Fuzzy Real Investment Valuation Model for Giga-Investments, and a Note on Giga-Investment Lifecycle and Valuation

Mikael Collan

Institute for Advanced Management Systems Research

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Mikael Collan
Åbo Akademi University,
Institute for Advanced Management Systems Research

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Abstract

Very large industrial real investments are different from financial investments and from small real investments, even so, their profitability is commonly valued with the same methods. A definition of a group of very large industrial real investments is made, by requiring three common characteristics. The decision support needs arising from these characteristics are discussed and a summary of existing methods to value and to provide decision support for large industrial investments is presented. A model built specifically to support investment decisions of very large industrial real investments and a numerical application of the model are presented. The model is discussed and commented. A note is made on an observation regarding the giga-investment lifecycle and its effect on giga-investment valuation.

Keywords: Large industrial investments, Profitability analysis, Fuzzy corporate finance

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1. Introduction

Power plants, paper machines, steel mills etc. are very large industrial real investments. They are examples of investments that require a large initial investment and have long economic lives. The profitability of these investments is dependent on the markets, i.e., these investments are made under uncertainty about the future. Very large industrial investments are important for the companies undertaking them in many ways. They may, e.g., lock the strategy of the company to the technology that is invested in for a long time and tie up a large amount of capital (that could be invested elsewhere). It may be impossible to find similar investments and circumstances for comparison purposes as the number of very large industrial investments is constrained by factors like demand, geographical location, and efficiency of technology. Because of their large size and often due to their technical nature very large industrial real investments take a long time to build. In their geographical area very large industrial real investments may have a commanding market position, or the market of a certain geographical areas may otherwise be closer to an oligopoly than a competitive market structure. Entry of a very large investment may also cause changes in the market structure, which may render it difficult to estimate future cash flows from the investment with stochastic statistical methods. All of the above translates to a set of difficult issues that managers responsible for large industrial investment decision making have to face. To make an investment decision managers need to be able to assess the profitability of investments and since large industrial investments are not straightforward from the point of view of valuation, it is the position of this paper that a new method to assist in the decision-making of very large industrial investments is needed.

A realistic approach that is based on existing methods and combines usability with innovation has been the basis for the starting the design of a new valuation model to support the decision making of very large industrial investments. To be able to model the profitability of very large industrial real investments we start by defining them. We select three characteristics to form the definition: large irreversible initial investment, long economic life (>5years), and a long time to build the investment (months). Very large industrial real investments that exhibit all these characteristics are, for the purposes of this research, called giga-investments. The three characteristics are selected based on a number of cases from Finnish heavy industry presented, e.g., in [BOD 00], [BLO 00], [PAL 00] and [LEI 01] and studied during a 4-year WAENO research program on the profitability of very large industrial investments. The selected three characteristics seem to be particularly common in the group of very large industrial investments and, therefore, can be assumed to be important to the profitability analysis of these investments in most of the cases.

The main purpose of this paper is to introduce a model designed for valuation of giga-investments and to support giga-investment decision-making, to present valuation issues arising from giga-investment characteristics, and to shortly introduce the state of the art of valuing large industrial real investments. In the following the three giga-investment characteristics are presented. Then the state of the art of valuing large industrial real investments is presented, followed by presentation of the FRIV model, designed to value giga-investments and to support giga-investment decision making. Finally a numerical example of using FRIV is presented and conclusions and discussion are made about the issues presented.

Irreversibility

Large initial investments in industrial production facilities and other industrial infrastructure are very often, if not always, irreversible. Giga-investments are by definition investments that are to a high degree irreversible. "Investments have a degree of irreversibility whenever they have attributes that make the capital specific to the firm, a product, or an industry, or else costly enough to move and relocate that the value of the capital becomes effectively tied to its original use." [BAR 94]. According to this definition most industrial investments are to a certain degree irreversible. Irreversibility is not always significant from the point of view of value. In situations where an investment is made under conditions of certainty irreversibility does not effect the value of the investment. Coupled with uncertainty irreversibility has an effect on investment value and thus becomes significant for analysis of profitability and the methods used in profitability analysis. This is investigated, e.g., in [DIX 94] where the authors provide a systematic theoretical approach to capital investment decisions stressing the uncertainty and irreversibility in most investment decisions. If the outcome of an irreversible investment is uncertain, then the possibility to postpone the investment is valuable.

Long building time

Giga-investments have a long building time (months) and the investment does not produce revenue during the period between the investment decision and the completion of the construction. During the building time the circumstances surrounding the investment may change, even significantly. This uncertainty is relevant to the decision-making. Interestingly the building time is not explicitly captured (modelled) by real option valuation or by the standard neoclassical methods, however, it is important to giga-investments.

Long economic life

The further in the future cash flows are projected to take place, the more difficult it is to forecast them accurately. Uncertainty about the accuracy of cash flow estimates increases with time, irrespective of how the prognoses are made, by expert opinion, or by using stochastic or other forecasting methods. The uncertainty of future investment cash flows comes from a large unspecified number of sources that grows with time and "As the complexity of the system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [ZAD 73]. The above quote is called Zadeh's principle of incompatibility and fits the problem a financial manager faces when she has to make profitability analyses based on prognoses about the future. Precise estimates about an uncertain future can cause a credibility problem; we can also call it a false sense of accuracy (or precision). This is highly relevant for giga-investments, because of their long economic lives and the need to estimate cash flows up to 60 years into the future.

2. The state of the art of valuing large industrial investments

This section presents the most commonly used investment valuation methods, discusses the valuation of potential with real options, discusses the importance of estimation uncertainty in valuation of investments and shortly covers the discipline of project finance.

Standard neoclassical methods

The three most commonly used (neoclassical) profitability analysis methods are NPV, IRR and Payback, of which NPV is the most important one [RYA 02] and [VER 01]. The theoretical framework underlying these methods is based on the theory of investment, pioneered, e.g., by Irving Fisher in the beginning of the 20th century. The original theory of investment does not consider irreversibility or uncertainty, this is reflected in that these models do not consider the value of waiting and the other forms of managerial flexibility. The failure to take into consideration the value of the possibility to postpone irreversible investments "undermines the theoretical foundation of standard neoclassical investment models, and invalidates the net present value rule as it is usually taught in business school..." [PIN 91] and means that "In industries with significant irreversible investments and uncertainty, these omissions mean that standard profitability measures will tend to give inappropriate indicators for investment and entry decisions..." [BAR94]. This means that giga-investment decisions are not optimally supported by standard profitability analysis methods.

Valuation of potential (managerial flexibility)

"An irreversible investment opportunity is much like a financial call option" [PIN 91] is the observation that has lead to real option valuation (ROV), the term was coined in [MYE 77] and since then a wide literature on the subject is available, see [GUI 99] and [GUI 04]. Valuing real investments with option valuation models like, e.g., with the Black and Scholes option pricing formula [BLA 73] and the CRR binomial option pricing [COX 79] is accepting the fact that the assumptions underlying the models may not be fully compatible with real investments. Because a number of different practices to use real option valuation models exist and no clear best practices have been established real option valuation practices have received criticism, e.g., [BOR 03].

The research into real option valuation models is ongoing and there are, among others, models that integrate real option valuation with soft computing techniques, e.g., fuzzy real option valuation models [ZME 01] and [COL 03a]. There are also initiatives to enhance investment decision support by making real option valuation more practical with DSS tools [ALC 03] and [COL 03b]. Despite criticism, real option valuation gives valuable support to decision making that standard neoclassical profitability analysis methods do not alone provide. There are a number of applications of real option valuation to support decision making of irreversible industrial investments, e.g., [MAJ 87] and [DIX 94].

Call option value is always positive or zero, however, the profitability of an investment can also be negative, this is why ROV is not used as such to value investments. By modifying the NPV rule from "Invest when the value of a unit of capital is at least as large as its purchase and installation cost" to "The value of the unit must exceed the purchase and installation cost, by an amount equal to the value of keeping the investment option alive" [PIN 91] we effectively include the option value of waiting in the profitability analysis. The above means that the positive potential of the investment is taken into consideration by including the real option value in the analysis. Every irreversible investment made under uncertainty can also turn negative, however, the negative potential of investments is not captured by real option valuation.

Giga-investments have a long building time (months), the investment is not productive during a period between the investment decision and the completion of the construction. During building the circumstances surrounding the investment may change, depending on the building time, even significantly. This uncertainty and the negative potential carried with it are not explicitly captured by real option valuation or by the standard neoclassical methods, however, it is important to giga-investments.

Capturing uncertainty

Everything that happens in the future is uncertain, because we cannot be certain about what will happen exactly. The further in the future cash flows are projected to take place, the more difficult it is to forecast them accurately. Uncertainty about the accuracy of cash flow estimates increases with time, irrespective of how the prognoses are made, by expert opinion, or by using stochastic or other forecasting methods. The uncertainty of future investment cash flows comes from a large unspecified number of sources that grows with time and "As the complexity of the system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [ZAD 73]. The above quote is called Zadeh's principle of incompatibility and fits the problem a financial manager faces when she has to make profitability analyses based on prognoses about the future. Precise estimates about an uncertain future can cause a credibility problem; we can also call it a false sense of accuracy (or precision). This is highly relevant for giga-investments, because of their long economic lives and the need to estimate cash flows up to 60 years into the future.

To support investment decision making the methods relying on exact estimates are commonly complemented by a set of project analysis methods to model the "what ifs". An alternative to having to use exact amounts, or estimates, in profitability analysis is to integrate the use of fuzzy numbers into profitability analysis methods to capture the perceived uncertainty[1] of future cash flow estimates. "One usually employs educated guesses, based on expected values or other statistical techniques, to obtain future cash flows and interest rates." [BUC 87] This approach resembles some methods using Monte Carlo Simulation (MCS), however, the randomly simulated cash flow distributions are replaced by possibility distributions (fuzzy numbers), based on the perception of uncertainty. While MCS based methods expect the probability of a future cash flow value to be randomly distributed between given (subjectively selected) minimum and maximum values, using possibility distributions is accepting the fact that it is possible to estimate the degree of the likelihood that a certain value occurs (and not only the minimum and the maximum values). Degree of possibility that a cash flow will have a certain value is given by the membership function of the fuzzy number that includes the set of possible cash flow values. Fuzzy sets do not follow bi-value logic that standard profitability analysis and project analysis methods use, they are based on a separate fuzzy arithmetic, see, e.g., [DUB 78] and [DUB 80]. There are existing fuzzy extensions of classical mathematics of finance including the most commonly used profitability analysis methods, e.g., [BUC 87], [LIC 88] and [KUC 00] and financial planning models based on the use of fuzzy logic [TAR 00]. Based on the earlier results it seems that using fuzzy mathematics in financial modelling is a possibility that may suit giga-investment decision making, because due to their long economic lives and irreversibility modelling forecasting uncertainty (inaccuracy) is relevant to giga-investment decision support.

Project finance

Project finance is a set of techniques designed for financing and managing large-scale capital-intensive projects and as an academic discipline investigates and develops these techniques. Project financing is most often understood as a way to finance a large investment where the revenues generated by the investment are used to repay the loans used for the financing of the project and the assets of the investment work as the collateral. In other words, the assets invested in are the motor of repayment and the guarantee for the financing.

Rationale for project financing is that it gives the possibility to spread the risk of large capital-intensive investments. Allocating the risk between number of parties makes it possible to initiate large investments in situations, where no single party is willing or able to carry all the risks and debt obligations connected to an investment. The type of financing used in large-scale projects may have an effect on the profitability of a project. The amount of leverage that a project has may change during the lifetime of the project, thus having an effect on the project risk. [EST 99] observes that changing leverage has an effect on the discount rate used in the calculation of present values of future cash flows, hence the discount rates used for investment cash flows should reflect the changes in the financing.

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[1] Uncertainty as seen by expert opinion, a measure of uncertainty, e.g., standard deviation, or some other measure (also the selection of the measure makes the perception always subjective)

Together the above four issues make the backbone of how large industrial real investments are valued and their profitability based investment decision-making supported. In the following a model that is based on the characteristics of giga-investments and the existing methods for valuation of large industrial real investments is presented.

3. Fuzzy real investment valuation model (FRIV)

The basis for building the model has been the observation that decision support offered to giga-investments by the standard profitability analysis methods is not optimal. Based on giga-investment characteristics and existing valuation methods the model has been built to consider the potential value of waiting important to irreversible investments, to include the uncertainty brought by time to build the investment, and to accept and model the perceived uncertainty of estimate accuracy.

The possibility to wait may be valuable to an investment, if waiting increases the value of the investment. This possible value increase by waiting is often called potential, however, it is possible that the value of an investment decreases during waiting. Potential from waiting is commonly modelled by real options valuation. Option valuation models, however, assume that the value of an option is always zero, or larger than zero, and hence do not take into consideration the possibility of a decrease in the investment value (negative potential). It is the intention of the FRIV model to show both, the potential and the negative potential to the decision maker, to avoid showing only the positive and thus, to avoid (showing) bias.

The potential and the negative potential are important, when there is time to wait and during the time the investment is being built. The potential and the negative potential are not symmetrical due to the fact that they are constructed differently, this resembles the separate (often different) upward and downward probabilities that are used in the CRR binomial option pricing model (commonly accepted to capture potential). The FRIV model considers the total potential (potential and negative potential) for initial costs and for the revenue stream generated by the investment separately. This is achieved for initial costs by multiplying the possibilistic standard deviation (of costs) with the possibilistic mean value and with the time to wait. For revenues the calculation is similar, but the time to build is added to the time to wait. The two potentials are added to the fuzzy present values of the initial costs and the revenues by using a heuristic context dependent operator that allows the potential to be distributed realistically. Total potential for the investment is captured by adding the fuzzy present values of initial costs and revenues, combined with their respective potentials, together to form the FRIV value.

The model separates between discount rates (crisp) for the initial costs (IC) and for the free cash flows (FCF) and between standard deviation of the IC and the FCF. Using separate (correct) discount rates for costs and revenues reflects the different risks for the different types of cash flows. Assessing different discount rates for each cash flow is supported by evidence from the project finance literature, because, e.g., the capital structure of the investment changes with time. Using separate standard deviations for cost and revenue cash flows is due to the fact that they may follow different markets and, de facto, have different volatility.

The model relies on fuzzy sets for the modelling of forecasting uncertainty (inaccuracy) and uses modern fuzzy methods to treat it in an innovative way. Possibilistic standard deviation for the costs and revenue is calculated from the aggregate fuzzy cash flow estimates making the volatility an internally determinable variable in the model. A fuzzy variable is included to handle possible costs or rewards arising from, e.g., strategic interactions (FMA/SMA). The expected value of the variable is zero. The model is suggested to be used with trapezoidal fuzzy numbers for increased usability.

Formula for the FRIV model is:

$$FRIV_t = R\eta E(R) * \sigma_R * (t + t_C) - C\eta E(C) * \sigma_C * t + \lambda_t \qquad [1]$$

where

$$R = \sum_{i=0}^{L} \frac{1}{(1+r_{Ri})^i} * R_i$$

Fuzzy present value of the fuzzy free cash flows from the project

$$C = \sum_{i=0}^{L} \frac{1}{(1+r_{Ci})^i} * C_i$$

Fuzzy present value of the fuzzy initial costs

Cash flow estimates R_i and C_i are fuzzy sets, discount rates r_{Ri} and r_{Ci} are considered crisp (single numbers) and are (can be) specific for each cash flow.

$$\sigma_C = \frac{\sqrt{\mathrm{var}_F(C_i)}}{E(C_i)}$$

$$\sigma_R = \frac{\sqrt{\mathrm{var}_F(R_i)}}{E(R_i)}$$

Possibilistic standard deviation of the fuzzy revenues (FCF), in %

The standard deviation is calculated separately for IC and FCF using possibilistic mean value and variance of fuzzy numbers as they are presented in [CAR 01].

$$\eta = \begin{cases} - \text{ when A(V)} < \text{E(A)} \\ + \text{ when A(V)} \geq \text{E(A)} \end{cases}$$

Heuristic operator

A heuristic context dependent operator is introduced to consistently treat the potential caused by the possibility to wait and the time to build. Using a possibilistic addition operator would result in a shift of the possibilistic expected value E(A) of the fuzzy set (move the set on the value axis), this effect is undesirable to our modelling purposes. By using the heuristic operator the possibilistic mean of the fuzzy set remains constant while the effect of the potential is divided to upward and downward potential (widening the set). See APPENDIX for a graphical presentation of the operator.

r_{Ri} = Discount rate specific to the free cash flows from the project (crisp number)

r_{Ci} = Discount rate specific to the initial cost cash flows (crisp number)

t = Time the possibility to invest is available (time to wait) (crisp number)

t_C = Time to the beginning of the revenue stream after entering the project (time to build the asset)
(crisp number)

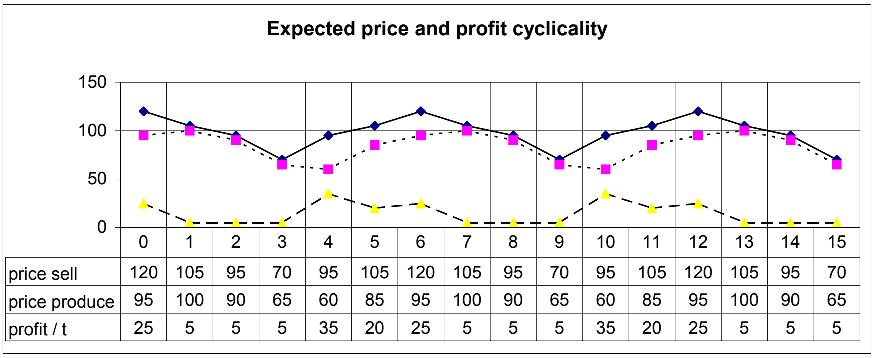
λ_t = External value created during waiting (fuzzy number)

The total potential of the investment captured by the FRIV is caused by the aggregate uncertainty during the time to wait and during the time to build. When the investment decision is made the investment cost is treated as contracted and the potential caused by the time to build comes from the revenues (FCF). It is important to note that if there is no time to wait and no building time, the FRIV collapses to fuzzy NPV (FNPV) added with λ_t. Further, if there is no uncertainty the result of the model is equal to NPV.

4. Numerical example

For a numerical example the context of the markets including the market estimates and expert assessments are presented and the characteristics of the investment explained. Then the FRIV calculations are made based on the cash flow estimates, results presented and discussed.

The business to which the investment is made is dependent on cyclical world market prices for the output product. The prices of the raw material(s) follow the market prices of the output product with a time lag. Because of the cycles, the timing of the investments made to the business is important, since wrong timing will cause the perceived investment to face falling prices for the output product, which may cause losses. The investment in question is a production facility with innovative, yet well-known to the investing company, technical characteristics that are not available to competitors. The economic life of the investment is assumed to be 15 years from now. This is based on the profitability estimates of the technology, not on the mechanical life of the machinery, i.e., if the investment was ready now it would produce the estimated cash flows for 15 years from now. After that the technology is too old to be used (this is partly a strategic quality choice). What this means in light of calculating the FRIV is that if the investment is postponed cash flows will be lost from the beginning of the investment. It is assumed that the cash flows are distributed homogeneously within any given year. The size of the initial investment is estimated at around € 180 million and the time to build the investment is 24 months. The building costs are estimated to be between 89 and 90 million for the first year and between 90 and 92 million for the second year of building. The company in question has been in business for 40 years and has industry experts with decades of experience in the markets and with similar investments in the past. The investment, if built, will have an effect on the local markets and a slight effect on the world markets, therefore the historical information about the markets is not fully relevant for decision making. Exhibit 1 shows the commonly expected[2] yearly prices of 1 ton of the output product, the production price (incl. the raw materials) for 1 ton of the product, and the derived estimated yearly profits for production of 1 ton of the product in question. The production capacity is 300 KT ad annum. The opportunity cost of capital for the FCF is determined by experts to be at the level of 12% and for the IC at the level of 7%, i.e., these will function as the discount rates for the IC and FCF cash flows (simplification to use a single discount rates).



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
price sell	120	105	95	70	95	105	120	105	95	70	95	105	120	105	95	70
price produce	95	100	90	65	60	85	95	100	90	65	60	85	95	100	90	65
profit / t	25	5	5	5	35	20	25	5	5	5	35	20	25	5	5	5

Exhibit 1: Expected market environment perceived to face the investment

The experts in the company that is contemplating investment into the production facility estimate the revenues (FCF) from the investment according to the perceived market environment and their personal subjective knowledge about the markets. They have given their expert opinions as intervals (fuzzy numbers) to include the subjectively perceived uncertainty that they see facing each of the cash flows. Exhibit 2 presents the expert opinions together with the FCF expectation derived from the commonly expected prices. The further into the future the cash flows need to be estimated the more uncertainty the experts seem to perceive.

There are no expected first or second mover advantages identified by the experts and the external value created during waiting is expected to be zero. Taking into consideration the fact that timing is of the essence and that the trend is now (time 0) downward sloping the managers wish to see what is the effect of postponing the building of the investment, the

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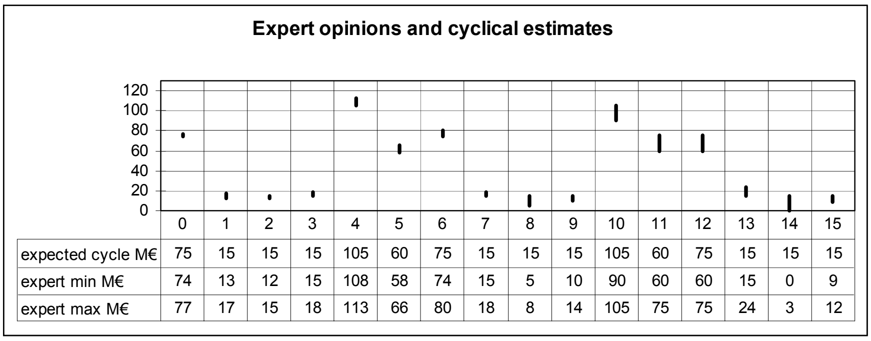
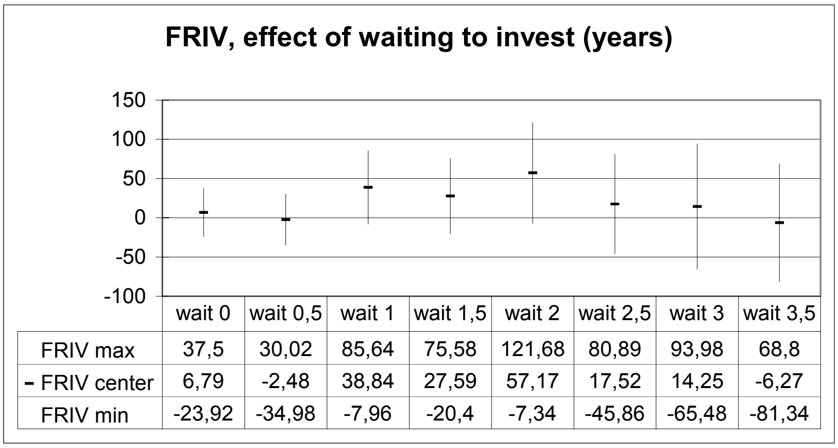
[2] The common industry expectation of the prices



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
expected cycle M€	75	15	15	15	105	60	75	15	15	15	105	60	75	15	15	15
expert min M€	74	13	12	15	108	58	74	15	5	10	90	60	60	15	0	9
expert max M€	77	17	15	18	113	66	80	18	8	14	105	75	75	24	3	12

Exhibit 2: Cash flow estimations made by experts, based on their perception of the future of the markets facing the investment and their expert knowledge about the uncertainty

The expectation is that if the investment is postponed the cash flows from the time during waiting will be lost, ceteris paribus. Exhibit 3 presents the FRIV values for the eight different possibilities (invest now to postpone until 3½ years). Not unexpectedly, it seems that the investment is most profitable, if postponed by two years. It is also visible from exhibit 3 that according to the FRIV the total potential of the investment grows the longer the investment is postponed. If managers decide to postpone the investment for two years and expect to enhance the profitability they also must face increased uncertainty (total potential) about the project outcome.



	wait 0	wait 0,5	wait 1	wait 1,5	wait 2	wait 2,5	wait 3	wait 3,5
FRIV max	37,5	30,02	85,64	75,58	121,68	80,89	93,98	68,8
- FRIV center	6,79	-2,48	38,84	27,59	57,17	17,52	14,25	-6,27
FRIV min	-23,92	-34,98	-7,96	-20,4	-7,34	-45,86	-65,48	-81,34

Exhibit 3: Effect of waiting to invest to the FRIV value

The fact that FRIV shows the increase of uncertainty (total potential) caused by waiting and by the time to build is highly relevant to giga-investments due to the fact that timing is often essential for large industrial investments. The model does not bias the information given to the decision-maker by showing only the positive potential, but clearly shows also the possible negative scenario as well. This is due to the fact that the cash flows are given as fuzzy numbers (include the perceived uncertainty as seen by the expert), as well as, to the fact that the added uncertainty caused by time to wait and by time to build is included in the result. A screenshot from an experimental excel based Interval FRIV Tool used to calculate the FRIV is available in the APPENDIX.

5. Conclusions and Discussion

The FRIV model is based on giga-investment characteristics and the observation that they can be modelled with methods from existing financial literature. From a model building point of view FRIV model integrates three different threads of real investment decision support modelling: modelling of profitability, modelling of uncertainty, and modelling of potentiality, each of which are commonly done separately by using three sets of complementary methods. It is also a clear attempt to make a context dependent model for a specific group of investments that demand custom investment decision support. The FRIV model offers a new approach to modelling negative and positive potential together, which can be said to be very close to the way a risk manager would consider an investment, i.e., the possible loss is clearly taken into consideration

The fact that the model has a similar construct to NPV and fuzzy NPV is helpful when it is showed to managers who are accustomed to these methods. The jump from NPV to FRIV may be easier to understand due to the simple overall construction of the model, than the jump from NPV to real option valuation. Still the model is able to capture potential in a similar way that binomial option valuation is used to capture it, when used for real option valuation.

The nature of the model is also dynamic, or at least the model gives the possibility to escape from the "plan and forget" paradigm of investment decision support. By this comment the author wishes to point to research on the advantages reached by a process of constantly gathering, analysing and integrating information to the investment planning process, e.g., [COL 02] and the emergence of methods to support this process, e.g., [WAL 00], [LIU 98] and [COL 03c]. The dynamic nature of the model is apparent in the use of fuzzy cash flow estimates and in the way the standard deviation is internally generated from future cash flow estimates, and hence new information about the cash flows has a direct effect on the FRIV result.

The heuristic context dependent operator used in the model makes the model escape some problems that using the standard possibilistic operators would yield, however, the heuristic operator is also a simplification of the reality, for it assumes the possibilistic mean value to divide the distribution of the potential. This may in some cases be inaccurate, a case-wise consideration to modify the operator may be needed. The term describing the value created during waiting and building is a simplistic one term aggregate representation of the net value created during waiting and building as a whole. This means that to be able to give a reasonable value for the variable demands a separate game theoretic consideration of the situation (investment) at hand.

Using the possibilistic standard deviation for estimation of potential from the time to wait and from the time to build may bias the value of the potential, because it is calculated from the aggregate values of IC and FCF. The actual uncertainty may be higher or lower than the modelled uncertainty. This effect may be significant for giga-investments, because of their long economic lives. It may be beneficial to investigate, case-by-case, how the standard deviation should be calculated for the time to wait and for the time to build. However, when considering competing investments it is important that the method used is uniform, the selection of the possibilistic standard deviation has been based on usability and robustness of the method. When considering competing investments valued with FRIV, the result (fuzzy number) makes it necessary to use some descriptive numbers, or defuzzification, for ranking the investment alternatives. This adds a step to using the FRIV, which crisp number NPV does not have.

The main contribution of the model is to offer specific investment decision support to very large industrial real investments that exhibit the giga-investment characteristics. This is a group of investments that has previously not been selected for special modelling, despite the fact that it clearly is a group of investments that has a high degree of importance for the companies undertaking them. Future research directions include testing the FRIV model with investment cases, empirically assessing the "goodness" of the model to provide decision support for very large industrial investments, and finding out managerial reactions to the type of decision support the model provides.

The author wishes to acknowledge the support of Peter Majlender.

6. Note on giga-investment lifecycle and valuation

To understand the lifecycle (and valuation) of giga-investments better, we can divide giga-investment life into three major stages that we call planning (stage), building (stage), and operation (stage), see exhibit 4. Planning stage is the time before the giga-investment decision, when the giga-investment, as a whole, can be viewed as an option (to invest). Both, investment cost and value are uncertain.

Building stage is the period, under which the giga-investment is under construction, after the irreversible giga-investment decision. The giga-investment cost is most often contracted, and can be understood nearly as fixed (certain), or having a low standard deviation. The value of the giga-investment is, however, uncertain. The building stage of giga-investments resembles a commodity ***forward*** contract, where the price is fixed, but market price is uncertain. To the best of our knowledge, this is an original observation, however, it is not in the scope of this research to investigate forward pricing of giga-investments.

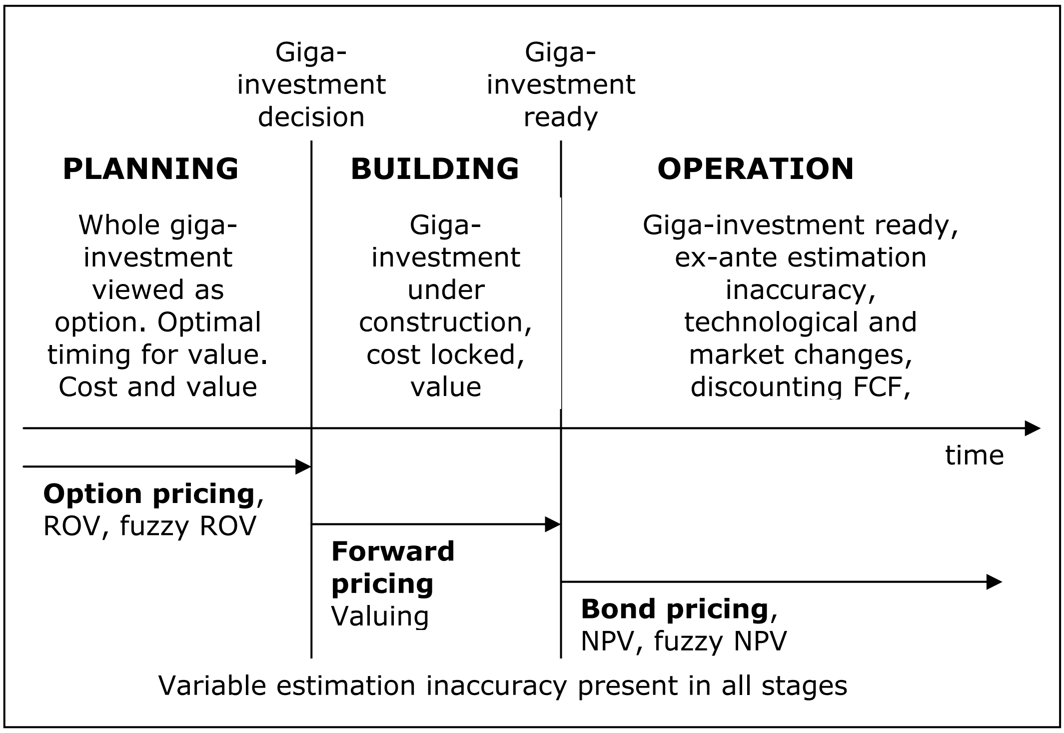


Exhibit 4. Giga-investment lifecycle presented in three stages, planning, building, and operation - resembling option, forward and bond pricing.

Operation stage is the stage after completion of the construction of the giga-investment, until the end of its economic life. The operation stage of the giga-investment resembles a bond contract, and is commonly valued with NPV. Estimation inaccuracy of the variables affecting the value of the giga-investment, is an issue that is present in all stages of the giga-investment lifecycle.

Valuation of the whole giga-investment lifecycle can be done by combining the valuation of the three giga-investment stages.

When designing models and methods to aid in the valuation and decision making of giga-investments it is important to take into consideration the issues arising from the giga-investment characteristics, and the issues arising from the giga-investment lifecycle.

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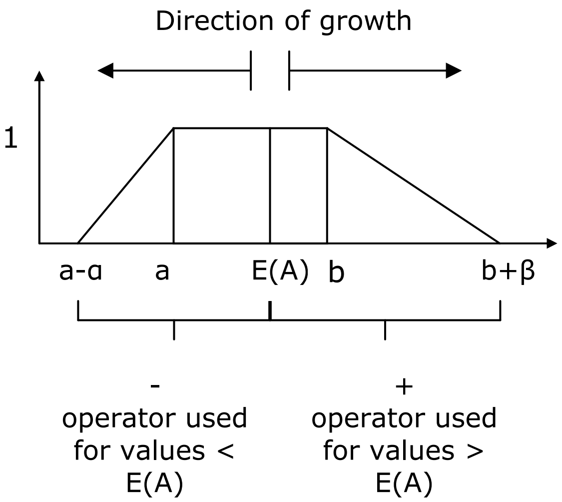
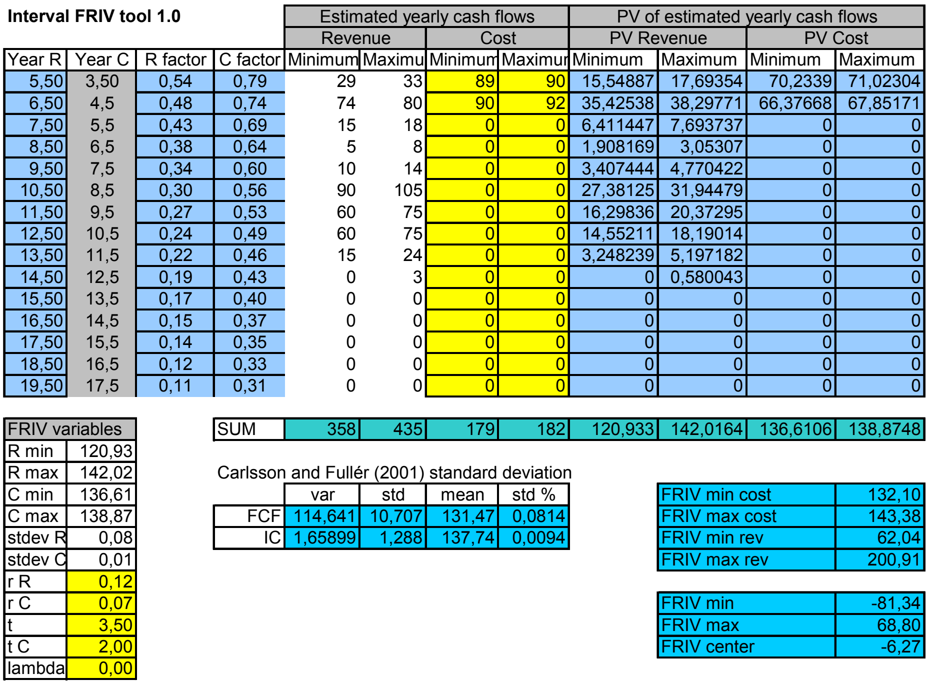
APPENDIX

Graphical illustration of how the heuristic operator works. Potential (crisp) is deducted from values below the possibilistic mean value E(A) and added to values above E(A).



Screenshot from an interval FRIV tool used for calculation



Interval FRIV tool 1.0

				Estimated yearly cash flows				PV of estimated yearly cash flows			
				Revenue		Cost		PV Revenue		PV Cost	
Year R	Year C	R factor	C factor	Minimum	Maximu	Minimum	Maximur	Minimum	Maximum	Minimum	Maximum
5,50	3,50	0,54	0,79	29	33	89	90	15,54887	17,69354	70,2339	71,02304
6,50	4,5	0,48	0,74	74	80	90	92	35,42538	38,29771	66,37668	67,85171
7,50	5,5	0,43	0,69	15	18	0	0	6,411447	7,693737	0	0
8,50	6,5	0,38	0,64	5	8	0	0	1,908169	3,05307	0	0
9,50	7,5	0,34	0,60	10	14	0	0	3,407444	4,770422	0	0
10,50	8,5	0,30	0,56	90	105	0	0	27,38125	31,94479	0	0
11,50	9,5	0,27	0,53	60	75	0	0	16,29836	20,37295	0	0
12,50	10,5	0,24	0,49	60	75	0	0	14,55211	18,19014	0	0
13,50	11,5	0,22	0,46	15	24	0	0	3,248239	5,197182	0	0
14,50	12,5	0,19	0,43	0	3	0	0	0	0,580043	0	0
15,50	13,5	0,17	0,40	0	0	0	0	0	0	0	0
16,50	14,5	0,15	0,37	0	0	0	0	0	0	0	0
17,50	15,5	0,14	0,35	0	0	0	0	0	0	0	0
18,50	16,5	0,12	0,33	0	0	0	0	0	0	0	0
19,50	17,5	0,11	0,31	0	0	0	0	0	0	0	0

SUM	358	435	179	182	120,933	142,0164	136,6106	138,8748

FRIV variables	
R min	120,93
R max	142,02
C min	136,61
C max	138,87
stdev R	0,08
stdev C	0,01
r R	0,12
r C	0,07
t	3,50
t C	2,00
lambda	0,00

Carlsson and Fullér (2001) standard deviation

	var	std	mean	std %
FCF	114,641	10,707	131,47	0,0814
IC	1,65899	1,288	137,74	0,0094

FRIV min cost	132,10
FRIV max cost	143,38
FRIV min rev	62,04
FRIV max rev	200,91

FRIV min	-81,34
FRIV max	68,80
FRIV center	-6,27