Beyond price discrimination: welfare under differential pricing when costs also differ

Yongmin Chen and Marius Schwartz

University of Colorado, Boulder, Georgetown University

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Yongmin Chen[†] and Marius Schwartz[‡]

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Abstract. We extend the analysis of monopoly third-degree price discrimination to the empirically important case where marginal costs also differ between markets. Differential pricing then reallocates output to the lower-cost markets, hence welfare can increase even if total output does not, unlike under pure price discrimination. To induce output reallocation the firm varies its prices but—again, unlike under pure price discrimination—with no upward bias in the average price. Due to this price dispersion, differential pricing motivated solely by cost differences will increase consumer surplus (and total welfare) for a broad class of demand functions. We also provide sufficient conditions for beneficial differential pricing in the hybrid case where both demand elasticities and marginal costs differ.

Keywords: price discrimination, differential pricing, price dispersion, add-on pricing.

[†]University of Colorado, Boulder, CO 80309 <yongmin.chen@colorado.edu>.

[‡]Georgetown University, Washington DC 20057 <mariusschwartz@mac.com>.

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1. INTRODUCTION

A longstanding question in economics concerns the welfare effects of uniform pricing by a monopolist compared to third-degree price discrimination—charging different prices to consumers in separate markets characterized by some exogenous signal about the elasticities of market demands. Originating with Pigou (1920) and Robinson (1933), the literature was extended by numerous authors including Schmalensee (1981), Varian (1985), Layson (1988), Schwartz (1990), Malueg (1993), Aguirre, Cowan and Vickers (2010), and Cowan (2012). A key maintained assumption is that total cost depends only on aggregate output and not on its allocation across markets. This paper extends that analysis by allowing marginal costs, as well as demand elasticities, to differ between markets.

The assumption that total cost is invariant to the allocation of output fits several classic examples of price discrimination, such as discounts to students or pensioners for non-personalized services, or the sale of intellectual-property goods or other low marginal-cost items to different geographic markets. But in many situations the costs of service differ. For instance, manufacturers often sell to heterogeneous distributors who perform varying ranges of wholesale functions that relieve the manufacturers of different costs. This made it difficult to distinguish price discrimination from cost-justified discounts under the Robinson-Patman Act, the main U.S. law governing price discrimination (Schwartz 1986).[1] As another example, book publishers sell both hardback and paperback editions that may implement quality-based price discrimination between customer groups but also entail different marginal costs.[2]

Another broad class of examples is add-on pricing. Sellers commonly offer a base good

[1] The Act only prohibits discrimination where it may substantially reduce competition among the purchasers, hence does not apply to final consumers. But its experience illustrates that price differences characterized as price discrimination often are accompanied by cost differences. The examples discussed next involve price differences to final consumers.

[2] By contrast, there are instances where quality-based price differences clearly entail price discrimination because the lower-quality imposes higher marginal costs on the seller, as occurs when the lower-quality products are purposefully "damaged" versions of the high quality products (McAfee and Deneckere 1996).

and optional add-on services that can only be consumed in conjunction with the base good (see, e.g., Ellison 2005). Airlines sell a ticket (the base good) and offer costly options such as booking by phone, checking a bag, or onboard meals and movies; manufacturers sell a product and offer technical support; hotels offer a room and extras such as phone service.[3] Importantly, some consumers take the optional items while others do not. If the seller charges an all-inclusive price (bundled pricing), this represents uniform pricing across consumer groups that impose different costs according to whether they use the add-ons or not. Moving to unbundled pricing by charging separate prices for the add-ons can implement cost-based pricing: airline passengers who check a bag and subject the airline to an additional cost pay a higher total price than those who do not. At the same time, unbundled pricing is often controversial among consumers because the prices for the add-ons may substantially exceed their incremental costs and be motivated at least in part by demand differences between the groups—add-on pricing may be a form of price discrimination.

To our knowledge, there has been scant analysis of welfare under differential pricing in the empirically important case where marginal costs differ. This paper addresses the gap. We adopt the standard setting where a monopolist serves two markets under uniform pricing, demand in each market is independent of price in the other market, and a move to differential pricing lowers the price in one market and raises it in the other.[4] But we allow different (though constant) marginal costs of serving each market. In this setting

[3] Airline revenues from various ad-on charges, known as ancillary fees, have been growing rapidly. For 47 of the world's largest airlines that collectively account for almost half of airline revenues globally their ancillary revenues rose from $13.5 bn in 2009 to $22 bn in 2010. Some budget carriers derive more than one third of their revenues from ancillary revenues (Michaels 2011). Ancillary revenues have continued to grow, and are projected to reach $36.1 bn in 2012 (BusinessWire.com 2012). Banks also have been imposing ancillary fees for various services such as cash withdrawals at ATMs, paper statements, and rush delivery for card replacement (Dash 2012).

[4] When one market is not served under uniform pricing, allowing discrimination may open up new markets and could yield a Pareto improvement (Hausman and MacKie-Mason 1988). With both markets served under uniform pricing, the assumption that discrimination will cause prices to move in opposite directions can fail if the profit function in at least one of the separate markets is not concave (Nahata et al. 1990, Malueg 1992), or if demand in each market also depends on the price in the other market (Layson 1998).

differential pricing will increase profit compared to uniform pricing by expanding the firm's pricing options, but the effect on aggregate consumer surplus across the markets and on total welfare—profit plus aggregate consumer surplus—is ambiguous a priori. Our focus is on characterizing broad demand conditions under which cost-based differential pricing benefits consumers and overall welfare and to highlight the contrast with pure price discrimination—differential pricing that is based solely on different demand elasticities.

A central result in the literature is that pure price discrimination can increase welfare only if total output rises, since discrimination misallocates a given total output between markets by inducing consumers to choose quantities at which their marginal valuations differ (Schmalensee 1981; Varian 1985; Schwartz 1990). When marginal costs differ, however, there is a new effect: differential pricing saves cost by reallocating output to lower-cost markets. Consequently, we show that welfare can easily rise even if total output does not.

Less obviously, differential pricing can increase also consumer surplus without raising total output. The mechanism is subtle, since the cost savings from output reallocation—which are the source of increased welfare when output does not rise—do not benefit consumers directly. Rather, consumers benefit because in order to shift output to the lower-cost market the firm must vary its prices and consumers gain from the resulting price dispersion. This cost-motivated price dispersion does not entail an upward bias in the weighted-average price across markets—in sharp contrast to pure price discrimination.

We begin our analysis by characterizing the monopolist's optimal uniform and differential prices in Section 2, which also provides bounds on the change in welfare and consumer surplus due to differential pricing. Section 3 analyzes Pigou's (1920) case of linear demands, extended to allow different (but constant) marginal costs of serving the markets. Differential pricing yields the same total output as uniform pricing, hence welfare must fall if costs do not differ (the standard welfare result). By contrast, we show that if markets differ only in their marginal costs of service then differential pricing will increase welfare—the cost savings outweigh the consumption misallocation—as well as consumer surplus. In the hybrid case, differential pricing is beneficial if the difference in the demand-elasticity parameter is not too large relative to the difference in marginal costs (Proposition 1).

In Section 4 we allow demands in the two markets to have any curvature, but assume they are proportional to each other thereby having equal elasticities at any common price, so as to isolate the welfare effects of differential pricing that is purely cost based. Consumer surplus then rises for a broad class of demand functions: those for which the pass-through rate from marginal cost to the monopoly price does not increase too fast[5] or, equivalently, the curvature of the inverse demands does not decrease too fast (Proposition 2). (When this condition is violated, however, differential pricing can reduce consumer surplus.) Overall welfare is shown to increase for a broader class of demand functions (Proposition 3). We contrast the conditions in Propositions 2 and 3 with their more stringent counterparts under pure price discrimination, identified in the comprehensive analyses by Aguirre, Cowan and Vickers (2010) for overall welfare and by Cowan (2012) for consumer surplus.

Section 5 extends the analysis to general demand functions. We provide sufficient conditions on the demand functions for differential pricing to improve consumer welfare (and hence also total welfare) if the difference in demands is not too large relative to the cost differences (Proposition 4), as with the hybrid case under linear demands. The basic policy message is unsurprising but worth reiterating: differential pricing deserves a considerably more favorable outlook when the price differences are plausibly motivated, wholly or in part, by cost differences. Section 6 presents the conclusions.

2. PRICING REGIMES AND WELFARE BOUNDS

Consider two markets, H and L, with strictly decreasing demand functions $q^H(p)$, $q^L(p)$ and inverse demands $p^H(q)$, $p^L(q)$. When not necessary, we omit the superscripts in these

[5] A non-increasing pass-through rate implies that differential pricing motivated solely by cost differences will not raise the weighted-average price, and therefore will increase aggregate consumer surplus. Even when the pass-through rate is increasing so that average price rises, consumer welfare may still increase because of the beneficial price dispersion. Pass-through by firms with market power was first analyzed by Cournot (1838). Bulow and Pfleiderer (1983) identify classes of demand functions with constant pass-through rates, while Weyl and Fabinger (2012) demonstrate the value of pass-through as an analytical device in numerous diverse settings.

functions. The markets can be supplied at constant marginal costs c_H and c_L.

Denote the prices in the two markets by p_H and p_L. Profits in the two markets are

$$\pi^i(p_i) = (p_i - c_i)\, q^i(p_i)\,, \text{ for } i = H, L,$$

and $\pi^i(p_i)$ is assumed to be strictly concave.

Under differential pricing, maximum profit in each market is achieved when $p_i = p_i^*$, where p_i^* satisfies

$$\pi^{i\prime} = q^i(p_i^*) + (p_i^* - c_i)\, q^{i\prime}(p_i^*) = 0.$$

We assume $p_H^* > p_L^*$. In Robinson's (1933) taxonomy, H is the "strong" market while L is the "weak" (though we allow the prices to differ also for cost reasons).

If the firm is constrained to charge a uniform price, we assume parameter values are such that both markets will be served (obtain positive outputs) at the optimal uniform price $\bar{p}$, which solves

$$\pi^{H\prime}(\bar{p}) + \pi^{L\prime}(\bar{p}) = 0.$$

The strict concavity of $\pi^i(p)$ and $p_H^* > p_L^*$ implies that $p_H^* > \bar{p} > p_L^*$, $\pi^{H\prime}(\bar{p}) > 0$, and $\pi^{L\prime}(\bar{p}) < 0$. Let $\Delta p_L = p_L^* - \bar{p} < 0$ and $\Delta p_H = p_H^* - \bar{p} > 0$. Also, let $\Delta q_L \equiv q^L(p_L^*) - q^L(\bar{p}) \equiv q_L^* - \bar{q}_L > 0$ and $\Delta q_H = q^H(p_H^*) - q^H(\bar{p}) \equiv q_H^* - \bar{q}_H < 0$.

Aggregate consumer surplus across the two markets, which we take as the measure of consumer welfare, is

$$S^* = \int_{p_H^*}^{\infty} q^H(x)\, dx + \int_{p_L^*}^{\infty} q^L(x)\, dx; \quad \overline{S} = \int_{\bar{p}}^{\infty} q^H(x)\, dx + \int_{\bar{p}}^{\infty} q^L(x)\, dx$$

under differential and uniform pricing, respectively. The change in consumer surplus due to differential pricing is

$$\Delta S \equiv S^* - \overline{S} = \int_{p_L^*}^{\bar{p}} q^L(x)\, dx - \int_{\bar{p}}^{p_H^*} q^H(x)\, dx, \tag{1}$$

which, together with $p_H^* > \bar{p} > p_L^*$, $\Delta p_L < 0$ and $\Delta p_H > 0$, immediately implies the following lower and upper bounds for ΔS:

$$-q^L(\bar{p})\,\Delta p_L - q^H(\bar{p})\,\Delta p_H < \Delta S < -q^L(p_L^*)\,\Delta p_L - q^H(p_H^*)\,\Delta p_H. \tag{2}$$

That is, with differential pricing that raises the price in market H and lowers it in market L, the change in consumer surplus is bounded below by the sum of price changes weighted by outputs at the original (uniform) price, and is bounded above by the sum of price changes weighted by outputs at the new (differential) prices. The result below, which follows immediately from (2), provides sufficient conditions for differential pricing to raise or lower aggregate consumer surplus:

Lemma 1 *(i)* $\Delta S > 0$ *if* $q^L(\bar{p})\Delta p_L + q^H(\bar{p})\Delta p_H \leq 0$, *and (ii)* $\Delta S < 0$ *if* $q^L(p_L^*)\Delta p_L + q^H(p_H^*)\Delta p_H \geq 0$.

The intuition for part (i) can be visualized by starting with the case $q^L(\bar{p})\Delta p_L + q^H(\bar{p})\Delta p_H = 0$. If both demand curves were vertical at the initial quantities, consumers' gain in market L would exactly offset the loss in market H. Since demands are downward-sloping, however, consumers in L gain more than $q^L(\bar{p})\Delta p_L$ by increasing the quantity purchased while consumers in H mitigate their loss by decreasing their quantity. Both of these quantity adjustments imply $\Delta S > 0$. If $q^L(\bar{p})\Delta p_L + q^H(\bar{p})\Delta p_H < 0$, then $\Delta S > 0$ even before considering the quantity adjustments. A similar argument explains part (ii), because if the price changes are weighted by the *new* quantities, $q^L(p_L^*)\Delta p_L$ will *overstate* the gain in L while $q^H(p_H^*)\Delta p_H$ will *understate* the loss in H.

Recalling that $\Delta p_L = p_L^* - \bar{p}$ and $\Delta p_H = p_H^* - \bar{p}$, the condition in Lemma 1(i) for consumer surplus to rise also can be expressed as

$$\Delta S > 0 \text{ if } \left(\frac{\bar{q}_L}{\bar{q}_L + \bar{q}_H}\right) p_L^* + \left(\frac{\bar{q}_H}{\bar{q}_L + \bar{q}_H}\right) p_H^* \leq \bar{p}. \quad (3)$$

That is, differential pricing raises aggregate consumer surplus across the two markets if the average of the new prices weighted by each market's share of the initial total output is no higher than the initial uniform price. This formulation highlights an important principle: Increased price dispersion that does not raise the weighted average price will benefit consumers overall, because they can advantageously adjust quantities by purchasing more where price falls and less where price rises.

Now consider total welfare, the sum of consumer surplus and profit: $W = S + \Pi$. Since differential pricing increases profit (by expanding the firm's pricing options) total welfare must rise if consumer surplus does not fall, but if consumer surplus falls the change in welfare is ambiguous. It will be useful also to analyze welfare directly without calculating profit and consumer surplus. Under differential pricing

$$W^* = \int_0^{q_L^*} \left[p^L(q) - c_L\right] dq + \int_0^{q_H^*} \left[p^H(q) - c_H\right] dq. \tag{4}$$

Welfare under uniform pricing, $\overline{W}$, is obtained by replacing q_L^* and q_H^* in W^* with $\bar{q}_L$ and $\bar{q}_H$. The change in total welfare from moving to differential pricing is

$$\Delta W = W^* - \overline{W} = \int_{\bar{q}_L}^{q_L^*} \left[p^L(q) - c_L\right] dq + \int_{\bar{q}_H}^{q_H^*} \left[p^H(q) - c_H\right] dq, \tag{5}$$

which, together with $\Delta q_L = q_L^* - \bar{q}_L > 0$ and $\Delta q_H = q_H^* - \bar{q}_H < 0$, immediately implies the following lower and upper bounds for ΔW:

$$(p_L^* - c_L)\,\Delta q_L + (p_H^* - c_H)\,\Delta q_H < \Delta W < (\bar{p} - c_L)\,\Delta q_L + (\bar{p} - c_H)\,\Delta q_H. \tag{6}$$

That is, the change in welfare is bounded below by the weighted sum of the output changes, using the markups at the new (differential) prices as weights; and it is bounded above also by the weighted sum of output changes, but using instead the markups at the original (uniform) price as weights.[6] From (6), we immediately have the following sufficient conditions for differential pricing to raise or lower total welfare:

Lemma 2 *(i)* $\Delta W > 0$ *if* $(p_L^* - c_L)\,\Delta q_L + (p_H^* - c_H)\,\Delta q_H \geq 0$, *and (ii)* $\Delta W < 0$ *if* $(\bar{p} - c_L)\,\Delta q_L + (\bar{p} - c_H)\,\Delta q_H \leq 0$.

As with Lemma 1, these results arise because demands are negatively sloped. In market L the average value to consumers of the output expansion Δq_L is below old uniform price $\bar{p}$ and above the new lower price p_L^*; while in market H the average value of the output reduction Δq_H is above $\bar{p}$ and below the new higher price p_H^*. Thus, $(p_L^* - c_L)\,\Delta q_L$ understates the welfare gain in market L and $(p_H^* - c_H)\,\Delta q_H$ overstates the loss in H, so welfare must rise if the sum of these terms is weakly positive (result *(i)*). Similarly, *(ii)* holds because

[6] Varian (1985) provides a similar expression for the case where marginal costs are equal.

$(\bar{p} - c_L)\,\Delta q_L$ overstates the gain in market L while $(\bar{p} - c_H)\,\Delta q_H$ understates the loss in market H.

The insight from the literature on price discrimination, that price discrimination reduces total welfare if total output does not increase, obtains as a special case of Lemma 2(ii) when $c_H = c_L$. When costs differ ($c_L < c_H$), part (i) of Lemma 2 implies:

Remark 1 *If differential pricing does not reduce total output compared to uniform pricing, then total welfare increases if the price-cost markup under differential pricing is weakly greater in the lower-cost than in the higher-cost market* ($p_L^* - c_L \geq p_H^* - c_H$).

Intuitively, the absolute price-cost margin (i.e., the marginal social value of output) under uniform pricing is higher in the lower-cost market L than in H ($\overline{p} - c_L > \overline{p} - c_H$), so welfare can be increased by reallocating some output to market L. Differential pricing induces such a reallocation, and if the margin in L remains no lower than in H then the entire reallocation is beneficial, hence welfare must increase if total output does not fall ($\Delta q_L \geq -\Delta q_H$).

To highlight the roles of output reallocation versus the change in total output, we use the mean value theorem to rewrite (5) as

$$\Delta W = \left[p^L(\xi_L) - c_L\right]\Delta q_L + \left[p^H(\xi_H) - c_H\right]\Delta q_H,$$

where $\xi_L \in (\bar{q}_L, q_L^*)$ and $\xi_H \in (q_H^*, \bar{q}_H)$ are constants, with $p^L(\xi_L) < \bar{p}$ and $p^H(\xi_H) > \bar{p}$ representing the average willingness to pay in market L and market H, respectively. Let $\Delta q \equiv \Delta q_L + \Delta q_H$. Then, with $\Delta q_H = \Delta q - \Delta q_L$, we have the following decomposition of the welfare change due to differential pricing:

$$\Delta W = \underbrace{\left[p^L(\xi_L) - p^H(\xi_H)\right]\Delta q_L}_{\text{consumption misallocation}} + \underbrace{(c_H - c_L)\,\Delta q_L}_{\text{cost saving}} + \underbrace{\left[p^H(\xi_H) - c_H\right]\Delta q}_{\text{output effect}}, \tag{7}$$

where the first term represents the reduction in consumers' total value due to reallocating output between markets starting at the efficient allocation under uniform pricing, the second term represents the cost savings from the same output reallocation to the lower-cost market, and the last term is the welfare effect due to the change in total output (which takes the sign of Δq since price exceeds marginal cost).[7]

We can combine the first two terms in (7) and call it the (output) reallocation effect, as opposed to the (change in) output effect:

$$\Delta W = \underbrace{\left[\left(p^L\left(\xi_L\right) - c_L\right) - \left(p^H\left(\xi_H\right) - c_H\right)\right]\Delta q_L}_{\text{reallocation effect}} + \underbrace{\left[p^H\left(\xi_H\right) - c_H\right]\Delta q}_{\text{output effect}}. \tag{8}$$

When output does not decrease ($\Delta q \geq 0$), differential pricing increases welfare if the *average* value net of cost of the reallocated output is higher in market L: $p^L\left(\xi_L\right) - c_L > p^H\left(\xi_H\right) - c_H$. This is a weaker condition than $p_L^* - c_L \geq p_H^* - c_H$ in Remark 1 (since $p^L\left(\xi_L\right) > p_L^*$ and $p^H\left(\xi_H\right) > p_H^*$),[8] but the latter condition may be more observable.

3. LINEAR DEMANDS

The case of linear demands highlights a sharp contrast between the welfare effects of price discrimination versus cost-based differential pricing. Relative to uniform pricing, pure price discrimination lowers consumer surplus and total welfare, whereas differential pricing that is motivated solely by cost differences will raise both.

Suppose that

$$p^i\left(q\right) = a_i - b_i q, \text{ where } a_i > c_i \text{ for } i = H, L.$$

Then, under differential pricing,

$$p_i^* = \frac{a_i + c_i}{2}; \quad q_i^* = \frac{a_i - c_i}{2b_i}; \quad \pi_i^* = \frac{\left(a_i - c_i\right)^2}{4b_i},$$

and $p_H^* > p_L^*$ requires that $\left(a_H - a_L\right) + \left(c_H - c_L\right) > 0$. Under uniform pricing, provided

[7] Alternatively, one can use the output change in market H and write $\Delta W = \left[p^L\left(\xi_L\right) - p^H\left(\xi_H\right)\right]\Delta q_H + \left(c_H - c_L\right)\Delta q_H + \left[p^L\left(\xi_L\right) - c_L\right]\Delta q$. Our decompositions are similar in spirit to expression (3) of Aguirre, Cowan and Vickers (2010), except that they consider infinitesimal changes in the allowable price difference and assume no cost savings.

[8] The average value to consumers of the reallocated output exceeds p_L^* in market L since output there rises and is less than p_H^* in market H since output falls. The condition in Remark 1 is therefore sufficient but not necessary for the output reallocation to be beneficial.

that both markets are served:

$$\bar{p} = \frac{(a_H + c_H)\, b_L + (a_L + c_L)\, b_H}{2\,(b_L + b_H)}; \quad \bar{q}_i = \frac{1}{b_i}\left[a_i - \frac{(a_H + c_H)\, b_L + (a_L + c_L)\, b_H}{2\,(b_L + b_H)}\right].$$

It follows that

$$q_H^* - \bar{q}_H = -\frac{a_H - a_L + c_H - c_L}{2\,(b_H + b_L)} < 0, \quad q_L^* - \bar{q}_L = \frac{a_H - a_L + c_H - c_L}{2\,(b_H + b_L)} > 0,$$

and $(q_H^* + q_L^*) - (\bar{q}_H + \bar{q}_L) = 0$. Pigou (1920) proved this equal outputs result when marginal cost depends only on the level of total output and not its allocation between markets. We showed that the result holds also when markets have different but constant marginal costs of serving them:

Remark 2 *If both markets have linear demands, constant but possibly different marginal costs, and would be served under uniform pricing, then total output will be the same under uniform or differential pricing.*

We now can readily compare the change in welfare moving from uniform to differential pricing in two polar cases: (i) the pure price discrimination scenario where demand elasticities differ but costs are equal ($a_H > a_L$, but $c_H = c_L$), versus (ii) equal demand elasticities but different costs ($a_H = a_L$, but $c_H > c_L$).[9]

Total Welfare. Since differential pricing leaves total output unchanged, the change in welfare is determined by the reallocation effect. When only demand elasticities differ, the reallocation effect is harmful since uniform pricing allocates output optimally while differential pricing misallocates consumption (see (7)). When only costs differ, uniform pricing misallocates output by under-supplying the lower-cost market L where the price-cost margin is higher ($\overline{p} - c_L > \overline{p} - c_H$). Differential pricing reallocates output to market L, and with linear demands the margin remains higher in market L also at the differential prices ($p_L^* - c_L = (a - c_L)/2 > (a - c_H)/2 = p_H^* - c_H$), implying from Remark 1 that welfare rises.

[9] Recall that with linear demand, the demand elasticity in market i equals $p/(a_i - p)$, hence depends only on the vertical intercept and not the slope.

Consumer Surplus. When only demand elasticities differ (i.e., $a_H > a_L$ but $c_L = c_H$), moving to differential pricing causes the sum of the price changes weighted by the *new* outputs to be positive,

$$q^L\left(p_L^*\right)\Delta p_L + q^H\left(p_H^*\right)\Delta p_H = -\frac{\left(a_H - a_L\right)\left(a_L - a_H\right)}{4\left(b_H + b_L\right)} > 0.$$

So from Lemma 1(*ii*), consumer surplus falls. By contrast, when only costs differ ($a_H = a_L$, but $c_H > c_L$),

$$q^H\left(\bar{p}\right)\Delta p_H + q^L\left(\bar{p}\right)\Delta p_L = \frac{\left(a_L - c_H - a_H + c_L\right)\left(a_L - a_H\right)}{2\left(b_H + b_L\right)} = 0,$$

so by Lemma 1(*i*), consumer surplus rises: the sum of the price changes weighted by the *initial* outputs is zero, hence the weighted average price equals the initial uniform price and consumers gain due to the price dispersion (recall (3)).

In the general case where both demand elasticities and costs may differ, from (1):

$$\Delta S = \frac{\left(a_H - a_L + c_H - c_L\right)\left[\left(c_H - c_L\right) - 3\left(a_H - a_L\right)\right]}{8\left(b_H + b_L\right)}.$$

It follows that

$$\Delta S > 0 \text{ if } a_H - a_L < \frac{c_H - c_L}{3}, \text{ and } \Delta S < 0 \text{ if } a_H - a_L > \frac{c_H - c_L}{3}. \tag{9}$$

Furthermore, since

$$\Delta \Pi = \frac{\left(a_H - c_H\right)^2}{4b_H} + \frac{\left(a_L - c_L\right)^2}{4b_L} - \bar{\pi}\left(\bar{p}\right) = \frac{\left(a_H - a_L + c_H - c_L\right)^2}{4\left(b_H + b_L\right)},$$

we have

$$\Delta W = \Delta S + \Delta \Pi = \frac{\left(a_H - a_L + c_H - c_L\right)\left[3\left(c_H - c_L\right) - \left(a_H - a_L\right)\right]}{8\left(b_H + b_L\right)}.$$

Thus,

$$\Delta W > 0 \text{ if } a_H - a_L < 3(c_H - c_L), \text{ and } \Delta W < 0 \text{ if } a_H - a_L > 3(c_H - c_L). \tag{10}$$

Letting a_i denote the "choke price" in market i (the vertical intercept of the demand curve), we summarize the above results as follows:

Proposition 1 *If both markets have linear demands, a move from uniform pricing to differential pricing has the following effects. (i) Total welfare increases (decreases) if the difference between markets in their choke prices is lower (higher) than three times the difference in costs* ($a_H - a_L < (>)\ 3(c_H - c_L)$)*.(ii) Consumer surplus increases (decreases) if the difference in choke prices is lower (higher) than one third of the cost difference* ($a_H - a_L < (>) \frac{c_H - c_L}{3}$)*.*

Therefore, differential pricing is beneficial when the difference in demand elasticities (which motivates third-degree price discrimination) is not too large relative to the difference in costs. The condition for welfare to rise is less stringent than for consumer surplus to rise, since differential pricing increases profit, so $\Delta S \geq 0$ implies $\Delta W > 0$ but not vice versa.[10]

4. EQUALLY ELASTIC DEMANDS

This section and the next extend the analysis beyond linear demand functions. For constant marginal cost c, the monopolist's profit under demand $q(p)$ is $\pi = q[p(q) - c]$. The monopoly price $p^*(c)$ satisfies $p(q) + qp'(q) - c = 0$. It will be useful for later analysis to let $\eta(p) = -pq'(p)/q$ be the price elasticity of demand (in absolute value); let $q^* = q(p^*(c))$; and let $\alpha = -pq''(p)/q'(p)$ and $\sigma = -qp''(q)/p'(q)$ be the curvatures (i.e., the elasticity of the slopes) of direct and inverse demand functions, respectively, where $\alpha \equiv \eta\sigma$.

The pass-through rate from marginal cost to the monopoly price also will prove useful. As noted by Bulow and Pfleiderer (1983), the pass-through rate equals the ratio of the slope of inverse demand to that of marginal revenue. Thus,

[10] The specific condition for welfare to rise, $a_H - a_L < 3(c_H - c_L)$, implies that the gap in margins between market H and L under differential pricing, $(p_H^* - c_H) - (p_L^* - c_L)$, is less than under uniform pricing, $(\overline{p} - c_L) - (\overline{p} - c_H) = c_H - c_L$. This requires the output reallocation to market H not to be so large as to create a greater (but opposite) discrepancy in price-cost margins than under uniform pricing. The condition for consumer surplus to rise, $a_H - a_L < (c_H - c_L)/3$, can be shown to imply that the weighted-average price under differential pricing is not sufficiently higher than the uniform price to outweigh consumers' gain from the price dispersion.

$$p^{*\prime}(c) = \frac{p'(q^*)}{2p'(q^*) + q^* p''(q^*)} = \frac{1}{2 - \sigma(q^*)} > 0, \tag{11}$$

where we maintain the standard assumption that the marginal revenue curve is downward-sloping, so that $2p'(q) + qp''(q) < 0$ and hence $2 - \sigma(q) > 0$. Thus,

$$p^{*\prime\prime}(c) = \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} q'(p^*) p^{*\prime}(c) \leq 0 \tag{12}$$

if and only if

$$\sigma'(q) \geq 0. \tag{13}$$

That is, the pass-through rate from marginal cost to the monopoly price will be non-increasing in marginal cost if and only if the curvature of the inverse demand is not decreasing in output (inverse demand is not less convex or more concave at higher q).

The curvature σ is non-decreasing for many common demand functions, including those that display constant pass-through rates (Bulow and Pfleiderer, 1983): (i) $p = a - bq^\delta$ for $\delta > 0$, which reduces to linear demand if $\delta = 1$, and whose pass-through rate is $p^{*\prime}(c) = 1/(1+\delta) \in (0,1)$; (ii) constant-elasticity demand functions $p = \beta q^{-1/\eta}$ for $\beta > 0$, $\eta > 1$, hence $p^{*\prime}(c) = \eta/(\eta - 1) > 1$; and (iii) $p = a - b \ln q$ for $a, b > 0$ and $q < \exp(a/b)$, which reduces to exponential demand ($q = e^{-\alpha p}$) if $a = 0$ and $\alpha = 1/b$, and whose pass-through rate is $p^{*\prime}(c) = 1$.

To isolate the role of pure cost differences, this section abstracts from price discrimination incentives by considering demand functions in the two markets that have equal elasticities at any common price. Equal elasticities require that demands be proportional to each other, which we express as $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda) q(p)$ so that $q^L = \frac{\lambda}{1-\lambda} q^H$, for $\lambda \in (0,1)$. A natural interpretation is that all consumers have identical demands $q(p)$ and λ and $(1-\lambda)$ are the shares of all consumers represented by market L and H, respectively. The function $q(p)$ can take a general form.

With proportional demands the monopolist's differential prices are given by the same function $p^*(c)$ but evaluated at the different costs: $p_L^* \equiv p^*(c_L)$, $p_H^* \equiv p^*(c_H)$. Let $\overline{c} \equiv \lambda c_L + (1-\lambda) c_H$. The optimal uniform price $\overline{p}$ maximizes $\pi(p) = \lambda(p - c_L) q(p) + (1-\lambda)(p-$

$c_H)q(p) = [p - \bar{c}]q(p)$. Thus, $\overline{p} \equiv p^*(\bar{c})$: the monopolist chooses its uniform price as though its marginal cost in both markets were $\bar{c}$, the average of the actual marginal costs weighted by each market's share of all consumers. It follows that $\lambda p_L^* + (1-\lambda)\, p_H^* \leq \overline{p}$, or differential pricing does not raise average price for the two market, if $p^*(c)$ is concave ($p^{*\prime\prime}(c) \leq 0$), i.e., if the pass-through rate is non-increasing.

Proportional demands further imply that aggregate consumer surplus at any pair of prices (p_L, p_H) equals $\lambda S(p_L) + (1-\lambda)\, S(p_H)$, i.e., consumer surplus in each market is obtained using the common function $S(p)$ derived from the base demand $q(p)$, but evaluated at that market's price and weighted by its share of consumers. Then, when $p^*(c)$ is concave (or if $\sigma'(q) \geq 0$ from (13)),

$$
\begin{aligned}
S^* &= \lambda S(p_L^*) + (1-\lambda)\, S(p_H^*) \\
&> S(\lambda p_L^* + (1-\lambda)\, p_H^*) \quad \text{(since } S(p) \text{ is convex)} \\
&\geq S(\overline{p}) \quad \text{(since } \lambda p_L^* + (1-\lambda)\, p_H^* \leq \overline{p}).
\end{aligned}
$$

That is, when $\sigma'(q) \geq 0$ or the pass-through rate is non-increasing, which ensures that average price is not higher under differential than under uniform pricing, the price dispersion caused by differential pricing must raise consumer welfare.

Even if differential pricing raises the average price somewhat, as occurs when $\sigma'(q) < 0$ (hence $p^{*\prime\prime}(c) > 0$), consumer welfare will still increase due to the gain from price dispersion if $\sigma(q)$ does not decrease too fast, or

$$
\sigma'(q) > -\frac{2-\sigma(q)}{q}, \tag{A1}
$$

where the right hand side is negative since $2 - \sigma(q) > 0$ from (11).

Proposition 2 *Assume $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda)\, q(p)$ for $\lambda \in (0,1)$. If (A1) holds, differential pricing increases consumer surplus relative to uniform pricing.*

Proof. First, we show that, if and only if (A1) holds, aggregate consumer surplus is a strictly convex function of constant marginal cost c. With demand $q(p)$, aggregate consumer surplus under $p^*(c)$ is

$$s(c) \equiv S(p^*(c)) = \int_{p^*(c)}^{\infty} q(x)\,dx.$$

Thus, $s'(c) = -q(p^*(c))\,p^{*\prime}(c)$ and

$$s''(c) = -q'(p^*(c))\left[p^{*\prime}(c)\right]^2 - q(p^*(c))\,p^{*\prime\prime}(c).$$

Using the expressions for $p^{*\prime}(c)$ and $p^{*\prime\prime}(c)$ from (11) and (12), we have $s''(c) > 0$ if and only if (A1) holds.

Second, consumer surplus under differential pricing (S^*) and under uniform pricing ($\bar{S}$) are ranked as follows:

$$\begin{aligned}
S^* &= \int_{p_L^*}^{\infty} \lambda q(x)\,dx + \int_{p_H^*}^{\infty} (1-\lambda)\,q(x)\,dx \\
&= \lambda s(c_L) + (1-\lambda)\,s(c_H) \\
&> s(\lambda c_L + (1-\lambda)\,c_H) \ \text{ (by the convexity of } s(c)\text{)} \\
&= S(p^*(\bar{c})) = \int_{\bar{p}}^{\infty} q(x)\,dx \\
&= \int_{\bar{p}}^{\infty} \lambda q(x)\,dx + \int_{\bar{p}}^{\infty} (1-\lambda)\,q(x)\,dx = \bar{S}.
\end{aligned}$$

■

We note that (A1) is a fairly tight sufficient condition for differential pricing to raise consumer welfare, in the sense that it is the necessary *and* sufficient condition for consumer surplus as a function of constant marginal cost, $s(c)$, to be strictly convex.[11]

Condition (A1) can be equivalently stated as $\eta > p^*(c)\,p^{*\prime\prime}(c) / \left[p^{*\prime}(c)\right]^2$, the assumption on the pass-through rate made in Cowan (2012, p. 335).[12] Cowan (2012) analyzes price changes due to pure price discrimination as if there were counterfactual changes in marginal

[11] If $s''(c)$ had a consistent sign over the relevant range of c, then (A1) would also be the necessary (and sufficient) condition for differential pricing to increase consumer welfare, but since in general $s''(c)$ may not have a consistent sign, (A1) is sufficient but may not be necessary.

[12] This assumption is satisfied by numerous demand functions, including those for which the pass-through rate is constant or decreasing.

costs.[13] Under the assumption on the pass-through rate or, equivalently, our (A1), he shows that discriminatory pricing will increase aggregate consumer surplus if, evaluated at the uniform price, the ratio of pass-through rate to price elasticity of demand is no lower in market L than in H (Cowan's Proposition 1(i)).[14] This turns out to be a rather restrictive condition. Cowan notes that "The set of demand functions whose shape alone implies that [consumer] surplus is higher with discrimination is small. The surprise, perhaps, is that it is non-empty."[15] Strikingly, when differential pricing is motivated solely by different costs instead of demand elasticities, our Proposition 2 shows that (A1) *alone* is sufficient to ensure that differential pricing will increase consumer welfare.[16]

Since Proposition 2 covers all demand functions with a constant pass-through rate, it includes proportional linear demands. Our Proposition 1, however, addressed linear demands that may differ also in their elasticity parameter ($a_H > a_L$, whereas proportional demands allow only the slope parameters b_L and b_H to differ).

Total welfare increases with differential pricing more often than does consumer surplus since welfare includes profits which necessarily rise. In fact, our welfare comparison will use

[13] The analogy holds because the monopolist's uniform price $\overline{p}$ would be its optimal price for each market if, counterfactually, it faced different costs in the two markets: $\widehat{c}_H = MR_H(q_H(\overline{p})) < MR_L(q_L(\overline{p})) = \widehat{c}_L$ instead of the common marginal cost c. Whereas under differential pricing the monopolist sets prices based on c and the different demand elasticities.

[14] Intuitively, differing elasticities create a bias for discrimination to raise the average price. In order to offset this bias the demand curvatures must be such that the monopolist has a stronger incentive to cut price in the market where its virtual marginal cost fell than to raise price in the other market.

[15] Specifically, his sufficient condition for consumer surplus to rise is only satisfied by two demand functions: logit demands with pass-through above one half and demand based on the Extreme Value distribution (Cowan, pp. 340-1).

[16] The contrast between cost-based versus elasticity-based differential pricing is also seen from Cowan's Proposition 1(ii) which provides sufficient conditions for consumer surplus to fall. One such case is concave demands in both markets with the same pass-through rate (Cowan, p. 339). That case falls within our Proposition 2, hence consumer surplus would increase when differential pricing is motivated purely by different costs.

the following sufficient condition:

$$\sigma'(q) \geq -\frac{[3-\sigma(q)][2-\sigma(q)]}{q}, \tag{A1'}$$

where $3 - \sigma(q) > 1$ since $2 - \sigma(q) > 0$ from (11). Thus condition (A1') relaxes (A1). Condition (A1') ensures that total welfare is a strictly convex function of marginal cost (whereas (A1) ensured the same for consumer surplus), yielding the following result whose proof is otherwise similar to that of Proposition 2 and therefore relegated to the Appendix.

Proposition 3 *Assume $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda) q(p)$ for $\lambda \in (0,1)$. If (A1') holds, differential pricing increases total welfare.*

In analyzing (pure) price discrimination, Aguirre, Cowan and Vickers (2010, ACV) assume an increasing ratio condition (IRC): $z(p) = \frac{p-c}{2-m\eta\sigma}$ strictly increases, where $m \equiv (p-c)/p$. ACV then show that price discrimination *reduces* welfare if the direct demand function in the strong market (our H) is at least as convex as in the weak market at the uniform price (ACV, Proposition 1). Since

$$z'(p) = \frac{2 - m\eta\sigma + (p-c)\left(\frac{d(m\eta)}{dp}\sigma + m\eta\sigma'(q)q'\right)}{(2-m\eta\sigma)^2},$$

$z'(p) > 0$ is equivalent to

$$\sigma'(q) < \frac{1}{-q'}\left[\frac{2-m\eta\sigma}{p-c} + \frac{d(m\eta)}{dp}\sigma\right]\frac{1}{m\eta},$$

which, provided $\frac{d(m\eta)}{dp} = -\frac{[q'+(p-c)q'']q-(p-c)[q']^2}{q^2} \geq 0$, is satisfied if $\sigma'(q)$ is not too positive.[17] Therefore, the IRC condition in ACV and our (A1') both can be satisfied if $\sigma(q)$ neither increases nor decreases too fast, which encompasses the important class of demand functions with a constant σ. However, in contrast to price discrimination, for these demand functions differential pricing based purely on cost differences will *increase* total welfare.

Under the IRC assumption, ACV's Proposition 2 shows that welfare is *higher* with discrimination if the discriminatory prices are not far apart and the inverse demand function

[17] From ACV, condition $z'(p) > 0$ holds for a large number of common demand functions, including linear, constant-elasticity, and exponential. IRC neither implies nor is implied by our (A1).

in the weak market is locally more convex than that in the strong market. However, our Proposition 3 shows that differential pricing motivated by cost differences increases welfare also for markets that have the *same* demand curvatures.

The remainder of this section further illustrates the channels by which differential pricing affects overall welfare and consumer surplus. With cost differences the output reallocation effect of differential pricing can be positive for welfare, which is ensured if the margin under differential pricing is no lower in market L than in H (Remark 1). This condition is met in the case of proportional demands if the pass-through rate does not exceed 1 over the relevant cost range, because $p^{*\prime}(c) \leq 1$ implies $p_H^* - p_L^* = \int_{c_L}^{c_H} p^{*\prime}(c)\, dc \leq \int_{c_L}^{c_H} dc = c_H - c_L$. (Conversely, $p_H^* - p_L^* > c_H - c_L$ if $p^{*\prime}(c) > 1$.)

Remark 3 *With proportional demands the output reallocation effect from differential pricing is positive if (but not only if) the pass-through rate does not exceed one over the range of marginal costs in the two markets: $p^{*\prime}(c) \leq 1$ for $c \in [c_L, c_H]$.*

Given a pass-through rate not exceeding one, differential pricing can be beneficial even if total output falls:

Example 1 *(Differential pricing reduces output but raises total welfare and consumer surplus.) Suppose $p = a - bq^\delta$, with $q = \left(\frac{a-p}{b}\right)^{1/\delta}$ and $\delta > 1$. For $c < a$, we have $p^*(c) = a - \frac{a-c}{\delta+1}$, $q^*(c) = \left(\frac{1}{b}\frac{a-c}{\delta+1}\right)^{1/\delta}$, so $q^*(c)$ is strictly concave when $\delta > 1$. Hence*

$$\Delta q = (q_L^* + q_H^*) - (\bar{q}_L + \bar{q}_H) = \lambda q^*(c_L) + (1-\lambda)\, q^*(c_H) - q^*(\lambda c_L + (1-\lambda)\, c_H) < 0,$$

so differential pricing reduces total output. However, this demand function satisfies (A1). Thus, differential pricing increases consumer surplus and, hence, also total welfare. Consumer surplus increases here because the weighted-average price is equal to the uniform price (since $p^{*\prime\prime}(c) = 0$), but differential pricing generates price dispersion which benefits consumers. Since total output falls the increase in welfare must come from the reallocation effect (recalling (8)). From Remark 3, the reallocation indeed is beneficial since the pass-through rate is less than one, $p^{*\prime}(c) = 1/(\delta+1)$, and in this case dominates the negative *output effect.*[18]

For proportional demands, although unusual, it is possible to find cases where (A1) does not hold and differential pricing *reduces* consumer surplus, as in the example below. (A second example is in the Appendix .) We have not been able, however, to find examples where differential pricing under pure cost differences reduces total welfare.

Example 2 *(Differential pricing reduces consumer surplus: logit demand.) Assume* $c_L = 0$, $c_H = 0.5$, $\lambda = 1/2$, *and logit demand*

$$q^L = \frac{1}{1+e^{p-a}} = q^H;\ p^L = a - \ln\frac{q}{1-q} = p^H.$$

Let $a = 8$*. Then* $p_L^* = 6.327$, $p_H^* = 6.409$, $\bar{p} = 6.367$; $q_L^* = 0.842$, $q_H^* = 0.831$, $\bar{q} = 0.837$. *Differential pricing in this case raises average price and lowers total output. It reduces consumer welfare:* $\Delta S = -8.59 \times 10^{-4}$; *but total welfare increases:* $\Delta W = 4.87 \times 10^{-4}$. *Notice that in this example, (A1) is violated when* $q > 0.5$, *but (A1') is satisfied for* $q < 1$ *(which is always true).*

5. GENERAL DEMANDS

When demand is linear in both markets Proposition 1 showed that if the cost difference is sufficiently large relative to the demand difference, differential pricing will increase both total welfare and consumer surplus. It is not obvious that this result extends to general demands, because as the cost difference grows the average price under differential pricing may rise faster than that under uniform pricing (as shown later in Example 3). To address the mixed case where there are differences both in general demand functions and in costs we develop an alternative analytical approach that more clearly disentangles their roles, and use it to derive a sufficient condition for differential pricing to raise consumer surplus, hence also total welfare.

[18] The reallocation is beneficial for any $\delta > 0$. If $\delta \leq 1$ (instead of > 1 as assumed thus far), then differential pricing would not lower total output, and the two effects would reinforce each other to increase total welfare.

Without loss of generality, let

$$c_H = c + t, \qquad c_L = c - t.$$

Then, $c_H - c_L = 2t$, which increases in t, and $c_H = c_L$ when $t = 0$. Thus, c is the average of the marginal costs and t measures the cost differential. For $i = H, L$, the monopoly price under differential pricing $p_i(t)$ satisfies $\pi^{i\prime}(p_i(t)) = 0$ or

$$q^i(p_i(t)) + [p_i(t) - c_i]\, q^{i\prime}(p_i(t)) = 0. \tag{14}$$

Define the monopoly price in each market when there is no cost difference as $p_i^0 \equiv p_i(0)$, and define $q_i(t) = q^i(p_i(t))$.

From (14), using $c_H = c + t$, we have

$$p_H'(t) = \frac{q^{H\prime}(p_H(t))}{\pi^{H\prime\prime}(p_H(t))} = \frac{1}{2 + \frac{[p_H(t) - c_H]}{p_H(t)} p_H(t) \frac{q^{H\prime\prime}(p_H(t))}{q^{H\prime}(p_H(t))}}.$$

Using the definitions of η, λ, and σ, recalling that $\frac{p^*(c)-c}{p^*(c)} = \frac{1}{\eta(p^*(c))}$, and noticing that $dc_L/dt = -dc_H/dt = -1$, we have

$$p_H'(t) = \frac{1}{2 - \sigma^H(q_H(t))} > 0; \qquad p_L'(t) = -\frac{1}{2 - \sigma^L(q_L(t))} < 0, \tag{15}$$

where $2 - \sigma^i(q_i(t)) > 0$ from (11).

Let $\bar{p}(t)$ be the monopoly uniform price, which solves

$$q^H(\bar{p}(t)) + [\bar{p}(t) - c - t]\, q^{H\prime}(\bar{p}(t)) + q^L(\bar{p}(t)) + [\bar{p}(t) - c + t]\, q^{L\prime}(\bar{p}(t)) = 0.$$

Then

$$\bar{p}'(t) = -\frac{q^{L\prime}(\bar{p}(t)) - q^{H\prime}(\bar{p}(t))}{\pi^{H\prime\prime}(\bar{p}(t)) + \pi^{L\prime\prime}(\bar{p}(t))}. \tag{16}$$

Since the denominator is negative, $sign\, \bar{p}'(t) = sign\, [q^{L\prime}(\bar{p}(t)) - q^{H\prime}(\bar{p}(t))]$. Thus, $\bar{p}'(t) > 0$ if at the initial uniform price the demand function q^L is less price sensitive (steeper) than is q^H. Intuitively, an increase in the cost difference t gives the monopolist an incentive to raise the output-mix ratio q_L/q_H. This requires increasing the uniform price, hence reducing total output, if q^L is steeper than q^H, and lowering price if q^H is steeper.

Define

$$\phi^i(q) \equiv \frac{q}{2-\sigma^i(q)}. \tag{17}$$

Then $\phi^{i\prime}(q) > 0$ for $i = H, L$ if and only if (A1) holds.

Using (1), we have

$$\begin{aligned}
\Delta S'(t) &\equiv S^{*\prime}(t) - \overline{S}'(t) \\
&= -q^L(p_L(t))\, p_L'(t) - q^H(p_H(t))\, p_H'(t) + \left[q^L(\bar{p}(t))\, \bar{p}'(t) + q^H(\bar{p}(t))\, \bar{p}'(t)\right].
\end{aligned}$$

Consumer welfare will increase faster under differential than under uniform pricing with a marginal increase in t if $\Delta S'(t) > 0$, and, for any given $t > 0$, consumer welfare will be higher under differential pricing if $\Delta S(t) > 0$.

Using (15) and (16), we can write $\Delta S'(t)$ as

$$\Delta S'(t) = \phi^L\left(q^L(p_L(t))\right) - \phi^H\left(q^H(p_H(t))\right) - \frac{\left[q^L(\bar{p}(t)) + q^H(\bar{p}(t))\right]\left[q^{L\prime}(\bar{p}(t)) - q^{H\prime}(\bar{p}(t))\right]}{\pi^{H\prime\prime}(\bar{p}(t)) + \pi^{L\prime\prime}(\bar{p}(t))}. \tag{18}$$

Proposition 4 *Under (A1), if (i) $q^{L\prime}(\bar{p}(t)) \geq q^{H\prime}(\bar{p}(t))$, (ii) $\phi^L\left(q^L(p_L(\delta_1))\right) \geq \phi^H\left(q^H(p_H(\delta_1))\right)$ for sufficiently small $\delta_1 \geq 0$, and (iii) $\Delta S(0) \geq -\delta_2$ for sufficiently small $\delta_2 \geq 0$, then there exists some $\hat{t} \geq 0$, with $\hat{t} = 0$ if $\delta_1 = \delta_2 = 0$, such that when $t > \hat{t}$, consumer surplus and total welfare are higher under differential pricing than under uniform pricing.*

Proof. From (18), when (i) holds, $\Delta S'(t) \geq \phi^L\left(q^L(p_L(t))\right) - \phi^H\left(q^H(p_H(t))\right) \equiv S^{*\prime}(t)$. Since $p_L'(t) < 0$ and $p_H'(t) > 0$, we have

$$S^{*\prime\prime}(t) = \phi^{L\prime}\left(q^L(p_L(t))\right) q^{L\prime}(p_L(t))\, p_L'(t) - \phi^{H\prime}\left(q^H(p_H(t))\right) q^{H\prime}(p_H(t))\, p_H'(t) > 0,$$

where $\phi^{i\prime}\left(q^L(p_L(t))\right) > 0$ by (A1). Hence $S^{*\prime}(t)$ is strictly increasing. Then, from (ii), $S^{*\prime}(t) > 0$ for $t > \delta_1$. Therefore, if $\delta_1 = \delta_2 = 0$, we have $\Delta S(0) \geq 0$, $\Delta S'(0) \geq 0$, and $\Delta S'(t) = \phi^L\left(q^L(p_L(t))\right) - \phi^H\left(q^H(p_H(t))\right) > 0$ for all $t > 0$. It follows that $\Delta S(t) > 0$ for all $t > 0$, or $\hat{t} = 0$.

Next suppose that $\delta_i > 0$ for at least one i. We can fix some $\hat{t} > 0$ such that when $t > \hat{t}$ and $\delta_1 < \hat{t}/2$:

$$\begin{aligned}
\Delta S(t) &= \int_0^t \Delta S'(x)\,dx + \Delta S(0) = \int_0^{\delta_1} \Delta S'(x)\,dx + \int_{\delta_1}^t \Delta S'(x)\,dx + \Delta S(0) \\
&\geq \int_0^{\delta_1} S^{*\prime}(x)\,dx + \int_{\delta_1}^{\hat{t}/2} S^{*\prime}(x)\,dx + \int_{\hat{t}/2}^{\hat{t}} S^{*\prime}(x)\,dx + \Delta S(0) \\
&> S^{*\prime}(0)\,\delta_1 + S^{*\prime}\left(\frac{\hat{t}}{2}\right)\frac{\hat{t}}{2} - \delta_2 \\
&> 0 \text{ when } \delta_1 \text{ and } \delta_2 \text{ are small enough.}
\end{aligned}$$

Finally, total welfare is also higher since $\Delta W(t) \geq \Delta S(t)$. ■

The sufficient conditions for differential pricing to benefit consumers under general demands include (A1), as with proportional demands, and three additional conditions whose roles are as follows. Condition (i) requires that $q^L(p)$ be at least as steep as $q^H(p)$ at the uniform price $(\bar{p}(t))$. This ensures that under uniform pricing a marginal increase in t does not reduce price, hence does not increase consumers surplus. Condition (ii) ensures that a marginal increase in t increases consumer surplus under differential pricing at some small t, and (A1) further ensures that consumer surplus will increase at an increasing rate. Hence, if consumer welfare is not too much lower under differential than under uniform pricing when there is no cost difference, which is ensured by (iii), consumer welfare will be higher under differential pricing if the cost difference is sufficiently high.

The conditions for Proposition 4 can be satisfied in many plausible situations, even when pure price discrimination ($c_H = c_L$) would reduce consumer welfare, as in many of the cases identified in Proposition 1 of ACV. For instance, the linear demands case of Section 3 is covered by Proposition 4.[19] Also, if demands are proportional, $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda)q(p)$, then for $\lambda \geq 1/2$ (market L is at least as large as H) one can verify

[19] Recall that $q^H = \frac{a_H - p}{b}$ and $q_L = \frac{a_L - p}{b}$, with $a_H > a_L$. Then, both (A1) and (i) are satisfied, with $p_H(t) = \frac{a_H + c + t}{2}$, and $p_L(t) = \frac{a_L + c - t}{2}$. Furthermore, both (ii) and (iii) hold for $\delta_1 = \frac{a_H - a_L}{2}$ and $\delta_2 = \frac{(a_H - a_L)}{8}$. Thus, if $t > \hat{t} = \frac{3}{2}(a_H - a_L)$—implying $(c_H - c_L) > 3(a_H - a_L)$, the condition in part (ii) of Proposition 1—then differential pricing increases consumer and total welfare, even though for linear demands pure price discrimination reduces consumer welfare.

that conditions (i)-(iii) are all satisfied with $\delta_1 = \delta_2 = 0$, so that under (A1) differential pricing increases consumer and total welfare.[20]

The next example shows that if the conditions in Proposition 4 are not met, then differential pricing can reduce total welfare (hence also consumer surplus) even as $c_H - c_L$ becomes arbitrarily large (subject to the constraint that both markets will still be served under uniform pricing).

Example 3 *(Differential pricing reduces welfare for any cost difference.) Suppose that* $c_L = 0$, $c_H \in (0,\ 0.539]$ *and demands are*

$$q^L = 2\,(1-p)\,; \quad q^H = e^{-2p},\ p^H = -\frac{1}{2}\ln q.$$

Then, both markets are served under uniform pricing if and only if $c_H \leq 0.539$, *and* $p_L^* = 0.5$; $p_H^*(c) = 0.5 + c_H$. *Notice that condition (i) in Proposition 4 is violated here since* $\bar{p}\,(t) \geq 0.5$ *and*

$$q^{L\prime}\,(p) = -1 < q^{H\prime}\,(p) = -2e^{-2p} \text{ for all } p \geq 0.5.$$

Thus, under uniform pricing $\bar{p}$ *would fall as the cost difference rises if average cost were kept constant. This force causes total welfare to be lower under differential than under uniform pricing in this example over the entire range of cost differences for which both markets are served. Table 1 illustrates this, where for convenience we have fixed* $c_L = 0$ *and considered increasing values of* c_H *(so that* $\bar{p}$ *increases, but less so than* $(p_L^* + p_H^*)\,/2$, *as average cost rises):*

Table 1. $c_L = 0,\ p_L^* = 0.5,\ p_H^* = 0.5 + c_H,\ \Delta q = q^* - \bar{q}$

c_H	0.1	0.2	0.3	0.4	0.5	0.539
$\bar{p}$	0.515 1	0.529 6	0.543 3	0.556 5	0.5691	0.573 9
$\frac{p_L^*+p_H^*}{2}$	0.5500	0.6000	0.6500	0.7000	0.7500	0.769 5
Δq	-0.0255	-0.0409	-0.0489	-0.0503	-0.0469	-0.044
ΔW	-0.010	-0.011	-0.004	-0.044	$-0.03\,7$	-0.034

[20] However, conditions (i)-(iii) are sufficient but not necessary since Proposition 2 showed that with proportional demands differential pricing increases consumer and social welfare under (A1) for all $\lambda \in (0, 1)$.

Interestingly, in Example 3 the allocation of output is efficient under differential pricing (and not under uniform), since the markups are equal in the two markets: $p_H^* - c_H = p_L^* - c_L = 0.5$. However, average price under differential pricing $(0.5 + c_H/2)$ exceeds the uniform price $\bar{p}$ for all values of c_H and output is lower, which reduces welfare despite the improved allocation. By contrast, differential pricing improved welfare in Example 1 that exhibited pure cost differences, even though output fell there as well (but average price under differential pricing equaled the uniform price for all cost values there). The added incentive to raise average price under differential pricing due to demand differences causes a stronger negative output effect here that outweighs the improved allocation.

The analysis in ACV uses the approach that varies the constraint $p_H - p_L \leq r$, where r is the price difference allowed. Proposition 2 in ACV gives a sufficient condition for price discrimination to increase total welfare, which can be satisfied *only if* inverse demand in the weak market is *more* convex than that in the strong market at the discriminatory prices and these prices are close to each other (ACV, p. 1606). Letting

$$z_i(p) \equiv \frac{(p - c_i)}{2 - m^i(p)\,\alpha^i(p)}, \tag{19}$$

where $m^i(p) \equiv \frac{p - c_i}{p}$ is the proportional price-cost margin (the Lerner index) and $\alpha^i(p) = -\frac{pq^{i\prime\prime}(p)}{q^{i\prime}(p)}$, we can extend ACV's Proposition 2 straightforwardly to our more general case where costs as well demands differ.

Remark 4 *Assume* $z_i'(p) > 0$. *If in addition*

$$\frac{p_L^* - c_L}{2 - \sigma^L(q_L^*)} \geq \frac{p_H^* - c_H}{2 - \sigma^H(q_H^*)}, \tag{20}$$

then differential pricing increases total welfare.

Since $p_L^* < p_H^*$, if costs are equal as in ACV, condition (20) can only be met if $\sigma^L(q_L^*) > \sigma^H(q_H^*)$: at the discriminatory prices, inverse demand is more convex in the weak market, which is needed for price discrimination to increase total output. This curvature condition, however, is not required for differential pricing to increase welfare when costs differ, since differential pricing can easily induce $p_L^* - c_L > p_H^* - c_H$, i.e., a positive reallocation effect in the strong sense relative to uniform pricing.

6. CONCLUSION

Prevailing economic analysis of third-degree price discrimination by a monopolist paints an ambivalent picture of its welfare effects relative to uniform pricing. In order for overall welfare to rise total output must expand, and without specific knowledge of the shapes of demand curves the literature yields no presumption about the change in output unless discrimination leads the firm to serve additional markets. Moreover, since discrimination raises profits, an increase in overall welfare is necessary but not sufficient for aggregate consumer surplus to rise.

This paper showed that judging differential pricing through the lens of pure price discrimination understates its beneficial role when price differences are motivated at least in part by differences in the costs of serving various markets. Differential pricing then saves costs by reallocating output to lower-cost markets, and can easily benefit consumers in the aggregate by creating price dispersion which—unlike pure price discrimination—does not come with a systematic bias for average price to rise.

One policy application involves the common and growing practice of add-on pricing or unbundling the pricing of various elements from the base good. This is sometimes decried as harmful to consumers based, perhaps implicitly, on a price discrimination view. Our analysis casts add-on pricing in a considerably more benign light when the add-on services entail significant incremental costs. A potential extension would be to analyze whether/how the beneficial aspects of differential pricing under different costs might extend beyond monopoly to imperfect competition, building on the analyses of oligopoly price discrimination (e.g., Holmes 1989 and Stole 2007).

APPENDIX

Example 4 *(Differential pricing reduces consumer surplus: concave demands over the relevant range.) Suppose that* $c_L = 0$, $c_H = 0.2$, *and*

$$p^L = (1 - \ln(q/10))^{\frac{1}{4}} = p^H; \; q^L = 10e^{\left(1-p^4\right)} = q^H.$$

Then $p_L^* = 0.707\,11$, $p_H^* = 0.762\,94$, $\bar{p} = 0.733\,5$; $q_L^* = 21.\,170$, $q_H^* = 1\,9.37\,1$, $\bar{q} = 20.\,351$. *(A1) is violated for* q^* *since*

$$\sigma'(q) < -\frac{[2 - \sigma(q)]}{q} \text{ if } q > 15.392.$$

$\Delta S = -0.037$; *hence aggregate consumer welfare is lower due to differential pricing. Here, the rise in the average price due to differential pricing,* $\frac{p_L^* + p_H^*}{2} = 0.735\,03 > \bar{p}$, *outweighs the consumer benefits from price dispersion. Total output falls:* $q_L^* + q_H^* = 40.541 < 2\bar{q} = 40.702$. *However, total welfare is still higher under differential pricing:* $\Delta W = 0.053 > 0$.

Proof of Proposition 3. First, we show that under (A1'), total welfare is a strictly convex function of constant marginal cost c. Total welfare under $p^*(c)$ is

$$w(c) \equiv W(p^*(c)) = \int_0^{q(p^*(c))} [p(x) - c]\, dx.$$

Thus

$$w'(c) = [p^*(c) - c]\, q'(p^*(c))\, p^{*\prime}(c) - q(p^*(c)).$$

From the first-order condition for $p^*(c)$, we have $[p^*(c) - c]\, q'(p^*(c)) = -q(p^*(c))$. Hence

$$w'(c) = -q(p^*(c))\, p^{*\prime}(c) - q(p^*(c)) = -q(p^*(c)) \left[p^{*\prime}(c) + 1\right].$$

It follows that

$$w''(c) = -q'(p^*(c))\, p^{*\prime}(c) \left[\frac{1}{2 - \sigma(q^*)} + 1\right] - q(p^*(c)) \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} q'(p^*)\, p^{*\prime}(c).$$

Therefore, $w''(c) > 0$ if

$$3 - \sigma(q^*) + q(p^*(c)) \frac{\sigma'(q^*)}{2 - \sigma(q^*)} > 0,$$

or if (A1') holds.

Next,

$$\begin{aligned}
W^* &= \lambda \int_0^{q(p^*(c_L))} [p(x) - c_L]\, dx + (1-\lambda) \int_0^{q(p^*(c_H))} [p(x) - c_H]\, dx \\
&= \lambda w(c_L) + (1-\lambda) w(c_H) \\
&> w(\lambda c_L + (1-\lambda) c_H) \text{ (by the convexity of } w(c)) \\
&= \int_0^{q(p^*(\bar{c}))} [p(x) - \bar{c}]\, dx \\
&= \lambda \int_0^{q(\bar{p})} [p(x) - c_L]\, dx + (1-\lambda) \int_0^{q(\bar{p})} [p(x) - c_H]\, dx = \bar{W}.
\end{aligned}$$

■

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