Predicting crises: Five essays on the mathematic prediction of economic and social crises

Albers, Scott

American College in Paris

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PREDICTING CRISSES:
Five Essays on the Mathematic Prediction of Economic and Social Crises

By Scott A. Albers
For Andrew, Alison and Rachel
INTRODUCTION TO THESE ESSAYS 
AND ACKNOWLEDGMENTS

This volume – Predicting Crisis: Five Essays on the Mathematic Prediction of Economic and Social Crises – is the first of three sets of essays. In this first set the economic and social history of the United States is shown to be a “system of movement,” i.e. a logical and mathematic progression of events which may be analyzed according to a set formula. The model proposed demonstrates that the citizen’s individual “consciousness” is writ large in the macro-economic statistics of this unique economy and thereby made available for inspection at other levels of reality.

The second volume – A Theory of Consciousness: Five Essays on the Structure of Thought – presents five other systems of movement, demonstrating that the pattern displayed in this first volume may be found at other levels of reality including set theory, chemistry, psychology, law and culture.

The third volume – A Theory of Reality: An Introduction to Oppositional Analysis – presents a formal theory of reality which incorporates the “consciousness” examined in the first two volumes as a formal “fifth dimension” of our experience. This is given in detail for music, and then used to re-analyze the economic material of the first and second volumes.

I would like especially to thank Kate Anderson, Department of Biochemistry and Microbiology, St. Andrew’s University, Edinburgh, Scotland, and my son Andrew Albers, Mathematics Department, Montana State University, Bozeman, for their interest, support and encouragement.

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All quotes used herein are believed to be within the public domain, used after obtaining the permission of the author or a fair use of scholarly material. If this should be in error please contact the author at: scott_albers@msn.com.

For the positions taken and the methods used herein I alone am responsible.
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1 The pagination of this volume is designed to permit a two-page pdf view of the material. At several points in these essays it is convenient to contrast the material on the left hand side of the screen with the right hand side.
PREFACE: PREDICTIONS

At the dawn of the last century Albert Einstein proposed that three dimensions of space and one dimension of time must be considered to understand the world around us. In 1919 Theodor Kaluza proposed an undiscovered “fifth dimension” which might be used to unify General Relativity with Maxwell’s equations for electro-magnetism.

As these ideas relate to the social sciences we must admit that the prevailing view of time, at least as it presents itself to the historian, the economist or the social scientist, is hardly better than “one damn thing after another.” However the growing body of historic, social and particularly economic data which has been collected over the past three centuries permits us to challenge this view.

On December 8, 2003 an early draft of this essay entitled “The Coming Panic of 2005” was sent to Mr. Jim Foley, head of the Montana office of Senator Max Baucus, chairman of the Senate Finance Committee. The abstract of the article states:

A 56-year spiral of American economic growth demonstrates the Fibonacci Series, thereby illustrating the mathematic and biologic relationship between the American economy and the natural phenomena underlying it. This spiral provides the basis for a prediction that the year 2005 will mark a tremendous diplomatic and financial panic throughout the world. The chief advantage of this approach is that it provides exact dates as to when change will occur, and hints as to what sort of change will occur. This approach anticipates that the years 1781, 1837, 1949 and 2005 will be analogous to one another, each year presenting a sudden, dramatic challenge to the United States. (emphasis supplied)

Ten months later, on September 17, 2004, the FBI also warned that a financial crisis was imminent. [http://www2.fbi.gov/congress/congress04/swecker100704.htm](http://www2.fbi.gov/congress/congress04/swecker100704.htm)

The potential impact of mortgage fraud on financial institutions and the stock market is clear. If fraudulent practices become systemic within the mortgage industry and mortgage fraud is allowed to become unrestrained, it will ultimately place financial institutions at risk and have adverse effects on the stock market. Investors may lose faith and require higher returns from mortgage backed securities. This may result in higher interest rates and fees paid by borrowers and limit the amount of investment funds available for mortgage loans.

Often times, mortgage loans are sold in secondary markets or are used by financial institutions as collateral for other investments. Repurchase agreements have been utilized by investors for protection against mortgage fraud. When loans sold in the secondary market default and have fraudulent or material misrepresentation, loans are repurchased by the lending financial institution based on a "repurchase agreement." As a result, these loans become a non-performing asset. In extreme fraud cases, the mortgage backed security is worthless. Mortgage fraud losses adversely affect loan loss reserves, profits, liquidity levels and capitalization ratios, ultimately affecting the soundness of the financial institution.
The first prediction above precedes by ten months the warning given by the FBI to Congress in September, 2004. Both warnings highlight the historic precedents and imminent nature of the crisis, i.e. 2005. Yet nothing was done to anticipate or to mitigate the forewarned and even expected “tremendous diplomatic and financial panic throughout the world.”

On November 4, 2008 Senator Barrack Obama was elected 44th President of the United States and its first African-American President in the midst of a devastating Global Financial Crisis, wars in Afghanistan and Iraq, and with the largest popular vote in American history. To put in perspective the significance of these predictions in light of subsequent experience, see Roberts, 2011.

How did the official leaders of capitalist economic strategy act before, during and after the Great Recession?

Before 2007, no official strategist of economic policy forecast any crisis. US Fed Chairman Greenspan in 2004 told us that “a national severe price distortion is most unlikely in real estate.” In 2006, he told us that “the worst may be over for housing,” just the housing bubble burst. US treasury secretary Hank Paulson said the crisis in the overall economy “appears to be contained,” March 2007.

During the crisis, in October 2008, the great financial maestro Greenspan told the US Congress, “I am in a state of shocked disbelief.” He was questioned: “In other words, you found that your view of the world, your ideology was not right, it was not working?” (House Oversight Committee Chair, Henry Waxman). “Absolutely, precisely, you know that’s precisely the reason I was shocked, because I have been going for 40 years or more with very considerable evidence that it was working exceptionally well.”

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2 This text is prepared in pdf format with a two-page per view in mind. In this fashion the left hand page relates to the right hand page.

1.1 The basis for predictions of crises

If the amendments to the federal constitution of the United States are tracked according to their placement in a unit circle, moving clock-wise in a 56-year cycle, we have the following. The portion in blue represents a 22-year “upswing” in American history, a period of increasing radicalization. The portion in pink represents a 22-year “downswing” in American history, a period of increasing conservatism. The two portions in purple represent two 6-year “transition periods” lodged between the two opposing and longer periods of time. (Albers & Albers, 2011)
If these same amendments are placed in a sine wave based upon the same unit circle we have the following. One can plainly see that the most significant amendments, and the vast majority of amendments, are associated with the upswing of the sine curve.

![Diagram 5-5. 56-Year Kondratiev Sine Wave with Amendments in Upswing and Downswing Periods](image)

This chart may be better viewed placed on end, as set forth on the following page. Note that the up-swing amendments change the character of the American history, whereas the down-swing amendments generally enshrine a previously established set of values.

On the basis of these regularities I argue that a form of consciousness may be found in American economic history, one which is both mathematically demonstrable and important. In this book I present a model of economic and political growth based upon systematic addition.

We begin with a philosophic model of trade (pp. 32-44);

aggregate this model over the course of year to state the real Gross National Product of the United States and its relationship to the rate of employment (pp. 45-60);

aggregate this model over the course of many years to find the growth of the United States stated as a natural “14-year octave” within real GNP data (pp. 61-89);

multiply this octave times two to find the 28-year natural rate of price fluctuation (pp. 90-110); and

multiply this octave times four to find the 56-year natural rate of political change (p. 90-110).

The final model (pp. 111-134) is the larger “fractal” of the model of trade which begins these essays, in essence demonstrating that the United States “trades” values over a period of time in much the same way the individual citizen trades goods and services for money on a personal basis.
Diagram 5-5.
56-Year Kondratiev Sine Wave with Amendments in Upswing and Downswing Periods

1776 - Declaration of Independence
1789 - Federal Constitution
1791 - Bill of Rights
1795 - 11th Amendment - Sovereign Immunity
1804 - 12th Amendment - Election of President

1865 - 13th Amendment - Abolish slavery
1868 - 14th Amendment - Due process, Equal protection
1870 - 15th Amendment - Franchise for former slaves

1913 - 16th Amendment - Income Tax
1913 - 17th Amendment - Direct election of Senators
1919 - 18th Amendment - Prohibition of liquor
1920 - 19th Amendment - Franchise for women

1933 - 20th Amendment - Congressional terms
1933 - 21st Amendment - Repeal

1951 - 22nd Amendment - Two terms for President

1961 - 23rd Amendment - Electors for Washington D.C.
1964 - 24th Amendment - Eliminate poll tax
1971 - 26th Amendment - Franchise at 18th Birthday

1992 - 27th Amendment - Congressional salary
1.2 The Objective Determination of a “Crisis”

The first four essays of this work lay the intellectual foundation for an extended discussion in Essay Five of the “Political Economy Wave,” pictured below. The x-axis values represent 20,454 days within a 56-year cycle, beginning with April 9, 1805, 1861, 1917, 1973 and, prospectively, 2029. Positive y-axis values are associated with thoughts in favor of political stability and negative y-axis values are associated with thoughts in favor of political change. After a careful review of all possible intersections of the waves which make up this model, the following points have been found to represent predictable dates of “crises,” or fundamental change, in the understandings and behavior of citizens within the United States. These points are:

(A) the beginning point,
(B) the first peak, wherein positive y-values reach their greatest level,
(E) the first trough,
(H) the second peak,
(M) the point wherein the wave passes from positive y-values to negative values,
(P) the second trough, wherein negative y-values reach their lowest point and
(X) the point wherein the wave passes from negative y-values to positive without the introduction of a new cosine wave.

INTRODUCTORY DIAGRAM 4. THE "POLITICAL ECONOMY WAVE"
The pink rectangle above represents the year 2005, a date associated with previous financial political-economic crises coming in 56-year increments in 1781, 1837, 1893, 1949 and – prospectively – 2005. The vertical red line represents the election of November, 2008 at which point Senator Barack Obama was elected President of the United States in the midst of a global financial crisis.

In this essay I present seven predictions which may be associated with the seven points given above – A, B, E, H, M, P, and X – vis-à-vis their association with the cover of TIME Magazine. These magazine covers represent a public statement of pressing events which is completely independent of this approach.

The magazine’s interpretation of the event is not in question. Of interest to these essays are the dates associated with the event in question and the bold, unequivocal and “for profit” manner in which the event is stated, i.e. “on the cover.” These covers are linked to this essay for ready reference.

Also provided are proposed correlations for much earlier dates at the same point of in the Political Economy Wave. Links are provided to internet articles pertinent to the crisis described. Predictions are listed in order of proximity to the date of the publication of these essays.

One of the most striking things about this approach is the interesting association of TIME Magazine covers – each of which conveys a serious, pressing and dramatic turn of events – with the dates provided. A review of the magazine covers linked to these essays, as well as those covers immediately preceding and subsequent to the dates given, may be sufficient to confirm the interest and potential usefulness of this approach.
INTRODUCTORY DIAGRAM 4.
THE "POLITICAL ECONOMY WAVE"
Point P. The second trough.

This point will next be reached between December 8 to 12, 2014.

The most recent dates associated with this point are December 8 to 12, 1958.
TIME National Affairs Articles:
Rising racial tension in the South and the threat of nuclear war.
http://www.time.com/time/covers/0,16641,19581208,00.html

Interpretation of the Political Economy Wave: Point P introduces Americans to their future, ultimately leading to the wholesale change to which it is directly related at Point A.

Important dates near December 8-12, 2014 at 56-year intervals are:

December 14, 1790: Alexander Hamilton proposes a National Bank, a perennial issue since that time, made vastly more complicated by the Louisiana Purchase of 1803.
http://www.civil-liberties.com/cases/bank.html

December 6-7, 1846: The Battle of San Pasqual. American troops reach California per annexation agreement ending Mexican-American War, leading to the California Gold Rush and the strengthening of Yankee non-slave commerce throughout the North.

December 7, 1902: British and German ultimatum ignites the Venezuelan Crisis, eventually leading President Roosevelt to the Roosevelt Corollary to the Monroe Doctrine. American imperialism in Latin America is thus codified and leads to an ever more expansive international involvement by the United States and ultimately entry into World War I.
<table>
<thead>
<tr>
<th>Point X.</th>
<th>The point wherein the wave passes from negative y-values to positive without the introduction of a new cosine wave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This point will next be reached on <strong>September 13, 2028.</strong></td>
<td></td>
</tr>
<tr>
<td>The most recent date associated with this point is September 13, 1972.</td>
<td></td>
</tr>
<tr>
<td>Interpretation of the Political Economy Wave: Point X anticipates the tremendous change which arrives at Point A.</td>
<td></td>
</tr>
<tr>
<td>Important dates near September 13, 2028 at 56-year intervals are:</td>
<td></td>
</tr>
<tr>
<td>September 13, 1804: Lewis and Clark in Brule County, South Dakota, travelling west, yet not reaching the terminus of the available settlements; <a href="http://www.lewisanndclarktrail.com/section2/sdcities/Chamberlain/history1.htm">http://www.lewisanndclarktrail.com/section2/sdcities/Chamberlain/history1.htm</a></td>
<td></td>
</tr>
<tr>
<td>September 13, 1860: Senator John Crittenden of Kentucky, a border state, declares that he would stand for the union even if Lincoln won. His Crittenden Compromise of December 18, 1860 was unsuccessful in averting civil war. He returned home and worked to keep Kentucky in the union. Two of his sons would become generals on opposing sides of the war. <a href="http://www.civilwar-online.com/2010/09/september-13-1860-senator-john-j.html">http://www.civilwar-online.com/2010/09/september-13-1860-senator-john-j.html</a> <a href="http://sunsite.utk.edu/civil-war/critten.html">http://sunsite.utk.edu/civil-war/critten.html</a></td>
<td></td>
</tr>
<tr>
<td>September 15, 1916: First tank used in World War I, a new military technology which American troops would face upon their entry into France and the European Theatre. <a href="http://www.eyewitnessstohistory.com/tank.htm">http://www.eyewitnessstohistory.com/tank.htm</a></td>
<td></td>
</tr>
</tbody>
</table>
INTRODUCTORY DIAGRAM 4.
THE "POLITICAL ECONOMY WAVE"

Combined Graph with Tail

- Sine Function
- Cosine Function
- Sum Functions
- Cosine Tail

Time (Days)

Constitutional Amendments

Declaration of Independence

Federal Constitution
Point A. The beginning point.

This point will next be reached on April 9, 2029.

The most recent date associated with this point is April 9, 1973.
TIME Cover: High inflation rates sweep the nation.
http://www.time.com/time/covers/0,16641,19730409,00.html

Interpretation of the Political Economy Wave: Point A ends all previous associations with previous historic development and begins something entirely new, hitherto untried and unknown.

Important dates near April 9, 2029 at 56-year intervals are:

April 6, 1805: Lewis and Clark depart Fort Mandan, North Dakota: “We were now about to penetrate a country at least 2000 miles in width upon which the foot of civilized man had never trodden.” http://www.infoplease.com/t/hist/lewis-clark-journal/day329.html


Of the 71 Time Magazine covers between points A (April 9, 1973) and Point B (September 16, 1974) 37 covers – slightly over half – specifically refer to events involving the Watergate scandal. Included in this sequence of events were the resignation of Vice President Agnew on October 10, 1973, the appointment of Gerald Ford as Vice President on December 6, 1973, the resignation of President Nixon and the beginning of the presidency of Gerald Ford on August 9, 1974.
Point B. The first peak, wherein the y-values reach their greatest level.

This point will next be reached between **September 16-October 1, 2030.**

The most recent dates associated with this point are **Sept. 16 – Oct. 1, 1974.**

TIME Cover: President Ford pardons ex-President Nixon for his role in the Watergate scandal.
http://www.time.com/time/cover/0,16641,19740916,00.html
http://www.time.com/time/cover/0,16641,19740923,00.html
http://www.time.com/time/cover/0,16641,19740930,00.html
http://www.time.com/time/cover/0,16641,19741007,00.html

Interpretation of the Political Economy Wave: Point B resolves the intense conflict / difficulty running between Points A and B.

Important dates near September 16-October 1, 2030 at 56-year intervals are:

September 23, 1806: Lewis and Clark complete their journey, arriving in St. Louis.
http://www.lewisandclarktrail.com/section1/mocities/St.Charles/1806history1.htm

September 17, 1862: Battle of Antietam, Union victory in “the bloodiest single day in American military history” giving Lincoln the military success he desired to prelude the Emancipation Proclamation and forestall British and French recognition of the Confederacy.
http://en.wikipedia.org/wiki/Battle_of_Antietam

August 18-September 16, 1918: The last stage of the Second Battle of the Marne, the Oise-Aisne Offensive, first use of independent American forces in Europe at St. Mihiel September 12-16, 1918, with the Americans freeing the St. Mihiel salient on September 16, 1918 and “the beginning of the end of the Great War” with an armistice declared 100 days later.
http://en.wikipedia.org/wiki/United_States_campaigns_in_World_War_I
INTRODUCTORY DIAGRAM 4.
THE "POLITICAL ECONOMY WAVE"
Point E.  The first trough.
This point will next be reached between June 28 – July 20, 2041.
The most recent dates associated with this point are June 28 – July 20, 1985.

TIME Cover:  Hijacking / Terrorism of TWA Flight 847 and the return of 151 of 152 hostages.
http://www.time.com/time/cover/0,16641,19850624,00.html
http://www.time.com/time/cover/0,16641,19850701,00.html
http://www.time.com/time/cover/0,16641,19850708,00.html
http://www.time.com/time/cover/0,16641,19850715,00.html

Interpretation of the Political Economy Wave:  Point E resolves long-standing prior difficulties with success.

Important dates near June 28 – July 20, 2041 at 56-year intervals are:


July 8, 1873:  the Modoc War, last of the Indian Wars in California and Oregon, resolves for California ongoing Indian conflicts ending with convictions and the death sentences of the Indians involved on July 8, 1873. http://en.wikipedia.org/wiki/Modoc_War

July 24, 1929:  the Kellogg-Briand Pact comes into effect, whereby signatory states promised not to use war to resolve "disputes or conflicts of whatever nature or of whatever origin they may be, which may arise among them." Parties failing to abide by this promise "should be denied the benefits furnished by this treaty.” This treaty resolves the legality of war for territorial aggrandizement, a law still binding law in the United States. http://en.wikipedia.org/wiki/Kellogg%E2%80%93Briand_Pact
Point H. The second peak. This point will next be reached between April 4 – April 14, 2050. The most recent dates associated with this point are April 4 – April 14, 1994.

http://www.time.com/time/covers/0,16641,19940404,00.html
http://www.time.com/time/covers/0,16641,19940411,00.html

Interpretation of the Political Economy Wave: Point H raises difficult international issues left unresolved, presaging troubles in the near future.

Important dates near April 4 – April 14 2050 at 56-year intervals are:

April 12, 1770: in light of the Boston Massacre of the previous month (March 5), Parliament repeals all duties imposed by the earlier Townshend Act except that on tea (April 12), thereby permitting further colonial dissent from English policies; http://en.wikipedia.org/wiki/Townshend_Acts

April 1, 1826: Samuel Morey, an American inventor, patents the first internal combustion engine, a central part of the modern economy. http://kinnexions.com/smlsource/samuel.htm

April 13, 1882: Anti-Semitic League formed in Prussia, a society dedicated to the expulsion of all Jews in Europe, raises the question of its possible success and under what terms; http://quod.lib.umich.edu/m/moajrn1/bac8387.0033.193/135:17?rgn=full+text;view=image

April 10, 1938: Plebiscite in Austria approves Anschluss by Germany by 99%, leading to the expansion of Nazi Germany on a wave of public enthusiasm. http://www.otr.com/Linz.html
INTRODUCTORY DIAGRAM 4.  
THE "POLITICAL ECONOMY WAVE"
Point M. The point wherein the wave passes from positive y-values to negative values. This point will next be reached on **February 21, 2059**. The most recent date associated with this point is February 21, 2003.

**TIME Cover:** President G. W. Bush’s effort to invade Iraq.  
[http://www.time.com/time/cover/0,16641,20030224,00.html](http://www.time.com/time/cover/0,16641,20030224,00.html)

Interpretation of the Political Economy Wave: Point M enormously and controversially expands the idea of American democracy and its applicability to new groups of people.

Important dates near February 22, 2059 at 56-year intervals are:

- **February 25, 1779:** Gen. G. W. Clark captures the British fort at Vincennes, Indiana, thereby doubling the colonial geographic area of the United States.  

- **February, 1835:** “Democracy in American” Volume I by Alexis de Tocqueville is published, a work still used to analyze American character and civic identity in the United States.  
  [http://www.loc.gov/loc/lcib/9712/kammen.html](http://www.loc.gov/loc/lcib/9712/kammen.html)

- **February 22-25, 1891:** First meeting of National Council of Women of the United States held in Washington, D. C. leading to the 19th (prohibition) and 20th (suffrage) Amendments.  

- **February 17, 1947:** The Voice of America begins broadcasts into Eastern Europe and the USSR.  
Conclusion

This model is derived entirely from an analysis of published data. It is intended as an empirical statement of the manner in which the political economy of the United States evolves over time.

The methodology presented in these essays has not been used previously in studies of economics. The economy of the United States is the sole topic herein inasmuch as:

1. the United States has not suffered from the invasions and border reductions which have typified virtually all other countries available for consideration, thereby permitting an equivalence between the data generated and the subject studied,

2. the economic data pertaining to the United States is long-standing, precise, self-consistent, authoritative and easily available and

3. the combination of a single political sovereignty with the right to tax, a national legal jurisdiction of arbitrary finality and a monetary / fiscal policy orchestrated by a single government have been central characteristics of the economic history of the United States from at least 1868.

In sum, this model singles out certain dates as carrying the weight of historic change. Not unlike a vibrating string wherein the tension of the string itself actually defines its own points of maximum stress, so do these conjoined waves appear to create moments of vital change in the body politic of the United States.

The purpose of this model is to demonstrate the periodicity of American economic and social history, and to provide a basis for its understanding in scientific terms.
INTRODUCTION: WHAT IS A “CRISIS”?

As used in these essays, the term “crisis” refers to a mathematically determined and yet fundamental date of change which forever alters all subsequent understanding of events. I propose to predict crucial dates of social and economic change using a mathematic model of nested waves repeated at five separate levels of society and as derived entirely from data on annual real GNP, prices and employment of the United States.

The model presented here is of five concentric unit circles which collectively provide patterns of economic and social development in the United States generated by trigonometric functions. In this model each “larger” radius states an aggregate of the preceding and smaller unit circles. Using this model I propose that it is possible to predict exactly when and what sort of crises are likely to occur within the economic and social structure of the United States for specific dates far distant in the future.

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4 Trigonometric ratios are derived from the “unit circle,” a mathematic device whereby the center point of a circle is placed at the origin (the “x y intercept”) of a Cartesian coordinate graph; an arbitrary value of “1” is given to the radius of a circle commensurate with the value “1” on the graph; and the measurement of the circumference of the circle of $2\pi$ is then charted on the graph. An infinite number of right triangles are circumscribed within this circle, taking the radius of the circle as a hypotenuse of a right triangle; and the placement of the right angle along the x-axis always moving in accord with a y-axis variable as located by the circumference of the circle. The Pythagorean Theorem states that the square of one side of a right triangle (the x value) plus the square of the second side (the y value) shall equal the square of the hypotenuse (the radius). Calculations from this arrangement are made systematic by the fact that the “radius / hypotenuse” of all triangles created is “1” and that the square of “1” is “1.” The study of these angles and the proportions of these sides is known as “trigonometry.”

5 Although the radius of the “unit circle” of trigonometry never varies from the ratio of 1 : $\pi$ at any point, the word “larger” here is meant simply as a demonstrative aid. From a strictly pedagogical point of view, this model proposes that the “larger” logical relationships within society are an aggregate of congruent “smaller” dimensions within the same society.

6 The “clock-wise” direction of movement around the unit circle and the “9:00 o’clock” place of beginning the analysis as used in these essays are opposite that taken in most trigonometry textbooks.
1.1 A brief description of these essays

1. The first essay introduces the “chooser – available choice” model. The “chooser – available choice” model is the initial “fractal” at the core of the entire system. The first unit circle (Introductory Diagram 3, in yellow) uses dichotomies of economic “actions” (“trading” vs. “keeping,” along the x-axis) and economic “thoughts” (y-axis) to state the essential elements underlying the simplest economic transaction. The radius of the unit circle represents the choice of the consumer as faced with ½ the circumference of the circle, the choices available. This “chooser – available choice” model is the underlying basis for the remaining “unit circles.”

This approach does not alter the trigonometric identities considered in the slightest and provides an approach to the measurement of time which is consistent with the sense of the hands of a clock.
2. The aggregate of all economic choices over the course of a single year is stated by the annual Gross National Product of the United States. The second essay aggregates the “chooser – available choice” model over the course of a year to derive Okun’s Law, a fundamental proportion of economics. (Introductory Diagram 3, in green)

The second essay presents Okun’s Law as the function of a ring torus, a three dimensional figure resembling an inner tube wherein a smaller circle (with radius “r” for “Minor Radius” representing the choices made by consumers) is swept around the axis of a larger circle (with radius “R” for “Major Radius” representing the annual rate of employment). A $1 : \pi$ relationship between the change in employment and change in annual GNP results, a figure strikingly consistent with the data available as to the economy of the United States.
3. The third essay aggregates these annual figures for real GNP through the use of the “GNP ratio,” i.e. the quotient of one year’s real GNP as divided by the real GNP of an earlier year. Using two different but compatible techniques we discover two interesting patterns presented by these ratios at different “spreads” of years, i.e. the time interval separating the numerator and denominator of the ratio itself.

The first of these is the “14-year octave.” (Introductory Diagram 3, in pink) This technique provides an unmistakable sense that a form of musical harmony governs the rate of production in the United States.

The second pattern of obvious interest is a pentagonal figure associated with the 11-year spread and an unusual pattern of dissonance. These two discoveries are described in more detail in the essay itself.
4. The fourth essay aggregates the 14-year octave in two larger octaves building upon it, i.e. the 28-year and 56-year octaves. These are presented as the largest of the unit circles (Introductory Diagram 3, in red and blue).

The fourth unit circle (in red) describes a damping cosine wave of prices over periods of 28-years, twice that of the 14-year octave.

The fifth unit circle (in blue) describes a 56-year sine wave of socio-political change, the “Kondratiev Wave,” at twice the length of the price wave.

The fourth essay proposes that these octaves work together to create a relationship between the economy of the United States and the Golden Mean, an ancient and important mathematic ratio of 1 : 1.6180… (given as the symbol φ, Albers & Albers, 2011) A 14-year period of economic development commensurate with the Golden Mean requires a steady-state rate of growth of exactly 3.4969 percent per year, a rate which matches quite well the available data. In this way the economic past is connected to the economic present as well as the economic future.

An appreciation of the role played by both π (essay two) and φ (essay four) within the economic statistics of the United States is essential to an understanding of economics and social dynamics.
5. The fifth essay proposes that the “Chooser – Available Choice” model, given for the decisions of a single person, is copied in the economy of the United States as a larger fractal of historic development.

To demonstrate this, the fifth essay adds together the 28-year “economy wave” (damping cosine curve) and a 56-year “political wave” or “Kondratiev Wave” into the 56-year Political Economy Wave by charting a 56-year (20,454 day) period of changes beginning on April 9, 1805, 1861, 1917 and 1973. The equation for this combined wave is the following:

\[ g(y) = 2 \left( \frac{1}{2} - \frac{2 \left( y + \frac{1}{4} \pi \right)}{\pi} \right) \sin \left( 2y + \frac{1}{2} \pi \right) + \sin(y) \]
Using the same color scheme given for the previous models we will examine in these essays the creation of the following mathematic wave running through American economic and social history.

![Combined Graph with Tail](image)

Through an understanding of this wave and its operation we may propose historic analogies and predict crises in accord with these analogies. These predictions may then be tested against events as they unfold.
Five Essays on the Mathematic Prediction of Economic and Social Crises

Essay 1:
The “Chooser – Available Choice” Model

1. Hypothesis

I propose that social law is the larger fractal of the human mind and that society is the set of fractals created by this fact. We begin with a discussion of the central core of this model, the “Chooser- Available Choice” model.

2. Methods

I introduce the Cartesian coordinate system to establish a plane between that which is experienced by human beings (the x-axis) and that which is known to human beings (the y-axis). Taking the knowledge of something as the exact equal of the experience of it, we propose an infinite set of squares representing the merger of these two parts of human existence in a simple geometric shape.

The jury trial is proposed as the human mind writ large. From this social fact a “unit circle” may be developed based upon a geometric graphing of “actions” (the x-axis) and “thoughts” (the y-axis) regarding trading vs. keeping a piece of property in the simplest possible exchange of goods.

3. Data

I present here a strictly philosophic argument underlying the rest of the essays.
4. Procedure

4.1 The Pythagorean Theorem in relation to Actions and Thoughts

Every jury trial in the United States begins with the proposition that all which is necessary for the jury to know for its decision will be presented to it as a part of the evidence in the trial, and that which is irrelevant will be kept away. In this fashion the jury’s personal experience of the evidence as presented in trial is contrasted with the decisions they make collectively as a result of their deliberations on that evidence.

We may build upon this social fact in the following manner.

For the purposes of these essays we will take as an axiomatic truth that all human life is based upon the presumed equivalence between that which we experience through the senses and that which we know to be real. If “that which we experience” is given the variable “X” and “that which we know to be real” is given the variable “Y”, we may state this equivalence as:

\[ X = Y. \]

If we place this equation in a Cartesian coordinate system, we have the following 45 degree angle line, beginning at \( x = 0, y = 0 \) and extending on toward and infinite number of associations.

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7 See Boswell, J. (1820). “After we came out of the church, we stood talking for some time together of Bishop Berkeley's ingenious sophistry to prove the nonexistence of matter, and that every thing in the universe is merely ideal. I observed, that though we are satisfied his doctrine is not true, it is impossible to refute it. I never shall forget the alacrity with which (Samuel) Johnson answered, striking his foot with mighty force against a large stone, till he rebounded from it -- "I refute it thus."

One might assert that the experience of reading a book and enjoying the imaginary world conveyed is not the same as “experiencing” or “knowing” anything about the world imagined.

Our point here is far more modest and direct. The “experience” referred in this essay is simply that of “reading the book” and the knowledge considered is simply that the person reading knows that he or she is reading a book. The equivalence understood between the experience of reading the book, and the knowledge that one is reading a book, is the equivalence with which we begin this analysis.
This picture represents the outcome of an infinite number of squares, wherein each corner point has a specific meaning. “X” represents our experience of something, “Y” represents our knowledge of the thing experienced, the point “(X, Y)” represents the interaction between our experience of the thing itself and our knowledge of the thing itself, and the origin of the graph “(0, 0)” represents the beginning association we make between experience and knowledge as fundamental assumptions of all inquiry.

![Diagram 1-3. Trigonometry of Experience and Knowledge](image)

### 4.2 Extension to the Jury Trial of a Criminal Case

In the United States the jury trial of a case is premised on this same equation “X = Y,” “experience” and “knowledge,” taken to the next higher social level of the jury. The jury’s reception and consideration of the evidence presented indicates that this small group is the expansion of the smaller individual and included minds. In the jury’s deliberation the jury demonstrates itself as being the larger, expanded, copied and congruent larger “fractal” of the individual mind.

This expansion has significant mathematic consequences.

As indicated at the outset, every jury trial in the United States begins with the proposition that all which is necessary for the jury to know for its decision will be presented to it as a part of the trial, and that which is irrelevant will be kept away. In this fashion the jury’s personal experience of the evidence as presented in trial represents the “X” of a trial proceeding.

The jury’s evaluation of this evidence as understood through the prism of their own life experiences is the “Y” of the trial proceeding, their collective knowledge of the facts presented.

The final verdict given by the jury states its evaluation of the association between the “X” of the trial (the evidence presented) with the “Y” of the trial (the jury’s evaluation of this evidence).
This simple model may be expanded upon.

The criminal law of the United States is based upon a dichotomy between the criminal act alleged to have been committed – the *actus reus* of the offense – and the mental intent – the *mens rea* of the offense – associated with the crime. For example, the act of killing someone is a homicide if done with the intent to kill the individual. If the killing was the result of recklessly driving in a crowded street, the crime is less because the evil of the intent to harm was less. Differences in the consequence to the Defendant can be quite significant, depending upon the nature of the criminal act and mental intent found by the jury.

If we let the “actus reus” of any given offense equal a particular number – for example, 5 – then the jury’s experience with the evidence presented as to the criminal act (X = 5) and the jury’s understanding of that evidence (Y = 5) may be given as a square, in blue below.

Similarly, if we let the “mens rea” of the same offense equal a different number – for example, 3 – then the jury’s experience with the evidence presented as to mental intent (X = 3) and the jury’s understanding of that evidence (Y = 3) may be given as a red square, in red below.
The culpability, if any, of the Defendant for a crime is given in accordance with the sum of these two elements of proof. The full experience and knowledge summarized by the case will equal the sum of these two squares. Stating the jury’s experience with the evidence of a criminal act as a positive distance “A” and the jury’s experience with the evidence of mental intent as a positive distance “B”, then the experience / knowledge represented by Culpability (C) associated with the verdict should equal the sum of these two things, or:

\[ A^2 + B^2 = C^2 \]

Geometrically, this equation may be portrayed with the proportions of the Pythagorean Theorem as follows.

The use of Euclidean Geometry as a basis for understanding social relationships leads to a form of trigonometry which describes the wave characteristics explored in later essays. In this essay we present that trigonometry, a trigonometry of time.
4.3 Neo-Classical Economics and its Relationship to Our Approach

We may explore the impact of this approach through the indifference curve of neo-classical economics. (Shapiro, 2012) These curves attempt to demonstrate the microeconomic relationship between consumer preferences as balanced between two different goods. The curve drawn represents the “indifference” for any consumer as to which combination of good is offered. (left, Diag. 1-14)

The “indifference curves” generated from this pair of dichotomies represents the willingness to trade one set of goods for different goods. As increasing levels of affluence at provided, a map of multiple curves becomes possible. (center Diag. 1-14) The indifference curves of two competing trading partners may be explored by inverting the curve of one of the partners. (right, Diag. 1-14) “Pareto optimality” refers to the qualitative evaluation of these relationships. Given an initial allocation of goods among a set of individuals, a change to a different allocation that makes at least one individual better off without making any other individual worse off is called a Pareto improvement. An allocation is defined as "Pareto efficient" or "Pareto optimal" when no further Pareto improvements can be made.

The thought process which pertains to a jury trial may be joined to our modern views of economics to create a foundation for our approach. We begin with the fact that American Society is governed by laws which state the rights of persons and parties within the jurisdiction of the United States and its member states. The right to a jury is guaranteed to every party before the courts, thereby permitting the party to have judgment declared by persons not directly connected with the political process. Empirically testable hypotheses may be structured upon these relationships as lawfully required.
From the economic point of view, there is no difference between stating that “John purchased x” and “John is guilty of purchasing x.” The relationship between the act and the thought which motivates the act, speaking economically, is the same as that of the court considering such an act criminally.

4.4 Extension to Micro-economics - The “Chooser – Available Choice” Model

Each of the points within the plane of an indifference curve – both those on the curves and those outside the curve – represents a given decision to trade or to keep various properties. If we contrast the actions of trading a good versus keeping that same good, a set of dichotomies may be constructed which maybe used to structure our understanding of economic development.

The first dichotomy – action, as comparable to the “actus reus” of criminal law – represents a tension between “Keeping” a particular good vs. “Trading” the good for something else. This is indicated in the circle below by the opposition of “Keep” at 3 o’clock and “Trade” at 9 o’clock. All economic life stems from the core principle that one may act freely in choosing either to keep a given property or to trade it for some other piece of property and that these transactions clearly affect the status of the property so owned or traded.
This is contrasted with a secondary dichotomy – thoughts, as comparable to the “mens rea” of criminal law – which represents a tension between one’s mental “thoughts in favor of keeping” and “thoughts in favor of trading” a particular property, located at 12 o’clock and 6 o’clock respectively in the circle below. These are the mental pre-dispositions of every owner towards keeping or trading a given piece of property for something else.

Using the Pythagorean Theorem to structure the sum total of possible permutations between the “Action” aspect of a purchase, and the “Thought” aspect of a decision to Purchase, we may structure every possible balancing of these two with the “Purchase” itself.
The Pythagorean relationships inherent in the association of Action and Thought as expressed previously create around the unit circle create an infinite set of mathematic relationships wherein the actual possibility of a Purchase is set as the sum of some combination of Action and Thought.

The unity of the underlying ego which selects these various points may be associated with the radius of this circle. If we give this radius the number “1” it represents the “unity” of the ego as a balancing radius between these two dichotomies of Action (“Trading” vs. Keeping”) and Thoughts (“Thoughts related to Trading the property,” “Thoughts related to Keeping the property”). An internal angle is thus constructed at the origin of the coordinate system.
4.5 Waves Arising From The “Chooser – Available Choice” Model

If we consider the side opposite the internal angle as divided by the hypotenuse of “1” we set up a set of fractions which may be charted against an x-axis wherein the circumference of the circle is superimposed upon the x-axis in divisions associated with $2\pi$. Beginning at 9 o’clock and moving clockwise, we have the following mathematic associations between various points along the unit circle, to wit, the sine curve.

The equation for this wave is:

$$g(y) = \sin(y)$$

There is only one point along the Unit Circle where Action is wholly aligned with Trading, i.e. the point at 9:00. All other points along the unit circle are similar to one another in that there is some “Y” component connected to some mental aspect of trading and/ or keeping the object in question. This mental aspect must include some possibility of cancelling the action contemplated. Consequently only at 9:00 o’clock is the possibility of a “Trade” wholly equivalent with Action, and at this point “Thought” is Zero and the Action Trading occurs.

Conversely at 3:00 o’clock the action undertaken is to “Keep” the property in question and the status quo is actively continued.
The unique aspect of this point at 9:00 o’clock creates an unavoidable change in the overall unit circle. The break which is presented at \((x = -1, y = 0)\) creates a new and unknown element in the unit circle itself. Once the trade is made, the situation is no longer the way it was. Something new has taken place.  

In contrast, when the x-axis is directed toward “Keeping” a particular good, the point at which Thought = 0 will be that point most dedicated in favor of the status quo.

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**Diagram 1-14. New Circle on Trade.**

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8 There is an analogy here to quantum mechanics in the “Schrodinger’s Cat Thought Experiment.” The second half of the third postulate of quantum mechanics states, roughly speaking, that observation changes the physical system. [http://vergil.chemistry.gatech.edu/notes/quantrev/node20.html](http://vergil.chemistry.gatech.edu/notes/quantrev/node20.html) A physical system exists in as many state as possible until it is observed. Once the observation has been made, it changes into another state, one which can be unique or not.

Until one opens the box, the cat is both dead and alive. Opening the box (observing the state of the cat), indicates which state it is, and so changes the state of the physical system. In this essay, trading equates with the observation. By analogy, stating that with trade “something new has happened” one would indicate that the wave function describing the state of the cat has changed.
The model will be referred to as the “chooser – available choice” model, as a way of presenting the unit circle and its radius of “1” – representing the “chooser” – and the number \( \pi \) – representing the “choices available” – in a simple and direct fashion. Our premise is that a radius originating at the center of the unit circle and moving toward any spot on the circle of possible choices divides the circle at a \( 1 : \pi \) ratio. Half of the circle constitutes “available choices” which will be associated with the point at which the radius and the circle intersect. This relationship will exclude an equivalent set of opposite choices on the opposing side of the circle.

In other words, one can not simultaneous trade a good and keep the same good, or vice versa. The possible choices which are available toward any particular goal are those which are not directly undermining of whatever goal is chosen. The choices which are not available are those which are in some negative value, or opposite position, from this chosen goal. This same dynamic applies to any point of psychological consideration along the unit circle.
CONCLUSION

We conclude that it is possible to construct a simple and mathematically straight-forward model of micro-economic choices which is completely in accord with the available evidence of social behavior as evidenced by universal and legally required social understandings.

By drafting the experience and knowledge of a jury as the larger “fractal” of the individual mind, we have the ability to state a pattern of “mind” itself which is both useful and concrete in its form.
Five Essays On the Mathematic Prediction Of Economic and Social Crises

Essay 2: Does Okun’s Law State A “π : 1” Relationship?

INTRODUCTION: WHAT IS “OKUN’S LAW”?

In the previous essay we discussed the first, central “unit circle” relating to the “chooser – available choice” model. We come now to the aggregate of this model, that “unit circle” wherein microeconomic decisions to trade or keep a given thing aggregate to form a year’s worth of such decisions. This aggregate is known as “annual real Gross National Product.” (Diagram below, in orange)

The Gross National Product of any country, in association its rate of employment, are two of the most important variables in economics.

The empirical relationship between production and employment in the United States is known to mainstream economics as “Okun’s Law.” “Okun’s Law” states that for every three percentage points of increase in real GNP the rate of employment will increase by one percentage point, and that decreases of both will take place in the same proportion. This 3 : 1 proportion is generally referred to using a double negative, i.e. an increase of three percent in real GNP will lead to a one percent decrease in the rate of unemployment. First stated by Arthur Okun, at the time senior economist of President Kennedy’s Council of Economic Advisors, the rule is “one of the most reliable empirical regularities in macroeconomics.” (Tobin, 1983)
The article “How Useful Is Okun’s Law?” by Dr. Edward Knotek, a senior researcher for the Kansas City Branch of the Federal Reserve, was published in the bank’s research journal Economic Review in the Fourth Quarter 2007. The data sets used in this essay cover a 60 year period of American economic history, i.e. the second quarter of 1947 through the third quarter of 2007.

In this article Dr. Knotek provides an overview of the ideas underlying Okun’s Law as well as the economic measurements which support it. According to this article at the present time Okun’s Law is an unexplained statistical regularity which must be considered more a “rule of thumb” rather than a formal law.

Our purpose in this essay is (1) to lay a theoretic foundation for an understanding of the regularity of Okun’s Law, as well as (2) to present our proof that the actual mathematic relationship between changes in the rate of unemployment and the rate of GNP growth is not a 3 : 1 relationship but a π : 1 relationship.

This essay will concentrate on three charts from Knotek 2007. Charts One and Two graph the quarterly and annual data sets supporting the regularity of the relationship between changes in the size of real GNP (x-axis) and the corresponding effect this has on the rate of employment (y-axis).

Dr. Knotek argues that the stability of the trend lines shown above masks the underlying dynamics of these relationships. Consequently Chart 3 also will be discussed (in Essay 5). He states:

One problem with a long time series – such as from 1948 to 2007 – is that history can hide changes in relationships. This is the case for Okun’s law. The previous section (Charts 1 and 2) found considerable similarities between Okun’s original estimate and an updated regression using a longer time series. This section shows that, when estimated over shorter time horizons, the relationship between changes in the unemployment rate and real output growth has varied considerably.

To capture this variation, this article uses a technique called rolling regressions. A rolling regression estimates a particular relation over many
different sample periods. Each regression produces a set of estimated coefficients. If the relationship is stable over time, then the estimated coefficients should be relatively similar from one regression to the next. Variations in the relation will appear as sizeable movements in the estimated coefficients.

![Diagram 2-3. Chart Three of "How Useful Is Okun's Law?"]

We seek here to augment this opinion with information from these essays.

**ECONOMIC METHODOLOGY**

1. **Hypothesis**

   United States annual real GNP may be represented geometrically as a torus the radius of which represents the rate of employment; one half the circumference of which represents the chronologic creation of real GNP over the course of a calendar year; the opposing half circumference of which represents the money price of each transaction making up that real GNP; and the cylindrical radius of which is set equal to the GNP per capita of the United States for the year in question. Using this model the relationship between an increase in the rate of employment and the size of real GNP is not 1: 3 but rather 1: \( \pi \).

2. **Method**

   We provide a description of the econometric measurements which may be expected to develop from the previous essays surrounding the relationship between economic choice, employment and real GNP. We then review the data sets used in Knotek 2007.
We conclude that there is strong support for the proposition that the 3:1 ratio presented by Okun’s Law is in reality a $\pi : 1$ ratio. We further suggest that the regularity of Okun’s Law and its central place in macroeconomics is due to the trigonometric relationship which this $\pi : 1$ ratio has on the entire spectrum of economic study.

3. Data

This essay is an extended comment on the measurements provided by Dr. Edward Knotek in his article “How Useful Is Okun’s Law?” published by the Federal Reserve of Kansas City in their publication Economic Review. The employment and GNP data used are those of the article has kindly been supplied by Dr. Knotek, and this has been cross-referenced with the original data kept by the United States Government.

4. Procedure.

4.1 U. S. Real Gross National Product:
The “Chooser - Available Choice” model in aggregate

The “chooser – available choice” model is the central point of departure for this model. If we invert this model such that the willingness to “trade” of one person meets the willingness to “trade” of a trading partner, we have a connection between two people indicating a mutual willingness to exchange goods or services with one another. (See discussion of Pareto efficiency in Essay One, lines 503-509) The willingness and ability of persons to trade goods and their services with one another is the foundation for the entire economy.

Let us begin with a proposed willingness of Farmer Jones to part with two cows in return for three horses. This willingness is met by Farmer Smith who is willing to trade three specific horses which he owns in return for two specific cows belonging to Farmer Jones.

The fact that these two farmers have met with a match which in their minds is favorable to both is indicated by the fact that both have extended the 9:00 axis “Action : Trade” towards one another. As a result of this trade, Farmer Jones’ two cows will be handed over to Farmer Smith, and Farmer Smith’s three horses will be handed over to Farmer Jones.
The following two circles simplify the basic ideas going into the above trade. Note that the early barter of horses for cows suggested by the circles below depicts trading at its most elementary level. Note that the trade itself must in some fashion state an improvement in the lives of the trading partners. Consequently the act of trading makes more efficient and useful the sum total of property within society because those who own the property are seeking ever more agreeable collections of that property by trading what they have for things which they desire but do not possess.

These trades represent a re-arrangement of property amongst those owning property. There is no “expansion” of the economy based upon this trade. However the usefulness of the property exchanged, in combination with the improved efficiency brought about by the trade, suggests that the natural rate of increase in any biologic organism – a farm, a household, a local market – will result from the full set of trades engaged in by all persons.

In short, the same property and the same traders exist after as well as before the trade. However the straight forward exchange of one set of property for another is conveyed by the model above.

There is no limit to the number of such trades which can be done over the course of any particular period of time. We may imagine two pipes running parallel, each suggesting the desire of one of two trading partners to enter into trade. Each trade may be listed in chronologic order and depicted as below.

As reliable currency enters into circulation persons engaged in trading have the further ability to makes trades of much greater complexity that a straight-forward barter. By saving the money obtained from prior trades people are able to amass a trading ability to trade which far exceeds the more clumsy and complicated trade of physical objects, herds of cattle or flocks of geese, etc.
The ability to trade goods and services for currency permits the evaluation of the worth of the trade itself in relative terms vis-à-vis all other trades, however subjective. A trade of $50 might represent an acre of land, a pair of mules, a suit of fine clothes or a suite of furniture. By “mirroring” the value of these various goods (or services), currency permits a much broader extent of trading and trading partners.

The pastel coloration below of the thing traded – money – is available to give a relative value to all the trades of an economy. These “trades” now become “sales,” i.e. the surrender of something in return for currency.

The chronology of the trade is given by the difference in color, the red trade being first, the yellow being second, the green third, the orange fourth, etc. The pastel coloration indicates that in this case Farmer Jones did not trade goods for goods but rather money for goods (or services).

The size of the trade in question, its monetary value, is indicated by the number of circles used. For example Farmer Smith’s trade of goods or services for money (three green circles) is three times as valuable in monetary terms as Farmer Brown’s trade of goods and serves for money (one red circle), Farmer Frederick’s trade of goods or services for money (one yellow circle) and Farmer Armstrong’s trade of goods or services for money (one orange circle).
If we set an arbitrary division of the stream of trade at a single 365-day year, we can place the monetary and the “real” aspects of these sales of goods and services as oppositions from one another. The result is a circle of such sales. The length of half the circle indicates the monetary value of each of the sales of goods or services included in the year. If the size of these transactions is copied into the length of the circuit itself, we have the following. Because the connection of any particular sale of a good or service to the year “1973” is no greater than any other trade, we draw here a circle, i.e. that geometric construct in which all points in a plane lie equidistant from a single point.

Diagram 2-8.
United States Annual Real GNP

Diagram showing the circle of sales with Farmer Brown, Farmer Fredericks, Farmer Smith, and Farmer Armstrong connected to the year 1973. The circle illustrates the monetary price paid for goods or services purchased and the goods or service included in annual real GNP.
The development of currency and its association with trade given above suggests that the “employment” necessary for Farmer Smith or Farmer Jones to possess trade-able items has now become the “employment” of Farmer Smith and Farmer Jones as engaged in these occupations. In this fashion the use of currency which has turned “trades” into “sales” is in a direct relationship to the rate of employment, i.e. that employment necessary to sustain the full scope of sales given above.

In the above diagram 2-8 we have used the GNP per capita of the United States as a radius “r” of the generating circle and the rate of unemployment as the radius “R” of the generating torus which swings the smaller circle in an arc around the center point “1973.”

If this relationship is stated geometrically, it would appear necessary that an increase in the rate of employment from one year to the next (R = the radius of the circle = 1) will correlate geometrically to a necessary increase in the size of GNP (Y = half circumference = \(\pi\)) at the necessary ratio of 1 : \(\pi\), as follows.

---

9 The 2010 real GNP for the United States was $2.27 trillion dollars in 1958 dollars with a population in the same year of 308,745,538 residents, for a GNP per capita of $7,355 per resident in 1958 prices. (See Essay Three, Data Set One, for figures as to real GNP. See 2010 Census for population figures.)

One might picture the relative size of these relationships by noting that if GNP per capita was set as the one inch radius of a pipe and the length of pipe set equal to U.S. real GNP, the pipe would run 406 miles (25,728,794 inches), roughly the distance from Chicago to Kansas City. To bend this pipe into the shape of half a circle would require a radius of 129 miles, roughly the distance from Washington D.C. to Philadelphia.

These proportions might be taken on a smaller scale. If a length of string representing 2010 real GNP was set equal to the length of a football field (3600 inches), the equivalent proportional thickness of the string would measure 0.00014 inches in a radial thickness. Spider silk measurements vary from 0.00012 to 0.00032 inches in diameter. The radius would run from the goal line to the 31.8 yard line.
4.2. Data Survey – Expected Findings

Let us examine how these relationships must be found in the available data.

1. The selection of a chain of economic transactions as a subset of an entire chain of such transactions is an action arbitrary in nature.

   ![Diagram 2-10. Chronologic Chain of Transactions]

2. The above points simply indicate an ongoing flow of transactions over time. If we denominate a certain subset of these transactions as occurring in “1973”, we draw an arbitrary distinction between them. This is the arbitrary and imposed similarity of a temporal association between these transactions as occurring within the time span of “1973.”

   ![Diagram 2-11. Selection of Transactions within "1973"]
3. As we associate these transactions with a period of time, none are more or less connected to 1973 than any of the others. Consequently there is an equivalence between all points listed as equidistant from a single point. This is conveyed by a diagram of the radius of \( \frac{1}{2} \) a circle (in blue), the other \( \frac{1}{2} \) being the monetary value of the particular transaction (in red). The relative value of these transactions vis-a-vis all other transactions is given by the length of the circumference taken up. A larger transaction occupies a larger portion of the circumference, smaller transactions occupy a lesser portion of the circumference.

![Diagram 2-12: Chronologic Set of Transactions](image)

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4.3 Necessary Linear Relationships

4. The relationship which we are anticipating is that a 1: π relationship will exist between a change in the rate of unemployment and the percent change in growth of GNP. As the rate of growth increases on the x-axis, the rate of unemployment will go down on the y-axis. Setting this relationship as a straight-forward linear relationship, we have the following.

![Diagram 2-13](image)

5. In order to establish a Golden Mean proportion in the economy the United States must possess a steady state rate of growth of approximately 3.4969% per year (see Essay Three and Four). As these associations are made with a new steady state rate of growth of 3.4969% per year, the new y-axis intercept is not “1” but $3.4969 / \pi = 1.1131227$.

![Diagram 2-14](image)
4.4 Knotek’s Presentation

6. We now review the econometrics presented in Knotek 2007.

Chart One (Knotek 2007) uses quarterly growth data which has been annualized. However quarterly employment data is not annualized.

7. We adjust the trend line for annualized quarterly data by multiplying quarterly employment data by four, thereby “annualizing” quarterly employment data. We thereby match annualized quarterly data on growth with “annualized” quarterly data on employment.

Throughout this essay the phrase “observed fully annualized quarterly trend line” will represent the trend line in Chart One wherein the data for quarterly employment data has been multiplied by four.
4.5 Relationship between Knotek 2007 and the “Chooser – Available Choice” Model

8. Charts One and Two of Knotek’s paper, below, provide the statistical trend lines of both quarterly and annual measurements of Okun’s Law, as follows.

9. Note that the slopes of the “observed annual” and “observed fully annualized quarterly” trend lines are 19.29 degrees and 15.136 degrees respectively.

The “chooser- available choice” model makes clear that the relationship between the radius and 1/2 the circumference of a circle is an important trigonometric identity to consider in relation to economic growth. The area of rectangle, $\pi \times 1$, figured through such an association is as follows, with a bisecting angle of 17.66 degrees.

The slope of the angle bisecting the angles given in Charts One and Two is 17.213 degrees, less than half a degree from the slope of 17.66 degrees of the “chooser – available choice” model.
10. The “Annual” y-intercept given in Knotek 2007 as 1.2091387. The “4 x Quarterly” intercept is 0.92376. The multiple of these two intercepts is 1.1169539.

If the steady state rate given for the Golden Mean proportion (Essay Four, median average growth rate of 3.4969) is divided by \( \pi \), the y-axis intercept is 1.1131227. The extraordinary proximity of these two intercepts to one another – the “Annual” and “4 X Quarterly” intercept on the left in red and the \( \varphi / \pi \) intercept on the right in blue – is demonstrated in the chart below.
In short at that place on Charts One and Two where change in the growth rate is zero (the y-axis), the “Annual” and “4 x Quarterly” y-axis intercepts are multiplicative inverses of one another as set about the line designated in green in Diagram 2-14.

11. Let us now turn to the x-intercept for annual data. The annual x intercept is 3.4551266. The x-intercept for the Golden Mean is 3.4969781.

As to this discrepancy, it must be pointed out that the data used in Knotek 2007 goes back only through 1947. This data misses all GNP values which are available for the period 1869 through 1946, a period of 78 years. During the early part of this period very large values were stated in GNP ratios. (See Essay Three) Thus the annual data used by Knotek must be slightly less than that generated by our analysis herein because it does not include the same set of figures for GNP, but only a subset.

Interestingly the x-axis intercept given for Annualized Quarterly Data is virtually the same as that anticipated by the Golden Mean analysis herein. This may be because the number of observations for quarterly values is four times that of observations for annual values. These values are 3.4969 (Golden Mean) vs. 3.4971 (“Rounded” Annualized Quarterly) and 3.4999 (Updated, Unrounded Annualized Quarterly)\(^\text{10}\).

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\(^{10}\) See also Essay Four. This last value of 3.4999 is quite close to the Median Average figured for the 14-year spread of 3.4995226.
CONCLUSION

The mathematic regularity of Okun’s Law over time, and the correlations noted, are possible only if the relationship between change in the rate of employment and the growth of GNP are connected by the geometry of a half-circle via the steady ratio of $1 : \pi$ as described herein.
INTRODUCTION: THE GNP RATIO, THE “AGGREGATE” OF ANNUAL REAL GNP

In this essay we will discuss the third of the rings of this model, a demonstration of the economic “octave” of U.S. real GNP as manifested at 14-year intervals.

We suggest in this essay that, just as the “Chooser-Available Choice” model is a mathematic “wave” which is in sync with the human mind, and just as Okun’s Law represents that mind in aggregate over the course of a single year, so may the growth of U.S. real GNP be considered a wave which is in sync with the annual growth of human development.
Introduction 1.1: Intervals in Music as an Analogy in Economics

It is well known that Pythagoras first developed the modal system of Western harmony upon noticing that a vibrating string, divided exactly in half, produced a pleasant, melodious sound, whereas even a slight alteration from the division of the string into perfect halves produced dissonant, unpleasant discarding sounds. From this a spectrum emerged—the eight tones of the ancient modal scale made famous by Pythagoras, and the thirteen half-tones of the modern chromatic scale made famous by J. S. Bach, each based upon the mathematic division of a vibrating string. Upon this modal system the entire spectrum of Western musical harmony has emerged.

Of importance for this paper, between solitary note Middle C and its octave there exist 14 separate “chords” or “intervals” which may be sounded.\footnote{Notice that I do not state that there exist 14 notes in the chromatic scale (there are 13, including both Middle C and its higher octave). Nor do I state that there are 14 half-tones (there are twelve). Rather I state that two instruments, playing each possibility of harmony between them and including both the solo note of one instrument played while its partner remains at rest, as well as the unison of both instruments on the same note, combine to create fourteen “intervals.”}

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Diagram 3-2. Intervals of the Chromatic Musical Scale} \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
\hline
\end{tabular}
\end{center}
Introduction 1.2: Intervals in Human Development

A similar span of fourteen distinct years of human growth may be explored as human development passes through childhood and reaches adolescence.

To connect this musical scale to the development of the human being, let us propose that a child is born at 1:00 a.m., January 1, 2000. On this day the child experiences his first New Year’s Day. From this point we may chart the chronologic sequence of his second, third, fourth, etc. New Year’s Day, as follows.

This counting of dates is to be distinguished from the counting of the child’s birthdays. To experience one's first birthday party, or second, or third, etc. is a celebration of developmental growth. Each year claimed by a new birthday arrives with the celebration of a new biologic level of accomplishment. This concept of biologic development may be placed along the y-axis as follows.
As demonstrated below, of the 15.6 million “regular secondary school students” in the United States in 2007-2008, 12.5 million (79.7%) were enrolled in school systems which ended primary school at eighth grade and began enrollment in secondary school at ninth grade. This break occurs generally at the age of 14. (total student population of these schools, including 9, 10, 11, 12 grade students in red lettering below).

<table>
<thead>
<tr>
<th>Student Population</th>
<th>School System</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total, all secondary schools (post-primary)</td>
<td>16,184,724</td>
</tr>
<tr>
<td>2. Total, all regular secondary schools</td>
<td>15,680,507</td>
</tr>
<tr>
<td>3. Grades 7 to 8 and 7 to 9</td>
<td>1,578,163</td>
</tr>
<tr>
<td>4. Grades 7 to 12</td>
<td>927,888</td>
</tr>
<tr>
<td>5. Grades 8 to 12</td>
<td>451,656</td>
</tr>
<tr>
<td>6. Grades 9 to 12</td>
<td>12,500,341</td>
</tr>
<tr>
<td>7. Grades 10 to 12</td>
<td>418,850</td>
</tr>
<tr>
<td>8. Other spans ending with Grade 12</td>
<td>41,545</td>
</tr>
<tr>
<td>9. Other grade spans</td>
<td>266,281</td>
</tr>
</tbody>
</table>

The en masse separation of primary and secondary education into two completely different school systems tracks the tremendous difference between the end of childhood (in aggregate at the age of 14) and the beginning of adolescence and onset of procreative capabilities (in aggregate at the age of 14). Certainly the popularity of alternative systems to the 9-12 scheme, as measured by student enrollment, leaves little doubt that the preferred transfer date for students from primary to secondary education is at the age of 14. Other ages for transfer to secondary enrollment are less popular by ratios of 13:1, 27:1, 29:1, 46:1 and 300:1.

<table>
<thead>
<tr>
<th>Student enrollment</th>
<th>Comparative size to enrollment in 9-12 system</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Grades 7 to 12</td>
<td>927,888</td>
</tr>
<tr>
<td>5. Grades 8 to 12</td>
<td>451,656</td>
</tr>
<tr>
<td>6. Grades 9 to 12</td>
<td>12,500,341</td>
</tr>
<tr>
<td>7. Grades 10 to 12</td>
<td>418,850</td>
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<tr>
<td>8. Other spans ending with Grade 12</td>
<td>41,545</td>
</tr>
<tr>
<td>9. Other grade spans</td>
<td>266,281</td>
</tr>
</tbody>
</table>

---

12 Taken from the Digest of Education Statistics, Table 99, Public secondary schools, by grade span, average school size and state or jurisdiction: 2007-2008, National Center for Education Statistics; and Enrollment of public secondary schools, by state, 2007-2008, collected at the request of the authors from the NCES on Friday, June 10, 2011. Data Set Four and Five are at the conclusion of this paper.

13 This approach may parallel studies emphasizing the role of learning in the structure of globalization. See e.g. Marchetti, C. (1980) and Devezas, T., et al. (2008:32) “The framework proposed by Devezas and Modelski opens the door to conceptualizing the emergence of world organization and, more recently of globalization, as a process of systemic learning, which leads in turn to the concept of a learning civilization.”
Viewed in aggregate, the 14\textsuperscript{th} year of life may be a fundamental biologic rhythm, one which lays through biologic fertility the economic basis for a 14-year spread in the higher social level of the Kondratiev Wave.

It is possible to find in these constantly recurring 14-year cycles a pattern of human development over time. In other words, there are “harmonies” within human productivity which – like the musical intervals above – may be calculated, studied and used.

**ECONOMIC METHODOLOGY**

1. **Hypothesis**

My hypothesis is that the 50-60 year Kondratiev Wave is in reality a wave form composed of a number of smaller well-defined parts. Possible wavelengths can be evaluated and distinguished from one another by examining the underlying ratios of real GNP in the United States over various “intervals of years” or “spreads of years” which make up the cycle itself.

2. **Methods**

The musical scale results in the comparison of notes to one another, giving birth to the idea of the musical concept of dissonance and harmony as found between the various musical intervals. Musical harmony and comparative dissonance may be modeled mathematically.

With the idea of a musical “interval” in mind, I place a consistent set of real GNP figures (1868 to present) in “GNP ratios,” i.e. the quotient of one year’s real GNP as divided by that of a former year. This method results in the discovery of an “octave” in the data when the numerator and denominator are separated by a period of 14 years. In other words, the notable lack of economic “dissonance” when an interval of 14 years is given as the “spread” of the ratio is surrounded by much greater levels of economic dissonance for ratios using 11, 12, 13, and 15, 16, and 17 years of separation in the ratio.

The shape of the economic “octave” is virtually identical to the musical “octave” as graphed mathematically.

3. **Data**

These essays use figures for U.S. real GNP data published by the United States Government, as described and located in the Appendices.
4. Procedure: Examine Ratios of un-averaged U.S. real GNP

4.1. The Musical Intervals of the Diatonic Scale

We begin with the observation that one way in which to understand the interaction of waves and their mathematic construction is through the generation of waves of sound in music. Using a musical analogy we may better understand the mathematic relationships between waves and their interaction.

When a musical scale is sounded it can be thought of (or realized) as a wave moving through the atmosphere in the form of a sound wave. The velocity of this pulse moving through the atmosphere is the same for all notes. However the wavelength of that pulse is inversely proportional to the frequency of its peaks and troughs. The longer the wavelength, the less frequent the peaks and troughs of the wave; the shorter the wavelength, the more frequently occur its peaks and troughs. A higher frequency corresponds to a higher sounding note, or a higher pitch. A lower frequency corresponds to a lower sounding note and a lower pitch.

The relationship between (pairs of) notes are intervals. Collections of intervals can be thought of as a musical scale. As presented below the frequency of a particular note is given as “x”. The frequencies of the other notes in the diatonic scale as based upon this note “x” are given as multiples of this first basic “root” note.

The chief interval is the octave, an even division of a vibrating string into two parts, thereby doubling the frequency of the new higher pitched note as compared with the first note of lower pitch. The further division of the string into other mathematic subdivisions results in a number of differently pitched musical scales. The diatonic scale pictured above is among the most common.

The speed of sound is the distance travelled during a unit of time by a sound wave propagating through an elastic medium. In dry air at 20 °C (68 °F), the speed of sound is 340 metres per second (1,115 ft/s). This is 1,236 kilometres per hour (768 mph), or about one kilometer in three seconds or approximately one mile in five seconds.
4.2. Ratios of GNP as Intervals of Economic Growth

As used in this paper, a “ratio of GNP” is a numeric fraction which takes as its numerator the real GNP of one year and takes as its denominator the real GNP of an earlier year. The comparison of a GNP ratio as created by “an interval of years” or “a spread” between economic figures represents the economic equivalent of a musical “interval” in the harmonic relationship between one note and a note distant from it in the musical scale. Music is the art of juxtaposition of these relationships; the intervals are not “all the same.”

From the point of view of existing economic theories, no useful information should emerge from the interplay of different intervals of time as rates of U.S. real GNP are compared to one another. Neoclassical economics – and all other forms of economics – considers the physical passage of time as an irrelevant or at least an un-described / unknown mechanism and the relationships between economic quantities in time have no meaning beyond the numeric result obtained.

From the biologic standpoint however mathematic ratios of real GNP taken at different years represent the production of between 2.6 million to 311 million people over 235+ years (1777-present) as they live their lives in patterns associated with family, pride, livelihood, health, rigid social forms, moral expectations, criminal laws, civil relationships and a host of other necessary and indeed vital social interactions and expectations. These relationships are the generating substrate of all economic measurement.

For example let us consider two ratios of GNP, 1933/1928 and 1943/1938, each composed of an interval of but five years between numerator and denominator. In billions of dollars these are:

<table>
<thead>
<tr>
<th>Years compared</th>
<th>GNP figures</th>
<th>#1 GNP ratio</th>
<th>Years compared</th>
<th>GNP figures</th>
<th>#2 GNP ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1933</td>
<td>169.5</td>
<td>0.887</td>
<td>1943</td>
<td>337.1</td>
<td>1.747</td>
</tr>
<tr>
<td>1928</td>
<td>190.9</td>
<td></td>
<td>1938</td>
<td>192.9</td>
<td></td>
</tr>
</tbody>
</table>

The first ratio, 0.887, a ratio less than one representing a decline in value over time, states in a single number the decline between the peak of the Roaring Twenties and the nadir of the Great Depression. The second ratio, 1.747, roughly double the preceding ratio, represents in a single number the explosion of production associated with the end of the Great Depression and the middle year of World War II. The entire sequence of numbers occupies but 15 years, 1928 through 1943.

Considered biologically, the measurement of the difference between “myself now” and “myself next year” is a significantly different biologic measurement than the measurement of “myself now” as against “myself 20 years hence.” Considered biologically, the relationship of one year’s GNP to the next year’s GNP is actually a measurement of life at one point as measured against life at a point separated by some specified number of years.

As figures for real GNP are put into ratios against one another, we have the beginning of a biologic approach to economics, something upon which a larger theory of economic biocomplexity might be proposed.
4.3. An Evaluation of Multiple Spreadsheets of all GNP Ratios

We examined “ratios of U.S. real GNP” in order to determine whether such sub-cycles may be demonstrated empirically. As mentioned previously, a ratio of GNP is a numeric fraction which takes as its numerator the real GNP of one year and takes as its denominator the real GNP of an earlier year. The term “ratio” suggests a proportion between these two numbers which, no matter how large, over time governs the general existence of the numbers themselves.

A typical Excel spreadsheet with this data is as follows.\(^{15}\)

The number of years between numerator and denominator – the “interval” between them – represents the passage of time. It is referred to herein as a “spread of years” or simply a “spread.” In order to establish the possible period of the sub-cycle we took ratios of GNP at different spreads of years and placed these ratios in Excel spreadsheets based upon the number of years in the spread.

For every year of the spread we constructed a single row within a spreadsheet. Because the data set is finite, a tighter spread between years results in a larger number of columns, and a broader spread between years results in a reduced number of columns.

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\(^{15}\) See Appendix for all spreadsheets used in this evaluation.
By way of example let us consider Column Four Row One of the 12 year spread. (See Diagram 1, Sample Spread Sheet.) This GNP ratio is 1916 / 1904, representing a spread of 12 years between the numerator and the denominator of the ratio. The US real GNP values for this fraction are 134.4 / 89.7 with a result of 1.49833. This ratio is placed in Column Four Row One in the 12-year spread spreadsheet.

The next ratio in the series, 1917 / 1905, or 135.2 / 96.3, gives the result of 1.40395. This is placed in Column Four Row Two of the 12-year spread spreadsheet.

This continues on for a period of 12 years, i.e. from 1916 through 1927. The final fraction in Column Four Row Twelve is 1927/1915, or 189.9 / 124.5, for a result of 1.5253. This result is placed in Column Four Row Twelve and the series continues on to the next column.

The next column, Column Five, begins in Row One with the ratio 1928 / 1916, for a ratio of 190.9 / 134.4 and a result of 1.42039. This is placed in Column Five Row One and the process continues. Notice that the numerator of the cell in Column Four Row One (“1916 = 134.4”) becomes the denominator of the cell immediately to the right, Column Five Row One.

An Excel spread sheet may be generated for any given spread of years using “Data Base 2 – U.S. Real GNP” as its foundation.16 17

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16 The data provided by the Federal Government commences with a series of GNP values for the nine year period of 1869-1877 of a single figure, i.e. 23.1. This is followed by an 11-year period of 1878-1888 of a single value, i.e. 42.4. We have extended this series back one year by giving the year 1868 the figure 23.1, thereby permitting the larger spreads to include data series dating back to 1868.

This has been helpful in that it allows the 14-year, 15-year, 16-year, 17-year and 18-year spreads to include both the most antique, as well as the most current data – through 2010 – in their spreadsheets. Given the significance of the 14-year spread as described in this paper, it has been important to use this 1868 value of 23.1 as the beginning point for each spreadsheet in an effort to provide uniformity in this approach.

17 The use of this spreadsheet is the direct progeny of the first spreadsheet used in the discovery of the Golden Mean as an operating mathematic structure in the economy of the United States. (see Albers and Albers, 2012.) The only columns considered in the evaluation of spreadsheets (plural) are those columns which are complete. Under this rule the final column of the 12-year spread spreadsheet presented here, 2002/1990 through 2010/1998, would not be counted in any evaluation of spreadsheets. In this way a consistency between “spreadsheets” qua spreadsheets is obtained which would be impossible under any other measurement.

On the other hand this rule has the effect of eliminating from consideration between the spreadsheets themselves a number of years of data, depending upon the common year with which one begins the creation of each of the spreadsheets, the number of years in the data set, the spreads used to create the various spreadsheets to be compared and the final date of the data set which is used.

An alternative method to these difficulties is presented at the end of this essay which permits all data in the set to be considered, and which permits the evaluation of very broad and very narrow “spreads” between data. A “fingerprint” is not generated by this method however (see 4.2.6). Both methods appear to be legitimate in that they both discover the “octave” of the economy of the United States at 14 years, although the specific method used varies slightly.
For every Row and for every Column in every spread sheet there exists a High Ratio and a Low Ratio. For example, in the Columns and Rows mentioned previously regarding the 12-year spread, we have the following:

12-year Spread, High
Row One 1880/1868 = 42.4/23.1 = 1.8354978
Row Two 1881/1869 = 42.4/23.1 = 1.8354978
Row Twelve 1951/1939 = 383.4/209.4 = 1.8309455
Column Four 1927/1915 = 189.9/124.5 = 1.5253012
Column Five 1928/1916 = 190.9/134.3 = 1.4203869

12-year Spread, Low
Row One 1940/1928 = 227.2/190.9 = 1.1901519
Row Two 1941/1929 = 263.7/203.6 = 1.2951866
Row Twelve 1939/1927 = 209.4/189.9 = 1.1026856
Column Four 1921/1909 = 127.8/116.8 = 1.0941781
Column Five 1938/1926 = 192.9/190.0 = 1.0152632

We noticed that High Averages represent ratios which contrast a very dynamic year of growth in the numerator with a previous year of very slow or depressed growth in the denominator. Conversely Low Averages contrast a year of slow or depressed growth in the numerator with a previous year of growth in the denominator.

The full range of these contrasts is as follows as to the 12-year spread.

[Diagram 3.7: Row and Column Dynamics for 12-Year Spread]
4.4. Examination of Row Dynamics

From the above charts it becomes clear that these spreadsheets are characterized by "Row Dynamics" and "Column Dynamics." From these dynamics we have calculated four additional points within both the Rows and the Columns of all spreadsheets. These are:

The "Mid-Range." The mid-range is the mid-point lying between the high and low ratios in the sample, i.e. the average of the highest and lowest numbers in the set: 
\[
\frac{(H + L)}{2}.
\]

The "Average" or "Arithmetic Mean." The sample mean is the sum of all the observations divided by the number of observations.

The "Median." The median is that number for which half the data is larger than it, and half the data is smaller. It is also called the 50\textsuperscript{th} percentile. If the data has an odd number of members, the median will be the number in the center of these members; if an even number of members, the median will be the mid-point between the two numbers closest to the center.

The "Median Average." The Median Average is the mid-point between the Median and the Average (Arithmetic Mean). It is figured as:
\[
\frac{(\text{Median} + \text{Average})}{2}
\]
and is the approximation used throughout this paper – in conjunction with the Midrange – as the best estimate of the dynamics within Rows and Columns.

The High, Midrange, Median Average and Low of Row Dynamics for each Excel spreadsheet may now be compared. The following points are made as to this approach.

1) In every Row there exists a Highest Average of the possible averages in the Row. This Highest Average represents the greatest margin of growth over decline for the time period of that spread for that Row. Conversely the Lowest Average represents the greatest depth of decline over growth for the time period of the spread for that Row.

2) The Midrange between the Highest Average and the Lowest Average is simply the arithmetic division of the distance between these two. It lies half-way between them in any given row. The Midrange represents the arbitrary balance between these two extremes for that Row in any given spread of years. The Midrange is completely independent of, and unconnected to, the Median Average of the Row, other than the fact that they both include the Highest Average and the Lowest Average in their calculus.

3) The Median Average states the accumulated “weight” of all the ratios in the row. It is unconnected to the Highest Average and the Lowest Average other than it includes both of them as a part of its calculation. It is completely independent of, and unconnected to, the Midrange value and does not take it directly into account in its calculus.
4) When a particular spread of years generates Rows which contain Midrange values and the Median Average values which are quite close to one another, the spread has established a relationship between the most basic ratios of the economy which is balanced and uniform. In the context of our search herein, we use the term “harmonic” to indicate this balance.

5) When a particular spread of years generates Rows which contain Midrange values and Median Average values which are at relatively great distances from one another, the spread has failed to establish a relationship between these basic ratios of the economy. By comparison to the other spreads, the particular spread in question is relatively unbalanced and not uniform. In the context of our search herein, we use the term “dissonant” to indicate this discord, turbulence or lack of harmony.¹⁸

¹⁸ A physical analogy may be helpful to follow the logic at this point. Imagine that a mother and three children are at a playground and that a see-saw is available. A “harmonious” or “balanced” see-saw might be characterized by a simple fulcrum with a board balanced upon it. This would be analogous to the “midrange” (the center of the board) and the “median average” (the weight as distributed on the board) of the spread being very proximate to one another and thereby balanced without further effort or conflict.

The manufacturers of see-saws know, however, that the balance of children and their parents are frequently not evenly distributed. For this reason they place beneath the board a metal set of arches which may be used to adjust the length of the see-saw vis-à-vis the fulcrum in aid of the balance itself. An imbalanced see-saw is characterized by more weight at one end of the see-saw than the other. In the analogy under these circumstances the center of the board (midrange of the spread) and the weight imposed upon the board (the median average of the spread) are far apart and a form of imbalance or “dissonance” must result.

A balance may be restored by the addition of weight on one end of the see-saw to bring the balanced weight back, or by the shifting of the board itself from the center point within the metal arches to a point “off-center.” This displacement of the fulcrum beneath the board is equal to the imbalance of weight above the board. In this essay we seek to measure the extent of this displacement as applied systematically to economics.
6) The implication is that when a given spread of years generates Midrange and Median Average values which are proximate to one another and therefore “harmonious” or “balanced,” some underlying pattern or overriding logic may be at work to create this harmony as opposed to a random and disconnected set of processes and their resulting discordant and dissonant variables.

4.5. “Midrange Minus Median Average”:
An Evaluation of Differing Levels of Dissonance

In order to examine these relationships more carefully and across all spreadsheets, the following program was constructed.

Diagram 1-12, left side, presents the Row Dynamics for the 12-year spread shown in Diagram 1-11. The x-axis indicates the row of the spreadsheet under consideration. The y-axis represents the figure presented by that row as its High, Low, Midrange or Median Average ratio.

Diagram 1-12, right side, presents the graph of the

\[
\text{x-axis} = \text{Row of the Spread} \\
\text{y-axis} = \text{Midrange minus Median Average}
\]

When the Median Average is greater than the Midrange, the score is negative; when the Median Average is less than the Midrange, the score is positive. The number along the x-axis again indicates the row of the spread sheet under consideration. The number along the y-axis represents an amount of difference between Midrange and Median Average as found in that row.
The effort to compare systematically the common characteristics of different spreads led us to invent four new terms. Referring to Diagram 1-12 above these are:

“General Dissonance.” The pale blue area running as a ribbon from left to right represents the notion of a “General Dissonance,” i.e. an arbitrary, acceptable distance between Median-Average and Midpoint. When a row possesses a Midrange and a Median Average which are in close proximity to one another, the distance between them will be found within the space designated by pale blue, “General Dissonance.” After reviewing all spreads of years, this number has been set at +/- 0.05 in as much as it appears applicable to all spreads of years as general field of activity.

“Used General Dissonance.” The amount of dark blue is termed “Used General Dissonance,” i.e. that portion of “General Dissonance” which is actually used by the given row in stating the distance between the Midrange and the Median Average, either as a positive or negative amount surrounding y = 0.

“Acute Dissonance.” The portion in red represents an “Acute Dissonance.” When the distance between Midrange and Median Average falls outside the arbitrarily stated “General Dissonance” the excess is given in red shading. If the distance between the Midrange and the Median Average of a row is great, the “Acute Dissonance” so stated will be signified by large areas of red shading. Lesser amounts of “Acute Dissonance” generate less red shading.

“Claimed Dissonance.” The pink portion running as a ribbon from left to right is “Claimed Dissonance,” i.e. that volume of spread between the high point of “Acute Dissonance” and the low point of “Acute Dissonance.” This is the range of values necessary to accommodate the entire spectrum of variation between these two extreme points.

We then compared all spreads of years, from the 7-year spread to the 18-year spread using the “Midrange Minus Median Average” formula. The data used in this formula is as follows.
4.6. The “Fingerprint” of Growth Rates

An important difficulty arises in this regard as each spreadsheet is composed of varying numbers of columns and rows. Consequently the frequency of repetition varies. The 18-year spread is 2.571 longer in duration than is the 7-year spread. This means that – taken to infinity – the 7-year spread may be anticipated to have 2.571 as many columns as the 18-year spread. Conversely, because the number of rows is always finite, the 18-year spread has approximately 2.5 as many rows as the 7-year spread.

In the chart below the number of years in the spread is equalized by stretching the horizontal frame so that all spreads between a 7-year and an 18-year spread take up the same total horizontal space. This balances large spreads (large number of rows, relatively few columns) with the smaller spreads (small number of rows, large number of columns).

One may notice above that some spreads have distinctly lower profiles as to claimed dissonance than the other spreads. In addition the pattern of peaks and troughs created by this method provides a distinct “fingerprint” for each growth rate, irrespective of the growth rate.
itself, as generated by this method for separate spreads of years. Note, in particular, the low profile of claimed and acute dissonance in the 14-year spread. Note also the sudden change to an entirely negative set of values with large levels of claimed dissonance found in the 11-year spread.¹⁹

4.7. Systematic Description of Claimed, Acute and Used General Dissonance

We examined this finding in more detail by comparing the numbers generated by these different spreads and associating them with one another in a more systematic way.

Each value given as the sum or difference for equation “Midrange Minus Median Average” may be divided into two parts, i.e. positive and negative values. These parts are further sub-divided by those values for this number which fall close to the y = 0 axis and inside the range of +/- 0.05. This range is referred to as “General Dissonance.” Values which fall outside this range are referred to as “Acute Dissonance.”

“Claimed Dissonance” locates the High and the Low extremes of the “Midrange Minus Median Average” for a given Row. Once we locate the point at which the Midrange most exceeds the Median Average (High), and the point at which the Midrange is most exceeded by the Median Average (Low), we may draw the y-axis distance between these two extremes (column 13). This is then taken as the boundary of a pink ribbon denoting “Claimed Dissonance” against the y-axis for the entire spread.

“Claimed Dissonance” is a measurement of the extent to which any given spread of years generates turbulence and discord between the Midrange and the Median Average. Like harmonies with discord between them, a high value for Claimed Dissonance indicates that the GNP ratio in question would not function well as a fundamental building block for an economic system, whereas low values for Claimed Dissonance provide the underlying balance necessary.

¹⁹ These two spreads of years have been found to have a remarkable connection to the Golden Mean in two different ways. The 56-year cycle which is developed from the 14-year spread (Essays 4 and 5) clearly generates ratios within the GNP of the United States which indicate that the Golden Mean is a constant operating force upon the development of the United States. On the other hand, the 11 year spread, as seen here, is characterized by a very negative relationship within its fingerprint, a feature which is completely unknown to the other spreads.

If one considers the 11–year spread in the context of a 56-year cycle (see Essay Five) one notices that at 55 years the 11-year spread has gone through five repetitions, while the 14 year spread has gone through almost four. The result is a pentagon shape in the economic data (See Essay 5) which is dramatically connected to the Golden Mean by way of the geometry of a pentagon.
“The Expansion – Contraction Fraction”

All of these figures fit into the broader scheme of our effort to compare spreadsheets. Toward this end we have developed “the expansion – contraction fraction,” i.e. that fraction which serves as a stretching or shrinking device to accomplish numerically for spreadsheets what stretching and shrinking the horizontal frame of graphs accomplished in Diagram 3.

By way of example, in order to make the distance for “Claimed Dissonance” for the seven year spread equal that of the “Claimed Dissonance” for the 18-year spread, it must expand 2.571 times. If we used the fraction 18/7 we would create this “expansion – contraction fraction” and thereby “stretch” the data for the seven year spread accordingly.

Such a fraction may be used to equalize all figures for all spreadsheets. For example, an “Acute Dissonance” at the 7-year spread sheet exists within a pattern of time which repeats itself 10 times in a 70 year span. An “Acute Dissonance” of an equivalent amount in an 18-year spreadsheet repeats under four times in the same 70 year span. The following fractions were used to multiply the spreadsheet data into numeric representations which would be equivalent.

<table>
<thead>
<tr>
<th>7-year spread</th>
<th>x</th>
<th>12/7</th>
<th>1.7142</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td>12/8</td>
<td>1.5000</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>12/9</td>
<td>1.3333</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>12/10</td>
<td>1.2000</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>12/11</td>
<td>1.0909</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>12/12</td>
<td>1.0000</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>12/13</td>
<td>0.9230</td>
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<td>12/14</td>
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<tr>
<td>15</td>
<td></td>
<td>12/15</td>
<td>0.8000</td>
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<tr>
<td>16</td>
<td></td>
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<td>0.7500</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>12/17</td>
<td>0.7058</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>12/18.</td>
<td>0.6666</td>
</tr>
</tbody>
</table>

4.8. The Economic Octave

As demonstrated below, a remarkable and unexpected result occurs when a ratio of real GNP possesses a numerator and the denominator separated by 14 years. At this span of time, the level of Acute Dissonance is the least of all ratios (0.151795) and the level of Claimed Dissonance is second-to-least (2.39229).

In addition, the spreads of three years before (11, 12, 13) and after (15, 16, 17) the 14-year spread generate the greatest amount of Claimed Dissonance, more than double that of the 14-year spread. One may demonstrate this conclusively by:

(1) setting out each spread in direct proportion to the others,

(2) repeating the spread as necessary to demonstrate the continual repetition of the spread itself over a given period of time, and

(3) measuring the area of “Claimed Dissonance” taken up by each spread for the same period of years.
In Diagram 3-10 each spread is set into the repetition necessary to complete a 36-year period of time. One can see the pattern of Claimed Dissonance building to the 13-year spread, then suddenly dropping at the 14-year spread, and then immediately returning to a very high level of Claimed Dissonance at the 15-year Spread.

This “piling on” of Claimed Dissonance immediately before and after the 14-year spread is the origin of our selection of the term “dissonant,” i.e. the sense that at the 14-year spread an almost acoustic “octave” is sounded against an underlying reality.

This is surrounded by discording, conflicting “harmonies” immediately preceding and following this spread which are out-of-harmony with this reality. ²⁰

²⁰ See e.g. William Sethares, Relating Tuning and Timbre, Experimental Musical Instruments: “To explain perceptions of musical intervals, Plomp and Levelt note that most traditional musical tones have a spectrum consisting of a root or fundamental frequency, and a series of sine wave partials that occur at integer multiples of the fundamental. Figure 2 depicts one such timbre. If this timbre is sounded at various intervals, the dissonance of the intervals can be calculated by adding up all of the dissonances between all pairs of partials. Carrying out this calculation for a range of intervals leads to the dissonance curve. For example, the dissonance curve formed by the timbre of figure 2 is shown below in figure 3.
In Diagram 3-11 the same effect is created with the numeric values of these ranges, as calculated with the “Expansion – Contraction Fraction” given above. The result is a rather vivid suggestion of the significance of an “octave” at the 14-year spread within a single set of data, i.e. the Real GNP of the United States, 1868 to present.

Observe that this curve contains major dips at many of the intervals of the 12 tone equal tempered scale. The most consonant interval is the unison, followed closely by the octave. Next is the fifth, followed by the fourth, the major third, the major sixth, and the minor third. These agree with standard musical usage and experience. Looking at the data more closely shows that the minima do not occur at exactly the scale steps of the 12 tone equal tempered scale. Rather, they occur at the “nearby” simple ratios 1:1, 2:1, 3:2, 4:3, 5:4, and 5:3 respectively, which are exactly the locations of notes in the “justly intoned” scales (see Wilkinson). Thus an argument based on tonal consonance is consistent with the use of just intonation (scales based on intervals with simple integer ratios), at least for harmonic timbres.”
The suggestion is that, just as an octave is created by the equal division of a vibrating string into two harmonic parts, and just as a slight variation from this even division between the perfect center of the vibrating string results in intolerable out-of-tune sense of dis-harmony, so does the use of a 14-year interval between years when measuring GNP values result in great sympathy and proximity between Midrange and Median Average values for the entire economy, unlike every other spread of years. And also like the vibrating string, the most out-of-tune dissonance occurs immediately surrounding the perfect division of the string, while tapering off as one takes distances further from the center.


If we consider the positive and the negative “General Dissonances” as a combined positive distance (absolute value), we can see that each spread of years comes to approximately the same amount of “General Dissonance” (dark blue columns below). However the 11 and 14 year spreads are quite different from the rest in that the first is almost entirely in the negative region, and the second is, by comparison to the others, relatively small.
If we look to the amount of “Acute Dissonance” which goes above and beyond the general dissonance of these two points we have the following. The diagram on the left represents the amount of dissonance created by the spread (absolute value), and the diagram on the right represents the amount of harmony of the spread, i.e. the difference between the greatest level of dissonance (13 year spread) and the year in question.

In both charts, the relative lack of dissonance in the 14 year spread, or conversely the striking harmony of the 14 year spread, is quite clear.

If we look at the combined total of these dissonances, we have an even stronger representation of that portion wherein harmony resides, as opposed to measurements of other spreads.
The charts above track the level of harmony/dissonance for twelve different spreads between years. It is quite clear that again the 14-year spread provides the most harmony and the least dissonance. Like a place on a ball bat where the “acoustics” of the bat provide a “sweet spot” where it is best to hit a baseball, the span of 14 years seems to bring with it a natural “sweet spot” in the harmonics of the economy.

By simply flipping the comparison, we can see the preferred harmony brought on by a 14 year spread between years with very little acute dissonance.
4.10. An Alternative Approach

As mentioned previously, the use of spreadsheets is problematic in several ways. First, the choice of a common date with which to begin all spreadsheets is an inherently arbitrary choice. Second in order to maintain the integrity of the “spreadsheet” approach, only columns which are complete within the spreadsheet have been used. The “fingerprint” thereby generated may be compared to other spreadsheets in a fashion which is consistent as to the method employed, i.e. “only completed columns will be considered.” However the exclusion of uncompleted columns for analysis means that each spreadsheet excludes data which others may or may not use. Consequently the data being considered is not consistent and the results may be suspect.

If this analysis is run through a computer program whereby the spreads themselves are considered independently of the spread sheets which are generated, we have the following. This method does not permit a comparison of “fingerprints” per spread. Note that the use of the “expansion contraction fraction” in this example does not result in the same graph as previously as to “claimed dissonance.” However an examination of the ranges without this alteration is quite similar in its finding of an octave without the expansion contraction fraction.
Using the computer program we are able to generate spreads of years with all available data in a consistent fashion. Although there are no “fingerprints” of different spreads to compare using this method, we are nevertheless enabled to see the same “octave” in the analysis generated.

This dissonance graphs are virtually identical.

And the extension of this method into multiple years demonstrates that the 14-year period appears to be of significance for at least two additional periods of 14 years.
Moreover we have in this method the ability to consider other spreads as well, for example, the fingerprint of the 11 year spread was entirely negative. If multiples of 11 years are considered we have the following. In the case of the 11, 22 and 33 years spreads a significant jump in dissonance is brought on, one which is perhaps consistent with the one-off aspect of the multiple 55 years spread.

4.11. What accounts for the relatively small dissonance of the 14-year spread?

If we place all of the row and column dynamic charts next to each other, we have the following. This chart clearly shows that a rhythm exists in the economy such that at the 14-year spread the highest maximum ratio and the deepest minimum ratio balance each other with such perfection that the resulting midranges and median averages cancel each other out leaving very little dissonance. In addition, the least maximum ratio and the least minimum ratio again balance each other out.

In each of the other spreads the peaks and troughs do not align against one another or cancel out.

For example, the 12 year spread aligns the least minimum ratio with the greatest maximum ratio, resulting in significant dissonance.

Conversely, the 15 year, 16 year and 17 year spreads present a highest maximum ratio which is clearly “out of sync” or “out of phase” with the deepest minimum ratio.

Consequently it is the discovery of a rhythm or a phase within the economy of the United States which accounts for this octave, an empirical discovery of importance to these essays.
CONCLUSION

In this paper I have suggested that significant evidence supports the proposition that the economy of the United States may be organized according to “octaves” of economic growth in connection with a 14-year spread between years. One possible basis underlying the 14-year period may be that this is the period of time necessary for human development to turn the individual citizen from an infant to a reproducing adult, a period which is indicated by the break which occurs in four-fifths of American lives, i.e. the end of grade school and the beginning of secondary education.

It may be helpful to state specifically several points which may be taken from this essay.

1. Just as bees are alive and contribute to the life of the larger hive, so must the hive have distinct similarities in time span and structure as imposed upon it by the biology of the bees themselves.

2. Just as human beings are subject to the requirements of their own biologic growth, so too is the economy of the United States the outgrowth of these human beings and their collective biologic forces, needs, limitations, etc.

3. If the human beings which make up the economy are alive, then the economy itself is a living thing, something with its own rhythm and pace. In this essay we have proposed to seek out that rhythm and that pace.

4. As the product of living human beings who mature and grow, give birth and die, at fixed stages of biologic development, the collective economic product created by these people, year after year, will demonstrate a “tree-ring” type of development over time.

5. Human biology regulates the productive growth of the United States and draws it into accord with its own rhythm and pace. When the economic growth of the United States is excessive it is balanced by naturally occurring economic depression at a span of 14-years hence.

6. An “octave” is sounded in economic data when measurements of GNP ratios are in accord with the underlying scheme of human development, i.e. when a congruent “pace” is located between biology of the small (individual human) and the biology of the large (American economic history); it “makes sense.” Dissonance, chaos, wrong-answers and misunderstanding are sounded when measurements of economic data conflict with this scheme or when the scheme is ignored entirely.

7. We find in these essays that harmony is noted in the data which is congruent with the biologic pace of human beings, when they are viewed in a fashion which is synchronous. We further note that disharmony is noted in the data when these two “paces” are not synchronous.
8. When the measurement of the economy takes into account the underlying biology of the economy, a picture of American economy history may be developed which is in accord with both the biology of the individual member as well as the larger and encompassing biology of the economy. When the measurement of the economy ignores the underlying biology of the economy, nothing but dissonance and chaos results.

This completes the central, third unit circle described at the beginning of these essay.

To the further elaboration of this “octave” approach as displayed in the fourth and fifth circles we now turn.²¹

²¹ The significance of a 14-year spread between years as a defining characteristic of the American economy finds at least tentative support in spectral analysis. See e.g. Korotayev and Tsirel, 2007:10. Note that in both charts provided, the 14-year span is the most significant point of balance between the two charts, no matter how adjusted. (as taken from)
INTRODUCTION: 28-YEAR AND 56-YEAR HARMONIC OCTAVES
BASED UPON THE 14-YEAR OCTAVE

We turn now to the fourth and fifth “octaves” presented at the beginning of these essays, i.e. the presence of a 28-year damping price wave as encompassed by a 56-year sine wave of political change, the “Kondratiev Wave.”

In his 1925 work *The Major Economic Cycles* Nikolai Kondratiev postulated a long-term wave running throughout the economic histories of various western countries of approximately 50 to 60 years. (Kondratiev, 1925) Kondratiev’s plan analyzed European and even global patterns of economic development with the thesis that democratic capitalism may possess the tools necessary to save itself from the inevitable self-destruction predicted by Marx

Kondratiev’s work originated in the dangerous political context of prior socialist discoveries (Van Gelderen (1913), DeWolff (1924) and Kautsky (1917)) and communist theories (e.g. Marx, Lenin, Trotsky, Stalin) as to the evils of capitalism and the nature of its inevitable demise. (Goldstein, 1988:30-31) Kondratiev’s suggestion that democratic capitalism might avoid such demise brought to him the censure of Stalin and death in a prison camp.

Orthodox economics, on the other hand, maintains an enormous breadth of opinion as to whether considerations of political policy must, or must not, be a part of doctrinal discipline. This paper
of his disciples. Kondratiev’s original plan (Korotayev & Tsirel, 2010) provided dates for “upswings,” “transition periods” and “downswings” which Joseph Schumpeter’s 1939 work Business Cycles acknowledged as significant to economics. (Schumpeter, 1939)

The academic search for evidence of “long waves” running through the economic history of various nation-states is long standing (Goldstein, 1988) and a central topic of heterodox economics. Indeed a 52-53 year cycle has been described in very extensive detail underlying the global meltdown (Korotayev and Tsirel, 2010) and incorporated into the study of the current revolutionary movements in the Middle East. (Tausch, 2011) As one modern researcher of Kondratiev Waves has remarked, “Altogether I think the idea of 55 year cycles in the behavior of our society is one of the most penetrating and useful in organizing social and economic facts.” (Marchetti, 1988:7) However the dating and even existence of these periods are controversial.

concludes that there is much in Kondratiev’s work which is directly applicable to the economic history of the United States, but does so without reference to Marx, et al.

See Goldstein, 1988:30: “The Kondratieff-Trotsky long-wave debate … revolved around the question of the stability of capitalism. Do ‘universal crises’ threaten the survival of capitalism (as Trotsky thought), or are they only a phase of a more stable capitalist dynamic (as Kondratieff argued)? Kondratieff, like Kautsky, presented a picture of capitalism as more stable over the long term than either Trotsky or Lenin saw it. This parallel between Kondratieff’s approach and that of the hated Kautsky may help to explain the very negative reception given to Kondratieff by his fellow Soviet Marxists.”

See Goldstein 1988:7. “Long waves (or Kondratieff cycles) are defined by alternating economic phases – an expansion phase (for which I will often use the more convenient term upswing) and a stagnation phase (which I will often call the downswing). These economic phase periods are not uniform in length or quality. The transition point from an expansion phase to a stagnation phase is called a peak, and that from stagnation to expansion is a trough. The long wave, which repeats roughly every fifty years, is synchronous across national borders, indicating that the alternative phases are a systemic-level phenomenon.”

These terms are used in Korotayev and Tsirel, 2010:1-2, et seq. but may hide a diversity of views in light of contrasting research. See e.g. Korotayev and Tsirel, 2010:1-6, Goldstein 1988. See also Coccia, M. 2010:730-738. “(T)here are different long-wave chronologies and certain timings of long waves are often better for some countries but not for the world as a whole... These different cycles "do not have a synchronized rhythm across countries...”

Orthodox economics rejects Kondratiev as a fallacy. See e.g. Rothbard, 1984. See also, e.g. Solomou, 1990:61. “(T)he evidence rejects the Kondratieff wave phasing of post-1850 economic growth. This conclusion is valid for all the national case studies examined here. Whether one takes the 1856-1913 or 1856-1973 a Kondratieff wave phasing cannot be supported. … (O)bserved variations do not follow a Kondratieff wave pattern.”

Mainstream analysis has focused rather on econometric measurements of other variables, i.e. the stochastic vs. deterministic effects governing the creation of real GNP itself. (See e.g. Nelson and Plosser, 1982) The distinction has been important for mainstream economics. (See e.g. Cochrane, 1988: “The distinction between a random walk … and a trend-stationary series … is extreme. Long-range forecasts of a random walk move one for one with shocks at each date, while long-range forecasts of a trend-stationary series do not change at all. There are two related ways to think about a series that lies between these two extremes.”)

The significance of this inquiry however may be questionable. (Sowell, 1992: “The fact that postwar GNP series cannot distinguish between a time trend and a unit root model has important implications for theoretical models of the economy. Attention should be given to models where both the policy and theoretical implications of interest are not sensitive to the model of the trend. Ideally we would like a model which implies the same results if the trend is modeled as either a time trend or a unit root. Until such models are developed, further attention should be given to new statistical techniques which focus on discovering the long-run behavior of time series.”)
Studies in globalization have attempted to merge evolutionary theories with fractal geometry, “emergence,” the study of complexity and a host of other mechanisms in explication of Kondratiev Waves. Calls for clarification have followed as to the methods, dates and theories surrounding “long waves.”

See e.g. Modelski, G. (2008:5) “(There are) two important implications of this evolutionary approach: first, there is reason to believe that an analysis drawing on evolutionary theory lends itself to modeling, simulation, and forecasting. Secondly, such an approach allows us to view globalization as an enterprise of the human species as a whole. … The emphasis is not on broad based accounts of the course of world affairs but, selectively, on processes that reshape the social (including economic, political, and cultural) organization of the human species; processes such as urbanization, economic growth, political reform and world organization, and the making of world opinion; and the innovations that animate these developments.

As to requirements for a theory of causation for long waves, see Louca, F. (1999). “According to Kuznets, two conditions had to be met in order to establish the credibility of the Long Wave program: (for the “weak version of the recurrence requirement”) one must prove (i) that the oscillations are general, and (ii) that there are either external factors or internal peculiarities within the economic system that create the recurrence (Kuznets, 1940:267). … A stronger version… means that the recurrence must conform to further definitions: a time variation in certain very precise limits and under well defined and stable causal relations – i.e. that the previous phase causes the next phase in the cycle or that sequence not only exists but also that causality can be exhaustively accounted for. This may be called the strong version of the recurrence requirement. … Rosenberg and Frischtak prolonged (the debate) by requiring the research programme on Long Waves to indicate a specific form of causality, timing, recurrence – precisely what was implied by Kuznets and Lange – and economy-wide repercussions of such fluctuations in order to be valid.”

See e.g. Devezas, T., Corredine, J. (2001) “… Complexity theory and nonlinearity are currently hot topics of interdisciplinary interest among the natural and social sciences, but still fall short of explaining the cyclic and evolutionary dynamics of society. … Although much has been published on K-waves, we must consider:

1 - a comprehensive and embracing theory of Kondratiev economic cycles still needs to be elaborated, while at least four major issues remain to be clarified:
   i - why is there disregard among many contemporary economists and social scientists, some of them even stubbornly rejecting the existence of these waves?
   ii - what is to be understood about the causality of the phenomenon - not just the mechanisms, but also the underlying causes?
   iii- why the half-century beat? and since when? (only after, or even before the Industrial Revolution?, and more: where did the clock come from?).
   iv- will there be more Kondratievs? Free-will or determinism? …

3 - The use of new tools of science mentioned above may lead us to a better understanding of the causality of the phenomenon. … But the question remains: is it something endogenous, inherent to social behavior of the human being? Or is there some kind of exogenous causality (external to human beings, even cosmic causes?). The understanding of all the above-mentioned aspects (not only in their economic character, but as a whole physical or social phenomenon), could contribute significantly to futures research, helping us trace the best trajectory through the coming millennium. …"
ECONOMIC METHODOLOGY

1. Hypothesis

Our hypothesis is that the 50-60 year Kondratiev Wave is in reality a wave form composed of a number of smaller well-defined parts. 28-year and 56-year wavelengths are found in the price data collected for the United States as larger “octaves” of the 14-year octave demonstrated in Essay Three.

2. Methods

We graph prices in the United States, 1800 – 1993, and find therein evidence of a damping cosine wave with a period of 28 years. This wave is repeated twice as nested within a sine wave of longer-term relationships over 56-years, a period of time associated with the Kondratiev or “long” wave.

We review inflation figures and suggest that both “endogenous” and “exogenous” arguments for causality of the Kondratiev Wave are valid and that both endogenous and exogenous factors are at work in the timing of the wave. The wave itself is endogenous to the United States but the context in which this wave appears as a historic trend is closely related to the exogenous actions of other nation states.

Finally we evaluate the operation of these two longer waves in a single circuit, finding therein an increase in U.S. real GNP every 14 years of 1.618590, a figure 0.034% greater than “the Golden Mean,” a famous mathematic constant of 1.618033…. This reference to the Golden Mean connects economics to Euclidean geometry and trigonometry. Using this constant as one governing the creation of the data itself we are able to calculate an expectation that the United States is controlled by a steady-state growth rate of between 3.4969% and 3.4995% per year on average.

3. Data

The price and production data collected in Data Sets 1 and 2 from Essay One are utilized in this Essay Two.
4. Procedure

4.1. Price Patterns

Having established that a 14-year sub-period may be important in the evaluation of the economic history of the United States, we now examine the price indexes for the United States between 1800 and 1994. The figures from “Data Set 1 – Prices” (as taken from Essay Three) are stated below (1) in 7-year running averages (red line, top graph, semi-logarithmic scale), and (2) the change between a given year’s seven-year average as divided by the average itself (blue line, bottom graph). The lower graph permits us to see the increasingly large inflationary price index values of later years (post-1966) as placed in a more consistent relationship with the preceding values of the series.
Note that the 56 year period (14 x 4 = 56) between peaks at 1861 through 1917 suggests the possibility that similar periods of time might connect other peak points of inflation. If a 14-year span (blue rectangles above) is drawn around the years 1805, 1861, 1917 and 1973 (each of which is separated by periods of 56 years), virtually all inflationary peaks are contained in a single model.

A damping wave, beginning with a peak in the blue-shaded areas, has been noticed three times in the course of American economic history in consideration of prices.
4.2. Models of Price Fluctuation

Regarding the diagram above, we may relate the foregoing chart of price fluctuations to US real GNP. If we divide a circle into 56-year rays, all things being equal, as the arrows of production move outward to meet the expectation of GNP per year (arrows of radii moving out from the center of the circle) this production should be met by uniform resistances (arrows moving toward the center of the circle) which balance the natural increase of production exactly. As will be discussed at greater length in Essay Three, a trigonometric sine wave may be used to represent the application of a steady, uniform growth rate over time to all angles of the circumference of the circle as indicated by 56 equal divisions.

Regarding the diagram above we might consider as well extreme yet periodic events which cause these spikes in inflation noticed in Diagram 4-3. If a particular period of time fails to offer uniform resistance to production, or if the strength of production for some reason is particularly strong, the inherent productivity of the citizenry will create a bulge in productivity which must then be balanced out by a depression at some other time in the course of the circuit. Only in this fashion can a constant of growth be maintained in the face of unequal strengths of production and resistance to production. A wave must then develop over time during which this bulge will even out as time goes on until the next (generally unexpected) opportunity for unusual productivity occurs. If this damping price wave is placed along an x-axis, we have the following.
4.3. Exogenous vs. Endogenous Causation

We deal here exclusively with the United States and the discovery of strong evidence that a 56-year Kondratiev Wave appears to have significant impact upon the US economy. A long-standing issue regarding the Kondratiev Wave is the causation of the wave itself. This debate centers largely upon the "exogenous" vs. "endogenous" nature of the cycle.

From the "exogenous" point of view, it is difficult to understand how events which occur with an apparently chaotic randomness outside the United States can affect the American economy with dependable regularity.

From the "endogenous" point of view, although a form of biologic regularity might be granted to the American economy, it remains difficult to explain how such internal developments might affect with the same regularity international events over which the United States has no control whatsoever.

There can be no question that political events in Europe and throughout the world have had much to do with the inauguration of these cycles. Nor can there be serious question that the relationship between the economic development of the United States and that of Europe must be explored. The problem appears to be that two distinct yet interacting levels of economic life must be considered, one national (American) and one European. These concerns are dealt with in our separate paper entitled "On Revolution and the Cultural Development of Europe: Toward a European "System of Movement." (unpublished at this time)

As to the United States considered endogenously and independently, we present as persuasive a 14 x 4 = 56 year cycle as found between the inauguration of the American Civil War and the entry of the United States into the First World War. As these relate to the "exogenous" / "endogenous" debate, the following points may be made.

1. The American Civil War began on April 12-13, 1861.

2. The First World War began in Europe on July 28, 1914.

3. 56 years after the inauguration of the American Civil War, almost to the day, the United States entered the First World War “by joint resolution of Congress on April 6, 1917” (as taken from the Peace Treaty of August 25, 1921, section 1, between the United States and Germany)
One can explore the "endogenous" vs. "exogenous" nature of the 56-year period by considering the price patterns within the United States leading up to the First World War. As taken from Data Set One, these are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Price</th>
<th>Change from previous year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1910</td>
<td>28.00</td>
<td></td>
</tr>
<tr>
<td>1911</td>
<td>28.00</td>
<td>+0.0</td>
</tr>
<tr>
<td>1912</td>
<td>29.00</td>
<td>+1.0</td>
</tr>
<tr>
<td>1913</td>
<td>29.70</td>
<td>+0.7</td>
</tr>
<tr>
<td>1914</td>
<td>30.10</td>
<td>+0.4</td>
</tr>
<tr>
<td>1915</td>
<td>30.40</td>
<td>+0.3</td>
</tr>
<tr>
<td>1916</td>
<td>32.70</td>
<td>+2.3</td>
</tr>
<tr>
<td>1917</td>
<td>38.40</td>
<td>+5.7</td>
</tr>
<tr>
<td>1918</td>
<td>45.10</td>
<td>+6.7</td>
</tr>
<tr>
<td>1919</td>
<td>51.80</td>
<td>+6.7</td>
</tr>
<tr>
<td>1920</td>
<td>60.00</td>
<td>+8.2</td>
</tr>
<tr>
<td>1921</td>
<td>53.60</td>
<td>- 6.4</td>
</tr>
<tr>
<td>1922</td>
<td>50.20</td>
<td>- 3.4</td>
</tr>
<tr>
<td>1923</td>
<td>51.10</td>
<td>+0.9</td>
</tr>
</tbody>
</table>

One can see from the above that the inauguration of World War I in Europe in 1914 did not impact dramatically upon the price structure of the United States. Examining the United States' price structure for the years of European conflict 1914, 1915 and 1916 (in blue) changes of 0.4 + 0.3 + 2.3 = 3.0 may be noted.

The American entry into World War I in 1917 is associated with a spike in prices for the years 1917, 1918 and 1919 (in red) for a total of 5.7 + 6.7+ 6.7 = 19.1, over six times the cumulative changes of the previous three years. This would indicate that the domestic decision to enter World War I had far more to do with the resulting inflation than did the existence of the war in Europe itself. Indeed this inflationary trend carried on beyond the 1919 Armistice, entering into a downward trend only in the year of the Peace Treaty with German in 1921.

The "exogenous" aspects of the analysis simply admit that at a European level, a vast war was occurring into which the United States ultimately was drawn. The "endogenous" aspects of the analysis insist that the United States was governed by its own internal development as to whether and when to join the conflict.
A similar point may be made with regard to the Vietnam War. Below are contrasted the steadily rising casualty counts for American soldiers 1956-1980 (as taken from the National Archives at http://www.archives.gov/research/military/vietnam-war/casualty-statistics.html) with the price index from Data Set 1. When changes to the price index exceed the fraction 1.06 they are highlighted in red.

We see below that the Vietnam War was not a strong inflationary factor throughout the years of its most ferocious conflict when the annual casualty count exceeded 1,000, i.e. between 1965-1971 (also highlighted in red). At no time during this period did the price index exceed a multiple of 1.06 over the previous year.

On the other hand as of 1973, a year when the annual casualty count had diminished to less than 200, the inflation rate suddenly increased by no less than a multiple of 1.06 for nine of the next ten years.

In a fashion similar to 1917, inflation during this period is associated with the United States passing through a particular phase of its development and is not directly connected with the previous existence of the War in Vietnam.

<table>
<thead>
<tr>
<th>Casualty count</th>
<th>Current year / Previous year</th>
<th>Price Index</th>
<th>Current year / Previous year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1956-1960</td>
<td>9</td>
<td>88.70</td>
<td></td>
</tr>
<tr>
<td>1961</td>
<td>16 + 1.77</td>
<td>89.60 1.0101</td>
<td></td>
</tr>
<tr>
<td>1962</td>
<td>52 + 3.25</td>
<td>90.60 1.0111</td>
<td></td>
</tr>
<tr>
<td>1963</td>
<td>118 + 2.26</td>
<td>91.70 1.0121</td>
<td></td>
</tr>
<tr>
<td>1964</td>
<td>206 + 1.74</td>
<td>92.90 1.0130</td>
<td></td>
</tr>
<tr>
<td>1965</td>
<td>1,863 + 9.04</td>
<td>94.50 1.0172</td>
<td></td>
</tr>
<tr>
<td>1966</td>
<td>6,143 + 3.29</td>
<td>97.20 1.0285</td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>11,153 + 1.81</td>
<td>100.00 1.0288</td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>16,592 + 1.48</td>
<td>104.20 1.0420</td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>11,616 + 0.70</td>
<td>109.80 1.0537</td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>6,081 + 0.52</td>
<td>116.30 1.0591</td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>2,357 + 0.38</td>
<td>121.50 1.0447</td>
<td></td>
</tr>
<tr>
<td>1972</td>
<td>641 + 0.27</td>
<td>125.40 1.0320</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>168 + 0.26</td>
<td>133.20 1.0622</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>178 + 1.05</td>
<td>147.90 1.1103</td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>161 + 0.90</td>
<td>161.40 1.0912</td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td>77 + 0.47</td>
<td>170.70 1.0576</td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>96 + 1.24</td>
<td>181.80 1.0650</td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>447 + 4.65</td>
<td>195.60 1.0759</td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>148 + 0.33</td>
<td>217.80 1.1134</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>26 + 0.17</td>
<td>247.20 1.1349</td>
<td></td>
</tr>
<tr>
<td>1981</td>
<td></td>
<td>272.70 1.1031</td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td></td>
<td>289.50 1.0616</td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td></td>
<td>298.80 1.0310</td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Placing in red inflation rates exceeding a multiple of 1.06 or greater from the previous year, we have:

<table>
<thead>
<tr>
<th>Year</th>
<th>Price Index from previous year</th>
<th>Year</th>
<th>Price Index from previous year</th>
<th>Year</th>
<th>Price Index from previous year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1854</td>
<td>27</td>
<td>1904</td>
<td>28</td>
<td>1954</td>
<td>27</td>
</tr>
<tr>
<td>1855</td>
<td>28</td>
<td>1905</td>
<td>28.00</td>
<td>1955</td>
<td>28.00</td>
</tr>
<tr>
<td>1856</td>
<td>27</td>
<td>1906</td>
<td>29.00</td>
<td>1956</td>
<td>29.00</td>
</tr>
<tr>
<td>1857</td>
<td>28</td>
<td>1907</td>
<td>29.70</td>
<td>1957</td>
<td>30.10</td>
</tr>
<tr>
<td>1858</td>
<td>26</td>
<td>1908</td>
<td>30.10</td>
<td>1958</td>
<td>30.40</td>
</tr>
<tr>
<td>1859</td>
<td>27</td>
<td>1909</td>
<td>30.40</td>
<td>1959</td>
<td>31.10</td>
</tr>
<tr>
<td>1860</td>
<td>27</td>
<td>1910</td>
<td>31.10</td>
<td>1960</td>
<td>32.70</td>
</tr>
</tbody>
</table>

56 years separates dates along a horizontal line. Given the striking inflationary trends noticed below the above horizontal line, we conclude that a 56-year Kondratiev Wave has much to offer in the analysis of decisions "endogenously" considered by the United States, while acknowledging the importance of the world wide "exogenous" factors which compel these decisions to be made.

### 4.4. Cumulative Change/Average Inflation Over A 56-Year Cycle

We then placed all change/average inflation (lower graph above) along a 56-year circuit shown below. In the following diagram 9 o’clock represents the midpoint of the cumulative average of all inflation along a 56 year cycle as contained within the blue rectangles above. (This is marked as “Year One” in Data Set 6.) 3 o’clock represents the midpoint of the cumulative average of all inflation rates 28 years later. (Line 29 in Data Set 6)
The circumference of each circle represents a positive increase in the cumulative change/average figure of 1/2 percent (for example, a change/average cumulative amount of 1805 + 1861 + 1917 + 1973 lying directly at 9 o’clock). Points found within the interior of the smallest circumference represent negative figures by a comparable amount.

The blue square below represents the four 14-year segments of time set forth in Diagrams 4-2 and 4-3. The blue-shaded rectangles previously given in Diagrams 4-2 and 4-3 are represented by the vertical left line segment (below). Taken together 4 x 14 periods of time create the 56 year circuit of time of this model. Note that the Great Depression of 1929-1940 is part of the deep indentation between axis 12 and 22, i.e. at the top horizontal of the blue square and interior to the smallest radii.
4.5. The GNP Spiral and the Median Average for Growth

We then placed the U.S. real GNP figures given in “Data Set 2 – U.S. Real GNP” in a 56 year circuit, with the four 14-year quarter cycles indicated in blue, to create “the GNP Spiral” below. The center of the spiral, beginning at axis 9 = 1869, represents the real Gross National Product for that year of 23.10 billion dollars in 1958 prices. The Gross National Product for subsequent years in real terms are given along each axis respectively, with each circle of circumference representing ten billion dollars of real GNP in 1958 prices. Each row of the 14-year spreadsheet is represented by a “cross” within the spiral. Row One of the spreadsheet is given below as the diagonal “cross” of the square, Row 8 of the spreadsheet is at the horizontal and vertical axes or “cross” of the square.

As can be seen from the 14-year spreadsheet (placed in the Appendix, we then:

1) figured the average for each row of the spreadsheet for a total of 14 averages (Column F),

2) figured the Median (1.617735) and Average (1.619446) of Column F, and

3) figured a final Median Average for the entire spreadsheet of 1.618590.

In all spreadsheets this set of calculations is termed a “circle analysis.” This nomenclature refers to the arrangement of Row Averages as points along the circumference of a circle, each one counted equally and but once toward a final Median Average of the spreadsheet.
4.6. The Golden Mean and the Structure of American Economic History

This number 1.618590, the final Median Average of rows, is 0.034% greater than the constant phi, 1.6180339… This constant, sometimes referred to as “the Golden Mean,” “the Golden Ratio” or “the Golden Section,” was defined circa 300 b.c. by Euclid of Alexandria, as follows:

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser. 

If line segment AB is set to 1, and if the line segment AC is in a Golden Mean relationship to AB, then line segment AC will equal 1.6180339… often referred to by the Greek letter phi, \( \phi \), small case.

This finding can be checked by creating the following graph wherein we:

a) indicate the spread between years which generates the ratio (presented below in the “# of years” first column),

b) set forth the Median Average for all ratios generated for any given spread of years (second column below),

c) figure the “absolute difference” and the “percentage difference” of these different Median Averages from phi (3rd and 4th columns below), and finally

d) state these differences as absolute values (5th and 6th columns below).

As mentioned in the text, a “circle analysis” counts each average of rows (column F) a single time toward a final Median Average for the entire spreadsheet. A “square analysis” counts the first row twice, and arrives at a slightly different number, one which is 0.0053% in proximity to the Golden Mean.

Euclid of Alexandria, Elements, Book VI, Definition 3, circa 300 b.c. A broad array of texts may be suggested describing the well-known associations between the Golden Mean and patterns discovered in Nature. See e.g. Livio, 2002; Skinner, 2006; Hemenway, 2005.

See the discussion of the Golden Mean in Essay One, lines 290-306.
This data is summarized in the bar graph below this data. This graph demonstrates that Median Average generated by a 14-year spread between years are closest to 1.6180339..., = $\phi$, or the Golden Mean.
As noted in Essay One, the final Median Average for the 14-year spread of 1.618590 was generated as a result of the following Row Dynamics, a pattern which had the least “Used General Dissonance,” the least “Acute Dissonance” and the second-to-least “Claimed Dissonance” of all spreads considered. As can be seen below quite clearly, and unlike the other spreads considered, when a high average of the row is reached it is immediately balanced by a low as determined from the approximate midpoint of the Golden Mean. In addition, as time has passed the American economy has steadily narrowed its focus to precisely this same single point.

![Diagram 4-10. Row and Column Dynamics: 14-Year Spread](image)

The last two columns of the Column Dynamic graphic represent a time period stretching from the end of Column 7 (1979) through the end of Column 9 (2007). During this period of time the economic volatility of previous years markedly narrowed. This finding is reflected in the graph below charting the volatility of the U. S. Gross Domestic Product and its abrupt lessening in 1984. (Summers, 2005)

![Diagram 4-10a. "The Great Moderation"](image)

Although hailed at the time as “The Great Moderation” and a possible sign of progress in economic understanding (e.g. Bernanke, 2004) post-Global Financial Crisis this view has come under attack. (e.g. Chomsky, 2011) Chart 2-10 demonstrates that a marked narrowing of volatility began two columns prior to 1979, i.e. beginning with the end of Column 5 (1951), named here “The Greater Moderation” by way of comparison. See Essay Five for further comments on this matter.
To figure the annual increase implied by the GNP Spiral, we may use the formula for simple interest compounded annually:

\[ FV = PV \times (1+r)^t \]

… state a present value (PV) of $1,000,000; a time period (t) of 14 years; and the future value (FV) as given below in proportion to the varying numbers derived in the GNP Spiral. These assumptions give us the following interest rates (r).

<table>
<thead>
<tr>
<th>Future Value</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>x= Circle Analysis: $1,618,590</td>
<td>interest rate is: 3.4995226</td>
</tr>
<tr>
<td>x= Golden Mean: $1,618,033</td>
<td>interest rate is: 3.4969781</td>
</tr>
</tbody>
</table>

These “interest rates” are the annual “rates of growth” necessary to obtain the various proportions of the GNP Spiral over time.\(^{33}\), \(^{34}\)

---

\(^{33}\) At least one reference – albeit atavistic – may be cited in support of a similarity between the large number of designs found in Nature which incorporate the Golden Mean (the galactic spiral, the Chambered Nautilus, seed pods of various plants, aspects of DNA, etc.) and the almost biologic dynamism of the GNP Spiral presented herein. (See e.g. Kahn, 1961:425) “(I)t … seems likely that Stalin’s caution (regarding antagonism toward the United States) did not stem from fear of the atomic bomb as a decisive weapon. What alarmed him about the United States was Detroit – not (the Strategic Air Command)! He appears to have felt very strongly that no sensible government tangles with a nation with a GNP of $300 billion a year. Luckily we had both assets – the bomb and the GNP – so that any difference between U.S. and Soviet calculations was not crucial.”

\(^{34}\) A surprisingly eclectic reading list may be constructed on possible parallels to the 56-year cycle suggested herein. These include: (1) the circular arrangement of 56 “Aubrey holes” at Stonehenge, (Cleal, et al. 1995); (2) price fluctuations predicted in 1875 by an Ohio farmer (Benner 1875); (3) business cycles of 56-years (Funk 1933); (4) astrologic cycles generally connected to the orbit of Saturn (Williams 1947, 1959, 1982); (5) an “energy use cycle” of 56-years (Stewart 1989); (6) the “Joseph Cycle” (Sim 2008) and (7) a compendium of geologic, weather, financial and other information (McMinn 2006, 2007, 2011). The Jewish festival Birkat Hakhammah “Blessing of the Sun” takes place every 28 years, most recently April 8, 2009. See also Tompkins (1976:282) “Hunab Ku, sole source of movement and measure, symbolized the universe for the Maya in the form of a circle with an inscribed square. The circle was the symbol of the infinite, the spiritual; the square of the material. Hunab Ku was thus a universal dynamism or that which motivates and stimulates life in its total manifestation as spirit and matter, the all in one.”
CONCLUSION

Referring once again to the definition of the Golden Mean, we have:

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

As described in this article, and in connection with the economic progress of the United States, the Golden Mean appears to tie the past (line segment BC) to the present (line segment AB) to the future (line segment AC) in a self-consistent and harmonic fashion. It is a mathematic statement of the historic identity of the United States itself, as moving from date to date in a coherent, repeating manner as connected to a 14-year spread between years and as nested as a quarter-cycle within a 56-year circuit of social time.

The 14-year interval of time which lays the foundation for the 14-year spread between numerator and denominator in ratios of GNP, like the musical interval of an octave, provides a framework within which this evolution of GNP may take place. Like the octave, it lays the essential mathematic relationship of the entire spectrum of harmonies of growth. This coincides with the 50-60 year period given by Kondratiev as the basis for his model.

There is at least a poetic similarity between the division of a line segment into past-present-future and the familial context underlying society itself wherein one’s parents (past) give birth to one’s self (present) as continued through one’s children (future). Inasmuch as each stage of this familial expansion of self begins with the onset of reproductive capacities at age 14, the GNP Spiral / classic Kondratiev Wave may form as a parallel to an underlying biologic pattern.
It appears to be very likely that this underlying geometry of “generational time” lays the foundation for the strict cyclical element of the Kondratiev Wave, one which is biologically driven but upon which an enormous host of other economic, social and political relationships float inter-connectedly.

One might bear in mind the sheer force of life which continually bears on this dynamic. If we imagine that this “life force” of the economy may be viewed physically at the graduation of a high-school class, we can see that the force of these repetitive 14-year periods is not limited to a single family unit but rather constitutes a continuing host of waves, each breaking into the future as a new, highly charged and hopeful high school graduation class.

Returning to the hypothetical child born on January 1, 2000, we can watch the cumulative force of this development. Below we see a straight-line development over time as represented by each high school class graduation date, beginning with the graduation date of said child at 2018 (in highlighted yellow below). Every graduation class possesses a 14-year wavelength sustaining it. And each class is like the others in that the persons graduating begin the ascent through the careers which they choose.
As a single life goes through the sequential 14-year periods of Primary School, Secondary School, Early Career, Mid-Career, Late Career and Retirement which are themselves complemented by similar high school class graduations, we have the following.

Diagram 4-12.
HIGH SCHOOL GRADUATION CLASSES
WITH 14-YEAR CYCLES

Mid-Career

Early Career

Secondary School

Birth, Infancy, Primary School
In conclusion it appears that the fundamental “octave” of life is the motion leading from birth to reproductive capacity (Diagram 3-12, pink “claimed dissonance” graph). This is encompassed by the dampening price wave (Diagram 4-3, 4-5), and the largest 56-year octave of the entire Kondratiev cycle (Diagram 4-4).

We note that the intermediate “octave” represented below by the damping wave of price change appears to connect the biologic human octave of 14 years to the larger 56-year octave of the Kondratiev Wave.

If we associate the “political” life of the United States with a 56-year Kondratiev Wave (the sine wave below in dark green), and the “economic” life of the United States with the damping price wave (the cosine wave below in red), we may construct a “political economy wave” (below in light green) by the simple addition of these two separate but inter-connected waves.

April 9 is used as the beginning date of this approach, i.e. the date mid-way between the April 12-13 date in 1861 beginning the American Civil War, and the April 6, 1917 date beginning the United States’ involvement in World War I. Figuring 20,453 days between these April 9, 1961 and April 9, 1917, this figure can be added to April 9, 1917 to obtain the date April 8, 1973. It can also be subtracted from April 1861 to obtain the date April 10, 1805.

It has been proved helpful to figure the length of the cycle as exactly 20,454 days, beginning with the dates of April 9 in the years 1805, 1861, 1917 and 1973, using a common “April 9” starting date. Despite this numeric inconsistency of one day, the cycles suggested by this approach are useful in that they track trigonometrically essential aspects of American economic history, as follows.

Diagram 1-2. Political - Economy Wave

In Essay Five we turn to the systematic study of these relationships.
INTRODUCTION

As mentioned in the Preface to these essays, based upon the analysis presented one can predict that a financial panic should be expected to take place in 2005, as based upon similar financial economic / political panics which occurred previously in the years 1781, 1837, 1893 and 1949, each date separated by 56 years.
The spreads noted in Essay Three may be graphed in association with their placement in the 56-year cycle. The red lines of association between dates of a spread are drawn by taking the first row of each spreadsheet and indicating the date associated with the denominator (in each case, 1868) and then drawing lines to the point associated with each subsequent denominator in the row.
This has the effect of creating a “clock of bio-complexity” in that, as one moves down the rows of the spreadsheets, the graph of the associated red lines of Diagram 5-2 move over one space moving clockwise.

Highlighted in yellow are the two spreads which appear most dramatically from the previous essays to evidence a relationship to the Golden Mean, these being the 14-year spread (Essays 3 and 4) and the pentagon pattern of the 11 year spread which is closely connected to the geometry of the Golden Mean as well and whose “fingerprint” was so strikingly different. Highlighted in blue are the remaining symmetries of the spreadsheets.

These essays propose that the waves described herein have a central place in the steady development of the United States over time, and that maintenance of fundamental proportions of this development require a continuing and predictable series of crises during which the electorate is forced to change existing attitudes and beliefs to accommodate new, developing standards.

As researchers we must be interested in the timing, meaning and prediction of these crises because they form a key element in the understanding of the economy itself. Had these warnings been heeded the crisis may have been better anticipated perhaps would have occurred at the time predicted and resolved along historic precedents. The global financial crisis of 2008, imploding just months before the election of United States Senator Barack Obama, meets the description of the panic predicted but as postponed – and thereby expanded – by a delay of three years.

The 56-year cycle may be used as a basis for comparison between spreadsheets creating, in effect, a biologic clock of time as operating on the history of the United States. The Golden Mean is displayed in the following pentagon, which matches the effect of the 11-year spread as transposed onto a 56-year cycle.

On March 7, 2012 Professor William Black, Associate Professor of Economics and Law at University of Missouri - Kansas City, criticized neo-classical economics in testimony before Congress on hearings related to the Global Financial Crisis. (Black, 2012) He states: Neo-Classical Economic Policies are Criminogenic: They Cause Control Fraud Epidemics.

Neo-classical economics (has) failed… to develop a coherent theory of fraud, bubbles, or financial crises (Black 2005). It (has) continued to rely on a single methodological approach (econometrics) that inherently produces the worst possible policy advice during the expansion phase of a bubble….
ECONOMIC METHODOLOGY

1. Hypothesis

Our hypothesis is that the 50-60 year Kondratiev Wave may be analyzed as a precise fluctuation in the consciousness of the American People over time. This is demonstrated by the fact that the Kondratiev Wave as described in these essays is the larger fractal of the “Chooser-Available Choice” model presented in the first essay. (See Diagrams 5-7 and 5-8.)

2. Methods

We review the previous four essays and develop a mathematics which accommodates the concepts involved.

3. Data

We recapitulate the materials of the previous essays, placing them in an effective working arrangement.

In the wake of the Global Financial Crisis, a large body of criticism of neoclassical macroeconomics and its various models may be cited in support of this view.

See e.g. Krugman, 2009:
http://www.nytimes.com/2009/09/06/magazine/06Economic-t.html?pagewanted=all\&_r=0
See also Solow, 2010.
A candid appraisal of graduate education in economics is found at Smith, 2011.

For neo-classical analysis of unemployment see Knotek (2007, Footnote 9): “(I)ntegrated models featuring monetary policy and unemployment are only now beginning to appear; see, for instance, Blanchard and Gali (2006). For the most part, New Keynesian dynamic stochastic general equilibrium models – including medium-scale models of the type developed by Smets and Wouters (2003) – have avoided unemployment per se. Moreover, see Shimer (2005) for evidence on the severe shortcoming of the models that do include unemployment.”
4. Procedure

4.1. The Kondratiev Wave as the Larger Fractal of the Chooser – Available Choice Model

We began the search for the fundamental root of this “clock of bio-complexity” underlying American Economic History with the “Chooser - Available Choice” Model.

The micro-economic approach to “trading” vs. ”keeping” a particular good may be expanded “fractal-like” in the consideration of the Kondratiev Wave as a method whereby society “trades” or “keeps” various social values over time. If the process whereby the Kondratiev Wave is constructed is congruent to – a “larger fractal” of – the micro-economic dynamics described previously, the similarities and interaction between the two models should assist in the understanding of both.

The coloration of the square-shaped timeline below surrounding the GNP Spiral provides the dates actually given by Kondratiev for periods of Phase A “upswing” (blue), Phase B “downswing” (pink), “transition” (purple). (Korotayev and Tsirel, 2010). The same coloration is used to figure a similar set of economic periods albeit based upon the GNP Spiral itself.

In short, the square timeline represents the Kondratiev wave as it relates to the GNP Spiral and the circular shading represents the GNP Spiral as it relates to Kondratiev wave.
We may chart the political development of the United States by observing that the amendment of the Federal Constitution appears to be closely related to the phases of the GNP Spiral – Classic Kondratiev scheme. Considering the pale blue “Phase A” region given above, we find a striking willingness to alter existing rules in favor of new methods and legal expectations. These include:

1) the Federal Constitution itself, ratified in 1788,
2) the Bill of Rights, the first ten Amendments, ratified in 1791, and
3) 11 additional Constitutional Amendments,

… for a total of 21 Amendments. Conversely only 3 Amendments are found in the downswing phase “Phase B” region, below in pink. The ratio of amendments between the phases is 7:1.

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37 Additional plans of other scholars are considered briefly in Albers & Albers (2011).
Amendments falling in the blue shaded area are far more fundamental to American constitutional law than those in the pink shaded area. Moreover the transition periods form an interesting unit. Amendment 22, prohibiting a single individual from serving more than two presidential terms, was aimed (by Republicans) at the four elections won by (Democrat) President Roosevelt. The 13th, 14th and 15th Civil War Amendments were clearly intended to consolidate Abolitionist, Western and Northern gains against the Southern slave holding class.
If the historic course of amendments is tracked according to their placement on a sine wave and color-coded in directly relationship with the foregoing cycle, we have the following.

The amendments which most dramatically affect the lives and legal history of the United States are clearly associated with the up-sweep of this sine curve. A la Kondratiev, “Phase A” Amendments were often the victories of hard-fought battles wherein the people of the United States did, indeed, save themselves from demise.
We may begin to connect the foregoing macro-economic statement of change with the neo-classical micro-economic dichotomies of “trading” vs. “keeping” a particular good if we color code the GNP Spiral and the Political Economy Wave as follows. As noted below, four distinct historic periods – expansion, hyper-inflation, collapse and consequence – may be considered.

![Diagram 5.6. Cumulative "Change/Average" Inflation](image)

- **Expansion**
- **Hyper-Inflation**
- **Collapse**
- **Consequences**

**Diagram Notes:**
- Lines in blue represent 14 year periods of time.
- Numbers in black represent separate and distinct rays along the 56-year circuit.
- Years in red indicate dates lying along the same axis.
- Each radius of circumference represents a positive increase of 1/2 percent in the cumulative inflation rate for that axis.
The nature of the upswing vs. downswing noticed by Kondratiev is directly related to the “chooser- available choices” model provided earlier in this essay. Let us first consider those portions of the trigonometric unit circle wherein the actions and thoughts in favor of trade (change) exceed those of keeping the property (stability). These are the amendments associated with this period of time.

**Diagram 5-7.**
**Change vs. Stability - Southwest Corner**

[Diagram showing the relationship between thought and action in the context of change vs. stability through the southwest corner of the trigonometric unit circle.]

1. Religion (Establishment and Free Exercise), Speech, Press, Assembly, Petition
2. Keep and Bear Arms
3. Quartering of Soldiers Prohibited
4. Search and Seizure, Warrants
5. Grand Juries and Indictments, Double Jeopardy, Due Process of Law, Just Compensation upon Exercise of Eminent Domain
6. Speedy / Public Trial, Local Venue, Indictment/Information, Confrontation of Witnesses, Subpoena Power to Accused, Assistance of Counsel
7. Jury Trial for Civil Suits
8. Prohibition on Excessive Bail, and Fines, Cruel and Unusual Punishments
9. Exemptions of Rights does not restrict “Others Retained By The People”
10. Powers reserved to the States or People

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The northeast half of this model is quite different.
Only two constitutional amendments fall within a ten year span of "Year 29," i.e. 3 o’clock. Amendment 22 restricts a president from serving more than 2 terms in office (axis 31=1951) and enshrines in law a tradition begun by George Washington 154 years earlier when in 1797 he refused to run for a third term in office. Amendment 27 prohibits laws affecting Congressional salary from taking effect until the beginning of the next session of Congress. This amendment was proposed September 25, 1789 and enacted 203 years later in May 1992.

We might also consider the two remaining Amendments on the right hand side of the cycle. Both enacted in 1933, Amendment 20 determined the dates of term commencements for Congress and the President and Amendment 21 repealed the federal prohibition on consumption of alcohol. Amendment 20 was a purely administrative and Amendment 21 returned the country to a well-established social norm.

It is of course possible to take any data set and superimpose upon it a spiral of any sort. The list of Amendments to the Federal Constitution is useful in this analysis because:

(1) each Amendment carries with it a specific date of adoption, thereby making placement in the cycle non-controversial,
(2) each Amendment engages the entire United States by virtue of the centrality of the Federal Constitution and the difficulties posed in their adoption,
(3) each Amendment declares in the clearest possible terms what is intended, albeit this interpretation remains subject to further interpretation by the courts, and
(4) each Amendment remains an influence upon continued American development. In many cases these Amendments are intended to direct the process of the economic future of the American people away from evils previously experienced (slavery, disenfranchisement of African-Americans, women and persons of draft age, resistance to federal taxation of income, addiction to alcohol, unjust use of governmental powers, etc.)
The numerous amendments on the left-hand side of the circuit above should be contrasted with one of the most fundamental documents of American economic history occurring on the right-hand side of the circuit, the Declaration of Independence of 1776. This document makes clear that the colonists did not perceive themselves as setting forth upon some new and novel declaration of rights. Rather they viewed themselves as collectively determined to continue to enjoy rights which they already possessed.

Regarding George III the colonists declared in their first five grievances:

He has refused his assent to laws, the most wholesome and necessary for the public good.

He has forbidden his governors to pass laws of immediate and pressing importance, unless suspended in their operation till his assent should be obtained; and when so suspended, he has utterly neglected to attend to them.

He has refused to pass other laws for the accommodation of large districts of people, unless those people would relinquish the right of representation in the legislature, a right inestimable to them and formidable to tyrants only.

He has called together legislative bodies at places unusual, uncomfortable, and distant from the depository of their public records, for the sole purpose of fatiguing them into compliance with his measures.

He has dissolved representative houses repeatedly, for opposing with manly firmness his invasions on the rights of the people.

The remainder of the Declaration of Independence describes in ever expanding detail the list of wrongs done by the king to his colonists. Each of these royal acts or omissions justified – at least in the minds of the signatory colonists – an immediate separation of the colonies from the crown in protection of long-held rights, customs and privileges.
4.3. Employment and Okun’s Law under this Model

One must ask the mechanism by which these various waves are kept in check. According to Knotek 2007 the statistical pattern viewed is a regularity of the data, with no explanation being given for the recurrence of these proportions over time.

On the contrary, we may examine the “random-ness” of the variables published in Knotek 2007 and find a strict congruence with the 56-year cycle described herein.

As we examine the coefficients of Okun’s Law under the analysis given so far and using the same color scheme for “Expansion” (blue), “Hyper-Inflation” (purple), “Collapse” (green) and “Consequences” (yellow) we have the following. Note below that the green portion “Collapse” has proven to be a formal historic period which is entirely predictable in nature. It would appear that the each of these sections of time have had a bearing on the manner in which Okun’s Law has functioned via a connection to the underlying geometries implied.

![Diagram 5-9. Rolling Regressions with Historic Context](image)
4.4. Speculation as to the nature of the cycle

We may now speculate as to the nature of the right-left division underlying the GNP Spiral.

For the purposes of this paper regarding American economic history, let us define a “Belief-system” as the constellation of ideas surrounding any principle of governance: a monarchy, the bourgeoisie, slavery, the relationship of labor to capital, etc. Second, let us define the term “Revolution” as a period of time when significant portions of a time-honored belief-systems are destroyed and when new and largely untried belief systems are inaugurated. Third, let us define in contradistinction to “Revolution” the term “Consolidation” as an opposing historical period in which honor or reverence are given to relatively recent belief-systems in a manner calculated to preserve and prolong them. It would appear that the left half of the circuit is “revolutionary” in character, while the right half is “consolidating” in character in the context of historic American belief systems.

In light of the numerous constitutional amendments adopted on the left-hand side of the circuit, and the virtual lack thereof on the right-hand side, let us label each of the segments of American History as follows:
Note in the above that as each period of consolidation has come to its close, the United States has very predictably experienced a complete meltdown of the economy. This occurred most recently in September through December of 2008, the last months of the terms of George W. Bush. Prior events of similar magnitude are:

1. The collapse of the colonial economy, circa 1781,
2. The Panic of 1837,
3. The Panic of 1893 and

Two unusual characteristics of the recent global meltdown should be pointed out. These are (1) the difficulty of “dating” the recent crisis, and (2) the delay of the expected time of crisis. Let us consider these important points briefly.

Diagram 5-11
Recent Global Financial Crisis

Evolution

Black Lines represent dates of previous financial crisis
Red Lines represent dates of recent Global Financial Crisis
Each of the previous dates of “meltdown” clearly corresponded with events between axes 33 and 34. A description of these crises may be given simply by citing textbooks of American History.

Colonial meltdown of 1781
“In 1764 Parliament had outlawed paper money in the colonies altogether. Independence ended this restriction, and both the Continental Congress and the states printed large amounts of money during the Revolution, with inflationary results. To cite some examples, the Continental dollar became utterly worthless by 1781, and Virginia eventually called in its paper money at 1,000 to 1.”

Panic of 1837
“In 1836 the second United States Bank automatically came to the end of its checkered career and the country under the inspiration of the new democracy entered an epoch of “wild cat” finance. The very next year (May, 1837), a terrible business depression fell like a blight upon the land, bringing as usual more suffering to farmers and mechanics than to the “rich and wellborn”; but this calamity was likewise attributed by the masses to the machinations of the money power rather than to the conduct of their hero, President Jackson. Nothing would induce them to retrace their steps. For three decades a union of the South and West prevented a restoration of the centralized banking system. Not until the planting statesmen withdrew from Congress and the storm of the Civil War swept minor gusts before it were the ravages wrought by Jackson repaired by the directors of affairs in Washington.”

Panic of 1893
“The (Cleveland) Administration was not three months old when a series of bank failure and industrial collapses inaugurated the panic of (February) 1893. The treasury’s gold reserve was depleted by an excess of imports and by liquidation of American securities in London after a panic there. Gold was subject to a steady drain by the monthly purchase of useless silver required by the Silver Purchase Act of 1890, and by the redemption of greenbacks which by law were promptly reissued and formed an “endless chain for conveying gold to Europe.”

40 Garraty, p. 795.
Reviewing the same axes for the years 1948-1949, we have, in addition to the creation of the Marshall Plan to rebuild post-war Europe (April 1948), the following:

1949
In 1949 a business recession occurred and prices declined slightly. (p. 819) … Further alarmed by the news, released in September 1949, that the Russians had produced an atomic bomb, Congress appropriated $1.5 billion to arm NATO and in 1951 General Eisenhower was recalled to active duty and placed in command of all NATO forces. (p. 785) … This (civil war in China) resulted in the total defeat of the nationalists; by the end of 1949 Mao ruled all China and Chiang’s shattered armies had fled to sanctuary on the island of Formosa, now called Taiwan. This loss of over half a billion souls to communism caused an outburst of indignation in the United States and deeply divided the American people. Critics claimed that Truman had not backed the nationalists strongly enough and that he had stupidly underestimated both Mao’s power and his dedication to the cause of world revolution. (p. 786)41

The recent Global Financial Crisis began when, in September 2004, the FBI reported that it had uncovered widespread fraud in the home mortgage market (axis 32). The date of this FBI report precedes the axes of the above mentioned crises, i.e. 1781, 1837, 1893 and 1948-1949, by a matter of months. However, and unlike previous crises, action to correct these frauds was not undertaken and the final implosion was delayed for four years, i.e. to September 2008, two months before the election of Barack Obama. Public reaction, not unlike previous moments along axis 33, has been extremely suspicious about the timing and origin of this world-wide panic. 42

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41 Garraty, p. 786.
42 See e.g. House Bill 3995, presented by Representative Kaptur, November 3, 2009:

“(4) Fraud also played a decisive role in the Savings and Loan crisis (of the late 1980s and early 1990s). The FBI and Justice Department made prosecuting those elite frauds among its highest priorities. This took a massive commitment of FBI resources, but it produced the most successful prosecution of an epidemic of elite fraud in history--over 1,000 ‘priority’ felony convictions of senior insiders, according to Professor William K. Black in his book ”The Best Way to Rob a Bank is to Own One’.

(5) However, the FBI, because of its crippling personnel limitations, has been unable to assign sufficient FBI agents to investigate the current global financial crisis. The FBI identified the mortgage fraud ‘epidemic’ in congressional testimony in September 2004. It had so few white-collar crime specialists available, however, that it was able to assign only 120 special agents to mortgage fraud cases--less than one-eighth the agents it found essential to respond adequately to the huge, but far smaller, Savings and Loan crisis.

(6) Given the magnitude of the financial crisis of 2008 and the resulting losses and billions of taxpayer dollars spent to keep the financial system from collapsing, the FBI should have no less than 1,000 agents to address corporate, securities, and mortgage fraud located across the country, and, in addition, more forensic experts and Federal prosecutors to uncover the crimes committed and bring the perpetrators to justice.”
To conclude our speculation as to the nature of this circuit brings us to a discussion of the current events of today. We are, today, at the dividing line within the yellow section marked below.

The yellow portion of the above represents the beginning of an evolving revolutionary trend starting in 2008. This correlates to an impressive extent with the current difficulties faced by the United States in the Middle East. Note that as of the date of the publication of this article, the United States has attempted to deal with a number of revolutionary changes throughout the Arab world.
These have included but are not limited to: Tunisia, Egypt, Libya, Bahrain, Yemen, Syria, Morocco and Algeria. These events have become known popularly as “The Arab Spring.” Chronologically, these were preceded by the 2009 Revolution in Iran. They have been joined since that time by protests, revolts and crackdowns in Tibet, China, England and Greece as well as a painful sovereign debt crisis in Europe with additional austerity measures generally anticipated. The fact that these events are taking place at the very beginning of the “Evolving Revolution” segment of American economic history may presage much greater events to come.

A strong correlation between the onset of inflation and the axes of this period has been described by this model. The graph above demonstrates the historic inflationary rise which typically accompanies this period of American economic history.

The amount of blue and purple given in the above development towards revolution represents inflation, the strength of which emerges most dramatically along the left-pointing axis at nine o’clock. These years represent very difficult times in the history of the United States – the coming of the war with Britain in 1812 during which the White House, the Capitol, the Library of Congress and the Treasury were burned to the ground (1814); the American Civil War beginning in 1861 ending in the assassination of President Lincoln in 1865; the First World War beginning for the United States in 1917; and the OPEC Embargo of 1973. This axis brings revolutionary times of great uncertainty, a forced re-reading of America’s place in world history.

4.5. Predictions which may be made from this model

We have mentioned that there is only one point along the Unit Circle where Action is wholly aligned with Trading, i.e. the point at 9:00. All other points along the unit circle are similar to one another in that there is some “Y” component connected to some mental aspect of trading and/ or keeping the object in question. This mental aspect must include some possibility of cancelling the action contemplated. Consequently only at 9:00 o’clock is the possibility of a “Trade” wholly equivalent with Action, and at this point “Thought” is Zero and the Action Trading occurs.
The unique aspect of this point along the circle creates an unavoidable change in the overall unit circle. The break which is presented at \((x = -1, y = 0)\) creates a new and unknown element in the unit circle itself. Once the trade is made, the situation is no longer the way it was. Something new has taken place.

In Essay Four we mentioned that

Regarding the diagram above we might consider as well extreme yet periodic events which cause these spikes in inflation noticed in Diagram 2-3. If a particular period of time fails to offer uniform resistance to production, or if the strength of production for some reason is particularly strong, the inherent productivity of the citizenry will create a bulge in productivity which must then be balanced out by a depression at some other time in the course of the circuit. Only in this fashion can a constant of growth be maintained in the face of unequal strengths of production and resistance to production. A wave must then develop over time during which this bulge will even out as time goes on until the next (generally unexpected) opportunity for unusual productivity occurs. If this damping price wave is placed along an x-axis, we have the following.

This same moment of trading, a crisis at the social level, was pointed out in Essay Part Fourne, as follows.
The development of a damping cosine curve is inherent in such a change, because through this new curve an adjustment is made to the underlying sine curve of established values.

One door into the study of these relationships is to consider more carefully the 56-year “Political Economy Wave” and its intersections. Keeping the peak of the damping cosine wave at the same level as that of the original sine wave (“1”) we may graph the damping to occur by halves. The equation which adds the previous sine curve together with this damping cosine curve is as follows:

\[
g(y) = 2 \left( \frac{1}{2} - \frac{2 \left( y + \frac{1}{4} \pi \right)}{\pi} \right) \sin \left( 2y + \frac{1}{2} \pi \right) + \sin(y)
\]
If we associate this graph with a period of 20,454 days, beginning April 9, 1973, we may chart the addition of these two waves together are as follows, with the peaks, troughs and intersections between the lines noted. These are:

- **(A)** the beginning point,
- **(B)** the first peak, wherein positive y-values is at its greatest point,
- **(E)** the first trough,
- **(H)** the second peak,
- **(M)** the point wherein the wave passes from positive y-values to negative values,
- **(P)** the second trough (P), wherein negative y-values is at its greatest point and
- **(X)** the point wherein the wave passes from negative y-values to positive without the introduction of a new cosine wave.

The red rectangle above represents the year 2005, a date associated with previous financial political-economic crises coming in 56-year increments in 1781, 1837, 1893 and 1949. The vertical red line represents the election of November, 2008 at which point Senator Barack Obama was elected President of the United States in the midst of a global financial crisis.

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43 The addition (in pale green below) of a pre-cursive half-cosine wave has been added to further the study of this wave as it relates to the possibility of the expression of this wave over time.
CONCLUSION

The reader may now return to the Preface where a brief synopsis was given as to the points calculated by this model reveal the presence of a “crisis” in the development of the economic, social and political history of the United States.

The techniques used in these essays give researchers into the economic development of the United States a clear set of empirical measurements to use in their studies. The value of these measurements is capable of testing by considering the accuracy of the predictions made.

If this model is indeed persuasive as to an understanding of social reality then it may be of assistance also in the understanding of the role played by consciousness in the physical sciences as well.

Scott Albers
Great Falls, Montana
December 11, 2012
Appendices

Charts One and Two of Knotek 2007 were based upon data which was constructed by manipulating data originally taken from the St. Louis Fed’s FRED database. The change in unemployment (du) is the December level of the unemployment rate in a given year minus the December level of the previous year, and the GDP growth rate (dy) is the percent change in GDP from the quarter of a given year over the fourth quarter of the previous year.

As to unemployment data, the Bureau of Economic Analysis now uses 2005 chain weights. For annual data, the change in the unemployment rate is the current December minus the previous December, and GDP growth is 100*((GDP in the fourth quarter of this year)/(GDP in the fourth quarter of last year)-1). For quarterly data, the change in the unemployment rate is the difference between subsequent quarterly averages, while the BEA gives a formula for computing seasonally adjusted annualized growth rates.
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**Monthly Unemployment, Bureau of Labor Statistics**

1. **Prices.**

For price data, 1800 through 1993, we used two sources.


2. The Consumer Price Index of 1997, also published by the United States Department of Commerce, continues this series by dividing the historic series by 3, or a multiple of 1/3.

In the following table, we begin with the Consumer Price Index listed in Series E 135-166 of the *Historical Statistics of the United States: Colonial Times to 1970, Part 1* (column 3) and compare this with the Consumer Price Index of 1960–1997. (column 1) The years of overlap clearly reduce the number for the historic series to a precise one-third of its value as the value given for the modern series. (column 2)

We then spliced these two series into a single data set for prices based upon the values given in the historic series. We continued this data set past 1970 by multiplying the modern number by 3 and including this value in the final data set. (column 4)

We then figured centered moving averages for seven-year periods for the entire series. In this format a price index is averaged for seven sequential years and the average is placed at the middle term, e.g. the price indices for 1870, 1871, 1872, 1873, 1874, 1875, 1876 are averaged and placed as the figure for 1873. The process then continues to the next seven-year series by dropping the first and adding the next year in the chronology and beginning the averaging again. The technical term for this alteration of the data is “smoothing.” (column 5)

We then found the annual change between 7-year running averages for each year, and placed these next to the centered moving average itself. (column 6)

We then divided the annual change in 7-year running averages for a given year by the 7-year running average for that year, to be denominated “Change / Average Inflation.” In this way the larger numbers for the Consumer Price Index found in later years were brought into conformity with the price patterns of prior years. (column 7)
Data Set 1 – Prices.
### Data Set 6 - Inflation: Cumulative Averages

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2. United States Real Gross National Product.

We located two sources for real US GNP.

1. Figures for U. S. Real GNP 1869-1970 may be found in the book *Historical Statistics of the United States: Colonial Times to 1970, Part 1*, published by the United States Department of Commerce. Series F 1-5 presents "Gross National Product" for the United States between the years 1869-1970 according to 1958 prices. The years 1869-1878, and 1879-1888 are given with decade averages of 23.1 billion and 42.4 billion dollars respectively.

2. Figures for U. S. Real GNP 1947-present are collected by the St. Louis Federal Reserve.44

In Data Set 2 we begin with figures from the United States Department of Commerce which has published one set of numbers based upon 1958 prices running from 1869 through 1970. (column 2) The St. Louis Federal Reserve has published a different sequence of numbers based upon 2005 prices extending between 1947 through to the present day. (column 7)

Splicing multiples are quite necessary when considering two different series each of which proposes to calculate U.S. Real GNP over different periods of time. To “splice” or to “graft” these two sets together is necessary if an extended series running from 1869 to the present day is to be obtained. There does not exist at the present time such a series published by the United States Government. Consequently our first step in the analysis is to construct such a series as the foundation of this approach.45

We considered two possible multiples with which to splice these two series of U.S. Real GNP figures together. The first possible splicing multiple is 5.881696, the average of all 23 multiples between 1947-1970. These are the years during which these two separate series overlap. (column 6) This number is problematic in that there is a clear drift from 1947 through 1970 toward higher multiples. Figures from 1947-1960 range from 5.646318 (1953) to 5.977644 (1958) and average at 5.8239423. Figures from 1961-1970 range a bit higher, i.e. from 5.907649 (1962) to 6.071220 (1965).

A second possible splicing multiple is 5.962552, the average of the final ten years of overlap, i.e. between 1961-1970. This multiple is the one used to splice these series in this paper as it is nearer in time to the eventual cutoff between the series and includes only multiples found in the later and more recent multiples. (column four)

For the purposes of the demonstration herein, more elaborate splicing techniques have not been deemed necessary. Data Set 2 figures an extended series for U.S. Real GNP in constant terms from 1868 to 2009. For the purposes of this paper only the second splicing multiple, 5.962552, will be used for calculations. (column 9)

Each spreadsheet is a mathematic arrangement of the figures in “Data Set 2 – U.S. Real GNP.”

---

44 These figures are available at: [http://research.stlouisfed.org/fred2/series/GNPC96](http://research.stlouisfed.org/fred2/series/GNPC96)

45 See e.g. Cochrane, 1988:902. “The presence of a splice in 1947 also does not drive the result. Every long series of GNP data contains at least one splice. The wide surveys used to construct later data are simply not available for earlier periods, so some projection using a restricted set of industries is unavoidable.”
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Data Set 2 – U.S. Real GNP
## 10-Year Spread

### 10-Year Ratios Based on Annual Real GDP Multiple 5.962552

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### Row Dynamics - 10 Year Spread

- Maximum Ratio of Row
- Minimum Ratio of Row
- Mid-Range Ratio of Row
- Average Ratio of Row
- Median Average

### Column Dynamics - 10 Year Spread

- Maximum Ratio of Column
- Minimum Ratio of Column
- Mid-Range Ratio of Column
- Average Ratio of Column
- Median Average

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### 11-Year Spread

#### 11-Year Ratios Based on Annual Real GNP; Multiple 5.902552

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#### Row Dynamics - 11 Year Spread

#### Column Dynamics - 11 Year Spread
### 12-Year Spread

#### 12 YEAR RATIOS BASED ON ANNUAL REAL GNP: MULTIPLE 5.962552

| Year | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 | Year 8 | Year 9 | Year 10 | Year 11 | Year 12 | Year 13 | Year 14 | Year 15 | Year 16 | Year 17 | Year 18 | Year 19 | Year 20 | Year 21 | Year 22 | Year 23 | Year 24 | Year 25 | Year 26 | Year 27 | Year 28 | Year 29 | Year 30 | Year 31 | Year 32 | Year 33 | Year 34 | Year 35 | Year 36 | Year 37 | Year 38 | Year 39 | Year 40 | Year 41 | Year 42 | Year 43 | Year 44 | Year 45 | Year 46 | Year 47 | Year 48 | Year 49 | Year 50 | Year 51 | Year 52 | Year 53 | Year 54 | Year 55 | Year 56 | Year 57 | Year 58 | Year 59 | Year 60 | Year 61 | Year 62 | Year 63 | Year 64 | Year 65 | Year 66 | Year 67 | Year 68 | Year 69 | Year 70 | Year 71 | Year 72 | Year 73 | Year 74 | Year 75 | Year 76 | Year 77 | Year 78 | Year 79 | Year 80 | Year 81 | Year 82 | Year 83 | Year 84 | Year 85 | Year 86 | Year 87 | Year 88 | Year 89 | Year 90 | Year 91 | Year 92 | Year 93 | Year 94 | Year 95 | Year 96 | Year 97 | Year 98 | Year 99 | Year 100 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|

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### Row Dynamics - 12 Year Spread

#### Column Dynamics - 12 Year Spread
### 13-Year Spread

**13 YEAR RATIOS BASED ON ANNUAL REAL GNP; MULTIPLE 5.962552**

|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|

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### Row Dynamics - 13 Year Spread

- **Maximum Ratio of Row:** 1.1678
- **Minimum Ratio of Row:** 1.1559
- **Mean Ratio of Row:** 1.1828
- **Median Ratio of Row:** 1.1828
- **Average Ratio of Row:** 1.1828
- **Minimum Average:** 1.1559

### Column Dynamics - 13 Year Spread

- **Maximum Ratio of Column:** 2.0280
- **Minimum Ratio of Column:** 1.1678
- **Mean Ratio of Column:** 1.5960
- **Median Ratio of Column:** 1.5960
- **Average Ratio of Column:** 1.5960
- **Minimum Average:** 1.1678
## 16-Year Spread

### 16 Year Ratios Based on Annual Real GNP; Multiple 5.962552

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### Row Dynamics - 16 Year Spread

- **Minimum Ratio of Row**: 0.00000
- **Maximum Ratio of Row**: 0.00000
- **Median Ratio of Row**: 0.00000
- **Average Ratio of Row**: 0.00000

### Column Dynamics - 16 Year Spread

- **Minimum Ratio of Column**: 0.00000
- **Maximum Ratio of Column**: 0.00000
- **Median Ratio of Column**: 0.00000
- **Average Ratio of Column**: 0.00000

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## 17-Year Spread

**17 Year Ratios Based on Annual Real GNP; Multiple 5.962552**

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### Row Dynamics - 17 Year Spread

- **Minimum Ratio of Row**: 0.00
- **Maximum Ratio of Row**: 1.02
- **Median Ratio of Row**: 0.89
- **Average Ratio of Row**: 0.89
- **Range Ratio of Row**: 1.02

### Column Dynamics - 17 Year Spread

- **Minimum Ratio of Column**: 0.00
- **Maximum Ratio of Column**: 1.02
- **Median Ratio of Column**: 0.89
- **Average Ratio of Column**: 0.89
- **Range Ratio of Column**: 1.02

---

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### 18-Year Spread

**18 Year Ratios Based on Annual Real GNP; Multiple 5.962552**

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**Row Dynamics - 18 Year Spread**

**Column Dynamics - 18 Year Spread**

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### Data Set 3 - Values for "Midrange Minus Median Average"

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#### Mid-range Minus Median Average

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## DATA SET FOUR.
### SCHOOL ENROLLMENT - STUDENT POPULATION

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**Source:** U.S. Department of Education, National Center for Education Statistics, Common Core of Data, 2007-08.
## Data Set Five.
### School Enrollment - School Type

<table>
<thead>
<tr>
<th>State or Jurisdiction</th>
<th>Total, by grade span</th>
<th>Average number of students per school</th>
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<td><strong>U.S. Virgin Islands</strong></td>
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</tbody>
</table>

**Table 99.** Public secondary schools, by grade span, average school size, and state or jurisdiction: 2007-08

**Notes:**

- Excludes vocational, special education, and alternative schools.
- Vocational schools are also included under appropriate grade spans.

**Description:**

- Average for schools reporting enrollment data. Enrollment data were available for 22,500 out of 24,626 public secondary schools in 2007-08.

**Source:** U.S. Department of Education, National Center for Education Statistics, Common Core of Data (CCD), "Public Elementary/Secondary School Universe Survey," 2007-08. (This table was prepared September 2009.)
REFERENCES


Euclid of Alexandria, Elements.


The Economist, February 1, 1992, pp. 52-53.


See also the figures for Real GNP, 1947 to present, maintained by the St. Louis Federal Reserve at http://research.stlouisfed.org/fred2/series/GNPC96.

American casualty counts during the Vietnam War are kept at the National Archives and may be found at http://www.archives.gov/research/military/vietnam-war/casualty-statistics.html.

House Bill 3995, presented by Representative Kaptur, November 3, 2009, 111th Congress, First Session.

Digest of Education Statistics, Table 99, Public secondary schools, by grade span, average school size and state or jurisdiction: 2007-2008, National Center for Education Statistics; and Enrollment of public secondary schools, by state, 2007-2008, collected at the request of the authors from the NCES on Friday, June 10, 2011.

Additional information regarding the Golden Mean association with quasi-crystals may be found at:

http://materials.technion.ac.il/shechtman.html
Lifshitz, R., Introduction to quasicrystals,
http://www.tau.ac.il/~ronlif/quasicrystals.html
Senechal, M. (2011), Quasicrystals gifts to mathematics,
http://www.youtube.com/watch?v=pjao3H4z7-g&feature=relmfu
Steuer, D. (2010) Fascinating quasicrystals,
http://www.youtube.com/watch?v=jM4AIipGOdk
Steinhardt, P. J. What are quasicrysstals”
http://www.physics.princeton.edu/~steinh/QuasiIntro.ppt
Press release:
http://www.i-sis.org.uk/Golden_Mean_Wins_Chemistry_Nobel_Prize.php