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Causes, Effects and implications for economic and social policy

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Abstract

The causes and effects of technical change are investigated in a multi-sector economy. The underlying modelling framework is a hybrid of Classical economic thinking as introduced by Ricardo (1821) and formalised by Sraffa (1960), and of Evolutionary economics following Schumpeter (1934) and Nelson & Winter (1982). The special case of one sector is elaborated at length, leading to several implications concerning economic and legal policy in the presence of ongoing technical change. This includes technological unemployment and technologically induced wage inequalities which are either temporary or persistent, and also the problem of effective demand in a dynamic economic environment is discussed. Within the model business cycles as a consequence of innovative general purpose technologies with subsequent technical progress can be illustrated.

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The causes and effects of technical change are investigated in a multi-sector economy. The underlying modelling framework is a hybrid of *Classical* economic thinking as introduced by Ricardo (1821) and formalised by Sraffa (1960), and of *Evolutionary* economics following Schumpeter (1934) and Nelson & Winter (1982). The special case of one sector is elaborated at length, leading to several implications concerning economic and legal policy in the presence of ongoing technical change. This includes technological unemployment and technologically induced wage inequalities which are either temporary or persistent, and also the problem of effective demand in a dynamic economic environment is discussed. Within the model business cycles as a consequence of innovative *general purpose technologies* with subsequent technical progress can be illustrated.

1 Introduction

Looking at economic growth patterns throughout the past centuries, one can identify a *take-off* of per-capita gross domestic product at the end of the 18th and the beginning of the 19th century, a time which is labelled *industrial revolution*. The term stems from the observation of a number of important inventions at that time, which fostered the subsequent development of technological knowledge and its application. These activities, including the respective institutional environment, are often put centre stage in explaining economic growth patterns of modern civilisation (e.g. Mokyr, 1992). Hence, one can conclude that technical progress is one of the driving forces of economic growth since the take-off of the industrial revolution. The causal relationship between technical change and economic growth challenged economists since Adam Smith's (1723–1790) *Inquiry into the Nature and Causes of the Wealth of Nations* (1776). It was David Ricardo (1772–1823) in his book *Principles of political economy and taxation* (1821), who made the *choice-of-technique* problem explicit, asking for the economic and social causes and effects concerning the use of different modes of production. His thinking was concerned with the *long-period position* of an economy. This was formalised by Piero Sraffa (1898–1983) in his influential book *Production of Commodities by Means of Commodities* (1960) and further elaborated by Kurz & Salvadori (1995). The complementary approach, dealing with short-run effects of technical change, is provided by *evolutionary economics* launched by Nelson & Winter (1982). This strand of economic literature heavily relies on the analysis of Joseph A. Schumpeter (1883–1950) concerning the economic effects of innovations (see Dopfer, 2005, for a recent stocktaking of this field of research). What gets apparent as a consequence of the *bounded rationality*-assumption of evolutionary economics is the time-lag between the emergence of some innovation and its adoption due to uncertainty. On empirical grounds, this effect is investigated by *diffusion research* (Rogers, 2003), which provides insights into the importance of the institutional setting influencing the emergence of innovations as well as the pace of the diffusion of some new technology. Diffusion research also acknowledges and studies the social consequences of this process, since economic dynamics triggered by technical change necessarily induce structural change within a social system.

This article adds to the existing literature on economic and social causes and consequences of the diffusion of innovations by setting up a theoretical framework to study inter-sector feedback effects of the emergence of *process innovations*. It connects the concept of a *long-period position* (Kurz & Salvadori, 1995) with evolutionary ideas provided by the concept of *replicator dynamics* as utilised by *evolutionary game theory* (Weibull, 1997; Metcalfe, 1998). The stated problem is closely related to Schumpeter's (1934) dynamic approach to economic development and is accomplished by a fusion of classical and evolutionary thinking as indicated by Kurz (2008). The discrete-time one-sector analysis of Steedman & Metcalfe (2011) provides the intellectual starting-point for this article, whereas the presented framework introduces an extension to multiple sectors in a time-continuous setting.

The proposed model is capable of explaining important economic and social effects of the diffusion of innovations. This includes the clarification of causes of sustained technological unemployment and its compensation, as well as the exposition of both transitory and permanent technologically induced wage inequality. Social and fiscal policy measures such as taxation and unemployment benefits are introduced and evaluated, as they can relieve the frictions resulting from wage inequality and unemployment. Furthermore, in a multi-sector setting frictions concerning effective demand necessarily arise out of the dynamics caused by technical change. Within the presented framework an analysis of the emergence of business cycles and recessions is provided.

To accomplish the announced program, the article proceeds in three steps. Firstly, the static framework of a multi-sector economy with heterogeneous labour is introduced in section 2 to clarify price and quantity determination in the long-period position. Next, section 3 augments the static model by introducing evolutionary aspects, including the diffusion of innovations through the economic system. The concluding section 4 applies the presented framework to the analysis of the emergence of social and economic frictions in the presence of technical change.

2 Static model

Technical change induces a non-trivial dynamic element into economic systems. To facilitate the understanding of this dynamism, first of all the static properties of the respective system (i.e. in the presence of fixed technologies) have to be scrutinised. The difference between these two methodological approaches, namely *static* and *dynamic* economic modelling, and their interdependence was stressed by Schumpeter inter alia in his influential book *Theorie der wirtschaftlichen Entwicklung* (1934, ch. 2). Following his suggestion and content-related approach, this section deals with static pre-work to pave the way for dynamic considerations in the following section 3. The partitioning into subsection 2.1 focusing on prices and quantities and subsection 2.2 addressing wages and wage inequalities is owed to the importance of economic growth on the one hand, and of questions concerning social inequality on the other hand.

2.1 Prices and quantities in the long-period position

To facilitate the understanding of economic dynamics in the presence of technical change, (Schumpeter, 1934, ch. 1) began his analysis with the *circular flow* as an abstract, static economic concept. A circular flow is defined by the three characteristic features *private property*, *division of labour* and *free markets*. These aspects are necessary to allow for a benchmark-description of *production* and of *barter*, which are the two basic kinds of economic activity. The latter arises out of the division of labour in the sense Schumpeter uses this concept, implying that different sectors exist, each one producing some specific good. The respective commodities are then traded between these sectors. Formalising this idea, a number $N \in \mathbb{N}_+$ of sectors exists, each sector $n \in [1, N]$ producing one specific product. Smith (1776, Book I, ch. I) also mentions the division of labour, but in a different context. He introduced it as a diversification of labour within the production process of a single sector. Hence, different skills are needed for some specific production process. Formally, $K \in \mathbb{N}_+$ skills exist, which in the argumentation of Smith (1776, Book I, ch. X) may be remunerated differently for several reasons, such as severeness or extraordinary skill. Wage premia can also be shown to exist on empirical grounds (see e.g. Grogger & Eide, 1995), and it is additionally justified by contemporary principle-agent theory (Bolton & Dewatripont, 2005). Relative wages in this case can be represented by some vector $\mathbf{u} \in \mathbb{R}_+^K$ with $\|\mathbf{u}\| = 1$, and by the overall wage level $w > 0$. Real wages are given by $\mathbf{w} = w\ \mathbf{u}$, with w_k denoting the remuneration of one unit of labour of skill k.

In the argumentation of Schumpeter, the circular flow is characterised by well-defined behaviour of all economic agents of the system, which accrues from habit formation. In this respect, not conscious economic calculation, but habitude build the basis for economic action. This is true both for producers as well as for consumers. The latter have to decide, which (aggregate) commodity bundle $\mathbf{y} \in \mathbb{R}_+^N$ they buy, where y_n denotes the final demand of good n. The firms have two different choices to make: Firstly, the technology they utilise, and secondly, the quantity of output they produce. These choices are co-determined by the economic environment, especially by prevailing prices. Hence, by constructing a static economic model, prices and quantities have to be chosen coherently. The classical view implies some circularity within the production process: Commodities are produced by means of commodities (Sraffa, 1960). Thus, the production process of some specific sector is characterised by coefficients a_{nm} $(n, m = 1, \ldots, N)$ of the *capital input matrix* $A \in \mathbb{R}_+^{N \times N}$, denoting the input of commodity m which is needed to produce (on average) one unit of output of good n. Obviously, $a_{nm} \in [0, 1]$ and $1 - \sum_m a_{nm} \geq 0$ has to hold. The coefficients l_{nk} $(n = 1, \ldots, N,\ k = 1, \ldots, K)$ of the *labour input matrix* $L \in \mathbb{R}_+^{N \times K}$ denote the amount of skill k-labour needed to produce one unit of good n.

To get coherent habits of firms, characterised by the modes of production (A, L) and output $\mathbf{x}$ (x_n is the total output of sector n), the *market clearing condition*

$$A^T\mathbf{x} + \mathbf{y} = \mathbf{x} \tag{1}$$

has to hold on the goods markets, and labour demand $L^T\mathbf{x}$ must not exceed the total workforce of the economy. The left-hand-side of equation (1) represents the partition of the aggregate expenditures of the households of the economy into savings $A^T\mathbf{x}$ (which equal investments into production) and final consumption $\mathbf{y}$.

Next, also prices have to be determined properly. Capital input $A^T\mathbf{x}$ is remunerated by some rate of profits $1+r$, which equals the prevailing rate of interest. In the present setting, r is exogenously given. It can be assumed to be determined by monetary policy or by financial market conditions. Wages $\mathbf{w}$ on the other hand are paid for labour input $L^T\mathbf{x}$. Costs of production are determined by prices of input factors, and prices of output equal costs of production. The price vector $\mathbf{p}$, with $(\mathbf{p})_n \equiv p_n$ denoting the price of commodity n, is therefore given by

$$(1+r)A\mathbf{p} + L\mathbf{w} = \mathbf{p} \tag{2}$$

if the period of production is normalised to one for all sectors. In monetary terms, the (aggregate) *budget contraint*

$$\mathbf{x}^T\mathbf{p} = \mathbf{x}^T\left[L\mathbf{w} + (1+r)A\mathbf{p}\right] = \mathbf{y}^T\mathbf{p} + \mathbf{x}^T A\mathbf{p} \tag{3}$$

has to hold. Equations (3) indicate that the nominal value $\mathbf{x}^T\mathbf{p}$ of total output is consumed by aggregate wage income $\mathbf{x}^T L\mathbf{w}$ and by aggregate capital income $(1{+}r)\mathbf{x}^T A\mathbf{p}$. Total expenditures of the economy are partitioned into *nominal gross domestic product* (nGDP) $\mathbf{y}^T\mathbf{p}$ and nominal value $\mathbf{x}^T A\mathbf{p}$ of capital investments, as indicated by the market clearing condition (1). On average, the rate of profits $1+r$ is equalised across sectors by means of a *no-arbitrage* argument, which prevails in the presence of free markets. Accordingly, both equity capital as well as debt capital have to yield the same returns. Both labour income $\mathbf{x}^T L\mathbf{w}$ as well as capital income $(1+r)\mathbf{x}^T A\mathbf{p}$ can be divided into consumption $\mathbf{y}^T\mathbf{p}$ and investment $\mathbf{x}^T A\mathbf{p}$. Hence, investment is provided by savings.

2.2 Wages and wage-inequality

Given relative wages $\mathbf{u}$ and the uniform rate of profit $1+r$, prices are derived from equation (2) as

$$\mathbf{p} = w\left[\mathbb{I} - (1+r)A\right]^{-1} L\mathbf{u} > \mathbf{0}. \tag{4}$$

Non-negativity of prices holds as a consequence of the *Perron-Frobenius Theorem* (Kurz & Salvadori, 1995, ch. A.3). Introducing a *numéraire* $\mathbf{d} \in \mathbb{R}_+^N$ by $\mathbf{d}^T\mathbf{p} = 1$, the wage level w can be determined from (4):

$$w = \frac{1}{\mathbf{d}^T\left[\mathbb{I} - (1+r)A\right]^{-1} L\mathbf{u}} \tag{5}$$

This is the $w - r$ relationship already discussed by Ricardo (1821) and explicated by (Kurz & Salvadori, 1995, p. 98).

Social inequality can be studied with respect to the distribution of wealth between labourers and capitalists or by investigating wage inequalities within the group of workers. Since in the present setting wage earners, by means of savings, also receive capital-income, the two groups cannot be separated straightforwardly and hence this article focusses on the latter. Let $\hat{\mathbf{u}} \in \mathbb{R}_+^{K+1}$ be the normalised vector (i.e. $\|\hat{u}\| = 1$) of wage income after tax deduction and subsidy payments, where the zeroth entry $\hat{u}_0$ (by abuse of mathematical notation) denotes unemployment payments, and $\hat{u}_k \geq \hat{u}_{k-1}$ for $i = 1, \ldots K$. If skill level k receives subsidies, then $u_k < \hat{u}_k$; if taxes have to be paid, then $u_k > \hat{u}_k$. Unemployment U is straightforwardly defined by

$$U = 1 - \sum_{i=k}^{K} \left(\mathbf{x}^T L\right)_k \geq 0,$$

if the work force is normalised to one (no population growth is considered). The GINI index as a measure of wage-inequality reads

$$GINI = 1 - \sum_{k=0}^{K} s_k \frac{\mu_k + \mu_{k-1}}{\mu_K}. \tag{6}$$

$s_k = \left(\mathbf{x}^T L\right)_k$ for $k = 1, \ldots, K$ denotes the amount of skill-k-labour which is employed, and $s_0 = U$. $\mu_k = w\sum_{i=0}^{k} \hat{u}_i s_i$ is the aggregate disposable income of all workers up to wage-level $k = 0, \ldots, K$, and $\mu_{-1} \equiv 0$.

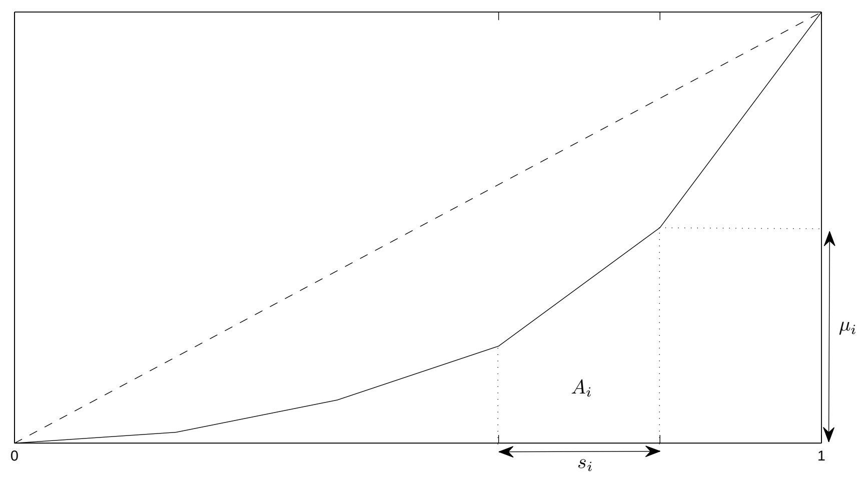

Figure 1: Calculating the GINI-coefficient in case of K wage levels.

Proof of equation (6). Calculating the areas $A_i = s_i\left(\mu_i + \mu_{i-1}\right)/2$ in figure 1, the GINI index is determined by

$$GINI = \frac{\frac{1}{2}\mu_K - \sum_{i=0}^{K} A_i}{\frac{1}{2}\mu_K}.$$

Rearranging terms yields the proof of expression (6). □

To get a first idea of the meaning of the general expression (6), it can be analysed for the special case of two different skills remunerated by $\mathbf{u} = (u_1, u_2)^T$. Full employment implies $s_1 + s_2 = 1$, leading to

$$GINI = (1 - s_2)\frac{(u-1)s_2}{1 + (u-1)s_2}. \tag{7}$$

$u - 1 = (u_2 - u_1)/u_1 > 1$ denotes the wage premium of skill 2. Higher wage premia therefore lead to higher inequality, as the GINI index in (7) is positively related to u. The case of total equality, defined by $GINI = 0$, can only be obtained for the trivial cases that all workers provide the same skill to the labour market, i.e. either $s_1 = 0$ or $s_2 = 0$, or if both skill levels are remunerated with the same wage, i.e. $u = 1$. Maximal inequality

$$GINI^* = \frac{\sqrt{u} - 1}{\sqrt{u} + 1}$$

can be calculated by differentiation of (7) with respect to s_2 and is obtained by a share

$$s_2^* = 1 - s_1^* = \frac{1}{\sqrt{u} + 1}$$

of workers employed at the higher wage level. Wage inequality in this case is technologically and politically co-determined, since s_2 is defined by the labour input matrix L, and $\mathbf{u}$ can be altered by re-distributional measures.

3 Evolutionary dynamics in the presence of technological change

Deviating from the static view of the economy in the preceding section, several different processes are introduced in every sector. Each process is characterised by its input coefficients and the respective costs of production. Firms producing cheaper than average will gain extra profits, and therefore grow, whereas non-innovative firms face ongoing losses, as prices converge towards the lower costs of production. Hence, firms are forced to be innovative or they are driven out of the market (*creative destruction*; Schumpeter, 1954, ch. 7). The respective model is formulated for a multi-sector economy in section 3.1, whereas the one-sector case is analysed in depth in section 3.2.

3.1 Evolutionary model of technological diffusion

As in section 2, let N be the number of different sectors and K the number of different skills. For each sector n a finite set $\mathcal{I}_n \subset \mathbb{N}_+$ of different production processes exists. Process $i_n \in \mathcal{I}_n$ produces some output $x_n^{i_n}$, and total output x_n of the sector is determined by

$$x_n = \sum_{i_n \in \mathcal{I}_n} x_n^{i_n}.$$

Once a new process is invented, $\mathcal{I}_n$ is enlarged by one element. An invention $j_n \in \mathcal{I}_n$ is an *innovation* if and only if $x_n^{j_n} > 0$. Each production process i_n of sector n is characterised by input vectors $\mathbf{a}_n^{i_n} \in \mathbb{R}_+^N$ and $\mathbf{l}_n^{i_n} \in \mathbb{R}_+^K$. $a_{nm}^{i_n} = (\mathbf{a}_n^{i_n})_m$ denotes the amount of commodity m utilised by process i_n to produce one unit of commodity n. Analogously, $l_{nk}^{i_n} = (\mathbf{l}_n^{i_n})_k \geq 0$ denotes labour-input of skill k for the unit-production of good n. Different processes are employed, and therefore also unit costs of production

$$c_n^{i_n} = (1+r)(\mathbf{a}_n^{i_n})^T \mathbf{p} + w\ (\mathbf{l}_n^{i_n})^T \mathbf{u} \tag{8}$$

of processes $i_n \in \mathcal{I}_n$ in sector n may differ. In the presence of free markets, only one single price p_n exists for the commodity produced in sector n. As an assumption, within each sector n the aggregate budget constraint

$$x_n p_n = \sum_{i_n \in \mathcal{I}_n} x_n^{i_n} c_n^{i_n} \tag{9}$$

holds, indicating that innovative firms can sell above their costs of production, whereas non-innovative firms have to sell with a loss. In terms of shares $q_n^{i_n} \equiv x_n^{i_n}/x_n$ of output of some specific process i_n, one gets the price p_n of commodity n determined by *average unit costs of production*, namely

$$p_n = \sum_{i_n \in \mathcal{I}_n} q_n^{i_n} c_n^{i_n}. \tag{10}$$

Inserting the individual costs of production (8) of the respective processes into the price equation (10) for all sectors n, one can collect terms and formally define some average technology

$$\begin{aligned} \bar{\mathbf{a}}_n &= \textstyle\sum_{i_n \in \mathcal{I}_n} q_n^{i_n} \mathbf{a}_n^{i_n} \\ \bar{\mathbf{l}}_n &= \textstyle\sum_{i_n \in \mathcal{I}_n} q_n^{i_n} \mathbf{l}_n^{i_n}, \end{aligned}$$

leading to

$$p_n = (1+r)(\bar{\mathbf{a}}_n)^T \mathbf{p} + w(t)(\bar{\mathbf{l}}_n)^T \mathbf{u}. \tag{11}$$

Combining all N sectors, the *mean technology* of the economy is represented by $\big(\bar{A}, \bar{L}\big)$, where $\bar{\mathbf{a}}_n^T$ is the n-th row of matrix $\bar{A} \in \mathbb{R}_+^{N \times N}$ and $\bar{\mathbf{l}}_n^T$ is the n-th row of matrix $\bar{L} \in \mathbb{R}_+^{N \times K}$. Equation (11) concisely can be written as

$$(1+r)\bar{A}\mathbf{p} + w\bar{L}\mathbf{u} = \mathbf{p}, \tag{12}$$

which is the dynamically determined price system, replacing equation (2) of the static model.

Technical progress can temporarily change the average rates of profit of the innovative sector. The respective extra profits $\rho_n^{i_n}$, which can be gained by technology i_n in sector n, are implicitly defined by

$$\big(1 + r + \rho_n^{i_n}\big) (\mathbf{a}_n^{i_n})^T \mathbf{p} + w(t)(\mathbf{l}_n^{i_n})^T \mathbf{u} = p_n. \tag{13}$$

These extra profits influence both the decision of firms which technology to use, and depending on the choice i_n of some firm it determines its growth potential. Hence, two channels can be identified of how the market share $q_n^{i_n}$ of some technology i_n within sector n changes over time.

Diffusion by choice: On the one hand, firms have the freedom to choose between processes given by $\mathcal{I}_n$. Uncertainty, financial constraints or ignorance prevent an instantaneous switch of all firms to the new technology. Hence, the diffusion of information and of confidence (Rogers, 2003) play a role in the process of the diffusion of some superior (cheaper) technology. This phenomenon is the underlying idea of the *Brass-model* (surveyed for example by Mahajan *et al.*, 2000), which in the presence of two technologies can be stated as

$$\dot{q}(t) = Q\ q(t)[1 - q(t)] + M\ [1 - q(t)], \tag{14}$$

if q denotes the share of output produced by the cheaper process. The second term on the right-hand-side of equation (14) indicates the influence of mass-media, calibrated by some parameter $M > 0$. It leads to a first innovative push within the system as a consequence of newly available information, and it fades out as more and more firms apply the best-practice technology. As soon as a positive share of output is produced by the innovative technology, information networks, characterised by some parameter Q, start to operate, since knowledge is passed on from innovative firms to those firms still applying the old technology. This is indicated by the first term on the right-hand-side of equation (14), which models *word-of-mouth* information diffusion by some epidemic mechanism. $q(t)[1 - q(t)]$ is the probability that two firms meet, where one is a laggard (sticking to one of the expensive processes) and one is an innovator (using the best-practice technology). With some probability $Q \in [0, 1]$ the non-innovative firm gets convinced to switch to the innovative process, if it meets an innovator.

More generally, assume that at each time t two infinitesimally small firms meet, with firm 1 using process i_n and firm 2 using process j_n, respectively. They know about their unit costs of production and about their extra profits (or losses) $\rho_n^{i_n}$ and $\rho_n^{j_n}$. Meeting each other leads to a transfer of knowledge of production process. Firm 1 will switch to process j_n if and only if it is convinced that extra profits can be increased by this action, i.e. if $\rho_n^{j_n} > \rho_n^{i_n}$. Let $f(\rho_n^{j_n} - \rho_n^{i_n})$ denote the probability with which firm one believes in $\rho_n^{j_n} > \rho_n^{i_n}$. Otherwise, firm two switches to process i_n, which induces a symmetric situation if beliefs are the same across firms. Since $q_n^{i_n}$ denotes the probability to draw a firm using process i_n out of the continuum of firms, one gets

$$\dot{q}_n^{i_n} = q_n^{i_n} \sum_{j_n \in \mathcal{I}_n} q_n^{j_n} \left[2\ f\left(\rho_n^{i_n} - \rho_n^{j_n}\right) - 1\right] \tag{15}$$

for all $i_n \in \mathcal{I}_n$. In case of firms with finite size, the right-hand-side of equation (15) denotes the transition rates of some *Markov jump process* (Rainer & Schütz, 2012).

Proof of equation (15). Since $q_n^{i_n}$ is the probability that firm 1 uses process i_n, $q_n^{j_n}$ is the probability that firm 2 uses process j_n, and $f(\rho_n^{j_n} - \rho_n^{i_n})$ is the probability that process i_n is superior to process j_n, equation (15) is a consequence of

$$\dot{q}_n^{i_n} = q_n^{i_n} \left[\sum_{j_n \in \mathcal{I}_n} q_n^{j_n} f\left(\rho_n^{i_n} - \rho_n^{j_n}\right) - \sum_{j_n \in \mathcal{I}_n} q_n^{j_n} \left(1 - f(\rho_n^{i_n} - \rho_n^{j_n})\right) \right].$$

□

Following Aoki & Yoshikawa (2006, ch. 3), it is outlined in Rainer & Schütz (2012, ch. 3.2) that if $\rho_n^{i_n} - \rho_n^{j_n}$ is normally distributed with variance σ, then

$$f(\rho_n^{i_n} - \rho_n^{j_n}) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\rho_n^{i_n} - \rho_n^{j_n}}{\sqrt{2}\sigma}\right)\right]. \tag{16}$$

With this specification, equations (15) yield a well-defined system of differential equations describing the diffusion process of different technologies as the result of interpersonal communication in the presence of uncertainty. For the special case of $\sigma = 0$, i.e. if $\rho_n^{j_n}$ is known to firm 1, and if i_n is the most profitable technology of the sector, equation (15) becomes the *logistic equation*

$$\dot{q}_n^{i_n} = q_n^{i_n}\left(1 - q_n^{i_n}\right) \tag{17}$$

as a special case of the Bass model (14) with $Q = 1$ and $M = 0$. Equation (17) holds, because $f(\rho_n^{j_n} - \rho_n^{i_n}) = 1$ for $i_n \neq j_n$ and $= 0$ for $i_n = j_n$. Otherwise, as long as profit differentials are small, i.e. $\rho_n^{i_n} \approx \rho_n^{j_n}$, and additionally uncertainty prevails (if σ is sufficiently large), then f in equation (16) can be approximated with sufficient accuracy by its first order Taylor expansion

$$f(\rho_n^{j_n} - \rho_n^{i_n}) \approx \frac{1}{2} + \frac{1}{\sigma\sqrt{2\pi}}(\rho_n^{i_n} - \rho_n^{j_n}).$$

The diffusion process (15) is then described by the system

$$\dot{q}_n^{i_n} = Q\ q_n^{i_n}\left(\rho_n^{i_n} - \bar{\rho}_n\right), \qquad Q = \frac{1}{\sigma\sqrt{\pi}}. \tag{18}$$

$\bar{\rho}_n = \sum_{i_n \in \mathcal{I}_n} q_n^{i_n} \rho_n^{i_n}$ denotes the *average profit* which can be attained in sector n as the result of innovative activity. Equations (18) provide a system of $\sum_{n=1}^{N} |\mathcal{I}_n|$ differential equations determining the diffusion of different technologies throughout the economic system, where one equation in each sector can be replaced by $\sum_{i_n \in \mathcal{I}_n} q_n^{i_n} = 1$.

The derivation of equation (18) solely was based on the assumption of contagion by means of knowledge transfer within a community of infinitesimally small firms, which decide according to their beliefs, characterised by variance σ, which technology to apply. However, this approach can be augmented by introducing q as an externality: The more firms use some technology, the more reliable the result is for others. This leads to a variance which depends on q, i.e. $\sigma = \sigma(q(t))$ with $\partial_q \sigma < 0$, and consequently $Q = Q(q(t))$ in equation (18). It was also assumed that the probability that two firms which meet each other are randomly drawn out of a common pool.

Diffusion by growth: The second way of thinking about diffusion of technologies is the consideration of growth possibilities of firms employing some cheaper process. To abstract from decision processes of firms, producers stick to the technology they use at time $t = 0$. Depending on whether extra profits are positive or negative, a firm grows or shrinks, depending on the technology it applies. Formally, the output of some process $i_n \in \mathcal{I}_n$ evolves according to

$$\frac{\dot{x}_n^{i_n}(t)}{x_n^{i_n}(t)} = \rho_n^{i_n}(t). \tag{19}$$

Consequently, from $\dot{x}_n = \sum_{i_n \in \mathcal{I}_n} \dot{x}_n^{i_n}$, one can derive the overall growth rate of sector n as

$$\frac{\dot{x}_n(t)}{x_n(t)} = \bar{\rho}_n(t). \tag{20}$$

By differentiation of $x_n q_n^{i_n} = x_n^{i_n}$ with respect to time and acknowledging equation (20), equation (19) can be rewritten in terms of growth rates of shares $q_n^{i_n}$ as

$$\frac{\dot{q}_n^{i_n}(t)}{q_n^{i_n}(t)} = \rho_n^{i_n}(t) - \bar{\rho}_n(t). \tag{21}$$

Equation (21), which is a result of firm growth, is similar (up to some factor Q) to equation (18), which arose out of decision processes of firms within a social communication network. One can keep this formal similarity in mind and potentially take some general proportionality factor Q into account, which influences the speed of diffusion. If the growth mechanism as outlined above is either not that pronounced (for example because R&D activity or organisational necessities use up some of the extra profits), or if it is more pronounced as the result of gold-rush moods as a result of major technological improvements, equation (21) can be replaced by its more general version (18) with $Q < 1$ or $Q > 1$, respectively.

Within the framework just introduced, equation (18) is taken as a starting point for analysing economic dynamics in the presence of technical change. They are of additional interest, since they straightforwardly resemble the *replicator dynamics* of *evolutionary game theory* (Weibull, 1997) and, hence, provide an intuitive interpretation of the model. In the biological analogy of equation (18), technologies i_n are different populations which compete in sector n for market shares $q_n^{i_n}$. Extra profits multiplied by some factor Q are a measure of the *fitness* of the respective population, whereas the growth rate of some population is given by the difference between its own fitness and the average fitness of the ecosystem.

3.2 A partial model: the case of one sector and two processes

Diffusion research as surveyed by Rogers (2003) deals with the time-path of the share of an innovation, which enters some economic system. It is therefore the natural field of application of the modelling framework of this article. As diffusion research is first and foremost concerned with single markets, in a first step the multi-sector model with multiple technologies is reduced to a one-sector model with only two technologies. An application of the multi-sector approach, which allows for the investigation of inter-sector feedback effects, is performed by Strohmaier & Rainer (2012). There the influence and the diffusion of new *general purpose technologies* (Helpman, 1998) on connected sectors are empirically analysed and theoretically substantiated by a two-sector version of the proposed model.

To begin with, take one sector and two processes (a^i, l^i) with $i = 1, 2$. Prices are normalised to $p = 1$, implicitly determining extra profits ρ^i $(i = 1, 2)$ by

$$(1 + r + \rho^i(t))a^i + w(t)l^i = 1. \tag{22}$$

For some arbitrary constant parameter Q, equation (18) reduces to

$$\frac{\dot{q}(t)}{q(t)} = Q\,(1 - q(t))\left(\rho^2(t) - \rho^1(t)\right). \tag{23}$$

Inserting ρ^i from equation (22) into equation (23), the share q of the new technology

$i = 2$ evolves according to

$$\dot{q}(t) = Q\ q(t)\,(1 - q(t)) \left[\left(\frac{1}{a^2} - \frac{1}{a^1} \right) + w(t) \left(\frac{l^1}{a^1} - \frac{l^2}{a^2} \right) \right]. \tag{24a}$$

From the $w - r$ relationship (5), wages are determined by

$$w(t) = \frac{1 - (1+r)\bar{a}(t)}{\bar{l}(t)} = \frac{1 + z(t) - (1+r)[a^1 + z(t)a^2]}{[l^1 + z(t)l^2]}$$

with the relative market share $z \equiv q/(1-q)$ of the innovative process as a new variable. Differential equation (24a) then reads

$$\frac{\dot{z}(t)}{z(t)} \frac{l^1 + z(t)l^2}{\mu^1 + z(t)\mu^2} = Q \tag{24b}$$

with the auxiliary parameter μ^i defined by

$$\frac{\mu^i}{l^i} = R_2 \left(1 - \frac{w_i}{W_2} \right) - R_1 \left(1 - \frac{w_i}{W_1} \right).$$

$1 + R_i \equiv 1/a^i$ is the maximal attainable rate of profit some process i can achieve if it is operated alone and if no wages are paid ($w = 0$). This can be seen by calculating r from the then valid price equation $(1+r)a^i = 1$ (Kurz & Salvadori, 1995, p. 46). $W_i \equiv (1 - a^i)/l^i$ is the maximal wage (for $r = 0$), and $w_i = (1 - (1+r)a^i)/l^i$ denotes the wage which prevails, if only process i is operated. The general solution $z(t)$ of the diffusion process can now implicitly be derived as

$$z(t) \left(\mu^1 + \mu^2 z \right)^D = Ce^{Qt}, \qquad D = \frac{\mu^1 l^2}{\mu^2 l^1} - 1, \qquad C = z_0(\mu^1 + \mu^2 z_0)^D, \tag{25a}$$

or, in terms of the share $q(t)$ of the innovative technology $i = 2$,

$$\frac{q(t)\bar{\mu}(t)}{(1 - q(t))^{1+D}} = Ce^{Qt} \tag{25b}$$

with $\bar{\mu} = (1-q)\mu^1 + q\mu^2$. $z_0 \equiv (1 - q_0)/q_0$ is the initial condition, where q_0 is the share of firms which adopt the new process at time $t = 0$.

Equations (25a) and (25b) in the interpretation of diffusion as a consecutive adoption of some new technology by firms show that increasing certainty about extra profits increases the speed of diffusion, since in this case $Q = 1/(\sigma\sqrt{\pi})$ also increases and therefore z respectively q grow at a faster pace.

To give a first intuition of the diffusion process in case of one sector and two processes, the special cases of labour saving technical progress by simultaneously using more capital

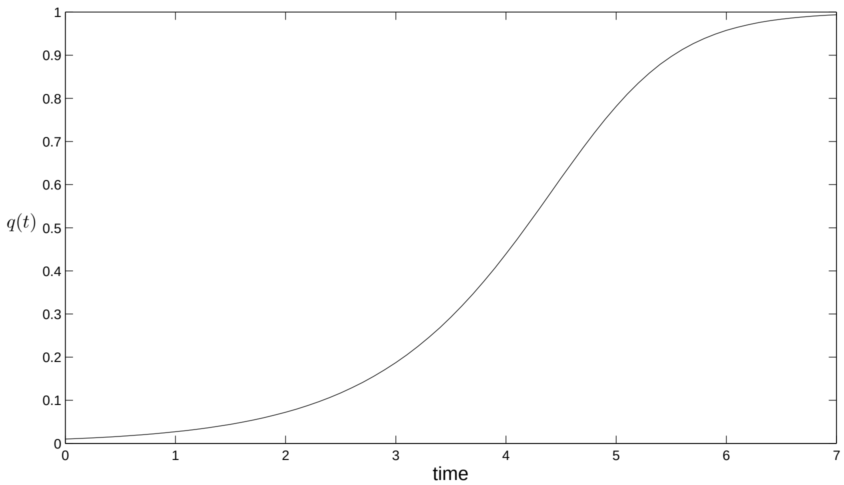

Figure 2: The diffusion of labour saving technical progress

($a^1 < a^2$ and $l^1 > l^2$) is illustrated in figure 2 with the special values $G = 1$, $a^1 = 0.3$, $a^2 = 0.4$, $l^1 = 0.3$, $l^2 = 0.2$, $r = 0.1$ and $q(0) = 0.01$. The time path of $q(t)$ with its slow start and sudden take-off including the flattening at the end of the diffusion process resembles the diffusion patterns as found by many cases of technology diffusion which were investigated by diffusion research (Rogers, 2003).

Total output x of the aggregate sector grows according to equation (20), i.e. $\dot{x}/x = \bar{\rho}$, and from market clearing (1) one gets $y(t) = (1 - \bar{a}(t))\, x(t)$, leading to the growth rate

$$\frac{\dot{y}}{y} = \bar{\rho} - \frac{q(1-q)}{q - \frac{1-a^1}{a^1-a^2}} \left[\left(\frac{1}{a^2} - \frac{1}{a^1} \right) + w \left(\frac{l^1}{a^1} - \frac{l^2}{a^2} \right) \right] \tag{26}$$

of GDP. Both growth rates are depicted for the just stated numerical case in figure 3. One striking result of this one-sector example is the negative growth rate in the presence of technical progress, as more capital input is needed. Positive growth rates can be obtained in case of pure labour-saving technical progress, as indicated by the right-hand-side of equation (26). That this scenario is realistic is suggested by the research on *general purpose technologies* (GPT), such as the steam engine, semiconductors or IT innovations. One special feature which can be observed is a downturn after a new GPT emerges, followed by sustained growth (Helpman, 1998). The respective explanation given by the proposed model is that first a GPT with more capital and less labour is implemented, and this new technology possibly leads to both labour-saving and capital-

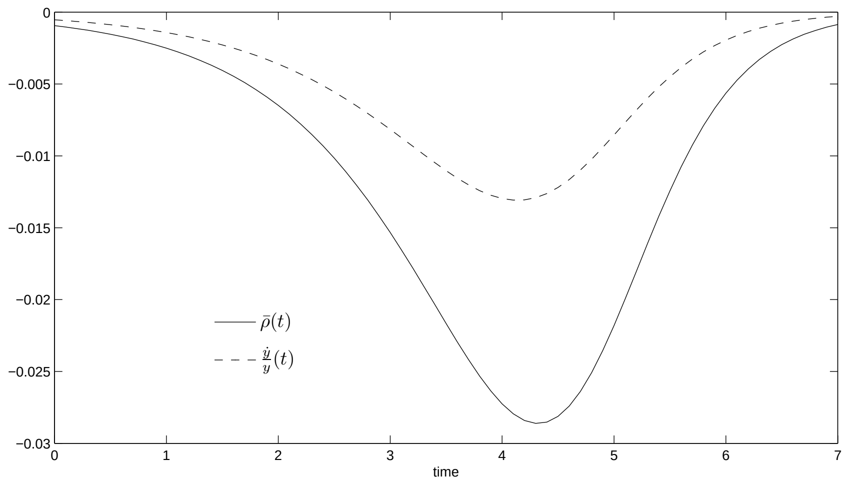

Figure 3: Negative GDP growth

saving technical progress. Hence, a one-time invention in some basic sector is capable of starting an avalanche of successive innovations in other sectors, which are both capital-saving and labour-saving. Figure 4 illustrates the situation of capital- and labour-saving technical progress with the same parameters as above, just interchanging a^1 and a^2.

If full employment is considered at $t = 0$, i.e. if $x(0)l^1 = 1$, then in the course of the diffusion process some positive *rate of unemployment* $u(t) = 1 - x(t)\bar{l}(t) > 0$ emerges, both because $x(t)$ shrinks and $\bar{l}(t)$ decreases in case of labour-saving technical progress. This kind of *technological unemployment* can be compensated either by accompanying technical progress which leads to positive growth, or by re-distributional measures of the government in terms of unemployment benefits. Taxing wage income with a fixed tax rate β and redistributing the tax revenues to the unemployed imply a disposable per-capita income of $(1 - \beta)w(t)$. The total amount $\beta w(t)(1 - u(t))$ of aggregate tax income of the government is distributed to the unemployed part $u(t)$ of the population, inducing a disposable income of $\beta w(t)(1 - u(t))/u(t)$ of the unemployed. Consequently, wage inequality as a result of labour-saving technical progress is indicated by the GINI index

$$GINI = u(t) - \beta\big(1 - u(t)\big).$$

This kind of wage inequality resulting from *long-run technological unemployment*, caused by labour-saving technical progress, contrasts the case of *transitional wage in-*

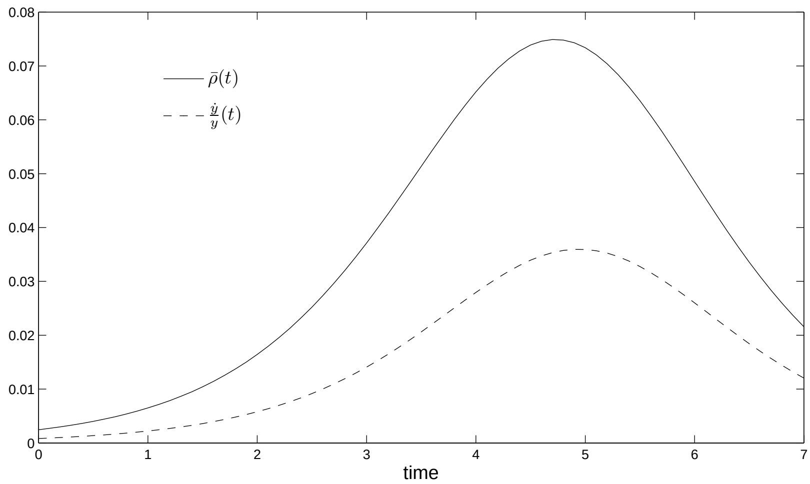

Figure 4: Positive GDP growth

equality, which arises if new skills are needed for the innovative process (see section 2.2 as well as Strohmaier & Rainer (2012), where an empirical analysis is included). In the latter case there is a rising GINI-coefficient, which again diminishes towards the completion of the diffusion process. Crucial for this result is the assumption that workers in the old process get lower wages compared to those employed in the new process. The wage premium and the fact that the innovation is not instantaneously adopted by all firms lead to the observation that wage inequality disappears only when the diffusion process is completed.

4 Conclusions

Technical change and social frictions: Technical circumstances in several ways influence the economic and social situation of the members of the system. As discussed at the end of section 2.2, wage inequality prevails as soon as some technology is used which employs workers of different skill levels. Depending on the wage premia of better paid workers and influenced by the relative amount of low-skill labour compared to high-skill labour, the GINI index varies and suggests government action in form of redistributional measures. This kind of social friction is owed to technical possibilities and cannot be solved by economic market forces. Indeed, as mentioned in the course of the discussion of this feature in section 2.2, contract theory (e.g. Bolton & Dewatripont, 2005) indicates that as soon as different skill levels are employed within one specific production process, wage differentials arise as a result of *moral hazard* and *adverse selection*. Another factor influencing wage differentials and, hence, wage inequality are variations of supply and demand of different skills, since not every worker can easily switch between different tasks according to labour demand. Higher wages for skills with excess demand are the consequence of this observation. Government action in both cases can be identified to lie within the educational system, providing labour supply for those skills, which are demanded by firms.

Another kind of long-run wage inequality arises in the presence of labour-saving technical progress, as discussed at the end of section 3.2. In the course of the diffusion of some labour-saving technology, assuming a constant labour force, less labour input implies a mismatch of aggregate labour supply and demand. Things get even worse, if, as investigated in section 3.2, capital intensity rises and, hence, total net output gets reduced (or not increased accordingly) in the course of the diffusion process (figure 3, see also Kalmbach & Kurz, 1992). The introduction of some new technology therefore may be a *step back*, to take a run-up to sustained economic growth, nourished by subsequent and sustained technical progress. Policy action in this case can be evaluated with respect to three aspects. Firstly, sustaining disposable income of the population can be one goal. This can be accomplished by redistribution, as wages are taxed and unemployment benefits are paid. A second focus can lie on the attempt to bring the unemployed back to work, which can be managed by overall economic growth or by a stimulation of labour intensive production processes. As the old process is labour intensive and the new process promotes growth, one policy advice would be the targeted prolongation of the diffusion process such that innovation-induced growth can compensate the technologically induced displacement of labour. Thirdly, technological wage inequality is only transitory, if a newly diffusing technology utilises differently remunerated labour compared to the old technique. As soon as the innovation takes over the market completely, the GINI index also vanishes again (Strohmaier & Rainer, 2012). In case of a single innovation the problem exists only temporarily, whereas it is a long-run phenomenon if there is ongoing technical progress within an economy. Social friction in this case can be relieved by introducing a progressive tax system (lowering the GINI index), or by accelerating the diffusion process to keep the timespan of inequality short.

The suggestions of how policy should influence the diffusion process of some innovation for the case of technological unemployment and for the case of transitory wage inequality are opposed to each other. Hence, it is important to identify the needs to act socially responsible, and it is another research task to identify possible ways to accelerate or to decelerate the diffusion of certain technologies. Additionally, one has to understand the relation between labour-saving technical progress and the necessity to enhance economic growth to keep social friction at a minimum. Institutions and the legal setting in this context are of importance. An appropriate patent law and bankruptcy law for example can enhance innovative behaviour as indicated by the discussion in section 3.1 of how uncertainty influences the diffusion of innovations. Van Waarden (2001) more specifically investigates the influence of law and, more generally, of institutions on the innovativeness of firms. Also the different strands of economic literature on Institutionalism as surveyed by Hall & Taylor (1996) are of interest in this context, as there is a strong relationship between technical change and the institutional setting of a society.

Technical change and uncleared markets: There is one crucial assumption of the proposed model of this article, which easily can be imagined to be violated, namely the dynamic version

$$\mathbf{y}(t) = \left(\mathbb{I} - \bar{A}^T(t)\right)\mathbf{x}(t)$$

of the market clearing condition (1). *Effective demand* in this context may pose a problem, if one sector n, supplying others with factors of production, innovates and changes its output x_n (especially if final demand y_n does not match the residual quantity $(\mathbb{I} - \bar{A}^T(t))_n x_n$). Even in the one-sector case of section 3.2 problems may arise if effective demand $\hat{y}$ does not meet effective supply y, i.e. if

$$\hat{y} < y = (1 - \bar{a}(t))x(t). \tag{27}$$

If inequality (27) holds for a sustained period of time, stock levels increase. This situation can be triggered by some gold-rush mood of the innovative sector, including *herd behaviour* as pointed out by Keynes (1936) and discussed by Scharfstein & Stein (1990). A bubble both in the real economy and on the financial markets is the result, and its burst leads to recession, depression or crisis depending on its intensity. One way out the dilemma of over-production is the opening up of new markets by means of exporting. If importing countries get in trouble (by whatever reason), the just stated situation of huge losses eventuate and may lead to a turmoil. To avoid this situation, governments have different tools at hand. Demand can be held high by greater government spending or by subsidies for the producers (or for consumers to increase their willingness to buy). But to solve this special crisis, a structural change within the producing industry has to take place simultaneously. Also importing countries can be subsidised by the innovative, exporting country, something which was done by the USA in the aftermath of World War II (WW II) by means of the *Marshall plan*, and it can be regarded as the underlying reason why China offers help to the struggling European Union in the present crisis. Both the USA after WW II as well as China at the beginning of the 21st century can be regarded as examples fitting this picture, which is reinforced by the observation that, once a sector of some country becomes an exporter as the result of past productivity gains, this situation prevails as the consequence of some *lock-in* effect. Therefore, even after completing the diffusion process of innovations the dependence of the now non-innovative sector on exports remains. This problem can be tackled for example by a directed downsizing of some sector. Hence, economic policy is not only necessary for holding output high, but also to reduce output in a socially tenable way in certain sectors, perhaps by fostering sectors with growing final demand to take the pressure off the labour market.

Concluding remarks: As a result of the above discussion, the emergence of major technical advances since the industrial revolution is understood as a source of both economic and social prosperity as well as of economic and social frictions. Purposeful policy measures can help promoting the former and curbing the latter. The presented modelling framework introduced in this article investigates the inter-sector consequences of the diffusion of innovations and therefore facilitates the evaluation of economic and social policy action, concerned with the effects of ongoing technical change.

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