ *observation* or *design* vector, T_t is an $m \times m$ *transition* matrix, a_t is an $m \times 1$ *state vector*, and m therefore denotes the number of elements in the state vector, i.e. the number of lags. In many state space models R_t in (3) is simply the identity matrix of order $m \times m$. However, in various models it is of order $m \times r$ with $r < m$, and consists of the first r columns of the identity matrix I_m. In this case R_t is called a *selection* matrix since it selects the rows of the state equation which have non-zero disturbance terms. Finally, the $r \times 1$ vector η_t contains the r state disturbances with zero means, and unknown variances collected in an $r \times r$ diagonal matrix Q_t.

The above model is estimated by the Kalman filter algorithm, which is a powerful estimator, consisting of predicting and updating equation (for more on Kalman filter, see Hamilton, 1994, Harvey, 1989 or Harvey, 1993).

The AR approximation to the Macedonian de-meaned, de-trended and seasonally-adjusted GDP (Y_t) is given by the Yule-Walker equations for fitting AR models, minimizing the Akaike Information Criterion. The chosen order of the AR process was 4. The results of the estimation of the model over the whole sample are shown below. Details are reserved for the Appendix.

$$Y_t = -0.113\,Y_{t-1} + 0.304\,Y_{t-2} + 0.003\,Y_{t-3} - 0.508\,Y_{t-4} + e_t \quad (4)$$

3.3. The "foreign demand" model

The *"foreign demand"* model is based on a simple and intuitive premise - domestic GDP depends on foreign GDP. This can occur through at least two channels - demand and expectations. Higher foreign GDP implies higher external ***demand*** for Macedonian products, which increases Macedonian exports, and consequently GDP. Alternatively, higher GDP abroad makes domestic economic agents ***expect*** that domestic income will be higher, which than translates into higher investment and consumption. The *"foreign demand"* model just exploits this empirical regularity without going into its underpinnings.

In terms of the model specification, this relationship is represented by including the foreign demand as an explanatory variable for the Macedonian GDP (see section "Data" for how the foreign demand variable is constructed). However, domestic GDP does not depend entirely on foreign GDP. It seems that part of domestic GDP is not influenced by foreign GDP movements. This is captured by including a lag of domestic GDP in the regression. The lag of the domestic GDP captures the inertia, i.e. the persistence that is observed in GDP, which may be due to habits in consumption, or expectations, or other factors. The regression specification is given below:

*dlog(gdp_mk) = a1 +a2*dlog(foreign_demand)+a3*dlog(gdp_mk(-1))* (5)

where dlog stands for the first difference of the logs of the variables, and both Macedonian GDP and foreign demand are seasonally-adjusted. The regression is estimated by Ordinary Least Squares, since both Macedonian GDP and the foreign demand are stationary. The results of the regression are given below:

*dlog(gdp_mk_sa) = 0.01+0.70 dlog(fordems)-0.27*dlog(gdp_mk_sa(-1))* (6)

Forecasting with this model requires assuming certain values for the future behavior of the foreign demand. In real time operation of this model forecasts for the foreign GDP are taken from some external source, like Consensus Forecast. In this forecasting exercise, however, actual data on foreign GDP were used. Details of the estimation are given in the Appendix.

3.4. The "GDP components" model

The "GDP components" model is a small, old-fashioned, structural model, which represents Macedonian GDP as a sum of its expenditure components (eq. 7) - private consumption ***(cons)***, government consumption ***(gov)***, gross investments ***(inv)***, exports of goods and services ***(exp)*** and imports of goods and services ***(imp)***. Every GDP component is modeled as a function of some explanatory variables (except government consumption, which is taken exogenously, from the projections of the budget). The structure of each equation is inspired from the economic theory, but, as in the previous case, the focus is on the forecasting accuracy, not on inference or analysis. ***Private consumption*** is modeled as a function of the income (average net wage multiplied with the number of employed persons), private transfers from abroad and the interest rate on credits (eq. 8). This structure of the equation corresponds to the standard consumption functions usually met in the literature. Our approach to modeling ***investments***, which are usually very difficult to model, was to break them down to components, and then to try to find variables that will explain the components to a reasonable extent. Hence, investments (eq. 9) are modeled as a function of the foreign direct investment, government capital expenditure, industrial production (capturing domestic private investments) and exports (capturing the rise/decline in the inventories as exports decrease/increase). It is the inclusion of exports that proved vital in obtaining a reasonable fit for the investments equation, which actually implies that investment dynamics is driven by the inventories. ***Exports*** (eq. 10) are modeled as a function of the foreign demand and the relative export prices (price of Macedonian exports, relative to world prices of the same products). ***Imports*** are modeled as a function of the private consumption, investments, government consumption and exports (eq. 11). The ***government consumption,*** as already mentioned, was taken as an exogenous variable, from the projections of the government budget.

In addition to these variables, the model includes equations for the wages, the number of employed persons and the industrial production. ***Wages*** are modeled as a function of the price level, the GDP and the employment (eq. 12), the number of ***employed persons*** depends on inertia and the GDP (eq. 13), and ***industrial production*** depends on its own lag and the foreign demand (eq. 14).

All the equations are specified in "dlog" form, i.e. the variables that enter the equations are the first differences of the logs of the original variables. Exception is the equation for wages and for employment, which are specified in a Vector Error Correction form. All the variables are seasonally-adjusted.

gdp_mk = cons + gov + inv + exp - imp (7)

*cons = f(wages*employed, transfers, interest_rate)* (8)

inv = f(gov_capital, FDI, industrial, exports) (9)

exp = f(fordem, relative_exp_price) (10)

imp = f(cons, inv, gov, exp, imp) (11)

wages = f(CPI, GDP, empolyed) (12)

employed = f(GDP) (13)

industrial = f(fordem) (14)

Thus, the model consists of eight equations (seven structural and one identity), eight endogenous variables and eight exogenous. Due to the interdependencies between the regressions, the model is estimated as a system, by the Seemingly Unrelated Regression method.

Forecasting with this model requires setting assumptions for the exogenous variables - interest rate, private transfers, government capital expenditure, FDI, foreign demand, relative export prices and government consumption. For some of these variables the assumptions are taken from projections of the responsible institutions (government consumption and government capital expenditure from the Ministry of Finance, transfers, FDI and interest rates from the Central Bank projections). For some of the variables (foreign demand, export prices) forecasts from external sources are taken (Consensus Economics and IMF).

Below we present the results of the model estimated through 2009q4.

*dlog(cons) = -0.00 + 0.4*dlog(wages*employed) + 0.07*dlog(transfers) + 0.04*dlog(transfers(-1)) - 0.02*d(interest_rate)* (15)

*dlog(inv) = 0.00 - 0.25*dlog(inv(-1)) + 0.01*dlog(FDI) + 0.05*dlog(FDI(-1)) + 0.00*dlog(FDI(-2)) - 0.76*dlog(exp) + 1.88*dlog(industrial) + 0.04*dlog(gov_capital)* (16)

*dlog(exp) = - 0.00 + 1.37*dlog(fordem) - 0.33*dlog(relative_exp_price)* (17)

*dlog(imp) = -0.01 + 0.97*dlog(cons) + 0.53*dlog(exp) + 0.12*dlog(exp(- 1)) + 0.27*dlog(inv) + 0.2*dlog(gov)* (18)

dlog(employed) = -0.18(log**(employed(- 1)) - 0.26*log(GDP(- 1)) - 10.42)** - 0.00*TR0204 + 0.37*dlog(employed(- 1)) + 0.07*dlog(GDP(- 3)) + 0.12*dlog(GDP(- 4))* (19)

*dlog(wages) = -0.11**(log(wages(- 1)) - 1.49*log(CPI(- 1)) - 0.63*log(GDP(- 1)) - 0.87*log(employed(- 1))** + 15.6) + 0.24*dlog(GDP(- 2)) + 0.37*dlog(GDP(- 3))* (20)

*dlog(industrial) = -0.01 - 0.37*dlog(industrial(- 1)) + 1.84*dlog(fordem)* (21)

3.5. The "static factor" model

The "static factor" model falls into the class of static factor models that were explained above. This model actually estimates an OLS regression between the first difference of the logged GDP and few principal components extracted from a dataset of 31 variables. The principal components were extracted as a weighed average of the series, i.e. through an eigendecomposition (spectral decomposition) of the sample covariance matrix. Principal component analysis, or factor analysis in general, which extracts information from a high number of variables, has become quite popular lately, not just for forecasting, but also for policy analysis (see for instance Bernanke et al 2005).

The dataset in our case consisted of 31 variables (see Table 1), which is much less than what is usually met in the literature. However, data availability is a big problem for Macedonia, and even collecting 31 variables for the period 1997-2009 is quite a laborious task. Furthermore, this is a first attempt at estimating this type of model for the case of Macedonia. Still, one must not forget that more is not necessarily better when working with factor models (see Bai and Ng, 2002, Watson, 2003, Boivin and Ng, 2006, Berhoumi et al, 2008).

Table 1: Variables included

1 VAT	17 Production of consumption goods
2 PPI (Producer Price Index)	18 Total deposits
3 Government capital expenditures	19 Real effective exchange rate
4 Foreign effective demand	20 Foreign direct investments
5 Completed construction works	21 Gross foreign reserves
6 Industrial production	22 Government revenues
7 Domestic CPI	23 Private transfers
8 Foreign effective CPI	24 M4 monetary aggregate
9 CB bills interest rate	25 Retail trade
10 Telecommunications	26 Wholesale trade
11 Credits to households	27 Exports of goods
12 Credits to firms	28 Imports of goods
13 Metals prices	29 Imports of consumption goods
14 Oil price	30 Imports of means of production
15 Average net wage	31 Employed persons
16 Production of capital goods	

All the variables were logged and differenced, to make them stationary. Regarding the selection of the principal components (PCs), we did not follow the recommendations in the literature. These recommendations basically state that the first few components, that explain most of the variation, should be retained (say, the first five PCs, or the PCs that explain 90% of the variation). However, our experience showed that following these rules results in worse forecasts. Thus, we first run an OLS regression between the GDP and all the PCs, and then retained only those PCs that were significant. In this way we ended up with 5 PCs (the third, the fifth, the sixth, the tenth and the eighteenth), that explained only a small bit of the sample variation, but proved to forecast the GDP much better than the PCs that explained most of the variation. The results of the model, estimated for the whole period, are presented in the Appendix.

Forecasting with this model requires setting assumptions for the factors for the forecast horizon. In the literature, this is usually done by assuming some time series model for the factors. In this pseudo-out-of-sample forecasting exercise, however, we did not set the future values of the factors by assumption, but we used the actual series of the factors, which means that we have assumed that our forecast of the future evolution of the factors has been perfect. As this seems highly unlikely, the forecasts from this model are likely to be worse than those that we obtained in this exercise (as a matter of fact, this holds for all our models, we just emphasize it here).

3.6. The "FAVAR" model

The "FAVAR" model is another model that is based on a principal component analysis. It estimates a VAR model between the GDP (i.e. the first difference of the logged GDP) and the same principal components from above. The VAR included the GDP and five principal components, with only 1 lag of every variable (including more lags failed to improve the forecasting performance). The results of the model, estimated for the whole period, are presented in the Appendix. As this model is essentially a VAR, forecasting with it does not require setting assumptions for the factors.

4. Data

We use a total of 31 series, covering roughly all areas of the economy. The series are from the official institutions - the State Statistical Office of the Republic of Macedonia, the Ministry of Finance of the Republic of Macedonia, the National Bank of the Republic of Macedonia, the IMF. All the series are in real terms. Those that were originally available as nominal series were deflated by the CPI index. The sample period spans from 1997q1 to 2009q4. The data sources for the variables are shown in Table 2.

The ***foreign demand*** variable is calculated as a weighted average of the GDP of nine major trading partners (Germany, Greece, Italy, Serbia, Belgium, Spain, Netherlands, Bulgaria and Croatia). The weights are obtained as normalized share of these countries in Macedonian exports in the period 2006-2009. These countries account for around 67% of Macedonian exports. ***The foreign effective CPI*** is calculated in the similar manner, as a weighted average of the CPI of the ten countries with highest share in the import of consumption goods (Serbia, Germany, Greece, Bulgaria, France, Italy, Austria, Slovenia, Croatia, United States). The weights are obtained from the normalized share of the countries in Macedonian imports of consumption goods for the period 2006-2009. ***Relative export prices*** are calculated as Macedonian export prices, relative to world prices of Macedonian exports. The products that were included in the world prices index include cotton, iron ore, lamb, nickel, steel, zinc and petrol. The weights are obtained from the normalized shares of these products in the total exports, and the prices for these products are from IMF and Bloomberg.

Table 2 – Data used and sources of data

Series	Source
Macedonian GDP	State Statistical Office
Private consumption	State Statistical Office
Gross investments	State Statistical Office
Government consumption	State Statistical Office
Exports of goods and services	State Statistical Office
Imports of goods and services	State Statistical Office
Interest rate on credits	National Bank of the Republic of Macedonia
Relative export prices	Compiled by NBRM, on data the State Staristical Office, IMF and Bloomberg
VAT	Ministry of finance
PPI (Producer Price Index)	State Statistical Office
Government capital expenditures	Ministry of finance
Foreign effective demand	Compiled by NBRM, on data from Eurostat and national statistical offices
Completed construction works	State Statistical Office
Industrial production	State Statistical Office
Domestic CPI	State Statistical Office
Foreign effective CPI	Compiled by NBRM, on data from Eurostat and national statistical offices
CB bills interest rate	National Bank of the Republic of Macedonia
Telecommunications	State Statistical Office
Credits to households	National Bank of the Republic of Macedonia
Credits to firms	National Bank of the Republic of Macedonia
Metals prices	IMF
Oil price	IMF
Average net wage	State Statistical Office
Production of capital goods	State Statistical Office
Production of consumption goods	State Statistical Office
Total deposits	National Bank of the Republic of Macedonia
Real effective exchange rate	National Bank of the Republic of Macedonia
Foreign direct investments	National Bank of the Republic of Macedonia
Gross foreign reserves	National Bank of the Republic of Macedonia
Government revenues	Ministry of finance
Private transfers	National Bank of the Republic of Macedonia
M4 monetary aggregate	National Bank of the Republic of Macedonia
Retail trade	State Statistical Office
Wholesale trade	State Statistical Office
Exports of goods	State Statistical Office
Imports of goods	State Statistical Office
Imports of consumption goods	State Statistical Office
Imports of means of production	State Statistical Office
Employed persons	State Statistical Office

5. Design of the forecast evaluation exercise

We carry out a "pseudo one quarter ahead" forecasting exercise, which means that we estimate the models up to a certain data point (e.g. 2003q4), and use the data that are available now (not that would have been available then), to forecast the next quarter (e.g. 2004q1). This means that actual realizations for the exogenous variables in the models are used, instead of assumptions (therefore, "pseudo"). The starting point in our evaluation is 2004q1, which means that we have 24 periods for forecasting.

We use two alternative criteria for comparing the models ***- the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) of the forecasts.***

The RMSE is calculated by the following formula:

$$RMSE = \sqrt{[\frac{1}{N}\sum_{i=1}^{N}(GDP_{for} - GDP_{act})^2]} \quad (17)$$

where N is the number of observations (in our case 24), GDP_{for} is the forecasted GDP and GDP_{act} is the actual (realized) GDP.

The RMSE is the most widely used criterion for assessing forecasts, but its weaknesses are also well known, especially its penalty for outliers. This is why we also use the Mean Absolute Error, which is calculated according to the formula:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}[abs(GDP_{for} - GDP_{act})] \quad (18)$$

Additionally, ***to see whether the forecast differences of the alternative models are significant, the Diebold-Mariano test*** (DM test) was carried out (see Diebold and Mariano, 1995, Harvey, Leybourne and Newbold, 1997). This test tests whether the forecast errors of two models are significantly different from each other. We compare the forecast errors of the different models with the forecast errors of the model with lowest RMSE and MAE. This actually means that we test whether the forecasts of the "best" model are better than the forecasts of the remaining models. Basis of the DM test is the sample mean of the observed loss differential series {dt : t=1, 2, …}.

Two time series of forecast errors are: $e_{i1},..., e_{iT}$ and $e_{j1},..., e_{jT}$. The quality of each forecast is evaluated by some loss function g of the forecast error.

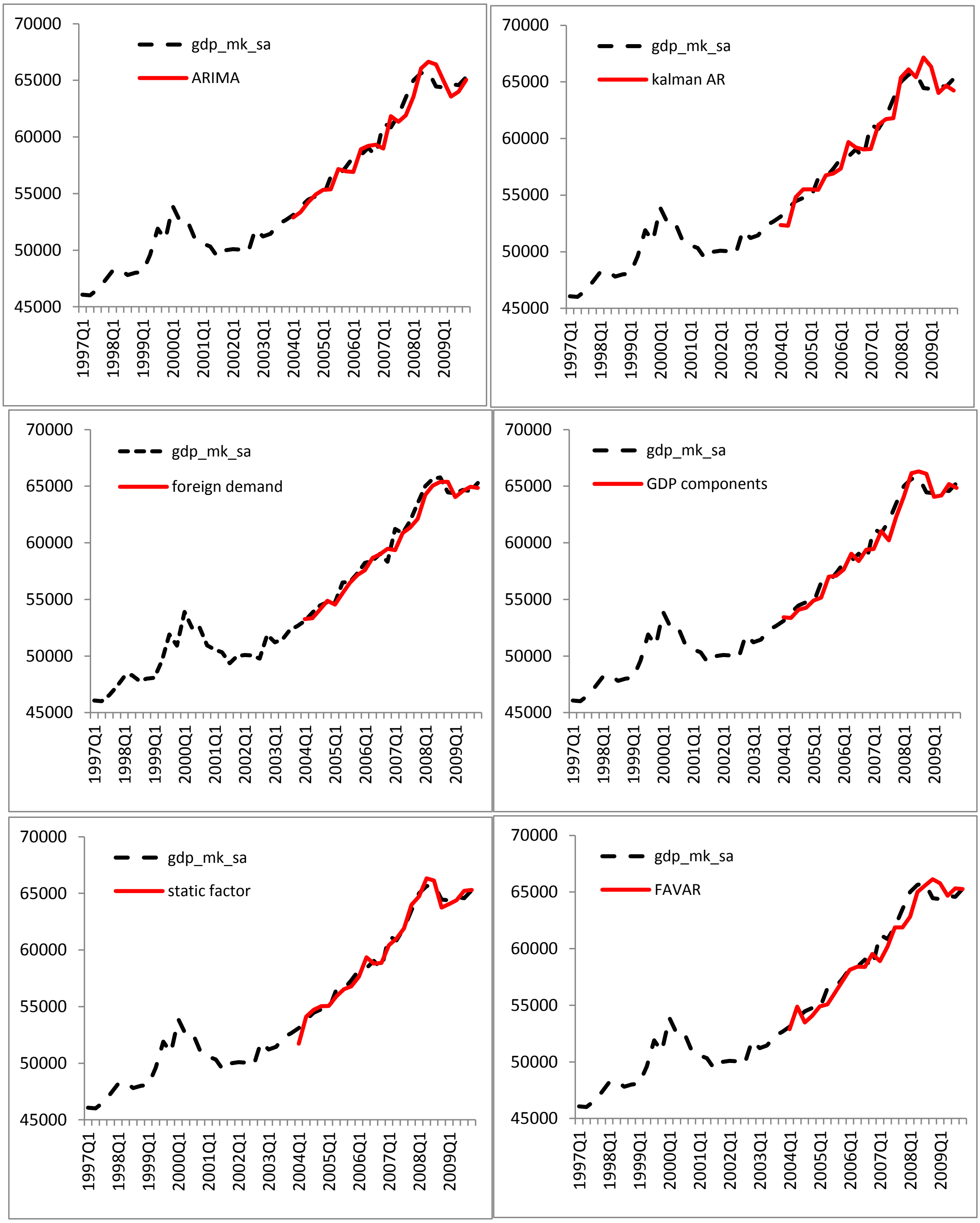
The null hypothesis of equal predictive accuracy is:

$E(d_t) = 0$ for all t where $d_t = g(e_{it}) - g(e_{jt})$

6. Results

Forecasts of the alternative models, compared to the actual GDP are shown on Figure 1. The forecasted, values, the forecast errors, and the RMSE and the MAE are presented in the Appendix.

Figure 1: Comparison of the forecasts of the GDP and the actual GDP



Comparison of the forecasting performances of the models, assessed by the RMSE and the MAE is shown in Table 3.

Table 3: RMSE and MAE of the six models

	ARIMA		**Kalman AR**		**Foreign demand**		**GDP components**		**Static factor**		**FAVAR**	
	Result	**Rank**	Result	**Rank**	Result	**Rank**	Result	**Rank**	Result	**Rank**	Result	**Rank**
RMSE	982	**4**	1106	**6**	687	**2**	856	**3**	546	**1**	1038	**5**
MAE	806	**5**	873	**6**	514	**2**	711	**3**	447	**1**	779	**4**
overall		***4***		***6***		***2***		***3***		***1***		***4***

The results show that according to the both RMSE and the MAE, the best performing model is the static factor model. The model based on the foreign GDP comes second according to the both RMSE and MAE. The small structural model "GDP components" is third overall, whereas "FAVAR" and "ARIMA" share the fourth position. The "kalman AR" model is last, performing worse even than the benchmark "ARIMA" model.

The "goodness" of the models can be also intuitively assessed by looking at the difference between the year-on-year growth rates of the GDP *implied by the model*, and the *realized* growth of the GDP. For illustration, for the "static factor" model, this difference is on average 0.8 percentage points (in absolute terms), for the "foreign demand" model it is 0.9, for the "GDP components" it is 1.2, for the "FAVAR" it is 1.3, for the "ARIMA" it is 1.4, and for the "Kalman AR" it is 1.5.

However, to assess the difference of the forecasting performance more rigorously, we computed the Diebold-Mariano (DM) test. This test tests if the forecasts from one model (in our case the "static factor" model) differ from the forecasts from some other model (in our case the remaining five models). The results of these tests are shown in Table 4.

Table 4: P values of the Diebold-Mariano tests

	ARIMA	**KALMAN AR**	**Foreign demand**	**GDP components**	**FAVAR**
p value of the Modified DM test for the ***absolut*** errors	0.01	0.00	0.54	0.04	0.04
p value of the Modified DM test for the ***squared*** errors	0.02	0.01	0.36	0.07	0.02

The results of the DM tests show that the forecasts of the "static factor" model are indeed statistically different at 10%, according to both the squared errors and absolute errors, from all the other models, except the "foreign demand" model. In other words, the forecasts of the "static factor" model are statistically better than these models. Only the "foreign demand" model is not statistically worse than the best model, the "static factor" model.

To summarize, the results of our forecasting exercise indicate that the "static factor" model gives most accurate forecasts, followed by the "foreign demand" and the "GDP components" models. The forecasts of the "static factor" model are statistically different from all the other models, except the “foreign demand” model.

7. Conclusion

Forecasting plays prominent role in the monetary policy decision making process - policy makers must know what the future is likely to be, in order to make the right moves. In this paper we presented six models that have been developed at the National Bank of the Republic of Macedonia, for short term forecasting of the Macedonian GDP, and compared their forecasting performances.

Our results indicate that models that incorporate more data (the ***static factor*** model) seem indeed significantly better at forecasting the Macedonian GDP than the other models. Only the simple ***foreign demand*** model provided forecasts that were not inferior to the forecasts given by the ***static factor*** model, suggesting that simplicity can have some virtues, too. The small structural ***GDP components*** model comes third, outperforming the time-series models and the sophisticated ***FAVAR*** model, reviving the interest in the long forgotten, structural, Cowles Commission-type models. The interest in this model is magnified by the fact that, differently from the other models, the ***GDP components*** model is able to tell the story of the GDP, not just to forecast it. Strangely, the sophisticated ***FAVAR*** model failed to perform better than the purely time series models, which is probably due to the fact that it does not incorporate contemporaneous information, only lagged.

Future work in this field should focus on improvement of the models that were evaluated in this occasion and on developing other state-of-the art models. The finding that the static factor model appeared best for forecasting the Macedonian GDP seems as a nice introduction for further work on the dynamic factor models, as those in Forni et al (2004) and Doz et al (2006). Also, future work must not neglect the Bayesian VAR models, which have been claimed to be particularly good at forecasting.

References

Barhoumi, K., Darné, O. and Ferrara, L. (2009). "Are disaggregate data useful for factor analysis in forecasting French GDP?," Documents de Travail 232, Banque de France.

Bai J. and Ng S. (2002). "Determining the number of factors in approximate factor models". *Econometrica*, 70, 191-221.

Bernanke, Ben, Jean Boivin and Piotr S. Eliasz (2005). "Measuring the Effects of Monetary Policy: A Factor-augmented Vector Autoregressive (FAVAR) Approach*," The Quarterly Journal of Economics*, MIT Press, vol. 120(1), pages 387-422, January.

Boivin J. and Ng S. (2006). "Are more data always better for factor analysis?", *Journal of Econometrics*, 132, 169-194.

Box, G. E. P. and Jenkins G.M. (1976). *Time Series Analysis: Forecasting and Control*, San Francisco CA, Holden Day.

Carriero, A, G. Kapetanios and M. Marcellino (2007). "Forecasting large datasets with reduced rank multivariate models", Working paper No. 617, Queen Mary University of London

Del Negro,Marco and F Schorfheide (2004): "Priors from equilibrium models for VARs*", International Economic Review*, Vol. 45, pp 643-673.

Diebold F.X. and Mariano R.S. (1995). "Comparing predictive accuracy". *Journal of Business and Economic Statistics*, 13, 253-265.

Doz C., Giannone D. and Reichlin L. (2006). "A quasi maximum likelihood approach for large approximate dynamic factor models". CEPR Discussion Paper No. 5724.

Doz C., Giannone D. and Reichlin L. (2007). "A two-step estimator for large approximate dynamic factor models based on Kalman filtering". CEPRDiscussion Paper No 6043.

Felix, R. M. and L. C. Nunes (2003). "Forecasting Euro Area Aggregates with Bayesian VAR and VECM Models". Working Paper 4-03, Banco De Portugal

Forni M., Hallin M., Lippi M. and Reichlin L. (2000). "The generalized dynamic factor model: Identification and estimation". *Review of Economics and Statistics*, 82, 540-554.

Forni M., Hallin M., Lippi M. and Reichlin L. (2004). "The generalized dynamic factor model: Consistency and rates". *Journal of Econometrics*, 119, 231-255.

Forni M., Hallin M., Lippi M. and Reichlin L. (2005). "The generalized dynamic factor model: One-sided estimation and forecasting". *Journal of the American Statistical Association,* 100, 830-840.

Goodfriend, Marvin (2007). "How the World Achieved Consensus on Monetary Policy," NBER Working Papers 13580, National Bureau of Economic Research, Inc.

Hamilton, James D. (1994), *"Time Series Analysis"*, Princeton University Press

Harvey, Andrew C. (1989), *"Forecasting, Structural Time Series Models and the Kalman Filter"*, Cambridge University Press

Harvey, Andrew C. (1993), *"Time Series Models"*, Harvester Wheatsheaf, Second Edition

Harvey, Andrew (2006). "Forecasting with Unobserved Components Time Series Models", in *Handbook of Economic Forecasting,* vol. 1, Elliott, G, C. W. J. Granger and A. Timmermann, eds. North Holland

Harvey, D. I., Leybourne, S. J., and Newbold, P. (1997), "Testing the Equality of Prediction Mean Squared Errors," *International Journal of Forecasting*, 13, 281-291.

Litterman, Robert B. (1986). "Forecasting With Bayesian Vector Autoregressions: Five Years of Experience." *Journal of Business and Economic Statistics*.January, 4:1, pp. 25–38.

Mankiw, N. Gregory (1991). "A Quick Refresher Course in Macroeconomics," NBER Working Papers 3256, National Bureau of Economic Research, Inc.

Mankiw, N. Gregory (2006)."The Macroeconomist as Scientist and Engineer," NBER Working Papers 12349, National Bureau of Economic Research, Inc.

Sims, Christopher A. (1980). "Macroeconomics and Reality," *Econometrica*, vol. 48(1), pages 1-48, January.

Sims, Christopher A. (1993). "A Nine Variable Probabilistic Macroeconomic Forecasting Model," *in NBER Studies in Business: Business Cycles, Indicators, and Forecasting*, Volume 28. James H. Stock and Mark W. Watson, eds. Chicago: University of Chicago Press, pp. 179–214.

Sims, Christopher A. and Tao Zha (1996). "Bayesian methods for dynamic multivariate models," Working Paper 96-13, Federal Reserve Bank of Atlanta.

Smets, F and R Wouters (2007): "Shocks and frictions in US business cycles: a Bayesian DSGE approach",*The American Economic Review*, Vol 97, No. 3, pp. 586-606.

Stock, James H., and Mark W. Watson (2001). "Vector Autoregressions," *Journal of Economic Perspectives*, Vol. 15 No. 4, pp. 101-115.

Stock, J.H. and M. Watson, (2002), "Forecasting using principal components from a large number of predictors". *Journal of the American Statistical Association* 97, 1167-1179.

Stock, James H., and Mark W. Watson (2006). "Forecasting with Many Predictors", in *Handbook of Economic Forecasting*, vol. 1, Elliott, G, C. W. J. Granger and A. Timmermann, eds. North Holland

Thomas Doan, Robert Litterman and Christopher Sims (1984). "Forecasting and conditional projection using realistic prior distributions", *Econometric Reviews* 1, pp. 1-100

Tovar, Camilo E. (2008), "DSGE Models and Central Banks", BIS Working Paper No. 258

Watson M. (2003). "Macroeconomic forecasting using many predictors". In Dewatripont M., Hansen L. and Turnovsky S. (eds), *Advances in Economics and Econometrics, Theory and Applications: Eighth World Congress of the Econometric Society*, Vol. III.

Woodford, M. (1999). "Revolution and Evolution in Twentieth-Century Macroeconomics," , Presented at a conference, *Frontiers of the Mind in the Twenty-First Century,* U.S. Library of Congress, Washington, D.C., June 1999.

APPENDIX

1. Details of the estimation of the "ARIMA" model

Correlograms of the Autocorrelation and the Partial Correlation function for dlog(GDP)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.221	-0.221	2.6327	0.105
		2	0.267	0.229	6.5582	0.038
		3	-0.032	0.071	6.6150	0.085
		4	-0.059	-0.126	6.8136	0.146
		5	-0.081	-0.140	7.1994	0.206
		6	-0.174	-0.186	9.0125	0.173
		7	0.143	0.160	10.272	0.174
		8	-0.139	0.010	11.486	0.176
		9	0.020	-0.116	11.512	0.242
		10	0.017	-0.024	11.531	0.318
		11	0.057	0.094	11.754	0.382
		12	-0.179	-0.192	13.972	0.302
		13	0.044	-0.060	14.108	0.366
		14	0.014	0.084	14.122	0.441
		15	-0.085	-0.038	14.667	0.476
		16	-0.052	-0.147	14.880	0.533
		17	0.098	0.074	15.648	0.549
		18	-0.129	-0.129	17.017	0.522
		19	0.032	-0.003	17.102	0.583
		20	-0.088	-0.088	17.780	0.602
		21	0.125	0.049	19.185	0.573
		22	-0.182	-0.135	22.277	0.443
		23	0.078	0.017	22.860	0.469
		24	-0.003	-0.077	22.860	0.528

Correlograms of the Autocorrelation and the Partial Correlation function of the residuals after fitting the ARMA model

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.018	-0.018	0.0157	
		2	0.062	0.062	0.2192	
		3	0.082	0.085	0.5813	
		4	0.059	0.059	0.7736	
		5	-0.062	-0.071	0.9897	
		6	-0.135	-0.155	2.0255	
		7	0.112	0.107	2.7634	0.096
		8	-0.243	-0.221	6.3038	0.043
		9	-0.090	-0.083	6.7992	0.079
		10	0.075	0.108	7.1556	0.128
		11	0.081	0.112	7.5854	0.181
		12	-0.224	-0.226	10.918	0.091
		13	-0.015	-0.037	10.933	0.142
		14	0.028	-0.045	10.988	0.202
		15	-0.133	-0.089	12.274	0.198
		16	-0.010	0.013	12.282	0.267
		17	0.146	0.151	13.928	0.237
		18	-0.109	-0.174	14.873	0.248
		19	-0.139	-0.082	16.466	0.225
		20	-0.033	-0.147	16.558	0.280

Results of the ARMA regression

Dependent Variable: D(LOG(GDP_MK_SA),1)
Method: Least Squares
Date: 06/04/10 Time: 12:55
Sample (adjusted): 1998Q1 2009Q4
Included observations: 48 after adjustments
Convergence achieved after 14 iterations
Backcast: 1997Q2 1997Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007472	0.002309	3.236552	0.0024
AR(1)	0.859789	0.158142	5.436808	0.0000
AR(2)	-0.336818	0.189657	-1.775936	0.0832
AR(3)	-0.258316	0.137855	-1.873820	0.0681
MA(1)	-1.247559	0.143895	-8.669929	0.0000
MA(2)	1.116452	0.047545	23.48208	0.0000
MA(3)	-0.105900	0.047551	-2.227103	0.0315

R-squared	0.391799	Mean dependent var	0.006712
Adjusted R-squared	0.302794	S.D. dependent var	0.018515
S.E. of regression	0.015460	Akaike info criterion	-5.367090
Sum squared resid	0.009799	Schwarz criterion	-5.094206
Log likelihood	135.8102	F-statistic	4.401989
Durbin-Watson stat	2.027246	Prob(F-statistic)	0.001603

Inverted AR Roots	.60+.62i	.60-.62i	-.34
Inverted MA Roots	.57+.82i	.57-.82i	.11

2. Details of the estimation of the "kalman AR" model

Forecasting Macedonian Real GDP using PROC STATESPACE *command in SAS*

The PROC STATESPACE command in SAS is fitting an AR approximation to the Macedonian log transformed, trend and seasonally adjusted real GDP time series. The sample period is *1997 Q1-2009 Q4*. We use the following SAS statements for the analysis:

In the **SAS Output,** the sequential construction of the state vector is shown, as well as the iterative steps of the likelihood maximization. In the SAS Output, you can observe the sample mean, $\bar{Y}$, and standard deviation and the sequence of AICs for up to ten AR lags. The smallest AIC in the list is -348.315, which occurs at lag 4. Thus, the initial AR approximation involves four lags and is given by:

$$Y_t = -0.113\, Y_{t-1} + 0.304\, Y_{t-2} + 0.003\, Y_{t-3} - 0.508\, Y_{t-4} + e_t$$

The AR approximation to the Macedonian log transformed, trend and seasonally adjusted real GDP time series is given by the Yule-Walker estimates for minimum AIC.

Note the canonical correlation analysis. Initially, consideration is given to adding $Y_{t+1|t}$ to the state vector containing Y_t. The canonical correlation, 0.557, is an estimate of the second-largest canonical correlation between the set of variables (Y_t, Y_{t+1}) and the set of variables $(Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4})$. The first canonical correlation is always 1 because both sets of variables contain Y_t. The question is whether 0.557 is an estimate of 0. PROC STATESPACE concludes that a correlation is 0 if DIC<0. In this case, DIC=9.447, so 0.557 is not an estimate of 0. This implies that the portion of Y_{t+1} that cannot be predicted from Y_t is correlated with the past of the time series and, thus, that $Y_{t+1|t}$ should be included in the state vector.

Now consider the portion of $Y_{t+2|t}$ that you cannot predict from Y_t and $Y_{t+1|t}$. If this portion is correlated with the past of the series, you can produce a better predictor of the future than one that uses only Y_t and $Y_{t+1|t}$. Add $Y_{t+2|t}$ to the state vector unless the third-highest canonical correlation between the set (Y_t, Y_{t+1}, Y_{t+2}) and the set $(Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4})$ is 0. The estimate of the third highest canonical correlation is 0.491. PROC STATESPACE assumes that 0.491 is not an estimate of 0 because DIC is positive (6.979). This implies that the portion of Y_{t+2} that cannot be predicted from Y_t and $Y_{t+1|t}$ is correlated with the past of the time series and, thus, that $Y_{t+2|t}$ should be included in the state vector.

Now consider the portion of $Y_{t+4|t}$ that you cannot predict from Y_t, $Y_{t+1|t}$, $Y_{t+2|t}$, and $Y_{t+3|t}$. If this portion is correlated with the past of the series, you can produce a better predictor of the future than one that uses only Y_t, $Y_{t+1|t}$, $Y_{t+2|t}$, and $Y_{t+3|t}$. Add $Y_{t+4|t}$ to the state vector unless the fifth-highest canonical correlation between the set $(Y_t, Y_{t+1}, Y_{t+2}, Y_{t+3}, Y_{t+4})$ and the set $(Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4})$ is 0. The estimate of the fifth highest canonical correlation is 0.195. PROC STATESPACE assumes that 0.195 is just an estimate of 0 because DIC is negative (–0.170). This means that once you have predicted Y_{t+4} from Y_t, $Y_{t+1|t}$, $Y_{t+2|t}$, and $Y_{t+3|t}$, you have the best predictor available. The past data do not improve the forecast. Thus, $Y_{t+4|t}$ is not added to the state vector.

Again, the two tests agree that $Y_{t+2|t}$ is a linear combination of Y_t and $Y_{t+1|t}$. Thus, the only information you need to predict arbitrarily far into the future is in

$$Z_t = (Y_t, Y_{t+1|t}, Y_{t+2|t}, Y_{t+3|t})'$$

$$Z_{t+1} = FZ_t + GE_{t+1}$$

PROC STATESPACE estimates these matrices to be initially

Estimate of Transition Matrix (F)			
0	1	0	0

0	0	1	0
0	0	0	1
-0.59418	-0.23015	0.516026	-0.02402

Input Matrix for Innovation (G)
1
-0.11271
0.316602
-0.06727

and finally

Estimate of Transition Matrix (F)			
0	1	0	0
0	0	1	0
0	0	0	1
-0.59001	-0.17733	0.564954	-0.07698

Input Matrix for Innovation (G)
1
-0.12813
0.226063
0.011788

The SAS System

GDP Data

The STATESPACE Procedure

Number of Observations	47

Variable	Mean	Standard Error	
yl	-0.00068	0.027189	Has been differenced. With period(s) = 1,4.

The SAS System

GDP Data

The STATESPACE Procedure

Information Criterion for Autoregressive Models										
Lag=0	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5	Lag=6	Lag=7	Lag=8	Lag=9	Lag=10
-338.863	-338.256	-337.991	-336.298	-348.315	-347.703	-346.65	-345.557	-344.627	-343.533	-342.22

Schematic Representation of Correlations											
Name/Lag	0	1	2	3	4	5	6	7	8	9	10
yl	+	.	.	.	-	.	-	.	.	.	.
+ is > 2*std error, - is < -2*std error, . is between											

Schematic Representation of Partial Autocorrelations										
Name/Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
yl	.	.	.	-	.	.	.	.	.	.
+ is > 2*std error, - is < -2*std error, . is between										

Yule-Walker Estimates for Minimum AIC				
	Lag=1	**Lag=2**	**Lag=3**	**Lag=4**
	yl	**yl**	**yl**	**yl**
yl	-0.11271	0.303899	0.00267	-0.50781

The SAS System

GDP Data

The STATESPACE Procedure

Canonical Correlations Analysis

yl(T;T)	yl(T+1;T)	Information Criterion	Chi Square	DF
1	0.556866	9.446759	16.70434	4

yl(T;T)	yl(T+1;T)	yl(T+2;T)	Information Criterion	Chi Square	DF
1	0.601716	0.49122	6.978829	12.56461	3

yl(T;T)	yl(T+1;T)	yl(T+2;T)	yl(T+3;T)	Information Criterion	Chi Square	DF
1	0.604144	0.580538	0.451283	6.703064	10.47534	2

yl(T;T)	yl(T+1;T)	yl(T+2;T)	yl(T+3;T)	yl(T+4;T)	Information Criterion	Chi Square	DF
1	0.615001	0.591845	0.461307	0.19539	-0.17053	1.810011	1

The SAS System

GDP Data

The STATESPACE Procedure

Selected Statespace Form and Preliminary Estimates

State Vector			
yl(T;T)	yl(T+1;T)	yl(T+2;T)	yl(T+3;T)

Estimate of Transition Matrix			
0	1	0	0
0	0	1	0
0	0	0	1
-0.59418	-0.23015	0.516026	-0.02402

Input Matrix for Innovation
1
-0.11271
0.316602
-0.06727

Variance Matrix for Innovation
0.00051

Iterative Fitting: Maximum Likelihood Estimation											
Iter	**Half**	**Determinant**	**Lambda**	**F(4,1)**	**F(4,2)**	**F(4,3)**	**F(4,4)**	**G(2,1)**	**G(3,1)**	**G(4,1)**	**Sigma (1,1)**
0	**0**	0.000491	0.1	-0.5941754	-0.2301465	0.51602566	-0.0240216	-0.1127096	0.31660203	-0.0672663	0.00049136
1	**0**	0.000482	0.01	-0.5916684	-0.1864302	0.53294756	-0.0641431	-0.140673	0.25479839	-0.0073138	0.00048229
2	**0**	0.000481	0.001	-0.5891268	-0.1823418	0.57714823	-0.0779961	-0.1269687	0.21896328	0.01400511	0.00048114
3	**2**	0.000481	0.01	-0.5899039	-0.1777236	0.56573684	-0.078991	-0.12838	0.22544475	0.01232445	0.00048111
4	**3**	0.000481	0.1	-0.5895385	-0.1802352	0.57142283	-0.0783735	-0.1277158	0.22212202	0.01337903	0.00048111
5	**4**	0.000481	1	-0.5897002	-0.1792755	0.56909932	-0.0783306	-0.1279763	0.22347246	0.01297301	0.0004811
6	**2**	0.000481	10	-0.5899663	-0.177618	0.56553156	-0.0772423	-0.1281494	0.22568735	0.01199901	0.00048108
7	**2**	0.000481	100	-0.5900057	-0.1773564	0.56501389	-0.0770058	-0.1281346	0.22602361	0.01181051	0.00048108
8	**2**	0.000481	1000	-0.5900099	-0.1773286	0.56495955	-0.0769798	-0.1281322	0.22605913	0.01178985	0.00048108
9	**2**	0.000481	1000	-0.5900103	-0.1773258	0.56495408	-0.0769772	-0.128132	0.2260627	0.01178777	0.00048108

Maximum likelihood estimation has converged.

The SAS System

GDP Data

The STATESPACE Procedure

Selected Statespace Form and Fitted Model

State Vector			
yl(T;T)	yl(T+1;T)	yl(T+2;T)	yl(T+3;T)

Estimate of Transition Matrix			
0	1	0	0
0	0	1	0
0	0	0	1
-0.59001	-0.17733	0.564954	-0.07698

Input Matrix for Innovation
1
-0.12813
0.226063
0.011788

Variance Matrix for Innovation
0.000481

Parameter Estimates			
Parameter	**Estimate**	**Standard Error**	**t Value**
F(4,1)	-0.59001	0.118794	-4.97
F(4,2)	-0.17733	0.195240	-0.91
F(4,3)	0.564954	0.189509	2.98
F(4,4)	-0.07698	0.196843	-0.39
G(2,1)	-0.12813	0.144339	-0.89
G(3,1)	0.226063	0.144425	1.57
G(4,1)	0.011788	0.148487	0.08

Covariance of Parameter Estimates							
	F(4,1)	**F(4,2)**	**F(4,3)**	**F(4,4)**	**G(2,1)**	**G(3,1)**	**G(4,1)**
F(4,1)	0.0141121	-.0003771	-.0054633	0.0008753	0.0038808	-.0051661	0.0027991
F(4,2)	-.0003771	0.0381188	0.0044175	-.0141118	-.0007358	0.0023076	-.0035737
F(4,3)	-.0054633	0.0044175	0.0359135	0.0039157	-.0011857	0.0016267	-.0064580
F(4,4)	0.0008753	-.0141118	0.0039157	0.0387471	0.0010867	-.0076021	0.0085658
G(2,1)	0.0038808	-.0007358	-.0011857	0.0010867	0.0208338	-.0024058	0.0055348
G(3,1)	-.0051661	0.0023076	0.0016267	-.0076021	-.0024058	0.0208585	-.0032701
G(4,1)	0.0027991	-.0035737	-.0064580	0.0085658	0.0055348	-.0032701	0.0220484

Correlation of Parameter Estimates							
	F(4,1)	**F(4,2)**	**F(4,3)**	**F(4,4)**	**G(2,1)**	**G(3,1)**	**G(4,1)**
F(4,1)	1.00000	-0.01626	-0.24268	0.03743	0.22633	-0.30111	0.15869
F(4,2)	-0.01626	1.00000	0.11939	-0.36719	-0.02611	0.08184	-0.12327
F(4,3)	-0.24268	0.11939	1.00000	0.10497	-0.04335	0.05943	-0.22950
F(4,4)	0.03743	-0.36719	0.10497	1.00000	0.03825	-0.26741	0.29306
G(2,1)	0.22633	-0.02611	-0.04335	0.03825	1.00000	-0.11541	0.25824
G(3,1)	-0.30111	0.08184	0.05943	-0.26741	-0.11541	1.00000	-0.15249
G(4,1)	0.15869	-0.12327	-0.22950	0.29306	0.25824	-0.15249	1.00000

3. Details of the estimation of the "foreign demand" model

Dependent Variable: DLOG(GDP_MK_SA)
Method: Least Squares
Date: 05/25/10 Time: 15:24
Sample (adjusted): 1997Q3 2009Q4
Included observations: 50 after adjustments
DLOG(GDP_MK_SA) = C(1) + 0.7*DLOG(FORDEM)+C(2)
*DLOG(GDP_MK_SA(-1))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.005030	0.002551	1.971635	0.0544
C(2)	-0.274442	0.132456	-2.071958	0.0437

R-squared	0.155382	Mean dependent var	0.006996
Adjusted R-squared	0.137786	S.D. dependent var	0.018194
S.E. of regression	0.016894	Akaike info criterion	-5.284535
Sum squared resid	0.013700	Schwarz criterion	-5.208054
Log likelihood	134.1134	Durbin-Watson stat	1.857482

Serial correlation test (p value)	0.43
White heteroskedasticity test (p value)	0.07
Jarque Bera normality test (p value)	0.63

ADF test (p value) 0.00

4. Details of the estimation of the "GDP components" model

DLOG(CONS_SA) = C(11) + 0.4*DLOG((WAGES/CPI_SA)*EMPLOYED_SA) + C(12)*DLOG(TRANSFERS_SA) + C(13)*D(INTEREST) + C(14)*DLOG(TRANSFERS_SA(-1))

DLOG(INV_SA) = C(21) + C(22)*DLOG(INV_SA(-1)) + C(23)*DLOG(FDI) + C(24)*DLOG(FDI(-1)) +C(25)*DLOG(FDI(-2)) + C(26)*DLOG(EXP_SA) + C(27)*DLOG(INDUSTRIAL_SA) +0.04*DLOG(GOV_CAPITAL_EXP_SA)

DLOG(EXP_SA) = C(31)*DLOG(FORDEM) + C(32)*DLOG(REL_EXP_PRICE_SA) + C(33)

DLOG(IMP_SA) = C(41) + C(42)*DLOG(CONS_SA) + C(43)*DLOG(EXP_SA) + C(44)*DLOG(EXP_SA(-1)) + C(45)*DLOG(INV_SA) + 0.2*DLOG(GOV_SA)

D(LOG(EMPLOYED_SA)) = c(51)*(LOG(EMPLOYED_SA(-1)) + c(52)*LOG(CONS_SA(-1)+INV_SA(-1)+GOV_SA(-1)+EXP_SA(-1)-IMP_SA(-1)) + c(53)) + C(54)*TR0204 + C(55)*DLOG(EMPLOYED_SA(-1)) + C(56)*DLOG(CONS_SA(-3)+INV_SA(-3)+GOV_SA(-3)+EXP_SA(-3)-IMP_SA(-3)) + C(57)*DLOG(CONS_SA(-4)+INV_SA(-4)+GOV_SA(-4)+EXP_SA(-4)-IMP_SA(-4))

D(LOG(WAGES)) = c(61)*(LOG(WAGES(-1)) + c(62)*LOG(CPI_SA(-1)) + c(63)*LOG(CONS_SA(-1)+INV_SA(-1)+GOV_SA(-1)+EXP_SA(-1)-IMP_SA(-1)) + c(64)*LOG(EMPLOYED_SA(-1)) + c(65)) + C(66)*DLOG(CONS_SA(-2)+INV_SA(-2)+GOV_SA(-2)+EXP_SA(-2)-IMP_SA(-2)) + C(67)*DLOG(CONS_SA(-3)+INV_SA(-3)+GOV_SA(-3)+EXP_SA(-3)-IMP_SA(-3))

DLOG(INDUSTRIAL_SA) = C(71) + C(72)*DLOG(INDUSTRIAL_SA(-1)) + C(73)*DLOG(FORDEM)

System: SY3
Estimation Method: Seemingly Unrelated Regression
Date: 07/21/10 Time: 17:04
Sample: 1998Q3 2010Q1
Included observations: 47
Total system (unbalanced) observations 324
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 6 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.003944	0.005338	-0.738796	0.4606
C(12)	0.068165	0.015538	4.387130	0.0000
C(13)	-0.019999	0.008720	-2.293464	0.0225
C(14)	0.040377	0.014969	2.697427	0.0074
C(21)	0.007999	0.026239	0.304863	0.7607
C(22)	-0.248715	0.130192	-1.910370	0.0571
C(23)	0.005324	0.036549	0.145680	0.8843
C(24)	0.046659	0.037679	1.238326	0.2166
C(25)	0.003323	0.037019	0.089759	0.9285
C(26)	-0.759328	0.433259	-1.752597	0.0807
C(27)	1.877553	0.488889	3.840451	0.0002
C(31)	1.365757	0.677051	2.017214	0.0446
C(32)	-0.329896	0.098845	-3.337493	0.0010
C(33)	-0.004579	0.008209	-0.557811	0.5774
C(41)	-0.006149	0.003276	-1.876874	0.0615
C(42)	0.969896	0.076406	12.69392	0.0000
C(43)	0.529337	0.055140	9.599922	0.0000
C(44)	0.116126	0.047784	2.430237	0.0157
C(45)	0.272952	0.014779	18.46851	0.0000
C(51)	-0.183625	0.051208	-3.585858	0.0004
C(52)	-0.262099	0.118750	-2.207144	0.0281
C(53)	-10.41576	1.297939	-8.024847	0.0000
C(54)	-0.002635	0.000615	-4.282052	0.0000
C(55)	0.371845	0.102591	3.624528	0.0003
C(56)	0.074310	0.111209	0.668199	0.5045
C(57)	0.123257	0.112245	1.098106	0.2731
C(61)	-0.114806	0.045662	-2.514265	0.0125
C(62)	-1.490506	0.362879	-4.107450	0.0001
C(63)	-0.633881	0.343721	-1.844171	0.0662
C(64)	-0.873725	0.562152	-1.554252	0.1212
C(65)	15.64585	6.029358	2.594945	0.0099
C(66)	0.240365	0.097511	2.464996	0.0143
C(67)	0.370040	0.101108	3.659832	0.0003
C(71)	-0.013129	0.007937	-1.654125	0.0992
C(72)	-0.371862	0.129997	-2.860552	0.0045
C(73)	1.840874	0.635077	2.898662	0.0040

Determinant residual covariance 1.25E-21

Equation: DLOG(CONS_SA) = C(11) + 0.4*DLOG((WAGES/CPI_SA)
*EMPLOYED_SA) + C(12)*DLOG(TRANSFERS_SA) +
C(13)*D(INTEREST) + C(14)*DLOG(TRANSFERS_SA(-1))
Observations: 47

R-squared	0.417947	Mean dependent var	0.008526
Adjusted R-squared	0.377339	S.D. dependent var	0.045852
S.E. of regression	0.036181	Sum squared resid	0.056290
Durbin-Watson stat	2.356952		

Equation: DLOG(INV_SA) = C(21) + C(22)
*DLOG(INV_SA(-1)) + C(23)*DLOG(FDI) +
C(24)*DLOG(FDI(-1)) +C(25)*DLOG(FDI(-2)) +
C(26)*DLOG(EXP_SA) + C(27)*DLOG(INDUSTRIAL_SA)
+0.04*DLOG(GOV_CAPITAL_EXP_SA)
Observations: 47

R-squared	0.229959	Mean dependent var	-0.000580
Adjusted R-squared	0.114452	S.D. dependent var	0.207796
S.E. of regression	0.195543	Sum squared resid	1.529484
Durbin-Watson stat	1.808834		

Equation: DLOG(EXP_SA) = C(31)*DLOG(FORDEM) + C(32)
*DLOG(REL_EXP_PRICE_SA) + C(33)
Observations: 47

R-squared	0.321038	Mean dependent var	0.003823
Adjusted R-squared	0.290176	S.D. dependent var	0.063355
S.E. of regression	0.053378	Sum squared resid	0.125363
Durbin-Watson stat	2.227935		

Equation: DLOG(IMP_SA) = C(41) + C(42)*DLOG(CONS_SA) +
C(43)*DLOG(EXP_SA) + C(44)*DLOG(EXP_SA(
-1)) + C(45)*DLOG(INV_SA) + 0.2
*DLOG(GOV_SA)
Observations: 47

R-squared	0.944616	Mean dependent var	0.004849
Adjusted R-squared	0.939342	S.D. dependent var	0.095105
S.E. of regression	0.023423	Sum squared resid	0.023044
Durbin-Watson stat	1.941593		

Equation: D(LOG(EMPLOYED_SA)) = C(51)*(LOG(EMPLOYED_SA(
-1)) + C(52)*LOG(CONS_SA(-1)+INV_SA(-1)
+GOV_SA(-1)+EXP_SA(-1)-IMP_SA(-1)) +
C(53)) + C(54)*TR0204 + C(55)*DLOG(EMPLOYED_SA(-1)) +
C(56)*DLOG(CONS_SA(-3)+INV_SA(-3)
+GOV_SA(-3)+EXP_SA(-3)-IMP_SA(-3)) + C(57)
*DLOG(CONS_SA(-4)+INV_SA(-4)+GOV_SA(
-4)+EXP_SA(-4)-IMP_SA(-4))
Observations: 44

R-squared	0.530394	Mean dependent var	0.002869
Adjusted R-squared	0.454242	S.D. dependent var	0.019332

S.E. of regression	0.014282	Sum squared resid	0.007547
Durbin-Watson stat	1.886804		

Equation: D(LOG(WAGES)) = C(61)*(LOG(WAGES(-1)) + C(62)
*LOG(CPI_SA(-1)) + C(63)*LOG(CONS_SA(-1)
+INV_SA(-1)+GOV_SA(-1)+EXP_SA(
-1)-IMP_SA(-1)) + C(64)*LOG(EMPLOYED_SA(-1)) +
C(65)) + C(66)*DLOG(CONS_SA(-2)+INV_SA(-2)
+GOV_SA(-2)+EXP_SA(-2)-IMP_SA(-2)) + C(67)
*DLOG(CONS_SA(-3)+INV_SA(-3)+GOV_SA(
-3)+EXP_SA(-3)-IMP_SA(-3))
Observations: 45

R-squared	0.457303	Mean dependent var	0.013815
Adjusted R-squared	0.371614	S.D. dependent var	0.014815
S.E. of regression	0.011744	Sum squared resid	0.005241
Durbin-Watson stat	2.229567		

Equation: DLOG(INDUSTRIAL_SA) = C(71) + C(72)
*DLOG(INDUSTRIAL_SA(-1)) + C(73)*DLOG(FORDEM)
Observations: 47

R-squared	0.242025	Mean dependent var	-0.003366
Adjusted R-squared	0.207572	S.D. dependent var	0.057571
S.E. of regression	0.051249	Sum squared resid	0.115564
Durbin-Watson stat	1.915142		

5. Details of the estimation of the "static factor" model

Dependent Variable: DLOG(GDP_MK_SA)
Method: Least Squares
Date: 06/04/10 Time: 12:52
Sample (adjusted): 1998Q1 2009Q4
Included observations: 48 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PC3	-0.018368	0.004323	-4.248843	0.0001
PC5	-0.036477	0.008025	-4.545231	0.0000
PC6	0.027485	0.008776	3.131809	0.0032
PC10	-0.039456	0.014080	-2.802277	0.0076
PC18	0.105964	0.036428	2.908856	0.0058
C	0.006712	0.001773	3.786594	0.0005

R-squared	0.606868	Mean dependent var	0.006712
Adjusted R-squared	0.560066	S.D. dependent var	0.018515
S.E. of regression	0.012281	Akaike info criterion	-5.845115
Sum squared resid	0.006334	Schwarz criterion	-5.611215
Log likelihood	146.2828	F-statistic	12.96684
Durbin-Watson stat	2.305369	Prob(F-statistic)	0.000000

6. Details of the estimation of the "FAVAR" model

Vector Autoregression Estimates
Date: 06/04/10 Time: 12:53
Sample (adjusted): 1998Q2 2009Q4
Included observations: 47 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	DLOG(GDP_MK_SA)	PC3	PC5	PC6	PC10	PC18
DLOG(GDP_MK_SA(-1))	0.026566	-0.980855	-2.000087	2.986917	-0.940205	-0.239389
	(0.22950)	(4.92619)	(2.77226)	(2.53581)	(1.41300)	(0.65304)
	[0.11575]	[-0.19911]	[-0.72147]	[1.17789]	[-0.66540]	[-0.36657]
PC3(-1)	0.004682	-0.439616	0.038758	0.028842	-0.041632	0.007165
	(0.00771)	(0.16553)	(0.09315)	(0.08521)	(0.04748)	(0.02194)
	[0.60716]	[-2.65577]	[0.41606]	[0.33849]	[-0.87683]	[0.32653]
PC5(-1)	0.015451	-0.235003	-0.363457	-0.140000	-0.036781	-0.036319
	(0.01465)	(0.31441)	(0.17694)	(0.16184)	(0.09018)	(0.04168)
	[1.05485]	[-0.74745]	[-2.05418]	[-0.86503]	[-0.40785]	[-0.87138]
PC6(-1)	0.002731	-0.134111	-0.144017	0.046252	0.194365	-8.77E-06
	(0.01467)	(0.31484)	(0.17718)	(0.16207)	(0.09031)	(0.04174)
	[0.18621]	[-0.42597]	[-0.81284]	[0.28539]	[2.15229]	[-0.00021]
PC10(-1)	0.038933	-0.566816	-0.032030	0.040695	-0.393806	-0.019497
	(0.02282)	(0.48991)	(0.27570)	(0.25219)	(0.14052)	(0.06495)
	[1.70580]	[-1.15698]	[-0.11618]	[0.16137]	[-2.80245]	[-0.30021]
PC18(-1)	-0.090897	0.542642	0.845623	-0.692943	0.298488	0.093706
	(0.05973)	(1.28215)	(0.72154)	(0.66000)	(0.36776)	(0.16997)
	[-1.52173]	[0.42323]	[1.17197]	[-1.04991]	[0.81163]	[0.55131]
C	0.006329	-0.003100	0.013902	-0.012554	0.013287	0.000821
	(0.00308)	(0.06611)	(0.03720)	(0.03403)	(0.01896)	(0.00876)
	[2.05498]	[-0.04689]	[0.37370]	[-0.36893]	[0.70072]	[0.09373]
R-squared	0.164673	0.226883	0.168211	0.134599	0.247278	0.034848
Adj. R-squared	0.039374	0.110916	0.043442	0.004789	0.134369	-0.109925
Sum sq. resids	0.013345	6.148522	1.947216	1.629230	0.505859	0.108052
S.E. equation	0.018265	0.392063	0.220636	0.201819	0.112457	0.051974
F-statistic	1.314238	1.956436	1.348182	1.036892	2.190075	0.240709
Log likelihood	125.2288	-18.89262	8.127948	12.31783	39.80354	76.07919
Akaike AIC	-5.031012	1.101814	-0.047998	-0.226291	-1.395895	-2.939540
Schwarz SC	-4.755458	1.377367	0.227556	0.049263	-1.120342	-2.663986
Mean dependent	0.006466	-0.007201	0.000505	0.005784	0.006270	-0.000867
S.D. dependent	0.018636	0.415799	0.225591	0.202304	0.120870	0.049333

Determinant resid covariance (dof adj.)	1.27E-12
Determinant resid covariance	4.81E-13
Log likelihood	266.3782
Akaike information criterion	-9.548008
Schwarz criterion	-7.894685

7. Actual GDP, forecasts from the different models, forecast errors, RMSE and MAE

	Actual GDP	Foreign demand			ARIMA			Static factor			FAVAR			GDP components			Kalman AR		
		GDP forecast	Squared error	Absolut error	GDP forecast	Squared error	Absolut error	GDP forecast	Squared error	Absolut error	GDP forecast	Squared error	Absolut error	GDP forecast	Squared error	Absolut error	GDP forecast	Squared error	Absolut error
2004Q1	53106.3	53256.1	22447.5	149.8	52894.7	44747.2	211.5	51723.4	1912426.9	1382.9	52894.3	44946.2	212.0	53414.9	95249.2	308.6	52346.0	578056.4	760.3
2004Q2	53829.2	53344.7	234826.2	484.6	53353.4	226460.5	475.9	54105.0	76044.3	275.8	54868.2	1079440.7	1039.0	53357.4	222678.9	471.9	52289.4	2371250.9	1539.9
2004Q3	54457.1	54108.9	121271.0	348.2	54228.6	52230.5	228.5	54725.9	72199.8	268.7	53481.4	952029.2	975.7	54083.3	139756.2	373.8	54822.3	133333.9	365.1
2004Q4	54760.8	54886.0	15663.6	125.2	54915.5	23933.5	154.7	55040.7	78324.1	279.9	54069.2	478290.6	691.6	54269.7	241204.3	491.1	55514.9	568646.3	754.1
2005Q1	54724.0	54540.2	33778.1	183.8	55339.8	379236.4	615.8	55046.9	104272.0	322.9	54905.3	32873.9	181.3	54891.4	28020.0	167.4	55504.6	609365.6	780.6
2005Q2	56486.0	55539.1	896558.1	946.9	55368.8	1248107.9	1117.2	55905.5	336942.5	580.5	55072.1	1998908.2	1413.8	55153.0	1776909.0	1333.0	55473.8	1024483.1	1012.2
2005Q3	56542.9	56481.4	3784.9	61.5	57167.5	390098.2	624.6	56524.2	349.0	18.7	56070.8	222917.7	472.1	57011.9	219893.8	468.9	56740.9	39175.6	197.9
2005Q4	57317.0	57168.5	22056.9	148.5	56968.4	121512.1	348.6	56783.5	284692.5	533.6	57123.0	37642.1	194.0	57089.0	51973.0	228.0	56915.5	161214.9	401.5
2006Q1	58218.3	57578.6	409320.0	639.8	56898.2	1742693.6	1320.1	57630.9	345122.4	587.5	58125.0	8720.0	93.4	57630.0	346180.7	588.4	57338.1	774810.5	880.2
2006Q2	58407.1	58664.2	66108.7	257.1	58919.0	261976.2	511.8	59355.9	900251.9	948.8	58402.5	21.4	4.6	59040.1	400658.7	633.0	59691.2	1648748.6	1284.0
2006Q3	59027.5	58991.4	1304.6	36.1	59203.3	30905.9	175.8	58792.6	55168.2	234.9	58385.6	412111.6	642.0	58393.8	401625.3	633.7	59209.6	33155.6	182.1
2006Q4	58327.2	59464.7	1293817.3	1137.5	59316.3	978379.9	989.1	58847.4	270588.1	520.2	59534.1	1456730.4	1207.0	59394.6	1139451.4	1067.5	59028.6	491984.2	701.4
2007Q1	61230.5	59353.8	3522232.9	1876.8	58981.7	5057377.1	2248.9	60404.9	681716.6	825.7	58889.6	5479818.8	2340.9	59442.0	3198665.3	1788.5	59064.1	4693294.5	2166.4
2007Q2	60847.0	60866.5	380.1	19.5	61829.5	965338.0	982.5	60999.3	23212.4	152.4	60173.8	453216.9	673.2	61026.5	32222.5	179.5	61197.4	122791.5	350.4
2007Q3	61930.7	61345.0	343012.4	585.7	61345.6	342380.2	585.1	61909.7	440.3	21.0	61876.8	2907.6	53.9	60223.1	2915872.5	1707.6	61707.4	49873.0	223.3
2007Q4	63486.7	62108.7	1898827.8	1378.0	61909.4	2487968.7	1577.3	63983.1	246373.6	496.4	61864.2	2632569.8	1622.5	62298.2	1412602.6	1188.5	61798.6	2849646.5	1688.1
2008Q1	64997.9	64227.9	592873.0	770.0	63543.1	2116275.7	1454.7	64730.2	71637.9	267.7	62819.8	4743956.2	2178.1	64087.5	828814.5	910.4	65373.6	141141.1	375.7
2008Q2	65642.6	65041.6	361113.5	600.9	66046.1	162798.4	403.5	66329.9	472467.6	687.4	65008.4	402206.1	634.2	66153.0	260572.3	510.5	66107.7	216339.2	465.1
2008Q3	65767.8	65369.9	158369.1	398.0	66642.1	764424.8	874.3	66134.3	134273.8	366.4	65584.7	33524.2	183.1	66303.4	286860.8	535.6	65420.8	120413.2	347.0
2008Q4	64445.9	65369.9	853715.7	924.0	66406.2	3842491.4	1960.2	63747.4	487989.7	698.6	66134.6	2851530.0	1688.6	66081.0	2673608.9	1635.1	67150.0	7311872.3	2704.0
2009Q1	64369.5	64029.8	115417.0	339.7	64941.4	327057.2	571.9	64056.5	98013.3	313.1	65782.1	1995379.9	1412.6	64060.4	95592.8	309.2	66343.2	3895409.5	1973.7
2009Q2	64663.6	64562.2	10271.8	101.3	63568.2	1199944.3	1095.4	64390.5	74588.9	273.1	64660.4	10.4	3.2	64181.2	232690.2	482.4	64010.4	426604.5	653.1
2009Q3	64575.1	64943.8	135949.4	368.7	63998.9	332025.8	576.2	65225.6	423102.4	650.5	65313.7	545564.2	738.6	65190.5	378758.0	615.4	64675.4	10062.7	100.3
2009Q4	65287.0	64840.1	199692.8	446.9	65044.2	58966.4	242.8	65304.1	293.8	17.1	65255.0	1023.4	32.0	64856.3	185502.5	430.7	64237.7	1100967.5	1049.3
RMSE			**686.6**			**982.3**			**545.8**			**1038.2**			**855.5**			**1106.3**	
MAE				**513.7**			**806.1**			**446.8**			**778.6**			**710.8**			**873.2**