# Programming identification criteria in simultaneous equation models

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**Abstract**

Examining the identification problem in the context of a linear econometric model can be a tedious task. The order condition of identifiability is an easy condition to compute, though difficult to remember. The application of the rank condition, due to its complicated definition and its computational demands, is time consuming and contains a high risk for errors. Furthermore, possible miscalculations could lead to wrong identification results, which cannot be revealed by other indications. Thus, a safe way to test identification criteria is to make use of computer software. Specialized econometric software can off-load some of the requested computations but the procedure of formation and verification of the identification criteria are still up to the user. In our identification study we use the program editor of a free computer algebra system, Xcas. We present a routine that tests various identification conditions and classifies the equations under study as «under-identified», «just-identified», «over-identified» and «unidentified», in just one entry.

**Keywords**: Simultaneous equation models; order condition of identifiability; rank condition of identifiability; computer algebra system Xcas.

**JEL Classification Codes**: C10; C30; C51; C63.

## 1. Introduction

Our identification study relates to models specifying a complete system of structural equations and in which a given set of endogenous and exogenous variables enters linearly. Koopmans (1949) describes a complete system as one in which there are as many equations as endogenous variables. From identification study of structural equations in simultaneous equation models, identities are excluded, since they are a priori identified equations. Koopmans (1949), Koopmans and Reiersol (1950) and Koopmans et al. (1950) established conditions for identification in linear simultaneous models.

The identifiability concept in econometrics is a matter of practical importance: an identification test is equivalent to the question whether parameters in structural equations of econometric models are uniquely defined. The identification problem is related to the uniqueness of the solution of the transformation from the reduced form parameters to the structural form parameters. The study of identifiability concept enables a researcher to justify certain estimation methods for the parameters of simultaneous equations.

Identification testing in econometrics is often a continuous request, as new linear restrictions on the parameters of structural equations are imposed or existing linear restrictions are redrawn. After such alterations, identification results based on the initial simultaneous equation model are not valid and must be recalculated. For the researcher looking for the proper parameter estimation for an econometric model, identification tests are part of an already complex process, either identification is checked numerically in terms of the possibility for IV estimation (McFadden, 1999) or via detailed order and rank conditions for identification.

Since traditional identification conditions constitute a subject in the fields of Linear Algebra, proper computer software could perform mathematical calculations. Hence, we envisioned a routine to provide the identification result instantly, in a black box mode. In this direction, it seemed convenient to work in the program editor of free software, as every user interested can have open access. We chose the programming environment of computer algebra system Xcas[1], since similar studies for economic applications have been made adequately in (Halkos and Tsilika, 2011; Halkos and Tsilika, 2012a, b).

## 2. Identifiability Analysis

Let y denotes a vector of endogenous variables, x a vector of exogenous variables and u the disturbance vector. Then a behavioral or structural simultaneous equations system can be presented as

$$y_t' A + x_t' B = u_t'$$

Where $y_t' = \left(y_{1t}, y_{2t}, \ldots, y_{Nt}\right)$ is a 1xN vector of endogenous variables, A is an NxN array of coefficients, $x_t' = \left(x_{n1}, x_{n2}, \ldots, x_{Mt}\right)$ is a 1xM vector of exogenous variables, B is a MxN array of coefficients and $u_t'$ is a 1xN vector of disturbances. The reduced form of this system is

$$y_t' = x_t' K + \varepsilon_t'$$

Where K=-BA$^{-1}$ and $\varepsilon_t' = u_t' A^{-1}$. If P is the NxN covariance matrix of $u_t$ then the covariance matrix of $\varepsilon_t$ equals to $A'^{-1} P A^{-1}$ with A being non-singular.

Restrictions have to be imposed on the coefficients of A and B arrays and even in the covariance matrix P for consistency in the estimation. For this reason we need

[1] Xcas is a Computer Algebra System available free in http://www-fourier.ujf-grenoble.fr/~parisse /giac.html

detailed order and rank conditions for identification from the structure of matrices A and B and the condition that KA+B=0 (McFadden, 1999).

The theoretical framework of our analysis sets the following assumptions.

- Identification concept is considered at the first order level, i.e. identification is based on the conditional expectation of the endogenous variables (Holly, 2012).
- Linear restrictions are imposed on the first order parameters (i.e. elements of A and B) of the same equation.
- The a priori constraints concern only the matrices A and B and not the variance-covariance matrix P.

The identification conditions for the case of restrictions on P are discussed in Fisher (1966), Hausman and Taylor (1980) and Wegge (1965).

A traditional identification study in linear models is related to two basic criteria of identifiability. A necessary (and not sufficient) condition for identifiability of a structural equation is the so-called *Order Condition*. A necessary and sufficient condition of identifiability is the so-called *Rank Condition*. The rank condition tells us whether the equation under consideration is identified or not, whereas the order condition tells us if it is exactly identified or overidentified. It is interesting to notice that different classification of the same equation can be given by order condition and rank condition. The accuracy of the identification specification in linear systems is a matter of the researcher.

## 2.1 The Order Condition of Identifiability

In a complete system of M simultaneous equations, in order for an equation to be identified, the number of predetermined variables excluded from the equation must not be less than the number of endogenous variables included in that equation less 1.

This is known as the *order condition of identifiability*. A mathematical formulation of the order condition is the following (Gujarati, 2003 p. 748):

- if K-k=m-1 the equation is just identified
- if K-k>m-1 the equation is overidentified,

where

K is the number of predetermined variables (including the constant term) in the model,

k is the number of predetermined variables in a given equation

M is the number of endogenous variables in the system and

m is the number of endogenous variables in a given equation.

Koopmans (1949, p. 135) rephrased the order condition in the following way: A necessary condition for the identifiability of a structural equation within a given linear model is that the number of variables excluded from that equation (or more generally the number of linear restrictions on the parameters of that equation) be at least equal to the number M of structural equations less one.

The order condition is not sufficient. It only states the minimal number of a priori information on the (first-order) parameters of an equation, for this equation to be identifiable (Holly, 2012).

## 2.2 The Rank Condition of Identifiability

Using the order condition helps us to check if sufficient variables have been omitted from the equation under examination, without checking the rest of the system. In this way we may face the problem of identifying a specific equation by excluding a certain variable, which however does not belong to any other equation of the system.

The rank condition checks both: the sufficient exclusion restrictions as well as that the omitted variables guarantee identification as they play an influential role in the model.

For estimation of a system it is necessary to have a scaling normalization for each equation. A necessary and sufficient condition for the identifiability of a structural equation within a linear model, restricted only by the exclusion of certain variables from certain equations, is that we can form at least one nonvanishing determinant of order M - 1 out of those coefficients, properly arranged, with which the variables excluded from that structural equation appear in the M - 1 other structural equations. That is, in a system of M endogenous variables in M equations, a specific equation is identified if and only if one nonzero determinant of order (M-1)(M-1) can be formed from the coefficients of the variables omitted from that equation but included in other equations of the system. This is known as the *rank condition of identifiability* (Koopmans, 1949 p. 135; Gujarati, 2003).[2]

A basic feature involved in the rank condition is the coefficient matrix $A_i$ (one for every structural equation) constructed from the coefficients of the variables (both endogenous and predetermined) excluded from that particular equation but included in the other equations of the model. $A_i$ has zero elements in the row of the i-th equation. For that reason, $rank(A_i) \leq M-1$.

In $A_i$, the number of columns is equal to the number of variables excluded from the i-th equation.

An equivalent reformulation of the rank criterion for identifiability of a given structural equation, in terms of coefficients of the reduced form, is to consider only those equations of the reduced form that solve for dependent variables, specified by the model as occurring in (strictly: as not excluded from) the structural equation in

[2] The proof can be found in Hood and Koopmans (1953) and Fisher (1966).

question. Now form the matrix $A_i$ of the coefficients, in these M equations, of those predetermined variables that are excluded by the model from the structural equation involved. A necessary and sufficient condition for the identifiability of that structural equation is that the rank of $A_i$ be equal to M-1 (Koopmans, 1949 p. 136). This is known as the Rank Condition on the Reduced Form.

### 2.3 The conditions for a strict specification of identifiability

Consider a system of M simultaneous equations. The general principles of identifiability of a structural equation in an M simultaneous equations system are (Gujarati, 2003 p. 753):

- if $K-k>m-1$ and the rank of the A matrix is M-1, the equation is overidentified
- If $K-k=m-1$ and the rank of the A matrix is M-1, the equation is exactly identified
- If $K-k\geq m-1$ and the rank of the A matrix is M-1, the equation is underidentified
- If $K-k<m-1$ the equation is unidentified. The rank of the A matrix now is bound to be less than M-1,

where as shown before, K is the number of predetermined variables (including the constant term) in the model, k is the number of predetermined variables in a given equation, M the number of endogenous variables in the system and m is the number of endogenous variables in a given equation.

## 3. Relative existing literature in identification

The rank and order conditions were first explored as conditions on a sub-matrix of the reduced form coefficient in Koopmans (1949), Koopmans and Reisersol (1950) and Wald (1950). These results were generalized by Fisher (1959, 1963) and extended to nonlinear systems (Fisher 1961, 1965). Wegge (1965) used the Jacobian matrix and Rothenberg (1973) used the information matrix in order to consider the identification of whole systems. Other similar studies are those of Bowden (1973), Richmond (1974), Kelly (1971, 1975) and Hausman (1983).

Harvey (1990, p.328) notes that «the order condition is usually sufficient to ensure identifiability and although it is important to be aware of the rank condition, a failure to verify it will rarely result to disaster». Similarly, Gujarati (2003 p. 753) claims that «when we talk about identification we mean exact identification or overidentification. There is no point in considering unidentified or underidentified equations because no matter how extensive the data is, the structural parameters cannot be estimated. However, parameters of overidentified as well as just identified equations can be estimated».

For large simultaneous-equation models, applying the rank condition is a formidable task. Various research papers deal with the problem of identification in the presence of autocorrelation (Hatanaka, 1975; Deistler, 1976, 1978; Deistlet and Schrader, 1979) or measurement error (Hsiao, 1976, 1977; Geraci, 1976) or errors in exogenous variables (Anderson and Hurwitz, 1949; Wiley, 1973). Examples can be found in Goldberger (1972, 1974) and Duncan and Featherman (1972). Finally, identification and estimation can be examined using Bayesian approaches (among others Zellner, 1971; Kadane, 1974; Drèze, 1974).[3]

In terms of nonparametric identification, Brown (1983) and Roehrig (1988) are widely cited in the literature (among others Newey et al. 1999; Angrist et al. 2000; Guerre et al. 2000; Brown and Wegkamp, 2002; Athey and Haile, 2002; Chesher, 2003; Matzkin, 2003; Newey and Powell, 2003; Benkard and Berry, 2006; Matzkin,

[3] For more details see Judge et al. (1985).

2008). The issue of identification in cointegrated systems can be found among others in Johansen (1995), Greenslade et al. (2002), Boswijk (2004) and Asteriou and Hall (2011).

## 4. Programming Identifiability Conditions in Xcas

For the application of the codes below, the researcher's responsibility is to specify the equations (equations) of the model, the variable vector with its endogenous variables (endogenous), the variable vector with its predetermined variables (exogenous) and the serial number (n) of the equation under consideration. The first equation of the system is indicated by setting n=0, the second equation of the system is indicated by setting n=1, etc. Our routine results in a classification of structural equations into categories, by testing three identification criteria.

In Xcas programming environment we create **pinakas** function, with arguments the equations (equations) of the model, the variable vector with its endogenous variables (endogenous), the variable vector with its predetermined variables (exogenous) and the serial number (n) of the equation under consideration. **pinakas** function generates the matrix with entries the coefficients of the variables (both endogenous and predetermined) excluded from the equation under consideration but included in the other equations of the model.

According to the rank condition, if the rank of that matrix is M-1 (where M is the number of equations), the corresponding equation is identified. For a direct answer to the rank condition, we define **rankcondition** function in Xcas, with arguments the equations (equations) of the model, the variable vector with its endogenous variables (endogenous), the variable vector with its predetermined variables (exogenous) and the serial number (n) of the equation under consideration. **rankcondition** function returns «identified» for identifiable equations and «unidentified» otherwise.

For a direct answer to the order condition, we define **ordercondition** function in Xcas, with arguments the equations (equations) of the model, the variable vector with its endogenous variables (endogenous), the variable vector with its predetermined variables (exogenous) and the serial number (n) of the equation under consideration. **ordercondition** function returns «exactly identified» and «overidentified». An overall test of the identifiability property, is made by **identificationtest** function. **identificationtest** function has the same arguments as **ordercondition** and **rankcondition** functions and separates equations of simultaneous equation models in categories of «overidentified», «exactly identified», «underidentified» and «unidentified».

The routine in Xcas is presented next.

```
var(endogenous,exogenous):=convert(endogenous union exogenous, list):;
m(equations,endogenous,exogenous):=syst2mat(equations,var(endogenous,exogen
ous)):;
pinakas(equations,endogenous,exogenous,n):=transpose(select(x->x!=0,
[seq(if(row(m(equations,endogenous,exogenous),n)[[k]]==0)col(m(equations,endo
genous,exogenous),k-1); ,k=1..length(var(endogenous,exogenous)))]))
:;
ordercondition(equations,endogenous,exogenous,n):=if(length(exogenous)-
count(x->x!=0,coeff(left(equations[[n+1]]),exogenous))+1=count(x-
>x!=0,coeff(left(equations[[n+1]]),endogenous))-2)"exactly identified";
else(if(length(exogenous)-count(x->x!=0,coeff(left(equations[[n+1]]),exogenous))
+1>count(x->x!=0,coeff(left(equations[[n+1]]),endogenous))-2) "overidentified";);
```

```
:;
rankcondition(equations,endogenous,exogenous,n):=if(rank(pinakas(equations,endogenous,exogenous,n))=length(equations)-1) "identified"; else "unidentified";
:;
identificationtest(equations,endogenous,exogenous,n):=if(rank(pinakas(equations,endogenous,exogenous,n))=length(equations)-1 and length(exogenous)-count(x->x!=0,coeff(left(equations[[n+1]]),exogenous))+1 =count(x->x!=0,coeff(left(equations[[n+1]]),endogenous))-2) "exactly identified"; else (if(rank(pinakas(equations,endogenous,exogenous,n))=length(equations)-1 and length(exogenous)-count(x->x!=0,coeff(left(equations[[n+1]]),exogenous))+1 >count(x->x!=0,coeff(left(equations[[n+1]]),endogenous))-2) "overidentified"; else (if(rank(pinakas(equations,endogenous,exogenous,n))<length(equations)-1 and length(exogenous)-count(x->x!=0,coeff(left(equations[[n+1]]),exogenous))+1 >=count(x->x!=0,coeff(left(equations[[n+1]]),endogenous))-2) "underidentified"; else "unidentified";););:;
```

The codes of **pinakas**, **rankcondition, ordercondition,** and **identificationtest** functions are saved in identificationtest.cxx program file. Working in any session, by writing in a commandline read("identificationtest.cxx") we can use **pinakas, rankcondition, ordercondition** and **identificationtest** functions.

*4.1 Some Examples*

Let us first apply our routine of identifiability criteria to a problem posed by Ezekiel and Klein and described in (Koopmans, 1949 p.138). The question is whether identifiability of the investment equation can be attained by the subdivision of the investment variable into separate categories of investment (that is, $I_1$ investment in plant and equipment, $I_2$ investment in housing, $I_3$ temporary investment corresponding to changes in consumer's credit and business inventories and $I_4$ quasi-investment corresponding to the net contribution from foreign trade and government budget). The system is the following:

$$
\begin{array}{llr}
S - I_1 - I_2 - I_3 - I_4 & = 0 & (1) \\
S \quad - a_1 Y - a_2 Y_{-1} \quad - a_0 & = u & (2) \\
I_1 \quad - b_1 Y - b_2 Y_{-1} \quad - b_0 & = v_1 & (3) \\
I_2 \quad - \gamma_1 \mathrm{Y} - \gamma_2 Y_{-1} - H \quad - \gamma_0 & = v_2 & (4) \\
I_3 \quad - \delta_1 \mathrm{Y} + \delta_1 Y_{-1} \quad - \delta_0 & = v_3 & (5) \\
I_4 - \varepsilon_1 \mathrm{Y} - \varepsilon_2 Y_{-1} \quad - E - \varepsilon_0 & = v_4 & (6)
\end{array}
$$

In the Ezekiel's model, variables $Y_{-1}, \mathrm{H}, \mathrm{E}$ are predetermined and $S, I_1, I_2, I_3, I_4, \mathrm{Y}$ are endogenous. No question of identifiability arises to the identity expressed by the equation (1) of the model.

The following results are generated in Xcas environment for equations (2-6) by our programmed functions. We first test the order condition. The results of Order Condition of identifiability for the second through the sixth equation are generated all together via Xcas built-in function seq, for n varying from 1 to 5:

```
seq(ordercondition([s-i1-i2-i3-i4=0,s-a1*y-a2*y1-a0=u,i1-b1*y-b2*y1-b0=v1,i2-
g1*y-g2*y1-h-g0=v2,i3-d1*y+d1*y1-d0=v3,i4-e1*y-e2*y1-e-e0=v4],[s,i1,i2,i3,i4,y],
[y1,h,e],n),n=1..5)
"overidentified","overidentified","exactly identified","overidentified","exactly
identified"
```

We test the rank condition. The results of Rank Condition of identifiability for the second through the sixth equation are generated all together via Xcas built-in function seq, for n varying from 1 to 5:

```
seq(rankcondition([s-i1-i2-i3-i4=0,s-a1*y-a2*y1-a0=u,i1-b1*y-b2*y1-b0=v1,i2-g1*y-g2*y1-h-g0=v2,i3-d1*y+d1*y1-d0=v3,i4-e1*y-e2*y1-e-e0=v4],[s,i1,i2,i3,i4,y],[y1,h,e] ,n),n=1..5)
"identified","identified","identified","identified","identified"
```

For a detailed analysis of the rank condition we can generate the coefficient matrices related to rank condition. The rank condition matrices for equations (2-6) of Ezekiel's model are:

```
seq(pinakas([s-i1-i2-i3-i4=0,s-a1*y-a2*y1-a0=u,i1-b1*y-b2*y1-b0=v1,i2-g1*y-g2*y1-h-g0=v2,i3-d1*y+d1*y1-d0=v3,i4-e1*y-e2*y1-e-e0=v4],[s,i1,i2,i3,i4,y],[y1,h,e] ,n),n=1..5)
```

$$\begin{bmatrix} -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We test the general principles of identifiability as discussed in section 2.3:

```
seq(identificationtest([s-i1-i2-i3-i4=0,s-a1*y-a2*y1-a0=u,i1-b1*y-b2*y1-b0=v1,i2-g1*y-g2*y1-h-g0=v2,i3-d1*y+d1*y1-d0=v3,i4-e1*y-e2*y1-e-e0=v4],[s,i1,i2,i3,i4,y],[y1,h,e] , n),n=1..5)
"overidentified","overidentified","exactly identified", "overidentified", "exactly identified"
```

Let us now consider the following hypothetical system of simultaneous equations in which the Y variables are endogenous and the X variables are predetermined.

$$\begin{aligned} &Y_{1t} - b_{10} - b_{12}Y_{2t} - b_{13}Y_{3t} - \gamma_{11}X_{1t} = u_{1t} \\ &Y_{2t} - b_{20} - b_{23}Y_{3t} - \gamma_{21}X_{1t} - \gamma_{22}X_{2t} = u_{2t} \\ &Y_{3t} - b_{30} - b_{31}Y_{1t} - \gamma_{31}X_{1t} - \gamma_{32}X_{2t} = u_{3t} \\ &Y_{4t} - b_{40} - b_{41}Y_{1t} - b_{42}Y_{2t} - \gamma_{43}X_{3t} = u_{4t} \end{aligned}$$

The results of Order Condition of identifiability in Xcas are:

```
seq(ordercondition([y1t-b10-b12*y2t-b13*y3t-g11*x1t=u1t,y2t-b20-b23*y3t-g21*x1t-g22*x2t=u2t,y3t-b30-b31*y1t-g31*x1t-g32*x2t=u3t,y4t-b40-b41*y1t-b42*y2t-g43*x3t=u4t], [y1t,y2t,y3t,y4t],[x1t,x2t,x3t],n),n=0..3)
"exactly identified","exactly identified","exactly identified","exactly identified"
```

The results of Rank Condition of identifiability in Xcas are:

```
seq(rankcondition([y1t-b10-b12*y2t-b13*y3t-g11*x1t=u1t,y2t-b20-b23*y3t-g21*x1t-g22*x2t=u2t,y3t-b30-b31*y1t-g31*x1t-g32*x2t=u3t,y4t-b40-b41*y1t-b42*y2t-g43*x3t=u4t], [y1t,y2t,y3t,y4t],[x1t,x2t,x3t],n),n=0..3)
"unidentified","unidentified","unidentified","identified"
```

For a detailed analysis of the rank condition we may generate the coefficient matrices related to rank condition:

```
seq(pinakas([y1t-b10-b12*y2t-b13*y3t-g11*x1t=u1t,y2t-b20-b23*y3t-g21*x1t-g22*x2t=u2t,y3t-b30-b31*y1t-g31*x1t-g32*x2t=u3t,y4t-b40-b41*y1t-b42*y2t-g43*x3t=u4t], [y1t,y2t,y3t,y4t,x1t,x2t,x3t],l),l=0..3)
```

$$\left( \begin{bmatrix} 0, & 0, & 0 \\ 0, & -g22, & 0 \\ 0, & -g32, & 0 \\ 1, & 0, & -g43 \end{bmatrix}, \begin{bmatrix} 1, & 0, & 0 \\ 0, & 0, & 0 \\ -b31, & 0, & 0 \\ -b41, & 1, & -g43 \end{bmatrix}, \begin{bmatrix} -b12, & 0, & 0 \\ 1, & 0, & 0 \\ 0, & 0, & 0 \\ -b42, & 1, & -g43 \end{bmatrix}, \begin{bmatrix} -b13, & -g11, & 0 \\ -b23, & -g21, & -g22 \\ 1, & -g31, & -g32 \\ 0, & 0, & 0 \end{bmatrix} \right)$$

The results of the general principles of identifiability in Xcas are:

```
seq(identificationtest([y1t-b10-b12*y2t-b13*y3t-g11*x1t=u1t,y2t-b20-b23*y3t-g21*x1t-g22*x2t=u2t,y3t-b30-b31*y1t-g31*x1t-g32*x2t=u3t,y4t-b40-b41*y1t-b42*y2t-g43*x3t=u4t],[y1t,y2t,y3t,y4t],[x1t,x2t,x3t],n),n=0..3)
"underidentified","underidentified","underidentified","exactly identified"
```

It is interesting to notice the different output of ordercondition and rankcondition functions. In this example it is safer to rely on identificationtest result.

## 5. Conclusions

Testing criteria for the identifiability of a structural equation in a linear model is a straightforward mathematical problem and needs a matrix algebra expert. The order condition of identifiability examines a relation among the number of predetermined variables in the model, the number of predetermined variables in a given equation and the number of endogenous variables in a given equation. The rank condition of identifiability is based upon a set of coefficient matrices, constructed by a complicated law of formation. A third identification test combines the two conditions of identifiability in order to give a strict specification of identifiability.

Considering the various difficulties arising in formulation and computations of identification study, the user turns to computer software to get the information included in classical conditions of identification. Typical versions of econometric and mathematical packages can only compute the rank of a matrix. The present work aims to supplement the limited contribution of computer software in the area of classical identification criteria.

With a routine in Xcas program editor, we give to Xcas users the choice to perform automated testing of the appropriate identification criteria. Since a variety of opinions is presented in the econometric literature for the role of each criterion, we programmed three different functions dedicated to three identification criteria: the order condition, the rank condition and a condition based on the combination of order and rank condition.

In particular, we programmed **rankcondition** function to classify structural equations of econometric models into categories according to the rank condition of identifiability, **ordercondition** function to classify structural equations of econometric models into categories according to the order condition of identifiability and **identificationtest** function for a detailed specification of identifiability. For pedagogical needs in educational practice, our programmed **pinakas** function, presents the matrix $A_i$ constructed from the coefficients of the variables (both endogenous and predetermined) excluded from the structural equation under study but included in the other equations of the model. The input of all functions needs basic information of the model, as the list of equations, the endogenous and the exogenous variable vectors.

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