Giga-Investments: Modelling the Valuation of Very Large Industrial Real Investments

Mikael Collan

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# TABLE OF CONTENTS

## PART I: SYNOPSIS

1. **Introduction**  
   1.1. Motivation for this Research  
   1.2. Outline of the Thesis

2. **Background**
   2.1. Very Large Industrial Real Investments (VLIRI)  
   2.2. State of the Art of Investment Profitability Analysis  
      2.2.1. Commonly Used Profitability Analysis (Investment Valuation) Methods  
      2.2.2. Decision Support Technology and Supplementary Capital Budgeting Tools  
   2.3. Valuing Managerial Flexibility with Option Valuation: Real Options  
   2.4. Modelling Imprecise (Financial) Information with Fuzzy Logic  
      2.4.1. Foundations of Fuzzy Sets  
   2.5. Summary of the Background for this Research

3. **Methodology**  
   3.1. Philosophical Position of the Research  
   3.2. Methods Used  
      3.2.1. Gaining an Understanding about Very Large Industrial Real Investments  
      3.2.2. Conceptualisation and Modelling

4. **Key Findings and Insights**  
   4.1. Defining Giga-Investments - Three Common Characteristics  
      4.1.1. A Long Economic Life  
      4.1.2. A High Degree of Irreversibility  
      4.1.3. A Long Building Time  
      4.1.4. Giga-Investment Valuation Issues and Lifecycle
4.2. Enhancing Black-Scholes Real Option Valuation for Giga-Investments
4.2.1. Fuzzy (Hybrid) Real Option Valuation Model (FROV)
4.3. A New Model for Valuation of Giga-Investments
4.3.1. Numerical Example of Giga-Investment Valuation with FRIV
4.3.2. Using New Methods for Giga-Investments
4.4. Dynamic Planning and Management of Giga-Investments
4.5. Original Publications

5. Summary, Conclusions, and Future Research
5.1. Summary of Goals and Contributions of this Research
5.2. Conclusions
5.2.1. Weaknesses of this Research
5.3. Future Research

6. References

APPENDIX 1: Four Historical Giga-Investment Cases
Case 1: Rautaruukki Oyj, Coking Plant, Raahed
Case 2: M-Real Oyj, paper machine 1 (PM1), Kirkniemi
Case 3: Outokumpu Harjavalta Metals Oy, Harjavalta
Case 4: Metsä-Serla Oy, pulp mill, Kaskinen

APPENDIX 2: Fuzzy Numbers and Uncertainty of Cash Flows

APPENDIX 3: On the Properties of the Heuristic Operator

APPENDIX 4: Calculation of FRIV with the values from the example in 4.3.1
PART II: ORIGINAL RESEARCH PAPERS


PART I: SYNOPSIS

*I no longer have any patience with natural scientists who imagine that they have some kind of patent on exactness which they have not licensed to the social science brethren.*

Herbert A. Simon *

* (Simon, 1991) p. 304
1. Introduction

1.1. Motivation for this Research

Valuation is at the heart of investment decision making. There are a number of factors that are important when investment decisions are made. However, the profitability of an investment is perhaps the single most important thing contributing to the decision. This applies to both, financial and real investments.

We can understand a real investment as an investment made in a single real asset, or in a single project, such as machinery or a plant. Financial investments are investments in securities that have a real asset, or more commonly, a collection of real assets (a company) underlying them. Generally, the same valuation models are used for both, financial and real assets.

Markets for selling investments in financial and real assets are different. If a company invests in financial assets and decides to reverse the investment decision, the financial assets can, with a very high probability, be sold on the financial markets. The price of the financial asset is determined in the markets existing for the asset in question, and the asset can be sold for that price. If a company invests in a real asset and decides to reverse the investment decision, it may be impossible to find a buyer for the asset.

This is especially the case when the investment has been made in specialised machinery, or some other uncommonly used real asset, or when the real investment is very large. In other words, real investments are to a larger extent irreversible than financial investments, and the more special or large the real investment is, the more difficult it is to reverse (the higher grade of irreversibility it has).

Hence, it seems that the markets of financial investments are different from the markets of real investments. This means that real assets are different from financial assets as investments, yet the valuation of real assets is generally made with the same models as the valuation of financial assets. It also seems that large real investments are different from small real investments, due to their higher grade of irreversibility. Very large real investments tie up a lot of capital and affect the future strategy of companies. The larger a real investments is, the more important it is to be able to value the investment correctly, because the companies are, de facto,
stuck with their very large real investments. An incorrect investment decision may cause these companies to lose significant amounts of money and to deteriorate in profitability. (Caves, Gale, and Porter, 1977) quote (Mancke, 1974) and state that "one of the most important determinants of many firms' ex post performance is their respective successes with large and ex-ante uncertain investments", they also point out that similar arguments appear in (Demsetz, 1973), (Bock and Farkas, 1969), and (McGee, 1974). Timing of investments is also an important issue in manufacturing industries. According to answers to a survey made among Irish manufacturing companies, timing of a large capital investment project will have a substantial effect on a firm's future prospects, (Driver and Whelan, 2001). From this background, very large industrial real investments can be said to be vital to the company performance.

The interest for valuing very large industrial real investments grew in the traditional industrial companies in Finland during the reference period (economic context) of this research, 1995-2004, during which the world experienced a boom of the so called dot-com companies. The projected, relatively low and historically decreasing, returns for the traditional industrial companies could not compete with the perceived high return expectations from the IT-oriented companies.\(^1\)

Due to the economic context, and the apparently low interest from the markets in funding their investments, industrial companies involved in making very large real investments have become more interested in models that are designed to value and support the investment decision making of very large industrial real investments. In the background of this interest was the suspicion that perhaps the models used for valuation of very large industrial real investments do not give the correct picture of their value. Interestingly, it seems that there are very few commonly available valuation models specifically designed for large real investments.

Very large industrial real investments are usually characterised by large sunken investment costs, which together with their large physical size and specialised equipment make the investments irreversible. Under such circumstances and, because the investments may be unique (there may be no historical information, or comparable investments available) uncertainty plays a key role in the decision making of large industrial investments; the

\(^1\) In hindsight we can say that the expectations from many dot-coms was fundamentally flawed, due to overly positive revenue expectations and partly from downplaying of the risks involved. A quick peek into the valuation of dot-coms can be found, e.g. in Desmet et al., 2000), and into errors in company valuation, e.g., in (Fernandez, 2003).
uncertainty about the profitability of the investment makes the investment a risk for the company. Managers need more information about the uncertainty and the effect of uncertainty to the value of very large industrial investments, this is why there is an interest in new methods that enhance the ex-ante profitability analysis of very large industrial real investments.

One way adding to the ex-ante analysis of real investments is to use real option valuation, based on the valuation of financial options, to capture the value of, e.g., waiting to invest. Real option valuation is a method that is also useful for large industrial real investments. Due to assumptions underlying the financial option pricing formulas (Black-Scholes) used in real option valuation, the usability of these models and real option valuation for these investments may be questioned. This thesis will present a fuzzy real option valuation method, based on the Black-Scholes formula that is designed to enhance real option valuation for very large industrial real investments.

An original fuzzy real investment valuation model, based on the characteristics of very large industrial real investments is introduced. This new model concentrates especially in the handling of uncertainty utilising fuzzy sets to capture cash flow uncertainty and relying on a forward looking, opposed to the commonly used backward looking, method of calculation of standard deviation for the estimation of the uncertainty of the investment.

This research will further show that we can (quite intuitively) divide the valuation of very large industrial real investments into three stages, which can be separately assessed, giving a new insight into the valuation of very large industrial real investments.

This research assumes that investment decisions are made by rational managers, i.e., the decisions are made systematically, based on the analyses on the profitability of the investment in question. Under this assumption better valuation models and better decision support tools that support systematic investment decisions will result in better decisions, and add to the wealth of the company. This research does not emphasise the organisational, psychological, and agency theory issues of decision making.
Based on the above discussion, the motivation for this research is to create new knowledge about very large industrial real investments, their valuation, and their decision support.

1.2. Outline of the Thesis

This thesis has been prepared as a collection of articles with the uniting theme of valuing and creating new knowledge about very large industrial real investments.

Within the framework of investment valuation the underlying hypothesis of this research is that a special group of investments, within the group of very large industrial real investments, can be defined and that the group has characteristics that need to, and can be, taken into consideration in their valuation. Methods can be developed and can be shown to represent extensions of, or enhancements to, existing investment models. We will argue that the models will enhance and improve the support for investment decisions. To investigate this hypothesis the goals of this research are:

(i) to understand and characterise very large industrial real investments
(ii) to, for the purposes of this research, create a definition of a special group of investments within the group of very large industrial real investments
(iii) to develop and select relevant constructs and methods to be used in valuation of the selected group of investments and of very large industrial real investments
(iv) to propose a framework for the valuation of the selected group of investments that is based on the constructs and methods proposed and selected

The four goals serve to enhance our knowledge of very large industrial real investments and should give us a framework for a tool that can be used in valuing them.

The thesis is divided into two parts. The first part is a synopsis of the research work. It winds together the different research issues and topics showing how they interconnect and form a whole. The second part presents the research papers on which this thesis draws on.

The rest of the first part of this thesis is structured as follows: Chapter 2 gives the background on which this research has been conducted, going
through the main contributing issues and disciplines that are relevant to this research.

Chapter 3 outlines the philosophical position the author has had during the whole of this research process and illustrates the research methods used in the different stages of this research.

Chapter 4 presents the main findings and insights of this research. The concepts, definitions, and theoretical constructs built for this research are presented, including presentations of the original models created.

Chapter 5 concludes on the findings of the research with reference to the valuation of very large industrial real investments and emphasises the main contributions. Limitations of the study are addressed and future research possibilities are suggested.
2. Background

This chapter will present the background on which this research is based. First, a short introduction to very large industrial real investments is made, where the characteristics that are relevant to valuation and profitability analysis of these investments are discussed and summarised. Then, a review of the state of the art of profitability analysis is made, with a presentation of the most commonly used capital budgeting methods, input variable value selection, supplementary capital budgeting tools, and the evolution of decision support technology for capital budgeting. Thirdly, modelling managerial flexibility and real option valuation are discussed, and finally a short presentation about modelling financial information with fuzzy logic is given, followed by a discussion about fuzzy sets and fuzzy numbers.

2.1. Very Large Industrial Real Investments (VLIRI)

Very large industrial real investments, abbreviated VLIRI, are usually investments that industrial companies make in their production facilities. Examples of very large industrial real investments are, e.g., nuclear and other power plants, mines and oilfields, steel mills, paper and pulp mills, and other large industrial production facilities. VLIRI are often important, not only to the companies undertaking them, but also to the society and the economy as a whole. Issues like environment, safety, and employment are not insignificant, when VLIRI are discussed. In the following discussion about VLIRI we will, however, concentrate on issues that have to do with the profitability and valuation of very large industrial real investments.

Initial investment size of the VLIRI varies, however, they are in the hundred million EUR (or USD) class\(^2\). The initial investment cost is most often a sunken cost. VLIRI are stationary investments and once their building has started, for their part, they lock the geographical strategy of the company undertaking them to the building location. The technology of the VLIRI is also most often fixed for a long time and often constrains, e.g., the production capacity and the production quality of the investment. "Investments have a degree of irreversibility whenever they have attributes

\(^2\) Initial investment costs used in the literature for VLIRI are e.g., USD 104 million (oil field) (Trigeorgis, 1995), USD 600 million (oil field) (Leslie and Michaels, 1997), FIM 1.4 billion (steel mill) (Blom, 2000), FIM 988 million (pulp mill) (Bodman, 2000), EUR 1.7-2.5 billion (nuclear power plant) (Teollisuuden Voima 2000), and USD 550 million (hydropower plant) (Keppo and Lu, 2003).
that make the capital specific to the firm, a product, or an industry, or else costly enough to move and relocate that the value of the capital becomes effectively tied to its original use" (Barham, Chavas, and Klemme, 1994). According to the above definition most VLIRI are, to a high degree, irreversible. This is also corroborated by the observation that there are no established markets for buying and selling VLIRI.

Building time of VLIRI can be long, ranging from months to several years, e.g., >2 years (paper machine) (Bodman, 2000), ~2 years (Palomäki, 2000), 3 years (coking plant) (Blom, 2000), 3 years (pulp mill) (Leivo, 2001), and 4-5 years (nuclear power plant) (Teollisuuden Voima, 2000). During a long building time there may be changes in the markets, in which the very large industrial real investment will operate after their completion, such changes can effect the profitability of these investments negatively or positively. A long building time may increase the difficulty of estimating the cash flows from a VLIRI, and thus increase the uncertainty of the investment.

VLIRI most often have long economic lives of 10-20 years, however, ranging up to 60-years (e.g., nuclear power plants (Teollisuuden Voima, 2000)), or even longer (e.g., mines). When the profitability of a VLIRI is analysed the long economic lives may cause problems in accurately estimating cost and revenue cash flows from the VLIRI, taking place far in the future. It is very likely that the markets change several times during the economic life or a VLIRI.

VLIRI may have an effect on their markets, e.g., "...in the case of electricity market the new power plant usually increases significantly the capacity in the market and, therefore, the investment decision may affect the electricity price" (Keppo and Lu, 2003). As a concrete example of an investment that steers the markets they present an analysis of a hydropower plant. Similar effects of market steering is visible also in the paper industry (fine paper and pulp) (Leivo, 2001) and (Bodman, 2000) and base metals industry (copper and steel) (Palomäki, 2000) and (Blom, 2000). This means that some VLIRI do not follow a price-taker behaviour, but there is a feedback loop between VLIRI and the markets into which they are built (Harris, 1978).

---

3 e.g., (Blom, 2000), (Bodman, 2000), (Palomäki, 2000), (Leivo 2001), and (Keppo and Lu, 2003)
Figure 2-1. VLIRI is a special case of investments. Some VLIRI can steer their markets: a feedback loop exists between the investment and its markets.

An investment that can affect the market prices by adjusting its production capacity, or by entry to the markets, gives the managers of such investments a chance to try to optimise the value of the investment by optimally adjusting production, or entry. The existence of such possibilities means that managers running a VLIRI with a possibility to steer the markets can make decisions based on information that is not stochastic (random). In cases where a very large industrial real investment can steer the markets there is an effect on the value of the investment (Keppo and Lu, 2003) and (Harris, 1978).

The choice of financing for a VLIRI may affect its value. Project finance is a set of techniques designed for financing and managing large-scale capital-intensive projects, and as an academic discipline investigates and develops these techniques. Project financing is most often understood as a way to finance large real investments, so that the revenues generated by the investment are used to repay the loans financing the project, and the assets of the investment work as collateral. Project financing is the financing of choice of many VLIRI, e.g., (Finnerty, 1996).

The rationale for project financing is that it gives the possibility to spread the risk of large capital-intensive investments. Allocating the risk between a number of parties makes it possible to initiate large investments in situations, where no single party is willing or able to carry all the risks and debt obligations connected to an investment. The type of financing used in large-scale projects may have an effect on the profitability of a project. The amount of leverage that a project has may change during the lifetime of the project, thus having an effect on the project risk. (Esty, 1999) observes that

---

4 Project financing includes techniques for setting up a special purpose project company for a project-financed investment. The techniques include methods for, e.g., feasibility studies, agreements between owners, contracts with project management, and others. Project financing is highly practice oriented.
changing leverage has an effect on the discount rate used in the calculation of present values of future cash flows, hence the discount rates used for investment cash flows should reflect the changes in the financing. Some companies use the weighted average cost of capital (WACC) as the discount rate for their investments, but it has been shown that it is incorrect to use WACC as discount rate, e.g., (Arditti, 1973), (Baranek, 1975), and (Brick and Thompson, 1978).

VLIRI are very large investments and thus are important to the companies undertaking them. When investment decisions on VLIRI are made a number of issues have to be taken into consideration. To shortly summarise some VLIRI characteristics important for valuation of VLIRI we can state that:

- they have a large initial investment, which is most often a sunken cost
- they are most often irreversible
- there are no markets for buying and selling VLIRI
- they often have a long building time
- they have a long economic life
- markets are likely to change during their economic life
- some VLIRI can steer their markets
- the VLIRI that steer their markets have strategic importance in their markets
- choice of financing may affect the profitability of VLIRI

The above characteristics make VLIRI different from financial investments and from other real investments.

Very large industrial real investments are made in the presence of uncertainty, a large part of which falls under the category of structural uncertainty that "is based on imperfect knowledge as to the structure the future can take" (Kyläheiko, Sandström, and Virkkunen, 2002). This is different from parametric uncertainty, under which the agent (in this case the manager) has certain knowledge about the structure of a decision problem, but is uncertain about the parameters of the problem (probabilities). The above means that decision makers and planners, responsible for very large industrial real investment decisions may find it to be impossible to, ex-ante, model the economic life of the VLIRI in a detailed way. This implies that when coping with VLIRI, decision makers are facing not only difficulties in estimating uncertain parameter values, but also uncertainty about the structure of the problem itself (in this case the analysis of the investment profitability). The structural uncertainty is
caused by a number of issues characteristic of the VLIRI, e.g., the markets in which they operate may change multiple times (new entry, technological changes).

In appendix 1 of this thesis, four historical investment cases of very large industrial real investments are presented for reference. The first case presents a coking plant investment in a highly uncertain and complex market environment by Rautaruukki Oyj, the second case goes through the economic life of a green field paper mill investment by M-Real Oyj, the third case illustrates the production expansion of nickel and copper production by Outokumpu Harjavalta Metals Oy, and finally the fourth case presents a green field pulp mill investment by Metsä-Serla Oy. The cases work as a background material for this research, and they all exhibit most of the VLIRI characteristics discussed above, including the uncertainty underlying the investment decisions that the cases present.

These cases are the result of the WAENO research program, for information see (Collan and Hirkman, 2003), and have been presented in (Blom, 2000), (Bodman, 2000), (Palomäki 2000), and (Leivo 2000).

2.2. State of the Art of Investment Profitability Analysis

Investment decision making by rational managers is based on the systematic analysis of the profitability of the investment. The result is a product of the methods used in the profitability analysis and of the supplementary capital budgeting tools. The technology utilised to do the analysis and to run the supplementary tools plays a role in the level of the decision support offered. Below we will review (discounted cash flow based) methods used for profitability analysis, selection of the input variable values, supplementary capital budgeting tools, the state of the technology used to support them, and draw some conclusions on the state of the art of investment profitability analysis.

2.2.1. Commonly Used Profitability Analysis (Investment Valuation) Methods

Neo-classical profitability analysis methods are the most commonly used profitability analysis methods today, they are methods that can be traced back to the neo-classical theoretical framework for investments and the theory of the firm, pioneered in the end of the 19th, and at the beginning of

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5 The term “neo-classical profitability analysis methods” is used in (Pindyck, 1991)
the 20th century, perhaps, most notably by Irving Fisher6. These profitability analysis methods are based on Fisher's observation that a dollar today is worth more than a dollar tomorrow, and use this as the basis for valuing investments. The methods that are based on Fisher's findings are today known as discounted cash flow (DCF) based methods. According to (Wing, 1965), not very much changed in the fundamentals of capital budgeting methods from 1915 to 1965. He bases his assessment on (Deventer, 1915) and states that it includes the basis for the DCF methods available per 1965 (discounting and opportunity cost of capital). According to a comparison of studies on the use of capital budgeting methods in (Ryan and Ryan, 2002) the most used methods in the 60's (2 studies) were the payback period method (see below) and accounting rate of return. The same states that the most popular methods in the 70's, 80's, and the 90's are also DCF based, and that "discounted capital budgeting methods are generally preferred over non-discounted techniques". (Thompson and Wong, 1991) state that "the discounted cash flow (DCF) approach to valuation is one of the most fundamental tools in finance." Based on the above, it seems that capital budgeting methods have changed fairly little from the beginning of the 20th century until today, and still are based on the DCF techniques.

As there are different profitability analysis methods based on the discounted cash flow (DCF) framework, and many versions of these; for the purposes of this research, we will concentrate on the three most commonly used methods (per ca. year 2000). According to (Ryan and Ryan, 2002) the most commonly used capital budgeting methods in Fortune 1000 companies are, in the following order, the net present value (NPV), internal rate of return (IRR) and the payback period (PB) methods. The same three are found to be the most commonly used methods in FT-SE 100 companies in the U.K. (Busby and Pitts, 1997), among about 600 companies in the Netherlands (Verbeeten, 2001), and in the HEX main list companies (86) in Finland (Collan and Långström, 2002). (Sandahl and Sjögren, 2003) find in their study of the largest Swedish companies that in "engineering industry" and in "investment" the same three methods are the most common. In "basic industries" the three are amongst the four most commonly used methods.

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6 Fisher made a definition of capital and discussed the concept of time value of capital (Fisher, 1896a. He developed also the concepts known as compounding and discounting (Fisher, 1896b) and (Fisher, 1907).
**Net present value method (NPV)**

Net present value (NPV) of an investment is the sum of the discounted present value of the initial investment cost (IC) and the estimated free cash flows (FCF) from the investment. A risk-adjusted discount rate is used. Investments with an NPV above zero are accepted. Early presentations of NPV can be found in (Dean, 1951) and (Bierman and Smidt, 1960), more recent presentations are available in standard textbooks on corporate finance, e.g., (Brealey and Myers, 2003).

**DEFINITION.** NPV is defined as,

\[
NPV = -\sum_{t=0}^{n} \frac{IC_t}{\prod_{j=0}^{t} (1 + r_{fj})} + \sum_{t=0}^{n} \frac{E(FCF_t)}{\prod_{j=0}^{t} (1 + k_{j})} \tag{1}
\]

where, \( t = \) time
- \( I_t = \) Investment at time \( t \) (initial investment)
- \( r_{fj} = \) Risk-free interest rate at time \( j \)
- \( E(FCF)_t = \) Expected cash flow at time \( t \)
- \( k_{j} = \) Discount rate at time \( j \)

Fuzzy versions of NPV are presented, e.g., in (Buckley, 1987) and in (Kuchta, 2000), which is a more thorough presentation. The fuzzy versions use fuzzy instead of crisp numbers for model variable values, fuzzy numbers are presented in section 2.4. of this thesis.

**Internal rate of return method (IRR)**

Internal rate of return (IRR) is the discount rate that gives NPV of zero. By comparing the IRR to a hurdle rate an investment is accepted, or discarded.

**DEFINITION.** IRR is the solution to the following equation,

\[
0 = -\sum_{t=0}^{n} \frac{IC_t}{\prod_{j=0}^{t} (1 + IRR)} + \sum_{t=0}^{n} \frac{E(FCF)_t}{\prod_{j=0}^{t} (1 + IRR)} \tag{2}
\]
Fuzzy versions of IRR are presented, e.g., in (Buckley, 1987), (Kuchta, 2000), and (Carlsson and Fullér, 1998), of which the last concentrates on fuzzy IRR.

**Payback method (PB)**

Payback method accepts an investment, if it pays back the initial investment cost (IC) before a selected cut-off date (has a payback time shorter than the cut-off time).

**DEFINITION.** Payback time (PBT) is defined as,

\[
PBT = \min \left\{ k : \sum_{i=0}^{k} FCF_i \geq 0, \infty \right\}
\]  

The length of the payback period may be used as a risk measure within the payback method; it can be assumed that the shorter the payback time, the lower the risk. One use for the payback period method is to compare two investments with approximately the same NPV. The investment with a shorter payback period generates the value faster, and may hence be more advantageous, ceteris paribus. Versions of PB that take the time value of money into consideration are also available. Fuzzy versions of PB are presented, e.g., in (Kuchta, 2000).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Method</th>
<th>Payback (PB)</th>
<th>Internal Rate of Return (IRR)</th>
<th>Net Present Value (NPV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment of FCF</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Time value of money</td>
<td>no / yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Method of coping with uncertainty</td>
<td>cut-off period</td>
<td>IRR decision rule</td>
<td>discount rate (if project specific rate used)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2-1.** Characteristics of payback, internal rate of return, and net present value

Table 2-1 presents the characteristics of the three most commonly used capital budgeting methods. Each of the three most commonly used discounted cash flow based profitability analysis methods provides a single criterion to summarise into a single figure the economic desirability of the proposed investment. Results from (Ryan and Ryan, 2002) and from (Verbeeten, 2001), nevertheless, suggest that companies do not limit the profitability analysis of their investments to analysis with one method, but they utilise a combination of methods to gain a more complete picture of
the profitability of the investment. This is easy to understand, when discussing VLIRI, because their large size and importance to companies justifies the additional resources needed for a more holistic analysis. This would also imply that rational managers rather want more than less information when making investment decisions.

**Input variable value selection**

The results from profitability analysis methods are dependent on the input variable values. In the three most commonly used methods presented above, the common input variables are the initial investment (IC) and the free cash flows (FCF). For NPV also the discount rate (opportunity cost of capital) needs to be selected. Below we will discuss ways to select the values for IC and FCF. The discount rate has already been shortly discussed in 2.1.

Determining the initial investment (IC) is a rather simple task for financial investments, because IC is the market price at the moment at which the investment is made. For small real investments the IC is the price at which the real good is bought and the cost of possible installation (which we assume is fairly simple to estimate accurately). For VLIRI the estimation of IC may not be equally straightforward, because the initial investment may be very large and the cost spread over a number of years. For example, for a nuclear power plant investment the building time is between 4-6 years and the initial investment cost depends on a number of contractors and suppliers (Teollisuuden Voima, 2000). Initial cost of large real investments is often slightly higher than the expected, and the investing companies usually prepare for this possibility, e.g. see (Palomäki, 2000).

For most investments, including VLIRI, the variability of FCF is dependent on the variability of markets (prices of the inputs into the production and the output products), therefore, the evolution of markets most likely has a high correlation with the evolution of the FCF of the investments. It is a fair assumption that by estimating the market movements one can estimate the FCF of the investment. It is not unusual that the estimation of future FCF is called cash flow forecasting.

There are a number of different ways to forecast, e.g., cash flows, (Armstrong and Crohman, 1972) classifies them into four different theoretical types according to whether they are subjective (judgmental, intuitive, implicit) or objective (process to obtain the forecast is well
specified) and whether they are naive (use data only on the dependent variable) or causal (consider many variables that may cause changes in the dependent variable). (Armstrong and Crohman, 1972) label the categories also as novice judgement (subjective-naive), extrapolation (objective-naive), expert judgement (subjective-causal), and econometric (objective-causal).

We exclude further elaboration on the novice judgement (subjective-naive) methods, methods that are not based on any systematic procedure, because we have assumed, in 1.1., rational managers and decision making, which is based on fact based systematic analysis. This has also been the position of (Armstrong and Crohman, 1972).

Extrapolation methods (objective-naive) are methods based on models that use the past history of a time series as a primary basis for constructing a qualitative forecasting model of its future (Fildes, 1979). There are many different extrapolation methods that can be divided into different groups, e.g., trend curve analysis, smoothing and Box-Jenkins methods, Bayesian forecasting, and adaptive models. The different types of models are suitable for different types of situations, but in general, trend curve analysis is often used for long term forecasting and the other models are perceived suitable for the shorter term (Fildes, 1979). The selection of the extrapolation method used is important for the credibility of the forecast.

There are enhancements to extrapolating methods available that use fuzzy set theory, further presented in 2.4., to enhance prediction accuracy, e.g., (Chen and Wang, 1999). The suitability of extrapolation methods for VLIRI may be questionable in some cases, due to the fact that there may not be a time series available for extrapolation.

Expert judgement (subjective-causal) forecasting methods can be divided into two main categories, separate judgmental forecasts and judgmental adjustment of forecasts (Bunn and Wright, 1991). Separate judgmental forecasting bases the forecast only on structured expert judgement.

Well known ways of separate judgmental forecasting are, e.g., round table discussions between experts to reach consensus over a forecast, and the delphi technique. The delphi technique, a set of procedures originally developed by The Rand Corporation in the late 1940's, is designed to obtain the most reliable consensus of opinion of a group of experts (Milkovich, Annoni, and Mahoney, 1972). Delphi is a series of intensive interrogations of each expert in a panel of experts, by using a set of
questionnaires. The procedure is designed to avoid direct confrontation between the experts, and is designed to be more conducive to independent thought and gradual formulation of a considered opinion (Dalkey, 1967). The experts are also asked to give their feedback on questions concerning issues close to, and relevant to the main question. For an example of the delphi method, e.g., (Milkovich, Annoni, and Mahoney, 1972) presents the use of the delphi procedures for manpower forecasting. It may be questioned if, e.g., the delphi method falls under the category of subjective methods, as the procedure itself is structured and transparent.

According to (Bunn and Wright, 1991) and based on, e.g., (McNees and Perna, 1981), (Corker, Holly, and Ellis, 1986), and (Turner, 1990), there are two main reasons to make judgmental adjustments to forecasts (made with econometric models, discussed below):

i) specification error: the model is not performing as well as expected and it makes sense to adjust the output rather than rebuild the whole model, and

ii) structural change: model performance is expected to be influenced by some extra-model factor or changes in the assumptions of the model.

Literature would seem to suggest that when experts are used in their familiar real-world forecasting context (in their specific area of expertise), then a judgmental adjustment process which is formalised as best practice will enhance the value of a good statistical model (Bunn and Wright, 1991). When judgmental adjustments are made, they should be made in a structured and transparent way, guidelines for such a structured way are given, e.g., in (Bunn and Wright, 1991).

It seems that the literature that compares judgmental with statistical models, discusses mainly cases where there is a possibility to build a reliable statistical (econometric) model and compares the models with judgmental forecasting. In cases where there is not enough background material for a reliable statistical model, it may very well be that judgement is the only reliable source of forecasts, or at least outperforms models. In such cases it would seem that separate judgmental forecasting would be more reliable than statistical models. However, when reliable statistical models can be built, it would seem that according to the literature they, together with judgmental adjustment, would yield the most reliable forecasts. For very large industrial real investments judgmental forecasting may be the most reliable way to go, for they may be unique investments with no established markets and their future is highly uncertain.
Judgmental forecasting has also shortcomings, e.g., when it is made by individuals not having comprehensive knowledge of the forecasted phenomenon (non-experts), the reliability of the forecast, and of the adjustments may be (is) compromised. Further, there may be biases attached to the individual experts, for each expert has a different basis of experience on which she produces her opinion.

Econometric (objective-causal) methods are based on econometric models, that are a way to characterise an economic or a behavioural system. They are typically aggregate linear or almost linear models with a well-defined stochastic structure. Model parameters are estimated from the data using well understood and statistically ‘optimal’ techniques based on these stochastic assumptions (Fildes, 1985). Econometric models are (should be) based on a definition of the studied system, and on analyses on the statistical characteristics of the variables, about the relationships and causality between the selected system variables, and on specification of the functional form of the model. A serious econometric model should also be tested. After the model is built it is used (a simulation is run) to generate a forecast. (Fildes, 1985) is a state of the art of econometric models that includes a comprehensive introduction to econometric model building and modelling strategies and a comparison of the forecasting accuracy of a number of (60) different econometric models, and also compares econometric methods with other methods.

Econometric models, like all models, have their shortcomings, which may have to do with, e.g., the available data to build the model: if there is not enough data about the studied system the model will not be able to reliably predict it. Also, "since identifying the appropriate model is, in part, a subjective process, it may be possible to fit more than one model that may seem to describe the data adequately" (Thompson and Wong, 1991). This means that a number of econometric models may, ex-ante, seem to fit a studied system. However, it may be impossible to know which one should be used for best ex-post results. Where the assumptions made in the econometric markets reflect reality, e.g., market movement is truly stochastic, and the model is built correctly to reflect the markets, econometric models can provide good forecasting accuracy. Indeed, it is the position of (Fildes, 1985) that “where data exists on the key explanatory variables in a problem, a causal model will usually outperform judgement”. (Armstrong and Crohman, 1972) have a similar view, however, no clear-cut answers are possible to the question, of which type of model is the most accurate (Fildes, 1985). In many occasions the assumptions of econometric models, even about uniform stock market
distributions, have been found to be inexact, e.g., see (Masoliver, Montero, and Porra, 2000), (Masoliver and Montero, 2001), and (Perello and Masoliver, 2002). Thus one is prompted to be cautious when using and selecting econometric models that have underlying assumptions.

In addition to the four types of methods classified by (Armstrong and Crohman, 1972) there are also some newer "intelligent" methods available for forecasting, e.g., artificial neural networks.

Artificial "neural network" technology was developed in an attempt to mimic the acquisition of knowledge and organisation skills of the human brain" (Wong and Selvi, 1998). An artificial neural network (ANN) builds models by using a simple computer emulation of biological neural systems. ANN attempts to learn patterns from data by sifting the data repeatedly, searching for relationships, automatically building models, and correcting the model’s own mistakes (Dhar and Stein, 1997). Even with incomplete and noisy data ANN can produce functioning prediction models (Li, 1994). ANN can be used to achieve high degrees of prediction accuracy, which makes them a suitable tool for forecasting. ANN has been used in forecasting, e.g., in finance and economics (Li, 1994), (Wong and Selvi, 1998), (Teräsvirta, van Dijk, and Medeiros, 2004) and in medicine (Nastac et al., 2004).

There are weaknesses in the use of ANN for prediction, these include, e.g., the fact that there are no structured methodologies available for choosing, developing, training and verifying an ANN (Li, 1994), but when an ANN is developed for a purpose it must be tailor made. To function well ANN needs comprehensive training data, if such data is not available it may be impossible to use ANN techniques. Even with its weaknesses ANN technology is feasible for business applications that require the solution of very complex system of equations, finding patterns from imperfect inputs, and adapting decisions to changing environment (Li, 1994).

Many different methods for forecasting are available, some of them are more suitable for forecasting FCF for VLIRI than others. It is not the intention of this research to advocate any methods of a particular type for input variable value selection, however, when forecasting FCF for VLIRI one will have to take into consideration the VLIRI characteristics and the restrictions they place on the method selection. (Armstrong and Crohman, 1972) states that “it is quite likely that combination of forecasts from different methods could lead to (still further) improvements [sic]" in forecast accuracy, a plausible statement, taking into consideration the
difficulties that forecasting FCF for VLIRI may face. Surveys of corporate forecasting practices show that most important forecasts involve judgement (Bunn and Wright, 1991). Indeed, based on (Soergel, 1983) and (Jenks, 1983), (Bunn and Wright, 1991) further states that "only judgement can anticipate one-time events such as extraordinary competitive developments." and that outputs from large econometric models are routinely subject to judgmental adjustments. It seems that combining different methods for better forecasting results is a rather commonplace practice in business.

In the next section we will discuss supplementary capital budgeting tools that are used to complement the investment decision support given by (the commonly used) profitability analysis methods, and look at evolution of investment decision support technology after the appearance of computers.

2.2.2. Decision Support Technology and Supplementary Capital Budgeting Tools

In addition to the different profitability analysis, or capital budgeting, methods there are additional techniques designed to support investment decision making and to enhance the analysis from profitability analysis methods, (Ryan and Ryan, 2002) calls them supplementary capital budgeting tools. The reason for the existence of the supplementary capital budgeting tools is that the discounted cash flow based profitability analysis methods do not integrate sufficient analysis of a number of important issues that may be relevant to the investment decision. For example, the original neo-classical theory of investment does not consider irreversibility of investments or the uncertainty of cash flow estimates. Because irreversibility is not taken into consideration, it automatically means that neo-classical models do not consider the value of waiting, or other forms of managerial flexibility. Fuzzy versions of the discounted cash flow based methods take into consideration the uncertainty of future investment cash flow estimates (see 4.4.). However, they too neglect the value of waiting.

The failure to take into consideration the value of the possibility to postpone irreversible investments "undermines the theoretical foundation of standard neo-classical investment models, and invalidates the net present value rule as it is usually taught in business school..." (Pindyck, 1991). "In industries with significant irreversible investments and uncertainty, these omissions mean that standard profitability measures will tend to give inappropriate indicators for investment and entry decisions..." (Barham, Chavas, and Klemme, 1994). This would indicate that NPV,
IRR, and PB used alone, are not suitable for profitability analysis of complex investments, e.g., VLIRI. Shortcomings of the most commonly used profitability analysis methods demand further analysis, which is made with supplementary capital budgeting tools. These tools include, among others, techniques to further analyse the risks and what-ifs, additional tools to assess the value of the project, and tools to graphically model the decision possibilities within a project.

Tools for further analysis of the risks and what-ifs are, e.g., sensitivity analysis and scenario analysis. Sensitivity analysis is used to analyse the effect of a change in one input variable at a time, e.g., cash flows, to the result from the profitability analysis method used, e.g., see (Learner, 1985) and (Borgonovo and Peccati, 2004). Usually a profitability analysis is run with expected (most likely) variable values, then variables perceived to be the most important for the overall result are tested with sensitivity analysis to see what are the, from the point of view of overall profitability, critical values for each variable (Wallace, 1998). Finding such critical values can also be called break-even analysis (Brealey and Myers, 2003), if the critical overall value used is NPV equal to zero. Break-even analysis is also commonly used, e.g., in risk management (Baker, Ponniah, and Smith, 1998).

Scenario analysis is based on a similar idea as sensitivity analysis, but the difference is that it allows for changes in more than one variable simultaneously. The analysis is commonly done on a number of likely scenarios, and shows how the profitability of the project would be affected, if they were to be realised (Brealey and Myers, 2003). Scenario analysis is also commonly used in risk management (Baker, Ponniah, and Smith, 1998).

Additional tools to assess the economic effect and the value of a project are, e.g., economic value added (EVA) and real option valuation (ROV).

Economic value added measures managerial effectiveness in a given year or period (net operating profit after taxes - after tax cost of capital required to support operations) (Ryan and Ryan, 2002). The insight that EVA gives, is the binding of managerial performance (and compensation) to the economic value that is added to the company by the investment decisions (made by managers). The managerial compensation can be designed in a way that optimises the selection of investments (Rogerson, 1997).
Real option valuation is the valuation of investments as options, or possibilities within investments (also called managerial flexibility) as options. ROV is discussed in more detail in section 2.3.

Decision tree is an example of a method for graphically and numerically modelling the sequential and contingent project decisions and their outcomes. Decision trees can be used to illustrate investment decisions or to analyse the optimal chain of investment decisions (stochastic decision trees). (Hespos and Strassman, 1965) presents the use of stochastic decision trees for analysis of investment decisions. Decision trees are often also used to model and to support real option valuation (Brealey and Myers, 2003).

According to (Ryan and Ryan, 2002) sensitivity analysis and scenario analysis are the most often used supplementary capital budgeting tools. (Verbeeten, 2001) finds that sensitivity analysis and scenario analysis are, according to managers, the two most important uncertainty analysis techniques in the capital budgeting process. His findings corroborate earlier findings from (Pike, 1996). When large investments are in question, it is likely that a selection of supplementary tools is used, because large investments justify the use of more resources for planning. This is definitely the case for the VLIRI, where uncertainties and what-ifs can play a major role in the overall analysis of an investment and real options may have a considerable value.

Evolution of decision support technology for capital budgeting

The available technology to support capital budgeting has evolved significantly during the last 40 years. The evolution of capital budgeting decision support technology has taken place hand in hand with the evolution of decision support systems (DSS) in general, which in turn have developed in pace with the development of computers. DSS are computer technology solutions that can be used to support complex decision making and problem solving. DSS are based on theoretical research on decision making by, e.g., Herbert Simon and technical research by, e.g., Gerrity and Ness in the late 1950's and early 1960's (Shim et al., 2002).

Since the 60's the investment decision support first changed from using mainframe computers for numerical optimisation with custom made code for each problem, and a separate expert to run the computer, to proprietary numerical analysis and simulation software run on mini computers. From there the evolution has been to commercial software, specialised in
investment profitability analysis and supplementary capital budgeting tools, including simulation, and graphical user interfaces run on work stations, to personal computers with multi purpose spread sheet software, with built-in profitability analysis functions and add-on packs and extensive graphics capabilities. Today, investment profitability analysis can be made, and supplementary capital budgeting tools used on laptop computers, virtually anywhere and any time by managers themselves. The ease of making profitability analysis has reached a level where making extensive analyses with common methods is available to everyone.

Advances in computing (computing power) do not only give better support to the use of existing methods, but have caused a number of new techniques to support investment profitability analysis to evolve, one of them is simulation.

Simulation is a technique that has developed with computers and software to become an important part in investment decision support. According to (Brealey and Myers, 2003) the use of simulation in capital budgeting was first advocated by David Hertz (Hertz, 1968) and the McKinsey and Company management consultants, however, (Hespos and Strassman, 1965) already mentions the use of simulation in capital budgeting. Be it as it may, simulation has been around in capital budgeting from the 1960's. A recent report from (Ryan and Ryan, 2002) shows that simulation is often used in about 20% of Fortune 1000 companies to assist in investment decisions.

Simulation is commonly used to open up the uncertainty surrounding investments (Salazar and Sen, 1968) by running it on capital budgeting models directly, or by using it to enhance the use of other supplementary capital budgeting tools, e.g., sensitivity analysis (Kleijnen, 2004) and decision trees (Hespos and Strassman, 1965). There are also supplementary capital budgeting analyses, fully based on simulation, e.g., Monte Carlo analysis (simulation), which enables the inspection of an entire distribution of simulated project outcomes. (Vose, 1996) is a guide to Monte Carlo simulation modelling.

In cases where there are more than one possible investment alternative, the selection of the best alternative is important, simulations can be used to support such selection by simulating different scenarios through existing valuation models, e.g., (Tugcu, 1983).
Data bases and data warehouses have increased the reliability and analysability of data, which together with analysis software have made, e.g., cash flow forecasting easier. Special forecasting software has brought both, extrapolative and econometric forecasting methods within the reach of managers, the use of specialised statisticians or econometricians is no longer absolutely necessary. (Fildes, 1988) presents a review of two forecasting software packages. Artificial Neural Networks have already been discussed above. ANN technology is a direct product of the advancement in computing and data analysis, the method cannot be used without a computer.

Forecasting accuracy can benefit from the increased availability of Information caused, e.g., by the "Internet revolution" and the World Wide Web, if it can be scanned in a meaningful way. There are intelligent software tools that can scan the internet according to the wishes of the manager, and collect and sort information into an easier-to-use format, saving the manager the trouble of scanning the business environment personally. These software tools are called intelligent scanning agents, e.g., see (Liu, 2000). The information gathered by intelligent scanning agents can be used by managers in adjusting cash flow estimates and as background information for investment decisions.

Such a chain of data collection, refining, and analysis is turning information into knowledge into profit (Liautaud and Hammond, 2000), and is sometimes called enterprise business intelligence. More information does not necessarily mean more knowledge, due to the fact that the amount of information available is huge, this situation is often called information overload. The ability of a company to sift out the relevant information from the huge information supply and turn the relevant information into knowledge, e.g., forecasts, may in the future become a substantial factor in creating a competitive advantage. One possible way of utilising such competitive advantage is making more informed and hence better investment decisions.

In the future, capital budgeting decision support systems will most probably be able to scan for, collect, and refine information automatically according to user preferences. To automatically apply the information to forecasting, profitability analysis methods, and supplementary capital budgeting tools and to present the results of the analysis in ways that are easy to understand, and that give high quality support to investment decision making. Perhaps the same systems will work for the whole economic life of the investment, gathering information, supporting
operational decision making, and automatically reporting on the profitability of the investment, replacing the need for profitability post-auditing.

Chapter 4.4. presents a novel, dynamic, approach to investment decision and management decision support, based on the present and partly on perceived future DSS technology. In the next chapter we will look at how we can, with option valuation models, value the possibility to postpone an investment and other types of managerial flexibility.

2.3. Valuing Managerial Flexibility with Option Valuation: Real Options

This section will first discuss the concept of managerial flexibility, and then shortly presents option valuation. After having shortly presented these two, the idea behind real option valuation is presented.

Managerial flexibility

Different possibilities that managers have to optimise the value of an investment are called managerial flexibility. Managerial flexibility, in a project, may exist, both, before and after investment. Examples of managerial flexibility are, e.g., possibilities to postpone an investment (before), possibilities to stage an investment (before), possibilities to expand and contract an investment (after), possibilities to shut down and restart an investment (after), possibilities to change input and output (after), and the possibility to abandon an investment (after) (Alcaraz and Heikkilä, 2002). These possibilities are valuable under conditions of incomplete reversibility (irreversibility) and uncertainty, e.g., if the outcome of an irreversible investment is uncertain, then the possibility to postpone the investment is valuable. (Dixit and Pindyck, 1994).

<table>
<thead>
<tr>
<th>Certainty</th>
<th>Uncertainty</th>
<th>Reversibility</th>
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<tbody>
<tr>
<td>No value</td>
<td>No value</td>
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<tr>
<td>No value</td>
<td>Value</td>
<td>Irreversible</td>
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Table 2.2. Value of the possibility to postpone an investment under different combinations of reversibility and certainty.

The higher the uncertainty about the future and the higher the level the irreversibility of the investment, the more the managerial flexibility is
worth (Copeland and Keenan, 1998). Value of managerial flexibility is further affected by how much room there is for managerial flexibility. Managerial flexibility, as defined above, are the possibilities that managers have at their disposal to affect (and optimise) the value of an investment. Financial options give their holder the right, but not the obligation to buy, or sell, an underlying security, i.e., they give their owner a possibility, which resembles managerial flexibility.

Modern option valuation

Modern option valuation experienced a breakthrough, in 1973 when Black and Scholes (Black and Scholes, 1973) presented their analytical option valuation model for European (financial) options, which was enhanced by Merton (Merton, 1973). After the Black and Scholes article a number of techniques for valuing European and American options have emerged. These techniques include lattice techniques (binomial and multinomial trees), e.g., (Cox and Ross, 1976), (Cox, Ross, and Rubinstein, 1979) and (Boyle, 1988), finite-difference methods, e.g., (Brennan and Schwartz, 1977), (Brennan and Schwartz, 1978), and (Schäider and Kandel, 1977), and quadrature methods, e.g., (Andricopoulos et al., 2003). The two most commonly used methods, to the best of our knowledge, are the Black and Scholes model and binomial option pricing.

The Black and Scholes model is based on a replication argument: the value of a call option is equal to the value of a combination of other instruments giving the same expected cash flows. The model uses a combination of lending and of buying the underlying stock in the future. There are three main sets of assumptions underlying the model, i) about interest rates, ii) about the volatility of the return from the underlying, and iii) about the markets.

i) interest rate is assumed to be the risk free rate of return (due to no arbitrage) and it is assumed to remain constant
ii) volatility of the return from the underlying is assumed to remain constant and is assumed to be deterministic
iii) the markets are assumed to be complete and efficient (no arbitrage), where assets are continuously traded, where assets can be split, where there are no taxes or transaction costs, and where asset prices follow geometric Brownian motion (GBM).

Under these assumptions the result from the Black and Scholes model is most accurate.
**Definition.** The Black and Scholes option valuation formula, as enhanced by Merton, calculates the call option value (V) as,

\[ V = S_0 e^{-rT} N(d_1) - X e^{-rT} N(d_2) \]  \[ \text{[4]} \]

where,

\[ d_1 = \frac{\ln \left( \frac{S_0}{X} \right) + \left( r - \delta + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \]  \[ \text{[5]} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]  \[ \text{[6]} \]

and

- \( S_0 \) = Stock price
- \( X \) = Exercise price
- \( T \) = Time to expiry
- \( \sigma \) = Volatility of the underlying stock
- \( d \) = Dividend payments
- \( r \) = Risk-free rate of return

Similar assumptions underlie both, the Black-Scholes model and the binomial model. The main difference is that the binomial model uses a discrete-time framework to trace the evolution of the underlying (markets) via a binomial lattice to approximate the continuous process used in the Black-Scholes model (geometric Brownian motion). In fact, for European options, result from the binomial option valuation converges to the result from the Black and Scholes option pricing formula (Benninga and Wiener 1997), i.e., the Black-Scholes model is a continuous time version of the binomial option valuation model. It has been the choice of the author to select the Black-Scholes option pricing formula to be the option valuation model to be used in this thesis. The selection is based on the fact that, to the best of our knowledge, the Black-Scholes formula is the most commonly used option pricing model and, because the Black-Scholes formula is the continuous time version of the binomial option valuation model (implicitly includes the binomial model), which is the other relevant model choice.
Real option valuation

Real option valuation (ROV) is based on the observation that the possibilities financial options give their holder resemble the possibilities found in real investments, i.e., managerial flexibility, e.g., "an irreversible investment opportunity is much like a financial call option" (Pindyck, 1991). Real option valuation is treating managerial flexibility as options and valuing managerial flexibility with option valuation models; the term, real options, was introduced in (Myers, 1977). Above, in 2.2.2., we have observed that real option valuation is a supplementary capital budgeting tool.

Using option valuation models designed originally for financial options in valuing managerial flexibility means that the model variables need to be adjusted for real investments. Figure 2-2. shows the analogy between the variables used for valuing financial and real options.

![The six levers of financial and real options](image)

**Figure 2-2.** Variables of financial and real options, figure from (Leslie and Michaels, 1997)

Assumptions that underlie option valuation models, underlie the same models also when they are applied to real option valuation. In fact, valuing real investments with option valuation models like, e.g., with the Black and Scholes option pricing formula and binomial option pricing is accepting the fact that the assumptions underlying the models may not be fully compatible with real investments. It is difficult to say to what degree the assumptions actually hold for real investments, but indeed, "...the assumptions of the Black-Scholes model seem somewhat restrictive when applied to real options" (Copeland and Keenan, 1998). If the underlying assumptions are not realistic, then "... most real options problems require
analysis that is capable of relaxing one or more of the standard Black-Scholes assumptions" (Copeland and Antikarov, 2001). Real option valuation has been criticised for the, sometimes unrealistic, assumptions and application of the models. (Borison, 2003) presents a critical discussion of the applicability of different approaches for applying real options, including an analysis of the validity of the assumptions of option valuation models.

The three main sets of assumptions (presented above) behind the Black-Scholes model can all be criticised for not holding for very large industrial real investments (VLIRI). The strongest criticisms can, perhaps, be made about the assumptions about the markets for the underlying. The Black-Scholes model assumes the markets to be complete and efficient; this does not hold for the, sometimes unique, very large industrial real investments. However, for real options within a very large industrial real investments the case is not equally clear. If the markets for VLIRI are non-existent or incomplete, the replication argument, fundamental to the model, can indeed be questioned.

Volatility of the return from the underlying is assumed, by the Black-Scholes model to be deterministic and to remain constant. It is ex-ante difficult to assess if the volatility of the return from a VLIRI will remain constant. The traditional way of deriving the standard deviation that is used with the Black-Scholes model uses historical data for derivation of the standard deviation, i.e., the derivation of the standard deviation from historical data is a backward-looking exercise. For VLIRI historical data may not be available. It is important to observe that when historical data is not available, the estimation of the standard deviation must be based on the expectations about the future, making it a forward-looking exercise. We have discussed in 2.1. that VLIRI are most often made under structural uncertainty, indicating that estimating the standard deviation, ex-ante, for VLIRI may not be deterministic. Indeed, it may not be imprudent to state that the ex-ante estimation of the volatility for a VLIRI could better be described as a contingent system, rather than a deterministic one. Yet, it is impossible to, ex-ante, definitively say if the standard deviation will remain constant or not.

The Black-Scholes model assumes that the interest rate will remain constant and is the risk free rate of interest. Very large industrial real investments have long economic lives, which means that it is quite probable that interest rates, in general, will change during their economic lives. However, as there is no single international benchmark for the risk
free rate of return (in practice the interest rate paid by short-term securities by large governments, e.g., the U.S. government, is considered to be a close substitute for the risk free rate of interest) it is difficult to conclude if the assumption about a constant interest rate holds. However, it can be argued that the interest rate, used in, e.g., financing of a VLIRI, is different from the risk free rate (or a close substitute), at least when the investing party is not a large government issuing short-term securities (at the close substitute interest rate).

This research does not make any definitive judgement on the criticism that can be made on the usability of the Black-Scholes formula for real option valuation. However, it is noted that there are a number of unresolved issues that need further study, before any definitive verdict on the usability of the model can be made.

Despite the criticism, real option valuation is a welcome addition to the method arsenal of profitability analysis, because discounted cash flow based valuation methods fail to take the value of managerial flexibility into consideration, jeopardising the correctness of the results obtained by using them (Barham, Chavas, and Klemme, 1994). Even if real option valuation may not be able to exactly, and exactly correctly, value managerial flexibility, it, nevertheless, is an established and systematic methodology to capture the value of managerial flexibility and to include it in the valuation of investments.

ROV alone is not used to value investments, because option value is always positive, or zero, and the profitability (value) of an investment can also be negative. By modifying the NPV rule from: "Invest when the value of a unit of capital is at least as large as its purchase and installation cost", to: "The value of the unit must exceed the purchase and installation cost, by an amount equal to the value of keeping the investment option alive" (Pindyck, 1991) we effectively include the option value (of waiting) in the valuation process. Managers view flexibility as an important factor in decision making, e.g., (Busby and Pitts, 1997) and (Collan and Långström, 2002), the above means that managerial flexibility of the investment is taken into consideration by including the real option value in the valuation.

Real option valuation is especially important, when the value of managerial flexibility is high, e.g., for investments that are highly irreversible, and made under considerable uncertainty, "the difference between ROV and other decision tools is substantial" (Copeland and Keenan, 1998).
There is an established, and growing, literature on valuation, application, characteristics, and methodologies of real options. For overviews of real options literature see, (Trigeorgis, 1995) for a thematically organised literature review, (Guimaraes Dias, 1999) for a note on the bibliographical evolution of real options, (Collan, Carlsson, and Majlender, 2003) for a short review on real options literature, and the World Wide Website of Marco Guimaraes Dias (Guimaraes Dias, 2004) for around 2000 references on real options. An annual international conference on real options has been held since 1997.

The roots of the real options literature can be said to reach back to the end of 1960’s, e.g., (Robicheck and Van Horne, 1967) and (Dyl and Long, 1969) discuss the value of the option to abandon. Above we have seen that (Myers, 1977) introduced the term real options. Real option literature can be divided roughly into two categories, general theory and application. Some topics on the general theory side have been (with selected references), e.g., entry and exit decisions (McDonald and Siegel, 1986), (Majd and Pindyck, 1987), (Dixit, 1989), (Berger, Ofek, and Swary, 1996), (Alvarez, 1999) and (Pennings and Lint, 2000), growth options (Kogut, 1991) and (Garner, Nam, and Ottoo, 2002), and the valuation of interrelated projects (Trigeorgis, 1993) and (Childs, Ott, and Triantis, 1998). Real option valuation has been applied notably to some specific types of industries and situations (with selected references), e.g., to petroleum (Ekern, 1985), (Paddock, Siegel, and Smith, 1988), and (Smit, 1996), mining (Cortazar and Casassus, 1998) and (Moel and Tuffano, 1999), natural resources in general (Brennan and Schwartz, 1985) and (Cortazar, Schwartz, and Salinas, 1998), R&D (Newton and Pearson, 1994), (Grenadier and Weiss, 1995), (Smith and Nau, 1995), (Faulkner, 1996), (Linth and Pennings, 1998) and (Childs, Ott, and Triantis, 2000), information technology (Benaroch and Kauffman, 1999), (Balasubramanian, Kulatilaka, and Storck, 2000), and (Campbell, 2002), and corporate strategy (Kulatilaka and Marks, 1988), (Bowman and Hurry, 1993) and (Das and Elango, 1995).

Very large industrial real investments (VLIRI) benefit from the analysis of value of managerial flexibility provided by real option valuation, because the managerial flexibility found in VLIRI, before and after investment, may be highly valuable due to the irreversibility of VLIRI, and due to their long economic lives. Indeed, many of the applied topics to which real option valuation has been used include VLIRI, e.g., petroleum extraction and mining.
2.4. Modelling Imprecise (Financial) Information with Fuzzy Logic

Finance concentrates on how economic actors allocate resources over time, i.e., defining investing in a general sense. (Tarrazo and Gutierrez, 2000) state that "the subject of finance is entirely forward-looking..." and that "every action in finance begins with an expectation, which can be of a qualitative nature, such as "the economy will be strong next quarter", or a quantitative one." This means that investment decisions are based on expectations about the future. According to Knight (Knight, 1921) uncertainty can stem from basically two sources. Firstly, all the states of the world may be known, but it is impossible to assign probabilities to these states. Secondly, neither the states of the world, nor the corresponding probabilities are (all) known. This means that most business decisions may be classified as decisions under uncertainty and that decisions based on expectations about the future are always to some degree uncertain, because "the future state of the environment is not known-to the decision maker at the time of the decision. Based on his past experience he can only estimate the likelihood of each of the states of the environment occurring." (Verbeeten, 2001).

Uncertainty also increases with time, because with time "the complexity of the system increases, (and) our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" (Zadeh, 1973). The above is called Zadeh's principle of incompatibility, and fits the problem a financial manager faces when she has to estimate, e.g., investment cash flows based on uncertain expectations about the future. Uncertainty about the accuracy of cash flow estimates increases the further in the future the cash flows take place, because the uncertainty of future investment cash flows comes from a large unspecified number of sources that grows with time; the complexity of the "system" increases.

According to Zadeh's principle of incompatibility uncertain future expectations cannot yield precise (certain) estimates of investment cash flows, and the further in the future the cash flows take place, the less precise the estimates can be. Turning this around means that using precise estimates of investment cash flows cannot give a correct picture of the uncertainty of the cash flow estimates, and the further away the cash flow takes place, the less correct the picture becomes. Precise estimates about an uncertain future can cause a credibility problem, which can also be called a
false sense of accuracy (or certainty). This credibility problem is especially relevant for investments with long economic lives.

If precise estimates cannot give a correct view of uncertainty, then the decision maker faces a dilemma; she must use precise estimates in her analysis and tolerate, from the point of view of uncertainty, incorrect answers, or use imprecise information and get correct, imprecise, answers.

The imprecision that investment decision makers must cope with can be characterised as follows, "...we could amass all the statistics concerning economic performance and put all the economics experts in the world in one room and we would still face many unknowns. This void of knowledge is what fuzzy logic researchers refer to as fuzziness" (Tarrazo and Gutierrez, 2000). Fuzziness is the imprecision that decision makers must cope with when making decisions under uncertainty. Fuzziness can be handled with fuzzy sets, presented in 2.4.1., and related fuzzy mathematics and theory.

2.4.1. Foundations of Fuzzy Sets

In classical set theory an element either belongs to a set or does not belong to a set, e.g., a colour is black or it is white, or "a future cash flow at year ten is x euros" or it is not. This type of bi-value, or true/false, logic is commonly used in financial applications. Bi-value logic, however, presents a problem, because as already observed above, financial decisions are most often made under uncertainty. Uncertainty means that it is impossible to give absolutely correct precise estimates of, e.g., a future cash flow. Fuzzy sets are sets that allow (have) gradation of belonging, such as all tones between black and white, or "a future cash flow at year ten is about x euros". This means that fuzzy sets can be used to formalise inaccuracy that exists in human decision making and as a representation of vague, uncertain or imprecise knowledge, which human reasoning is especially adaptive to. "Fuzzy set-based methodologies blur the traditional line between qualitative and quantitative analysis, since the modelling may reflect more the type of information that is available rather than researchers' preferences" (Tarrazo, 1997) and indeed in economics "The use of fuzzy subsets theory leads to results that could not be obtained by classical methods." (Ponsard, 1988).

The origins of fuzzy sets date back to an article by Lotfi Zadeh (Zadeh, 1965) where he developed an algebra for what he called fuzzy sets. This algebra was created to handle imprecise elements in our decision making processes, and is the formal body of theory that allows the treatment of
practically all decisions in an uncertain environment. "Informally, a fuzzy set is a class of object in which there is no sharp boundary between those objects that belong to the class and those that do not" (Bellman and Zadeh, 1970).

**Definition.** Let \( X = \{x\} \) denote a collection of objects (points) denoted generically by \( x \). Then a fuzzy set \( A \) in \( X \) is a set of ordered pairs

\[
A = \{(x, \mu_A(x))\}, \quad x \in X
\]  

where \( \mu_A(x) \) is termed the grade of membership of \( x \) in \( A \), and \( \mu_A : X \rightarrow M \) is a function from \( X \) to a space \( M \) called the membership space. When \( M \) contains only two points, 0 and 1, \( A \) is nonfuzzy and its membership function becomes identical with the characteristic function of a nonfuzzy set. This means that crisp sets are a subset of fuzzy sets.

A fuzzy number is a normal, convex fuzzy set whose referential set is the real numbers \( X \in \mathbb{R} \). Fuzzy set theory uses fuzzy numbers to quantify subjective fuzzy observations or estimates. Such subjective observations or estimates can be, e.g., estimates of future cash flows from an investment. To estimate future cash flows and discount rates "One usually employs educated guesses, based on expected values or other statistical techniques" (Buckley, 1987), which is consistent with the use of fuzzy numbers.

In practical applications the most used fuzzy numbers are trapezoidal and triangular fuzzy numbers. They are used, because they make many operations possible and are intuitively understandable and interpretable.

**Definition.** A fuzzy set \( A \in F \) is called a trapezoidal fuzzy number with core \([a,b]\), left width \( \alpha \) and right width \( \beta \) if its membership function has the following form

\[
A(t) = \begin{cases} 
1 - \frac{a-t}{\alpha} & \text{if } a - \alpha \leq t < a \\
1 & \text{if } a \leq t \leq b \\
1 - \frac{t-b}{\beta} & \text{if } b < t \leq b + \beta \\
0 & \text{if } t \in [a-\alpha, b + \beta]
\end{cases}
\]  

[8]
and we use the notation $A=(a,b,\alpha,\beta)$. It can easily be shown that the support of $A$ is $(a-\alpha, b+\beta)$. It can be noted that triangular fuzzy numbers are a special case of trapezoidal fuzzy numbers.

![Trapezoidal fuzzy number, figure from (Fuller, 1998).](image)

Initially the research interest on fuzzy sets concentrated on its logical and mathematical foundations and information theory aspects. Later on many application areas of fuzzy logic have gained interest among researchers, these include, e.g., control (robotics, automation, tracking, consumer electronics), information systems (DBMS, information retrieval), and pattern recognition (image processing, machine vision). Established text books on fuzzy sets and their applications are, e.g., (Dubois and Prade, 1980), (Kaufmann and Gupta, 1985), (Kaufmann and Gupta, 1986), (Klir and Folger, 1988), (Zimmerman, 1992), and (Carlsson and Fuller, 2001b). There are also a number of journals that cover the different aspects of fuzzy logic, e.g., International Journal of Fuzzy Sets and Systems, International Journal of Approximate Reasoning, and IEEE Transactions on Fuzzy Systems, these form the body of the literature on fuzzy logic.

"Given the amount of uncertainty and imprecision dealt with in economics and business management, one would have expected the first applications of fuzzy sets to be in those areas" (Tarrazo and Gutierrez, 2000). Even if this has not been the case, there is an increasing interest in using fuzzy logic in financial applications and a growing literature on applying fuzzy sets and fuzzy logic in economics and finance. Some early works include, e.g., (Ponsard, 1985) and (Ponsard, 1988), where he discusses the use of fuzzy sets in economic models and presents fuzzy mathematical models that deal with economic choice, these concentrate mainly on economic equilibrium models. (Buckley, 1987) concentrates on presenting applications of fuzzy sets to financial mathematics, he presents models for fuzzy future and present values, discusses fuzzy cash flows, and presents fuzzy net present value (NPV) and internal rate of return (IRR) models. (Li Calzi, 1990) proposes and discusses some general rules for how fuzzy
financial models should be constructed, he, however, has some questionable remarks on using DCF based capital budgeting methods. (Buckley, 1992) discusses solving fuzzy equations in economics and finance, he substitutes crisp parameters with uncertainty with triangular fuzzy numbers. There are a number of more recent works that discuss fuzzy versions of common capital budgeting methods, and relevant to this research, that have been presented in chapter 2.2. of this thesis, together with presentations of the capital budgeting methods themselves. Indeed, fuzzy mathematics is compatible with the discounted cash flow based methods of profitability analysis.

2.5. Summary of the Background for this Research

According to the above, the state of the art of investment profitability analysis in companies seems to be:

i) use of a combination of neo-classical capital budgeting methods (NPV, IRR, PB), with

ii) a combination of supplementary capital budgeting tools:
common use of sensitivity analysis and scenario analysis for uncertainty, not so often the use of real option valuation for managerial flexibility

iii) based on (a combination of forecasting methods) judgement or judgementally adjusted input variable value forecasts, with crisp numbers, sometimes supported by foresight

iv) commonly supported by a spread sheet based computerised DSS

In academia, it seems, the research on supporting profitability analysis has concentrated on forecasting and a number of supplementary capital budgeting tools, including real options, which have an established literature. Interestingly, the attempts to create novel profitability analysis methods to replace the old discounted cash flow based methods have been very few.

Use of fuzzy logic in capital budgeting and profitability analysis is a new idea, and has been around from the late 1980's, fuzzy versions of the most commonly used profitability analysis methods are available, even if they are not widely used in companies. Fuzzy logic is commonly used in a number of fields in engineering and intelligent computing.
There seem to be relatively few works on very large industrial real investments (VLIRI) available, yet it is clear that they are different from small real investments and from financial investments. Discounted cash flow based (neo-classical) profitability analysis methods cannot provide sufficient analysis for VLIRI due to the fact that VLIRI have value and profitability affecting characteristics that are not taken into consideration by them. Using extrapolative, or econometric methods to provide cash flow forecasts for VLIRI may suffer from the fact that there may not exist data on which these forecasting methods can be credibly built on and used.

Use of the most commonly used supplementary capital budgeting tools, sensitivity and scenario analysis, does help, however, still it seems that for VLIRI judgement and judgmental adjustment of forecasts is needed. Using fuzzy sets for inclusion of estimation inaccuracy seems to fit VLIRI, because of their long economic lives and the complex and uncertain environments in which they need to operate. Real option valuation, i.e., valuing managerial flexibility is important for VLIRI, due to the fact that flexibility within VLIRI and, e.g., the possibility to wait to invest may have a considerable value for VLIRI.
3. Methodology

This chapter will discuss the philosophical position of the research, describes the research methods used, and ties the research papers presented in the second part of this thesis with the methodologies used.

3.1. Philosophical Position of the Research

This research is based on previous action research by the author, which has been the basis on which a pre-understanding of reality has been formed. The pre-understanding of reality has helped the author to construct the research to be context-free. Valuation of investments is relativist, due to the fact that inputs into valuation are subject to subjective selection, which makes the result of the valuation subjective. This research aims to create new knowledge about very large industrial real investments and to identify suitable techniques and methods for their valuation. Very large industrial real investments and their characteristics are verifiable and observable, the analysis about the valuation needs based on the characteristics can be validated, and as a result the aim of the research can be said to bring valuation from a relativist closer to a context-free epistemology.

The ontology of this research is natural. It is an important issue in understanding this research: the author accepts that valuation based on estimates about the future, as an issue, has an existential view of reality, because of the subjectivity of inputs to valuation models (inputs are as they are perceived by the individual). Yet this research has a natural view of reality, for the effort is to try to create methods to make the valuation issues as they are perceived by individuals to be understood by others as well. The logic of this research is partly reductionist and partly interpretive. The construction of the models is reductionist, as it builds from existing knowledge and the parts of the models can each be explained by looking at the state of the art of valuation models, together the parts explain the whole model. The research as a whole is interpretive, for it is constrained by the fact that the research is done for, and about the large industrial investments, and as such is not directly applicable to investments in general.

Validity of the work is originally based on an inductive approach, the observations made from a number of cases about (common factors for) large industrial investments have lead to the need to create a (more narrow) definition for a special group of VLIRI. The definition is based on
commonly acceptable and observable (validatable) characteristics, which can be empirically proved or disproved (falsified) by investigating more cases. Based on the inductive selection of the definition, research has been conducted on what the definition means in terms of modelling valuation methods. In this way the research is developing theory and as the construct (artefact) of the theory development, new models are created.

Access to detailed information about large industrial investments has been reached by working together with Finnish companies involved in paper, metal, energy, and base metal industries, within a framework of a four-year research program. The companies involved in the research program have initiated a number of large industrial investments and have industry expertise, the people involved in the decision making and planning have been involved in the project, bringing access to their expertise on the researched issues. The access to first hand information, and experience, has brought the necessary pre-understanding for building a further understanding on the issues discussed in this research, surrounding giga-investment valuation, and in enhancing and creating valuation models. There is an existing literature on the general theory of investment. However, there is no specific theory for large industrial investments. Valuation models used for real investments are mostly based on models originally designed for valuation of financial instruments. Specific models for real investments are rare, and there are no well-known models for valuation of large industrial investments.

The quality of this research can be judged from looking at the internal and external validity of this research, and from examining the validity of the constructs from this research (Bryman, 1988). Internal validity can be assessed by examining the cause and effect relationship between the characteristics of large industrial investments and the conclusions drawn about the effect of these to investment valuation and valuation modelling. The external validity is the general applicability of the conclusions and the constructs (models) to VLIRI, and how well they reflect reality. The validity of the models and other constructs can be tested by assessing the extent to which they are able to measure what they are intended to measure (Eisenhardt, 1989). Such test may be done, e.g., by comparing ex-post profitability of an investment to the ex-ante valuation of the investment.

The paradigm within which this research has been conducted accepts the use of vague information as a basis for decision making. This is reflected by the fact that we do not categorically question the usability of fuzzy mathematics and possibility theory in modelling of vague information, and
in the case of giga-investments uncertainty existing in estimations of future events and it being modelled with the help of fuzzy numbers. The dominant paradigm of this research is to explicitly accept that it is impossible to, ex-ante, exactly know what will happen in the future. This is different from the reigning paradigm in the field of investment valuation, which is based on probabilistic mathematical theory drawing from stochastic processes to explicitly explain the phenomenon of forecasting the future, and as a result claims to reach explicit precise certain information about the future.

It is the position of the author that reality exists as truth, but we can apprehend it only imperfectly, a good fit to the paradigm of the research. We can reach objectivity, and value it, by achieving an overall resemblance of the findings to pre-existing knowledge.

On the whole the philosophical position of this research resembles the constructive approach presented in (Kasanen, Lukka, and Siitonen, 1993).

3.2. Methods Used

Conducting this research can be divided into two parts, gaining new knowledge about very large industrial real investments and conceptualising and modelling, based on the knowledge gained.

3.2.1. Gaining an Understanding about Very Large Industrial Real Investments

To gain an understanding of the reality of valuing large industrial investments the author was involved for three years, as a researcher, in the four-year WAENO research program on the profitability of very large industrial real investments. For more information on the WAENO research program, e.g., see (Collan and Hirkman, 2003). During that time the author was involved in hands-on work with four large industrial companies that had invested in VLIRI. The author worked on ex-post valuation of a number of historical very large industrial investment cases, using the most commonly used capital budgeting methods (NPV, IRR, and Payback) and real option valuation.

Within the WAENO program the author was also involved as a team member responsible for the ex-ante profitability analysis of a VLIRI. The work included a complete profitability analysis of the investment,
including gathering of data for the calculations, estimation of future cash flows based on expert opinions, construction of calculation worksheets for the analysis, and using the most commonly used profitability analysis methods and real option valuation. Fuzzy cash flow estimation was also used, together with, fuzzy versions of commonly used profitability analysis methods. The involvement of the author in the valuation of VLIRI can be loosely described as actions research, as described, e.g., in (Gummeson, 2000) and (Coughlan and Coghlan, 2002), however, the author did not keep a research journal, but acted as a responsible team member.

Based on the experiences from working in a team analysing the profitability of a VLIRI, on the experiences of the WAENO research program, and on four master's theses, (Bodman, 2000), (Palomäki, 2000), (Blom, 2000, and (Leivo, 2001), and interviews with experts from the respective companies, the author has written four historical case descriptions, available in appendix 1, to illustrate the reality and the nature of very large industrial investments. The cases work as an introduction to understanding what VLIRI are, and have been written in a way that illuminates, not only the information relevant for quantitative valuation procedures, but also the surrounding institutional and market environments of these historical cases. The author has presented some of the cases in international events, e.g., in the International Real Options Workshop in Turku, of which the author also edited the proceedings, (Collan, 2002). The cases have not been used to attempt generalisation, they have been used to present the problem domain.

The author made an exploratory survey about how Finnish companies deal with flexibility in capital budgeting (Collan and Långström, 2002). The purpose of the exploratory survey was to get a preliminary knowledge about the interest of Finnish companies and managers to value and model managerial flexibility, and about the methods they utilise to value their investments. A return rate of 42% was achieved and the exploratory results were in line with an earlier English study (Busby and Pitts, 1997), showing that also Finnish managers seem to perceive flexibility as valuable in investments. No generalisation, based on the exploratory results was attempted.

The survey was conducted by sending a questionnaire to 86 Finnish companies listed in the Helsinki Exchanges (HEX) main list (all the companies listed). The HEX main list was selected, as it consists of the largest companies in Finland and it was not unreasonable to expect that the companies listed would have existing procedures for investment profitability analysis, and would thus have better than average knowledge of real investments. The questionnaire consisted of four pages and included thirteen questions, based on an earlier, similar, survey conducted in England (Busby and Pitts, 1997).
The work the author did within the WAENO project as a researcher, and as a team member in an, ex-ante, investment profitability analysis, together with an exploratory survey form the basis for the knowledge about VLIRI that this research is based on.

3.2.2. Conceptualisation and Modelling

As already stated in 1.1. the motivation of this research is to create a new knowledge about very large industrial investments, and their decision support. The motivation is to create this knowledge for the scientific community, not only for the researcher making the research. To be able to convey the knowledge that the researcher has gained, in a rather novel research environment (the environment of very large industrial real investments) the researcher needs to create concepts that he can use to transmit the knowledge to others. Creating new concepts is meaningful, when usable concepts do not exist, if there are existing, usable concepts, then they should be used. This has been one of the principles used, when creating concepts for this research. Creation of concepts and a conceptual framework of new and existing usable concepts has been one of the most important methodological issues in this research, because the concepts and the conceptual frameworks created have been used as a basis for modelling the observed phenomena. It has been of special attention to this research that the concepts and conceptual frameworks give the most correct possible, and the most accurate possible picture of reality, and the problems that have been observed. In this research both, concepts connected to taxonomy of investments (papers 1, 2, and 4) and new "paradigms" for managing investments (papers 3, 4) have been created.

The scope of the modelling problems has been limited by the conceptual definitions, of the phenomena that are being modelled. E.g., the concept of giga-investments, presented in paper 1, creates the boundaries, within which, the giga-investment valuation modelling is done. The quantitative modelling done, is based on observations about reality, and based on the existing and created concepts and conceptual frameworks.

Papers 1 and 4 present an original model for valuing giga-investments. The model has been built by partly using constructs from existing valuation methods and partly by utilising original constructs, e.g., the model utilises a proposed heuristic fuzzy operator that has been created to reflect reality better.
Paper 2 presents a new real option valuation model that utilises existing, advanced constructs of fuzzy mathematics to model the observed issues from giga-investments, to better reflect reality in Black-Scholes real option valuation of giga-investments, i.e., the model is an enhancement of an existing model.

Paper 5 presents a heuristic tool to include more information to existing cash flow estimates, that is based on and supports a new "paradigm" created and discussed in papers 3 and 4, and at the same time presents a simplistic model for an observed problem.

The approach taken in this research to create quantitative models according to the created and existing usable concepts, adhering to observed reality, and solving existing observed problems, can be understood within, and is very similar to, the quantitative modelling framework presented in (Mitroff et al., 1974) and quantitative modelling methodologies discussed, e.g., in (Bertrand and Fransoo, 2002).

All of the above is in concert with the constructive approach presented in (Kasanen, Lukka, and Siitonen, 1993), which states that "the constructive approach means problem solving through the construction of models, diagrams, plans organizations, etc.". Indeed, the way this research has been conducted is analogous with the research process that is presented in (Kasanen, Lukka, and Siitonen, 1993) as being the division of a constructive research process into phases.

This research does not attempt to generalise the results to any larger population of real investments. It is, nevertheless, highly probable that the results generally apply to VLIRI. Further testing of the generality and the fit of the models is to be done in the future.
4. Key Findings and Insights

This chapter will present the key findings of this research. The chapter is divided into five sections, which all present the contributions of this research. The first section presents a conceptual construction: a definition for a special group of very large industrial real investments (VLIRI). The second section presents a fuzzy real option valuation model (FROV). The third section presents an original profitability analysis model (FRIV) specifically designed for giga-investments, utilising fuzzy sets, advanced fuzzy constructs, and capturing potential. The fourth section discusses a number of issues on giga-investment decision support and new approaches to giga-investment management. Finally, the fifth section shortly presents the original research papers.

4.1. Defining Giga-Investments - Three Common Characteristics

Very large industrial real investments (VLIRI) are a fairly homogenous subgroup of investments, within the set of real investments. Their characteristics are, however, not homogenous enough to explicitly define the group of VLIRI. Common characteristics among the VLIRI, nevertheless, exist that can be used as a basis for a narrower definition. We have earlier, in 2.1., presented characteristics of VLIRI and have found that VLIRI are most often irreversible, due to a number of reasons (e.g., large size, sunken costs, fixed location, no established markets, and fixed technology), that they most often have a long building time (months, sometimes years), and that they commonly have long economic lives (more than ten years). The nature of the markets in which VLIRI are made is uncertain and characterised by structural uncertainties that cannot be often exactly modelled. VLIRI also have other characteristics, e.g., some VLIRI can steer their markets, however, not all VLIRI exhibit them. Based on this we give a definition of giga-investments as a sub group of VLIRI.

**DEFINITION.** For the purposes of this research *giga-investments* are defined as very large industrial real investments that exhibit all the following characteristics:

i) a long economic life  
ii) a high degree of irreversibility  
iii) a long building time

The definition of giga-investments does not rule out VLIRI exhibiting other characteristics, than the giga-investment characteristics. VLIRI that
do not exhibit all giga-investment characteristics are, however, not giga-investments.

The three defining characteristics of giga-investments define a special group within the group of VLIRI, they also define a basis for valuation, and profitability analysis of giga-investments. The value of giga-investments is dependent on the three characteristics we have defined. Value of giga-investments is directly related to the giga-investment characteristics, but managers, responsible for giga-investment valuation and decision making, partly perceive the value of giga-investments indirectly, via valuation models. Investment valuation models are based on quantifying the effects that investment characteristics have on the value of the investment, thus we can expect that the more accurate the representation of the model of the effects, the more accurate the valuation results from the model.

The valuation models are built according to, and based on the experts' (model builders') understanding and perception of the reality surrounding investments. Hence, the models reflect the assumptions, modelling choices, and simplifications of reality that builders of the models have made. From the point of view of rational decision making, based on objective information, it is not unfounded to expect that models that present reality more accurately are preferred to less accurate models.

In the following we will go through the three giga-investment characteristics, discuss shortly how each characteristic affects the value and the valuation of giga-investments, and observe the possible application of existing methods and constructs to valuing giga-investments. Then a short summary of the issues arising from the characteristics is made, together with a discussion about the lifecycle of giga-investments.

4.1.1. A Long Economic Life

"Economic life" of an investment means the time that the investment generates cash flow(s), or other value that can be expressed in cash flow terms. Giga-investments are built for long term performance, and commonly have economic lives of over ten years, as discussed in 2.2., this means that they generate value for a long time.

Drawing on the neo-classical thinking presented in 2.3., "a dollar tomorrow is worth less than a dollar today". This implies that there is a time value to money, and to any other generated value that can be expressed as a cash flow. Time value of money is commonly, e.g., NPV and IRR, taken into
consideration by using discounting. Discounting is the main component of modern bond pricing. Because the economic life of giga-investments is long, it is important to take the time value of money into consideration when valuing giga-investments. The effect of (a long) time (on cash flow value) is a directly perceivable effect from a giga-investment characteristic to the value of the giga-investment.

As we have discussed in 2.1., 2.2., and analysed in 2.4., the further in the future an event takes place, the more difficult it may be to accurately estimate its effect on an investment, i.e. it becomes more difficult to accurately estimate, e.g., the size of a cash flow. A long economic life implies that (a part of) the investment cash flows take place far in the future, therefore, these cash flows are difficult to accurately estimate due to difficulties in accurately estimating future events. Accuracy of estimation is an important issue in giga-investment valuation. The lack of accuracy of estimation, per se, does not affect the value of giga-investments. However, it limits our possibilities to accurately identify what the value of the giga-investment is. The inaccuracy of future (giga-)investment variables can come from a number of separate sources, and combinations of sources. (Miller, 1992) divides overall uncertainty facing managers to uncertainty from the general environment, the industry, and from organizational factors. (Verbeeten, 2001) presents how different components of uncertainty affect capital budgeting practices. As discussed in 2.1., during a long economic life giga-investments may encounter changes, e.g., in their market environments, in their financing, and in their technology. “As the time span increases, it is more likely that large changes will occur in the environment” (Armstrong and Crohman, 1972). All unexpected changes that happen during the economic life of giga-investments work to increase the uncertainty and the difficulty in, ex-ante, accurately perceiving the value of giga-investments.

Because it is difficult to accurately estimate, or perceive, the future variable values of a giga-investment at any time, ex-ante, models that value giga-investments and show the realistic, inaccurate, perception of the giga-investment variable values, and giga-investment value, should be preferred. Difficulty of accurately estimating giga-investment value is an indirect, via human perception, effect of a giga-investment characteristic to giga-investment valuation.

Section 2.4. discusses the use of fuzzy logic in correctly modelling imprecise (inaccurate) financial information. Section 2.2. presents fuzzy versions of discounted cash flow based profitability analysis methods that
utilise fuzzy mathematics in investment valuation. We can conclude that use of fuzzy logic in presenting imprecise information within the giga-investment valuation framework is possible, and an established methodology.

As a further note, it is important to observe that the issue of long economic life has to do with the perceived long economic life of the giga-investment. The actual giga-investment may be abandoned after a shorter-than-expected time, however, the original perception of the investment, at the planning stage, is that it will have a long economic life, which has the effects on the perceived value and valuation discussed above.

4.1.2. A High Degree of Irreversibility

Large initial investments in industrial production facilities, and other industrial infrastructure, are very often, if not always, irreversible, as already observed in 2.1. Giga-investments are, by definition, very large industrial real investments that are, to a high degree, irreversible. Irreversibility, in the context of giga-investments, means that after the initial decision to (giga-)invest is made, it is impossible to reverse the decision without any loss. In practice the irreversibility of giga-investments is caused, as discussed in 2.1., by the fact that there are no markets for giga-investments (they cannot be easily sold), and because they are geographically and technologically rigid. Examples of four such investments are presented in appendix 1 of this thesis. Each one of the investments presented has sunken costs and is geographically fixed to the location into which it has been built.

As discussed in 2.3., irreversibility alone is not significant from the point of view of giga-investment value. However, when coupled with uncertainty about the future, irreversibility causes managerial flexibility to become valuable. Above, and in 2.1., we have established that uncertainty about the future exists for VLIRI, and hence, for giga-investments, and we have defined giga-investments as VLIRI with a high degree of irreversibility. This means that managerial flexibility, when planning and within giga-investments is valuable.

In 2.3. we have discussed the different types of managerial flexibility, and seen that managerial flexibility may exist before and after the investment decision. For giga-investments, due to their large initial capital investment and their, often strategic, importance, the managerial flexibility available before investment (e.g. possibility to postpone the investment) may be
more important in the planning phase of the investment. According to real options theory, before the investment decision, the whole investment can be viewed as an (one large) option. After the investment decision the managerial flexibility can be viewed as (smaller) options within the giga-investment (and the option to abandon the giga-investment). During the time the option to postpone the giga-investment is available the value of the investment may go up, or down. This means that there is both, upward and downward potential in the value of the planned giga-investment.

We have observed in 2.3. that managerial flexibility resembles options and can be modelled and valued with option valuation models, i.e., real option valuation. We can conclude that it is important to take the value of managerial flexibility into consideration when valuing giga-investments, and that real option valuation is an established methodology to do so.

4.1.3. A Long Building Time

We have seen in 2.1. that building times of VLIRI generally range from months to years, e.g., they most often have long building times. Giga-investments are defined as VLIRI that have long building times. Because giga-investments are (by definition) to a high degree irreversible, their investment cost after the investment can be viewed as fixed to a high degree. This view is supported by the fact that large industrial construction projects are most often fixed by contracts. When the investment decision is made the contracts for building the investment are drawn, and fix the commitments of both parties to deliver the contracted: the investing firm pays the investment cost, and the builders deliver the giga-investment. In appendix 1 of this thesis four investments with a long building time are presented.

Even if the building cost (investment cost) is contracted and can be viewed as fixed to a high degree, the value of the giga-investment is uncertain. This is, naturally, due to the fact that the value of the giga-investment depends on the uncertain value from the giga-investment that can be expressed in the form of cash flows (for all the economic life of the giga-investment). The increased uncertainty caused by the (long) time to build the giga-investment is due to, e.g., possible changes in the markets in which the completed giga-investment will operate. In other words, during the building time, the investment cost can be viewed as fixed, but the FCF from the giga-investment are uncertain.
The above means that the building time of a giga-investment resembles, e.g., a commodity forward contract, which fixes the price of a commodity at a future (maturity) date, but the value of the commodity fluctuates according to the market price of the commodity during the time to maturity of the forward. We can expect that as option valuation can be harnessed to model managerial flexibility, forward valuation models can be used to value the building time of giga-investments. The possibility that the value of the giga-investments goes up or down, during the building time can be also called potential: upward and downward potential.

We need to further observe that the building time should (obviously) be taken into consideration when calculating the time value of future cash flows from giga-investments.

We can conclude that the long building time of giga-investments is significant to giga-investment uncertainty and value, and should be taken into consideration when modelling and valuing giga-investments. It can be expected that as the building time of giga-investments resembles some forward contracts, existing models for valuing forwards can be utilised in modelling and valuation of giga-investments. During the building time the giga-investment value may fluctuate, there is potential for both upward and downward movements. Similar discussion applies to the inaccuracy of being able to estimate the value of the giga-investment at the completion of building, as was made about the accuracy in estimation of variable values in 4.1.1., i.e., we conclude that fuzzy mathematics are useful also, when modelling and valuing the building time of giga-investments.

4.1.4. Giga-investment Valuation Issues and Lifecycle

The three giga-investment characteristics give rise to a number of issues that directly, or indirectly, affect the value and valuation of giga-investments, see table 4-1. (and the discussion above). Time value of money is important due to the long economic life of giga-investments, and is also important when taking the long building time into consideration. Inaccuracy in being able to estimate values for variables is important to the valuation of giga-investments, and is relevant because of their long economic life and because of their long building time, the estimation inaccuracy is also relevant when valuing managerial flexibility. Being able to value managerial flexibility is important to giga-investments, because of the high degree of irreversibility, which is present simultaneously with uncertainty. Methods to take these into consideration, and suitable for giga-investments, are fuzzy sets, discounting based valuation, real option
valuation, valuation of potential, (and perhaps) forward valuation, and fuzzy versions of the above.

| A long economic life                  | time value of money important: discounting (NPV) |
|                                      | lack of accuracy in variable estimation causes inaccuracy of value estimation: fuzzy logic |
| A high degree of irreversibility     | together with uncertainty has effect on value, because causes managerial flexibility to become valuable: real option valuation |
|                                      | large sunken cost |
| A long building time                 | fixed costs, uncertain value: resembles forward contracts, potential up and down |
|                                      | effect on time value of future (uncertain) cash flows: discounting and fuzzy logic |

**Table 4-1.** Giga-investment characteristics and some effects on giga-investment value and valuation.

To understand the lifecycle (and valuation) of giga-investments better, we can divide giga-investment life into three major stages that we call planning (stage), building (stage), and operation (stage), see figure 4-1.

The planning stage is the time before the giga-investment decision, when the giga-investment, as a whole, can be viewed as an option (to invest). Both, investment cost and value are uncertain. The building stage is the period, during which the giga-investment is under construction, after the irreversible giga-investment decision. The giga-investment cost is most often contracted, and can be understood nearly as fixed (certain), or having a low standard deviation.

The value of the giga-investment is, however, uncertain. The building stage of giga-investments resembles a commodity forward contract, where the price is fixed, but market price is uncertain. Operation stage is the stage after completion of the construction of the giga-investment, until the end of its economic life. The operation stage of the giga-investment resembles a bond contract, and is commonly valued with NPV. In addition to the forecasted FCF of the investment there is additional value from the
managerial flexibility within the investment, e.g., manufacturing flexibility (Bengtsson, 2001). This managerial flexibility can be modelled with real option valuation and can be called operational options, e.g., (Nembhard, Shi, and Aktan, 2001). Estimation inaccuracy of the variables affecting the value of the giga-investment, is an issue that is present in all stages of the giga-investment lifecycle.

![Figure 4-1. Giga-investment lifecycle presented in three stages, planning, building, and operation - resembling option, forward, and bond pricing.](image)

When designing models and methods to aid in the valuation and decision making of giga-investments it is important to take into consideration the issues arising from the giga-investment characteristics, and the issues arising from the giga-investment lifecycle.

### 4.2. Enhancing Black-Scholes Real Option Valuation for Giga-Investments

We have seen in section 2.3. that real option valuation, used together with NPV, captures the value of flexibility in investments. We have seen in
section 2.4. that to be able to model imprecision in investment decision making we can use fuzzy sets, and in 2.2. that fuzzy versions of discounted cash flow based profitability analysis methods, including NPV, are available. In 4.1. we have defined giga-investments, and concluded that based on their characteristics giga-investment valuation benefits from being able to take both, variable estimate imprecision and managerial flexibility into consideration.

To be able to analyse the profitability of giga-investments in a way that takes into consideration the estimated imprecision (with fuzzy logic) and the managerial flexibility (with real options) in giga-investments, we need a fuzzy real option valuation model. Such a model can be used together with a fuzzy NPV model for profitability analysis of giga-investments. In the following we will shortly describe some existing models for fuzzy option valuation, and then present a fuzzy (hybrid) real option valuation model (FROV).

There are some, but only a few, published works on using fuzzy sets in option pricing. (Muzzioli and Torricelli, 2000a) present a model (based on the standard binomial option pricing) for pricing an option with a fuzzy payoff. They present a one-period model and use triangular fuzzy numbers. In (Muzzioli and Torricelli, 2000b) they present a fuzzy, binomial-tree based multi-period, option valuation model using triangular fuzzy numbers. The work generalises the standard binomial option-valuation model, and can most probably be utilised in real option valuation.

Fuzzy option pricing is used to value firm equity with the Black-Scholes formula in (Zmeskal, 2001), where he applies a fuzzy-stochastic method, presented in (Wang and Qiao, 1993), combining probability theory and fuzzy sets to model the uncertain valuation environment (markets). The paper concentrates on the option valuation of the equity of a firm, however, the principles assumed and used may be applicable to situations of managerial flexibility (real options). (Yoshida, 2003) presents a fuzzy model that uses the Black-Scholes formula to value financial options of the European type in a fuzzy (market) environment. The work is based on fuzzy stochastic process to model uncertainties in the financial markets. The work does not mention real option valuation.

Above we have established the need for a fuzzy real option valuation model suitable for giga-investments, and looked at some existing works that combine fuzzy sets with option pricing. Next we will see a fuzzy real option valuation model designed for giga-investments.
4.2.1. Fuzzy (Hybrid) Real Option Valuation Model (FROV)

The Black-Scholes option pricing model is, to the best of our knowledge, the best known and most widely used model for real option valuation. For these reasons we select the Black-Scholes model (Black and Scholes, 1993), as enhanced by Merton (Merton, 1973), as the starting point for our fuzzy real option valuation model (FROV). We use the construct of the Black-Scholes model, however, we have three enhancements to the construct and the application of the model:

i) We use fuzzy variables to replace the crisp variables for the present value of the expected cash flows ($S_0$), and for the present value of the expected costs ($X$), used in the original model.

ii) We use a new approach in calculating the volatility (standard deviation) used in the formula, and in the treatment of the fuzzy variables within the model.

We calculate the standard deviation used in our model from the fuzzy present value of the free cash flows from the giga-investment, using a method developed in (Carlsson, Fuller, and Majlender, 2001) for calculating possibilistic variance and standard deviation for fuzzy numbers. This is different from the calculation of standard deviation used in the original Black-Scholes formula. This method allows the market information from the experts, who have contributed to the cash flow estimates, to be included in the calculation of the standard deviation. This may enhance the usability of the model for giga-investments.

We use the possibilistic mean value of $S_0$ and $X$ (crisp), as defined in (Carlsson and Fuller, 2001a), within the model for calculation of $d_1$ and $d_2$.

iii) We suggest the estimation of (fuzzy) present values of the expected revenues and costs be done by using judgmental forecasting (expert-opinion), because, in our opinion, it suits giga-investments better, than using (only) stochastic methods.

By this we mean that judgement is used for estimation and adjustment of the fuzzy future cash flow estimates for the giga-investment. From these fuzzy future estimated cash flows the
present value is calculated, and aggregated to form the $S_0$ and $X$ used in the model.

We base our belief that using judgemental methods suits giga-investments better, than using only stochastic methods, on the fact that giga-investments have long (or very long) economic lives, and that stochastic (econometric) methods commonly fail to produce reliable results on the long term, e.g., (Shnaider and Kandel, 1989). Indeed, "... some economists, based on human reasoning and only relatively limited data and without the support of econometric models, were more accurate in predicting the timing and the intensity of the turning points of the economy. This is possibly because their reasoning was not constrained by the results generated by the econometric models" (Shnaider and Kandel, 1989). It is also our experience that the firms making giga-investments have the best experts on their planned giga-investments in their employment, and as "quite often in finance future cash amounts and interest rates are estimated. One usually employs educated guesses, based on expected values or other statistical techniques..." (Buckley, 1987), it is not unreasonable to expect that the result obtained with judgmental methods may be better, than the result obtained with stochastic methods.

Furthermore, if future cash flows for giga-investments are estimated by experts, taking into consideration all information about the future they possess, then the estimates reflect the future information and are forward looking, even if the experts would base their estimates on their past experience, or on econometric models. Stochastic methods rely only on past data.

There are also some practical considerations that speak for the approach: simplicity of the approach makes it usable in the industry, unlike, e.g., jump models that "... are more difficult to implement in the everyday industry practice (Keppo and Lu, 2003)".

Under these circumstances we define our model as (Carlsson, Fuller, and Majlender, 2001) and (Collan, Carlsson, and Majlender, 2003):

**DEFINITION.** We suggest the following formula for computing the fuzzy real option value (FROV):
\[ \text{FROV} = S_0 e^{-rT} N(d_1) - X e^{-rT} N(d_2) \]  

where,

\[ d_1 = \frac{\ln \left( \frac{E(S_0)}{E(X)} \right) + (r - \delta + \sigma^2 / 2)T}{\sigma \sqrt{T}}, \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

and \( S_0 = \) Present value of the expected free cash flows (fuzzy)
\( X = \) Present value of the expected costs (fuzzy)
\( E(S_0) = \) The possibilistic mean value of the present value of expected cash flows (crisp)
\( E(X) = \) The possibilistic mean value of expected costs (crisp)
\( \sigma = \) Possibilistic standard deviation of the present value of expected cash flows (crisp)
\( T = \) Time to expiry of the real option (crisp)
\( \delta = \) The value lost over the duration of the option (crisp)
\( r = \) The annualized continuously compounded rate on a safe asset (crisp)

The output from the model is a fuzzy number, which can be used together with the fuzzy NPV value to assist in the profitability analysis of giga-investments.

If the estimation of the expected fuzzy cash flows are done by judgmental methods, and the standard deviation used in the model is calculated from the present value of the fuzzy expected cash flows, then the standard deviation used will reflect the volatility of the cash flows, as it is seen by the experts. This makes also the calculation of standard deviation a forward-looking exercise.

It is interesting to compare the FROV model with the presented option valuation methods using fuzzy sets; for a short comparison we select the fuzzy-stochastic model presented in (Zmeskal, 2001). The two approaches differ quite a lot from each other, indeed, FROV is designed to value real options and the fuzzy-stochastic model for valuing the firm equity. Still,
both use fuzzy sets in option valuation, and some choices of modelling can be compared.

The treatment and derivation of the standard deviation used in the models is different; (Zmeskal, 2001) uses a fuzzification of the commonly used implied volatility, or historical volatility (actually it is not stated which, only that the volatility measure is fuzzy). The FROV model expects that the market uncertainty is captured by the originally fuzzy expert opinions and is found in the fuzzy cash flow forecasts, deriving volatility from them. This is, (Zmeskal, 2001) model relies on past data, and FROV model relies on forward-looking data. The use of the models is different, valuation of firm equity and valuation of a future investment, hence the difference in volatility measures may only reflect the different realities of these two situations.

(Zmeskal, 2001) replaces the geometric Brownian motion (GBM) used in the original Black-Scholes formula (and in the FROV model) with a fuzzy-stochastic methodology to include the market uncertainty in the valuation. A fuzzy-stochastic methodology allows the use of fuzzy variables also in the modelling of the markets, this may enhance the usability and credibility of the model.

The FROV model uses two fuzzy variables and derives volatility internally, the (Zmeskal, 2001) model has all variables fuzzy. The models are different, and without empirical testing it is not possible to draw definitive conclusions about their valuation abilities. The more forward looking approach of FROV model would seems to capture the uncertainty of giga-investments better, whereas, the methods used in (Zmeskal, 2001) may be more suitable for valuation of firm equity as an option. Applying a fuzzy-stochastic method to enhance the FRIV is an interesting future research opportunity.

The FROV model is presented, together with a numerical example, in (Collan, Carlsson, and Majlender, 2003), and is based on earlier work developed in, e.g., (Carlsson and Fuller, 2000) and (Carlsson, Fuller, and Majlender, 2001).

The FROV model can be used together with fuzzy NPV to aid in the real investment decision making. It enables the use of the same fuzzy cash flow estimates that are used for the fuzzy NPV calculation and provides a forward looking approach to calculating the standard deviation used in the real option valuation. The FROV model brings more realistic support to
giga-investments than the traditionally non-fuzzy real option valuation methods, because there is less need to make oversimplifying assumptions about the uncertainty of variable values, see appendix 2 for an example on how fuzzy numbers capture uncertainty.

4.3. A New Model for Valuation of Giga-Investments

We concluded in 4.1.4. that when designing models and methods to value giga-investments it is important to take into consideration the valuation issues arising from the giga-investment characteristics and from the giga-investment lifecycle. We can further state, that if it is possible to take into consideration also additional very large industrial real investment (VLIRI) characteristics (valuation issues), that are not giga-investment characteristics, but are relevant for some VLIRI, it is positive for the applicability (and accuracy) of giga-investment valuation models.

From the giga-investment characteristics follows that a model designed for giga-investment valuation should take into consideration, as discussed in 4.1.1., 4.1.2., and 4.1.3. (and can take into consideration with):

i) Time value of money (depreciation)
ii) Lack of accuracy in variable estimation (fuzzy sets)
iii) Value of managerial flexibility
iv) Effect of time to build on the time value of money (depreciation)
v) Upward and downward potential in the value of the giga-investment

From the giga-investment lifecycle follows that we can look at giga-investment valuation (and profitability analysis) in three stages, planning, building, and operation. Each of these stages are different, as observed in 4.1.4., and demands different types of valuation to reflect reality (the stage of giga-investment lifecycle).

i) In the operational stage the giga-investment cost is certain (historical), and time value of money and lack of accuracy in variable estimation make the free cash flow (giga-investment value) estimation uncertain (fuzzy). The value of operational managerial flexibility (within the investment), e.g., options to switch inputs/outputs, options to expand, and option to abandon, can be assessed separately (fuzzy real option valuation), and the value added, and “refreshed” to the cash flow estimations as it changes.
There is no option value of waiting to invest, and there is no potential from the building time.

ii) In the building stage, when the (irreversible) giga-investment decision has been made, the investment cost can be viewed as fixed to a high degree, however, the value of the giga-investment is uncertain (fuzzy). The value of operational managerial flexibility (within the investment) can be assessed separately (fuzzy real option valuation), and its value added, and “refreshed” with the cash flow estimations as it changes. There is no option value of waiting to invest, however, there is still potential (negative and positive), because during the building time the states of nature may change, changing the value of the irreversible investment.

iii) Before the giga-investment decision is made, the whole giga-investment can be seen as an option. This means that the option to giga-invest, is the option to begin the building stage of the giga-investment. The investment cost is uncertain and the giga-investment value is uncertain. The potential during the time to build needs to be taken into consideration.

Based on the three giga-investment characteristics, and the valuation issues arising from them, and the stage of lifecycle of the giga-investment, presented in 4.1.4., the discussion above, and taking also into consideration,

i) different discount rates for free cash flows and initial costs
ii) different standard deviation for free cash flows and initial costs

We propose a definition for a new model for valuation of giga-investments as,

**DEFINITION.** Fuzzy real investment value (FRIV) is calculated as:

$$FRIV_i = R \eta E(R) * \sigma_R * (t + t_C) - C \eta E(C) * \sigma_C * t$$  \[11\]

where,

$$R = \sum_{i=0}^{L} \frac{1}{(1+r_{ri})^i} * R_i$$  \ Fuzzy present value of the fuzzy free cash flows from the project

$$C = \sum_{i=0}^{L} \frac{1}{(1+r_{ci})^i} * C_i$$  \ Fuzzy present value of the fuzzy initial cost
The FRIV model does not take a position on how the FCF and IC cash flows are forecasted, different forecasting methods are presented in 2.2.1., in 4.2.1. we suggest the use of judgmental forecasting for the estimation of cash flows, the same suggestion applies for the FRIV model. The model allows the use of imprecise cash flow estimates, discussed in 2.4., and suggests the use of fuzzy numbers, presented in 2.4.1., for FCF and IC cash flow estimates. We further suggest the use of trapezoidal fuzzy numbers, defined in 2.4.1., because they simplify calculations. The calculation of present value is presented in 2.2.1., with references to works on the calculation of the fuzzy present value.

\[ r_{RI} = \text{Discount rate specific to each free cash flow from the project (crisp number)} \]

\[ r_{CI} = \text{Discount rate specific to each initial cost cash flow (crisp number)} \]

The discount rate is separately determined for each FCF and for the IC cash flow. Using separate series of discount rates for FCF and IC cash flows is different in practice from the common use of a single series of discount rates for the investment as a whole, as is done, e.g., in the net present value (NPV) and Black-Scholes based real option valuation (ROV) methods, presented in 2.2.1. and 2.3. In practice it may be very difficult to estimate the series of separate discount rates, and it may be practical to use only two discount rates, one for all FCF and one for all IC cash flows. Analogous practical operation for NPV and Black-Scholes based ROV methods is to use a single discount rate. Determination of discount rates is shortly discussed in 2.1. Fuzzy and crisp numbers can be used together, discount rates to be used in the model are proposed to be crisp to simplify calculation, it is however, possible to use also fuzzy discount rates, e.g., (Kuchta, 2000).

\[ E(R) \text{ and } E(C) = \text{Possibilistic mean (expected) value of } R \text{ and } C \text{ (R and C fuzzy)} \]

Calculation of the possibilistic mean (expected) value of fuzzy numbers is developed in (Carlsson and Fuller, 2001a). For trapezoidal fuzzy numbers, suggested to be used with the model, the possibilistic mean value is defined as,

\[
E(A) = \frac{a + b}{2} + \frac{\beta - \alpha}{6} \tag{12}
\]
if A is a trapezoidal fuzzy number, $A=(a,b,\alpha,\beta)$.

$$\sigma_R = \frac{\sqrt{\text{var}_f(R_i)}}{E(R_i)}$$ Possibilistic standard deviation of the fuzzy revenues (FCF), in % (crisp)

$$\sigma_C = \frac{\sqrt{\text{var}_f(C_i)}}{E(C_i)}$$ Possibilistic standard deviation of the fuzzy initial costs (IC), in % (crisp)

The standard deviation is calculated separately for R and for C, from the IC and FCF cash flows, and is given as a percentage. Determination of possibilistic variance and standard deviation of fuzzy numbers is developed in (Carlsson and Fuller, 2001a). For trapezoidal fuzzy numbers the possibilistic variance (of a trapezoidal fuzzy number A) is defined as:

$$\sigma^2(A) = \frac{(b-a)^2}{4} + \frac{(b-a)(\alpha + \beta)}{6} + \frac{(\alpha + \beta)^2}{24}$$ [13]

The possibilistic standard deviation of A is the square root of the possibilistic variance. The possibilistic standard deviation (of a trapezoidal fuzzy number A) as a percentage, presented, e.g., in (Carlsson and Fuller, 2000), is defined as:

$$\sigma(A) = \frac{\sqrt{\sigma^2(A)}}{E(A)}$$ [14]

In practice it may be difficult to determine a usable standard deviation for the investment from the standard deviations of the individual FCF and IC cash flows. It makes sense in many cases to replace the calculation of the standard deviation from the individual standard deviations by calculating the standard deviation from the aggregates, R and C (present values of R and C). By calculating the standard deviation from the aggregate cash flows simplifies the calculations, but accepts the fact that in some cases the aggregate standard deviation may differ from a standard deviation calculated from individual cash flow standard deviations.

Separate standard deviations are calculated for the revenues and the initial investment. This is different from, e.g., the Black-Scholes based ROV methods, where one standard deviation is used for the investment as a whole. The standard deviations for the FCF and IC cash flows may be significantly different, because FCF and IC cash flows are governed by
different markets. It is not implausible to expect that standard deviations for different markets can be different.

Possibilistic standard deviation for the costs and revenue is calculated from the fuzzy cash flow estimates making the volatility an internally determinable variable in the model, e.g., (Carlsson and Fuller, 2001a). Although it is hardly ever explicitly mentioned, it is common practice with option valuation simulation models to determine stochastic volatility internally, e.g., (Fouque, Papanicolaou, and Sirkar, 1999) and (Fouque and Tullie, 2002). This is different from the calculation of standard deviation used, e.g., in the original Black-Scholes formula. The approach is forward looking, because the standard deviation is calculated from the future cash flow estimates. The FRIV model does not require the use of any specific market model for the generation of the future cash flow estimates, this means that the best fitting market model can be selected (different from, e.g., the Black-Scholes model). We have proposed the use of judgmental forecasting for giga-investments, because of a number of reasons (see 4.2.1.). Standard deviation calculated from the judgementally forecasted future cash flows will take into consideration the knowledge of the experts about the uncertainty and complexity facing the giga-investment. We feel that this is an advantage over models that are limited to a single market model that makes FRIV a more suitable tool for giga-investments.

\[
\eta = \begin{cases} 
- & \text{when } A(V) < E(A) \\
+ & \text{when } A(V) \geq E(A) 
\end{cases} \quad \text{Heuristic operator for addition of potential (uncertainty)}
\]

A new heuristic operator is introduced. The operator utilises heuristically two extended algebraic operations on fuzzy numbers: the operation used for the values of the fuzzy number A smaller than the possibilistic mean value of A, A(V) < E(A), is subtraction, and for the A(V) > E(A), addition. Figure 4-2 is a graphical illustration of the heuristic operator. The heuristic operator makes operations on one number, i.e., it does not add two numbers in the common sense. The operator adds information to the number to which the operations are made, information brought by the "added" number is distributed around the possibilistic mean of the original number. This is different from the effect of the fuzzy addition operator. Using crisp numbers in forecasting future cash flow estimates is a simplification of the complexity of future uncertainties, also using (trapezoidal) fuzzy numbers simplifies the reality, however, to a lesser extent. Introducing changes in uncertainty or accuracy of perception to
future cash flow estimates given as (trapezoidal) fuzzy numbers is not straightforward and needs further study. The heuristic operator is a placeholder for methods that in the future will be able to consistently treat changes in uncertainty and complexity of future cash flow estimates.

The operator causes an unfortunate discontinuity in the resulting fuzzy number. It is likely that this discontinuity issue can be resolved in the future with fuzzy arithmetic operators. More about the operator, and possible remedy for the discontinuity issue are presented in appendix 3.

**Figure 4-2.** Graphical illustration of how the heuristic operator works. The potential (or added uncertainty) is distributed around the possibilistic mean value $E(A)$ stretching the fuzzy number.

$t =$ Time the possibility to invest is available (time to wait) (crisp number)

$t_C =$ Time to the beginning of the revenue stream after entering the project (building time) (crisp number)

*The time the possibility to invest is available* is the time before the investment decision is made and the decision-maker has the possibility to wait to invest. During waiting the decision-maker can learn and gain new information about the investment, e.g., see in what direction the output product markets are evolving; markets can evolve to the negative or the positive direction (negative and positive potential). The potential is generated during the time the possibility to invest is available. The model suggests the use of a crisp estimate for simplicity. We have discussed the value of the possibility to wait to invest (postpone investment) in 2.3.
Building time is the time it takes for the investment to be built. In 2.1, we have discussed the long building times for VLIRI. Potential is generated during the time the investment is being built. We suggest the use of a crisp estimate for simplicity.

Potential for the investment is proposed to be calculated by multiplying the expected present value of the FCF and the IC for the investment with the standard deviation of the FCF and the IC with the time the potential is generated. The potential generated by (the markets for) the FCF is calculated as $E(R) \times \sigma_R \times (t + t_c)$ and the potential generated by (the markets for) the IC as $E(C) \times \sigma_C \times (t)$.

The model proposes the separate calculation of the potential caused by the markets for the FCF, e.g., the output product markets and for the markets for the IC, e.g., building costs. This is because:

i) The amount of potential caused by (the markets for) the FCF and (the markets for) the IC is different.

The potential caused by the FCF and the IC (markets) is different, because, as discussed above, the standard deviation of the FCF and the IC cash flows is most often different. It seems that for very large industrial real investments the uncertainty facing the IC cash flows is much lower than the uncertainty facing the FCF, i.e., the potential caused by the FCF (markets) is higher due to the higher estimated standard deviation of the FCF.

ii) The time for which potential is generated by (the markets for) the FCF and the IC is not symmetric.

The generation of potential during the time to wait and during the building time are not symmetric. During the time the possibility to invest is available, the potential is generated by both, (the markets for) the FCF and (the markets for) the IC cash flows; the markets may evolve positively or negatively, causing positive and negative potential.

During the building time, after the investment decision is made, the model assumes that IC cash flows are contracted and fixed, e.g., paid up front, and no longer dependent on the markets for the IC cash flows (remaining uncertainty is included in the fuzzy cash flow estimates). According to the model the markets for the IC cash
flows do not generate potential during the building time. Potential is still generated during building time by the markets for the FCF, because the production has not begun and we don't know the exact market price, or the evolution of the market prices until the time the production is started.

NPV or fuzzy NPV do not take potential into consideration. The proposed calculation of potential differs from the way potential (value of the managerial flexibility to wait; possibility to postpone investment) is calculated with the real option valuation, using the Black-Scholes formula:

i) Upward and downward potential (the possibility that the market evolution is positive, or negative) are both included. Real option valuation, based on option valuation models assumes that the value of the potential is always at least zero. The fuzzy real investment valuation model (FRIV) considers also the possibility of a decrease in the (giga-) investment value (negative potential) during the time the possibility to invest is available. The FRIV model takes into consideration that the value of the giga-investment may decrease during the time to wait, and the initial investment cost may increase simultaneously, causing a negative change in the profitability of the investment, and thus captures also the negative potential. The model is based on the principle, that positive and negative potential are shown to the decision-maker, who will then make the decision. The potential and the negative potential are important, when there is time to wait and during the building time.

ii) The potential generated by (the markets for) the FCF and (the markets for) the IC cash flows are calculated separately. Real option valuation uses the same (a single) standard deviation for the FCF and the IC cash flows, i.e., they are assumed to be the same markets.

iii) If there is no time to wait, but there is a building time, the fuzzy real investment model (FRIV) proposes that some potential (from the FCF) still exists. Real option valuation, or the fuzzy real option valuation model (FROV) presented in 4.2.1., do not recognise the potential caused by the building time. If there is no time to wait and no building time, then the results from the fuzzy real investment valuation model and the fuzzy real option valuation model both collapse to fuzzy net present value. Further if there is no perceived
inaccuracy in the forecasted cash flows, the FRIIV and the FROV collapse to NPV. Commonly used, crisp (expected value based), Black-Scholes real option valuation collapses directly to NPV when there is no time to wait.

iv) If there is no perceived inaccuracy (uncertainty) in the forecasted cash flow estimates, there is no potential (the value of the possibility to postpone an investment under different combinations of reversibility and certainty are presented in table 2-2.). This is caused by the fact that the model proposes the calculation of the standard deviation used from the forecasted fuzzy cash flow estimates that include the uncertainty of the cash flows. Real option valuation commonly utilises methods for the estimation of standard deviation that do not take into consideration the perceived uncertainty of the future cash flows, but is based on the expected values of the cash flow estimates. The fuzzy real option valuation model presented in 4.2.1. uses the same approach as the fuzzy real investment valuation model. If the fuzzy real investment valuation model is, for some reason, used with crisp numbers, then also the standard deviation must be calculated by using an other than the suggested method.

v) The fuzzy real investment valuation model is not tied to a single market model. Because the standard deviation utilised in the model is derived from the cash flow estimates, the selection of the market model that is used to forecast the future cash flows is up to the decision-maker. The Black-Scholes option pricing model uses a single market model. The calculation of the potential is affected by the selection of the market model used, e.g., in cases where markets can be steered by large actors (some very large industrial real investments, as discussed in 2.1.), being unable to select an appropriate market model may create problems of credibility.

The FRIIV model adds the potential from the FCF to the present value of the FCF from the project (revenues) and the potential from the IC to the present value of the IC from the project (costs) with the heuristic operator $\eta$. If the model is used as proposed the potential is given as a crisp number.

After the potential from the FCF is added to the present value of the revenues and the potential from the IC is added to the present value of the costs, the cost-with-potential is deducted from the revenue-with-potential to reach the fuzzy real investment value (FRIIV value). The resulting FRIIV
value is the range of possible value outcomes from the planned giga-
investment that includes the uncertainty of cash flows and the positive and
negative potential generated by the time to wait and the building time.

NPV is a simplified method to assess the value of the investments, real
option valuation of managerial flexibility (potential) and using fuzzy
number estimates in capturing the complexities and uncertainties of the
future are efforts to bring valuation closer to reality. The fuzzy real
investment valuation model is an effort to bring the valuation of giga-
investments still closer to reality, by further including in the valuation the
effect of some special characteristics of these investments.

4.3.1. Numerical Example of Giga-Investment Valuation with FRIV

First, to illustrate the example, the context of the markets including the
market estimates and expert assessments for the example are presented,
and the characteristics of the investment explained. Then the FRIV
calculations are made, based on the cash flow estimates and the results
presented and shortly discussed.

The business to which the investment is made is dependent on cyclical
world market prices for the output product (like many VLIRI are). The
prices of the raw material(s) follow the market prices of the output product
with a time lag. Because of the cycles, the timing of the investments made
to the business is important, since wrong timing will cause the perceived

![Figure 4-3](image_url)

**Figure 4-3.** Expected market environment perceived to face the investment

investment to face falling prices for the output product, which may cause
losses. The investment in question is a production facility with innovative,
yet to the investing company well-known, technical characteristics that are
not available to competitors. The economic life of the investment is
assumed to be 15 years from now. This is based on the profitability estimates of the technology, not on the mechanical life of the machinery, i.e., if the investment was ready now it would produce the estimated cash flows for 15 years from now. After that the technology is too old to be used (this is partly a strategic quality choice). What this means in light of calculating the FRIV is that if the investment is postponed cash flows will be lost from the beginning of the investment. It is assumed that the cash flows are distributed homogeneously within any given year. The size of the initial investment is estimated at around € 180 million and the time to build the investment is 24 months.

The building costs are estimated to be between € 89 and € 90 million for the first year and between € 90 and € 92 million for the second year of building. The company in question has been in business for 40 years and has industry experts with decades of experience in the markets and with similar investments in the past. The investment, if built, will have an effect on the local markets and a slight effect on the world markets, therefore, the historical information about the markets is not fully relevant for decision making. Figure 4-3 shows the commonly expected yearly prices of 1 ton of the output product, the production price (incl. the raw materials) for 1 ton of the product, and the derived estimated yearly profits for production of 1 ton of the product in question. The production capacity is 300 KT ad annum. The opportunity cost of capital for the FCF is determined by experts to be at the level of 12% and for the IC at the level of 7%, i.e., these will function as the discount rates for the IC and FCF cash flows (simplification to use single discount rates).

The experts in the company that is contemplating investment into the production facility estimate the revenues (FCF) from the investment according to the perceived market environment and their personal subjective knowledge about the markets. They have given their expert opinions as intervals (fuzzy numbers) to include the subjectively perceived uncertainty that they see facing each of the cash flows. Figure 4-4 presents the expert opinions together with the FCF expectation derived from the commonly expected prices. The further into the future the cash flows need to be estimated the more uncertainty the experts seem to perceive.

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8 The common industry expectation of the prices
Figure 4-4. Cash flow estimations made by experts, based on their perception of the future of the markets facing the investment and their expert knowledge about the uncertainty.

Taking into consideration the fact that timing is of the essence and that the trend is now (time 0) downward sloping, the managers wish to see what is the effect of postponing the building of the investment, the postponement is considered at 6 month intervals until 3½ years. The expectation is that if the investment is postponed, the cash flows from the time during waiting will be lost, ceteris paribus. Figure 4-5 presents the FRIV values for the eight different possibilities (invest now to postpone until 3½ years). Not unexpectedly, it seems that the investment is most profitable if postponed by two years. It is also visible from figure 4-5 that according to the FRIV the total potential of the investment grows the longer the investment is postponed. If managers decide to postpone the investment for

**Figure 4-5.** Effect of waiting to invest to the FRIV value.
two years and expect to enhance the profitability, they must face increased uncertainty (total potential) about the project outcome.

The fact that FRIV shows the increase of uncertainty (total potential) caused by waiting and by the time to build is highly relevant to giga-investments due to the fact that timing is often essential for large industrial investments. The model does not bias the information given to the decision maker by showing only the positive potential, but shows clearly the possible negative scenario as well.

In the calculation of the FRIV the standard deviations for the FCF and the IC are calculated from the present value of the FCF (R) and the present value of the IC (C) for simplicity. Figure 4-5. does not include the discontinuity of the support of the FRIV value. A screenshot of the detailed calculation of FRIV, with values given in this example, is given for clarity in appendix 4. A simple spreadsheet model of the FRIV is available at request from the author.

4.3.2. Using New Methods for Giga-Investments

We suggest the following valuation principles for valuation of giga-investments, according to the stage of the giga-investment lifecycle, using fuzzy real option valuation (FROV) and the fuzzy real investment valuation model (FRIV), or using a combination of fuzzy real option valuation, fuzzy net present value (FNPV) and (fuzzy) forward pricing, as follows:

i) For giga-investment valuation in the operation stage use:

The fuzzy real investment valuation model and separately assess the (fuzzy) present value of managerial flexibility within the investment, i.e., operational options (fuzzy real option valuation of the operational options (FROV-O)), and add the result from the fuzzy real investment valuation model and the FROV-O value,

or use fuzzy net present value (FNPV), and separately assess (fuzzy) the present value of FROV-O, and add them up

For a presentation of flexibility and real options within a manufacturing investment (operational options, FROV-O) see, e.g., (Bengtsson, 2001).

ii) For giga-investment valuation in the building stage:
calculate the FRIV, separately assess the present value of the operational options (FROV-O), and add them up,

or calculate the FNPV, separately assess present value of FROV-O, and calculate the forward value of the giga-investment using a (fuzzy) forward pricing model. Add the fuzzy NPV, the present value of FROV-O, and the (fuzzy) forward value together to get the giga-investment value.

For giga-investment valuation in the planning stage:

calculate the fuzzy real investment value, separately assess the fuzzy present value of the operational options, and add them up,

or, calculate the fuzzy net present value, separately assess the fuzzy present value of the operational options, calculate the fuzzy forward value, assess the value of the managerial flexibility before investment, i.e., the value of waiting (FROV-P), with the fuzzy real option valuation model, using fuzzy net present value less the present value of investment cost added with the (fuzzy) forward value as the present value of free cash flows in the fuzzy real option valuation formula. Add together the fuzzy net present value, the fuzzy value of the operational options, the (fuzzy) forward value, and the value of waiting (FROV-P), to calculate the value of the giga-investment in the planning stage.

This suggests that the giga-investment decision rule becomes:

“Invest when fuzzy real investment value added with the fuzzy present value of the operational options is positive”, or “invest when the fuzzy net present value, added with the fuzzy present value of the operational options, and (fuzzy) forward value exceeds the initial investment cost, by an amount higher than the value of keeping the option to giga-invest alive”.

We must observe, that discussing (fuzzy) forward valuation suitable for giga-investments has not been within the scope of this research, however, we suggest the (existence of the) possibility of using (fuzzy) forward valuation for giga-investments in combination with the fuzzy real option valuation and fuzzy net present value as a possibility for valuing giga-investments. To the best of our knowledge, there have not been attempts to value the
building stage of real investments with forward valuation, or fuzzy forward valuation.

<table>
<thead>
<tr>
<th>Giga-investment decision</th>
<th>Giga-investment ready</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PLANNING</strong></td>
<td><strong>BUILDING</strong></td>
</tr>
<tr>
<td>FRIV + FROV-O</td>
<td>FRIV + FROV-O</td>
</tr>
<tr>
<td><strong>or</strong> FROV-P + fuzzy forward pricing + FNPV + FROV-O</td>
<td><strong>or</strong> fuzzy forward pricing + FNPV + FROV-O</td>
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</tbody>
</table>

**Figure 4-6.** Giga-investment valuation in planning, building, and operation stages

We can see from figure 4-6, and from the discussion above that the fuzzy real investment valuation model can be used in all the different stages of giga-investment lifecycle for valuation of giga-investments. This seems to be simpler than the other suggested, combined approach.

4.4. Dynamic Planning and Management of Giga-Investments

Giga-investments with their special characteristics, especially their long economic life, require the strategies of managers to stay focused on optimising the giga-investment according to the changes in the markets, which hence need to be monitored. We have above discussed real options and managerial flexibility and noted that they also require monitoring of the markets. Indeed both, the planning by real options and their management require knowledge about the present market situation and about the future changes. This section will shortly present the innovative notion of dynamic planning and management that is based on the suspicion that the two, planning and management information needs are to a high degree compatible. Further, these needs can be supported with decision support systems.
A foundation for a dynamic approach is laid by looking at giga-investment valuation issues, followed by a discussion about the reasons for, and the benefits from such an approach. Supporting dynamic planning and management of investments is also discussed.

**Basis for the approach**

As an input to valuation models, at any given time, we use variable values that are based on our perception of the states of nature, facing the giga-investment in the future, at that given time. We have observed in 2.4. that the further in the future an event takes place, the more difficult it may be to accurately perceive and estimate the event. If the future event is, e.g., something that can be measured in cash flow terms, then the above implies that the closer we get to a cash flow, as time passes, ceteris paribus, the easier it becomes to perceive and estimate that same event accurately.

This means that as time passes, our perception of the variable values that we use (should use) as input to valuation models changes, because our perception of the states of nature facing the giga-investment changes. Our perception of the states of nature may change because of increased perception of accuracy, or because of changes in our perception about, e.g., technology and markets. This means that our original, or earlier, perception of the variable values (made when the event was further away in the future, or according to the old information), may be different from our new perception. When our perception of variable values, affecting the value of the giga-investment changes, then our perception of the value of the giga-investment should also change. Putting the above in another way is to say that when there are changes in the investment environment, they should be reflected as changes in the investment valuation calculations. Taking the changes in the investment environment into consideration can be done by adjusting the corresponding values of the inputs to the selected valuation models.

Perceived changes may be based on different types of information, which we can for simplicity divide in two categories, quantitative and qualitative. Quantitative information, as understood in this context, is information that is rather easy to quantify or is already in a numerical format, and its effect on valuation is not difficult to update. An example of a quantitative information, in this context is, e.g., an expected negative change of 10% in production, for two months, due to maintenance. Qualitative information, as understood in this context, is information that is not always easy to represent as, or transform to, numerical data, and can be based on loosely
structured information. An example of qualitative information, in this context would be, e.g., an observation that Asian sales of a certain product are rumoured to increase.

Both, qualitative and quantitative information may be highly relevant to decision making, and hence it is important to be able to include both, when adjusting input values. Updating fuzzy variable values with a heuristic method, when new qualitative information arrives, is discussed in (Collan and Majlender, 2003). Combining fuzzy numbers can be harnessed for information updating purposes, e.g., see (Bardossy, Duckstein, and Bogardi, 1993) and (Lotan, 1997).

Planning and management of giga-investments that is based on continuous updating of variable values, and updating of the giga-investment value, according to changing information is, for the purposes of this research, called "dynamic planning and management". If the valuation is updated and "real time" value and other information of the investment is available, then decisions made under dynamic planning and management are decisions that are made with the latest available information. Under the assumption that decisions made, based on the latest available information, are rational (optimise wealth), then it is not implausible to expect that these decisions are closer to optimal (better), than decisions that are not made with the latest information, ceteris paribus, at any given time.

**Supporting dynamic planning and management**

We can expect that dynamic planning and management of giga-investments is easier to implement, if the planning and operational management personnel work together. Unfortunately this is often not the case (McIntyre and Coldhurst, 1987), but the planning and management of investments is often separated. Such separation may cause problems and overlap in the tasks of the persons responsible for planning, operational management, and analysis. Separation of planning and management can be eased, e.g., with a decision support system (DSS) that has been built for this purpose (Collan, 2004b), or by improving the organisation of market forecasting within companies (Fildes and Hastings, 1994).

When one examines suggested structures for the organisation of capital budgeting process\(^9\), e.g., (Pinches, 1982), (Mukherjee and Henderson, 1982), (Neale and Holmes, 1990) divides the process in to three stages: planning, implementation, and control. (Maccarone, 1996) divides it to: strategic planning, authorization, implementation and control, and post-auditing. The others are on similar
1987), (Neale and Holmes, 1990), (Maccarone, 1996), and (Azzone and Maccarone, 2001), and the three stages of the giga-investment lifecycle presented in 4.1.4., one notices that dynamic planning and management, and a DSS built for dynamic planning and management, would support all the stages of giga-investment lifecycle, and the suggested structures. In fact, many phases of the capital budgeting process, e.g., post-auditing and strategic management, could greatly benefit from such a system, e.g., (Collan, 2004b) and (Kivijärvi and Tuominen, 1999).

We have already observed, in 2.3., that managerial flexibility makes a part of the value of giga-investments, and concluded, in 4.3.2., that managerial flexibility is important for valuation of giga-investments during the whole life cycle of the investment. Id est, the value of the giga-investment changes, when the value of the managerial flexibility, i.e., real options available, before and within the investment changes. We can interpret this so that real options in particular benefit from a dynamic approach to planning and management, and from a DSS supporting such an approach, e.g., see (Alcaraz and Heikkilä, 2003), (Collan and Liu, 2003), and (Heikkilä, 2002).

According to the above discussion it is safe to say that giga-investments benefit from constant surveillance of the environment, in which the giga-investment exists, or is planned to be built. (Aguilar, 1967) calls this constant surveillance "environmental scanning". To support environmental scanning, intelligent software agents can be used to reduce, e.g., the workload of managers (analysts), continuously gathering data, by automating many of the scanning tasks (Liu, 2000). Intelligent software agents can also, most probably, be harnessed to automate some of the information updating tasks, discussed above. (Collan and Liu, 2003) presents a framework for a capital budgeting support system that uses intelligent software agents to support real option based, dynamic planning and management of large real investments (giga-investments).

Dynamic planning and management is most likely something that some managers are already engaged in, however, it lacks a holistic theoretical base. The paradigm, within which, companies and academia seem to be operating, is one of separation of planning and operational management (of investments), and in which strategic management is sometimes understood as yet another separate issue within companies. Perhaps, in the future, we will see a change of paradigm, and perhaps the change will be fuelled by
decision support systems capable of undoing organisational separation of planning and management.

4.5. Original Publications

This section will summarise the six original research papers and discuss how they support and contribute to the results described above.

Paper 1, (Collan, 2004a)


The background of the paper is that very large industrial real investments (VLIRI) are a group of investments that have not thoroughly been investigated, and are commonly valued by using discounted cash flow based capital budgeting methods, discussed in 2.1. Because of a number of reasons, mainly special characteristics of these very large investments, it is reasonable to expect that the valuation decision support that discounted cash flow based (neo-classical) capital budgeting methods offer is not optimal for these investments. The problem area that the paper concentrates on is the identifying the special characteristics of VLIRI and finding some defining common characteristics that can be used to classify VLIRI.

A definition of "giga-investments" is made by demanding from VLIRI three common characteristics i) long economic life ii) irreversibility iii) long building time, presented in 4.1. After the definition of giga-investments the paper proposes a framework for a method for valuation of giga-investments, called FRIV, discussed in 4.3. The FRIV method utilises fuzzy inputs for forecasted cash flows and introduces the need for an advanced operator that will take into consideration the need of adding information to a fuzzy number in the context of complex uncertain future cash flows, discussed in 4.3. and in appendix 3.

The paper concludes that the proposed model integrates profitability analysis, modelling of uncertainty, and modelling of potentiality, each of which are important for giga-investment capital budgeting decision support.
Paper 2, (Collan, Carlsson, and Majlender, 2003)


Focus of the paper is presenting a hybrid fuzzy Black-Sholes real option valuation model, in the context of very large industrial real investments. The model (FROV) is presented also in 4.2.1. The problem area the paper is interested in is "how to enhance real option valuation for very large industrial real investments" (the term giga-investments, presented in 4.1., is presented in the paper). The paper observes that it is difficult to accurately forecast variable values for giga-investments due to the complexities and uncertainty surrounding their, far away, future, and that such inaccuracies can be modelled with fuzzy sets. The same observations are made in 2.4. and 4.1.1. The paper concludes further that real options seem to be usable for the valuation giga-investments, conclusion made in 4.1.2. and based on 2.3., and based on a literature review observes that there do not seem to be real option valuation models that are usable with fuzzy number inputs.

To enable the use of fuzzy sets with Black-Scholes real option valuation the paper then presents a hybrid fuzzy approach to Black-Scholes real option valuation (FROV). The usability of the model is further elaborated by deriving an optimal investment strategy by using the presented FROV model according to a model by Benaroch and Kaufman (Benaroch and Kauffman, 2000). The model uses fuzzy inputs for some variables and utilises a new method for calculation of the standard deviation, presented also in 4.4. The paper illustrates the usability of the model with a numerical example. The paper goes further to discuss some methods to reduce the uncertainty of fuzzy numbers; vertical reduction, and horizontal reduction. These methods can be utilised to make a fuzzy set more "compact", when fuzzy sets, obtained as a result from, e.g., the FROV model, are too wide and thus unusable.

The paper concludes that as the model is able to use fuzzy forecasting estimates as an input it enhances the real option valuation of giga-investments by being able to handle fuzzy sets that can be used to take into consideration the complexities and uncertainty of far away future cash flows.

The paper discusses extending the tools used in planning and profitability analysis of investments to the management stage of (the same) investments. The paper proposes that this can be done with advanced DSS that overcome possible organisational gaps. The context area of the paper is the organisation of decision making support and management processes in the modern company.

The problem that the paper presents is that commonly the planning of investments (profitability analysis) is separated from the operational management of (the same) investments, i.e., separation of planning and management. The problem extends also to the separation of analysis functions (performed by, e.g., market analysts) from the operational management of investments, and sometimes also from the planning of investments. The problem of organisational separation causes loss of information when an investment evolves from the planning stage (planners) to the operational stage (managers). The cause of the problem can be understood by looking at planning and management as static blocks, each optimising according to their own criteria.

The paper proposes that the problem of organisational separation of planning and management, and the information loss occurring as the project moves from planners to operational managers, can be alleviated, if not annihilated, with a suitable approach that can be supported by a DSS. This approach is introduced in the paper as "dynamic management" and ties the planning stage (planners) to the operational management stage (managers), the basis of this approach are presented in 4.4. The paper illustrates with a case about a fictional DSS built to support the dynamic management approach.

The proposed approach and a supporting DSS would be evolutionary steps in the decision support methodology and technology for both, capital budgeting, discussed in 2.2.2., and management of investments.

The implications of the dynamic management approach on post-auditing are discussed. The paper concludes that if a supporting DSS is in place,
and is equipped with advanced database functionality, the profitability post-auditing of investments would be greatly enhanced in many ways. Some strategic planning views of such an approach and DSS are also discussed.

The paper concludes with a discussion of a selection of problems and proposals to remedy them with advanced methods from the domains of capital budgeting and of decision support systems.

**Paper 4, (Collan and Liu, 2003)**

Collan, M. and Liu, S., 2003, Fuzzy logic and intelligent agents: towards the next step of capital budgeting decision support, Industrial Management & Data Systems, 103, 6, pp. 410-422

The paper discusses a framework for integrating the use of intelligent scanning software agents with real options based investment decision support tools that use fuzzy inputs. The subject area of the paper is support technology for capital budgeting, discussed in 2.2.2. The framework, presented in the paper, is built in the context of very large real investments and the main problem area that the paper concentrates on are the difficulties in gathering, pre-processing, and utilising the large amounts of data the Internet offers to support investment decision-making and management. The problem is approached first, by shortly discussing the decision support needs of planning and management of very large investments and then, by introducing real option valuation, presented in 2.3. This is followed by a presentation of the use of fuzzy logic in capital budgeting, discussed in 2.4. Intelligent software agents are discussed in the context of the possibilities they can offer in automating and enhancing environmental scanning, discussed in 2.2.2. Environmental scanning can be used to support capital budgeting and management decisions, and is an integral part of the dynamic planning and management approach, presented in 4.4.

The framework is presented by utilising a process view, i.e., a model for an agent based capital budgeting DSS is described, after which the parts of the system are separately examined. The described system includes a scanning agent that automates the environmental scanning procedure (from both, the intranet and the Internet) and feeds the gathered information to a database. Another software agent, called for the purposes of the paper, "interpretation agent" generates structured information from the gathered data, generating event information and trends. This data is fed to a third
intelligent software agent, "option watcher" that identifies real options and generates decision trees based on the options identified, according to a predefined option logic. The decision trees can be then presented to the decision-maker for confirmation, modification, or to be discarded. "project reviewer" is a fourth type of intelligent software agent within the system. It monitors the project databases and reviews updated project status, also feeding the "option watcher", i.e., the "option watcher" can be triggered by the data coming from the "project reviewer", or from the "interpretation agent". The decision trees generated by the "option watcher", after the control by the decision-maker, are inputted to the fifth and final intelligent software agent in the system, "option analyser". The "option analyser" evaluates the different options by calculating values for each option and for each decision tree, enhancing the automation of the task of running profitability analysis. The "option analyser" can also include an output in plain language and graphics that further enhances the decision support capabilities of the described DSS. The framework presented is a possible direction, in which the evolution capital budgeting decision support technology can turn, and already partly has turned. It is also a concrete example of how the dynamic approach to planning and management, discussed in 4.4., can be conceptualised in terms of processes. The paper concludes by discussing the managerial implications of an agent based DSS.

Paper 5. (Collan and Majlender, 2003)


Based on a discussion on capital budgeting and use of fuzzy numbers in input value selection, the paper develops a simple method to adjust (fuzzy) cash flow forecasts as new information arrives. The problem that the paper addresses is in substance the same problem that is discussed in 4.3. in connection with the heuristic operator used in the FRIV model and further developed in appendix 3. The paper discusses the integration of qualitative, in the case of the paper, market trend information into the cash flow forecasts. The presented method is a way to heuristically allocate new information into the tails of trapezoidal fuzzy cash flow estimates. The method presented in the paper does not necessarily require new information to be in a quantitative (numerical) form. Including qualitative
information in decision making is important from the point of view of the quality, correctness, and availability of information to the decision-makers, especially, when the information that is needed is about events far away in the future. Giga-investments, as discussed in 4.1, are investments that have such informational needs. In 2.2.1., we have discussed judgmental adjustment of forecasts, and the method presented in the paper is a simplified approach to conceptualise the judgmental adjustment of cash flow estimates. The paper discusses the addition of new information, as it arrives, implying an ongoing environmental scanning, and thus supports the dynamic planning approach discussed in 4.4.

**Paper 6, (Collan, 2004c)**


The paper is an extension of the ideas presented in paper 1. The context of the paper is modelling and valuation of very large industrial real investments. The paper bases the definition giga-investments on a detailed presentation of the three giga-investment characteristics, done in 4.1. Then a review of the state of the art of valuing large industrial investments is made by discussing standard capital budgeting methods (discussed in 2.2.1.), valuation of potential (discussed in 2.3.), capturing uncertainty present in forecasting cash flow estimates (discussed in 2.4.), and by discussing project finance. A fuzzy real investment valuation model (FRIV) is proposed with a short discussion on the underlying modelling assumptions. The model presented is the same as in paper 1 and in 4.3.

The paper elaborates the used of the FRIV model with a numerical example of an industrial production investment, first describing the context of the investment and the background data useful for calculation and then presenting the results. A screenshot of a spreadsheet based calculation tool is presented in the appendix of the paper. Substantially the same numerical example and screenshot are presented in 4.3.1. and in appendix 4.

The paper concludes by discussing the characteristics of the model and the dynamic nature of the model that is compatible with the dynamic planning and management approach proposed for giga-investments in 4.4. Finally the paper presents a note on the three-stage giga-investment lifecycle and valuation, including the observation that in the building stage the giga-investment valuation resembles the valuation of financial forward
contracts. A similar observation has lead to the valuation of managerial flexibility with methods originally designed for the valuation of financial options. This has been discussed in 4.1.4. and in 4.3.2.
5. Summary, Conclusions, and Future Research

This section will present a summary of the contributions of this research, conclude on the findings and the weaknesses of the research, and suggest avenues for future research within the scope of this research and beyond.

5.1. Summary of Goals and Contributions of This Research

In 1.1. we have defined the motivation of this research to be the creation of new knowledge about very large industrial real investments (giga-investments), their valuation, and their decision support. In 1.2. we have defined the goals of this research, based on a hypothesis that a group within very large industrial real investments can be defined, and that the group has characteristics that need to, and can be, taken into consideration in their valuation. The goals of this research were:

(i) to understand and characterise very large industrial real investments
(ii) to, for the purposes of this research, create a definition of a special group of investments within the group of very large industrial real investments
(iii) to develop and select relevant constructs and methods to be used in valuation of the selected group of investments and of very large industrial real investments
(iv) to propose a framework for the valuation of the selected group of investments that is based on the constructs and methods proposed and selected

In the following we will summarise the contributions of this research, goal by goal, and discuss whether the findings are enough to conclude that very large industrial real investments have special characteristics

(i) We have described very large industrial real investments (VLIRI) and their characteristics. We have presented historical cases of VLIRI for further understanding and accounts of actual events (appendix 1). We have discussed the lifecycle of very large industrial real investments. All in all, we have presented an understandable picture of very large industrial real investments and their characteristics, and how this research views and presents them.
(ii) We have defined giga-investments as a special group of very large industrial real investments by defining them as VLIRI that exhibit three characteristics commonly found among VLIRI. We have explored these characteristics and analysed their effect on giga-investment valuation. We have defined the phenomenon (giga-investments) on which we have concentrated this research.

(iii) We have reviewed a number of existing capital budgeting methods, modelling of imprecise information, and modelling of managerial flexibility, and assessed their suitability for valuation of giga-investments. We have established connections between the giga-investment characteristics and some established valuation models that are, or can be, used in the valuation of such characteristics, we have proposed suitable existing methods for valuation of giga-investments. We have discussed some new approaches for the valuation of giga-investments, and presented a hybrid fuzzy real option valuation model.

(iv) We have proposed a construct for a customised valuation model for giga-investments based on our analysis on the giga-investment characteristics and on existing capital budgeting methods. We have selected, according to our understanding, the most suitable methods for the model, and applied them to suit the giga-investment lifecycle. We have observed a need for methods to consistently treat changes in uncertainty and complexity of future variable value estimates and propose a developed fuzzy operator to act as a place holder for such methods that can handle added information in a, for giga-investments, suitable way.

Based on the above, we conclude that our research hypothesis is true, for we have defined a group (of investments) within very large industrial investments according to characteristics that, we have shown, need to and can be taken into consideration in the valuation of these investments.

We have also fulfilled the motivation of this research. This research has widened the knowledge about very large industrial real investments (giga-investments) through the discussion and offered new possibilities and knowledge about their valuation through presentations of (new) valuation models. Knowledge about the decision support needs of giga-investments has been widened through discussion and analysis of the suitability of a dynamic approach to planning and management, and the possibilities to
support such an approach with decision support systems. Some practical applications for such support have also been suggested.

5.2. Conclusions

This research has first presented a picture of very large industrial real investments, safe to say, a group of investments that has not received a lot of attention from researchers. The notion of understanding very large industrial real investments as a special group within the group of real investments is a new approach.

This research has concentrated on investment valuation issues, and has taken the approach of defining the research scope by defining a set of investments within the group of very large industrial real investments, to simplify the complexity. The group of giga-investments has been analysed and their valuation needs discussed. It was revealed that existing investment valuation methods, if not incompatible, are sub-optimal for giga-investments. This research has not gone as far as to say that this analysis is true for very large industrial investments in general, but it applies to giga-investments. In this research contributions have been made to the valuation of giga-investments by enhancing existing models to suit giga-investments better and by proposing a framework for a new model, based on giga-investments, for their valuation.

Observation that the giga-investment lifecycle, which this research models in three stages, as opposed to the commonly used two-stage investment lifecycle models, may play a key role in the correct valuation of giga-investments, is, to the best of our knowledge, an original contribution to the knowledge about very large industrial real investments.

Giga-investments seem to benefit from a dynamic approach to planning and management, which is based on continuous environmental scanning and updating of information. This is true for a number of reasons, of which the least is not the dynamic nature of options that are found within investments. This research has looked into how a dynamic approach to planning and management benefits organisations to overcome separation of planning and management, and how, if properly supported, this can be done effectively with a help of intelligent decision support systems. The whole dynamic approach to planning and management is a fairly new approach and this research has contributed to, perhaps, changing the way
we think about planning and management of very large industrial real investments.

5.2.1. Weaknesses of this Research

When viewing this research critically from the point of view of research methodology, one may criticise the fact that the background material used for the initial investigations could have been wider than the four presented cases, and that the action research part of the research should have been more thoroughly elaborated. This would be a just comment. However, it was the decision of the author to present the results of the action research and cases, the VLIRI characteristics, in a looser format. This was selected, because it allows falsification by empirical investigation. The presentation of the basis for the definition of giga-investments, by using these characteristics, would then also be falsifiable. Yet, some may disagree on the wisdom of these choices, and a more thorough investigation (empirical) into very large industrial real investments, and their characteristics might have made the disagreement go away.

One singular point that can be raised is the suspicion of the suitability of the Black-Scholes option-pricing model for the valuation of real options. This has been discussed in the thesis and the author does not draw any definite conclusions as to whether the use of the model is justified or not. However, not being able to definitely make a judgement about the usability of the model is a known weakness of this thesis. In all fairness it needs to be observed that proof on the matter has as of yet eluded the scientific community.

Another point where criticism can be justifiably levelled is the usage of different mathematical models in valuing giga-investments. Has the author created models that correctly picture reality? This research does not fully test the created models, therefore, such judgement cannot be made on the basis of this research. Further systematic empirical research is be needed. This may, by some, be a significant weakness of this research. The author acknowledges this fact, and agrees that it is a weakness. However, the motivation of this research has been to create a new knowledge about very large industrial real investments and to show that it is possible to model them. The correctness of the models, when taking the point of view of this research, is an important issue. However, it is not the most important issue, which is to contribute new knowledge. Still, the author believes that the models presented enhance our ability to correctly value giga-investments.
This research has not emphasised the organisational, psychological, and agency problems of investment decision making. It has been the choice of the author to omit these issues, even if they unarguably are interesting and important issues when decision making is investigated.

Some might argue that the presentation of the dynamic approach to planning and management has been short, and deserves much wider attention. While the author agrees, it has not been the motivation of this research to concentrate on the decision support system aspects of giga-investments, but to focus on gaining new knowledge about them and valuing them. It is, however, a just critique of this research that when an issue is as interesting as the decision support of giga-investments is, it should merit more attention, than the introduction of this research is able to provide. This research has, nevertheless, shown an intuitive approach to decision support that seems to be more suitable for giga-investments than the currently used approaches.

5.3. Future Research

As the "evening is still young" for the academic research on very large industrial real investments, there is a wide scope for future research. In fact, it is relatively easy to point out some future steps in this research.

The work done in this research can be continued at least in the following ways:

i) The models created need to be further tested with investment data, both the FROV and FRIV models are fairly new and should be further tested.

ii) A background data base of giga-investments and very large industrial real investments can be built for building a basis for generalisation of the model for a larger population of investments, and testing the validity of the giga-investment definition.

iii) Further study on the suitability of the Black-Scholes option pricing formula for real option valuation is needed. The problems that the strict assumptions behind the formula face when contrasted with reality can, perhaps, be solved, but further study on the subject is clearly necessary.
iv) Forward pricing of the investment value during building time is an exciting new research issue that is of great interest from the point of view of modelling investments and from the point of view of correctly valuing large industrial real investments. Creation of fuzzy forward pricing model for compatibility with giga-investments is also an interesting future research possibility.

v) Integration of qualitative and other loosely structured information into valuation models is a very challenging issue and merits further study.

vi) Automated environmental scanning with, e.g., intelligent software agents, and building of a prototype decision support system for the dynamic approach of planning and management is an interesting and challenging future research arena.

There are undoubtedly numerous other very interesting avenues for future research. However, those listed above are the ones of most interest according to the author at the present time.
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APPENDIX 1: Four Historical Giga-Investment Cases

Summary of the cases

Case 1: Rautaruukki Oyj Coking Plant, Raahe
Case 2: M-Real Oyj, Paper Machine 1 (PM 1), Kirkniemi
Case 3: Outokumpu Oyj, Harjavalta Metals, Harjavalta
Case 4: Metsä-Serla Oy, Pulp Mill, Kaskinen

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry / Sector</td>
<td>Metal / Steel</td>
<td>Forest / Paper</td>
<td>Metal / Base Metal</td>
<td>Forest / Pulp</td>
</tr>
<tr>
<td>Size of investment</td>
<td>FIM 900M</td>
<td>FIM &gt;600M</td>
<td>FIM 1874M</td>
<td>FIM 955M</td>
</tr>
<tr>
<td></td>
<td>FIM 660M</td>
<td>FIM 190M</td>
<td>FIM 1874M</td>
<td>FIM 955M</td>
</tr>
<tr>
<td></td>
<td>FIM 200M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year of investment / Economic life per 2004</td>
<td>1984 / 20 years</td>
<td>1963 / 41 years</td>
<td>1995 / 9 years</td>
<td>1974 / 30 years</td>
</tr>
<tr>
<td>Building time</td>
<td>3 years</td>
<td>~ 3 years</td>
<td>~3 years</td>
<td>~ 3 years</td>
</tr>
<tr>
<td>Markets or technology have changed during the economic life</td>
<td>Yes, USSR collapse</td>
<td>Yes, Product change</td>
<td>Yes, Market prices</td>
<td>Yes, New technology</td>
</tr>
</tbody>
</table>

In the following the four cases, described in the table above, are presented. They are giga-investments that Finnish industrial companies have made, and that exhibit the giga-investment characteristics. The investment decision for each of the cases was done under uncertainty about the future, the behaviour of the markets and the technological advances of the future were unknown at the time of the investment decision.
Case 1: Rautaruukki Oyj, Coking Plant, Raahe

In the beginning of the 1980's Rautaruukki Oyj had an existing steel mill in Raahe, Western Finland. The steel mill did not include an integrated coking plant (as steel mills usually do), because the coke needed in the steel production was acquired through long-term contracts with coke producers in the Soviet Union. The price of coke from the Soviet Union was relatively low, negotiated to be near the cost of production (this was possible, because of the special trade relationship between Finland and the Soviet Union).

The arrangement had worked fine for the first 20 years of the life of the steel mill, however, by the end of 1970's the reliability of coke deliveries, of which the steel production was dependent on, begun to deteriorate. This was due to transportation and production problems in the Soviet Union. On some occasions Rautaruukki Oyj had to buy coke from the world markets at the market price, to secure the steel production, when deliveries had not arrived in time from the Soviet Union. The price for the coke bought from the world markets was considerably higher, than the price for the coke from the Soviet Union. On the background of increased and still increasing insecurity about the availability of coke from the Soviet Union and about the world market price of coke, Rautaruukki Oyj decided to build a coking plant to complement the steel production.

The coking plant investment was of strategic importance, as the uncertainty about the availability and price of coke was a risk to the profitability and continuation steel production. Rautaruukki Oyj was at that time a state owned company, which meant that the return on investment requirement did not play as large a role as in privately owned companies. The opportunity cost of capital was (artificially) considered low. This was done, because for state-owned companies raising financing was not a capital markets related problem, but the capital came from the state of Finland (tax-payers). This also had the effect that investments were more easily considered profitable, as the state-owned companies often used a low opportunity cost of capital due to the lower risk, caused by the risk free financing. In addition, the Finnish economy was highly regulated until the end of 1980’s, which meant that capital markets were not functioning freely.

In the beginning of the 1980's availability of coke on the world markets had deteriorated, meaning that if deliveries from the Soviet Union experienced problems, Rautaruukki Oyj would have to pay a relatively
high price for coke to keep its steel production at an unchanged level. The building of the coking plant would mean that Rautaruukki could reduce the amount of coke bought from the Soviet Union and at the same time shield itself against the market price risk of coke. The raw material for a coking plant is coal, which is not a scarce raw material (significantly lower risk).

On this background of financing possibilities and a scenario of growing risks the decision to build the coking plant was made in October 1984. After a three year building period, in October 1987, the coking plant became operational. The initial investment in the Plant was FIM 900 million (~150 M€). The production capacity of the plant was 475Kt of coke per year, which covered 60% of the demand for coke in the steel production. A coking plant produces coke-gas as a secondary product in the production process. The gas can be used to power the steel production. The energy needed, previously produced by using oil, could be fully replaced with energy generated by using coke-gas, thus generating savings. The planning, building know-how, and the machinery of the coking plant were mostly bought from Russian companies, due to Finnish inexperience with coke production. When the plant was built, it was the first coking plant in Finland and the technology was at an internationally high level.

In the planning and building phases, possibilities to expand production were taken into consideration, e.g., the coal handling facilities were designed to be able to handle double the production volume originally built to the plant. This meant investing more initially, to support a possible expansion in the future.

In 1990 a decision was made to start the planning of an expansion to the coking plant. The reason for the expansion was the still further deteriorating coke deliveries from the Soviet Union. World market price of coke had also risen.

The Finnish economy had little earlier gone through a deregulation, and the basis on which profitability was to be examined had (also) changed. As mentioned above, the original plant infrastructure was designed and built to be functional with larger capacities, causing a reduction in the cost of the following expansion investment. The investment cost of the expansion was FIM 660 million (€ 110M). The expansion became operational in 1992, and the production of the plant rose to 940Kt annually. After the expansion, the coking plant produces all of the coke needed in the steel production, the unit has become self-sufficient.
The strategic change brought by the giga-investment was a change from a unit fully dependent on coke deliveries from the Soviet Union and the world markets for coke, to a unit totally insulated from coke market risk. This change in risk was brought about by the change in the input raw material from coke to coal. The overall market risk was reduced, as coal is not as scarce as coke. The output steel markets were unaffected by the investment. The unexpectedly fast collapse of the Soviet Union in the beginning of the 1990's, and the following time of uncertainty in Russia, proved the timing of the building of the addition to the coking plant to be strategically excellent. The investment saved Rautaruukki from a need to buy up to 40% of the coke needed in the steel production from the world markets. The value of being able to avoid the losses caused by coke scarcity must have been difficult to evaluate ex-ante, however, ex-post it is clear that the value was high.

Because the investment was an addition to an already existing steel plant it is not straightforward to analyse the profitability of the investment separately from the profitability of the steel plant. The giga-investment in the coking plant can be viewed as a part of the larger investment (the steel mill), with an option to postpone the investment partly (the coking plant). The option to build later was exercised, when the price of uncertainty became too high.
Calculation of the ex-post profitability of the plant must take into consideration the changes in the input and the output markets and the changes in the circumstances affecting financing. Also other circumstances that were the reality at the time the plant was planned and built must also be taken into consideration. The change in the situation and the need to use different decision making criteria for the follow up add-on investment make the profitability assessment even more difficult. The case is been presented in the M.Sc. thesis of Linda Blom (2000).
Case 2: M-Real Oyj, paper machine 1 (PM1), Kirkniemi

The forest owners of Metsäliitto-yhtymä decided that it is important to build paper industry in the South of Finland. Behind the investment decision were expectations of the growth of the world (European) fine paper consumption and the lack of production capacity at the time. A two-phase plan was made to build a paper mill into Kirkniemi. In the first phase, one paper machine was to be built (PM1), which would then be followed by another machine (PM2). The unit was to reach profitability, when both paper machines were operational.

The markets for fine paper in the beginning of the 1960's were somewhat different from the markets today, because the market price was a result of an oligopoly price setting, something we would today call a cartel. This meant that the price of fine paper was, de facto, set at a profitable level from the point of view of the producers. The overall profitability of paper production would, therefore, be determined by the amount of produced. As the price of fine paper was, in practice, fixed, high quality production would not be rewarded with a higher price. However, producing high quality fine paper meant that production could be run at full capacity - this would be possible, because the markets would absorb all of the high quality paper before buying lower quality paper (for the same price).

Based on the strategies and the basic industry set-up (actual circumstances on the markets) the planning for the mill (PM1) was done during the years 1961 and 1962. The plans considered building of both machines (1 and 2) onto the same site. Building the infrastructure for the mill started in the late 1963. There was no operational paper mill in the area, and the Kirkniemi mill was built as a green field investment to an area surrounded by forest. The initial planned capacity of the PM1 was 80-100Kt annually. There were also plans to build a pulp factory in connection with the mill.

Building of the infrastructure and the PM1 was completed, and paper production started in the beginning of 1966, after more than 2 years of construction. Most of the investments were done with domestic machinery, based on the most up to date technology (the best technology available at the time), and by domestic contractors. The idea with using the best possible technology was to be able to produce high quality paper, to be able to be run at full capacity from the start of the completion of the PM 1. The initial investment in the paper mill was >FIM 600 million (~€100M).
PM 1 started to produce newsprint paper (MF) at a 100Kt yearly. The speed of the machine was 800m/min, and the weight of the paper produced 50g/m² (weight is important in paper markets, because paper is sold by weight). Already in 1968 the production was changed to periodicals paper (SC), which meant also some technical additions to the machine to polish the paper and to change the speed of the machine, the weight of the paper remained unchanged.

The mill was expanded in 1972 by building the other paper machine, PM2, to complement the first machine. The rationale of the second machine was based on the market reality of overall profits being maximised by large production quantities.

PM1 was fully renewed in 1981-1982 and the type of paper produced was again changed. The change was due to a shift in market focus, and a decline in the profitability of the old product. The new type of paper, called WSOP, was of a higher quality and based on a technology developed in Kirkniemi. Cost of the renewing of the machine was about FIM 190 million (€ 32M). The yearly capacity of the machine rose to 150Kt and the production speed to 1100m/min. Also the weight of the paper produced changed to be 50 - 80 g/m². A calculated risk was taken to start the production of a paper type, previously untested on the markets. The product proved to be a success due to its high quality, this made it possible for Metsäliitto to sell the product more easily, and to be able to run at full capacity.

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<td>planning</td>
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<td>of the</td>
<td>FIM 600</td>
<td>started</td>
<td>changed</td>
<td>FIM 190</td>
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<tr>
<td>investment</td>
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**Figure A-2.** Major events in the economic life of PM 1 during the period 1961 - 1995

By the beginning of the 1990's the WSOP product had reached the end of its life span. Another large change was made to PM1 in 1994-1995, when online-coating was added to the machine and the produced product changed again. The cost of the changes was about FIM 200 million (€ 34M). The new product is a film coated offset paper (FCO). The yearly capacity of the machine rose to 160Kt and the speed to 1200m/min. The weight of the produced paper changed to 36-57g/m².
During the lifetime of PM1 the weights of the papers produced have become lower and the production technology has changed dramatically three times. In addition to the larger technical investments there have been a number of maintenance, and minor efficiency enhancing investments. The lifetime of a paper mill is over thirty years, during which numerous technical improvements are made to the machines, and the type of product is changed, according to profitability and demand in the markets. Each implementation of new technology to a paper machine carries a risk, because it is very unprofitable to shut down the machine, even if it is for a short time. Each technological implementation carries a risk also, because it is uncertain if production can be started again fast, and if the optimal production levels can be reached immediately.

Paper prices are cyclical. The cycles are generally caused by over-investment and the resulting over-capacity in the markets. The industrial environment has changed a lot from the 1960's, when the mill was originally built. The markets have become highly competitive and profitability is determined by the margins made on the paper. The paper producers follow world market prices closely, and a single producer can not steer the world market price, however, with innovative technology, a paper mill can produce world market quality at a lower price and create profits. Paper markets products are standardised, which means that being able to sell higher quality at the world market price gives the producers of high quality paper a competitive advantage (even if the price is market determined), because they can run their mills at full capacity. On the raw materials side the paper producers face risk from the pulp prices.

The ex-post profitability of the PM1 has been quite difficult to assess exactly, due to different hinders. Some of the original materials have been destroyed and the accounting procedures have changed three times. The case has been presented in the M.Sc. thesis of Kari Leivo, Leivo (2001).
Case 3: Outokumpu Harjavalta Metals Oy, Harjavalta

In the late 1980's Outokumpu consortium decided to launch investigations into expanding their production of base metals, especially nickel. The investigations concentrated on utilisation of the nickel deposits found in Mt. Keath, Australia. After a number of candidate locations were analysed Harjavalta in Finland was chosen as the site for the expansion, because the possibility to utilise novel production technologies for nickel, and to enlarge the existing production capacity for copper. Together these were projected to enhance profitability.

Outokumpu had made a 10-year raw material delivery contract for nickel with the owners of the Mt. Keith deposits, which insulated the raw material deliveries from the changes in the world markets. World market prices for copper and nickel are dependent on the product demand and inventory stocks, which affect the output volumes of the producers. The raw materials and output markets seem to be cyclical and the amplitude of the cycles is affected by speculative demand on the products. The cyclical market prices, however, do not affect profitability of Outokumpu directly, as there are long term contracts with the producers of raw materials. Outokumpu operates in a highly competitive environment where the cost of production is one of the most important factors affecting profitability.

Planning of the investment was done during the years 1992 and 1993 and the project was approved in May of 1993. The planning revealed that using the whole capacity of the plant for nickel production would not be profitable and copper production would have to be continued and enhanced with new technology to reach profitability. An organisational regrouping of the plant would be made to enhance the productivity and profitability of the copper and the nickel production. Financing arrangements played a role in the speed of implementation of

Figure A-3. The focus of the giga-investment in the production chain

Raw materials, copper & nickel ore. → Production, unit cost and efficiency → Copper and nickel metal markets.

- Long term contracts with producers (Mt. Keith), currency risk
- Investment to realise economies of scale and an efficiency program
- Volatile world markets largest source of risk, currency risk

113
the plans, contrary to original expectations, financing was not needed from the outside, due to a better than expected financial position of the consortium.

The investment project consisted of six subprojects, totalling an investment of FIM 1874 million (€ 312M), and the production facilities were finished during the years 1993 - 1995. The following production goals were placed on the project: annual production of 165Kt of primary copper, 125Kt of cathode copper (together about 2% of world production) and 55Kt nickel (about 3% of world production). During the investment a decision was made to produce a further 40Kt of nickel brickets annually. The economic life of the production facilities is more than twenty years.

The organisational changes that were planned to bring cost benefits in production did not materialise as well as predicted. The profitability of the investment was not as good as projected, even if the production goals were reached. The main reason for the lower than expected profitability of the investment has been the adverse development of world market prices for the output products: world market prices for nickel did not live up to the expectations. Also the FIM/USD exchange rate caused profits to be lower than expected. The case has been presented in the M.Sc. thesis of Timo Palomäki, Palomäki (2000).

On a further note on the case, Outokumpu has sold their operations in Harjavalta to a Swedish company. The nickel plant was already earlier sold to an American company.
Case 4: Metsä-Serla Oy, pulp mill, Kaskinen

The idea to build a pulp mill in the Bothnia region in the beginning of the 1970's came from the discontent of the local forest owners; there was no large-scale forest industry in the region and the forest owners had difficulties in realising their capital from the forests. This was the political reason for starting the planning of a new pulp mill in Kaskinen. The project was supported also by the Finnish political leadership. The demand for pulp within Finland had also increased. The Bank of Finland had expressed a position that it would give a financing permission to build only one pulp mill in the area; the time was of high financial regulation in Finland. This meant that the decision to build the mill in Kaskinen would undo any other plans to build a mill in the same area. Oy Metsä-Botnia Ab was founded in March 1973 to be the mother-company for the pulp mill project. The planning of the project took place in 1973 and the investment decision was made in 1974. Some concerns were voiced about the profitability of the plant due to the fact that it was not integrated with a paper mill, and thus would not provide economics of integration. Some critical comments by the Bank of Finland were also made about the sufficient availability of the raw material to the pulp mill.

The mill was a green field investment, the building site was in the middle of a forest, without any existing infrastructure. However, the placement of the investment was supported by a set of favourable logistic and geographical conditions (the closeness of a sea-port, a near by river, closeness of a railway line and roads, and the availability of a motivated local working force). At the time of the building of the mill inflation caused the investment cost to rise (inflation >10% p.a.) and the construction companies were not willing to negotiate on the prices. The time of investment was not opportune from the point of view of investment costs.

The projected investment cost was FIM 852 (€ 125M), which included an FIM 80 million (€ 13M) buffer that was to take into consideration changes in the financial climate. The realised initial investment on the mill was FIM 883 million (€ 130M), which was 3.9% over the projected cost. The mill started its operation in 1977, and at that time the pulp markets in Finland were flooded with Swedish pulp that had been produced to boost Swedish employment, even as the demand was low. This meant that the mill become operational at a very bad time. The difficult period in the markets lasted about six months causing the financial situation of the
company to be very tight and causing a lot of voices criticising the sensibility of the investment.

The good quality of the pulp produced in Kaskinen helped the mill to overcome the problems in the beginning, the mill could be run at full capacity, because the pulp produced was absorbed by the markets. High quality pulp gives advantages in the later stages of paper production. Due to professional planning of the mill and the good working force, the production rose steadily and the profitability increased. In 1979, when the downward trend in pulp prices had turned to an upward trend, the annual production had already risen over the planned capacity and was 274Kt. It rose further to 287Kt in 1980 and to 303Kt in 1981. Because of the high pulp quality the mill could be run at full capacity, and thus produced maximum profitability. The cost of wood has been the single largest cost group during the lifetime of the mill, for the first 23 years of operation the cost of wood raw material for the pulp production has been FIM 7787 million (€ 1113M). The original plan of the forest owners of the region to activate their forest holdings to a productive use has been realised, as 75% of the raw wood utilised in the plant has come from the region, creating wealth to the forest owners.

The mill has undergone investments in production technology that have enhanced the operations and rising the capacity of the plant. In total the investment costs in the plant operations have been FIM 955,1 million
(€158M), which is roughly FIM 100 million (€ 17M) more than the originally planned. The correct timing of the additional investments has added to the profitability of the mill, which would otherwise have slowly deteriorated. The plant managers were able to time the enlargement and technology investments before upswings on the pulp market, which has meant that there was demand for the maximal capacity of the mill. The profitability of the mill is highly correlated with the price of pulp, which has evolved in cycles of about six years. The fact that a large part of the production of the mill has been bought by shareholders of the mill (paper manufacturers) has helped the mill overcome tough times.

The continuous demand of the pulp from the mill by the shareholders has been a guarantee to the continuity of the operation of the mill. This demand has created a better environment for additional investments in the production, which in turn have helped with future profitability and keeping of the competitive capability of the old pulp mill.

The economic life of the pulp mill in Kaskinen is estimated to be around 30 to 35 years and the mill has now been running for two thirds of its economic life. Legislation and changes in the environmental norms have had unexpected effects on the economic life of some parts of the investment and costly changes have been necessary. The costs of environmental investments in the mill have been FIM 430 million (€ 72M) during the first 23 years of operation. The case has been presented in the M.Sc. thesis of Jukka Bodman, Bodman (2000).

On a further note on the case, M-Real Oyj has decided to expand the pulp production in Kaskinen by building a new pulp mill.
APPENDIX 2: Fuzzy Numbers and Uncertainty of Cash Flows

The effect of changes in cash flow uncertainty is presented below with an example about how hedging changes the support of a fuzzy number. Fuzzy numbers can be changed in a similar way according to changes in our perception of future cash flow uncertainty (accuracy). The example also elaborates the difference in using fuzzy numbers and single numbers (crisp numbers) in estimation of cash flows:

AN EXAMPLE. A production process relies on two raw materials Z and X. The expected price of Z is \( V_e(Z) = 20 \) unit and the estimate using a fuzzy number for the price of Z is \( V_f(Z) = [16, 24] \) (price of Z can be between 16 and 22). The expected price of X is \( V_e(X) = 30 \) unit and the estimate by using a fuzzy number for the price of X is \( V_f(X) = [28, 32] \). The production process uses 10 units of both Z and X to produce one output. First, we decide to lock the price of Z to 20 by making a contract with the producer of Z. Second, we decide to lock also X with a contract to 30. Below the three cases, before hedging, after hedging Z, and after hedging Z and X shown as presented with single numbers, fuzzy numbers, and graphically.

The different cases with single numbers: \( V_e(Z) = 20 \) unit, \( V_e(X) = 30 \) unit

<table>
<thead>
<tr>
<th></th>
<th>Before hedging Z</th>
<th>After hedging Z</th>
<th>After hedging Z and X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20<em>10 + 30</em>10 = 500</td>
<td>20<em>10 + 30</em>10 = 500</td>
<td>20<em>10 + 30</em>10 = 500</td>
</tr>
</tbody>
</table>

*Observation: NO DIFFERENCE*

The different cases with fuzzy numbers: \( V_f(Z) = [16, 24] \), \( V_f(X) = [28, 32] \)

|                      | Before hedging Z  \( [16, 24]*10 \) + \( [28, 32]*10 = [440, 560] \) | After hedging Z \( 20*10 + [28, 32]*10 = [480, 520] \) | After hedging Z and X \( 20*10 + 30*10 = 500 \) |

*Observation: THE FUZZY NUMBER CHANGES WITH UNCERTAINTY*
The different cases presented with fuzzy numbers graphically:
APPENDIX 3: On the Properties of the Heuristic Operator $\eta$

We denote trapezoidal fuzzy numbers as $A=(a,b,\alpha,\beta)$, where $A$ is a trapezoidal fuzzy number (we have defined trapezoidal fuzzy numbers in section 2.4.1.).

With this notation and by the extension principle, the extended algebraic addition operation for addition of a trapezoidal fuzzy numbers with a constant is expressed as follows:

$$k \oplus A = (k+a,k+b,k+\alpha,k+\beta)$$

if $k$ is an ordinary number (a constant).

The heuristic operator $\eta$, presented in 4.3., for inclusion of uncertainty or potential (measured quantitatively or qualitatively) as a constant in a trapezoidal fuzzy number is expressed with the same notation as:

$$k \eta A = (-k+a,k+b,k+\alpha,k+\beta),$$

when $a \leq E(A) \leq b$

$$k \eta A = (k+a,k+b,-k+\alpha \text{ or } k+\alpha,k+\beta),$$

when $a > E(A)$, $b>E(A)$, and where $-k+\alpha$ for values of $A$ between $a-\alpha$ and $a < E(A)$, and $k+\alpha$ for values of $A$ between $a-\alpha$ and $a > E(A)$

$$k \eta A = (-k+a,-k+b,k+\alpha,-k+\beta \text{ or } k+\beta)$$

when $a<E(A)$, $b<E(A)$, and where $-k+\beta$ for values of $A$ between $b$ and $b+\beta < E(A)$, and $k+\beta$ for values of $A$ between $b$ and $b+\beta > E(A)$

We may be able to solve the unfortunate issue of the discontinuity caused by the proposed heuristic operator $\eta$ with another proposed heuristic operator, $\eta_1$, also designed for addition of uncertainty (potential) to one fuzzy number by "stretching the fuzzy number". The heuristic operator $\eta_1$ is presented only as an illustration and is expressed as:

$$k \eta_1 A = (k+Da,k+Db,k+\alpha,k+\beta)$$
if uncertainty is given as an ordinary number $k$, $D_a$ is the distance between $E(A)$ and $a_n$, and $D_b$ is the distance between $E(A)$ and $b$.

The difference between the addition operator $\oplus$ and the heuristic operators $\eta$ and the proposed heuristic operator $\eta_1$ for addition of uncertainty is that, the addition operator $\oplus$ adds two numbers together, while the heuristic operator $\eta$ and the proposed heuristic operator $\eta_1$ do operations on one number.

The heuristic operator $\eta$ and the proposed heuristic operator $\eta_1$ increase the possibilistic standard deviation of the number on which the operation is made, and distribute the "added" information (number) around the expected value of the original number. We can say that the proposed heuristic operator $\eta_1$ stretches the original trapezoidal fuzzy number. It is to be noted that the selection of the possibilistic mean value to be the reference point for the heuristic operator $\eta$ and for the proposed heuristic operator $\eta_1$ is subjective.

**Numerical examples of operations with the operators $\oplus$, $\eta$, and $\eta_1$**

We assume that $A=(10,20,5,5)$ and that $k=10$, then

$k \oplus A = (20,30,15,15)$

$k \eta A = (0,30,15,15)$

$k \eta_1 A = (0,30,15,15)$

**Graphical examples of operations with the operators $\oplus$, $\eta$, and $\eta_1$**

$k \oplus A = (20,30,15,15)$

$k \eta A = (5,25,15,15)$
$k \eta_1 \ A = (0,30,15,15) + 10 =$
APPENDIX 4: Calculation of FRIV with values from the example in 4.3.1

This screenshot is for the calculation of FRIV for the case that we wait 3,5 years until we do the initial investment.

The screenshot presenting a FRIV calculation with the input values for the numerical example presented in 4.3.1., for the case when we wait 3,5 years for to invest.

The resulting FRIV value is between the minimum of €-81,34M and the maximum of €68,80M, the center value is €-6,27M, this is presented graphically and numerically in Figure 4-5. under the wait 3,5 sign.
PART II: ORIGINAL RESEARCH PAPERS
Paper 2

Paper 3

Paper 4

Collan, M. and Liu, S., 2003, Fuzzy logic and intelligent agents: towards the next step of capital budgeting decision support, Industrial Management and Data Systems, Vol. 103, No. 6, pp. 410-422
Paper 5

Turku Centre for Computer Science
TUCS Dissertations

22. Timo Kaukoranta, Iterative and Hierarchical Methods for Codebook Generation in Vector Quantization
24. Linas Laibinis, Mechanised Formal Reasoning About Modular Programs
25. Shuhua Liu, Improving Executive Support in Strategic Scanning with Software Agent Systems
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37. Vesa Halava, The Post Correspondence Problem for Marked Morphisms
38. Ion Petre, Commutation Problems on Sets of Words and Formal Power Series
39. Vladimir Kvassov, Information Technology and the Productivity of Managerial Work
40. Franck Tétard, Managers, Fragmentation of Working Time, and Information Systems
41. Jan Manuch, Defect Theorems and Infinite Words
42. Kalle Ranto, Z-,Goethals Codes, Decoding and Designs
43. Arto Lepistö, On Relations between Local and Global Periodicity
44. Mikael Collan, Studies on Boolean Functions Related to Quantum Computing
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50. Dirk Nowotka, Periodicity and Unbordered Factors of Words
51. Attila Gyenesei, Discovering Frequent Fuzzy Patterns in Relations of Quantitative Attributes
52. Petteri Kaitovaaara, Packaging of IT Services – Conceptual and Empirical Studies
53. Petri Rosendahl, Niho Type Cross-Correlation Functions and Related Equations
54. Péter Majlender, A Normative Approach to Possibility Theory and Soft Decision Support
56. Tomas Eklund, The Self-Organizing Map in Financial Benchmarking
57. Mikael Collan, Giga-Investments: Modelling the Valuation of Very Large Industrial Real Investments