Geometrical exposition of structural axiomatic economics (I): Fundamentals

Kakarot-Handtke, Egmont

University of Stuttgart, Institute of Economics and Law

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Egmont Kakarot-Handtke*

Abstract
Behavioral assumptions are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. Part (I) of the structural axiomatic analysis submits three nonbehavioral axioms as groundwork and applies them to the simplest possible case of the pure consumption economy. The geometrical analysis makes the interrelations between income, profit and employment under the conditions of market clearing and budget balancing immediately evident. Part (II) applies the differentiated axiom set to the analysis of qualitative and temporal aggregation.

JEL D00, E00

Keywords new framework of concepts; structure-centric; axiom set; supersymmetry; general equilibrium; dimensionless variables; income; profit; distributed profit; retained profit; full employment; money; credit; Say’s Law

*Affiliation: University of Stuttgart, Institute of Economics and Law, Keplerstrasse 17, D-70174 Stuttgart. Correspondence address: AXEC, Egmont Kakarot-Handtke, Hohenzollernstraße 11, D-80801 München, Germany, e-mail: handtke@axec.de
The task of theoretical economics is to create a mental map of the whole economy without firsthand experience.

And in the social sciences it is even more obvious than in the natural sciences that we cannot see and observe our objects before we have thought about them. For most of the objects of social science, if not all of them, are abstract objects; they are theoretical constructions. (Popper, 1960, p. 135), original emphasis

That is, one has to leap from commonplace economics which trades in easy to grasp phenomena on a small scale to an extremely abstract set of foundational propositions about the economy as a whole.

Since, therefore, it is vain to hope that truth can be arrived at, either in Political Economy or in any other department of the social science, while we look at the facts in the concrete, clothed in all the complexity with which nature has surrounded them, and endeavor to elicit a general law by a process of induction from a comparison of details; there remains no other method than the à priori one, or that of “abstract speculation.” (Mill, 2004, p. 113-114)

The set of foundational propositions has to reduce the vast complexity of the real thing to almost nothing. From this almost-nothingness the real world complexity then has to be logically reconstructed.

Each theory starts from a small set of foundational ‘hypotheses or axioms or postulates or assumptions or even principles’ (Schumpeter, 1994, p. 15). General equilibrium theory rests on a set of behavioral axioms (Arrow and Hahn, 1991, p. v). This approach is known to be in need of a re-design because ‘anything based on this mock-up is unlikely to fly’ (Hahn, 1981, p. 1036).

The standard set of behavioral axioms is in the present paper at first replaced by structural axioms. By choosing objective structural relationships as axioms behavioral hypotheses are not ruled out. On the contrary, the structural axiom set is open to any behavioral assumption and not restricted to the standard optimization calculus (for details see 2011b). Hence it is analytically possible to dislocate human behavior from the center of the domain to the periphery. This opens new vistas.

The methodological case for structural axiomatization has been made at length elsewhere (2012). With the basic understanding that an objective formal foundation is reasonable as well as desirable the minimalistic structural frame is set up in Section 1. In Section 2 the axiom set is made geometrically concrete and the economic implications are elaborated. Reproducibility is ascertained with the conditions of market clearing and budget balancing. This structural supersymmetry and its implications for a move from underemployment to full employment is discussed in Section 3. Section 4 explains the existence and magnitude of profit. In Section 5 the relation between profit and distributed profit is clarified and in Section 6 the complementarity of retained profit and saving is established. Money and credit round off the picture in Section 7. Section 8 concludes.
1 Axioms and definitions

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. At first the period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product.\footnote{“The often heard rule that concepts are to be defined before they are used in a discussion is much too simple minded pre-Hilbertian. The only way to arrive at coherent languages is to set up axiomatic systems implicitly defining the basic concepts.” (Schmiechen, 2009, p. 344)}

Total income of the household sector $Y$ in period $t$ is the sum of wage income, i.e. the product of wage rate $W$ and working hours $L$, and distributed profit, i.e. the product of dividend $D$ and the number of shares $N$.

\[
Y = WL + DN \quad |t\]

(1)

Output of the business sector $O$ is the product of productivity $R$ and working hours.

\[
O = RL \quad |t\]

(2)

Consumption expenditures $C$ of the household sector is the product of price $P$ and quantity bought $X$.

\[
C = PX \quad |t\]

(3)

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other state activity. Albeit quite obvious, it is worth to emphasize that all axiomatic variables are measurable in principle. No nonempirical concepts like equilibrium, rationality or perfect competition are put into the premises.

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (5) wage income $Y_W$ and distributed profit income $Y_D$ is defined:

\[
Y_W \equiv WL \quad Y_D \equiv DN \quad |t.\]

(4)

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Formal axiomatic systems must be interpreted in some domain . . . to become an empirical science. (Boylan and O’Gorman, 1995, p. 198)

The economic interpretation is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and \textit{distributed profit} and not of wage income and profit. Profit and distributed profit are quite different things that have to be thoroughly kept apart.
2 An open opening

Figure 1 makes the structural axioms immediately concrete. The diagram looks like the familiar Cartesian coordinates. However, since there is no use for negative values the four axes represent the positive rational values of the variables employment \( L \), income \( Y \), consumption expenditures \( C \), quantity bought \( X \) and output \( O \), respectively. The bisecting line in the northwestern quadrant mirrors income from the horizontal to the vertical axis. This facilitates the direct comparison of the nominal values of income and consumption expenditures. The quadrants are numbered according to the axioms they enclose.

![Diagram](image)

**Figure 1:** At first the product market is not cleared and the household sector’s budget is not balanced

In the 1\textsuperscript{st} quadrant wage income \( Y_W \) is given as product of wage rate \( W \) and working hours \( L \). The wage rate is equal to the tangent function of the angle \( \alpha \) at \( L = 1 \). Angles are not needed for our analysis hence they are denoted with the respective economic variables that are used for geometrical multiplication. Total income \( Y \) consists at the beginning only of wage income \( Y_W \). Distributed profit \( Y_D \) as constituent of the 1\textsuperscript{st} axiom (1) has been set to zero and shall be considered separately in Section 5.

In the 2\textsuperscript{nd} quadrant output \( O \) is given as product of productivity \( R \) and working hours \( L \). The productivity is determined by the underlying production process and may vary with labor input. The 2\textsuperscript{nd} axiom (2) should therefore not be interpreted as a linear production function. It is compatible with increasing, constant and decreasing returns. Employment and productivity changes can always be treated...
separately and then combined; it is much like vector decomposition and addition as shown in Figure 2.

![Figure 2: Tracking of an arbitrary production function with the 2nd axiom](image)

Since we are not a priori wedded to the marginal principle there is no need to confine the analysis to decreasing returns. It should be noted in passing that any approach that operates solely with decreasing returns cannot claim to be general.

In the 3rd quadrant consumption expenditures $C$ is given as geometrical product of price $P$ and quantity bought $X$.

Since the quantity produced $O$ in Figure 1 is larger than the quantity bought $X$ the firm that at the moment represents the entire business sector has an unsold quantity $\Delta O$ left over at period end which has to be taken into stocks. The change of inventory in period $t$ is defined as:

$$\Delta O \equiv O - X \mid t.$$  \hspace{1cm} (5)

In the period under consideration the product market is not cleared. Consumption expenditures $C$ in the 1st quadrant is less than income $Y$, that is, the households save. Financial saving is defined as:

$$\Delta S_{f/t} \equiv Y - C \text{ here } Y = Y_W \mid t.$$ \hspace{1cm} (6)

In the period under consideration the household sector’s budget is not balanced. Is this configuration realistic? Of course. The households can save whenever and whatever they want and it is a normal incident that a firm takes part of current output

\hspace{1cm} $^2$ For the treatment of nonfinancial saving see (2011a, Sec. 4.2).
into stocks. All depends on what happens in the subsequent periods. If the firm sells the quantity $\Delta O$ in the next period in addition to the current output the inventory vanishes again. Hence, seen over two periods, the product market is cleared (see Part (II)). Problems arise, though, if the stock of unsold products accumulates over a longer time span, that is, if the configuration of Figure 1 is identically reproduced in subsequent periods. It is similar with saving. If the households dissave in the next period their budget is balanced over two periods.

Problems arise in the pure consumption economy if the households stubbornly go on with saving because this drives firms to the brink of bankruptcy. The business sector’s financial profit in period $t$ is defined with (7) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures $C$ – and costs – here identical with wage income $Y_W$:

$$\Delta Q_{fi} \equiv C - Y_W \mid t.$$  

(7)

Since distributed profit in (1) is zero we have in this simple case $Y = Y_W$. Hence from (7) in combination with (6) follows:

$$\Delta Q_{fi} \equiv C - Y \Rightarrow \Delta Q_{fi} \equiv -\Delta S_{fi} \mid t.$$  

(8)

The business sector makes a financial loss in the period under consideration which is equal to the financial saving of the household sector. This is certainly not a healthy situation over a longer time span. What is worse, the business sector cannot do much to change the situation because the households are perfectly free in their decision to spend or to save their income in the current period.

The definition of financial profit as such does, of course, not explain how profit comes about. A definition’s role is restricted to the exact specification of what one is talking about. In the following we shall determine the overall conditions that enable the firms to make a profit, that is, we shall answer the question how it comes about that revenues could be greater than wage costs. This is an old chestnut of political economy which has never been answered satisfactorily (Mirowski, 1986, p. 234).

There is, to be sure, not much use in speculating about what the households or our single firm would or could do in the situation that is given with Figure 1 or to introduce some convenient behavioral assumptions that make that household saving is always zero in the pure consumption economy. What can be done, though, is to determine the structural conditions for a reproducible period configuration.

### 3 Reproducibility, employment, and Say’s Law

After the implementation of the conditions of market clearing $X = O$ and budget balancing $C = Y$ (at the moment $Y = Y_W$) the pure consumption economy looks as shown in Figure 3. This configuration is referred to as supersymmetric. It is

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3 Profits from changes in the value of nonfinancial assets are neglected here. For details about effects of changes of inventory on profits see (2011a).

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important to notice that the two conditions are not a constituent part of the axiom set but an – in principle – arbitrary addendum.

![Diagram](image.png)

**Figure 3:** Implementation of the conditions of product market clearing and budget balancing

The market clearing price then follows from (3) and (1) as:

\[ P = \frac{W}{R} \]

if \( X = O, C = Y \) here \( Y = Y_W \). \( t. \) \( (9) \)

The market clearing price \( P \) is equal to unit wage costs \( \frac{W}{R} \). Hence profit per unit of output is zero and therefore overall profit is zero. This configuration is reproducible. The firm sells its period output completely and fully recoups its wage costs. It is worth emphasizing that the market clearing price is unequivocally determined by the three axioms and the supersymmetry conditions. It is therefore impossible to add independent demand and supply functions. There is simply no formal room left for additional behavioral assumptions or some occult market forces that equalize price and unit wage costs.

The market clearing price in (9) is independent of employment. Hence, if employment \( L \) changes while wage rate \( W \) and productivity \( R \) remain unaltered then the price \( P \) remains constant. This case is depicted in Figure 4.

It is therefore possible that the economy moves from underemployment \( L_u \) to full employment \( L_f \) without any change of wage rate and price if the productivity is given. If productivity and wage rate change on the move to full employment this affects only the market clearing price as given by (9). Since financial profit is zero
under the condition of balanced budget it is of no consequence for the business sector whether the economy operates at full employment or underemployment. Profit is zero in both cases. Business could therefore be indifferent about various employment levels. A wage rate reduction is no precondition for attaining full employment, it would only lower the market clearing price. From (9) follows immediately for the real wage:

$$\frac{W}{P} = R | t. \quad (10)$$

Whatever happens to the wage rate is of no consequence for the real wage which is invariably equal to the productivity. The latter, in turn, is objectively determined by the production process. Under the condition of increasing returns the move from underemployment to full employment entails an increasing real wage. With a constant productivity in the relevant range the real wage does not change at all. Under this condition the move to full employment is indifferent for the already employed and beneficial for the hitherto unemployed.

Under the conditions of product market clearing and budget balancing a move from underemployment to full employment that is perfectly indifferent for both the already employed wage earners and the business sector presupposes a constant productivity. In this benchmark case wage rate changes in either direction are immaterial. Wage stickiness does not play any role. If the business sector is not indifferent and does not move to a higher employment level unless profit is greater than zero then full employment is unattainable, that is, the business...
sector prevents a Pareto-optimal employment expansion. The supersymmetric consumption economy is reproducible at any employment level; because of the indifference of all employment levels the notion of equilibrium or disequilibrium has no meaning for the business sector. Since wage rate changes are of no consequence under the condition of product market clearing and budget balancing the cause of unemployment must be located in the business sector.

Say’s Law is famous for its richness – or vagueness – of meanings (Sowell, 2008, p. 1), (Baumol, 1977). We are now in the position to reconfigure it for the pure consumption economy in structural axiomatic terms. It is easy to see that Figure 4 represents the simplest case of ‘supply creates its own demand’. There is no inherent limit to production. Productivity may grow with a breathtaking rate at any given employment level and wage rate without causing a glut. There exists always a price in the product market that fulfills the conditions of market clearing and budget balancing. At this price profit is zero and workers absorb the whole output. From the statement that this configuration is feasible does, of course, not follow that it will be automatically realized. We have only determined the market clearing price but not demonstrated how it could be established (for details see 2011b, Sec. 6). All that can be said at the moment is that perfect flexibility of the product price is a necessary condition.

Another interpretation of Say’s Law is ‘that there is no obstacle to full employment’ (Keynes, 1973, p. 26), (Kates, 1998, pp. 144-145). Figure 4 shows that this is in fact the case. It is not necessary that the wage rate is flexible. All that is necessary is that the business sector expands employment at the going wage rate until the labor market is cleared. No matter how productivity develops on the way to full employment, the profit for the business sector as a whole is always zero. Because of the principle of indifference the business sector has no good reason not to establish full employment. The real wage may be higher or lower at full employment, the wage earners invariably absorb the whole output. A trade-off would arises only if the real wage at full employment were below the margin of subsistence.

From the standpoint of the structural axiomatic approach Say’s Law is not a law in the familiar sense but a feasible economic configuration with unique properties. These properties include in the simplest case: full employment, market clearing, budget balancing, zero profit, and the equality of real wage and productivity. This configuration is objectively reproducible but it is improbable that it will ever be realized. Say’s Law is rather a benchmark or desideratum.

4 The logical emergence of profit

Profit has been defined with (7). In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

\[ \Delta Q_f \equiv PX - WL \mid t. \]

Using the first axiom (1) one gets alternatively:
\[ \Delta Q_{fi} = C - Y + Y_D \mid t. \] (12)

The three definitions are formally equivalent. Profit can be seen from different perspectives. Taken together, the three perspectives make a comprehensive view. If distributed profit \( Y_D \) is set to zero in (12), then profit or loss of the business sector is determined solely by consumption expenditures and wage income. For the business sector as a whole to make a profit consumption expenditures \( C \) have in the simplest case to be greater than wage income \( Y_W \) as shown in Figure 5.

\[ \Delta Q_{fi} = C - Y + Y_D \mid t. \] (12)

The price \( P \) is determined by the axioms (1) to (3) and the condition of market clearing:

\[ P = \frac{C}{RL} \text{ if } X = O \mid t. \] (13)

We define the expenditure ratio \( \rho_E \) as:

\[ \rho_E = \frac{C}{Y} \mid t. \] (14)

Together with (13) this yields the market clearing price in the general form:

\[ P = \rho_E \frac{W}{R} \text{ if } X = O \mid t. \] (15)

The market clearing price is higher or lower than unit wage costs depending on the expenditure ratio \( \rho_E \). In Figure 5 the profit per unit is positive because \( \rho_E > 1 \).
So that profit comes into existence for the first time in the pure consumption economy the household sector must run a deficit at least in one period. As long as the households spend their wage incomes fully the business sector will not make a loss but it will not see any profits either.

The logical explanation of profit therefore consists in: the business sector’s revenues can only be greater than costs if, in the simplest of all possible cases, consumption expenditures are greater than wage income. The existence of profit for the economy as a whole does neither depend on the working hours, the wage rate nor on productivity. Variations of these variables are compensated for by the market clearing price. Profit, to be sure, does not come from profit maximizing behavior.

Some economists start from the seemingly innocuous presumption that the value of output is equal to the value of incomes. Figure 5 makes it immediately clear that this presumption is unfortunate because it holds only in the special case of budget balancing. To start with this assumption therefore amounts to an analytical self-lock-in. There is nothing in the real world that makes the equal-value presumption come true. The general case is $\rho_E \neq 1$. That branches of theoretical economics that are built on the implicit assumption $\rho_E = 1$ are inoperative.

The household sector’s initial deficit in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy (for the inclusion of money and banking see 2011c; 2011d).

Mention should be made that neither neoclassicals nor Keynesians ever came to grips with profit (Desai, 2008), (Tómasson and Bezemer, 2010).

5 Distributed profit and profit

Once profit has come into existence for the first time (that is: logically – a historical account is a quite different matter) the business sector has the option to distribute or to retain it. This in turn has an effect of profit. This effect is captured by (12) but it is invisible in (11). Both equations, though, are formally equivalent. Profit distribution and full spending out of distributed profit is depicted in Figure 6.

If the household sector’s budget is balanced, i.e. if consumption expenditures are equal to total income in (12) then profit is, as a corollary, equal to distributed profit:

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad \Rightarrow \quad \Delta Q_{fi} = Y_D \quad \text{if} \quad C = Y \quad |t. \quad (16)$$

The market clearing price follows from the axioms (1) to (3) and is given by:

$$P = \frac{W}{R} + \frac{Y_D}{RL} \quad \text{if} \quad X = O, C = Y \quad |t. \quad (17)$$

The market clearing price is higher than unit wage costs in the supersymmetric case if distributed profit is greater than zero. Given the amount of distributed profit
Figure 6: Distributed profit and spending out of distributed profit under the condition of market clearing and budget balancing

as well as wage rate and productivity the price varies with employment. With increasing employment the market clearing price falls.

The determinants of profit look essentially different depending on the perspective. For the firm price $P$, quantity $X$, wage rate $W$, and employment $L$ in (11) appear to be all important for profit; under the broader perspective of (12) these variables play no independent role. The profit definition provokes a cognitive dissonance between the micro and the macro view. It is therefore worthwhile to realize that equations (7), (11), (12) are not only equivalent but indeed indispensable for a consistent view of profit.

Profit $\Delta Q_f$ and distributed profit $Y_D$ is clearly distinguishable. The latter is a flow of income from the business to the household sector analogous to wage income. By contrast, profit is the difference of flows within the business sector. Profit is not connected to a factor input. So far, we have labor input as the sole factor of production and wage income as the corresponding factor remuneration. Since the factor capital is nonexistent in the pure consumption economy, profit cannot be assigned to it in functional terms. And since profit cannot be counted as factor income there is no place for it in the theory of income distribution.

The individual firm is blind to the structural relationship that is given by (12). On the firm’s level profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices. These factors play a role when it comes to the distribution of profits.
between firms and these phenomena become visible when similar firms of an industry are compared. Because of this, it is not wise to take the considerations of the individual firm’s management as analytical starting-point and then to generalize. The microeconomic approach is inherently prone to the fallacy of composition.

Under the condition $C = Y$ financial profit $\Delta Q_{fi}$ is according to (16) numerically equal to distributed profit $Y_D$ as in Figure 6. The fundamental difference between the two variables does not catch the eye in this limiting case. The equality of profit and distributed profit is an implicit feature of equilibrium models. These have no counterpart in reality. In the real world holds $C \neq Y$, therefore profit and distributed profit are never equal. Models that are based on the familiar definition total income $\equiv$ wages + profits are erroneous because profit and distributed profit is not the same thing.

6 Retained profit and saving

Profits can either be distributed or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit $\Delta Q_{re}$ is defined for the business sector as a whole as the difference between profit and distributed profit in period $t$:

$$\Delta Q_{re} \equiv \Delta Q_{fi} - Y_D \quad \Rightarrow \quad \Delta Q_{re} \equiv C - Y \mid t.$$  \hspace{1cm} (18)

Retained profit is, due to (12), equal to the difference of consumption expenditures and total income as shown in Figure 7.

Financial saving is given by (6) as the difference of income and consumption expenditures. In combination with (18) follows:

$$\Delta Q_{re} \equiv -\Delta S_{fi} \mid t.$$  \hspace{1cm} (19)

Financial saving and retained profit always move in opposite directions. Let us call this the complementarity corollary because it follows directly from the definitions themselves. The corollary asserts that the complementary notion to saving is not investment but negative retained profit. Positive retained profit is the complementary of dissaving. Since there is neither capital nor inventory investment in the pure consumption economy with market clearing the familiar IS-equilibrium evidently cannot hold. This crucial point, though, is worth its own thorough analysis (for details see 2011e).

7 Money, credit, and transaction balances

In order to reduce the monetary phenomena to the essentials it is supposed that all money transactions are carried out costlessly by the central bank (for transaction costs see 2011c, Sec. 4). The household and business sector’s respective stock of money then takes the form of current deposits or current overdrafts.
In Figure 8 the money transactions between the household and the business sector are mapped onto the supersymmetric four quadrant diagram of Figure 3 which is reproduced for comparison in the left panel. The 1st axiom is then geometrically replaced in the right panel by the pattern of cumulated wage payments. Both curves start at the origin and end at period income $Y$. The cumulated payment pattern provides additional information about the money transactions in period $t$.

The 3rd axiom is first mirrored into the 2nd quadrant and then replaced by the pattern of cumulated consumption expenditures which are here assumed to be equally distributed over the whole period. Geometrically there is, in this simple case, no difference between the 3rd axiom and the cumulated expenditure curve. Independently of the expenditure pattern, which might be quite irregular, the curve of cumulated expenditures always starts at the origin and ends at $C$.

By sequencing the initially given period length of one year into subperiods the patterns of wage payments and consumption expenditures that are displayed in the right panel of Figure 8 give rise to the changing stock of money along the downward pointing axis. It is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At the end of each subperiod, and therefore also at the end of the year, the household sector’s stock of money is zero. Money is present and absent depending on the time frame of observation.
If income is higher than consumption expenditures the household sector’s stock of money increases. The change in period $t$ is defined as:

$$\Delta \bar{M}_H = m Y - C \mid t.$$ \hspace{1cm} (20)

The identity sign’s superscript $m$ indicates that the definition refers to the monetary sphere.

The stock of money $\bar{M}_H$ at the end $\bar{t}$ of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{M}_H = \sum_{t=1}^{\bar{t}} \Delta \bar{M}_H + \bar{M}_H \mid_0 \bar{t}.$$ \hspace{1cm} (21)

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta \bar{M}_B = m C - Y \mid t.$$ \hspace{1cm} (22)

The business sector’s stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{M}_B = \sum_{t=1}^{\bar{t}} \Delta \bar{M}_B + \bar{M}_B \mid_0 \bar{t}.$$ \hspace{1cm} (23)

Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (21) the current overdrafts of the business sector are of equal amount according to (23), and vice versa. Each sector’s stock of money is either positive or negative. Money and credit are at first symmetrical. From the
central bank’s perspective the quantity of money at the end of an arbitrary number of periods is then given by the absolute value either from (21) or (23):

$$\bar{M} \equiv \left| \sum_{t=1}^{\infty} \Delta \bar{M}_{H:B_t} \right| \text{ if } \bar{M}_{H:B_0} = 0 \ | t. \quad (24)$$

The quantity of money is always $\geq 0$ and follows directly from the axioms. It is assumed at first that the central bank plays an accommodative role and simply supports the autonomous market transactions between the household and the business sector. For the time being, money is the dependent variable.

The first half of the household sector’s two-period transaction pattern in Figure 9a is nothing other than the decoupled and counter-clockwise rotated zigzag curve of Figure 8.

Figure 9: Household sector’s transaction pattern for different nominal incomes in two periods and the resulting average stock of transaction money as seen from the central bank; the business sector’s pattern is perfectly symmetrical under the given conditions.

In period $2$ the wage rate and the price is doubled. Since no cash balances are carried forward from one period to the next, there results no real balance effect provided the doubling takes place exactly at the beginning of period $2$.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 9a translates into the average amount of current deposits in Figure 9b. This average stock of transaction money depends on income according to the transaction equation

$$\hat{M}_T \equiv \kappa Y \ | t. \quad (25)$$

Different transaction patterns are characterized by different numerical values of the transaction pattern index $\kappa$. The index in turn depends on the pattern of wage payments and consumption expenditures for a defined minimum subperiod of length $d$, e.g. a day. The cash transactions in $d$ are expressed as a fraction $\alpha$ of period income $Y$ respectively $\beta$ of consumption expenditures $C$.
\begin{align*}
  Y_{td} &\equiv \alpha_{td} Y_t \\
  C_{td} &\equiv \beta_{td} C_t.
\end{align*}
(26)

The expenditure ratio \( \rho_E \) has been defined with (14) as:

\[ \rho_E \equiv \frac{C}{Y} \mid t. \]  
(27)

The change of the household sector’s stock of money in \( d \) is then analogous to (20) given by:

\[ \Delta \bar{\bar{M}}_{Hd} \equiv \left( \alpha_{td} Y_t - \beta_{td} \rho_{Et} Y_t \right). \]  
(28)

The cumulated stock of money at the end of an arbitrary number of subperiods \( d \) is defined analogous to (21) as the sum of previous changes of the stock plus the initial endowment:

\[ \bar{\bar{M}}_{Hd} \equiv \sum_{d=1}^{d} \Delta \bar{\bar{M}}_{Hd} + \bar{\bar{M}}_{H0}. \]  
(29)

With an initial endowment of zero this reduces, after the insertion of (28), to:

\[ \bar{\bar{M}}_{Hd} \equiv Y_t \sum_{d=1}^{d} \left( \alpha_{td} - \beta_{td} \rho_{Et} \right). \]  
(30)

From the perspective of the central bank the current deposits of either the household or the business sector in subperiod \( d \) are given by:

\[ \bar{\bar{M}}_{td} \equiv Y_t \sum_{d=1}^{d} \left( \alpha_{td} - \beta_{td} \rho_{Et} \right). \]  
(31)

From the stock of deposits in each subperiod \( d \) as given by (31) one arrives at the average stock of transaction money as follows:

\[ \bar{\bar{M}}_{Y_t} \equiv \frac{\bar{\bar{M}}_{t_1} + \ldots + \bar{\bar{M}}_{t_d}}{d} \Rightarrow \bar{\bar{M}}_{Y_t} \equiv \kappa_t Y_t \]  
(32)

with \( \kappa_t \) depending on \( \alpha_{td}, \beta_{td}, \rho_{Et} \)

If, in the limiting case, \( \rho_{Et} = 1 \) and \( \alpha_{td} = \beta_{td} \) then the stock of money in each subperiod \( d \) is zero and by consequence the average stock of transaction money vanishes. In this case wage payments and consumption expenditures are perfectly synchronous and the transaction pattern index \( \kappa_t \) is zero. This, of course, does not happen in the normal course of events, hence the transaction balances are different from zero for any realistic subperiod of the minimum length \( d \).

Figure 10 gives a complete picture of the supersymmetric consumption economy including the average stock of transaction money. It should be noted that the quantity of money \( \bar{\bar{M}} \) is again zero at the end of period \( t \) if \( \rho_{Et} = 1 \) according to (24). The
average stock of transaction money $\hat{M}_T$ is $> 0$ if $E_r = 1$ and $\alpha_t \neq \beta_t$. The $\neq$-sign provides the ultimate rationale for positive transaction balances. It is easy to see from Figure 10 that the average stock of transaction money (but not the quantity of money at period end) doubles if the wage rate and the market clearing price double according to (9).

![Figure 10](image.png)

**Figure 10:** Representation of the supersymmetric consumption economy including the average stock of transaction money $\hat{M}_T$

The four quadrant positive rational diagram, $4QPR$-diagram for short, replaces the rather simplistic demand–supply schedules as the easy to handle geometrical standard tool of analysis. The $4QPR$-diagram is objective, that is, free of any behavioral assumptions, and therefore truly general. The characteristic of the supersymmetric scheme is that it is reproducible for an indefinite time span provided there are no exogenous restrictions. Insofar, the supersymmetric scheme is indeed different from all other possible configurations of the structural axiom set. There is no reason, though, to expect that the realization of product market clearing, budget balancing and full employment in any period $t$ is more probable than any other structural configuration.

### 8 Conclusion

If we ask, ‘What is the most adequate model of behaviour for economics?’ we implicitly assume that economics actually needs a model of behaviour; hence, we already assume psychologism of a kind. (Hudík, 2011, p. 147)
Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Neither are they needed. The present paper excludes psychologism and suggests three nonbehavioral axioms as groundwork for the consistent reconstruction of the elementary consumption economy. The main results of the rather straightforward geometrical analysis are:

- The consumption economy that is at first given with the bare set of structural axioms is not reproducible over a longer time span. This is the original and general case.

- The addition of the conditions of market clearing and budget balancing to the axiom set yields a reproducible consumption economy with indifferent employment levels for the business sector.

- The market clearing price is unequivocally determined by the axiom set and the supersymmetry conditions. It is therefore impossible to add independent demand and supply functions. There is no formal room left for behavioral assumptions.

- Under the condition of supersymmetry wage rate changes in any direction are of no consequence for the move from underemployment to full employment.

- The logical explanation of profit consists in: revenues can only be greater than costs if, in the simplest of all possible cases, consumption expenditures are greater than wage income.

- Under the condition of budget balancing profit is numerically equal to distributed profit. In the real world the household sector’s budget is not balanced, therefore profit and distributed profit are never equal.

- Models that are based on the familiar definition total income $\equiv$ wages + profits are erroneous because profit and distributed profit is not the same thing.

- The complementary notion to saving is not investment but negative retained profit. Positive retained profit is the complementary of dissaving.

- The four quadrant diagram represents the structural axiom set geometrically and replaces the demand–supply schedules as easy to handle standard tool of analysis.
References


