On the (IR) Relevance of Monetary Aggregate Targeting in Pakistan: An Eclectic View

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On the (IR) Relevance of Monetary Aggregate Targeting in Pakistan: An Eclectic View

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ABSTRACT

This study investigates and searches for a stable money demand function for Pakistan's economy, where monetary aggregate is considered as the nominal anchor. The stability of the money demand has been the focus of numerous debates due to evolving financial innovations and regulations. Earlier studies in Pakistan on the subject provide conflicting explanations due to inadequate specification and imprecise estimation of money demand. However, this study finds that money demand in Pakistan is stable, if specified properly. Therefore, for developing countries, like Pakistan, targeting of monetary aggregates or responding to deviations from the desirable path is important for effective implementation and communication of monetary policy stance and it should remain, if not primary, an auxiliary target in the monetary policy framework.

JEL Classification: C20, E12, E41, E5
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1. INTRODUCTION

The role of monetary aggregate(s) as a virtue has long been debated in the conduct of monetary policy. It is generally believed that money has important information contents about underlying contemporaneous state of the economy, which helps in predicting some discernible monetary facts. And most of the central bank has used monetary aggregates as an intermediate target in their monetary policy framework. However, usefulness of monetary aggregates diminished, particularly during early 1990s, when the money demand functions subject to structural changes, see for instance, Mishkin (2000) and Woodford (1998, 2008).

It is generally argued that for a successful monetary aggregate targeting a stable, or at least predictable, relationship between money growth and inflation is necessary. However, this relationship becomes more obscure, particularly in advanced economies in 1990s, with the evolution of financial sector.[1] It is argued that financial deregulation and innovation significantly changed the preferences of household and financial sector and thus destabilized the money demand function [Arrau *et al*, (1995), Bernanke, (2006)].[2] As a consequence, many developed and developing countries changed their nominal anchor and switched from monetary aggregate targeting regime to inflation targeting or price level targeting.

Nevertheless, monetary aggregates contain important information and its significance in monetary policy framework cannot be ignored. Detailed assessment of monetary trends and its relationships with the goal variables (output growth and inflation) provide useful information regarding demand pressures in the economy (Hall *et al*., 2012). Keeping in view of this relative importance, we are motivated to analyze the specification and stability of the money demand function in the context of Pakistan. The core functional specification of money demand is derived from a set of intertemporal optimal decisions made by households and firms, in a Dynamic Stochastic General Equilibrium (DSGE) setting. This specification is then estimated empirically using various econometric techniques. While, in this stage, various other potential determinants of money demand are also investigated in terms of goodness of fit. This is because of the preferences of both households and firms, to some extent, changed and operational scope of financial sector widened during the last two decades.[3] However, since developments of structured financial instruments are still in evolution phase. So, it is necessary to analyze different empirical specifications for robustness check.

Another important empirical contribution of this paper is to provide eclectic stability analysis both in terms of model and parameter. As, in case of Pakistan, a vast body of literature estimated empirically the money demand function and look for stability of money demand and velocity of money. However, the results are mixed. Some of the studies find a robust relationship between money and the goal variables, Qayyum (2005), Omer (2010) and Azim, *et al*., (2010) while other find unstable money demand function, Moinuddin (2009), Omer and Saqib (2009). It is quite interesting that with almost the same specification and same

[1] When the households were started to invest in bond and mutual funds.

[2] Lieberman (1977) has argued that "increased use of credit, better synchronization of receipts and expenditures, reduced mail float, more intensive use of money substitutes, and more efficient payments mechanisms will tend to permanently decrease the transaction demand for money over time".

[3] See, Financial Stability Review (various issues) by State Bank of Pakistan.

methodology but with different sample range the researchers arrived at conflicting results.[4] In addition, earlier studies do talk about model stability but remain salient regarding evaluation of parameter stability of the money demand. It is worth noting that among other variables, consistency of interest rate sensitivity is important for stability of the money demand; changing interest rate sensitivity over time would mean that the money demand estimate in one period would not be able to predict well in other time, Mishkin (1995).

There are number of ways, where stability of money demand can be determined empirically. Earlier studies show that mere existence of long-run relationship between monetary aggregates and its determinants is the sign of stability of the money demand. However, this argument does not qualify for stability and requires more statistical test to determine, whether both long-run and short-run elasticities are stable over time, Bahmani-Oskooee and Rehman (2010). These stability tests include recursive estimation of coefficients and analyzing it evolution. If the coefficients display significant variations, both in magnitude and sign, as more information added in the sample then it indicates instability. It is also argued that the problem of instability may not be due to the incorrect specification of long-run function but it may arise due to inadequate modeling of short-term dynamics, see for example, Laidler (1993). Therefore, it is important that the money demand function should correctly specify both in the long-term and in the short-term.

Another way to determine money demand stability is to analyze velocity of monetary aggregates. Omer and Saqib (2009) modeled velocity of money (M0, M1, M2) in a univariate fashion, and argued that each series of monetary aggregate is not mean-reverting and integrated of high order (I(1)), thus concluding non-stationarity of velocities at level which signify instability in money demand. However, this analysis does not qualify to determine money demand stability as most of economic time-series depicting trend and thus non-stationarity and more importantly the latter could be due to structural changes[5], [an empirical point highlighted by Ericsson *et al*, (1997)]. Therefore, if empirical specification is modeled properly, then the result could be different.

Next section discusses the theoretical model of the money demand function. Section III, delineates stylized facts of the monetary aggregates during 1992-2011. Section IV, describes data and different variables used in the analysis. Section V, discusses empirical estimation and results of the money demand function. Section-VI, describes parameter stability. Section VII, narrates the simulation and the last section concludes.

2. SOME STYLIZED MONETARY FACTS IN PAKISTAN

Before going to investigate model specifications and technical details, it is important to take a visual look on various temporal developments of selected macro/monetary economic indicators of Pakistan. **Figure-1** shows trend in broad money (M2) and consumer price index (CPI), (the

[4] Qayyum, (2005) and Moinuddin (2009) used real M2, real GDP, call money rate for period of 1960-99 and 1974-06 respectively.

[5] During the last couple of decades, Pakistan's economy particularly financial sector, went under some structural changes, which include opening of equity market, long-term and short-term bond market for government securities, introduction of foreign currency account, liberalization (to some extent) of external account, opening of new domestic and foreign private banks etc.

variables are mean adjusted and in log form-in order to bring on one scale). During the last two decades, nominal money increased around 14 fold while consumer prices rise 4 times to the level of FY91, witnessing significant increase in broad money (M2).

Figure 1: Trend in Nominal M2 and CPI (mean adjusted & in log)

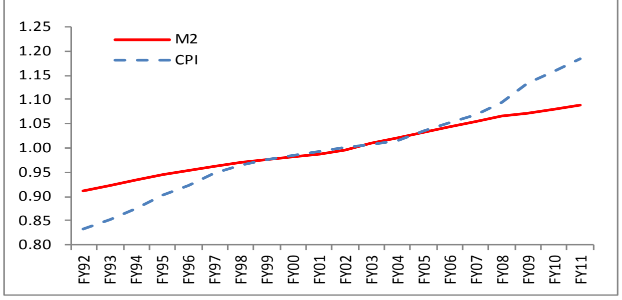


Figure 2: Trend in Real M2 and Real GDP (mean adjusted & in log)

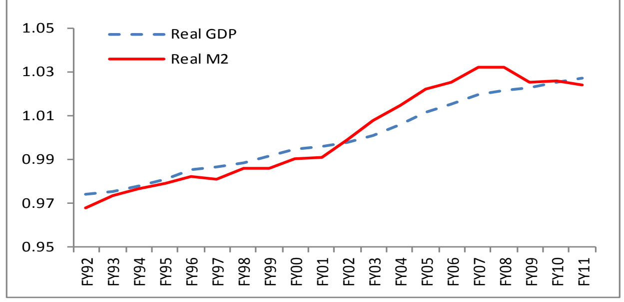


Figure 3: Growth in Nominal M2 and CPI Inflation

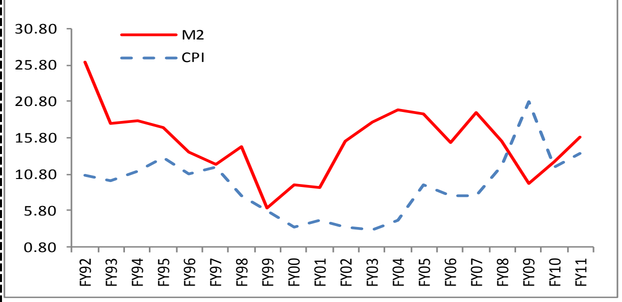


Figure 4: Trend in Lending rate and Velocity of M2

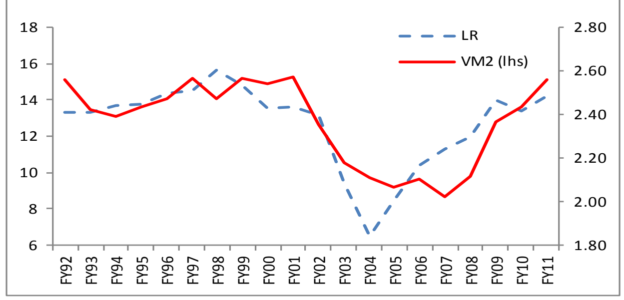


Figure 5: Growth in Nominal M2 and Velocity of M2

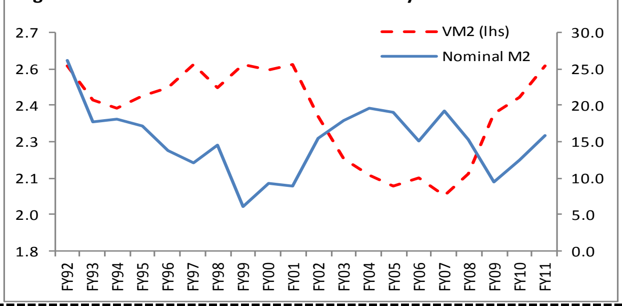
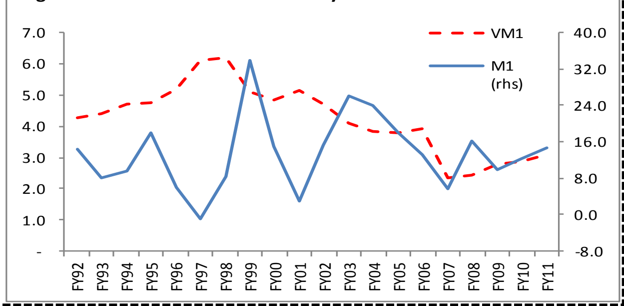


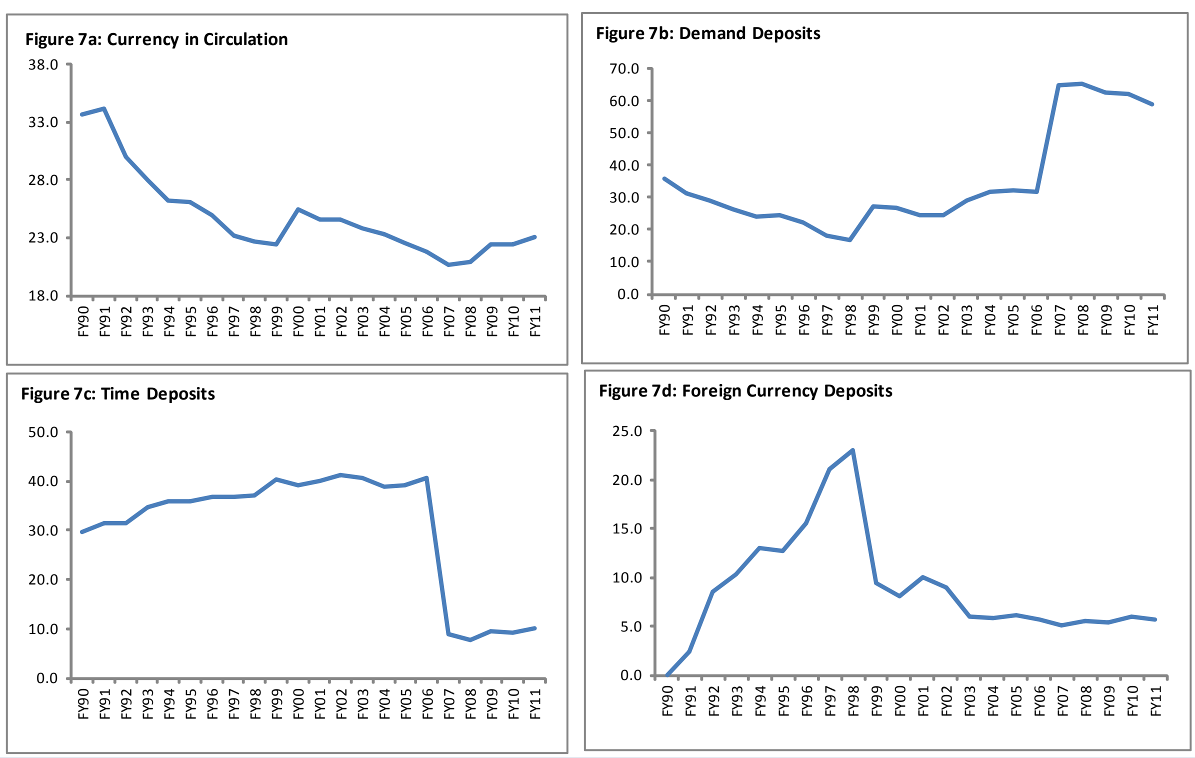
Figure 6: Growth in M1 and Velocity of M1



From **Figure-2**, the real money increase in parallel to real GDP, however, it gain pace in 2001 and onwards mainly due to significant increase in foreign inflows (though sterilized some inflows by SBP through FY04) and subdued inflation. And this phenomenal increase in nominal and real money has had serious repercussion to the economy in term of high inflation and protracted low economic growth later on. **Figure-3** shows that consumer price inflation stabilized when the monetary growth was controlled (particularly in 1995, when money targeting institutionalized); however, significant growth in nominal money in earlier 2000s induced inflationary pressure in the coming period. During the last decade, nominal money increased by 15.2 percent per annum on average, while CPI-inflation witnessed an increase of 8.4 percent per annum on average. However, inflationary pressures were more pronounced since FY08 due to frequent recourse of government borrowing from SBP for budgetary support (inflationary in nature) couple with exorbitant rise in international commodity prices in FY08, particularly energy prices, and erosion of domestic currency. Money velocity (VM2) fell moderately from 2.6 in FY01 to around 2.0 in FY07 and interest rate almost touches the bottom in FY04 (see **Figure-4**). Interestingly, velocity of narrow money depicts more volatility than the velocity of broad money due to impulsive nature of demand deposits and currency in circulation (see **Figure 5 and 6**).

In Pakistan, broad money (M2) include currency in circulation, deposit with SBP, demand and time deposits and foreign currency deposits. **Figure-7** shows the contribution of four major components of broad money, where currency in circulation (CIC) depicts a declining share from little over 34 percent in 1991 to 22 percent in 2010. The contribution of Demand deposits (DD) in broad money also shows decline during 90s, however, it bottom out in 1998 after freezing of resident foreign currency (RFCDs) accounts. Earlier decline in CIC and DD is primarily due to introduction of RFCDs in 1991, where it shares in broad money increased to 23 percent in 1998. However, subsequent to nuclear detonation, foreign currency accounts were frozen due to fear of foreign sanction and dread of capital flight. On the other hand, time deposits increase moderately over time. The data indicates a clear shift from time deposits into demand deposits in 2007 due to change in classification, where deposits with 6-month to one year tenor that was part of time deposits previously, are now reported in demand deposits. Both demand and time deposits contribute around 75 percent in broad money.

Figure 7: Percentage Contribution in Broad Money (M2)



3. THEORETICAL FOUNDATION OF MONEY DEMAND FUNCTION

This section deals with the micro-foundation of standard money demand function. To derive theoretical specification, we have considered a typical New-Keynesian DSGE model as in Gali (2008) and Walsh (2010). The model setup starts with the optimal decision of the households, who maximize their intertemporal utility function subject to life time budget constraint. The utility function depends on consumption, C_t, leisure, $(1 - l_t)$, and real money balances $\frac{M_t}{P_t}$. Firms on the other hand produce output by using labor as standard input of production and try to maximize profit in a monopolistic competitive fashion. Other agents include central bank which conducts monetary policy and government which deals fiscal policy related issues. As we are interested on the theoretical specification of money demand, so we start with household optimization. The lifetime utility function is given as:

$$U_0 = E_t \sum_{i=0}^{\infty} \beta^i u \left[C_{t+i}, (1 - l_{t+i}), \frac{M_{t+i}}{P_{t+i}} \right]$$

Where, $\beta \in (0,1)$, is subjective discount factor. For analytical simplicity, utility function is assumed to be separable and its specification is given as:

$$u \left[C_{t+i}, (1 - l_{t+i}), \frac{M_{t+i}}{P_{t+i}} \right] = \frac{(C_{t+i})^{1-\gamma}}{1-\gamma} + \frac{(1 - l_{t+i})^{1-\upsilon}}{1-\upsilon} + \frac{\xi_{m,t+i}}{1-\omega} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-\omega}$$

Where, $\gamma > 0$, is the parameter of risk aversion, $\upsilon \in (0,1)$, is inverse elasticity of labor supply, $\omega \in (0,1)$, is inverse of interest elasticity of money demand and $\xi_{m,t}$, is a stochastic shock to money demand. Other usual assumption regarding above utility function are: $\frac{\partial u}{\partial(\circ)} > 0$ and $\frac{\partial^2 u}{\partial(\circ)^2} < 0$, which imply that it is increasing but diminishing over time, in each of its arguments. Households want to maximize this utility function subject to following budget constraint:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} l_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t$$

Where, P_t, denotes general price level, B_t, is interest bearing assets with i_t, as nominal gross returns on assets at time t. $W_t l_t$,is nominal labor income and Π_t, is real dividend.

The household optimization process solves the following problem as:

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \left[\begin{array}{l} \left\{ \frac{(C_{t+i})^{1-\gamma}}{1-\gamma} + \frac{(1 - l_{t+i})^{1-\upsilon}}{1-\upsilon} + \frac{\xi_{m,t+i}}{1-\omega} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-\omega} \right\} \\ +\lambda_{t+i} \left\{ \begin{array}{c} \frac{W_{t+i}}{P_{t+i}} l_{t+i} + \frac{M_{t-1+i}}{P_{t+i}} + (1 + i_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} + \Pi_{t+i} \\ -C_{t+i} + \frac{M_{t+i}}{P_{t+i}} + \frac{B_{t+i}}{P_{t+i}} \end{array} \right\} \end{array} \right]$$

Where, λ_t, is the Lagrange-multiplier associated with household budget constraint. The solution to the above optimization process yields the following first order conditions (FOCs):

$$C_t^{-\gamma} = \lambda_t$$

$$\xi_{m,t}\left(\frac{M_t}{P_t}\right)^{-\omega} = \lambda_t\left(1-\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right)$$

$$(1-l_t)^{-\upsilon} = \frac{W_t}{P_t}\lambda_t$$

$$1 = \beta(1+i_t)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right)$$

We can define gross inflation rate: $\pi_{t+1} = (P_{t+1}-P_t)/P_t$. Solving above FOCs simultaneously yield two important results.

The intertemporal Euler equation of consumption:

$$1 = \beta\frac{(1+i_t)}{(1+\pi_{t+1})}E_t\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}$$

The non-linear money demand function:

$$\xi_{m,t}\left(\frac{M_t}{P_t}\right)^{-\omega} = C_t{}^{-\gamma}\left(\frac{i_t}{1+i_t}\right)$$

As economy wide aggregate resource constraint in the model can be written as: $Y_t = C_t$. Therefore, the final non-linear version of money demand function can be simplified as:

$$\xi_{m,t}\left(\frac{M_t}{P_t}\right)^{-\omega} = Y_t{}^{-\gamma}\left(\frac{i_t}{1+i_t}\right)$$

For empirical estimation purpose, one may consider the linear version of above money demand function. So, it can be log-linearize around the deterministic steady-state, using first order Taylor-approximation, any linearized variable can be defined as: $\tilde{x}_t = \log(x_t) - \log(\bar{x})$.

$$1 + \tilde{\xi}_{m,t} + (-\omega)\widetilde{\left(\frac{M_t}{P_t}\right)} = 1 - \gamma\widetilde{Y_t} + \tilde{\imath}_t$$

Or,

$$\widetilde{\left(\frac{M_t}{P_t}\right)} = \frac{\gamma}{\omega}(\widetilde{Y_t}) - \frac{1}{\omega}(\tilde{\imath}_{t)} + \frac{1}{\omega}(\tilde{\xi}_{m,t})$$

If we consider, $\varphi_1 = \frac{\gamma}{\omega}$, $\varphi_2 = -\frac{1}{\omega}$ and $\tilde{\varepsilon}_{m,t} = \frac{1}{\omega}\left(\tilde{\xi}_{m,t}\right)$, where, φ_1 is output elasticity of money demand according restriction on deep parameters, it is appear to be positive and φ_2 which is negative. It signifies that money demand is positively related with output and negatively with nominal interest rates. The final simplified linear version of money demand with the inclusion of an interest term, φ_0 for econometric consideration, can be given as:

$$\widetilde{\left(\frac{M_t}{P_t}\right)} = \varphi_0 + \varphi_1(\widetilde{Y_t}) + \varphi_2(\tilde{\imath}_{t)} + \tilde{\varepsilon}_{m,t}$$

This specification is also consistent with the basic version of Friedman's money demand function. However, Milton Friedman and their followers, also consider other potential determinants of money demand as: $M_d/P = f(W, r - r^e, \pi^e, h)$, where M_d/P is real money balances; W is wealth; r is interest rate; r^e is expected change in interest rate; π^e is expected change in prices level; h is ratio of human and non-human wealth. But since, in practice, it is difficult to determine wealth in the economy, therefore, scale variable (mostly real GDP or in some instance real consumption) is used as a proxy as we also observed in the final specification of above micro-founded model. In empirical work, there is widespread agreement on the choice of scale variable although various theories on money demand have highlighted the importance of different scale variables. For instance, transaction related theories suggest real income, while portfolio approaches emphasis financial wealth Calza *et al.*, (2001). Regarding empirical estimate of income elasticity (i.e. responsiveness of demand for money to changes in real income), the quantity theory suggests one to one relationship between real balances (M2) and real income, while 0.5 according to Baumol-Tobin model for transaction demand for money Calza, *et al.*, (2001). However, in developing countries, income elasticity is much higher (more than one percent) mainly due to insufficient avenues for alternative financial assets and pervasive monetization of economy, Adam *et al*., (2010). In some of empirical studies, stock prices or its volatility is used as additional variable in the money demand function that substantiates the wealth effect, Bruggerman, *et al*., (2003).

Similarly, economic theory also considers interest rate an important variable that reflect the opportunity cost of holding money, but it provides little guidance to select an appropriate interest rate (Laidler, 1993). Therefore, empirical literature uses a variety of interest rates, including short-term market or bond rate, long-term rate, rate of return on alternative financial assets. In portfolio decision making, the economic agent treats a variety of assets as alternate to money, therefore, a wide spectrum of rate of return(s) should be included in the money demand function (Heller and Khan, 1979). However, it raises some statistical issues (i.e. most of interest rates are co-linear) and complicates the estimation of the model. Since most of interest rates move in more or less same direction therefore, researcher should be restricted to limited rate of returns Calza *et al*. (2001). In case of developing countries, it is argued that since the financial market (particularly long-term bond market and equity market) is not fully developed, therefore, the money demand function should include short term market interest rate. Further, economic agent could choose short-term financial assets, as an alternate to money, in high inflationary environments while opting for long-term assets when economic conditions are stable and predictive.

It is also argued that since a significant portion of monetary aggregates (M2 or M3) are remunerative (include demand and time deposits, foreign currency deposit, various saving schemes) therefore own rate of return of monetary aggregates cannot be ignored. Bruggerman *et al*., (2003) uses weighted average rate of return on different components of M2/M3 for calculation of own rate for each country of the Euro area. It is generally expected that the coefficient of own rate of interest should take a non-negative value i.e. an increase in own rate raise demand for real balances (Laidler, 1993). However, it is argued that a tight monetary stance may raises the own rate of interest which in turn increases the demand for money that seems contradictory to the essence of monetary policy Calza *et al*, (2001). Therefore, most studies use interest rate spreads (market rate minus own rate or bond rate minus own rate) instead of including only the own rate.

In money demand function, the expected change in price level enters as opportunity cost of holding money along with nominal interest rate. Importantly, change in price level effect the rate of returns on inventory of goods as high expected inflation induces the economic agent to shift from money to goods (i.e. stocking inventory) due to high profit incentives. However, Golineli, *et al*, (2002) do not include inflation as measure of opportunity cost of holding money in their long-run specification of the money demand function and argued that it has no additional explanatory content on money demand compared to long-term interest rate.

In other specifications exchange rate (or depreciation) is also included in the money demand function in order to capture the effect of currency substitution. The exchange rate depreciation may have positive or negative effect on real balances, Bahmani-Oskooee and Rehman (2010). Since the depreciation of domestic currency increase the value of foreign assets in term of domestic currency and if it is perceived an increase in wealth then it has positive impact on real balance. However, if depreciations enhance expectations for further depreciations then currency substitutions may take place and thus reduces real money balances Bahmani-Oskooee and Sungwon (2010). The exchange rate variable is important in open economies' money demand function where claims denominated in foreign currency are high and currency conversion is prevalent.

Since the contribution of this paper is fairly empirical. So after careful identification of theoretical specification with potential determinants of money demand (both suggested by a micro-founded model and empirical literature), this study attempts to search for stable money demand functions by applying various econometrics techniques for Pakistan's economy. These techniques include residual based co-integration approach, auto regressive distributed lag (ARDL) modeling approach, recursive Johansen co-integration approach and Bayesian estimation approach with Markov-Chain Monte-Carlo (MCMC) Simulations. The next section outlines the empirical setup.

4. THE EMPIRICAL SETUP

As this study estimates money demand for Pakistan with different specification in light of theoretical micro-founded model and empirical literature. So we apply various econometric techniques to analyze, whether money demand is stable or not. More specifically, we have considered residual based co-integration approach as suggested by Engle and Granger (1987), ARDL approach as suggested by Pesaran and Shin (1995) and Pessaran *et al.,* (2001), recursive Johansen co-integration approach as suggested by Johansen (1988) and Johansen and Juselius (1990) and Bayesian estimation approach with MCMC Simulations as suggested by Canova (2007).

The variables include in all different empirical specifications are: nominal and real money (M2), industrial production index (IPI), real gross domestic product (Real GDP), consumer price index (CPI), expected inflation, weighted average deposit rate as own rate of broad money, weighted average 6-month market treasury bills (MTBs), 10-year bond rate (FIB & PIB), short-term interest rate spread (6-month MTB minus deposit rate), long-term interest rate spread (10-year bond rate minus deposit rate), weighted averaged lending rate, exchange rate, expected depreciation of exchange rate. Detail description of each variable and its data source is provided in table A1 of appendix-A. All variables are expressed in log form except interest rate, depreciation of exchange rate and inflation rate. Augmented Dickey-Fuller (ADF) is used to check stationarity of the variables (see, Appendix-A**, Table-A2**).

4.1 Engle-Granger Modeling Setup

From the theoretical model, as derived in section 3, we have the following standard linear specification of money demand function:

$$\widetilde{\left(\frac{M_t}{P_t}\right)} = \varphi_0 + \varphi_1(\widetilde{Y_t}) + \varphi_2(\tilde{\imath}_{t)} + \tilde{\varepsilon}_{m,t}$$

This specification suggests that real money demand depends on real output and nominal interest rate, where φ_1 ad φ_2 are elasticity parameters of money demand with respect to output and interest rate. However, in practice econometrician have attempted to estimate both nominal and real versions of money demand functions. The nominal version of money demand function considers actual inflation as an explanatory variable, whereas real money demand function takes expected inflation as a possible determinant to tackle future expectations. Further, recent literature has argued that due to structural changes in currency and deposits in developing economies, the income elasticity of money demand turns out to be high [see for instance, Adam *et al*, (2010)]. In order to tackle this possible empirical issue, we have constructed an index of structural change based on principle component technique. This index includes the effect of five variables, services and manufacturing as percent of GDP, imports and investment as percent of GDP, government consumptions as percent of GDP and credit to private sector as percent of broad money (M2). This variable is then augmented as an explanatory variable in both the regressions of money demand. Thus we have the following four specifications:

Nominal Money Demand:

$$\widetilde{(M_t)} = \varphi_0^{S1} + \varphi_1^{S1}(\widetilde{Y_t}) + \varphi_2^{S1}(\tilde{\imath}_t) + \varphi_3^{S1}(\tilde{\pi}_t) + \tilde{\varepsilon}_{M,t}^{S1} \quad \text{(EG - S1)}$$

$$\widetilde{(M_t)} = \varphi_0^{S2} + \varphi_1^{S2}(\widetilde{Y_t}) + \varphi_2^{S2}(\tilde{\imath}_t) + \varphi_3^{S2}(\tilde{\pi}_t) + \varphi_4^{S2}(\widetilde{StructChange_t}) + \tilde{\varepsilon}_{M,t}^{S2} \quad \text{(EG – S2)}$$

Real Money Demand:

$$\widetilde{\left(\frac{M_t}{P_t}\right)} = \varphi_0^{S3} + \varphi_1^{S3}(\widetilde{Y_t}) + \varphi_2^{S3}(\tilde{\imath}_t) + \varphi_3^{S3}(\tilde{\pi}_t^e) + \tilde{\varepsilon}_{m,t}^{S3} \quad \text{(EG – S3)}$$

$$\widetilde{\left(\frac{M_t}{P_t}\right)} = \varphi_0^{S4} + \varphi_1^{S4}(\widetilde{Y_t}) + \varphi_2^{S4}(\tilde{\imath}_t) + \varphi_3^{S4}(\tilde{\pi}_t^e) + \varphi_4^{S4}(\widetilde{StructChange_t}) + \tilde{\varepsilon}_{m,t}^{S4} \quad \text{(EG – S4)}$$

Where, as usual, [φ_1^{S1}, φ_1^{S2}, φ_1^{S3}, and φ_1^{S4}] >0 and [φ_2^{S1}, φ_2^{S2}, φ_2^{S3}, and φ_2^{S4}] <0 are long run elasticites of income and interest rate and can be estimated using static ordinary least square (static OLS) methodology. In order to test the long run dynamics, first we need to check stationarity of each variable. Once it is confirm that selected series are integrated and have same order, the Engle-Granger test of co-integration is then applicable. In the next stage this test checks the stationarity of estimated residuals of each specification. If estimated residual turns out to be stationary at level then it concludes that long-run dynamics exists. Using Granger representation theorem, any co-integrated regression can be map into its error-correction mechanism (ECM), which deals with the short-run dynamics.

In general, ECM version of any above specification can be written as:

$$\Delta\tilde{z}_t = \alpha + \beta\widetilde{ECM}_{t-1} + \sum_{i=1}^{p-1}\gamma_i\Delta\widetilde{z_{t-i}} + \sum_{i=1}^{p-1}\Psi_i\Delta\widetilde{Y_{t-i}} + \tilde{\varepsilon}_{z,t}$$

Where, $\beta \in (-1,0)$, is short run correction parameter?

One important shortcoming of Engle-Granger co-integration test based on static OLS (SOLS) is that the estimates have an asymptotic distribution that is generally non-Gaussian and exhibiting large sample bias. Since conventional testing procedures are not valid unless modified substantially, SOLS is generally inappropriate to make inference on the co-integrating vector. For this reason, we use ARDL, VAR models for co-integration and Bayesian simulation approaches.

4.2 ARDL Modeling Setup

Pesaran and Shin (1995) and Pessaran *et al.,* (2001) developed Bound test using ARDL to find long-run relationship between the variables irrespective of order of integration[6]. This technique does not require pre-testing of unit roots of the variables as ADF unit root test become redundant in presence of structural breaks. It allows co-integration relationship to be estimated by OLS, once the lag order to the model identified. Therefore, in general we can write the VAR(p) model as:

$$\tilde{Y}_t = \boldsymbol{b} + \boldsymbol{c}t + \sum_{i=1}^{p}\Phi_i\tilde{Y}_{t-i} + \tilde{\varepsilon}_{Y,t}$$

Where, $\tilde{Y}_t$ represents a vector of variables. The above expression can be written as a simple Vector ECM as:

$$\Delta\tilde{Y}_t = \boldsymbol{B} + \boldsymbol{C}t + \boldsymbol{\Theta}\tilde{Y}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\tilde{Y}_{t-i} + \tilde{\varepsilon}_{Y,t}$$

Where, $\boldsymbol{\Theta} = -(\mathrm{I}_{k+1} - \sum_{i=1}^{p}\Phi_i)$ and $\Gamma_i = -\sum_{j=i+1}^{p}\Phi_j$, i=1,…,p-1; are the (k+1)×(k+1) matrices of the long run multipliers and the short run dynamic coefficients. By making the assumption that there is only one long run relationship amongst the variables, Pesaran and Shin (1995) and Pessaran *et al.,* (2001) focus on the VECM equation and partition $\tilde{Y}_t$ into a dependant variable, $\tilde{z}_t$ and a set of forcing variables, $\widetilde{X_t}$. Under such conditions, the matrices $\mathbf{B}$, $\mathbf{C}$, $\boldsymbol{\Gamma}$ and, most importantly, $\boldsymbol{\Theta}$, the long run multiplier matrix can also be partitioned conformably with the partitioning of $\tilde{Y}_t$.

[6] This technique is applicable for variables that are either stationary at level or integrated of order one. However, relationship becomes explosive when one or all variables are of high order i.e. I (2).

$$\boldsymbol{\Theta} = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \boldsymbol{\Gamma} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}$$

The key assumption, that $\widetilde{X_t}$ is long run forcing for $\widetilde{z_t}$, then implies that the vector, θ_{21}=0, that is that there is no feedback from the level of $\widetilde{z_t}$ on $\Delta\widetilde{X_t}$. As a result, the conditional model for $\Delta\widetilde{z_t}$ and $\Delta\widetilde{X_t}$ can be written as:

$$\Delta\widetilde{z_t} = b_1 + c_1 t + \theta_{11}\tilde{z}_{t-1} + \theta_{12}\tilde{X}_{t-1} + \sum_{i=1}^{p-1} \gamma_{11,i}\Delta\tilde{z}_{t-i} + \sum_{i=1}^{p-1} \gamma_{12,i}\Delta\tilde{X}_{t-i} + \tilde{\varepsilon}_{1,t}$$

$$\Delta\widetilde{X_t} = b_2 + c_2 t + \theta_{22}\tilde{X}_{t-1} + \sum_{i=1}^{p-1} \gamma_{21,i}\Delta\tilde{z}_{t-i} + \sum_{i=1}^{p-1} \gamma_{22,i}\Delta\tilde{X}_{t-i} + \tilde{\varepsilon}_{2,t}$$

Under standard assumptions about the error terms in above representations,[7] Pessaran *et al.*, (2001) re-write the system as:

$$\Delta\widetilde{z_t} = \alpha_1 + \alpha_2 t + \phi\tilde{z}_{t-1} + \delta\tilde{X}_{t-1} + \sum_{i=1}^{p-1} \upsilon_i\Delta\tilde{z}_{t-i} + \sum_{i=1}^{p-1} \varphi_i\Delta\tilde{X}_{t-i} + \tilde{\varepsilon}_t$$

Which, they term as unrestricted error correction model. Note that in this expression, a long run relationship will exist amongst the level variables if the two parameters ϕ and δ are both non zero in which case, for the long run solution, we obtain:

$$\widetilde{z_t} = -\frac{\alpha_1}{\phi} - \frac{\alpha_2}{\phi}t - \frac{\delta}{\phi}\tilde{X}_t$$

Pesaran, *et al*., (2001) choose to test the hypothesis of no long run relationship between $\widetilde{z_t}$ and $\widetilde{X_t}$ by testing the joint hypothesis that $\phi = \delta = 0$ in the context of above VECM. The test they develop is a bounds type test, with a lower bound calculated on the basis that the variables in $\widetilde{X_t}$ are I(0) and an upper bound on the basis that they are I(1). They provide critical values for this bounds test from an extensive set of stochastic simulations under differing assumptions regarding the appropriate inclusion of deterministic variables in the ECM. If the calculated test statistic (which is a standard F test for testing the null hypothesis that the coefficients on the lagged levels, terms are jointly equal to zero) lies above the upper bound, the result is conclusive and implies that a long run relationship does exist between the variables. If the test statistic lies within the bounds, no conclusion can be drawn without prior knowledge of the time series properties of the variables. In this case, standard methods of testing have to be applied. If the test statistic lies below the lower bound, no long run relationship exists.

For estimation of money demand function, we have following general form:

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[7] Essentially that they are independently normally distributed with a positive definite variance covariance matrix.

$$\Delta\left(\widetilde{\frac{M_t}{P_t}}\right) = \alpha_1 + \alpha_2 t + \phi\left(\widetilde{\frac{M_{t-1}}{P_{t-1}}}\right) + \delta\tilde{X}_{t-1} + \sum_{i=1}^{p-1} \upsilon_i \Delta\left(\widetilde{\frac{M_{t-i}}{P_{t-i}}}\right) + \sum_{i=1}^{p-1} \varphi_i \Delta\tilde{X}_{t-i} + \tilde{\varepsilon}_t$$

Where, $\widetilde{X_t}$ is a set of exogenous variables. We have attempted seven alternative specification of ARDL model to test underline stability hypothesis. The main objective is to correctly identify real money demand function, and to determine long-run relationship vis-à-vis short-run dynamic. The vector of exogenous variables in each specification is listed below:

$\widetilde{X_t} = [Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Inflation]$ (ARDL-S1)
$\widetilde{X_t} = [Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Inflation; Exch.Rate]$ (ARDL-S2)
$\widetilde{X_t} = [Industiral.Output;\ 6-month.TBill.Rate;\ Expected\ Inflation]$ (ARDL-S3)
$\widetilde{X_t} = [Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Expected\ Inflation; Exch.Rate]$ (ARDL-S4)
$\widetilde{X_t} = [Industiral.Output;\ Own.Rate; 6-month.TBill.Rate;\ Inflation]$ (ARDL-S5)
$\widetilde{X_t} = [Industiral.Output;\ Own.Rate; 6-month.TBill.Rate;\ 10-year.bond.Rate;\ Inflation]$ (ARDL-S6)
$\widetilde{X_t} = [Industiral.Output;\ Risk\ Premium;\ Expected\ Inflation; Exch.Rate]$ (ARDL-S7)

These specifications consider quarterly data from 1991q1 to 2011q4 of variables industrial production index (IPI) as scale variable, opportunity cost variable includes CPI-inflation (QoQ), exchange rate and variant interest rate i.e. lending rate, 6-month treasury bond rate, long-term rate (weighted average of 10-year FIB/PIB), short-term interest rate spread and long-term interest rate spread are included. Weighted average deposit rate as own rate of broad money (M2) is also considered. A Schwartz Bayesian criterion (SBC) is applied for the selection of lag order for each model.

4.3 Johensan Modeling Setup

This setup is n extension of Engle-Granger modeling framework in a VAR fashion with a similar assumption of same order of integration of all the series. Further, the starting point is quite similar with ARDL setup with the construction of a VAR(p) model as:

$$\widetilde{Y_t} = \boldsymbol{b} + \boldsymbol{c}t + \sum_{i=1}^{p} \Phi_i \widetilde{Y}_{t-i} + \tilde{\varepsilon}_{Y,t}$$

Where, $\widetilde{Y_t}$ represents a vector of variables with same order of integration (let us consider, I(1)). The above expression can be written as a simple Vector ECM as:

$$\Delta\widetilde{Y_t} = \boldsymbol{B} + \boldsymbol{C}t + \boldsymbol{\Theta}\widetilde{Y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\widetilde{Y}_{t-i} + \tilde{\varepsilon}_{Y,t}$$

Where, $\boldsymbol{\Theta} = -(\mathrm{I}_{k+1} - \sum_{i=1}^{p} \Phi_i)$ and $\Gamma_i = -\sum_{j=i+1}^{p} \Phi_j$, i=1,…,p-1; are the (k+1)×(k+1) matrices of the long run multipliers and the short run dynamic coefficients. Since, $\Delta\widetilde{Y}_{t-i}$, …, $\Delta\widetilde{Y}_{t-i+1}$ are I(0) but $\widetilde{Y}_{t-1}$, is I(1). So in order to do this equation consistent, $\boldsymbol{\Gamma}_i$ should not be of full rank. Consider its rank is r<(k+1). We can decompose matrix, $\boldsymbol{\Theta} = \boldsymbol{\alpha\beta}'$, where, $\boldsymbol{\alpha}$ is (k+1) x r matrix of error correction terms (speed of adjustment parameters) and $\boldsymbol{\beta}'$ is r x (k+1) matrix of coefficients of co-integrating vector. Now, we need to estimate two residual series, first by regressing, $\Delta\widetilde{Y}_t$ on $\sum_{i=1}^{p-1}\mathbf{\Gamma}_i\Delta\widetilde{Y}_{t-i}$ and name it $\xi_{o,t}$ and second by regressing $\widetilde{Y}_t$ on $\sum_{i=1}^{p-1}\mathbf{\Gamma}_i\Delta\widetilde{Y}_{t-i}$ and name it, $\xi_{i,t}$. Now, in next step, we form a regression on estimated residuals as: $\xi_{o,t} = \boldsymbol{\alpha\beta}'\xi_{i,t} + \varsigma_t$. The variance/co-variance matrix of this regression can be written as:

$$\Omega = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix}$$

Where, $S_{11,}$ is sum of square of $\xi_{i,t}$, $S_{00,}$ is sum of square of $\xi_{o,t}$ and S_{01}, is Sum of Product of $\xi_{o,t}$ and $\xi_{i,t}$. It is important to note that OLS is not applicable to estimate VAR because of Cross equation restriction. So, there is need to use maximum likelihood technique. The maximum of the likelihood function is obtained by solving the Eigen-value problem as:

$$\left|S_{10}S_{00}^{-1}S_{01} - \lambda S_{11}\right| = 0$$

This is equivalent to find Eigen-value of: $\left|S_{11}^{-1}S_{10}S_{00}^{-1}S_{01} - \lambda\right| = 0$. We can obtain from here the Eigen-values which are the roots of this equation. These roots are called γ-canonical correlation between $\xi_{o,t}$ and $\xi_{i,t}$. Thus the maximum of likelihood is given by:

$$-2logLmax\left\{\alpha N\sum_{i=1}^{k+1}\ln(1-\lambda_i)\right\}$$

Using this expression, we can define trace test-statistic and maximum Eigen-value as:

$$\lambda_{trace} = -N\sum_{i=1}^{k+1}\ln(1-\hat{\lambda}_i)$$

and

$$\lambda_{max}(r, r+1) = -N\ln(1-\hat{\lambda}_{r+1})$$

The null hypothesis of maximum Eigen-value test is γ-co-integrating vector(s), whereas, the alternative hypothesis is γ+1 co-integrating vector(s).

To apply this modeling procedure on the estimation and stability of money demand function, we have considered seven alternative specifications. These are listed below:

$\widetilde{Y}_t = [real.m2; Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Inflation]$ (Johansen-S1)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Expected\ Inflation]$ (Johansen-S2)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ 6-month.TBill.Rate;\ Expected\ Inflation]$ (Johansen-S3)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ 6-month.TBill.Rate;\ Expected\ Inflation; Exch.Rate]$ (Johansen-S4)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ Weighted\ Avg.Ledning\ Rate;\ Inflation, Exch.Rate]$ (Johansen-S5)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ Risk\ Premium;\ Inflation]$ (Johansen-S6)
$\widetilde{Y}_t = [real.m2; Industiral.Output;\ Risk\ Premium;\ Expected\ Inflation; Exch.Rate]$ (Johansen-S7)

For estimation of short run dynamics of money demand function, we may extract VECM model(s) in each specification, for which the depended variable is real money demand as:

$$\Delta\left(\widetilde{\frac{M_t}{P_t}}\right) = \alpha_1 + \alpha_2 t + \phi\left(\widetilde{\frac{M_{t-1}}{P_{t-1}}}\right) + \delta\tilde{X}_{t-1} + \sum_{i=1}^{p-1} \upsilon_i \Delta\left(\widetilde{\frac{M_{t-i}}{P_{t-i}}}\right) + \sum_{i=1}^{p-1} \varphi_i \Delta\tilde{X}_{t-i} + \tilde{\varepsilon}_t$$

4.4 Bayesian Modeling Setup

In order to estimate money demand function using Bayesian estimation approach, we have considered the complete New-Keyensian monetary model in log-linearzed form as derived in section-3.[8] This model consists of six structural equations, (i) Forward-looking IS-Equation; (ii) New-Keynesian Phillips Curve; (iii) production function; (iv) money demand equation; (v) evolution of natural rate of interest and (vi) Taylor-type monetary policy rule. Furthermore, three stochastic shocks, (i) productivity shock; (ii) money demand shock and (iii) monetary policy shock are taken into account. The outline of NK model is given as:

$$\tilde{y}_t = E(\tilde{y}_{t+1}) - \frac{1}{\gamma}(\tilde{\imath}_t - E\tilde{\pi}_{t+1} - \tilde{r}_t^n) \quad \text{[NKIS]}$$
$$\tilde{\pi}_t = \beta E(\tilde{\pi}_{t+1}) + \kappa\tilde{y}_t \quad \text{[NKPC]}$$

Where, $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is slope of NKPC.

$$\tilde{r}_t^n = \rho + \gamma\psi(\rho_a - 1)\tilde{a}_t \quad \text{[Eq. of Natural Rate of interest]}$$
$$\tilde{y}_t = \tilde{a}_t + (1-\alpha)\tilde{l}_t \quad \text{[Eq. of production function]}$$
$$\tilde{m}_t = \varphi_1(\tilde{Y}_t) + \varphi_2(\tilde{\imath}_t) + \tilde{\varepsilon}_{m,t} \quad \text{[Eq. of real money demand]}$$
$$\tilde{\imath}_t = \rho + \chi_1(\tilde{\pi}_t) + \chi_2(\tilde{Y}_t) + \tilde{\varepsilon}_{i,t} \quad \text{[Monetary policy rule]}$$

$$\tilde{a}_t = \rho_a \tilde{a}_{t-1} + \xi_{a,t} \quad \text{[Productivity Shock]}$$
$$\tilde{\varepsilon}_{m,t} = \rho_m \tilde{\varepsilon}_{m,t-1} + \xi_{m,t} \quad \text{[Money demand shock]}$$
$$\tilde{\varepsilon}_{i,t} = \rho_i \tilde{\varepsilon}_{i,t-1} + \xi_{i,t} \quad \text{[Monetary policy Shock]}$$

Following Canova (2007), we try to fit out the above referenced model, which consists in placing a prior distribution $p(\Gamma)$ on structural parameters, Γ, the estimate of which are then updated using the data Y^T according to the Bayes-rule:

$$p\left(\frac{\Gamma}{Y^T}\right) = \frac{p\left(\frac{Y^T}{\Gamma}\right)}{p(Y^T)} \propto L\left(\frac{\Gamma}{Y^T}\right)p(\Gamma)$$

Where $p\left(\frac{Y^T}{\Gamma}\right) = L\left(\frac{\Gamma}{Y^T}\right)$ is the likelihood function, $p\left(\frac{\Gamma}{Y^T}\right)$ is the posterior distribution of parameters and $p(Y^T)$ is the marginal likelihood defined as:

$$p(Y^T) = \int p\left(\frac{Y^T}{\Gamma}\right)p(\Gamma)d\Gamma$$

[8] For simplicity, we have not provided the micro-foundations of supply side (firms and cost channel of monetary policy). These derivations can be available upon request.

Any NK-model forms a linear system with rational expectations, the solution to which is of the form:

$$R_t = B_1(\Gamma)R_{t-1} + B_2(\Gamma)\mu_t$$
$$\mu_t = B_3(\Gamma)\mu_{t-1} + B_4(\Gamma)\varepsilon_t$$

Where R_t is a vector of endogenous variables, μ_t is a vector of stochastic disturbances and ε_t is a vector of innovations to stochastic shocks and coefficient matrices A_i depending on the parameters of the model. The measurement equations linking observable variables used in the estimation with endogenous variables can be written as: $Y^T = C.R_t$, where, C is the deterministic matrix. The equations expressions form the state-space representation of the model. The likelihood of which can be evaluated using Kalman filter. The analytical solution of the whole system may not be obtain in general, however the sequence of posterior draws can be obtain using Markov-Chain-Monte-Carlo (MCMC) simulation methodology. This methodology is briefly discussed in Gelman *et al.* (2006) and Koopman *et al.* (2007). Finally, the random walk Metropolis-Hastings algorithm is used to generate Morkov-Chains (MC) for the model parameters.

After getting Bayesian estimation results, we use Global Sensitivity Analysis (GSA) toolkit, [9] to assess the fitness and stability of model structural parameters. This toolkit consists of MATLAB routines, which used Smirnov-test for stability analysis. Ratto (2008) provided detail discussion on using this toolkit with various applied examples.

5 THE RESULTS AND DISCUSSION

This section provides and interprets estimation results based on four alternative methodologies as outlined in the previous section.

5.1 Estimation Results of Engle-Granger Approach

In this approach, SOLS is used to estimate real and nominal money demand. Simple analysis shows that income elasticity is more than unity for both real and nominal money demand suggesting relatively high flow of money in the economy (see, Appendix B, **Table B1**). It is also argued that high income elasticity may due to structural change in the economy that resulted into high changes in currency and deposits (Adam *et al*, 2010). In order to capture structural change, principle component is devised that include the effect of five variables i.e. services and manufacturing as percent of GDP, imports and investment as percent of GDP, government consumptions as percent of GDP and credit to private sector as percent of broad money (M2)[10] (see, Appendix F, **Figure(s) F1a to F1e**). Income elasticity decreased with the inclusion of first principle component (though marginally). However, it coefficient is not

[9] http://www.dynare.org
[10] Choice of these variables is due to transaction-intensive nature and it reflects change in demand for liquid services (Adam *et al*, 2010).

significantly different from zero that signify a limited structural change over the period[11] (see Appendix F, **Figure F1f**). On the other hand, price elasticity of nominal money is close to unity reflecting one to one relationship between GDP deflator and M2.

To measure opportunity cost of money demand, weighted average lending rate is used, which is, in fact, weak proxy for rate of return on alternate assets because it represents interest rate on money. However, this variable is introduced due to its availability for entire period of analysis. The interest rate coefficient is correctly signed but its magnitude is small. Inflation representing the opportunity cost is also included in real balances (see Appendix B, **TableB1**, column-3 and 4), which has small effect. Residual from each equation is test for stationarity by using ADF-test and the null hypothesis of non-stationarity is rejected at 1, 5, and 10 percent level.

Dynamic error correction model is estimated using difference variables (with two lag) and lag of estimated error term. Error correction term is highly significant and correctly signed. High magnitude of EC term suggests faster adjustment towards long run equilibrium (see Appendix B, **Table-B2**). Strong inertial effect has shown by real and nominal money (**Table-B2**, column 1 to 4). The short-run effect of real output is high however it is marginally significant. On the other hand, short-term dynamic effect of opportunity cost is not significantly different from zero.

Study also estimates real and nominal money using quarterly data from 1992q1 to 2011q2 (see Appendix B, **Table-B3**). Since, quarterly data on real GDP is not available, therefore average of monthly industrial production index (IPI) used instead. In nominal demand function, coefficient of IPI is very low, signifying that it capturing small share of real output (around 15 to 18 percent). Coefficients of consumer price index (CPI) and lending rate is quite significant and having expected signs. However, estimated model, using static OLS, is spurious and suffering from serial correlation and hetroskedasticity.

5.2 Co-integration Results based on ARDL Approach

This approach considers seven alternative model specification of real money demand function. In all estimated models, the coefficient of scale variable is correctly signed and significant, however, the magnitude is less than unity as expected (see Appendix C, **Table C1**). In model-1 (M-1 in **Table-C1**), inflation rate representing opportunity cost with expected sign with lower magnitude. Lending rate is used to proxy the rate of return on alternate assets, though correctly signed but not significantly different from zero. During early 90s, financial sector underwent many changes that also include introduction of foreign currency account. This provide an alternate avenue to park money, therefore, exchange rate has a role to play in affecting money demand. In Model-2 (M-2 in **Table-C1**), money demand function is estimated using exchange rate as additional variable, which is correctly signed but highly significant. The underlying reason of insignificant could be volatility in exchange rate and couple with freezing of foreign

[11] The result may be different if include principle components that reflects supply side changes. As during the last two decades, particularly since 2000, the reach of financial sector in Pakistan has increased (in term of more branches, entrance of foreign banks, privatization of public banks, internet banking, ATMS, development of equity and bond market, initiation of micro-finance enterprises/banks etc.) that significantly reduces financial cost.

currency account in 1998 and huge inflows of foreign currency in 2001 onwards. Therefore, the interventions in the foreign exchange market have dampened the impact of exchange rate.

In most of the money demand equations, short-term bond rate (government's market treasury bills) is used to signify alternate to money holding. Therefore, weighted average rate of 6-month T-bills rate (M-3 in **Table-C1**) instead of lending rate is included, which is yield expected sign but insignificant (rationale, not use by economic agent until 2010).

With evolving financial sector reforms, particularly in late 90s, initiation of fiscal and monetary coordination - though not very effective until now- and frequent monetary policy communication has changed the economic perspective of economic agent (*Become* more rationalized and feeding-in future economic outlook in making present decision). Cognizant to changing behavior of economic agent, money demand function is augmented with expected inflation and expected depreciation of exchange rate as potential opportunity cost variables (see M-4 in **Table-C1**), where long-run coefficient of both variables have expected signs, and highly significant. This may also explain the Central Bank behavior of frequent intervention in foreign exchange market for stability of exchange rate.

Since most part of broad money is remunerative, therefore own rate of M2 cannot be ignored. A weighted averaged deposit rate is used to capture own rate of M2 (see Appendix C, **Table C1**, M-5 and M-6). Own rate of M2 is not only have inappropriate sign but also insignificant in both models. Hypothetically, the coefficient of own rate should be positively related, which implies an increase in own rate should increase demand for money (Laidler, 1993). The underlying reasons of non-responsiveness of own rate is due to sluggish movements of deposits rates in the lowest panel of interest rates corridor i.e. deposit rate is very low (until recently when minimum floor introduced in 2005) and hardly move with other market interest rate (Khan and khan, 2010).

Long-term bond market also provides alternate avenue to transaction demand for money, therefore, weighted average rate of FIBs and PIBs is include in our specification to examine its impact on money demand (see Appendix C, **Table-C1**, M-6). Though the coefficient on long-term bond has expected sign but not significantly different from zero. It is due to fact that long-term bond market in Pakistan is shallow and restricted, only few large banks and financial companies are allowed to transact government bond.

Since most of market interest rate moved in tandem, and it is argued that including each interest rate variable in money demand function may complicate the model and would difficult to interpret. Therefore, short-term (6-month T-bills minus deposit rate) and long-term (10-year FIBs/PIBs rate minus deposit rate) spread are included to capture the effect of own rate on broad money and opportunity cost (see Appendix C, **Table C1**, M-7). The coefficients of interest rate spreads have expected sign but insignificant.

Short-run dynamics of each model are also estimated to examine the process of convergence towards long-run path. The coefficient of error correction term is significant with appropriate signed and shows moderate speed of adjustment. Most of variables in (see Appendix C, **Table C2**, M-1 to M-4) are significant at conventional level and indicate convergence to equilibrium once deviated from long run path. On the other hand, the coefficients of own rate, short-term, long term rate and interest rate spread are insignificant (see Appendix C, **Table-C2**, M-5 to M-7).

5.3 Co-integration Results based on Johansen Approach

In order to test co-integration, unrestricted VAR models with various specification of nominal and real money demand are estimated. At the outset, variables are checked for unit root and Augmented Dickey-Fuller (ADF) confirms that all variables are non-stationary at level while become stationary with first difference, hence are integrated of order one I(1) (see Appendix-A, **Table A2**). For model with nominal money as dependent variable, the lag length selected by SBC and AIC is p=3 and P=4 respectively. By using quarterly data from 1992q1 to 2011q4 and lag length of three is chosen to maintain parsimonious selection. On the other hand, in the models with real money, lag length of four is chosen using SBC (see Appendix-D, **Table D1** and **D2**).

In Johansen co integration model, long run determinants of nominal M2 are industrial production (IPI), consumer price index (CPI) and lending rate, whereas for real money, the determinants included are IPI, expected inflation and lending rate. The short run dynamic are captured by taking the quarterly changes. The dynamism is introduced by incorporating the past changes of each economic determinant in explaining the M2 growth. In the short run, the determinants of nominal and real M2 growth are the last quarter values of economic growth, inflation and changes in interest rates. Further, past deviations of M2 from its stable long run path is also incorporated as an explanatory variable.

Co-integration relationship is determined by using trace and maximum Eigenvalue statistics. However, it is important to make assumptions regarding deterministic trend specification and drift term before estimating the rank. In macroeconomic literature, two specifications are more common i.e. restricted intercept without deterministic trend in co-integration relationship and unrestricted intercept with linear deterministic trend in short run equations. We used both specifications in our analysis.

Using specification of unrestricted intercept and linear trend in co-integration equation for nominal money, both the trace and maximum Eigenvalue statistics show one co-integrating vector. Considering one co-integrating vector, the estimated long-run relationship of nominal money is as following:

$$\log(M2) = \underset{(0.15)}{1.21}*\text{Log}(IPI) + \underset{(0.27)}{1.88}*\text{Log}(CPI) - \underset{(0.006)}{0.03}*\text{Lending rate} \; \underset{(0.006)}{-0.02}*\text{trend} + 5.55$$

Coefficients of all the variables are significant (see, standard error in parenthesis) and have expected signs. Income elasticity shows same magnitude as expected i.e. more than unity. The estimated empirical realization of adjustment parameter ($\hat{\alpha}$) to long-run equilibrium has following values:

$$\hat{\alpha} = \begin{bmatrix} -0.150\ (0.049) \\ 0.351\ (0.129) \\ -0.075\ (0.019) \\ -3.785\ (1.161) \end{bmatrix}$$

The first element of the column shows error correction parameter of estimated money demand function, which indicates high speed of adjustment towards equilibrium once shocked. The adjustment parameter slightly changed when short-run dynamic equations of nominal money and other variables re-parameterized to estimate projections using parsimonious relationship. Further, each equation is extended by incorporating seasonal variables and volatility of oil prices in order to include short-term effects of oil change and seasonal demand for money.

Similarly, for real money balances, with the specification of unrestricted intercept without linear deterministic trend, both trace and Eigenvalue statistics indicate one co-integrating vector and their long-run relationship is as following:

$$\log(\text{real M2}) = \underset{(0.21)}{0.88}*\log(\text{IPI}) - \underset{(0.02)}{0.08}*\text{Exp(inflation)} - \underset{(0.04)}{0.16}*\text{Lending rate} + 13.75$$

Coefficients of all the variables are significant (see, standard error in parenthesis) and have expected signs. The vector of adjustment parameter is as following:

$$\hat{\alpha} = \begin{bmatrix} -0.023 (0.008) \\ -0.073\ (0.019) \\ -0.804\ (0.543) \\ 0.015\ (0.210) \end{bmatrix}$$

The error correction of real money is very low and indicating a slow speed of adjustment to long-run equilibrium. However, the error correction terms for IPI and expected inflation are relatively high thus showing high speed of adjustments. ADF test indicates that residuals of short-run equations are stationary and normal.

A variety of models of real money demand with different specifications are also estimated (see Appendix D, **Table D3**). Lag length of unrestricted VAR model is based on SBC. In **Model-1 to 3**, weighted average rate of 6-month T-Bills is used instead of lending rate, as it represent short-term bond rate. The coefficient of MTB is turned out highly significant, however, with incorrect sign. In **Model-4**, exchange rate variable is included and trace statistics indicate two co-integrating equations. Although, all the long-run coefficients are significantly different from zero, however, effects are contrary to the theory. In **Model-5**, MTB is replaced with lending rate as opportunity cost variable and exchange rate also included. The coefficient of opportunity cost variables turn out to be highly significant and having expected sign, however the effect of exchange rate is more pronounced i.e. one percent increase in exchange rate (depreciation) would increase real money more than 9 percent. Nevertheless, positive expected inflation contradicts the theory. **Model-6** includes variable of risk premium, which is the difference of lending rate and risk free rate (MTB). Coefficient of risk premium shows expected sign but insignificant at conventional level. This model is extended by including exchange rate **(Model-7),** where all the variables, except exchange rate, are significantly different from zero and having expected signs. Further, the adjustment parameters for real money demand in all these models are either very low or explosive thus signifying a little or non-convergence to long-run equilibrium path.

5.4 Bayesian Estimation Results

Bayesian estimation approach use prior information regarding key structural parameters before taking model and data into simulation stage. According to the Canova (2007) these priors reflect researcher confidence about the likely location of structural parameter of the model. In practice, priors are chosen based on observation, facts and from existing empirical literature.

For our study, two parameters α (share of capital in production function) and β (subjective discount factor) are fixed[12] at 0.46 and 0.99. The parameter value of discount factor (β) is set in order to obtain historical mean of the nominal interest rate in the steady state. Following Haider and Khan (2008) the degree of price stickiness (θ) is assumed to be 0.74. This value is also consistent with the latest survey based finding of optimal pricing behavior of firm in Pakistan (see, Choudhary, *et al.*, (2011)). Elasticity parameter of money demand with respect to output (φ_1) is taken as 0.86 whereas, with respect to interest rate (φ_2) is -0.018. These values are consistent with ARDL long-run estimates (see, Appendix C, **Table C1**). Prior information about other selected parameter is given in Table-E1 of Appendix E.

After selection of priors, we apply Bayesian simulation algorithms by combining the likelihood distribution which leads to an analytically-intractable posterior density. In order to sample from the posterior, random walk Metropolis-Hastings algorithm is used to generate 150,000 draws from the posteriors. We reported the posterior results (parameter estimates) in the second column of Table-E2 of Appendix E. Furthermore, Figure E1 and E2 of Appendix E display kernel estimates of the priors and the posteriors of each parameter. These results show that prior and posterior means are not away from each other. To some extent, it confirms the stability of money demand parameter estimates. However, we also used various formal tests of parameter stability and results are discussed in the next section.

5.5 Parameter Stability Tests

As we observed in the estimation stage, the elasticity parameter of money demand with respect to interest rate is sensitive to alternative specification of money demand. Therefore, there is need to test parameter stability over time. To perform this task, best models are selected from each individual approach. Ideally, parameter stability can be checked by using different methods, which include empirical realization of income elasticity and opportunity cost variables with changing sample period or recursive estimation of coefficients of each model. And to see model stability, CUMUS and CUMUS-square of residual are applied.

Accordingly, first we select **Model-2** in the co-integrated VAR specification. This model is re-estimated by using Johansen procedure with 4 lag-length and trend specification but changing sample period i.e. ends of each fiscal year from 2004 to 2011. The results of this recursive estimation show that income elasticity and opportunity cost coefficients change significantly, both in magnitude and signs, with the changing sample period that signifying

[12] These fixed parameters are also known as stick priors in Bayesian sense.

unstable money demand function (see Appendix D, **Table D5**). However, when applied the same procedure on extended model i.e. **Model-7**, the income elasticity and opportunity cost coefficients are not only correctly signed but also steady over the time. This confirms that money demand function with this specification is stable. Further, the trace and maximum Eigenvalue posit one or two co-integrating relationship. Figure - (**F3a and F3b**) of Appendix F shows co-integration relation estimated with Johansen's procedure using Model - 2 & 7, where former signifies instability while latter indicate stability i.e. mean reverting properties are visible from graphical representation.

Alternatively, parameter consistency also checked by recursively estimating the coefficient of money demand function, where coefficient of each variables ($\hat{\beta}_{rt}$) estimated by adding more data to the equation (flexible window). Figure – (**F4a to F4d**) (see Appendix F), show recursively estimated log-run coefficient of **Model-7** using Johansen co-integration**.** To avoid the large uncertainty associated with the initial estimates, just slightly less than half of estimates for each coefficient are displayed (from Q3-FY06 to Q2-FY11). **Figure F4a** depicts stable income elasticity (around 0.7 percent) over changing sample period. **Figure F4b** shows recursive estimates of expected inflation, where its coefficient remained highly stable during initial period, however, later on it shows significant variation and jumped markedly in FY09. In fact, this was the period when economic conditions weaken considerable due to uncertain domestic environment couple with external financial crisis and unexpected increase in international commodity and energy prices that seeped into high domestic inflation. On the other hand, recursive estimates of exchange rate and risk premium remained more or less stable with slight variation in FY09 and on wards. We also recursively estimated the coefficients of other co-integration model, however the coefficients are not stable and display large variations.

Further, model stability is also tested using CUSUM of both residual and square residual. We estimated the residuals recursively of all ARDL Models, where the statics of CUSUM of residual of **Model-4** lie within 5 percent significant level (see Appendix F, Figure F2). It is important to note that ARDL model is slightly different from estimated Model-7 using Johansen co-integration, where exchange rate variable in latter replaced with expected depreciation of exchange rate as it is stationary at level and ARDL procedure can be applied irrespective of zero or one order of integration. And variable of risk premium is replaced with lending rate variables. Both variables in ARDL model represent opportunity cost of money holding.

Finally, parameter stability of money demand in NK-DSGE model specification is assessed using global sensitivity toolkit. This toolkit uses Smirnov-test of stability, which shows the significance of for individual parameter into the whole model. Furthermore, cumulative plots for stability and instability behavior provide us useful information for the fitness of each structural parameter. Figure– F5 and F6 (see, Appendix F) show that all model structural parameters of money demand are stable and properly fitter with respect to the data.

6 CONCLUDING REMARKS

This study tries to investigate money demand function for Pakistan, where monetary aggregates were considered as nominal anchor to the economy. Importantly, monetary aggregates, as operational and intermediate target, were contributing significantly in the implementation and communication of monetary policy stance. However, stability of money demand has long been argued with evolving financial innovations and regulations. Earlier studies on the subject provide conflicting explanation due to inadequate specification and imprecise empirical estimation of the money demand function. This study finds that money demand in Pakistan is stable if correctly specified (see ARDL **Model -4**, unrestricted co-integrated VAR **Model-7** and NK-DSGE Model) and concluded that monetary aggregates should remain, if not primary, as supportive target in the monetary policy framework. Though financial innovations have changed the preferences of economic agent (money holder) in developed countries, however, it has limited impact on the economic behavior in developing country like Pakistan.

APPENDIX-A

Table A1: Variables and Data Sources

Variables	Source
Nominal M2	Statistical Bulletins, SBP (Various Issues)
Real GDP (Annual data)	Pakistan Economic Survey, MOF, (Various Issues)
Industrial production index (IPI)	Statistical Bulletins, SBP (Various Issues)
Consumer price index (CPI)	Statistical Bulletins, SBP (Various Issues)
Inflation (Percent Change in CPI)	Statistical Bulletins, SBP (Various Issues)
Exchange Rate	Statistical Bulletins, SBP (Various Issues)
ER App/dep (+/-)	Statistical Bulletins, SBP (Various Issues)
Lending rate	Statistical Bulletins, SBP (Various Issues)
Own rate (wt. avg. deposit rate)	Statistical Bulletins, SBP (Various Issues)
Short-term rate (wt. avg. 6-month MTB)	Statistical Bulletins, SBP (Various Issues)
Long-term rate (wt. avg. 10-year FIB/PIB)	Statistical Bulletins, SBP (Various Issues)
Risk premium (Lending rate - MTB)	Statistical Bulletins, SBP (Various Issues)
short-term spread (6-m MTB - Deposit rate)	Statistical Bulletins, SBP (Various Issues)
long-term spread (10-Y bond - deposit rate)	Statistical Bulletins, SBP (Various Issues)

Note: SBP: = State Bank of Pakistan; MOF: = Ministry of Finance

Table A2: Unit Root test

Variables	Augmented Dickey-Fuller Test			Dickey-Fuller GLS Test		
	Level	Difference	Order of integration	Level	Difference	Order of integration
Nominal M2	-0.423	-2.004**	I(1)	-0.423	-2.004**	I(1)
Real M2	-1.810	-8.811*	I(1)	-0.848	-8.702*	I(1)
real GDP (Annual data)	-0.724	-6.366*	I(1)	0.749	-6.156*	I(1)
Industrial production index (IPI)	-1.934	-13.432*	I(1)	-1.012	-7.278*	I(1)
Consumer price index (CPI)	1.567	-3.6315*	I(1)	0.977	-3.284*	I(1)
Inflation	-2.556	-5.545*	I(1)	-0.978	-3.380*	I(1)
log(Exchange Rate)	-0.300	-3.435**	I(1)	0.161	-3.393*	I(1)
ER depreciation	-3.435**	-	I(0)	-3.416*	-	I(0)
Lending rate	-1.483	-3.369*	I(1)	-1.452	-2.977*	I(1)
Own rate (wt. avg. deposit rate)	-1.838	-2.228	I(1)	-0.527	-2.047**	I(1)
Short-term rate (wt. avg. 6-month MTB)	-1.274	-3.916*	I(1)	-1.341	-2.605**	I(1)
Long-term rate (wt. avg. 10-year FIB/PIB)	-1.942	-3.490**	I(1)	-1.481	-3.532*	I(1)
Risk premium (Lending rate - MTB)	-1.913	-5.380*	I(1)	-0.801	-3.592*	I(1)
short-term spread (6-m MTB - Deposit rate)	-2.49156	-6.779*	I(1)	-2.270**	-	I(0)
long-term spread (10-Y bond - deposit rate)	-2.356	-5.715*	I(1)	-2.011**	-	I(0)

Note: Level of significance, * at 1%, ** at 5% and *** at 10%; Critical values are from Mackinon (1996); SBC is used for lag selection ; Test estimation include intercept; All variables are in log except variant of interest rate, exchange rate depreciation and inflation.

APPENDIX-B: Static OLS Modeling Results

Table B1: Long-run Static Estimation of Money Demand (based on annual data)

Dependent variable: Log of Broad money (Sample range 1978 to 2011)				
Regressors	Nominal Money		Real Money	
	(1)	(2)	(3)	(4)
Log(GDP real)	1.24 (7.54)	1.13 (7.32)	1.26 (59.35)	1.14 (24.2)
log(GDP deflator)	0.98 (9.61)	1.00 (10.31)		
Lending rate	-0.02 (-2.79)	-0.01 (-2.08)	-0.01 (-2.18)	-0.01 (-1.81)
Inflation (averaged)			-0.01 (-2.65)	-0.002 (-1.0)
Structural changes		0.03 (0.22)		0.03 (1.58)
Constant	-8.96 (-4.31)	-7.48 (-3.87)	-4.55 (-14.9)	-2.92 (-4.32)
adj. R^2	0.99	0.99	0.99	0.99
DW	1.25	1.07	0.81	0.90
ADF of Residual	-4.22*	-3.3**	-2.74***	-2.97**
Serial corr. / Hetro.	yes	Yes	Yes	yes

Notes: t-statistics in parenthesis; *, **, *** significant at 1%, 5% & 10% respectively; First principle component represent structural change.

Table B2: Error Correction Models of Broad Money (based on annual data sample: 1973- 2011)

Dependent Variable / Regression	D(Log(M2)		D(Log(real M2)	
	(1)	(2)	(3)	(4)
$ECM_{(t-1)}$	-0.63 (-4.49)	-0.75 (-3.65)	-0.50 (-4.10)	-0.61 (-4.18)
D(LOG(M2(-1)))	0.64 (3.91)	0.70 (3.64)		
D(LOG(M2(-2)))	0.19 (1.29)	0.28 (1.42)		
D(LOG(real M2(-1)))			060 (4.80)	064 (3.91)
D(LOG(real M2(-2)))			0.18 (1.61)	
D(LOG(YR))	0.75 (2.77)	0.54 (1.41)	0.56 (2.15)	1.20 (3.25)
D(INF)			-0.01 (-5.30)	-0.01 (-4.36)
D(LR(-1))		-0.01 (-1.52)		
D(LR(-2))	0.01 (1.49)	0.01 (1.84)	0.01 (1.19)	0.01 (1.71)
D(Structural change)		0.05 (1.73)		
D(Structural change(-2)				0.01 (0.63)
Constant	-0.01 (-0.40)	-0.03 (-0.77)	-0.01 (-0.78)	-0.04 (-1.92)
Adj. R^2	0.43	0.39	0.65	0.67
SE	0.03	0.03	0.03	0.03
F-statistic	5.61 [0.001]	3.53 [0.01]	11.41 [0.00]	10.43 [0.00]
DW	1.87	1.95	1.97	

Notes: t-statistics in parenthesis; Structural change is first principle component

Table B3: Long-run Static Estimation of Money Demand (based on quarterly data)

Dependent variable: Log of M2 (Sample range 1992q1 to 2011q4)				
Regressors	Nominal Money		Real Money	
	(1)	(2)	(3)	(4)
Log(IPI)	0.35	0.35	0.98	0.53
	(6.61)	(6.13)	(22.48)	(9.53)
log(CPI)	1.53	1.53		
	(37.44)	(16.89)		
Lending rate	-0.02	-0.02	-0.01	-0.02
	(-7.63)	(-7.19)	(-0.89)	(-4.66)
Inflation (Q/Q)			-0.01	-0.004
			(-1.78)	(-1.75)
log(ER)		0.01		0.47
		(0.94)		(9.53)
Constant	5.84	5.84	9.64	10.1
	(41.43)	(38.08)	(40.06)	(59.84)
adj. R^2	0.99	0.99	0.89	0.95
DW	0.74	0.75	1.45	1.10
ADF of Residual	-4.22*	-4.28*	-2.74*	-5.5*
Standard error	0.07	0.07	0.12	0.08
Serial corr. / Hetro.	yes	yes	yes	yes

Notes: t-statistics in parenthesis; *, ** , *** significant at 1%, 5% & 10%

APPENDIX-C: ARDL Modeling Results

Table C1: Long-run Relationship of Real Money- ARDL Approach

Dependent variable log Real M2 (Sample range 1992q1-2011q4)								
Variables		M-1	M-2	M-3	M-4	M-5	M-6	M-7
log(IPI)		0.86 (7.91)	0.69 (6.74)	0.92 (7.70)	0.88 (12.59)	0.87 (6.77)	0.83 (6.41)	0.93 (13.04)
Lending rate		-0.02 (-1.51)	-0.03 (-3.89)		-0.02 (-1.89)			
Inflation		-0.02 (-1.85)	-0.003 (-0.43)	-0.02 (-1.37)		-.0.02 (-1.38)	-0.01 (-1.23)	
Expected inflation					-0.02 (-2.73)			-0.02 (-1.82)
log(Exchange Rate)			0.22 (1.98)					
Expected Depreciation of ER					0.03 (2.18)			0.04 (2.40)
Own rate (wt. avg. deposit rate)						-0.04 (-0.76)	0.01 (0.17)	
Short-term rate (wt. avg. 6-month MTB)				-0.02 (-1.37)		-0.01 (-0.38)	0.003 (0.12)	
Long-term rate (wt. avg. 10-year FIB/PIB)							-0.03 (-1.05)	
short-term spread (6-m MTB - Deposit rate)								
long-term spread (10-Y bond - deposit rate)								
Risk Premium (lending rate-6m MTB)								0.01 (0.53)
Constant		11.75 (20.45)	11.48 (28.30)	11.36 (20.82)	11.62 (30.70)	11.61 (19.07)	11.94 (18.16)	11.18 (35.59)
F-statistics [probability]		4.552 [0.001]	4.150 [0.003]	3.369 [0.014]	5.9502 [0.000]	3.805 [0.004]	3.452 [0.005]	4.526 [0.002]
Bound test	Upper value	4.378	4.049	4.378	4.049	4.049	3.805	4.049
	Lower value	3.793	2.85	3.793	2.85	2.85	2.649	2.85
lag-length selection	SBC & AIC	1-lag & 6-lags	1-lag & 5-lags	1-lag & 5-lags	1-lag & 5-lags	1-lag & 5-lags	1-lag & 7-lags	1-lag & 5-lags
ARDL lag selection		(1,0,0,0)	(3,0,0,1,1)	(2,0,1,0)	(3,0,0,0,1)	(1,0,0,0,0)	(1,0,0,0,0,0)	(1,0,0,0)
observations		79	77	79	77	72	72	77

Note: The SBC is used to select the optimum number of lag in the ARDL model.

Table C2: Short-run Error Correction Model

Dependent variable Δlog Real M2 (Sample range 1992q1-2011q4)

Variables	M-1	M-2	M-3	M-4	M-5	M-6	M-7
ECM $_{(-1)}$	-0.10 (-3.49)	-0.17 (-4.55)	-0.09 (-2.65)	-0.12 (-4.92)	-0.10 (-2.71)	-0.11 (-2.89)	-0.11 (-4.53)
DLIPIA	0.09 (2.91)	0.11 (4.77)	0.09 (2.58)	0.11 (4.36)	0.09 (2.51)	0.09 (2.59)	0.11 (4.14)
DLR	-0.002 (-1.61)	-0.004 (-3.09)		-0.002 (-1.84)			
DINF	-0.002 (-1.89)	-0.01 (-3.23)	-0.002 (-1.57)		-0.002 (-1.57)	-0.002 (-1.37)	
D Exp(INF)				-0.002 (-2.80)			-0.002 (-1.90)
DLER		-0.29 (2.70)					
D EXP(ER)				0.001 (0.46)			0.001 (0.87)
D(own rate)					-0.003 (-0.77)	0.001 (0.17)	
Dshot-term (6-months MTBs)			-0.001 (-1.21)		-0.000 (-0.004)	0.0002 (0.12)	
D long-term (10-year bond rate)						-0.004 (-1.02)	
dLRM21		-0.45 (-4.41)		-0.49 (-4.77)			-0.42 (-4.24)
dLRM22		0.29 (3.01)		0.24 (2.48)			0.32 (3.47)
d(risk premium)							0.001 (0.53)
Constant	1.21 (3.73)	1.91 (4.81)	1.07 (2.68)	1.43 (5.12)	1.14 (2.78)	1.29 (2.96)	1.27 (4.70)
Adj. R^2	0.15	0.56	0.11	0.53	0.10	0.1	0.51
F-statistics	F(4,74) 7.35[0.003]	F(5,69) 15.4[0.00]	F(4,68) 3.18[0.02]	F(7,69) 13.46[0.00]	F(5,66) 2.60[0.033]	F(6,65) 2.34[0.014]	F(7,69) 12.45[0.00]

Table C3: Forecasting of Broad Money

	M1	**M-2**	**M-3**	**M-4**	**M-5**	**M-6**	**M-7**
Q1-FY11	2.58%	0.77%	3.23%	0.49%	3.00%	3.08%	1.12%
Q2-FY11	1.88%	6.08%	2.76%	6.74%	2.63%	2.16%	7.1%
Q3-FY11	2.95%	2.54%	2.18%	0.80%	2.05%	1.71%	1.4%
Q4-FY11	2.21%	3.48%	2.23%	4.19%	2.15%	1.82%	4.77%
FY11	10.0%	13.43%	10.82%	12.66%	10.20%	9.07%	15.11%

Note: for forecasting purpose, we use actual data of CPI averaged inflation (13.7), Averaged lending rate (13.9 %), IPI growth (0.4 %) , Expected depreciation (-0.8%), M2 growth (15.89) for FY11;

APPENDIX-D: Johansen Modeling Results

Table D1: VAR Lag Order Selection Criteria

Endogenous variables: LOG(M2) LOG(IPIA) LOG(CPIA) LR

Exogenous variables: Constant; sample 1991q1 to 2011q4; observations: 74

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-67.973	NA	0.000	2.027	2.155	2.078
1	352.744	782.178	0.000	-9.373	-8.736	-9.120
2	440.986	154.112	0.000	-11.408	-10.261	-10.952
3	475.577	56.515	0.000	-11.932	-10.275*	-11.273
4	508.812	50.556*	4.94e-11*	-12.417*	-10.250	-11.556*
5	520.439	16.375	0.000	-12.294	-9.617	-11.230
6	527.591	9.268	0.000	-12.045	-8.858	-10.778
7	548.447	24.675	0.000	-12.182	-8.485	-10.712

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level); HQ: Hannan-Quinn information criteria
FPE: Final prediction error; AIC: Akaike information criterion; SC: Schwarz information criterion

Table D2: VAR Lag Order Selection Criteria

Endogenous variables: LOG(RM2) LOG(IPIA) INF(1) LR

Exogenous variables: Constant; Sample 1992q1 2011q4; observations: 74

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-336.383	NA	0.171486	9.588	9.716	9.639
1	7.800	639.890	1.66E-05	0.344	0.981	0.597
2	81.685	129.038	3.26E-06	-1.287	-0.140	-0.831
3	111.799	49.201	2.22E-06	-1.684	-0.027	-1.025
4	154.695	65.250*	1.06e-06*	-2.442	-0.275*	-1.580*
5	171.856	24.171	1.06E-06	-2.475*	0.202	-1.410
6	181.227	12.142	1.35E-06	-2.288	0.899	-1.021
7	194.098	15.228	1.59E-06	-2.200	1.497	-0.730

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level); HQ: Hannan-Quinn information criteria
FPE: Final prediction error; AIC: Akaike information criterion; SC: Schwarz information criterion

Table D3: Money Demand function with different specifications

Dependent variable: Log(real M2); Sample: 1992q1 to 2011q2

Model	Estimated long-run relation	SBC/AIC lag-length	Co-integration: Trend specifications	trace	Max Eigenvalue
1	log(M2) = 1.21*Log(IPI) + 1.88*Log(CPI) - 0.03*Lending rate -0.02*trend + 5.55	2 & 4	unrestricted intercept and with linear deterministic trend	1	1
2	log(real M2) = 0.88*log(IPI) – 0.08*Exp(inflation) – 0.16*Lending rate + 13.75	4 & 5	restricted intercept & without trend	**1**	**1**
3	log(RM2)=1.13*LOG(IPI) + 0.05*Exp(Inflation) + 0.06*MTB + 9.05	4 & 4	restricted intercept & without trend	1	0
4	log(RM2)=-1.53*LOG(IPI) +2.52*ER + 0.07*Exp(Inflation) - 0.02*MTB - 0.05*trend -0.07	2 & 7	unrestricted intercept and with linear deterministic trend	2	1
5	log(RM2)=1.25*LOG(IPI) + 0.02*Exp(Inflation) + 1.16*log(ER) - 0.02*LR - 0.02*trend - 5.99	2 & 5	unrestricted intercept and with linear deterministic trend	1	1
6	log(RM2)=0.48*LOG(IPI) + 0.002*Exp(Inflation) - 0.01*Risk +0.01*trend+ 12.4	3 & 5	unrestricted intercept and with linear deterministic trend	1	2
7	log(RM2)=1.38*LOG(IPI) - 0.08*Exp(Inflation) - 0.34*log(ER)- 0.11*Risk +11.23	2 & 7	restricted intercept and without linear deterministic trend	1	2

Note: weighted average rate of 6-month Market Treasury Bills (MTB); Lending rate (LR); Averaged Exchange Rate (ER); Risk=LR-MTB; Industrial Production Index (IPI); CPI-Inflation

Table D4: Estimation of Money Demand- Johansen Procedure

Sample-end period	η_{ipi}	$\eta_{E(inf)}$	η_{LR}	Co-integration	
				trace	Max Eigen Value
FY05-Q4	-3.10 (-1.58)	-0.02 (-0.76)	-0.21 (-4.08)	1	1
FY06-Q4	-2.06 (-0.92)	-0.04 (-1.18)	-0.31 (-3.97)	3	1
FY07-Q4	-0.13 (-0.05)	-0.11 (-1.57)	-0.48 (-4.94)	4	1
FY08-Q4	-0.19 (-0.32)	-0.002 (-0.09)	-0.11 (-4.98)	1	1
FY09-Q4	0.45 (1.68)	0.02 (-3.28)	-0.04 (4.18)	1	1
FY10-Q4	-0.41 (-0.38)	0.08 (3.33)	0..20 (4.35)	1	1
FY11-Q4	0.88 (4.27)	-0.08 (-3.69)	-0.16 (-4.45)	2	1

Table D5: Estimation of Money Demand-Johansen Procedure

Sample-end period	η_{ipi}	$\eta_{E(inf)}$	η_{ER}	η_{risk}	Co-integration	
					trace	Max Eigen Value
FY05-Q4	0.78 (6.17)	-0.01 (-2.17)	0.31 (3.14)	-0.04 (-5.54)	1	1
FY06-Q4	0.66 (6.90)	-0.01 (-2.01)	0.36 (3.74)	-0.05 (-5.75)	1	1
FY07-Q4	0.68 (9.03)	-0.01 (-2.26)	0.34 (3.91)	-0.04 (-6.05)	1	1
FY08-Q4	0.65 (9.59)	-0.01 (-1.98)	0.36 (4.51)	-0.04 (-5.68)	1	1
FY09-Q4	0.65 (12.97)	-	0.47 (8.38)	-0.03 (-5.20)	2	2
FY10-Q4	0.84 (7.18)	-0.02 (-3.84)	0.26 (2.20)	-0.05 (-4.68)	1	1
FY11-Q4	1.35 (6.56)	-0.08 (-6.64)	0.30 (1.41)	-0.09 (-4.35)	1	2

APPENDIX-E: Bayesian Modeling Results

Table E1: Benchmark Prior Estimates

Parameters	Description	Benchmark Priors	Source
α	Share of capital in production function	0. 46	Haider and Khan (2008)
β	Subjective Discount Factor	0.99	Ahmed et al. (2011)
ρ	Real interest rate in the steady state	0.025	Author Calculations
θ	Measure of price stickiness	0.75	Haider and Khan (2008)
κ	Slope Coefficient in NKPC	$\frac{(1-\theta)(1-\beta\theta)}{\theta}$	Haider and Khan (2008)
γ	Parameter of Risk Aversion	0.587	Ahmed et al. (2011)
φ_1	Output elasticity of money demand	0.860	Author Calculations
φ_2	interest elasticity of money demand	-0.018	Author Calculations
χ_1	Sensitivity of the central bank with respect to inflation	1.2	Author Calculations
χ_2	Sensitivity of the central bank with respect to output	0.31	Author Calculations
ρ_a	Persistence of the technology shock	0.97	Author Calculations
ρ_m	Persistence of the money demand shock	0.47	Author Calculations
ρ_i	Persistence of the monetary policy shock	0.32	Author Calculations

Table E2: Model Prior and Posterior Distribution Results

	Prior Distributions			Posterior Distribution			
Parameters	Distribution	Mean	Std.Dev	Distribution	Mean	5% Percentile	95% Percentile
φ_1	gamma	0.860	0.045	gamma	0.859	0.798	0.917
φ_2	gamma	-0.018	0.005	gamma	-0.024	-0.026	-0.009

Note: The posterior mean of all the estimation parameters are delivered by a 150,000 runs of Metropolis-Hastings algorithm. We have used MATLAB toolbox Dynare 4.1 for this simulation purpose.

Figure E1: Bayesian Prior vs. Posterior Distribution Plots

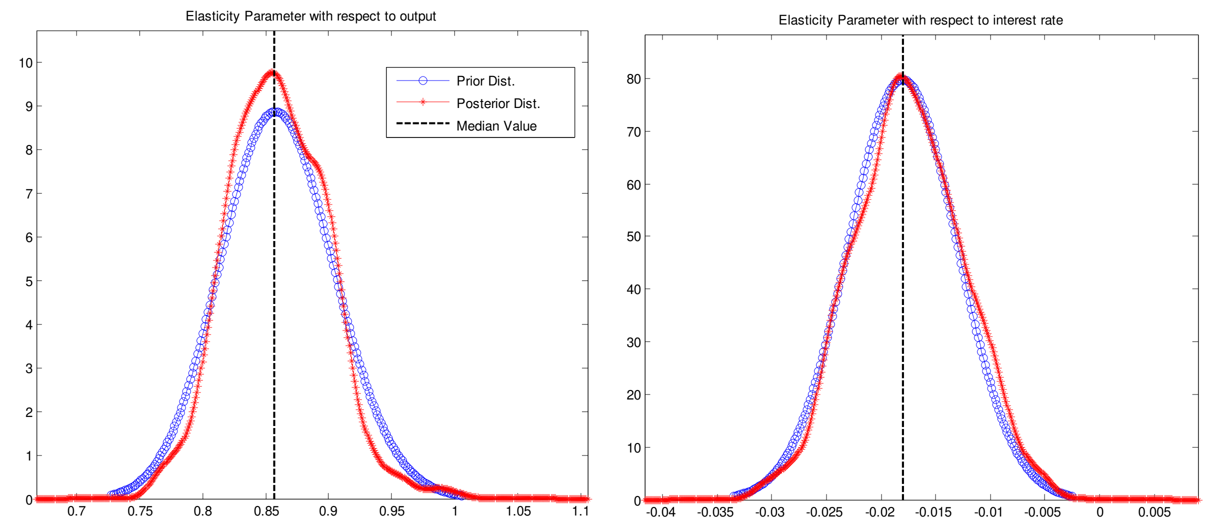
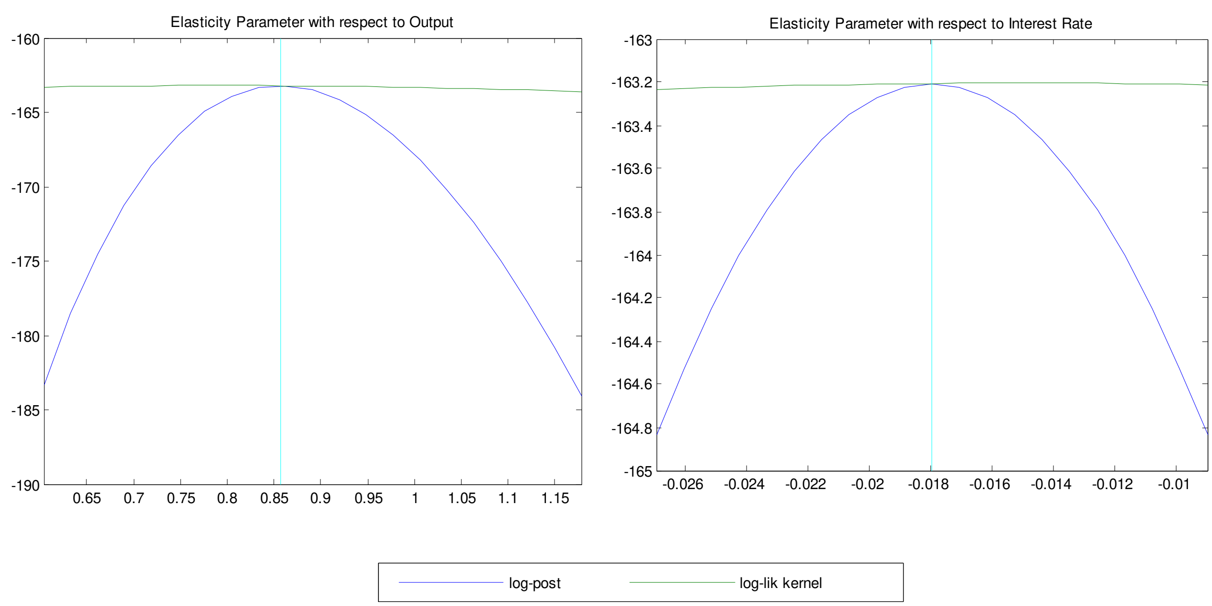


Figure E2: Distribution plots of Bayesian Posterior Kernel and Log-likelihood



APPENDIX-F: Stability Results

Figure F1: Trend in Economic Activity and First Principle Component

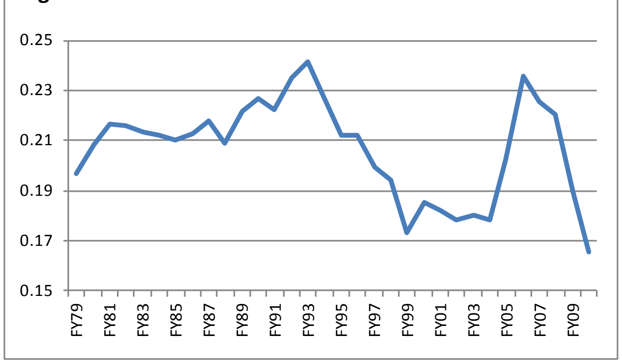


Figure F1a: Investment as % of GDP

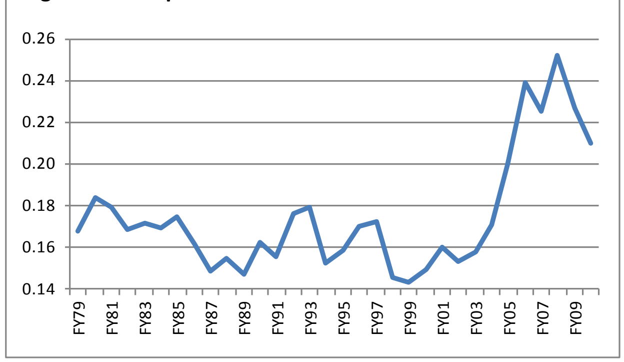


Figure F1b: Imports as % of GDP

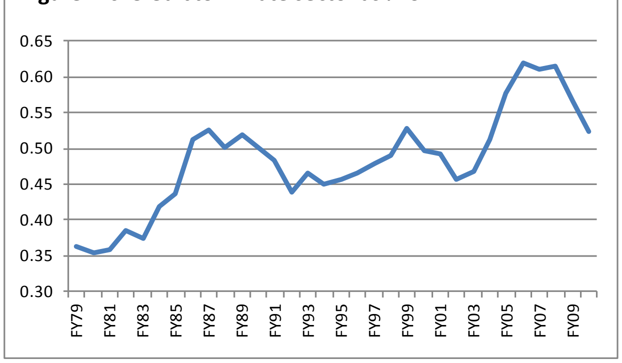


Figure F1c: Credit to Private sector as % of M2

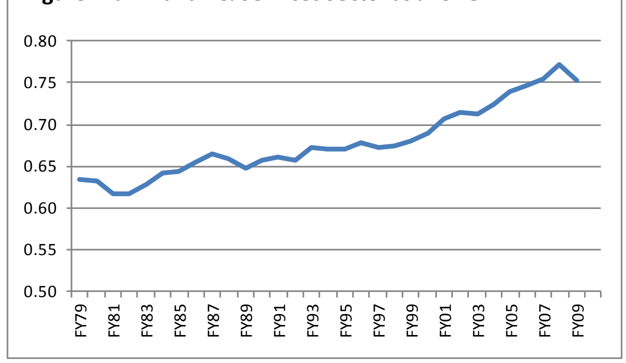


Figure F1d: Manuf. & Services Sector as % of GDP

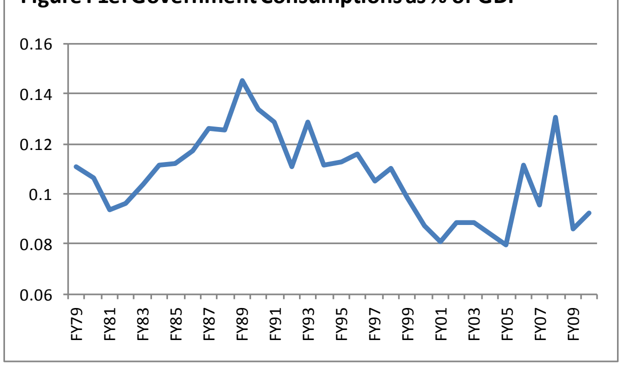


Figure F1e: Government Consumptions as % of GDP

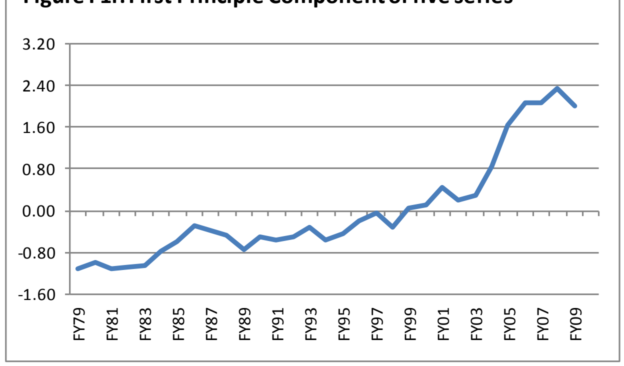
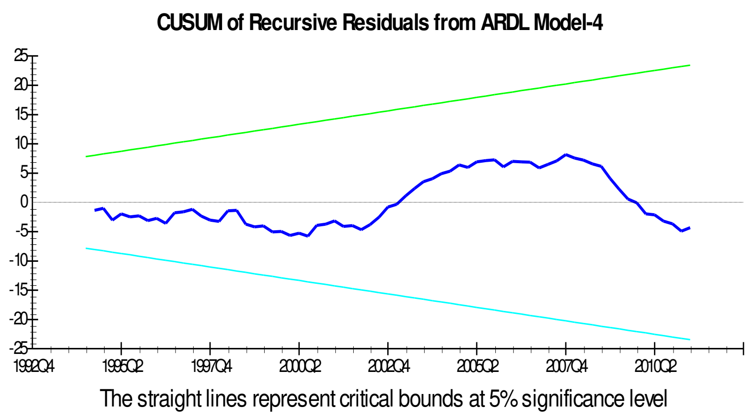


Figure F1f: First Principle Component of five series

Figure F2: CUSUM and CUMUS Square of ARDL Model -4



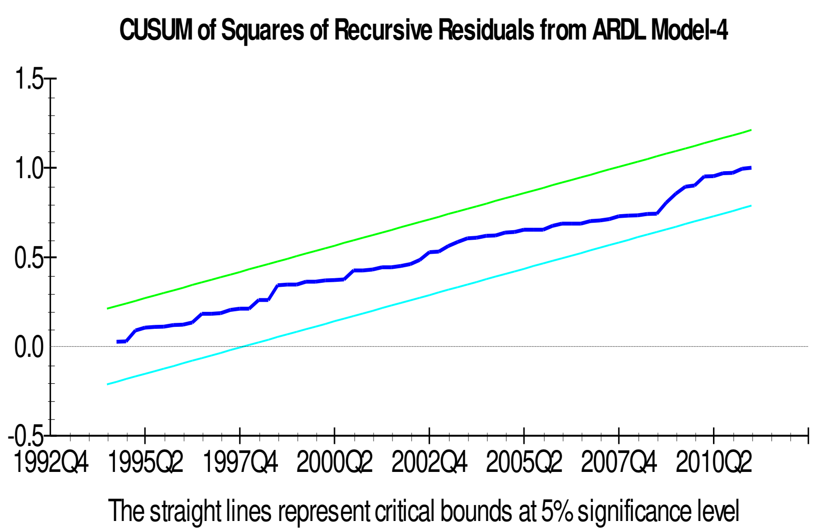
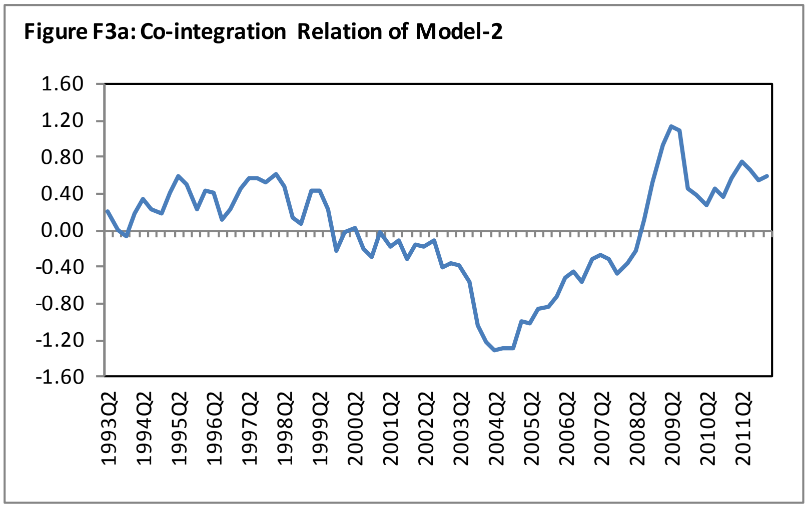


Figure F3: Co-integration relation of Model-2 and 7 (Johansen procedure)



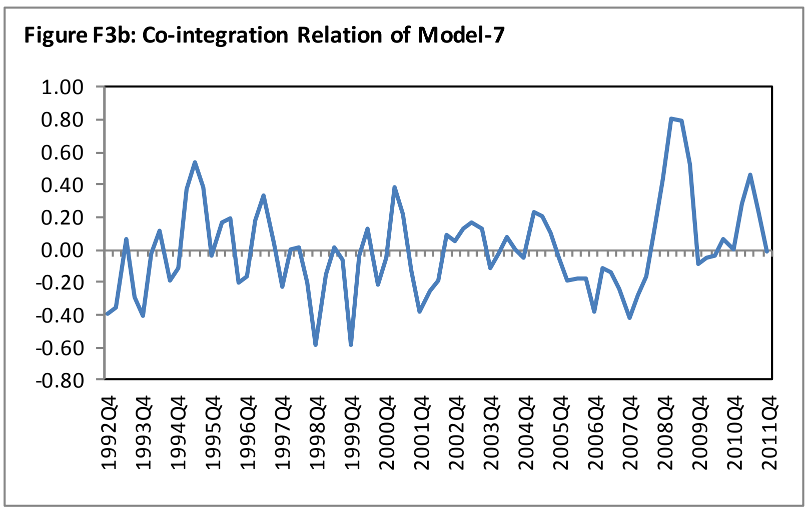


Figure F4: Recursive stability estimates of Model-7 (Johansen procedure)

Figure F4a.

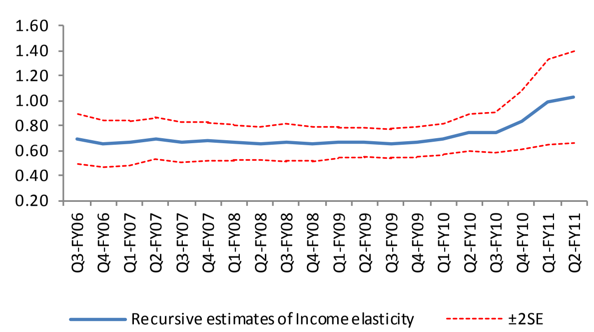


Figure F4b.

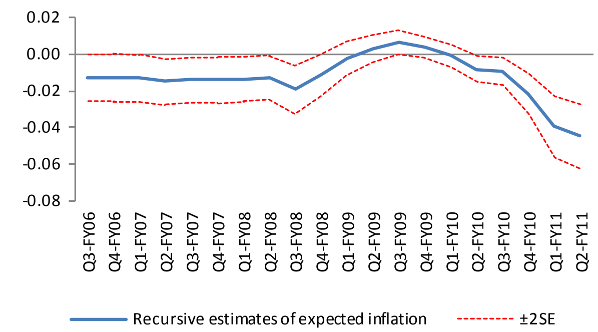


Figure F4c.

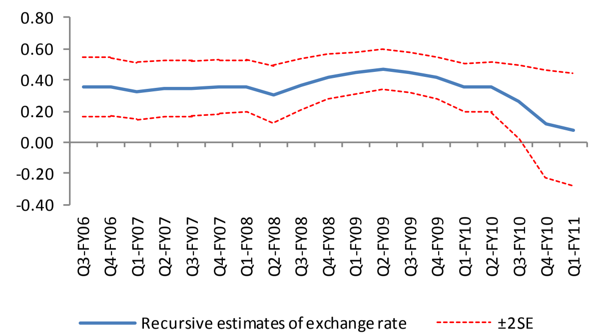


Figure F4d.

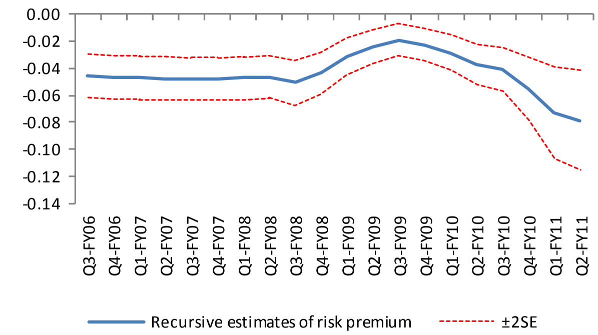


Figure F5: Bayesian Reduce form Screening of parameters

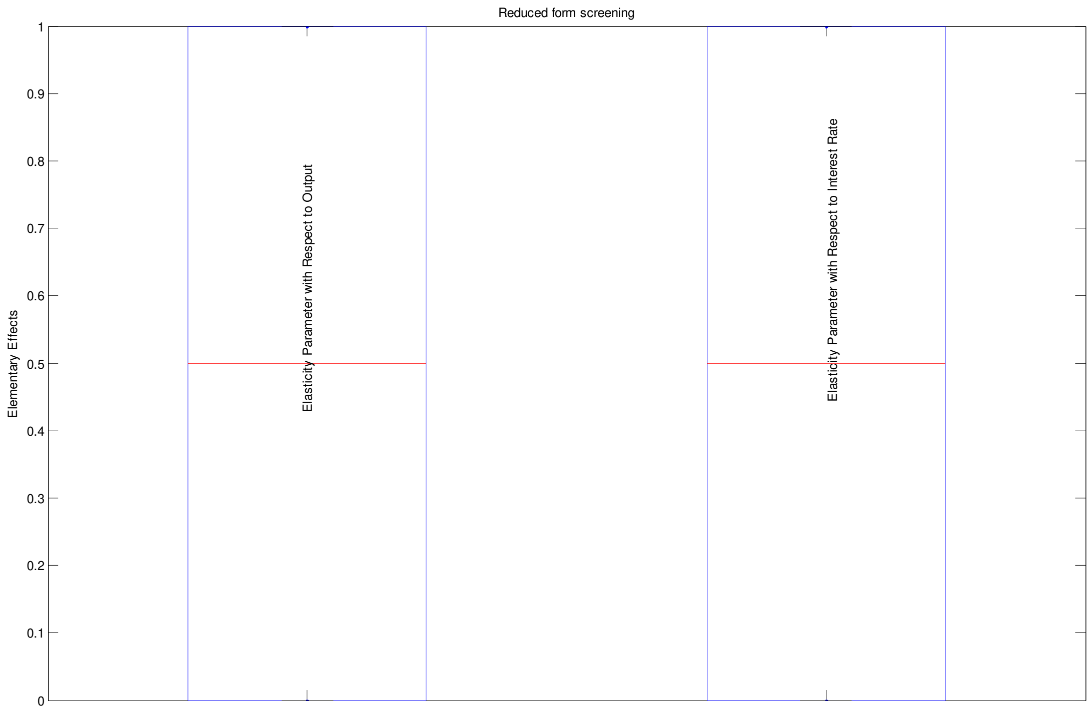
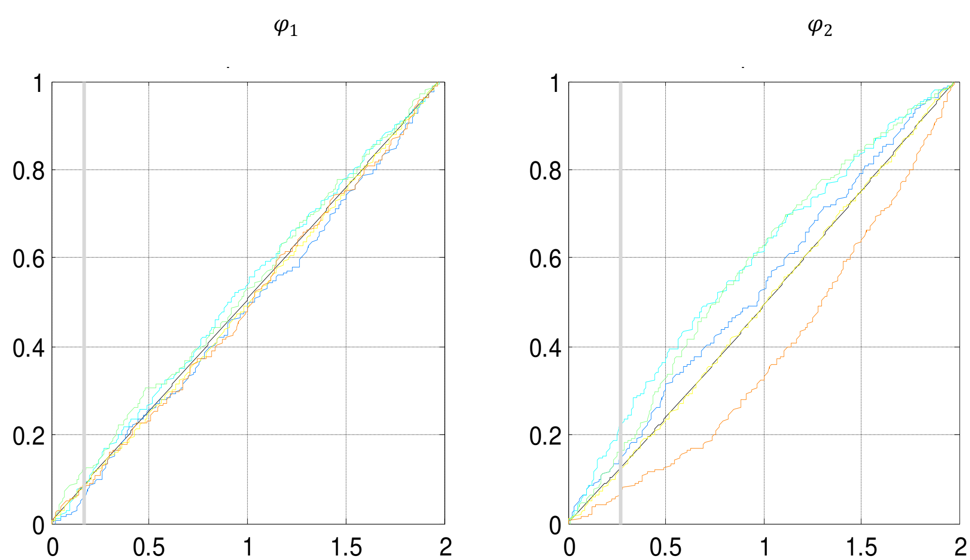


Figure F6: Bayesian Posterior Stability Plots using GSA toolkit [13]



[13] Since we have used different specification of nominal interest rate, therefore, we find different stability path of parameter φ_1 and φ_2.

APPENDIX-G: Selected Empirical Literature Review

SR	Study	Coverage of Variable	Functional Form	Findings
1	Akhtar (1974) Sample Space :1951-70	*M1* *M2* *Interest rates* *Real GDP* *Inflation*	Log Linear	Interest rates and income are found the most important determinant of money demand functions. The long term interest rates are significance and robust with unit elasticity of income.
2	Abe *et al.*, (1975) Sample Space :1951-70	M1 Per capita M2 Per capita GDP per capita Interest rates Inflation	Log Linear	The authors estimated the various models for different definitions of interest rates and national income. The main determinants of M1 and M2 are Interest rate, Inflation and Income. The interest rate and inflation rates are statistically significant with negative sign. And income is statistically significant with positive sign. .
3	Khan (1980) Sample Space :1960-78	M1 M2 Income (measured) Income (Permanent) Interest rates Bank Branches	Log Linear	The author concludes that income, rate of interest on time deposit, inflation and degree of monetization are the important explanatory variables which explain almost 99 percent of the variation in the demand for money. There is no evidence for the hypothesis that the permanent income is better explanatory variable than the measured income.
4	Khan (1982) Sample Space 1960-78	M1 M2 Income (measured) Income (Permanent) Interest rates	Log Linear	This study is conducted for six developing countries including Pakistan. Importantly, the study does not alter the main findings of Khan (1980). However, it is found that the permanent income and expected inflation are better explanatory variable, in the case of Sri Lanka, as compare to measured income and actual inflation respectively.
5	Nisar and Aslam (1983) Sample Space 1960-79	M1 M2 GNP Interest Rates	Log linear	Various models are estimated with alternative measure of interest rates and money stocks. The major conclusion, however, is that the interests rate are significant and robust with unit money elasticity.
6	Ahmad and Khan (1990) Sample Space :1960-87	M1 M2 Real GDP Interest Rate	Log Linear & time varying parametric approach	The article claims that the money demand function remain robust up till 1980 and unstable thereafter.
7	Hossain (1994) Sample space 1951-91	M1 M2 Real GDP Yield on Govt bond Market call rate Inflation	Log Linear	The author found a stable money demand function through the methodology of Johenson cointegration tests. Furthermore, the author found that the interest rate have the significant negative impact on the money stock with unit elasticity of income..
8	Qayyum (1998)	Real Money Demand Real income Yield on long term Govt bond Measured inflation Seasonal dummies	Error Correction Model	The long run money-income-proportionality hypothesis is accepted.
9	Qayyum (2005)	M2 Nominal Income Interest rates Inflation	Johenson Conitegration And Dynamic Error Correction	The article found that the rate of inflation is an important determinant of money demand. The analysis reveals that the rates of interest, market rate, and bond yield are important for the long-run money demand behavior.

10	Moinuddin (2009)	Real M2 Real GDP Real Interest Rates	Log linear	The money demand function is unstable in Pakistan; therefore monetary aggregate targeting is not suitable.
11	Omer and Saqib (2009)	M2 Real GDP Inflation	ARDL	The quantity theory is an inadequate explanation of inflation, income velocity of money is unstable, and money is endogenous. These results suggest rethinking on monetary targeting strategy in Pakistan.
12	Narayan *et al*., (2009) ·	M2 Yd ER Interest rates (foreign and Domestics)	panel cointegration and panel long-run estimation	The article finds that the panel Granger causality test suggests short-run causality running from all variables, except foreign interest rate, to money demand. Furthermore the money demand functions are stable except for Nepal. .
13	Azim *et al*., (2010) Sample space: 1973-2007	Broad Money (M2) Real GDP Inflation Exchange Rate	ARDL	There exist a long-run relationship between broad money and the goal variables. And the money demand function stable, in case of Pakistan, using CUSUM and CUSUM square test.
14	Omer (2010) Sample space: 1975-2006	Reserve Money (M0) Narrow Money (M1) Broad Money (M2) Velocity of money (M0, M1 and M2) Call money rate Nominal GDP Per capita income CPI-inflation	ARDL	Velocity of base and broad money is insensitive to interest rate changes, while responsive to income and business cycle fluctuations. On the other hand, velocity of narrow money (M1) depends on interest rate changes, income and business cycle fluctuation. Money velocities of all the three models are stable using CUSUM and CUSUM square test and conclude that money demand is stable

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