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Abstract

In the housing markets, three basic facts have been repeatedly reported by empirical studies: the existence of price dispersion, the positive correlation between housing price and time-on-the-market, and between housing price and trading volume. Since housing markets are also characterised by a decentralised framework of exchange with important search and matching frictions, this paper examines whether the baseline search and matching model, i.e. the Mortensen-Pissarides model, can account for these three basic facts. We find that the behaviour of the housing market reflected in the above empirical findings can be addressed adequately by the standard matching framework.

Keywords: matching models, time-on-the-market, housing price dispersion, trading volume, search and matching frictions.

JEL Classification: R21, R31, J63

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1. INTRODUCTION

Housing markets are characterised by a decentralised exchange framework with important search and matching frictions. It has, in fact, been acknowledged that housing markets clear not only through price but also through the time and money that a buyer and a seller spend on the market. Consequently, the search and matching approach is widely used even in the real estate market (see section 2).

Furthermore, three basic facts have been repeatedly reported: (a) the positive correlation between housing price and time-on-the-market (see Leung, Leong and Chan, 2002; Anglin et al. 2003; Merlo and Ortalo-Magne, 2004, among others);(b) the positive correlation between housing price and trading volume (see Leung, Lau and Leong, 2002; Fisher et al., 2003, among others); (c) the existence of price dispersion. Although price dispersion research is more commonly found in studies of non-durable consumption goods (see Baye et al., 2006), price dispersion studies on durable and resaleable goods such as real estate are also growing rapidly (for an overview see Leung, Leong and Wong, 2006). Price dispersion (or price volatility) is probably the most important distinctive feature of housing markets. It refers to the phenomenon of selling two houses with very similar attributes and in near locations at the same time but at very different prices. In a nutshell, the variance in house prices cannot be attributed completely to the heterogeneous nature of real estate. Remaining price differentials are in fact empirically non negligible. A significant part of housing price dispersion is basically due to the heterogeneity of buyers and sellers, in particular their sustained search costs (see e.g. Leung and Zhang, 2011). Vukina and Zheng (2010) find very strong empirical support for the theoretical prediction that bargaining with search costs explains price dispersion in the agricultural market.

The main aim of this paper is to show that the matching framework is able to explain the basic facts of housing markets without any significant deviation from the baseline model (see e.g. the textbook by Pissarides, 2000). Precisely, we develop a housing market matching model in which a seller can become a buyer and vice versa. Hence, the proposed work takes the distinctive feature of the considered market into account, since buyers today are potential sellers tomorrow (Leung, Leong and Wong,

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1 The time-on-the-market, i.e. the time it takes to sell a property, measures the degree of illiquidity of the real estate asset and is a fundamental characteristic differentiating real estate from financial assets.
and most houses are bought by those who already own one, and most houses are sold by those wanting to buy another house (Janssen et al., 1994). Furthermore, in this model, price dispersion exists only assuming a different number of houses per capita. Also, this simple theoretical model is able to explain, in a straightforward manner, two other well-known empirical regularities, namely the positive correlation between housing price and time-on-the-market, and between housing price and trading volume. Therefore, this paper clearly shows that the behaviour of the housing market, reflected in the above empirical findings, can be addressed adequately by the standard matching framework.²

The rest of the paper is organised as follows: section 2 briefly reviews the literature which makes use of search and matching models to study the housing market; section 3 presents the housing market matching model; while section 4 concludes the work.

2. Literature review

This paper belongs to the recent and growing literature that uses search and matching models to explain the behaviour of housing markets. The first search model of the housing market is Wheaton’s (1990). Since then, several papers have developed models to analyse the formation process of prices in housing markets with search/matching/trading frictions (Krainer, 2001; Albrecht et al., 2007, 2009; Caplin and Leahy, 2008; Novy-Marx, 2009; Ngai and Tenreyro, 2009; Diaz and Jerez, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012).

Furthermore, recent search and matching models of the housing market (Diaz and Jerez, 2009; Novy-Marx, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012) adopt an aggregate matching function and some of them also focus on the role of market tightness in determining the probability of matching between the parties. This is in line with the standard matching approach (see Pissarides, 2000). The main difference between our model and those in the quoted studies is that we closely track the standard matching framework à la

² Although this approach is commonly used in the labour market, Wasmer and Weil (2004) show that it can also be used to describe matching difficulties between financial backers (banks) and firms.
Mortensen-Pissarides without any significant deviation from the baseline model. For example, Diaz and Jerez (2009), Novy-Marx (2009), Genesove and Han (2010), Leung and Zhang (2011), and Peterson (2012) define the market tightness from a buyer perspective, i.e. housing market tightness is the ratio of buyers to sellers. Instead, we prefer to use the standard definition of tightness, thus considering the ratio of vacant houses to home seekers (the buyers). In the labour market, in fact, tightness is the ratio of job vacancies to job seekers.

Among this literature, our model is most related to the competitive search framework developed by Leung and Zhang (2011), since it aims to explain the three basic facts of the housing market. In Leung and Zhang (2011), a necessary condition for explaining the housing market facts is the heterogeneity on the seller’s and/or the buyer's side, which generates corresponding submarkets. Precisely, Leung and Zhang (2011) focus on one-side heterogeneity and assume that sellers are different in terms of their waiting costs for selling the house, where buyers are free to enter either submarket. However, in their model the reservation value of a buyer is exogenous and sellers commit to “stay” in one of the submarkets. Unlike Leung and Zhang (2011), we develop a matching model which is consistent with both a single housing market and different search-housing markets with heterogeneous fundamentals. Furthermore, in our model the free-entry or zero-profit condition for sellers à la Pissarides, rather than the buyer's free entry assumption used by Leung and Zhang (2011), allows to obtain a solution which characterises the direct relationship between market tightness and house price. The free-entry condition for sellers is also used by Albrecht et al. (2009) to endogenise housing market tightness. Nevertheless, in their model, search is directed rather than random, houses are sold by auction rather than by bargaining and sellers post prices to attract buyers.

3 Sellers with higher waiting costs (the so-called impatient or "fire-sale" sellers) are willing to accept lower prices, which attract a larger number of buyers so that the house can be sold faster. However, patient sellers (sellers with lower waiting costs) may find it profitable to enter that sub-market.

4 In Leung and Zhang (2011), the equilibrium is in fact determined by a system of three equations in three unknowns, where the value of seller, the value of buyer and the house price depend on market tightness. As a result, with a fixed entry value for the buyers and a fixed number of sellers, they first solve the market tightness, and then the seller value and the house price. Indeed, also in Genesove and Han (2010) there is a constant value for the buyer’s search and an infinite supply of buyers, thus assuming that buyers can choose among a large number of markets, while sellers are tied to a specific market.
3. A Matching Model of Housing Market

3.1 The hypotheses of the model

We adopt a standard matching framework à la Mortensen-Pissarides (see e.g. Pissarides, 2000) with random search and prices determined by Nash bargaining. The random matching assumption is absolutely compatible with a market where the formal distinction between the demand and supply side is very subtle; whereas, bargaining is a natural outcome of decentralised markets for heterogeneous goods.

The economy is populated by buyers \((b)\) and sellers \((s)\) who hold a certain number of homes \((h)\). Precisely, sellers hold \(h > 1\) houses of which \(h - 1\) are on the market: hence, vacancies \((v)\) are simply given by \(v = (h - 1) \cdot s\). Instead, buyers expend costly search effort to find a new or better house: in fact, they already hold a house, i.e. \(h = 1\), and there are no homeless persons, namely buyers with \(h = 0\). In the model, it is therefore possible that a buyer can become a seller and vice versa. Indeed, a buyer becomes a seller after buying another house.\(^5\)

Since we are interested in selling price, the market of reference is the homeownership market rather than the rental market. In this way, if a contract is legally binding (as hypothesised) it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up. In matching model jargon this means that the destruction rate of a specific buyer-seller match does not exist. As a result, the value of an occupied home for a seller is simply given by the selling price and, therefore, the expected values of a vacant house \((V)\) and of finding a house \((H)\) are the following:\(^6\)

\[
rv = -a + q(\theta) \cdot [P - V] \\
rH = -e + g(\theta) \cdot [x - H - P]
\]

where \(\theta \equiv v/b\) is the “overall” housing market tightness from the sellers’ standpoint; while \(q(\theta)\) and \(g(\theta)\) are, respectively, the (instantaneous) probability of filling a vacant house and of finding a home. The standard hypothesis of constant returns to

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\(^5\) As will be clear later, buyers get utility from the house. Hence, on the one hand, buyers may have incentive for buying second home (when the selling price decreases) and later selling their home (when the selling price increases); on the other hand, however, it is not optimal for the buyer to sell before buying (second home) because the utility flows will be zero.

\(^6\) Time is continuous; individuals are risk neutral, live infinitely and discount future payoffs at the exogenous interest rate \(r > 0\). As usual in matching-type models, the analysis is restricted to the stationary state in which the values of the variables are not subject to further changes over time.
scale in the matching function, \( m = m(v, b) \), is adopted (see Pissarides, 2000; Petrongolo and Pissarides, 2001), since it is also used in the recent search models of the housing market (see Diaz and Jerez, 2009; Novy-Marx, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012). Hence, the properties of these functions are straightforward: \( q'(\theta) < 0 \), \( g'(\theta) > 0 \), \( \lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty \), and \( \lim_{\theta \to 0} g(\theta) = \lim_{\theta \to \infty} q(\theta) = 0 \). Finally, the term \( a \) represents the cost flows sustained by sellers for the advertisement of vacancies; whereas, \( e \) represents the effort flows in monetary terms made by buyers to find and visit the largest possible number of houses. If a contract is stipulated, the buyer gets a linear benefit \( x \) from the property, which coincides with the value of the house (abandoning the home searching value) and pays the sale price \( P \) to the seller (who abandons the value of finding another buyer). The value of the house depends on the housing characteristics and it does not depend on the buyer's tastes.

### 3.2 The equilibrium

In the housing market with search frictions, the endogenous variables that are determined simultaneously at equilibrium are market tightness (\( \theta \)) and sale price (\( P \)).

The customary long-term equilibrium condition, namely the “zero-profit” or “free-entry” condition, normally used in the matching models (see Pissarides, 2000) yields the first key relationship of the model, in which market tensions are a positive function of price. In fact, using the condition \( V = 0 \) in [1], we obtain:

\[
\alpha/P = q(\theta) \Rightarrow q(\theta)^{-1} = P/\alpha
\]

with \( \partial q(\theta) / \partial P > 0 \), since \( q(\theta)^{-1} = 1/q(\theta) \) is increasing in \( \theta \). This positive relationship is very intuitive: in fact, if the price increases, more vacant houses will be on the market.

Instead, the generalised Nash bargaining solution, usually used for decentralised markets, allows the sale price \( P \) to be obtained through the optimal subdivision of surplus deriving from a successful match. The surplus is defined as the

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7 By definition, markets with frictions require positive and finite tightness, i.e. \( 0 < \theta < \infty \), since for \( \theta = 0 \) the vacancies are always filled, whereas for \( \theta = \infty \) the home-seekers immediately find a vacant house.

8 Also in Albrecht et al. (2007) and Leung and Zhang (2011) the value of the house is independent of agent types. Intuitively, the value of the house, and thus the buyer’s benefit, can be higher or lower according to the mix of desired and undesired housing characteristics.
sum of the seller’s and buyer’s value when the trade takes place, net of the respective external options, i.e. the value of continuing to search:

\[
\text{surplus} = (P-V) + (x-P-H) = x - H
\]

The price is then obtained by solving the following optimisation condition (recall that in equilibrium \( V = 0 \)):

\[
P = arg\max \left\{ (P-V)^{1-H} \cdot (x-H-P) \right\} \Rightarrow P = \frac{V}{1-H} \cdot (x-H-P) \Rightarrow P = V \cdot (x-H)
\]

where \( 0 < \gamma < 1 \) is the share of bargaining power of sellers. Entering into a contractual agreement obviously implies that the surplus is always positive, i.e. \( x > H, \forall \theta \). This realistic condition on the buyers’ side also ensures that the price is positive. Simple manipulations yield the equation for the selling price:

\[
P = \frac{V \cdot (r + e)}{r + g(\theta) \cdot (1-\gamma)}
\]

As market tensions increase, the probability of finding a home increases, and the sale price decreases; hence, we obtain the second key relationship of the model: \( \partial P/\partial \theta < 0 \). In short, if the market tightness increases, the effect of the well-known congestion externalities on the demand side (see Pissarides, 2000) will lower the price.

**Figure 1. Equilibrium price and market tightness**
Given the properties of the matching probabilities, it is straightforward to obtain from equation [3] that when $P$ tends to zero (infinity), $\theta$ tends to zero (infinity), since $q(\theta)$ tends to infinity (zero). Consequently, given the negative slope of equation [4], with positive intercept, i.e. $\lim_{\theta \to 0} P = \gamma \cdot (x + e/r)$, and the fact that the selling price is always positive, only one long term equilibrium deriving from the intersection of the two curves exists in the model (see point A in Figure 1).

Finally, the optimal number of houses per capita $h$ is obtained by the maximisation of the expected overall profit, namely the profit arising from the sale of all houses on the market. Since the value of an occupied home for a seller is simply given by the selling price, the expected overall profit to maximise is the following:

$$\max_{h} r\Pi = (h - 1) \cdot P$$

$$\Rightarrow P + (h - 1) \cdot \frac{\partial P}{\partial \theta} = 0 \Rightarrow (h - 1) = -\left(\frac{P}{\partial P/\partial \theta}\right) > 0 \quad [5]$$

An increase in vacant houses on the market, in fact, reduces the selling price. It follows that the number of houses on the market, $(h - 1)$, is always positive, namely the number of houses held by sellers, $h$, is always higher than 1.

Eventually, in order to close the model, we normalise the population in the housing market to the unit, $1 = s + b$, i.e. a person is either a seller or a buyer, but not both, at any point in time. As a result, given the equilibrium value of market tightness and price, $P^*$ and $\theta^*$, we find the optimal number of houses per capita, $h$, and after the stock of sellers and buyers.\(^9\)

### 3.3 The trade-off between house prices and time-on-the-market

The free-entry condition implies a trade-off between the housing price and the speed of sale for the seller. In fact, with a probability of filling a vacant house of $q(\theta)$, the expected time-on-the-market is $q(\theta)^{-1}$. As a result, from [3] there is a positive correlation between housing price and the time-on-the-market, since a higher price requires a longer time to sell a house (as pointed out by Leung, Leong and Chan, 2002; Anglin et al. 2003; Merlo and Ortalo-Magne, 2004; Leung and Zhang, 2011).

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\(^9\) Indeed, equations [3] and [4] together with equation [5] and the definitions of market tightness, vacancies and the condition $1 = s + b$, define a system of six equations in six unknowns ($P$, $\theta$, $h$, $v$, $s$, $b$).
Indeed, by combining equations [3] and [4], this model is able to reproduce the observed joint behaviour of prices and time-on-the-market: in fact, the house with a higher selling price has a longer time on the market (see equation [3]), but, *ceteris paribus*, as shown by equation [4], the longer the time-on-the-market the lower the sale price (see Krainer, 2001; Merlo and Ortalo-Magne, 2004; Leung and Zhang, 2011; Diaz and Jerez, 2009), since both \( q(\theta)^{-1} \) and \( g(\theta) \) are increasing in \( \theta \).

Consequently, the first proposition can be stated:

*Proposition 1: The standard matching model extended to the housing market is able to mimic the trade-off between selling price and time-on-the-market.*

### 3.4 Matching rate and trading volume

From equation [4], the selling price clearly depends on the bargaining power of the parties. Also, the selling price crucially depends on the search costs of buyers and sellers. In particular, from [4] it is straightforward to obtain that an increase in the search effort of buyers increases the selling price, since a higher \( e \) implies a more eager buyer. As regards the effect of advertising vacancies on the selling price, an increase in \( a \) decreases market tightness \( \theta \), which in turn increases the selling price (since \( g(\theta) \) is lower). In short, an increase in the seller’s search cost also leads to an increase in the selling price.

Intuitively, the trading volume for a given period, i.e. the number of contracts traded during a given period, is given by the matching function/rate (see Leung and Zhang, 2011). Since the search intensity and the cost of advertising vacancies may be seen as parameters of technological change in the matching function (see Pissarides, p. 124, 2000), it is straightforward to include the search cost/effort of sellers and buyers in the matching function, i.e. \( m = m[a \cdot v, e \cdot b] \), with \( \theta = a \cdot v / e \cdot b \).\(^{10}\) Indeed, on the one hand, the search process involves costs; on the other, those costs allow the matching probability to increase. Hence, an increase in the search effort or in advertising vacancies will increase the matching rate \( m \). As a result, the model can also explain the positive relationship between housing price and trading volume, since an increase

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\(^{10}\) In our simple specification, the level/cost of advertising vacancies and the search effort/cost of buyers are exogenous. However, the search intensity decision may be endogenised (see e.g. Pissarides, 2000; Yashiv, 2007).
in the search costs of buyers and sellers increases both the selling price and the matching rate. This theoretical result is in line with the empirical works of Fisher et al. (2003) and Leung, Lau and Leong (2002).

This result can be summarized in the following proposition:

**PROPOSITION 2:** In the baseline Mortensen-Pissarides model of the housing market we can find a positive correlation between house prices and trading volume.

### 3.5 Number of houses per capita and price dispersion

From the maximisation of the expected overall profit, we get a unique value of $h$ for each seller. However, in the real world the number of houses per capita $h$ is not the same among sellers. Indeed, it also depends on external factors as legacies, business cycle, property tax, location, etc.

Thus, we distinguish between the *ex-ante* value of $h$ and its *ex-post* (optimal) value. Precisely, the number of houses per capita *ex-ante* ranges between 2 and a maximum value $n$, i.e. $2 \leq h \leq n$. It follows that the number of vacant houses on the market is different according to the value of $n$:\(^{11}\)

\[
v = \frac{s}{n-1} \cdot \sum_{h=2}^{n} (h-1)
\]

As a result, different equilibrium values of market tightness and price are obtained. Indeed, a market with a larger number of sellers and/or vacant houses will have in equilibrium a higher value of market tightness, since

\[
\theta^* = \frac{v}{b} = \left[ \frac{s}{n-1} \cdot \sum_{h=2}^{n} (h-1) \right]/b; \text{ while, the opposite is true in the case of a market with a lower number of sellers/vacancies. Therefore, housing prices would be different even for identical or similar houses, i.e. houses which have identical or similar housing characteristics and thus give the same buyers' benefit } x.
\]

Thus, the following proposition applies:

**PROPOSITION 3:** Price dispersion exists in the basic model à la Mortensen-Pissarides only assuming different number of houses per capita in the housing market.

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\(^{11}\) For the sake of simplicity, we assume that the share of sellers does not vary across $h$, namely $s(h) = s/(n-1), \forall h$. 

Eventually, from [5] different equilibrium values of market tightness and price imply different (ex-post) optimal values of $h$.

4. CONCLUSIONS

Housing markets are characterised by a decentralised framework of exchange with important search and matching frictions. Furthermore, three basic facts have been repeatedly reported by empirical studies: 1) the variance in house prices cannot be completely attributed to the heterogeneous nature of real estate and the residual price volatility is empirically non negligible; 2) the positive relationship between housing price and the number of contracts traded during a given period (the trading volume); 3) the trade-off between housing price and the speed of sale for the seller. This theoretical paper clearly shows that the behaviour of housing markets, reflected in the above empirical findings, can be addressed adequately by the standard matching framework à la Mortensen-Pissarides without any significant deviation from the baseline model.

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