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Kevin Mongeon and Jason Winfree

University of New Haven, University of Michigan


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Kevin Mongeon\textsuperscript{2}    Jason A. Winfree\textsuperscript{3}

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\textsuperscript{2}Department of Sport Management/Hospitality and Tourism Management, University of New Haven, Maxcy 114; 300 Boston Post Rd. West Haven Ct. 06516, kmongeon@newhaven.edu

\textsuperscript{3}contact author: Program in Sport Management, University of Michigan, Observatory Lodge, 1402 Washington Heights, Ann Arbor, Michigan 48109, Phone: 734-647-5424, Fax: 734-647-2808, jwinfree@umich.edu
Abstract

Although many sports leagues are viewed as monopolies, research suggests that some economic competition exists between teams in different sports leagues. If fans make consumption choices based on the quality of all teams that are present in their region, then economic competition and ownership structure can impact an owner’s incentive to invest in talent. This article examines differences between monopolists, duopolists and cross-ownership. Consumer preferences and fan loyalty are allowed to vary across sports, and the winning percentages of teams in other leagues affects demand. Our model shows that economic competition results in an ambiguous level of investment compared to a monopolist. A firm that engages in cross-ownership will invest less in talent compared to a duopolist, but the difference in profits is ambiguous. League policies are studied and are shown to affect the quality of teams in other leagues.

Keywords: Sports Leagues, Talent Investment, Ownership Structures
JEL classification: L83
1 Introduction

Much has been made about the effects of monopoly power in professional sports leagues. Major League Baseball (MLB) has antitrust exemption and the other North American major professional leagues enjoy similar market structures. Most teams in these leagues have gained regional monopoly power by eliminating local competition in the same league. This has inspired a substantial body of research that focuses on player talent investment by teams and how league policies change these investments and the competitive balance of the league. However, if game attendees, television viewers, and advertisers are choosing between teams in different sports, then leagues do have economic competitors from other leagues. Furthermore, it is not uncommon for European football fans to be choosing between multiple teams in the same sport in the same region, but in different leagues.

Given that there is economic competition between sports leagues, this study examines the impact that the quality of teams in other leagues has on an owner’s incentive to invest in talent under various circumstances. In addition, we model the effect of a single owner owning two teams in separate leagues (cross-ownership) and also the effects of league policies across different leagues. Consistent with economic intuition, we find that cross-ownership reduces an owner’s incentive to invest in talent. However, not as intuitive is that economic competition can either increase or decrease team quality.

Most analysts have argued that an increase in competition will lead to a decrease in investment. For instance, both former MLB commissioner Bowie Kuhn and MLB’s Blue Ribbon Panel Report recommended putting a third team in New York to help competitive balance (Zimbalist 2003). As our paper shows, economic competition could increase the incentive to win for large market teams under certain conditions since fans would have more alternatives. Even less intuitive is that cross-ownership may result in lower profit levels compared to duopolists. That is, if there are two teams in a market, but in different leagues, it may be profitable to have two separate owners. This result occurs because other teams in the league, but outside the region, may increase their talent investment
under cross-ownership. Regarding league policies, our paper shows that under certain conditions, an increase in a salary cap in one league can decrease talent levels of some teams in other leagues and that revenue sharing directly mitigates some of the effects of cross-ownership.

Sports leagues have been modeled as revenue being determined by winning and talent (El Hodiri and Quirk (1971), Quirk and El Hodiri (1974), Fort and Quirk (1995), Vrooman (1995), Szymanski (2003), Szymanski and Kesenne (2004)). These models have substantially progressed the understanding of the behavior of professional sport leagues as well as provided insights into the effects of league policies such as salary caps and revenue sharing on profits, competitive balance, and player salaries. We build upon these models by including the effect of economic competition of the quality from teams in other leagues into the owner’s objective function. Because of this, the common ownership of teams across leagues and the cross-league effects of league policies become important.

The study of economic competition within and across professional sports leagues is a growing area of empirical research.\(^1\) Dealing with competition within a league, Winfree et al. (2004) show that the presence of MLB teams decreased the attendance of other MLB teams in the region. Evidence also suggests that when two teams are located in the same region fans are responsive to the quality changes of the other team in the region (Miller, 2006). This implies that fans have various levels of loyalty toward sports teams. Furthermore, Henrickson (forthcoming) found that teams price strategically based on the presence of other sports franchises and that competition can affect team location decisions.

There is also evidence that fans substitute between teams in other leagues. Researchers have used the 2004-05 National Hockey League (NHL) lockout as a natural experiment to examine how fans substituted to other leagues. Winfree and Fort (2008) find that in the absence of the NHL, minor league hockey teams located in the same city as an NHL team increased their attendance by 9 percent (although statistically in-

\(^1\)While not focusing directly on fans switching between teams, there is also a literature on fan loyalty (Wakefield and Sloan (1995), Dawson and Downward (2000), Depken II (2000), and Depken II (2001)).
significant) and junior league hockey teams by 19.9 percent. Rascher et al. (2009) found that minor league hockey teams increased attendance an average of 2 percent during the lockout. Both Winfree (2009a) and Rascher et al. (2009) show that a substitution effect also exists between National Basketball Association (NBA) teams and NHL teams. In the year of the lockout, attendance of NBA teams increased by 3 to 4 percent in cities where NHL teams were located. However, since NHL revenues are smaller, the increase in attendance in the NBA represented about 6-7 percent of NHL revenues. Winfree (2009b) finds that over the past decade when teams from other major North American sports leagues exited a market, incumbent teams increased short run attendance by 5.5 percent on average. Most recently, Robinson (forthcoming) gives evidence that European football fans will switch allegiances. It is often the case that fans are switching allegiances between teams in the same sport but different leagues given the promotion and relegation system in European football in which teams often switch leagues. Furthermore, it is often the case that teams in different leagues, but in the same region, often compete for advertising, sponsorship, and luxury suite sales from companies. These revenue sources also depend on the relative quality of the teams.

If fan allegiances are slow to change, one might expect the long run fan substitution to be greater than short run fan substitution. Table 1 shows the average local broadcast ratings for most NBA teams from the 1999-2000 season to the 2004-2005 season. This gives evidence of much higher television ratings in markets without other major sports competitors. Further, more sophisticated statistical analyses have been done showing that demand for teams for both attendance and media is higher in markets without other sports teams (Paul, 2003; Tainsky, 2010; Mongeon and Winfree, forthcoming).

Given that cross-league substitution effects exist, the study of the cross-ownership of teams in different leagues becomes important area of analysis. Most cross-ownership involves teams in separate leagues in the same city. However, the National Football League (NFL) is an exception. While NFL owners may own other major sports franchises, they may not do so in an NFL market. Therefore, we use MLB, NBA, and the NHL for
a benchmark. Table 2 shows cross-ownership groups between teams in MLB, NBA, and the NHL.\(^2\) Of the 24 cases of cross-ownership that we have identified, only one is between teams that are located in different regions.\(^3\) To provide a relative measure of the prevalence of cross-ownership, Figure 1 depicts the number of markets with more than one major league team and the number of cross-owned firms over time.\(^4\) The number of cities that contain more than one team changes over time from league expansion and team relocations. Figure 1 shows that the number of markets with one team from each of the three leagues increased from 7 to 14 from 1970 to 2007, while, the amount of cross-ownership increased from 2 to 9.\(^5\)

The study of the effects of league policies on an owner’s incentive to invest in talent is not new to the literature. However, to our knowledge, no one has examined the effects of a league policy on another league. If some fans are choosing between teams in different leagues based on their quality, and league policies affect team quality level, then leagues and teams should be aware of the policies of other leagues.

The rest of the article proceeds as follows: section 2 sets up team models and compares the different incentives for owners to invest in talent across different market structures. Section 3 examines the effect of cross-ownership. Section 4 determines the impact of league policies in alternate leagues, section 5 gives an example with functional forms, and section 6 concludes and discusses some implications.

\(^2\)Many of the ownership groups presented in Table 2 also own teams beyond the leagues of MLB, NBA, and the NHL, including minor or junior league teams, Major League Soccer (MLS) teams, and Arena Football League (AFL) teams. For conciseness, teams from these leagues are not included in the table. Since larger markets have more teams, cross-ownership is more common in the larger markets.

\(^3\)From this point forward, when we refer to cross-ownership we are implying the cross-ownership of teams that are located in the same city.

\(^4\)For example, two cross-owned teams are counted as one cross-owned firm.

\(^5\)While the amount of cross-ownership has increased over the last few decades it is not necessarily a result of anti-competitive behavior. For example, it may be more efficient to own multiple teams. Although beyond the scope of this article, vertical relationships exist between teams and media entities, concession companies or other vertically related firms as well.
2 Talent investment with competition across leagues

We first consider a two-team league monopoly as a base case. We define a monopolist as being the only team, in that league or otherwise, in the region. In this model, talent investment by each team determines the winning percentage for each team, which in turn determines revenue. Also, the winning percentages of both teams in a league must add up to one. The profit function for a monopolist is given by,

\[
\pi_{a1} = R_{a1}(w_{a1}(t_{a1}, t_{a2})) - t_{a1} \tag{1}
\]

and the profit function for the other team in the league is given by,

\[
\pi_{a2} = R_{a2}(w_{a2}(t_{a2}, t_{a1})) - t_{a2} \tag{2}
\]

Where \( R_{a1} \) is the revenue of team 1 in league \( a \), \( w \) represents winning percentage and \( t \) is the talent investment. We assume that revenue increases when winning increases, \( \frac{dR_{a1}}{dw_{a1}} > 0 \), winning increases (or does not decrease) when talent investment increases, \( \frac{\partial w_{a1}}{\partial t_{a1}} \geq 0 \), but at a decreasing rate, \( \frac{\partial^2 w_{a1}}{\partial t_{a1}^2} < 0 \), and winning decreases when the opposing team increases talent investment, \( \frac{\partial w_{a1}}{\partial t_{a2}} < 0 \). Also, if one team does not invest in talent then the other team wins all of the games \( (w_{a1}(t_{a1}, 0) = 1 \text{ implying } \frac{\partial w_{a1}}{\partial t_{a1}} = 0 \text{ when } t_{a2} = 0) \), but some investment will lead to a winning percentage greater than zero (if \( t_{a1} > 0 \text{ and } t_{a2} > 0 \text{ then } 0 < w_{a1} < 1 \)). We further assume that the marginal impact of talent investment on revenue is positive, but at a decreasing rate so that \( \frac{\partial R_{a1}}{\partial t_{a1}} > 0 \), and \( \frac{\partial^2 R_{a1}}{\partial t_{a1}^2} < 0 \). The first order conditions for both teams are,

\[
\frac{\partial \pi_{a1}}{\partial t_{a1}} = \frac{dR_{a1}}{dw_{a1}} \frac{\partial w_{a1}}{\partial t_{a1}} - 1 = 0 \tag{3}
\]

and

\[
\frac{\partial \pi_{a2}}{\partial t_{a2}} = \frac{dR_{a2}}{dw_{a2}} \frac{\partial w_{a2}}{\partial t_{a2}} - 1 = 0 \tag{4}
\]
Equations (3) and (4) imply that both teams will invest in talent to the point where the contribution of talent on revenue equals the cost of talent and in equilibrium, 

\[ \frac{dR_{a1}}{dwa_{a1}} = \frac{dR_{a2}}{dwa_{a2}}. \]

From this point on we do make the assumption that if winning has a greater impact on total revenue, that is if \( \frac{dTR_{a1}}{dwa_{a1}} \) shifts up, then the talent investment for that team will increase and the team will win more games. However, it is possible that this is not the case. Appendix A shows under what conditions an increase in \( \frac{dTR_{a1}}{dwa_{a1}} \) leads to an increase in winning percentage.

We now consider the duopolist’s case where there are two teams from different leagues in the market. In this case, some fans’ tastes are diverse, so that there are both sport-specific fans and general sports fans. Sport-specific fans only consume a specific sport and their purchasing decisions are based entirely on the quality of that specific team, regardless of the presence of a competing team in the market. In contrast, general sports fans will potentially consume any sport that is in the market and their consumption choice depends on the quality of all of the teams in the market. We assume that there are two teams in each league. However, for simplicity, we assume that team 1 in each league is in the same market, but team 2 in league \( a \) and team 2 in league \( b \) are in different markets and do not compete with each other economically.

For the duopolist, the function \( \gamma \) represents the proportion of total fans relative to the monopoly case. It is a closed bounded set between zero and one and depends on the winning percentages of both teams in the market. Therefore, \( \gamma_{a1}(w_{a1}(t_{a1}, t_{a2}), w_{b1}(t_{b1}, t_{b2}), R_{a1}) \)

\[ \text{There has been some debate in the sports economics literature about how talent investment affects the investment of the other teams in the league (Szymanski (2004), Eckard(2006), Szymanski (2006)). In particular, North American sports leagues are considered to have a fixed supply of talent (with some exceptions such as Major League Soccer), while European leagues have an elastic supply. Therefore, if this model is applied to European style leagues, talent investment could simply be considered talent. That is, an increase in talent investment by one team does not change the talent level of the other team. In North American style leagues, it is assumed that the two teams invest in talent, which is distributed between the two teams, and then determines winning percentages. Either way, winning is a function of talent investment.} \]

\[ \text{Up to this point we have denoted the monopolist’s revenue as } R. \text{ However, later in the paper team revenue is compared to the monopolist’s revenue and is a percentage of } R. \text{ Therefore we define total revenue as all revenue generated by that team.} \]

\[ \text{These fans could also be considered league-specific fans.} \]
represents total revenue. If the leagues are not economic competitors, then $\gamma = 1$. However, if we assume the leagues are economic competitors, $\gamma$ is a non-decreasing function of the team’s winning percentage, $\frac{\partial \gamma}{\partial w_{a1}} \geq 0$, and a non-increasing function of the other team’s winning percentage, $\frac{\partial \gamma}{\partial w_{b1}} \leq 0$.

The profit function for the duopolist is

$$\pi_{a1} = \gamma_{a1}(w_{a1}(t_{a1}, t_{a2}), (w_{b1}(t_{b1}, t_{b2}))R_{a1}(w_{a1}(t_{a1}, t_{a2})) - t_{a1} \tag{5}$$

And the first order condition is given by,

$$\frac{\partial \pi_{a1}}{\partial t_{a1}} = \left( \frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1} + \gamma_{a1} \frac{dR_{a1}}{dw_{a1}} \right) \frac{\partial w_{a1}}{\partial t_{a1}} - 1 = 0 \tag{6}$$

The change in total revenue from winning for the duopolist is divided into two parts, $\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1}$ and $\gamma_{a1} \frac{dR_{a1}}{dw_{a1}}$. The product $\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1}$ is the revenue gained from the additional non-sport specific fans that attend the game due to a one-unit increase in the team’s own winning percentage. The product $\gamma_{a1} \frac{dR_{a1}}{dw_{a1}}$ is the additional revenue generated from a one-unit increase in winning percentage from fans that are not switching leagues. The sum of the two products, $\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1} + \gamma_{a1} \frac{dR_{a1}}{dw_{a1}}$, represents a more general form of the marginal revenue than the monopolist’s. If $\gamma = 1$ and $\frac{\partial \gamma_{a1}}{\partial w_{a1}} = 0$, then equation (3) is equivalent to equation (6).

A team entering a market leads to an ambiguous effect on talent investment for the incumbent team. If a team enters a market and the fan base of market 1 is simply divided into loyal fans of team $a$ and loyal fans of team $b$, $\frac{\partial \gamma_{a1}}{\partial w_{a1}} = 0$ and $\gamma < 1$, then, essentially, the incumbent team has become a smaller market team. However, if instead the two teams are competing for non-loyal fans, the teams might have a stronger incentive to invest in talent, which is somewhat different than conventional wisdom.\footnote{Although we are modeling different leagues, the same intuition holds for teams in the same market in the same league. It is often argued that putting a third baseball team in New York will help Major League Baseball’s competitive balance since it would decrease the talent investment of the New York Yankees (Zimbalist 2003).} Comparing equations (6)
and (3), if $\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1} > (1 - \gamma_{a1}) \frac{dR_{a1}}{dw_{a1}}$ then the duopolist will invest more in talent than the monopolist since the marginal revenue from talent investment has increased. However, $(1 - \gamma_{a1}) \frac{dR_{a1}}{dw_{a1}}$ can be either greater or less than $\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1}$. This results leads to the following proposition:

**Proposition 1** If fans are loyal, then owners will invest less in talent in the presence of another league. If fans are not loyal, then the effect of the presence of another team on talent investment is ambiguous.

It should also be noted that loyalty in this case means something relatively specific. In this case, loyalty implies that a fan will only be a fan of one team in a two team market, but in the absence of their favorite team, they will be a fan of the other team.

We now examine the effect of a marginal change of talent investment by team 1 in league $b$ on team 1 in league $a$. Differentiating equation (6) gives us,

$$\frac{\partial^2 \pi_{a1}}{\partial t_{a1}\partial t_{b1}} = \frac{\partial w_{a1}}{\partial t_{a1}} \frac{\partial w_{b1}}{\partial t_{b1}} \left( \frac{\partial^2 \gamma_{a1}}{\partial w_{a1}\partial w_{b1}} R_{a1} + \frac{\partial \gamma_{a1}}{\partial w_{b1}} dR_{a1} \right)$$

(7)

The sign of equation (7) is ambiguous and introduces a strategic effect$^{10}$, $\frac{\partial^2 \gamma_{a1}}{\partial w_{a1}\partial w_{b1}}$. Little can be said about the sign or magnitude of $\frac{\partial^2 \gamma_{a1}}{\partial w_{a1}\partial w_{b1}}$ without a functional form.$^{11}$ However, if the preceding term is not sufficiently large, an increase in the economic competitor’s talent investment will cause a decrease in the team’s marginal revenue from talent, thereby decreasing their talent level.$^{12}$

The two-league two-team model also introduces an indirect effect. Changes in the investment of talent of team 2 in league $b$ (in a third market separate from team 2 in league $a$) will affect the winning percent of team 1 in league $b$, which, in turn, will affect the revenue of team 1 in league $a$. A marginal increase in talent of the team in market 2 in

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$^{10}$Strategic effects are defined by Tirole (1988).

$^{11}$If the logistic form were imposed on $\gamma_{a1}$ such that $\gamma_{a1} = \frac{f(w_{a1})}{f(w_{a1}) + f(w_{b1})}$, then $\frac{\partial^2 \gamma_{a1}}{\partial w_{a1}\partial w_{b1}} = \frac{f_{a1}' f_{b1}' (f_{a1} - f_{b1})}{(f_{a1} + f_{b1})^2}$ which is greater than zero if and only if $f_{a1} > f_{b1}$.

$^{12}$Talent acts as strategic substitutes if $\frac{\partial^2 \gamma_{a1}}{\partial w_{a1}\partial w_{a1}} < -\frac{\partial \gamma_{a1}}{\partial w_{a1}} \frac{dR_{a1}}{dw_{a1}}$ and strategic complements otherwise.
league $b$ affects team 1 in league $a$ in the following way,

$$\frac{\partial^2 \pi_{a1}}{\partial t_{a1} \partial t_{b2}} = \frac{\partial w_{a1}}{\partial t_{a1}} \frac{\partial w_{b1}}{\partial t_{b2}} \left( \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} R_{a1} + \frac{\partial \gamma_{a1}}{\partial w_{b1}} \frac{dR_{a1}}{dw_{a1}} \right)$$

Equation (8) has one term that is different than equation (7), $\frac{\partial w_{b1}}{\partial t_{b2}}$ replaces $\frac{\partial w_{a1}}{\partial t_{b1}}$ which are opposite in sign. Therefore, the sign of equation (8) is the opposite as the sign in equation (7).

3 Cross-ownership

As stated, cross-ownership is a common ownership among competing teams located within the same region. If team 1 in league $a$ and team 1 in league $b$ are owned by the same owner, the profit function for a cross-owned firm is given by,

$$\pi_{a1,b1} = \gamma_{a1}(w_{a1}(t_{a1}, t_{a2}), (w_{b1}(t_{b1}, t_{b2}))R_{a1}(w_{a1}(t_{a1}, t_{a2})) - t_{a1} +$$

$$\gamma_{b1}(w_{b1}(t_{b1}, t_{b2}), (w_{a1}(t_{a1}, t_{a2}))R_{b1}(w_{b1}(t_{b1}, t_{b2})) - t_{b1}$$

Equation (9)

The revenue and cost of talent of team $b$ is included in the profit function. The first order condition is given by,

$$\frac{\partial \pi_{a1,b1}}{\partial t_{a1}} = \left( \frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1} + \gamma_{a1} \frac{dR_{a1}}{dw_{a1}} + \frac{\partial \gamma_{b1}}{\partial w_{a1}} R_{b1} \right) \frac{\partial w_{a1}}{\partial t_{a1}} - 1 = 0$$

Equation (10)

Compared to the duopolist, the cross-owner’s first order condition includes an additional term, $\frac{\partial \gamma_{b1}}{\partial w_{a1}} R_{b1}$. Since that term is non-positive, this implies the cross-owner will invest the same or less in talent compared to the duopolist. An important change of the cross-owned firm is: in solving the joint profit maximization problem, the externalities between the teams are eliminated. This captures the reduction in revenue for team 1 in league $b$ from the decrease in fans due to the marginal increase in winning percentage from team 1 in league $a$. 
Proposition 2 The cross-owned firm will invest less in talent than the duopolist if there is any substitutability between teams in the same market.

For a similar reason as the duopolist the cross-owned firm’s level of talent compared to the monopolist is ambiguous. However, since the talent level of the cross-owned teams is less than the duopolist’s talent level, cross-ownership does make it more likely that investment is lower than the monopolist’s investment. Comparing equations (10) and (3), if \((1 - \gamma)\frac{dR_{a1}}{dw_{a1}} > \frac{\partial R_{a1}}{\partial w_{a1}} R_{a1} + \frac{\partial R_{b1}}{\partial w_{a1}} R_{b1}\) then the monopolist will have a higher talent level, otherwise the cross-owner will have a higher level of talent investment.

However, given that team 2 in league \(a\) and team 2 in league \(b\) will change their investment in talent, the change in profits from a duopolist to a cross-owner is ambiguous. If the cross-owned teams are in a large market and reduce their investment in talent when they become cross-owned, the other teams in the league may have an incentive to increase their talent level. So, while the cross-owner would essentially have a regional monopoly, there are strategic substitutes in the form of talent investment for other teams in the league. This is analogous to having losses from horizontal mergers. Salant et al. (1983) showed that in a three firm market, if two firms merge, the third will respond by increasing output and thus decreasing profits of the merged firm. Similarly, if a team is cross-owned, other teams in the league might respond by increasing their talent level due to decrease in talent of the cross-owned firm.

Proposition 3 The effect of cross-ownership, compared to a duopolist, on profits is ambiguous.

4 League Policies

4.1 Salary Cap

Given that the winning percentages of teams in other leagues are in the owner’s objective function, talent levels across leagues are linked. The purpose of this section is to determine
the conditions under which league policies affect other leagues’ talent levels. For example, when the NHL first introduced the salary cap in 2005-06, teams were forced to keep salaries under $39 million. The salary cap for 2009-10 was $56.8 million. Do the changes in the salary cap that the NHL implemented after the 2004-05 lockout affect the quality of teams in the NBA? In this context, a salary cap is a limit on the investment in talent.\footnote{Investment in talent can be very broad and represent such things as payroll and/or player development. For simplicity, we assume that the cap is on investment in talent, but typically a cap is on payroll. If the cap is not on the total investment in talent it will have a mitigating effect.}

To analyze the effects of a salary cap on the quality of teams in other leagues, we assume that a salary cap is present in a league and determine the impact of a marginal change in the salary cap. The following three cases exist when analyzing a salary cap.

1. The salary cap is not binding on any team. This scenario is trivial and not considered.

2. The salary cap is binding on all teams in the same league. In this case, both teams are expected to win half of their games.\footnote{We assume the contest success function is the same for both teams.} If the cap is binding on both teams, then there is no effect on winning percentages from a marginal change in a cap, and therefore no effect on the quality of teams in other leagues. This scenario is not considered.

3. The salary cap is only binding on the large market team. In this case, the salary cap will affect the quality of both teams in the league and therefore the quality of teams in other leagues. This scenario is considered because of its affects across leagues.

So, for our purposes, we assume that the cap is only binding on the large market team. We first explore the case of a duopoly with no cross-ownership. Given that team 1 in each league is in the same market, we assume that they are in the large market and are impacted by the salary cap. Further, we will assume the salary cap is on team 1 in league \( b \) and look at the impact on team 1 in league \( a \). Since we assume the cap is binding, the mathematical representation of the binding cap is, \( \frac{dt_1}{dCAP} = 1 \). Therefore, the marginal effect of a salary cap on a team in a different league located in the large market is,

\[
\frac{\partial^2 \pi_{a1}}{\partial t_{a1} \partial CAP} = \frac{\partial^2 \pi_{a1}}{\partial t_{a1} \partial t_{b1}} \frac{dt_{b1}}{dCAP} = \frac{\partial w_{a1}}{\partial t_{a1}} \frac{\partial w_{b1}}{\partial t_{b1}} \left( \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} R_{a1} + \frac{\partial \gamma_{a1}}{\partial w_{b1}} \frac{dR_{a1}}{dw_{a1}} \right) \tag{11}
\]
This is completely analogous to equation (7). In fact, a marginal increase in a binding cap will be the same as a marginal increase in talent. Therefore, as long as \( \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} \) is not sufficiently large, an increase in the salary cap for league \( b \) will cause a decrease in talent investment for the team in league \( a \). Conversely, implementing a salary cap in league \( b \) will increase investment for teams in league \( a \) in the same markets assuming that \( \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} \) is not too large.

**Proposition 4** If a competing team in the same market but a different league incurs a salary cap, then a team will increase their talent level if strategic effects are not large.

The case of a salary cap with cross-ownership is similar to the case of a salary cap in a duopoly. Again, assuming the cap is binding so that \( \frac{dt_{b1}}{dt_{CAP}} = 1 \), the marginal effect of an increase in a salary cap on a cross-owned team in a different league located in the large market is,

\[
\frac{\partial^2 \pi_{a1,b1}}{\partial t_{a1} \partial t_{b1}} = \frac{\partial^2 \pi_{a1,b1}}{\partial t_{a1} \partial t_{b1}} - \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} R_{a1} + \frac{\partial \gamma_{a1}}{\partial w_{a1}} dR_{a1} + \frac{\partial \gamma_{b1}}{\partial w_{a1}} dR_{b1}
\]

which is similar to equation (11).

### 4.2 Revenue Sharing

Next we will examine the impact that revenue sharing has on other leagues. The result that revenue sharing reduces the incentive to invest in talent for all teams within a league is well established. However, the effect of revenue sharing on competing leagues is yet to be explored.

We assume that league \( b \) has a revenue sharing policy and we first examine the effects on league \( b \). The profit functions for the two teams in league \( b \) are,\(^{15}\)

\[
\pi_{b1} = \alpha \gamma_{b1}(w_{b1}(t_{b1}, t_{b2}), w_{a1}(t_{a1}, t_{a2}))R_{b1}(w_{b1}(t_{b1}, t_{b2})) + (1 - \alpha)R_{b2}(w_{b2}(t_{b2}, t_{b1})) - t_{b1}
\]

\(^{15}\)We have assumed that team 2 in league \( b \) is a monopolist, therefore, they have no \( \gamma \) function.
\[ \pi_{b2} = (1 - \alpha) \gamma_{b1}(w_{b1}(t_{b1}, t_{b2}), w_{a1}(t_{a1}, t_{a2})) R_{b1}(w_{b1}(t_{b1}, t_{b2})) + \alpha R_{b2}(w_{b2}(t_{b2}, t_{b1})) - t_{b2} \] (14)

where \( \alpha \in (.5, 1) \) is the proportion of an owner’s revenue that is retained by the owner and pays \( 1 - \alpha \) to their opponents. The first order conditions are,

\[ \frac{\partial \pi_{b1}}{\partial t_{b1}} = \alpha \frac{\partial w_{b1}}{\partial t_{b1}} \left( \frac{\partial \gamma_{b1}}{\partial w_{b1}} R_{b1} + \gamma_{b1} \frac{dR_{b1}}{dw_{b1}} \right) + (1 - \alpha) \left( \frac{dR_{b2}}{dw_{b2}} \frac{\partial w_{b2}}{\partial t_{b1}} \right) - 1 = 0 \] (15)

and

\[ \frac{\partial \pi_{b2}}{\partial t_{b2}} = (1 - \alpha) \frac{\partial w_{b1}}{\partial t_{b2}} \left( \frac{\partial \gamma_{b1}}{\partial w_{b1}} R_{b1} + \gamma_{b1} \frac{dR_{b1}}{dw_{b1}} \right) + \alpha \left( \frac{dR_{b2}}{dw_{b2}} \frac{\partial w_{b2}}{\partial t_{b2}} \right) - 1 = 0 \] (16)

Given that in a two-team league model, \( \frac{\partial w_{b1}}{\partial t_{b1}} = - \frac{\partial w_{b2}}{\partial t_{b1}} \) and \( \frac{\partial w_{b2}}{\partial t_{b2}} = - \frac{\partial w_{b1}}{\partial t_{b2}} \), the following equilibrium condition is obtained.

\[ \left( \frac{\partial \gamma_{b1}}{\partial w_{b1}} R_{b1} + \gamma_{b1} \frac{dR_{b1}}{dw_{b1}} \right) \left( \alpha \frac{\partial w_{b1}}{\partial t_{b1}} + (1 - \alpha) \frac{\partial w_{b2}}{\partial t_{b2}} \right) = \frac{dR_{b2}}{dw_{b2}} \left( \alpha \frac{\partial w_{b2}}{\partial t_{b2}} + (1 - \alpha) \frac{\partial w_{b1}}{\partial t_{b1}} \right) \] (17)

As previously discussed in equation (6), the first term in (17) is the duopolist’s marginal revenue of winning. Therefore, if we denote \( \frac{dT R_{b1}}{dw_{b1}} = \frac{\partial w_{b1}}{\partial t_{b1}} R_{b1} + \gamma_{b1} \frac{dR_{b1}}{dw_{b1}} \), then equation (17) can be written as,

\[ \frac{dT R_{b1}}{dw_{b1}} \left( \alpha \frac{\partial w_{b1}}{\partial t_{b1}} + (1 - \alpha) \frac{\partial w_{b2}}{\partial t_{b2}} \right) = \frac{dR_{b2}}{dw_{b2}} \left( \alpha \frac{\partial w_{b2}}{\partial t_{b2}} + (1 - \alpha) \frac{\partial w_{b1}}{\partial t_{b1}} \right) \] (18)

Equation (18) represents the equilibrium condition for the winning percent in the two-team league. If team 1 is the large market team, then decreasing returns to investment implies that \( \frac{\partial w_{b1}}{\partial t_{b1}} < \frac{\partial w_{b2}}{\partial t_{b2}} \). Therefore, an increase in revenue sharing (a decrease in \( \alpha \)) results in \( \frac{dT R_{b1}}{dw_{b1}} \left( \alpha \frac{\partial w_{b1}}{\partial t_{b1}} + (1 - \alpha) \frac{\partial w_{b2}}{\partial t_{b2}} \right) > \frac{dR_{b2}}{dw_{b2}} \left( \alpha \frac{\partial w_{b2}}{\partial t_{b2}} + (1 - \alpha) \frac{\partial w_{b1}}{\partial t_{b1}} \right) \). Given diminishing returns to talent on revenues, an increase (decrease) in talent for the large (small) market team will make the LHS (RHS) smaller (larger). Consequently, the large market team
will improve relative to the small market team.\footnote{Syzmanski (2004) has a similar conclusion using functional forms.}

We now focus on the effect of revenue sharing on other leagues. If teams \( a1 \) and \( b1 \) are in the large market, then \( \frac{dw_{a1}}{da} < 0 \), implying that revenue sharing in league \( b \) will increase the winning percentage for team \( b1 \). Therefore, revenue sharing in league \( b \) will have the same qualitative effect on the large market team as an increase in talent of team \( b1 \); which is the same condition as in equation (7). That is, assuming

\[
\frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial w_{b1}} R_{a1} < -\frac{\partial \gamma_{a1}}{\partial w_{b1}} \frac{dR_{a1}}{dw_{a1}},
\]

and team 1 is in the large market, then revenue sharing in league \( b \) will increase in winning percentage for team 1 in league \( b \) and a decrease in winning percentage and talent investment for team 1 in league \( a \).

We now move on to revenue sharing with cross-ownership. If revenue sharing exists in league \( b \), then the corresponding profit function for the cross-owned team is given by,

\[
\pi_{a1,b1} = \gamma_{a1}(w_{a1}(t_{a1}, t_{a2}), (w_{b1}(t_{b1}, t_{b2}))) R_{a1}(w_{a1}(t_{a1}, t_{a2})) - t_{a1} + \alpha \gamma_{b1}(w_{b1}(t_{b1}, t_{b2}), (w_{a1}(t_{a1}, t_{a2}))) R_{b1}(w_{b1}(t_{b1}, t_{b2})) + (1 - \alpha) R_{b2}(w_{b2}(t_{b2}, t_{b1})) - t_{b1}
\]

(19)

The first order condition for team \( a \) is given by,

\[
\frac{\partial \pi_{a1,b1}}{\partial t_{a1}} = \left( \frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{a1} + \gamma_{a1} \frac{dR_{a1}}{dw_{a1}} + \alpha \frac{\partial \gamma_{b1}}{\partial w_{a1}} R_{b1} \right) \frac{\partial w_{a1}}{\partial t_{a1}} - 1 = 0
\]

(20)

The direct effect of revenue sharing is derived by comparing the first order conditions of the cross-owned firm with (equation (20)) and without (10) revenue sharing. While cross-owners will invest less in talent then duopolists, the effect is mitigated with revenue sharing.

However, there is also an indirect effect of team 2 in league \( b \) also investing less. Revenue sharing also has a strategic effect by altering the quality of the team in the other market, the cross-owned team, by affecting the team’s marginal revenue in the
following way

\[
\frac{\partial^2 \pi_{a1,b1}}{\partial a_1 \partial \alpha} = \frac{\partial w_{a1}}{\partial a_1} \left( \frac{\partial w_{a1}}{\partial b_1} \frac{\partial b_1}{\partial \alpha} + \frac{\partial w_{a1}}{\partial b_2} \frac{\partial b_2}{\partial \alpha} \right) + \alpha \frac{\partial^2 \gamma_{a1}}{\partial w_{a1} \partial b_1} R_{a1} + \frac{\partial w_{a1}}{\partial a_1} \frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{b1}
\]

The term \(\frac{\partial w_{a1}}{\partial b_1} \frac{\partial b_1}{\partial \alpha} + \frac{\partial w_{a1}}{\partial b_2} \frac{\partial b_2}{\partial \alpha}\) is the effect of revenue sharing on the winning percentage of team 1 in league b. Continuing to assume team 1 is the large market team, then \(\frac{\partial w_{a1}}{\partial b_1} \frac{\partial b_1}{\partial \alpha} + \frac{\partial w_{a1}}{\partial b_2} \frac{\partial b_2}{\partial \alpha} < 0\), or in words, less revenue sharing leads to a lower winning percentage for team 1 in league b. However, if the effect on winning percentages in league b is small, then the term \(\frac{\partial \gamma_{a1}}{\partial w_{a1}} R_{b1}\) will make equation (21) negative. This is because revenue sharing decreases the incentive to not take away fans from team 1 in league b.

**Proposition 5** If a league implements revenue sharing and it has a small effect on the winning percentages of that league, then revenue sharing will lead to higher levels of investment for cross-owners for teams not in that league.

### 5 An Example

In this section we give an example with functional forms. Suppose for the large market monopolist the profit is given by \(\pi_{a1} = \sigma_{a1} \left( \frac{t_{a1}}{t_{a1} + t_{a2}} \right) - t_{a1}\) where \(w_{a1} = \frac{t_{a1}}{t_{a1} + t_{a2}}\) and \(\sigma_{a1}\) is a constant greater than one. For the small market team we assume that profit is given by \(\pi_{a2} = \frac{t_{a2}}{t_{a1} + t_{a2}} - t_{a2}\). After taking first order conditions and solving for each team’s talent, winning, and profit in terms of the exogenous parameter, we find that \(t_{a1} = \frac{1}{(1 + \frac{1}{\sigma_{a1}})^{m}}\), \(w_{a1} = \frac{\sigma_{a1}}{1 + \sigma_{a1}}\), \(w_{a2} = \frac{1}{1 + \sigma_{a1}}\), \(\pi_{a1} = \frac{\sigma_{a1}^{3}}{(1 + \sigma_{a1})^{3}}\) and \(\pi_{b1} = \frac{1}{(1 + \sigma_{a1})^{3}}\).

Now suppose the large market team is a duopolist and their profit function is given by, \(\pi_{a1} = \left[ \phi_{1} + \phi_{2} \left( \frac{t_{a1}}{t_{a1} + t_{a2}} - w_{b1} \right) \right] \sigma_{a1} \left( \frac{t_{a1}}{t_{a1} + t_{a2}} \right) - t_{a1}\) where \(w_{b1}\) represents the winning percentage of the team in the same market, but different league. We will assume the profit function of team 2 in league \(a\) does not change. In this case, the winning percentage of the duopolist is given by, \(w_{a1} = \frac{2\sigma_{a1}\phi_{2} - \phi_{1} + w_{b1}\phi_{2} - 1 + \sqrt{(2\sigma_{a1}\phi_{2} - \phi_{1} + w_{b1}\phi_{2} - 1)^{2} + 8\sigma_{a1}^{2}\phi_{2}(\phi_{1} - w_{b1}\phi_{2})}}{4\sigma_{a1}\phi_{2}}\).
In the cross-ownership case where there is one owner of both teams in the market, the profit function is given by
\[ \pi_a = \left[ \phi_1 + \phi_2 \left( \frac{t_{a1}}{t_{a1} + t_{b1}} - \frac{t_{b1}}{t_{a1} + t_{b1}} \right) \right] \sigma_a \left( \frac{t_{a1}}{t_{a1} + t_{a2}} \right) - t_{a1} + \left[ \phi_3 + \phi_4 \left( \frac{t_{b1}}{t_{b1} + t_{b2}} - \frac{t_{a1}}{t_{a1} + t_{a2}} \right) \right] \sigma_b \left( \frac{t_{b1}}{t_{b1} + t_{b2}} \right) - t_{b1} \] and the winning percentage is given by,
\[ w_{a1} = \frac{\sigma_a \left( 2 \phi_2 - \phi_1 + w_{b1} \phi_2 \right) + \sigma_b w_{b1} \phi_4 - 1}{\sqrt{\left[ \sigma_a \left( 2 \phi_2 - \phi_1 + w_{b1} \phi_2 \right) + \sigma_b w_{b1} \phi_4 - 1 \right]^2 + 8 \sigma_a \phi_2 \left( \sigma_a \left( \phi_1 - w_{b1} \phi_2 \right) - \sigma_b w_{b1} \phi_4 \right)}} \]
These mathematical derivations are given in Appendix B.

Figure 2 shows the equilibrium winning percentage for a large market team under a monopoly, duopoly and cross-ownership for various values of \( w_{b1} \). With these parameter values the monopolist invests less in talent than the duopolist or cross-owners if \( w_{b1} \) is low and the reverse is true if \( w_{b1} \) is high. Furthermore, the duopolist always invests at least as much as the cross-owner showing that the cross-owner has less of an incentive to invest in talent. Given our specifications, this difference is magnified when \( w_{b1} \) is high.

While \( w_{b1} \) is irrelevant for the monopolist, and can be treated as exogenous for the duopolist, it is not exogenous for the cross-owner. Therefore, if we make the further assumption that the two leagues are completely symmetrical, we can derive talent levels, winning percentages and profit levels for all given structures. If the two leagues are in fact symmetrical, the equilibria in Figure 2 would be where the lines cross a 45 degree line. Table 3 gives values for the various equilibria. Of note is the fact that profit from one team is actually higher for the duopolist (1.228) compared to the cross-owner (1.196). This is because talent levels increase for teams outside the region.

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Figure 3 and Table 3 also give results when team 1 in league \( a \) and team 1 in league \( b \) are in a small market. In this case the monopolist always invests more than the duopolist and cross-owner. Also, profit increases with cross-ownership compared to the duopolist.

6 Conclusions

This paper shows that if sports fans make consumption choices based on the quality of all of the teams located in their markets that indirect competition and the ownership structure alters an owner’s incentive to invest in talent. We find that more loyal fans
reduce an owner’s incentive to invest in talent. In addition, economic competition results in ambiguous levels of talent investment. Also, a firm that is engaged in cross-ownership (owning two teams in different leagues in the same market) will invest less in talent compared to a duoplist, but the difference in profits is ambiguous. In addition, league policies can have an effect on the quality of teams in other leagues and revenue sharing mitigates some of the dulling effects that cross-ownership has on an owner’s incentive to invest in talent.

This analysis gives insight into how to generate more competitive balance. For example, supposing MLB wanted more competitive balance, some outcomes may work better than others. While it is often assumed that putting a third team in New York would help balance by lowering the quality of the incumbent New York teams, that completely depends on the nature of the baseball fans there. However, if the NFL lifted their salary cap (and assuming New York football teams would then improve), this would help the balance in MLB. Furthermore, revenue sharing in the NBA might help if it increases the quality of basketball teams in New York. Finally, if New York baseball teams were jointly owned with New York NHL teams, it might help the balance of MLB.

Extensions to this work are plenty. The opportunity for empirical work is clear. The effect of cross-ownership on league expansion and relocation is an important issue to be fully understood by economists, owners, league policy makers, and governments. A model can be developed to show that both the number of teams in a league and the location of teams are influenced by cross-ownership. In addition, the model in this paper can be extended to analyze the effects of indirect competition and ownership structure on competitive balance and players’ salaries. Further study intended to gain understanding into the pricing effects and efficiency gains caused by cross-ownership is worthwhile. Models that examine the effects of ownership structures in professional sports, beyond cross-ownership, would also be welcome additions to the literature. Finally, the study of indirect substitutes might be more important in sports than other industries because territorial rights have eliminated almost all direct competition.
A Appendix

In Appendix A, we examine the effect of an increase in the marginal revenue on winning. In a two team league, assume the profit functions are,

\[ \pi_1 = R_1(m, w_1(t_1, t_2)) - t_1 \]  

and

\[ \pi_2 = R_2(w_2(t_2, t_1)) - t_2 \]

where \( R \) is revenue, \( m \) is a variable representing the market, \( w \) is the team’s winning percentage and \( t \) represents talent investment. We further suppose that team 2’s market does not change, but for team 1, an increase in \( m \) leads to an increase in team 1’s marginal revenue curve. As in the rest of the paper, we assume \( \frac{\partial R}{\partial w} > 0 \), \( \frac{\partial w}{\partial t} > 0 \), \( \frac{\partial^2 w}{\partial t^2} < 0 \), \( \frac{\partial w}{\partial t_j} < 0 \), \( \frac{\partial R_i}{\partial t_i} > 0 \), and \( \frac{\partial^2 R_i}{\partial t_i^2} < 0 \). The first order conditions are,

\[ \frac{\partial \pi_1}{\partial t_1} = MR_1(m, w_1(t_1, t_2)) \frac{\partial w_1}{\partial t_1} - 1 = 0 \]  

and

\[ \frac{\partial \pi_2}{\partial t_2} = MR_2(w_2(t_2, t_1)) \frac{\partial w_2}{\partial t_2} - 1 = 0 \]

Where \( MR \) is the marginal revenue function, \( \frac{dR}{dw} \), and is a function of the team’s winning percentage, and in team 1’s case, an increasing function in \( m \), so that \( \frac{dMR_1}{dm} > 0 \). Totally differentiating (24) and (25) gives us

\[ MR_1 \left[ \frac{\partial^2 w_1}{\partial t_1^2} dt_1 + \frac{\partial^2 w_1}{\partial t_1 \partial t_2} dt_2 \right] + \frac{\partial w_1}{\partial t_1} \left[ \frac{\partial MR_1}{\partial m} dm + \frac{\partial MR_1}{\partial w_1} \left( \frac{\partial w_1}{\partial t_1} dt_1 + \frac{\partial w_1}{\partial t_2} dt_2 \right) \right] = 0 \]  

(26)

and

\[ MR_2 \left[ \frac{\partial^2 w_2}{\partial t_2^2} dt_2 + \frac{\partial^2 w_2}{\partial t_2 \partial t_1} dt_1 \right] + \frac{\partial w_2}{\partial t_2} \frac{\partial MR_2}{\partial w_2} \left( \frac{\partial w_2}{\partial t_2} dt_2 + \frac{\partial w_2}{\partial t_1} dt_1 \right) = 0 \]  

(27)
From these equations we find that

$$\frac{dt_1}{dm} = -\frac{\partial w_1}{\partial t_1} \frac{\partial^2 R_1}{\partial t_1^2} - \frac{\partial^2 R_1}{\partial t_1 \partial t_2^2}$$

and

$$\frac{dt_2}{dm} = -\frac{\partial w_1}{\partial t_1} \frac{\partial^2 R_2}{\partial t_2^2} - \frac{\partial^2 R_2}{\partial t_1 \partial t_1^2}$$

where $\frac{\partial^2 R_i}{\partial t_i^2} = \left(\frac{\partial w_i}{\partial t_i}\right)^2 M R_i \frac{\partial^2 w_i}{\partial t_i^2}$ and $\frac{\partial^2 R_i}{\partial t_i \partial t_j} = M R_i \frac{\partial^2 w_i}{\partial t_i \partial t_j} + \frac{\partial M R_i}{\partial w_i} \frac{\partial w_i}{\partial t_i} \frac{\partial w_i}{\partial t_j}$. Since $\frac{dw_1}{dm} = \frac{\partial w_1}{\partial t_1} \frac{dt_1}{dm} + \frac{\partial w_1}{\partial t_2} \frac{dt_2}{dm}$, this implies that,

$$\frac{dw_1}{dm} = \frac{\partial M R_i}{\partial m} \frac{\partial w_1}{\partial t_1} \left[ \frac{\partial w_1}{\partial t_1} \frac{\partial^2 R_2}{\partial t_1^2} + \frac{\partial^2 R_2}{\partial t_1 \partial t_2} \frac{\partial w_2}{\partial t_2} - \frac{\partial^2 R_2}{\partial t_2^2} \frac{\partial^2 R_2}{\partial t_1 \partial t_1} \right]$$

The sign of equation (30) is ambiguous. However, if the talent investment of team 1 does not change the marginal benefit of talent investment for team 2, $\frac{\partial^2 R_2}{\partial t_2 \partial t_1} = 0$, then an increase in team 1’s marginal revenue will increase their winning percentage.
B Appendix

We assume the duopolist’s profit function is given by,

$$\pi_a = \left[ \phi_1 + \phi_2 \left( \frac{t_{a1}}{t_{a1} + t_{a2}} - w_{b1} \right) \right] \sigma_{a1} \left( \frac{t_{a1}}{t_{a1} + t_{a2}} \right) - t_{a1}$$ \hspace{1cm} (31)

and the profit function for the other team in the same league is given by

$$\pi_{a2} = \left( \frac{t_{a2}}{t_{a1} + t_{a2}} \right) - t_{a2}$$ \hspace{1cm} (32)

The first order conditions are given by

$$\frac{\partial \pi_{a1}}{\partial t_{a1}} = \sigma_{a1} \frac{t_{a2}}{(t_{a1} + t_{a2})^2} \left[ 2\phi_2 \frac{t_{a1}}{t_{a1} + t_{a2}} + \phi_1 - \phi_2 w_{b1} \right] - 1 = 0$$ \hspace{1cm} (33)

and

$$\frac{\partial \pi_{a2}}{\partial t_{a2}} = \frac{t_{a1}}{(t_{a1} + t_{a2})^2} - 1 = 0$$ \hspace{1cm} (34)

Given the first order conditions and nature of the contest success function, we know that

$$t_{a1} = (t_{a1} + t_{a2})^2, \ w_{a1} = t_{a1} + t_{a2} \text{ and } w_{a1} + w_{a2} = 1.$$ Using this and setting the first order conditions equal to each other gives us,

$$[-2\sigma_{a1} \phi_2] w_{a1}^2 + [2\sigma_{a1} \phi_2 - \phi_1 + w_{b1} \phi_2 - 1] w_{a1} + [\phi_1 - w_{b1} \phi_2] = 0$$ \hspace{1cm} (35)

and using the quadratic formula gives us

$$w_{a1} = \frac{2\sigma_{a1} \phi_2 - \phi_1 + w_{b1} \phi_2 - 1 + \sqrt{(2\sigma_{a1} \phi_2 - \phi_1 + w_{b1} \phi_2 - 1)^2 + 8\sigma_{a1}^2 \phi_2 (\phi_1 - w_{b1} \phi_2)}}{4\sigma_{a1} \phi_2}$$ \hspace{1cm} (36)

(adding the root instead of subtracting it gives negative winning percentages in some cases).
In the cross-ownership case the profit function is given by

\[
\pi_{a1} = \left[ \phi_1 + \phi_2 \left( \frac{t_{a1}}{t_{a1} + t_{a2}} - \frac{t_{b1}}{t_{b1} + t_{b2}} \right) \right] \sigma_{a1} \left( \frac{t_{a1}}{t_{a1} + t_{a2}} \right) - t_{a1} + \\
\left[ \phi_3 + \phi_4 \left( \frac{t_{b1}}{t_{b1} + t_{b2}} - \frac{t_{a1}}{t_{a1} + t_{a2}} \right) \right] \sigma_{b1} \left( \frac{t_{b1}}{t_{b1} + t_{b2}} \right) - t_{b1}
\]

and the profit function for the other team in the same league is given by

\[
\pi_{a2} = \left( \frac{t_{a2}}{t_{a1} + t_{a2}} \right) - t_{a2}
\]

The first order conditions are given by

\[
\frac{\partial \pi_{a1}}{\partial t_{a1}} = \sigma_{a1} \frac{t_{a2}}{(t_{a1} + t_{a2})^2} \left[ 2\phi_2 t_{a1} t_{a2} + \phi_1 - \phi_2 w_{b1} \right] - 1 - w_{b1} \sigma_{b1} \phi_4 \frac{t_{a2}}{(t_{a1} + t_{a2})^2} = 0
\]

and

\[
\frac{\partial \pi_{a2}}{\partial t_{a2}} = \frac{t_{a1}}{(t_{a1} + t_{a2})^2} - 1 = 0
\]

Again, given that \( t_{a1} = (t_{a1} + t_{a2})^2 \), \( w_{a1} = t_{a1} + t_{a2} \) and \( w_{a1} + w_{a2} = 1 \), using this and setting the first order conditions equal to each other gives us,

\[
-2\sigma_{a1} \phi_2 \left( w_{a1}^2 + \sigma_{a1} (2\phi_2 - \phi_1 + w_{b1} \phi_2) + w_{b1} \sigma_{b1} \phi_4 - 1 \right) w_{a1} + \sigma_{a1} (\phi_1 - w_{b1} \phi_2) - w_{b1} \sigma_{b1} \phi_4 = 0
\]

and using the quadratic formula gives us

\[
w_{a1} = \frac{\sigma_{a1} (2\phi_2 - \phi_1 + w_{b1} \phi_2) + \sigma_{b1} \phi_4 - 1}{4\sigma_{a1} \phi_2} + \frac{\sqrt{\sigma_{a1} (2\phi_2 - \phi_1 + w_{b1} \phi_2) + \sigma_{b1} \phi_4 - 1}^2 + 8\sigma_{a1} \phi_2 (\sigma_{a1} (\phi_1 - w_{b1} \phi_2) - \sigma_{b1} \phi_4)}{4\sigma_{a1} \phi_2}
\]
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*Competitors represents the number of other MLB, NBA, NFL, and NHL teams in the market.  
**Competitor and population data for Charlotte/New Orleans refers to New Orleans.
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Table 3: Equilibria under various ownership structures and parameters

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*The profit for the cross-owner is only profit from one of the teams.
Figure 1: The number of markets with at least 1 team from each league (MLB, NBA, and the NHL) and number of cross-owned firms.
Figure 2: The monopolist’s, duopolist’s and cross-owned firm’s winning percentages $(\sigma_{a1} = 3, \sigma_{b1} = 3, \phi_1 = .8, \phi_2 = .2, \phi_4 = .2)$
Figure 3: The monopolist’s, duopolist’s and cross-owned firm’s winning percentages 
($\sigma_{a1} = .5, \sigma_{b1} = .5, \phi_1 = .8, \phi_2 = .2, \phi_4 = .2$)

![Graph showing winning percentages for different firms with specified parameters.]