A Simple Interest Rate Model with Unobserved Components: The Role of the Interbank Reference Rate

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Abstract

In this study, we theoretically investigate the potential role of the reference rate in stabilizing or destabilizing an interbank market with an environment where individual banks cannot fully identify the nature of underlying shocks affecting their interbank transactions. We show that a noise-free reference rate based on a sufficient number of sample transactions can help to make the market interest rate less volatile, whereas the stabilizing effects of the reference rate are significantly reduced if the reported interest rates contain some noisy components. Nevertheless, by increasing the number of sample transactions reflected in the reference rate, the adverse effects of the noise can be mitigated (or eliminated) provided the noise is idiosyncratic to individual transactions. However, if the noise is common to multiple transactions, then the adverse effects of the noisy reference rate cannot be reduced simply by increasing the number of sample transactions. This suggests that the noise in the interest rates reported by just a few of large banks can end up making the entire market more volatile, thereby impairing the transmission mechanism of monetary policy.

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1 Introduction

The role of the reference rate in interbank markets has become a major concern for various agents active in financial markets in light of the recent LIBOR manipulation problem. Policymakers the world over are already debating possible measures to the reliability and efficacy of interbank reference rates, but numerous issues have yet to be fully addressed by the existing literature, including (i) the mechanism by which the reference rate contributes to financial markets stabilization, (ii) conditions under which the reference rate might actually serve to destabilize financial markets, (iii) the specific properties that might allow a reference rate to help stabilize financial markets and thereby enhance the effectiveness of monetary policy.

In this study, we theoretically investigate the potential role of the reference rate in stabilizing or destabilizing financial markets, which is evaluated in terms of the volatility of interbank interest rates. To this end, we introduce a simple interest rate model in which individual banks cannot fully identify the nature of the underlying shocks that affect individual interbank transactions. More specifically, we assume that the banks engaged in individual interbank transactions cannot distinguish whether the underlying shocks are specific to individual transactions or broadly influential to the entire interbank market.

This kind of imperfect information environment is analogous to the famous island economy model developed by Phelps (1970) and Lucas (1972, 1973) in the context of the output-inflation tradeoffs and the effectiveness of monetary policy. We believe that our setup is useful to gain insights into the potential role of the reference rate, since its most fundamental one is considered to be the information it provides to individual banks. That is, the reference rate should ideally inform individual banks of the aggregate financial conditions that exist throughout the entire interbank market, thereby helping these banks to set more appropriate interest rates in individual interbank transactions and making the average interbank interest rate less volatile. Our framework allows for the reference rate to play this sort of role, and also enables us to investigate how the existence of distortions (or noise) in the reference rate can have a destabilizing impact.[1]

Our analysis based on a simple model of the interbank market contributes to the academic literature by shedding light on the essential role of the reference rate in financial market stability.[2,3] However, our potential contribution is by no means limited to the issue of financial stability. A

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[1] Our study is similar to the argument of Morris and Shin (2002) in the sense that market participants react to the noise included in the common signals. However, our framework is simpler than theirs because we do not introduce any strategic complementarity among market participants.

[2] Although our theoretical setup is new to the literature, a previous empirical study provided by Angelini, Nobili, Picillo (2011), who investigate interest rate determinants for individual transactions in the Italian interbank market, does present an empirical model in which individual interbank interest rates are affected by both bank-specific factors and aggregate factors.

[3] Some previous studies, such as Freixas and Holthausen (2004) and Heider, Hoerova, and Holthausen (2009), construct theoretical models to explain the mechanisms determining the interest rates in interbank markets. However, our study differs in that we pay particular attention to the role of the reference rate in interbank markets.

properly functioning interbank market is vital to the effectiveness of monetary policy transmission. In our model, the market interest rate should be perfectly correlated with the central bank's interest rate if information is perfect, while the average interest rate in the interbank market will tend to deviate from the policy rate in an imperfect information world. We show that the existence of a noise-free reference rate based on a sufficient number of sample transactions leads to an increased correlation between the interbank rate and the policy rate, thereby enhancing the effectiveness of monetary policy transmission.

This study is obviously not an exhaustive investigation of topics relating to the reference rate. For example, the recent LIBOR problem highlights a possible incentive for banks to manipulate interest rates through their reporting. Our analysis assumes that noise in the reported interest rates is purely exogenous, leaving the possibility of endogenous noise (manipulation) to other studies such as Ewerhart et al. (2007).[4] In addition, we have limited our focus to interbank markets. From a macroeconomic perspective, the impact of interbank market stability (or instability) on the aggregate economy, through the linkage between the interbank market and other macroeconomic activities, should be examined in the context of dynamic general equilibrium (DSGE) models, as suggested by Woodford (2010).[5] The ramifications of the spread between the interbank interest rates and the policy rate for monetary policymakers are also worthy of investigation. Finally, our analysis does not consider potential problems arising from the "offshore" nature of LIBOR. Reasons why offshore rates are used as the reference rates in many developed countries are briefly explained in some studies, but a deeper analysis of potential pitfalls should be the subject of future research.[6]

The remainder of this study is organized as follows. In Section 2, we present our simple interest rate model with unobserved components. In Section 3, we provide a benchmark analysis under the assumption that individual banks can observe each component separately. In Section 4, we examine how the results differ when individual banks are unable to observe each component separately. In Section 5, we investigate the role of the reference rate in stabilizing the market interest rate. In Section 6, we examine the implications of the noise in the reference rate. In Section 7, we present some numerical examples to illustrate the role of the reference rate and the implications of noise in the reference rate. Section 8 concludes our analysis.

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[4] Empirical studies examining the possibility of LIBOR manipulation during global financial crisis include Gyntelberg and Wooldridge (2008), Abrantes-Metz et al. (2012), and Kuo, Skeie, and Vickery (2012).

[5] Sudo (2012) explores the roles played by the interbank reference rate in business cycle fluctuations using a DSGE model with credit frictions, which is developed and estimated by Muto, Sudo, and Yoneyama (2012).

[6] Gyntelberg and Wooldridge (2008) present some potential reasons why the offshore rate is preferred as the reference rate, including (i) fewer regulatory distortions, (ii) greater liquidity (for example, in the London and Singapore markets), and (iii) greater diversity of participants in offshore markets (making them less vulnerable to the actions of a few large institutions).

2 Model

We consider a situation in which each bank cannot fully identify the sources of underlying shocks that have an influence on individual interbank transactions. Suppose that there are an infinite number of transactions (indexed by $j = 1, \cdots, \infty$) in an interbank market. The interest rate for the jth transaction is determined by the following two equations:

$$i_t^j = i_t^p + E_t^j(\omega_t^j - \mu_t), \tag{1}$$

$$\omega_t^j = \mu_t + \varepsilon_t^j, \tag{2}$$

where i_t^j is the interest rate for the jth transaction and i_t^p is the policy interest rate. ω_t^j is the fundamental factor influencing on the jth transaction (such as funding liquidity and counterparty credit risk), μ_t is the aggregate component of ω_t^j, and ε_t^j is the individual component of ω_t^j.[7]

(1) indicates that the banks in the jth transaction set the interest rate by adjusting the policy interest rate using their estimate of the deviation of factor ω_t^j from its aggregate component: $E_t^j(\omega_t^j - \mu_t)$.[8],[9] (2) indicates that the fundamental factor for the jth transaction consists of a shock influencing on the entire interbank market (μ_t) and a shock specific to the jth transaction (ε_t^j). These equations capture a situation in which the banks identify the individual shocks in the jth transaction and then take account of these shocks in determining their individual interest rate.

This kind of framework with imperfect information is analogous to the famous island economy model introduced by Phelps (1970) and Lucas (1972, 1973) in the context of the output-inflation tradeoffs and the effectiveness of monetary policy. Although our study is the first to apply this setup to interbank markets, the idea that individual interest rates in interbank markets are determined by both bank-specific and aggregate factors is shared with a recent empirical study by Angelini, Nobili, and Picillo (2011).

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[7] In this study we do not specify the fundamental factors which determine the spread between interbank interest rates and the policy interest rate. The issue of whether liquidity factors or counterparty risk factors are dominant as the determinants of LIBOR-OIS spread, especially after the onset of financial crisis, is examined by many authors, such as Michaud and Upper (2008), Taylor and Williams (2009), and Gefang, Koop and Potter (2011).

[8] We assume that the aggregate component is accommodated by the policy interest rate, although private agents are unable to infer the value of μ_t from movements in the policy rate due to the complexity of its determination mechanism. Consideration of market participants' inferences from the policy rate is left for future research. Another important issue worthy of examination is the influence of any central bank misperception of the aggregate component. From a macroeconomic perspective, Muto (2013) examines the impact of central bank transparency regarding the bank's views on the aggregate state of economy in a framework that allows for misperceptions on the part of both the central bank and private agents.

[9] It is quite straightforward to introduce a constant (or purely exogenous) component of the spread between the interbank interest rate and the policy interest rate, although we have refrained from doing so here.

For simplicity, we assume that each shock follows the i.i.d. normal distribution:

$$\mu_t \quad \sim \quad N(0, \sigma_\mu^2), \tag{3}$$

$$\varepsilon_t^j \quad \sim \quad N(0, \sigma_\varepsilon^2). \tag{4}$$

In the following argument we assume that the banks in the jth transaction observe the realization of ω_t^j. Therefore, (1) can be rewritten as follows:

$$i_t^j = i_t^p + \omega_t^j - E_t^j \mu_t, \tag{5}$$

However, the banks do not necessarily observe the values of μ_t and ε_t^j separately, although they do know that these shocks follow the processes of (3) and (4) respectively.[10]

3   When each component is observable

For benchmarking purposes we begin by assuming that the banks in the jth transaction can observe the realizations of μ_t and ε_t^j separately. Then the banks' estimate of the aggregate component $(E_t^j \mu_t)$ is exactly the same as the true value of μ_t. Therefore, from (2) and (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + \varepsilon_t^j. \tag{6}$$

The overall average of the interest rates (hereafter the "market interest rate"), denoted by i_t^m, is computed as follows by summing up the individual interest rates:

$$i_t^m = \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^n i_t^j = i_t^p + \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^n \varepsilon_t^j = i_t^p. \tag{7}$$

The spread between the market interest rate and the policy interest rate, denoted by i_t^s, is zero:

$$i_t^s \equiv i_t^m - i_t^p = 0. \tag{8}$$

The variance of the spread is also zero:

$$Var(i_t^s) = 0. \tag{9}$$

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[10] We also assume that all banks observe the policy rate level i_t^P.

4   When each component is unobservable

Next we assume that the banks in the jth transaction cannot observe the realizations of μ_t and ε_t^j separately and only observe the sum of these components: ω_t^j. In this situation, the banks need to make an estimate of the aggregate component μ_t in order to determine interest rates for individual transactions.

Suppose that the banks know the variances of μ_t and ε_t^j: σ_μ^2 and σ_ε^2. Then standard statistical inference yields the following estimator of μ_t:

$$E_t^j \mu_t = \theta \omega_t^j, \qquad \text{where} \ \ \theta \equiv \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2}. \tag{10}$$

By substituting (10) into (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + (1 - \theta)\omega_t^j = i_t^p + (1 - \theta)(\mu_t + \varepsilon_t^j). \tag{11}$$

By summing up the individual interest rates, the market interest rate is computed as follows:

$$i_t^m = \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^n i_t^j = i_t^p + (1 - \theta)\mu_t + (1 - \theta) \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^n \varepsilon_t^j = i_t^p + (1 - \theta)\mu_t. \tag{12}$$

Therefore, the spread i_t^s is non-zero:

$$i_t^s = i_t^m - i_t^p = (1 - \theta)\mu_t. \tag{13}$$

The variance of the spread is also non-zero[11]:

$$Var(i_t^s) = (1 - \theta)^2 \sigma_\mu^2. \tag{14}$$

This result indicates that the market interest rate will tend to be more volatile when banks cannot precisely identify the underlying shocks to their individual transactions.

5   The role of the reference rate

So far we have implicitly assumed that individual banks are unable to observe the market interest rate i_t^m. This is a quite realistic assumption since individual players in a real-world interbank market can rarely observe the overall average of the interest rate across all interbank transactions. However, banks do usually observe a sample average for some fraction of interbank transactions due

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[11] As shown in (7), the market interest rate coincides with the policy rate if each shock is observable. Therefore, the variance of the spread between the market interest rate and the policy rate is a good measure of the severity of distortions in the interbank market arising from informational imperfections.

to the existence of reference rates, such as LIBOR. Here we consider the role of interbank reference rates.

Suppose that there exists an institution which organizes an interbank market. The institution collects data on interest rates used in interbank transactions and provides these data to individual banks. Specifically, the institution selects q numbers of transactions and averages the interest rates used in these transactions:

$$i_t^q \equiv \frac{1}{q} \sum_{k=1}^{q} i_t^k. \tag{15}$$

The institution then announces the value of i_t^q to individual banks as the reference rate of interbank interest rates.[12]

Define ω_t^q as the fundamental factor corresponding to i_t^q:

$$\omega_t^q \equiv \frac{1}{q} \sum_{k=1}^{q} \omega_t^k = \mu_t + \overline{\varepsilon}_t^q, \tag{16}$$

where $\overline{\varepsilon}_t^q$ is defined as $\overline{\varepsilon}_t^q \equiv \frac{1}{q} \sum_{k=1}^{q} \varepsilon_t^k$.

Note that the distribution of $\overline{\varepsilon}_t^q$ is as follows:

$$\overline{\varepsilon}_t^q \sim N(0, \sigma_\varepsilon^2/q). \tag{17}$$

Here we assume that individual banks can acquire the knowledge of ω_t^q by observing the value of i_t^q. This means that, owing to the presence of the reference rate, individual banks can utilize the information on ω_t^q to infer the value of aggregate component μ_t.[13]

The banks know that ω_t^q consists of μ_t and $\overline{\varepsilon}_t^q$, but they do not observe each component separately. Based on (3) and (17), statistical inference yields the following estimator of μ_t:

$$E_t^j \mu_t = \eta \omega_t^q, \qquad \text{where} \quad \eta \equiv \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q}. \tag{18}$$

Note that the coefficient η is larger than θ when q is greater than unity. By substituting (18)

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[12] We assume that every bank not selected in the sample transactions observes the same value of i_t^q. In addition, we assume that a bank selected in the sample transactions observes all sample interest rates except for the bank's own interest rate. This means that the sample size reflected in the reference rate is q for the former bank and $q-1$ for the latter bank. This difference suggests that the estimators of μ_t made by these banks are slightly different. However, since there exist an infinite number of transactions in the interbank market, the impact of this difference on the entire market can be regarded as negligible and an explicit consideration of this difference does not alter the essence of our argument.

[13] For simplicity, we assume that individual banks use only the information of ω_t^q. In the Appendix, we confirm that the results are essentially the same even if banks use both ω_t^j and ω_t^q.

into (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + \omega_t^j - \eta\omega_t^q. \tag{19}$$

From (15) and (19), the average interest rate across the sample of q transactions is

$$i_t^q = \frac{1}{q}\sum_{k=1}^{q}(i_t^p + \omega_t^k - \eta\omega_t^q) = i_t^p + (1-\eta)\omega_t^q. \tag{20}$$

Therefore, as we assumed above, if the market-organizing institution provides data on i_t^q to individual banks, these banks can obtain the value of ω_t^q, via the simple calculation $(i_t^q - i_t^p)/(1-\eta) = \omega_t^q$.

From (2), (16), and (19), the market interest rate is

$$\begin{aligned}
i_t^m &= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}i_t^j = \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}(i_t^p + \omega_t^j - \eta\omega_t^q) \\
&= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}(i_t^p + \mu_t + \varepsilon_t^j - \eta\mu_t - \eta\bar{\varepsilon}_t^q) \\
&= i_t^p + (1-\eta)\mu_t + \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\varepsilon_t^j - \eta\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\bar{\varepsilon}_t^q \\
&= i_t^p + (1-\eta)\mu_t - \eta\bar{\varepsilon}_t^q. \tag{21}
\end{aligned}$$

Therefore, the spread between the market interest rate and the policy rate is

$$i_t^s = i_t^m - i_t^p = (1-\eta)\mu_t - \eta\bar{\varepsilon}_t^q. \tag{22}$$

The variance of the spread is

$$Var(i_t^s) = (1-\eta)^2\sigma_\mu^2 + \eta^2\sigma_\varepsilon^2/q. \tag{23}$$

Therefore, the variance of the spread depends on the number of sample transaction (q). If q approaches infinity, $Var(i_t^s)$ converges to zero:

$$\lim_{q\to\infty}Var(i_t^s) = \lim_{q\to\infty}\left[\left(1 - \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q}\right)^2\sigma_\mu^2 + \left(\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q}\right)^2\sigma_\varepsilon^2/q\right] = 0. \tag{24}$$

This means that, if the reference rate reflects the interest rates across all interbank transactions, then the volatility of the market interest rate due to informational imperfections is completely eliminated. In this case, the reference rate provides a perfectly accurate information on the aggregate shock. Then, the market interest rate is perfectly stabilized, as in the case of Section 3 where

individual banks can observe each shock separately.

In another limiting case of $q = 1$, however, η is equal to θ and $Var(i_t^s)$ is calculated as below:

$$Var(i_t^s) = (1 - \theta)^2 \sigma_\mu^2 + \theta^2 \sigma_\varepsilon^2. \tag{25}$$

By comparing (25) with (14), we find that $Var(i_t^s)$ in this section is larger than those presented in Section 4. This might be somewhat counterintuitive since it indicates that the existence of the reference rate makes the market interest rate more volatile. This phenomenon occurs because individual components included in the reference rate do not completely cancel each other out when q is very small. Since every individual bank observes the same reference rate, the averaged individual component included in the reference rate ($\bar{\varepsilon}_t^q$) affects each individual interest rate to an equal degree. This distorting effect of the reference rate is particularly large when the number of sample transactions is extremely small.

However, such an effect can be reduced by increasing the number of sample transactions. This can be confirmed by rewriting (23) as below:

$$Var(i_t^s) = \frac{\sigma_\mu^2 \sigma_\varepsilon^2}{q \sigma_\mu^2 + \sigma_\varepsilon^2}. \tag{26}$$

Thus, $Var(i_t^s)$ is monotonically decreasing with q. Therefore, by increasing the number of q, $Var(i_t^s)$ can be decreased. A simple calculation confirms that $Var(i_t^s)$ in this section is smaller than the corresponding value in Section 4, if and only if q is larger than a threshold value:

$$q > 2 + \frac{\sigma_\mu^2}{\sigma_\varepsilon^2}. \tag{27}$$

Therefore, when the sample number is sufficiently large, the reference rate helps to make the market interest rate less volatile. The mechanism is that the reference rate based on a sufficient number of sample transactions provides good information on the aggregate shock which is influential to the entire interbank market. This helps individual banks to identify more accurately the sources of shocks arising in the interbank market and to set individual interest rates at more appropriate levels.

6   Noise in the reference rate

In the previous section, we showed that the existence of a reference rate can reduce the volatility of the market interest rate. What if the reference rate includes some noisy components? The recent LIBOR problem suggests that this is a very real possibility, in which case the reference rate might not correctly reflect the interest rates actually used in interbank transactions.

In this section we consider the impact of the noise in the reference rate from the perspective of interbank market stability. In doing so, we make allowance for two different kinds of noise: (i) idiosyncratic noise, which is specific to individual transactions, and (ii) common noise, which has an influence on all transactions.

6.1 The case of idiosyncratic noise

As in Section 5, we assume that a market-organizing institution collects the data on interest rates used in individual interbank transactions. Here we also assume that the reported data for the interest rate in the kth transaction (denoted by $\widehat{i}_t^k$) includes idiosyncratic noise ν_t^k:

$$\widehat{i}_t^k = i_t^k + \nu_t^k, \tag{28}$$

where this noise follows the i.i.d. normal distribution:

$$\nu_t^k \sim N(0, \sigma_\nu^2). \tag{29}$$

The institution provides the average of the reported (not necessarily actual) interest rates:

$$\widehat{i}_t^q \equiv \frac{1}{q} \sum_{k=1}^q \widehat{i}_t^k = \frac{1}{q} \sum_{k=1}^q i_t^k + \overline{\nu}_t^q, \tag{30}$$

where $\overline{\nu}_t^q$ is defined as

$$\overline{\nu}_t^q \equiv \frac{1}{q} \sum_{k=1}^q \nu_t^k. \tag{31}$$

The distribution of $\overline{\nu}_t^q$ is as follows:

$$\overline{\nu}_t^q \sim N(0, \sigma_\nu^2/q). \tag{32}$$

Define $\widehat{\omega}_t^q$ as the fundamental factor corresponding to $\widehat{i}_t^q$:

$$\widehat{\omega}_t^q \equiv \mu_t + \overline{\varepsilon}_t^q + \overline{\nu}_t^q. \tag{33}$$

We assume that individual banks can acquire the knowledge of $\widehat{\omega}_t^q$ by observing the value of $\widehat{i}_t^q$. The banks know that $\widehat{\omega}_t^q$ consists of μ_t, $\overline{\varepsilon}_t^q$, and $\overline{\nu}_t^q$, but they do not observe each component separately. Based on (3) (17), and (32), statistical inference yields the following estimator of μ_t:

$$E_t^j \mu_t = \delta \widehat{\omega}_t^q, \qquad \text{where} \quad \delta \equiv \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q + \sigma_\nu^2/q}. \tag{34}$$

By substituting (34) into (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + \omega_t^j - \delta\widehat{\omega}_t^q. \tag{35}$$

From (30), (33), and (35), the average of the reported interest rates is

$$
\begin{aligned}
\widehat{i}_t^q &= \frac{1}{q}\sum_{k=1}^{q}(i_t^p + \omega_t^k - \delta\widehat{\omega}_t^q) + \overline{\nu}_t^q \\
&= i_t^p + (1-\delta)\widehat{\omega}_t^q.
\end{aligned}
\tag{36}
$$

Therefore, as we assumed above, if the market-organizing institution provides the data of $\widehat{i}_t^q$ to individual banks, the banks can obtain the value of $\widehat{\omega}_t^q$ via the simple calculation $(\widehat{i}_t^q - i_t^p)/(1-\delta) = \widehat{\omega}_t^q$.

From (2), (33), and (35), the market interest rate is

$$
\begin{aligned}
i_t^m &= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}i_t^j = \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}(i_t^p + \omega_t^j - \delta\widehat{\omega}_t^q) \\
&= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}(i_t^p + \mu_t + \varepsilon_t^j - \delta\mu_t - \delta\overline{\varepsilon}_t^q - \delta\overline{\nu}_t^q) \\
&= i_t^p + (1-\delta)\mu_t - \delta\overline{\varepsilon}_t^q - \delta\overline{\nu}_t^q.
\end{aligned}
\tag{37}
$$

Therefore, the spread between the market interest rate and the policy rate is

$$i_t^s = i_t^m - i_t^p = (1-\delta)\mu_t - \delta\overline{\varepsilon}_t^q - \delta\overline{\nu}_t^q. \tag{38}$$

The variance of the spread is

$$Var(i_t^s) = (1-\delta)^2\sigma_\mu^2 + \delta^2(\sigma_\varepsilon^2 + \sigma_\nu^2)/q. \tag{39}$$

We can rewrite (39) as follows:

$$Var(i_t^s) = \frac{\sigma_\mu^2(\sigma_\varepsilon^2 + \sigma_\nu^2)}{q\sigma_\mu^2 + \sigma_\varepsilon^2 + \sigma_\nu^2}. \tag{40}$$

By comparing (26) and (40), we find that, for any value of $q > 0$, $Var(i_t^s)$ is larger when the idiosyncratic noise is present ($\sigma_\nu^2 > 0$) rather than when it is absent ($\sigma_\nu^2 = 0$). Therefore, idiosyncratic noise makes the market interest rate more volatile.

However, the size of the destabilizing effect depends on the number of sample transactions (q).

In the limiting case of $q \to \infty$, the variance of the spread converges to zero:

$$\lim_{q \to \infty} Var(i_t^s) = \lim_{q \to \infty} \left[\left(1 - \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q + \sigma_\nu^2/q} \right)^2 \sigma_\mu^2 + \left(\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q + \sigma_\nu^2/q} \right)^2 (\sigma_\varepsilon^2 + \sigma_\nu^2)/q \right] = 0. \tag{41}$$

Therefore, even if idiosyncratic noise is included in the reported interest rates, the market interest rate can be made less volatile by basing the reference rate on a larger number of sample transactions.

6.2 The case of common noise

Next we assume that the reported interest rates include common noise ξ_t, which has an influence on all transactions:

$$\widehat{i}_t^k = i_t^k + \xi_t, \tag{42}$$

where this noise follows the i.i.d. normal distribution:

$$\xi_t \sim N(0, \sigma_\xi^2). \tag{43}$$

The institution provides the average of the reported interest rates to individual banks:

$$\widehat{i}_t^q \equiv \frac{1}{q} \sum_{k=1}^{q} \widehat{i}_t^k = \frac{1}{q} \sum_{k=1}^{q} i_t^k + \xi_t. \tag{44}$$

In this case, the fundamental factor corresponding to $\widehat{i}_t^q$ is

$$\widehat{\omega}_t^q \equiv \mu_t + \bar{\varepsilon}_t^q + \xi_t. \tag{45}$$

We assume that individual banks can acquire the knowledge of $\widehat{\omega}_t^q$ by observing the value of $\widehat{i}_t^q$. The banks know that $\widehat{\omega}_t^q$ consists of μ_t, $\bar{\varepsilon}_t^q$, and ξ_t, but they do not observe each component separately. Based on (3), (17), and (43), statistical inference yields the following estimator of μ_t:

$$E_t^j \mu_t = \chi \widehat{\omega}_t^q, \qquad \text{where} \quad \chi \equiv \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2/q + \sigma_\xi^2}. \tag{46}$$

By substituting (46) into (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + \omega_t^j - \chi \widehat{\omega}_t^q. \tag{47}$$

From (44), (45), and (47), the average of the reported interest rates is

$$
\begin{aligned}
\widehat{i}_t^q &= \frac{1}{q} \sum_{k=1}^{q} (i_t^p + \omega_t^j - \chi \widehat{\omega}_t^q) + \xi_t \\
&= i_t^p + (1 - \chi) \widehat{\omega}_t^q. \tag{48}
\end{aligned}
$$

Therefore, as we assumed above, if the market-organizing institution provides the data on $\widehat{i}_t^q$ to individual banks, the banks can obtain the value of $\widehat{\omega}_t^q$ by calculating $(\widehat{i}_t^q - i_t^p)/(1 - \chi) = \widehat{\omega}_t^q$.

From (2), (45), and (47), the market interest rate is

$$
\begin{aligned}
i_t^m &= \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} i_t^j = \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} (i_t^p + \omega_t^j - \chi \widehat{\omega}_t^q) \\
&= \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} (i_t^p + \mu_t + \varepsilon_t^j - \chi \mu_t - \chi \bar{\varepsilon}_t^q - \chi \xi_t) \\
&= i_t^p + (1 - \chi) \mu_t - \chi \bar{\varepsilon}_t^q - \chi \xi_t. \tag{49}
\end{aligned}
$$

The spread between the market interest rate and the policy rate is

$$
i_t^s = i_t^m - i_t^p = (1 - \chi) \mu_t - \chi \bar{\varepsilon}_t^q - \chi \xi_t. \tag{50}
$$

The variance of the spread is

$$
Var(i_t^s) = (1 - \chi)^2 \sigma_\mu^2 + \chi^2 (\sigma_\varepsilon^2 / q + \sigma_\xi^2). \tag{51}
$$

Therefore, the variance of the spread depends on the sample number of transactions. However, in the presence of common noise in the reference rate, the volatility of the market interest rate cannot be eliminated simply by increasing the number of transactions:

$$
\begin{aligned}
\lim_{q \to \infty} Var(i_t^s) &= \lim_{q \to \infty} \left[(1 - \chi)^2 \sigma_\mu^2 + \chi^2 (\sigma_\varepsilon^2 / q + \sigma_\xi^2) \right] \\
&= \lim_{q \to \infty} \left[\left(1 - \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2 / q + \sigma_\xi^2} \right)^2 \sigma_\mu^2 + \left(\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2 / q + \sigma_\xi^2} \right)^2 (\sigma_\varepsilon^2 / q + \sigma_\xi^2) \right] \\
&= \left(\frac{\sigma_\xi^2}{\sigma_\mu^2 + \sigma_\xi^2} \right)^2 \sigma_\mu^2 + \left(\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2} \right)^2 \sigma_\xi^2. \tag{52}
\end{aligned}
$$

Therefore, when the noise is common to multiple transactions, the variance of the spread does not necessarily converge to zero, even if the number of sample transactions becomes quite large. This result is obtained by the following two reasons. First, the value of χ does not approach unity even if q goes to infinity, since the upper limit of χ is $\sigma_\mu^2 / (\sigma_\mu^2 + \sigma_\xi^2)$, which is strictly smaller than unity. As a result, the first term of (51) does not approach zero. Second, as indicated by the third

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term of (50), the value of ξ_t is not cancelled out when we sum up the individual interest rates because of the inherent nature of common noise. This result suggests that the existence of the reference rate does not necessarily help to reduce the volatility of the market interest rate in cases where noise has a broader (or even market-wide) impact.

7   Numerical examples

So far we have explained the role of the reference rate and the implications of the noise in the reference rate. Our theoretical results suggest that the influence of the reference rate on the volatility of the market interest rate depends on (i) the number of interbank transaction reports reflected in the reference rate (q), (ii) the variances of shocks (σ_μ^2 and σ_ε^2), and (iii) the variances of noise (σ_ν^2 and σ_ξ^2). We now present some numerical examples in the hope of illustrating these relationships. For our benchmark example we set the variances of shocks and noise to $\sigma_\mu^2 = \sigma_\varepsilon^2 = \sigma_\nu^2 = \sigma_\xi^2 = 1.0$. We then calculate the variance of the interest rate spread ($Var(i_t^s)$) for various values of q, the number of interbank transaction reports reflected in the reference rate.

Figure 1 shows the relationship between $Var(i_t^s)$ and q for the situations examined in Section 3, 4, 5, 6.1, and 6.2, while Figure 2 shows the corresponding level of parameters of inference, such as θ, η, δ, and χ. In the case of Section 3, where banks can distinguish between aggregate shocks and individual shocks, $Var(i_t^s)$ is equal to zero for all values of q. In the case of Section 4, where banks are unable to observe each shock separately, $Var(i_t^s)$ is not equal to zero. As shown in Figure 2, the parameter of inference (θ) does not depend on q. Consequently, $Var(i_t^s)$ is positively constant (0.25 in this benchmark example) for all values of q. In the case of Section 5, the reference rate exists and is noise-free. When q is extremely small, such as $q = 1$ or 2, $Var(i_t^s)$ is larger than 0.25, which is the corresponding value in Section 4. However, since $Var(i_t^s)$ is monotonically decreasing with q, $Var(i_t^s)$ is less than 0.25 for sufficiently large values of q (specifically, $q > 4$ in this numerical example), and converges to zero when q approaches infinity. This illustrates the stabilizing effects of the reference rate.

However, if the reference rate contains noisy components, the stabilizing effects of the reference rate are significantly reduced. In the case of Section 6.1 when the reference rate includes idiosyncratic noise, $Var(i_t^s)$ is larger than those in the case of Section 5, for a given value of q. Nevertheless, since the noise is idiosyncratic to individual transactions, the volatility of the market interest rate can be reduced by increasing q. However, in the case of common noise, which is examined in Section 6.2, $Var(i_t^s)$ does not converge to zero even if q approaches infinity. By increasing the value of q, the first term of (51) converges to 0.25. However, because of the presence of the third term of (51), $Var(i_t^s)$ is strictly larger than 0.25, which is the corresponding level in Section 4.

Figure 3 presents a sensitivity analysis with respect to the variances of fundamental shocks. Panel (a) shows the case in which the variance of the aggregate shock is larger than the benchmark

value ($\sigma_\mu^2 = 5.0$). For a given value of q, the parameters of inference (θ, η, δ, and χ) are larger than those in the benchmark example. Then, $Var(i_t^s)$ is lower than the benchmark value in the case of Section 4. However, in the cases of Section 5, 6.1, and 6.2, $Var(i_t^s)$ is larger than the benchmark value. This result can be understood as follows. In the presence of the reference rate, if σ_μ^2 is large (compared to σ_ε^2), banks set the inference parameter at a higher value, because banks know that the reference rate contains more accurate information on the aggregate component. Higher inference parameters translate into higher final terms in (23), (39), and (51), thereby magnifying the volatility of the market interest rate. In the case of Section 4 where the reference rate is non-existent, this effect is absent. Then the rise of σ_μ^2 simply increases the inference parameter (θ), thereby lowering $Var(i_t^s)$.

Panel (b) shows the case in which the variance of the idiosyncratic shock is larger than the benchmark value ($\sigma_\varepsilon^2 = 5.0$). For a given value of q, the parameters of inference (θ, η, δ, and χ) are smaller than those in the benchmark example. Then, $Var(i_t^s)$ is larger than the benchmark level in all situations. In the case of Section 4, lower inference parameter (θ) leads to higher $Var(i_t^s)$, as shown in (14). This effect also exists in other cases, as shown in the first terms of (23), (39), and (51). In addition, larger σ_ε^2 magnifies $Var(i_t^s)$ by increasing the final terms in (23), (39), and (51). As a result, for a given value of q, larger σ_ε^2 leads to higher $Var(i_t^s)$ in all situations.

Figure 4 presents a sensitivity analysis with respect to the variances of noise in the reported interest rates. Panel (a) shows the case in which the variance of idiosyncratic noise (σ_ν^2) is larger than the benchmark level. This rise in σ_ν^2 simply lowers the parameter of inference in Section 6.1 (δ), for a given value of q. Then, $Var(i_t^s)$ is larger than in the benchmark example. Panel (b) shows the case in which the variance of common noise is larger than the benchmark level. The rise in σ_ξ^2 simply lowers the parameter of inference in Section 6.2 (χ), for a given value of q. Then, $Var(i_t^s)$ is larger than in the benchmark example. These results indicate that the market interest rate becomes more volatile for large variances in either idiosyncratic noise or common noise.

8 Conclusion

In this study, we have theoretically investigated the potential role of the reference rate in stabilizing or destabilizing an interbank market with an environment where individual banks cannot fully identify the nature of underlying shocks affecting their interbank transactions. We have shown that a noise-free reference rate based on a sufficient number of sample transactions can help to make the market interest rate less volatile, whereas the stabilizing effects of the reference rate are significantly reduced if the reported interest rates contain some noisy components. Nevertheless, by increasing the number of sample transactions reflected in the reference rate, the adverse effects of the noise can be mitigated (or eliminated) provided the noise is idiosyncratic to individual transactions. However, if the noise is common to multiple transactions, then the adverse effects of the noisy reference rate

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cannot be reduced simply by increasing the number of sample transactions. This suggests that the noise in the interest rates reported by just a few of large banks can end up making the entire market more volatile, thereby lowering the correlation between the interbank interest rate and the central bank's policy rate, and consequently impairing the transmission mechanism of monetary policy.

Appendix: When banks use ω_t^j and ω_t^q to infer the aggregate component

In Section 5, we have assumed that individual banks only use the information obtainable from the reference rate (ω_t^q) to infer the aggregate component (μ_t). Here we assume that banks additionally use the information on the fundamental shock in their individual transaction (ω_t^j). This means that the banks' estimator of μ_t takes the following form:

$$E_t^j \mu_t = \eta_1 \omega_t^j + \eta_2 \omega_t^q. \tag{A1}$$

Then, from (2), (4), (16), (17), and (A1), the parameters of η_1 and η_2 are computed as follows:

$$\eta_1 = \frac{\sigma_\mu^2}{(1+q)\sigma_\mu^2 + \sigma_\varepsilon^2}, \tag{A2}$$

$$\eta_2 = \frac{q\sigma_\mu^2}{(1+q)\sigma_\mu^2 + \sigma_\varepsilon^2}. \tag{A3}$$

By substituting (A1) into (5), the interest rate in the jth transaction is determined as follows:

$$i_t^j = i_t^p + (1 - \eta_1)\omega_t^j - \eta_2 \omega_t^q. \tag{A4}$$

The average interest rate across this sample of transactions is

$$i_t^q = \frac{1}{q}\sum_{k=1}^{q}\left[i_t^p + (1-\eta_1)\omega_t^k - \eta_2\omega_t^q\right] = i_t^p + (1 - \eta_1 - \eta_2)\omega_t^q. \tag{A5}$$

Therefore, if the market-organizing institution provides the data of i_t^q to individual banks, the banks can obtain the value of ω_t^q, from the simple calculation $(i_t^q - i_t^p)/(1 - \eta_1 - \eta_2) = \omega_t^q$.

From (2), (16), and (A4), the market interest rate is

$$
\begin{aligned}
i_t^m &= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n} i_t^j = \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\left[i_t^p + (1-\eta_1)\omega_t^j - \eta_2\omega_t^q\right] \\
&= \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\left[i_t^p + (1-\eta_1)(\mu_t + \varepsilon_t^j) - \eta_2(\mu_t + \bar{\varepsilon}_t^q)\right] \\
&= i_t^p + (1 - \eta_1 - \eta_2)\mu_t - \eta_2\bar{\varepsilon}_t^q.
\end{aligned}
\tag{A6}
$$

Therefore, the spread between the market interest rate and the policy rate is

$$i_t^s = i_t^m - i_t^p = (1 - \eta_1 - \eta_2)\mu_t - \eta_2 \overline{\varepsilon}_t^q. \tag{A7}$$

The variance of this spread is

$$Var(i_t^s) = (1 - \eta_1 - \eta_2)^2 \sigma_\mu^2 + \eta_2^2 \sigma_\varepsilon^2/q. \tag{A8}$$

In the limiting case of $q \to \infty$, the volatility of the spread converges to zero:

$$\lim_{q \to \infty} Var(i_t^s) = \lim_{q \to \infty} \left(1 - \frac{(1+q)\sigma_\mu^2}{(1+q)\sigma_\mu^2 + \sigma_\varepsilon^2}\right)^2 \sigma_\mu^2 + \left(\frac{q\sigma_\mu^2}{(1+q)\sigma_\mu^2 + \sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2/q = 0. \tag{A9}$$

This result is the same as in Section 5.

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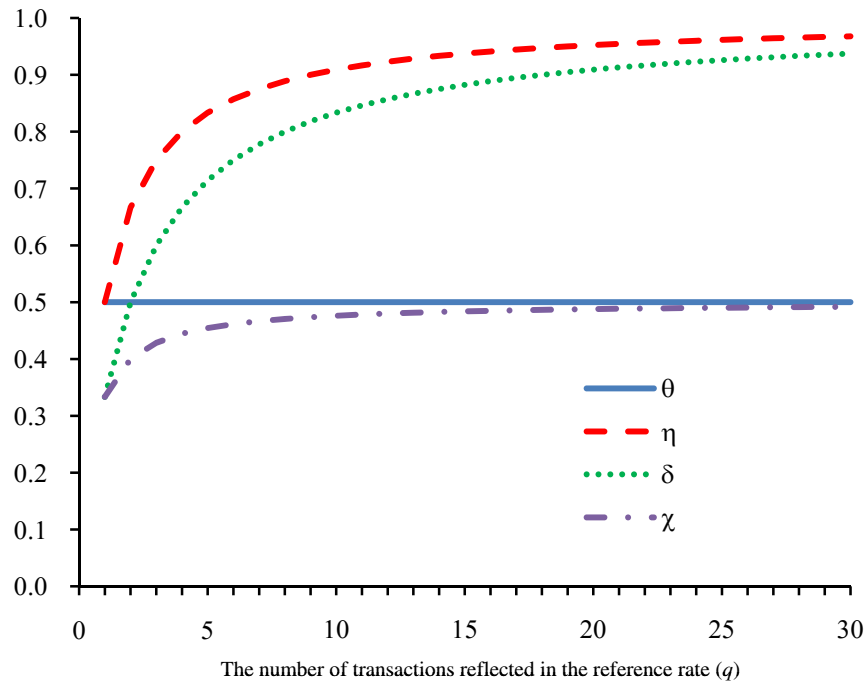
The volatility of the
interest rate spread (*Var* (i^s_t))
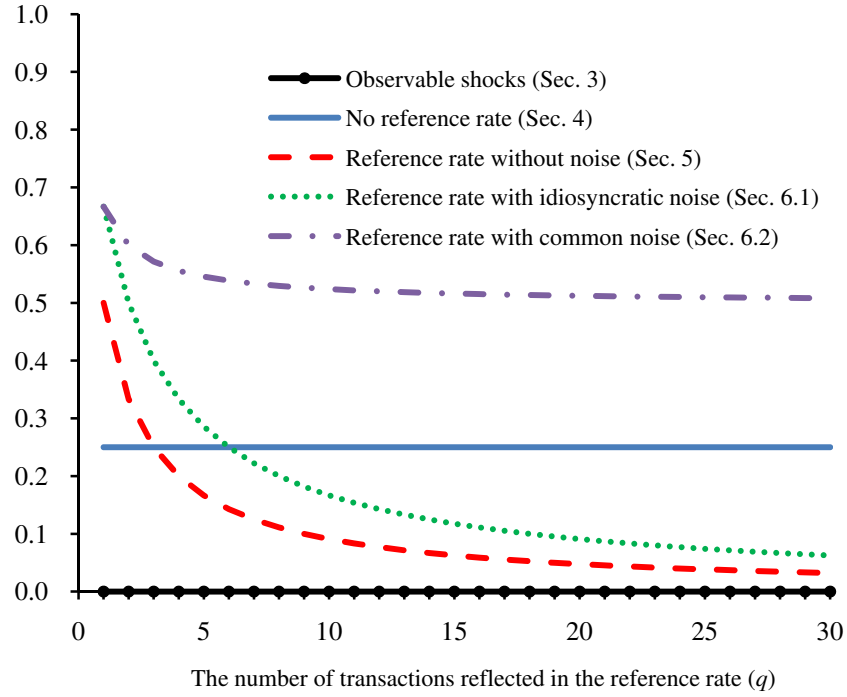
Note: The parameter values are set as: $\sigma_\mu^2=\sigma_\varepsilon^2=\sigma_v^2=\sigma_\xi^2=1.0$.

Figure 1: The volatility of the interest rate spread and the number of transactions reflected in the reference rate

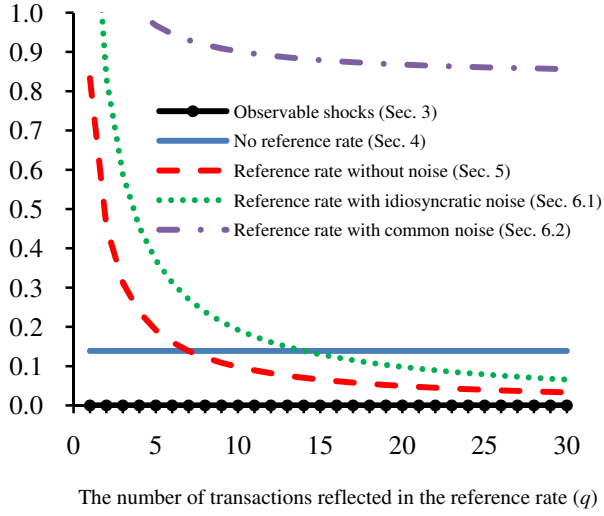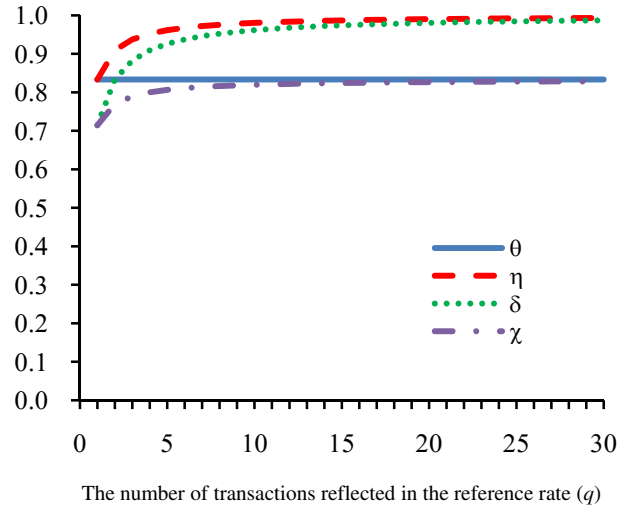Parameters of inference on
aggregate components

Note: The parameter values are set as: $\sigma_\mu^2 = \sigma_\varepsilon^2 = \sigma_\nu^2 = \sigma_\xi^2 = 1.0$.

Figure 2: Parameters of inference and the number of transactions reflected in the reference rate

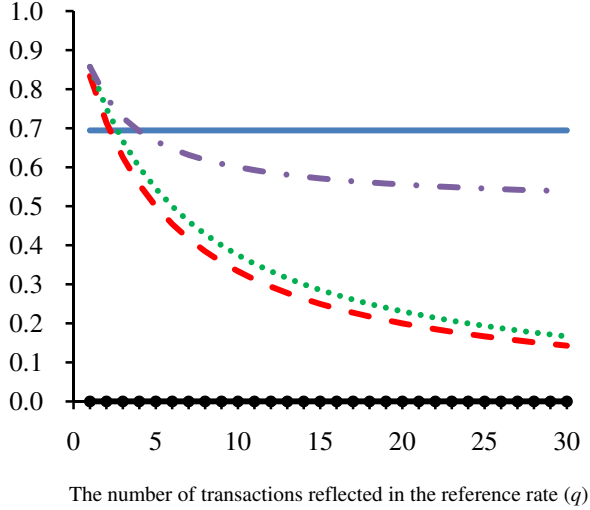The volatility of the
interest rate spread ($Var\ (i^s_t)$)
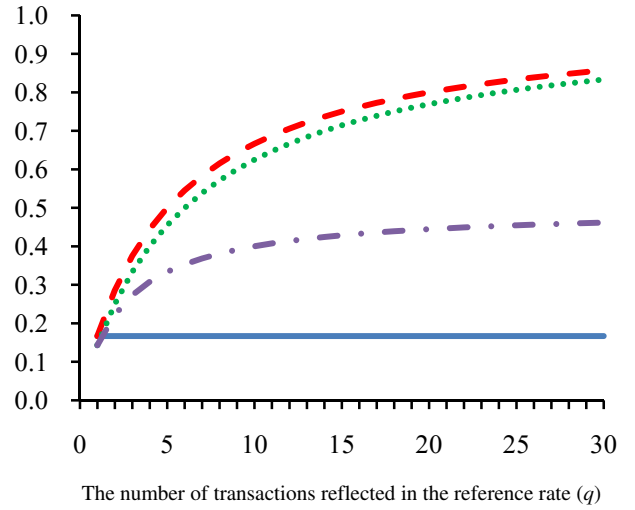
Parameters of inference on
aggregate components

- ●—— Observable shocks (Sec. 3)
- —— No reference rate (Sec. 4)
- – – Reference rate without noise (Sec. 5)
- ⋯⋯ Reference rate with idiosyncratic noise (Sec. 6.1)
- –·– Reference rate with common noise (Sec. 6.2)

The number of transactions reflected in the reference rate (q)

- —— θ
- – – η
- ⋯⋯ δ
- –·– χ

The number of transactions reflected in the reference rate (q)

(a) $\sigma_\mu^2 = 5.0$, $\sigma_\varepsilon^2 = \sigma_\nu^2 = \sigma_\xi^2 = 1.0$

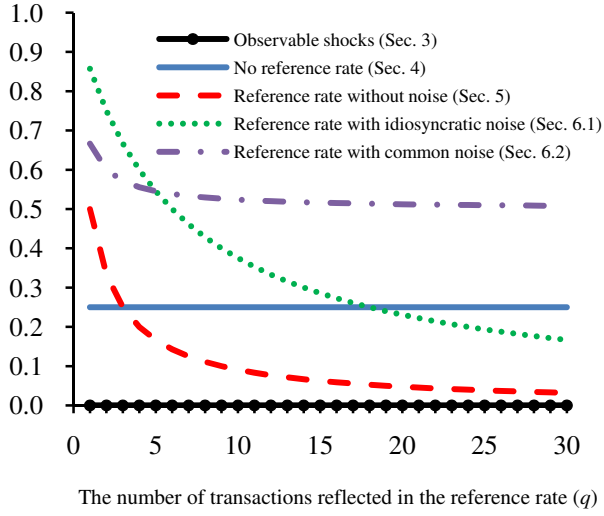The volatility of the
interest rate spread ($Var\ (i^s_t)$)

Parameters of inference on
aggregate components

The number of transactions reflected in the reference rate (q)

The number of transactions reflected in the reference rate (q)

(b) $\sigma_\mu^2 = 1.0$, $\sigma_\varepsilon^2 = 5.0$, $\sigma_\nu^2 = \sigma_\xi^2 = 1.0$

Figure 3: Sensitivity analysis with respect to shock variances
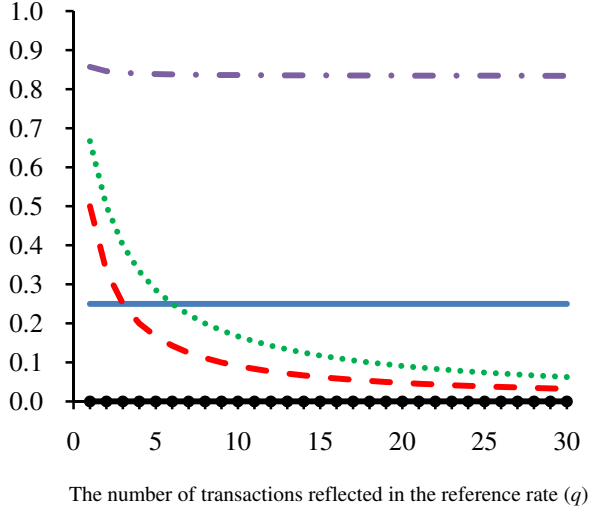
The volatility of the
interest rate spread (Var (i^s_t))
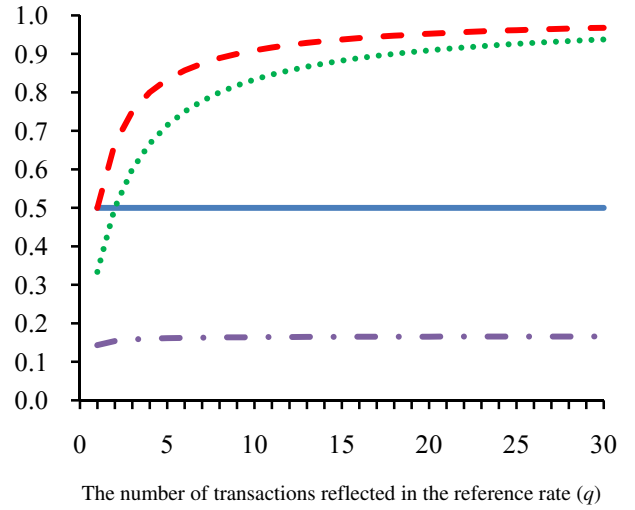
Parameters of inference on
aggregate components

Legend (left chart, top):
- Observable shocks (Sec. 3)
- No reference rate (Sec. 4)
- Reference rate without noise (Sec. 5)
- Reference rate with idiosyncratic noise (Sec. 6.1)
- Reference rate with common noise (Sec. 6.2)

Legend (right chart, top):
- θ
- η
- δ
- χ

The number of transactions reflected in the reference rate (q)

The number of transactions reflected in the reference rate (q)

(a) $\sigma_\mu^2=\sigma_\varepsilon^2=1.0$, $\sigma_\nu^2=5.0$, $\sigma_\xi^2=1.0$

The volatility of the
interest rate spread (Var (i^s_t))

Parameters of inference on
aggregate components

The number of transactions reflected in the reference rate (q)

The number of transactions reflected in the reference rate (q)
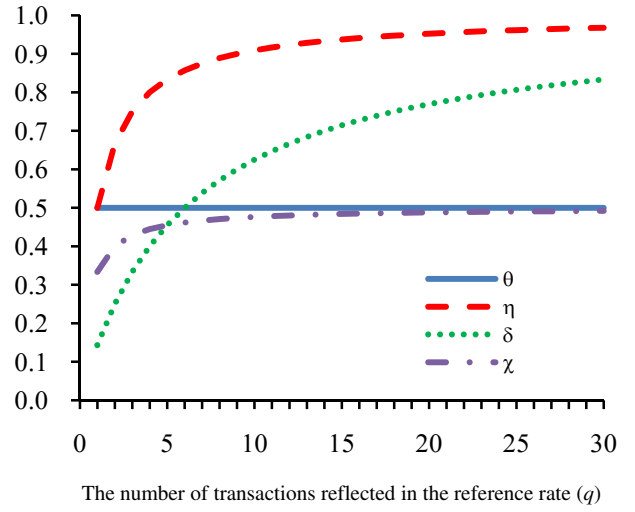
(b) $\sigma_\mu^2=\sigma_\varepsilon^2=\sigma_\nu^2=1.0$, $\sigma_\xi^2=5.0$

Figure 4: Sensitivity analysis with respect to noise variances