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Abstract. We propose a model of coopetitive-game (of normal-form type) and devote it to Greek crisis, conceiving this model at a macro level, with the aim of rebalancing the current account of Greece. We construct the game trying to represent feasible scenarios of the strategic interaction between Greece and Germany. We shall suggest - after a deep study of our sample - feasible transferable utility solutions, in a properly coopetitive perspective, for the divergent interests of Greece and Germany.

1. Introduction

The main purpose of our contribution is to explore solutions for the Greek crisis, aware of the difficulties affecting the Greek economy. Although Greece has a GDP that reaches only 2 per cent of total GDP of the whole euro area [IMF, 2011], the Greek crisis is creating a lot of troubles in Europe and all over the world. Despite the financial aid programs that have been devised to help Greece by the euro area authorities and IMF Greece is still on the brink of collapse both for economic and political reasons. In the present work we propose an original economic coopetitive model applied to the Greek crisis, which aims at rebalancing the current account of Greece. This model, based on normal form game theory and conceived at a macro level, aims at suggesting feasible solutions in a coopetitive perspective for the divergent interests, which drive the economic policies of the countries in the euro area. In the model we consider only two countries: Greece and Germany. So, we propose a model that looks for a win-win solution. A win-win solution is the outcome of a game, which is designed in a way that all participants can profit from it in one way or the other. In our model the win-win solution entails that Germany should contribute to re-balance its trade surplus with respect to Greece, since Germany is the second world's exporter and also needs that the euro area be a financially stable region (Boone, Johnson, 2012). Indeed, we are aware that this is a mere hypothesis and that our framework of coopetition represents a normative model.

2. The coopetitive model

In the present model we apply the notion of coopetition, which was devised at micro-economic level for strategic management solutions by Brandenburger and Nalebuff [1995], who suggest to consider also a cooperative behavior to achieve a win-win outcome for both players. The coopetitive solution provided in our economic model is based on a set of three strategies: two are the classically non-cooperative strategies, the third one is a cooperative strategy. For Greece the two variables in our model are investments and exports, since this country must concentrate on them to improve the structure of production and its competitiveness, but also shift its aggregate demand towards a higher growth path in the medium term [Schilirò, 2011]. Thus Greece should focus on innovative investments, specially investments in knowledge [Schilirò, 2010], to change and improve its production structure and to increase productivity and, as a result of that, its competitiveness will improve. For Germany, on the other hand, the strategic variables in our model are private consumption and imports. The coopetitive variable (or shared variable) in the model is represented by the export of Greek goods to Germany. So in this situation Germany agrees to purchase a certain amount of goods imported from Greece, this shared variable, decided together by Greece and Germany, becomes the main instrumental variable of the model; consequently Greece will increase its exports by selling more products to Germany. The final result will be that Greece find itself in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries and, in addition, there will be the important advantage of a greater stability within the euro area. We have already devised a coopetitive model at a macroeconomic level [Carfì, Schilirò, 2011]. In that model [2011] we developed a coopetitive game by excluding the mutual influence of the actions (or strategies) for the two players. This choice has allowed us to greatly simplify the model, secondly it has highlighted the coopetitive aspect, although at the expense of the classical feature of game theory. In the present model, instead, we continue to highlight the coopetitive strategy in its cooperative dimension, represented by the shared variable (identified in the export of Greek goods to Germany), but, in addition, we reintroduce the classical strategic interaction between the two players. This generalization of the model allows us to reach a family of competitive solutions à la Nash from which to choose the win-win solution.

3. The mathematical model

We propose a coopetitive-game $G:=(f,>)$ (concept introduced in [Carfì, 2012]) in which:

1. *Germany stimulates the domestic demand and re-balances its trade surplus in favor of Greece;*
2. *Greece, in declining competitiveness of its products and with small exports, aims at growth by new investments and increasing exports firstly towards Germany.*

Our model G is a normative model:

it imposes a priori conditions to be respected, by contract, to enlarge the possible outcomes of both countries;
it shows appropriate win-win strategies, by considering both competitive and cooperative behaviors, simultaneously;
it gives appropriate fair divisions of the win-win payoffs.

Strategy spaces of G, where $f:E\times F\times C\rightarrow \mathbf{R}^2$ is the payoff-function, are:

1. the strategy-set E of Germany, set of all possible private-consumptions of Germany;
2. the strategy-set of Greece F, set of all possible investments of Greece;
3. a shared strategy-set C, whose elements z are amounts of Greek products to be imported into Germany, by contract.

Strategies in C are chosen, cooperatively, by the two countries.

3.1 Strategies

In the model G:

1. we consider an interaction between the two countries also at the level of their non-cooperative strategies;
2. we assume that Greece diminishes its wages and contains its home-consumption.

Specific strategic assumptions. We assume that:

1. $E:=[0, 3]$, set of possible German consumptions (in monetary unit u_1);
2. $F:=E$, set of possible Greek investments (in monetary unit u_2);
3. $C:=[0,2]$, set of possible amounts of Greek exports imported by Germany (in monetary unit u_3).

3.2 Germany Payoff-function

We assume Germany-payoff $f_1:E^2\times C\rightarrow \mathbf{R}$ is its aggregate demand $f_1=2+C_1+I_1+X_1-M_1$, where:

1. 2 is the government-spending (constant in our interaction);
2. German private-consumption C_1 is the first-projection of $S:=E^2\times C$, defined by $C_1(x,y,z):=x$, for any x in E (i.e. German-consumptions are first-components of 3-strategies in S);
3. gross-investment I_1 is constant on S and, by translation, supposed equal zero;
4. export X_1 is defined by $X_1(x,y,z)=-y/3$, for every possible investment y in innovative-technology (export X_1 is a strictly-decreasing reaction-function to Greek investments;
5. import M_1 is the third-projection of S, namely $M_1(x,y,z)=z$, for every cooperative-strategy z of C.

Concluding, Germany payoff-function is

$$f_1:S\rightarrow \mathbf{R}:f_1(x,y,z)=2+x-y/3-z,$$

for every (x,y,z) in S.

3.3 Greece Payoff-function

We assume Greece payoff-function is its aggregate-demand $f_2=2+C_2+I_2+X_2-M_2$, where:

1. private-consumption function C_2, by contract, depends on triples in S and defined by $C_2(x,y,z):=-2x/3$;
2. investment-function $I_2:S\rightarrow \mathbf{R}$ is defined by $I_2(x,y,z):=y+nz$, for every (x,y,z) in S (see [Carfi, Schilirò, 2011] for justification);
3. export-function X_2 is given by $X_2(x,y,z):=z+my$, for every (x,y,z) in S (see [Carfi, Schilirò, 2011] for justification);
4. import-function M_2 is independent on triples in S, so constant and, by translation, equal zero.

So, Greece reduces its private consumption by 2x/3, for each possible German-consumption x.

Concluding, Greece payoff-function is

$$f_2:S\rightarrow \mathbf{R}:f_2(x,y,z)=2-2x/3+(1+m)y+(1+n)z,$$

for every (x,y,z) in S.

3.4 Payoff-function of G

Coopetitive-game G has payoff-function

$$f:S\rightarrow \mathbf{R}^2:f(x,y,z)=(2,2)+(x-y/3,-2x/3+(1+m)y)+z(-1,1+n),$$

for every (x,y,z) in S.

3.5   Study of G

Fixed z in C, the section game $G(z) = (p(z),>)$, with payoff-function

$$p(z):E^2 \rightarrow \mathbf{R}:p(z)(x,y) := f(x,y,z),$$

is translation of game $G(0)$ by the "cooperative" vector $v(z):=z(-1,1+n)$. So, we study the initial game $G(0)$ and we translate any information of $G(0)$, by the vectors $v(z)$, to obtain the corresponding information on $G(z)$. Strategy square E^2 of $G(0)$ has vertices $(0,0)$, $3e_1$, $(3,3)$ and $3e_2$, where e_1 is $(1,0)$ and e_2 is $(0,1)$.

We assume factors m,n non-negative and equal, respectively, (for simplicity) to 0 and $1/2$.

3.6   Payoff-space of $G(0)$

To determine the payoff-space of the affine-game $G(0)$, we transform the vertices of the strategy-square ($G(0)$ is invertible and its critical zone is empty). The payoff-space boundary of the game $G(0)$ is the parallelogram with vertices $f(0,0)$, $f(3,0)$, $f(3,3)$ and $f(0,3)$ (see fig.1).
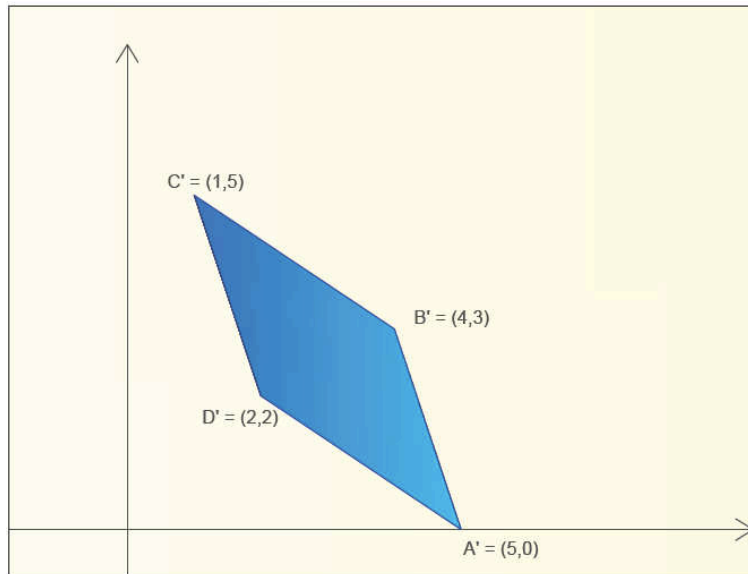

Fig.1 $p_0(E^2)$

The Nash-equilibrium is the bi-strategy $(3,3)$. Indeed, f_1 and f_2 are affine and increasing in the first and second argument, respectively.

3.7   Payoff-space of G

Image of f is union of images $im(p_z)$, with z in C: the hexagon with vertices $p_0(0,0)$, $p_0(3,0)$, $p_0(3,3)$ and their translations by $v(2)$ (see fig.2).
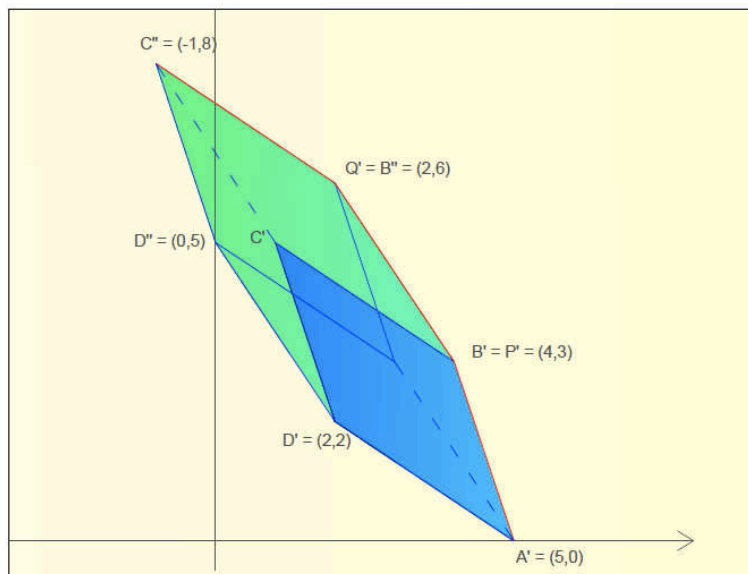

Fig.2 f(S)

3.8 Sup-boundary of G

The sup-boundary of f(S) is the union of segments [A′,B′], [P′,Q′] and [Q′,C″], where P′=f(3,3,0) and Q′=P′+v(2). Absolute slopes of [A′,B′], [P′,Q′] are strictly greater than 1, so, the aggregate-payoff $f_1 + f_2$ of G is not-constant on the sup-boundary of G.

Classic bargaining-solutions. The Nash-bargaining solution on f(S), with respect to the infimum of the sup-boundary and the Kalai-Smorodinsky (K-S) solution, with respect to the infimum and supremum of sup-boundary, are refused by Germany: they are collectively (TU) better than the Nash-payoff of G(0) but they are disadvantageous for Germany. These solutions are rebalancing solutions, but not implementable.

3.9 TU win-win solutions

We obtain win-win solutions by transferable-utility (TU) methods. Indeed, Q′=(2,6) is the point of maximum collective gain on f(S). We propose a rebalancing win-win coopetitive-solution, relative to maximum gain for Greece, as it follows:

1. let s be the portion of TU sup-boundary M:=Q′+$\mathbf{R}$(1,−1), contained in the strip determined by the lines P′+$\mathbf{R}e_1$ and C″+$\mathbf{R}e_1$, straight lines of Nash-gain for Greece in G(0) and of maximum gain for Greece in G, respectively;
2. we consider the K-S segment s′ of vertices B′ - Nash-payoff of the game G(0) - and the supremum (5,6) of the segment s;
3. our rebalancing-payoff is the point K, intersection of segments s and s′, i.e. the solution of the bargaining-problem (s,(B′,sup s)).

Fig. 3 shows the above extended K-S solution K and the K-S solution K′ of the classic bargaining-problem (M,B′). The distribution K is a rebalancing solution in favor of Greece more than the classic solution K′.
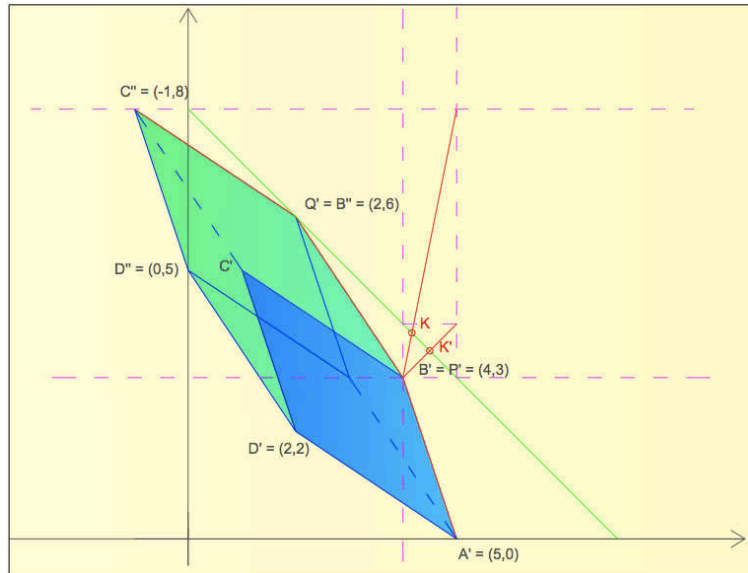

Fig.3 win-win solutions of G.

Rebalancing win-win solution relative to the maximum Nash-gain for Greece. We propose a more realistic rebalancing win-win coopetitive-solution, relative to the maximum Nash-gain for Greece:

1. let s be the part of TU sup-boundary M:=Q′+$\mathbf{R}$(1,−1), contained in the strip determined by P′+$\mathbf{R}e_1$ and Q′+$\mathbf{R}e_1$, straight lines of Greece Nash-gain in G(0) and of maximum Greece Nash-gain in G, respectively;
2. consider the K-S segment s′ of vertices B′ - Nash-payoff of G(0) - and the supremum (5,6) of s;
3. our rebalancing compromise is the point K, intersection of s and s′, i.e. the K-S solution of the bargaining-problem (s, (B′, sup s)).

Fig.4 shows the above K-S solution K and the classic solution K′. The new K is **more realistic than the previous one**.
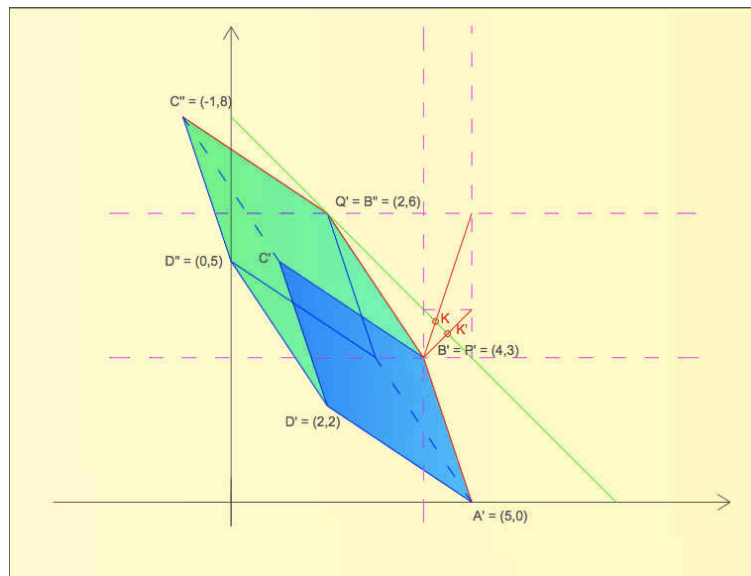
Figure 4. Realistic win-win solution of G.

The K-S solution K is a win-win solution, with respect to the initial gain B′: also Germany increases its initial profit from coopetition.

Win-win strategies. The win-win payoff K can be obtained in a properly TU-coopetitive fashion, as it follows:

1. players agree on the cooperative strategy 2 of the common set C;
2. players implement Nash-strategies in game G(2), *competing à la Nash*; the Nash-equilibrium of game G(2) is the bi-strategy (3,3);
3. players share the "social pie" $(f_1+f_2)(3,3,2)$, in a TU cooperative fashion (by contract) according to the decomposition K.

4. Conclusions

We stress that:

1. our coopetitive-game is a normative model;
2. we pointed out win-win strategies, in a properly TU-coopetitive perspective, for Greece and Germany.

We propose:

1. a properly-coopetitive solution (bad for Germany): K-S solution on the coopetitive Nash-path (set of all Nash-equilibria of G);
2. TU properly-coopetitive solutions, rebalancing the current account of Greece and also convenient for Germany;
3. extended Kalai-Smorodinsky methods, to determine rebalancing partitions on the TU sup-boundary of the coopetitive-game G.

Our model shows:

1. how "enlarge the pie and share it fairly";
2. win-win and rebalancing outcomes, for the two countries, within a coopetitive game-path;
3. "fair" amounts of Greek exports which Germany must import, to rebalance the current account of Greece;
4. optimal Greek investments necessary to contribute to the growth and stability of both economies.

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