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ABSTRACT

This paper examines the effects of ignored inefficiencies on the reliability of sustainability indicators and effectiveness of investment in resource-based economies. A model of a social planner does not include some phenomena that may influence the path of utility. These unspecified phenomena may cause inefficiency of the economy. In order to simulate this natural discrepancy between theory and real life, this study assumes that the planner applies the policies developed for an efficient (undistorted) model, whereas the real economy is distorted by some neglected effects that can influence utility, production, the balance equation, and the dynamics of the reserve. The resulting inefficiency affects the dependence of current utility change on investment. The analysis shows that, for sustainability in the presence of inefficiency, first, institutional and resource policies may become more important than investments; and secondly, it is preferable to underextract a natural resource under uncertainties in production possibilities and damages from economic activities. An inadequate accounting system, underestimated production possibilities, and insecure property rights are considered as examples of disregarded inefficiencies.

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Abbreviations CUC – Current utility change; GI – Genuine investment; HR – Hotelling rule; DHS – Dasgupta-Heal-Solow

1. Introduction

Sustainability of real economies is always evaluated in the presence of uncertainties in future production possibilities and in various distortions such as institutional imperfections, economic wastefulness, and damages from economic activities to utility and production. These uncertainties lead to errors in the decisions of a social planner and cause deviations from an efficient and optimal path of economic development. Since practical policies are based on simplified models, inefficiencies always exist in real economies, and it is important to learn how they may affect the reliability of sustainability indicators and sustainability policies developed for simplified models.

This paper is organized as follows: Section 2 identifies the place of this study in the literature; Section 3 describes the approach to modelling a dynamically inefficient "distorted" economy and derives the main theoretical results; Section 4 illustrates these results with the examples of various types of inefficient economies; Section 5 shows the dependence of sustainability evaluation on the specification of the model and the form of indicator, using the example of Russian oil extraction; and Section 6 concludes.

2. Contribution of this study

The literature on sustainability evaluation of resource-based economies offers an indicator of sustainable development, called genuine (net) saving or genuine investment (GI), which is equal to increase in man-made capital minus resource depletion. This indicator was developed in the studies of the change in the current (present) value of consumption or utility at a specific moment in time in dynamically efficient, optimal or competitive economies. Straightforward application of these results to real-world resource economies may create the impression that a non-declining path of utility can be achieved by investment policy only, regardless of other factors that may affect sustainability.

The current paper extends some of the theoretical studies by assuming that a planner, due to imperfections in knowledge or in institutions, uses the policies developed for a simplified (undistorted) model that deviates from the real economy. This natural discrepancy between a model and real life results in inefficiency and, in some cases, unsustainability of the economy. Moreover, the sustainability policies developed for an undistorted model may be inapplicable to an inefficient unsustainable economy because a feasible investment providing a non-declining path of utility may not exist. In these cases, the inefficiency must be reduced first, for example, a planner should correct institutions or the accounting system, and only then the "undistorted" sustainability policies can be applied.

The idea of developing sustainability indicators stems from the result of Hartwick (1977). The "invest resource rent" rule (zero GI), offered in this paper, addresses the problem formulated in Solow (1974) for the Dasgupta-Heal-Solow (DHS)[1] model of a maximin-optimal resource-based economy satisfying the standard Hotelling rule (HR) as a necessary condition of dynamic efficiency. For this model, zero GI with resource depletion measured in market prices leads to constant per capita consumption over time. Dixit et al. (1980) extended the Hartwick rule by showing that, for a more general production function in a competitive economy with multiple assets, GI that is constant over time in present prices is a necessary and sufficient condition for a constant path of utility.[2] Dasgupta and Heal (1979, pp. 303-306), Hamilton and Hartwick (2005), and Hamilton et al. (2006) analyzed the link between GI in current prices and current change in per capita consumption. Hamilton and Withagen (2007) derived the result of Dixit et al. (1980), as well as the result of Hamilton and Hartwick (2005), in a more general setting (for multiple consumption good and internalized externalities), showing that instantaneous utility increases if and only if GI decreases in present prices.

Various forms of the indicator GI developed for efficient economies have been used for practical evaluations of sustainability. For example, Pearce and Atkinson (1993) offered a simple indicator of weak sustainability[3] based on the assertion that "an economy is sustainable if it saves more than the combined depreciation on the two forms of capital" (man-made and natural). A variant of this indicator, modified for open economies, has been

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[1] This model with the Cobb-Douglas production function, which includes a nonrenewable resource as a factor, was developed in the works of Dasgupta and Heal (1974) and Solow (1974).

[2] Constant investment in present prices means that investment in current prices is growing with the rate of discount.

[3] Weak sustainability of growth (development) is defined in Pezzey (1992) as non-decreasing per capita consumption (utility).

developed in Proops et al. (1999). These indicators were used in both papers to classify a number of countries into sustainable and unsustainable. Hamilton and Clemens (1999) developed a theory of genuine saving by adding the investment in human capital to traditional net savings and subtracting the value of resource depletion and environmental damage. The value of genuine saving was offered as an indicator of sustainability, and this indicator was used for comparing sustainability of a wide range of developing countries. A review of empirical work on sustainability evaluation can be found in Hamilton (2010). As Hamilton and Hartwick (2005, p. 615) noted, "the magnitude of 'net investment' or 'genuine savings' has become a central focus in the measurement of the sustainability of an economy."

While most of the above mentioned theoretical results were obtained for optimal or competitive economies, Dasgupta and Mäler (2000) and Arrow et al. (2003), developing a theory of sustainable resource use in imperfect economies, proved that the accounting (or shadow) price[4] of a natural resource can be considerably higher than the market price, implying that the investment of the market resource rent and even the entire marked-valued output into man-made capital can be insufficient to compensate for damages caused by the resource extraction. In other words, GI in shadow prices may be negative despite any effort in saving, suggesting that, for some inefficient economies, institutional and resource policies are prerequisites of sustainability. Unfortunately, accurate shadow prices are not observable in real economies due to uncertainties in population growth, damages from economic activities, and future production possibilities (resource reserves estimates and rates of technical change). These uncertainties usually are dealt with by stochastic models in theory and

[4] The shadow price of the resource shows the change in the social (intergenerational) welfare when the resource stock is changed by one unit.

optimistic-pessimistic scenarios in practice. Both approaches result in some errors and, therefore, in inefficiencies. Asheim (2010), showing that the value of GI “cannot serve as a reliable indicator of sustainability,” noted also that inefficiency may further loosen the link between GI and sustainability.

The current paper continues the studies of sustainability indicators and policies in imperfect economies by examining the effects of disregarded inefficiencies. Proposition 1 provides the link between “undistorted” GI and current utility change (CUC) in a dynamically inefficient economy. This result extends Proposition 1 of Hamilton and Hartwick (2005), and shows that 1) CUC may be determined only by the influence of inefficiency when this influence is not close to zero; and 2) resource economies can be classified by importance of investment, resource, or institutional policies for CUC. This study (Corollary 1) extends also (to a more general form of inefficiency) the effect noticed by van der Ploeg (2011) that insecure property rights require more effort in investment to maintain constant per capita consumption. Corollary 1 adds, in particular, that underestimation of future production possibilities requires less investment than in the “undistorted” case.

The examples of distortions (Section 4) include 1) a resource-augmenting technical change that distorts the dynamics of the stock and leads to a sustainable but dynamically inefficient economy when a planner does not take it into account; 2) inadequate accounting system that ignores the damages from the resource use to utility and production, which leads to inefficiency and unsustainability; and 3) insecure property rights that also cause inefficiency and unsustainability, unless corrected by institutional reforms and resource policies.

The results of this study illustrate that, for sustainability, it is preferable to underestimate future production possibilities and overestimate damages since this policy of extra caution can reduce irreversible losses. Of course, this policy may lead to dynamic inefficiency caused by an overly conservative resource policy, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

Besides disregarded inefficiencies, the study shows how sustainability evaluation may depend on the specification of the same effects in the model and on the form of the indicator (Section 5). An indicator linked to CUC may be, by construction, insensitive to the changes in the long-term ability of an economy to maintain non-declining utility. On the other hand, an indicator that shows long-term sustainability, also by construction, may be insensitive to CUC, also leading to violation of sustainability at the initial point. Therefore, various indicators may complement each other. The dependence of sustainability evaluation on the model and on the form of the indicator is illustrated with the example of Russian oil extraction.

3. Investment and growth in inefficient economy

In order to define a distorted economy, it is instructive to introduce first a "perfect" or undistorted optimizing economy. Following Hamilton and Hartwick (2005, p. 618), assume that the economy is closed, time t is continuous, consumption is aggregated into a single good C, labour is fixed, so that output $Q(t) = F(K,R)$ depends on man-made capital $K(t)$ and the resource flow $R(t) = -\dot{S}(t)$, where $S(t)$ is the current resource stock ($\dot{S} := dS/dt$). The technology is stationary (F does not depend explicitly on t).

A number of studies, which results were used for practical evaluation of sustainability, assume that the economy satisfies the following:

- $F(K,R)$ is a regular production function that (a) denotes the maximum output for the given K and R, and (b) satisfies the Inada conditions, in particular $F_R > 0$ (*resource productiveness*), where $F_R := \partial F / \partial R$;
- output Q equals $F(K,R)$ (static efficiency);[5]
- the balance equation holds: $C + \dot{K} = F(K,R) - \delta K$, where $\dot{K}$ is investment and δK with $\delta = const$ is capital decay (non-wastefulness);
- the standard HR $\dot{F}_R = rF_R$ [6] holds as a necessary condition of dynamic efficiency;
- the economy (a planner) maximizes a (social) welfare function by choosing the paths of $\dot{K}$ and R (optimality).

In the real world, however, the resource use can be

- non-productive ($F_R = 0$) or counter-productive ($F_R < 0$);[7]
- productive, but static-inefficient ($Q < F(K,R)$);
- productive, static-efficient, but wasteful ($C + \dot{K} < F(K,R) - \delta K$);
- productive, static-efficient, non-wasteful, but dynamically inefficient;
- productive, non-wasteful, efficient, but not optimal.

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[5] Conventionally, efficiency is defined via the Pareto-optimality. Some studies, for example Hurwicz (1960), called this notion non-wastefulness.

[6] Here, $r(t) := F_K(t) - \delta$ is the market interest rate.

[7] The resource use is counter-productive when the decline in the resource stock results in the decline of output, for example, as a result of a wildfire or oil spill.

This paper assumes that there is a vector $\mathbf{D}(t) = (D_1(t), D_2(t), D_3(t), D_4(t))$, called *distortion*, which components are the distortions in

$$\text{production:} \quad F = F(K, R, D_1), \tag{1}$$

$$\text{social utility:} \quad U = U(C, D_2), \tag{2}$$

$$\text{the balance equation:} \quad \dot{K} = F(K, R, D_1) - C - \delta K - D_3, \tag{3}$$

$$\text{the dynamics of the stock:} \quad \dot{S} = -R + D_4. \tag{4}$$

The distortions may include imperfections, externalities, and any effects (including favourable for sustainability) that cause violation of the standard HR.

To illustrate the claims of this study, it is enough to accept the following

Assumptions: 1) $\mathbf{D}$ depends on parameters that do not depend on time explicitly[8] or

2) $\mathbf{D}$ depends only on the extracted amount $X(t) = \int_0^t R(\xi)d\xi = S_0 - S(t) + \int_0^t D_4(\xi)d\xi$.

Hence, $\mathbf{D}$ do not depend here on R,[9] K, and C. For example, D_1 and D_2 may result from irreversible damages caused by economic activities (stock externalities, for example, due to climate change); D_3 may stand for the growing cost of extraction (best-

[8] $\mathbf{D}$ may depend, for example, on the parameters that express institutional imperfections.

[9] In fact, $\mathbf{D}$ may depend on R, for example, when damage includes opportunity cost (Gaudet et al., 2006), or when damage is partly reversible. Then formula (6) below is more complicated, which does not alter the conclusions of the paper. $\mathbf{D}$ may also depend on the non-extracted resource, for example, when the stock has an amenity value (D'Autume, Schubert, 2008). Then, expressing this value in terms of utility, the problem can be reformulated by introducing the damage from the resource extraction. The latter approach may be more precise, since the uncertainty in the extracted stock is less than in the remaining. A review of studies with the modified HR is, for example, in Gaudet (2007).

quality stock extracted first) or for static inefficiency and (or) for wastefulness of the economy ($D_3 > 0$); D_4 may be the productivity of the stock-augmenting investment, which is, first, growing with the extraction due to learning-by-doing and eventually declining due to the scarcity of the resource.

Let D_5 be a deviation of the ratio $\dot{F}_R / F_R$ from a dynamically efficient path. Then the following result holds.

Lemma 1. *In economy (1)-(4),*

$$\dot{F}_R = [v(t) + \tau(t)]F_R, \tag{5}$$

where $v(t) := F_K - \delta$,[10] $\tau(t)$[11] $= \tau[\mathbf{D}(t)] := D_5 - \{\, \partial D_4 / \partial X$

$$+ [(U_{D_2} / U_C)\, \partial D_2 / \partial X + F_{D_1}\, \partial D_1 / \partial X - \partial D_3 / \partial X] / F_R \}\, \partial X / \partial S, \tag{6}$$

and $D_5 = 0$ *if the economy is dynamically efficient.*

Proof is in Appendix 1.

In this framework, dynamic efficiency is a relative notion. The planner's optimal path may be dynamically inefficient with respect to a first-best solution, for example, because the planner underestimates future production possibilities and considers the first-best path as infeasible (Section 4.1). The planner's path may also be inefficient when the planner ignores some effects while estimating social progress. In the latter case, the planner may even consider the first-best path as inefficient due to the difference between the units of measure for utility in the planner's and the first-best solutions (Section 4.2).

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[10] $v(t)$ is the market interest rate only with no distortion.

[11] $\tau(t)$ is the additive HR modifier or the influence of $\mathbf{D}$. This influence can be expressed in a multiplicative form: $\dot{F}_R = v\eta[\mathbf{D}]F_R$, where $\eta[\mathbf{D}] := 1 + \tau[\mathbf{D}]/v$. With no distortion, $\tau = 0$.

Deviation D_5 may depend on $\mathbf{D}$, for example on D_4, when a planner does not take into account resource-augmenting investments ($D_5 = (\partial D_4/\partial X)\cdot(\partial X/\partial S)$, Section 4.1), or on D_1 and D_2 when the planner ignores the damages from the resource extraction (Section 4.2). In some cases, however, D_5 may not depend on $\mathbf{D}$, and instead, both D_5 and $\mathbf{D}$ may be determined by the same phenomena, for example, imperfect institutions (Section 4.3).

Genuine investment (GI) defined in Hamilton and Hartwick (2005) is

$$G(t) := \dot{K}(t) + \dot{S}(t)F_R(t), \tag{7}$$

which equals $\dot{K}(t) - R(t)F_R(t)$ in the undistorted economy. This measure includes not only the investment into man-made capital $\dot{K}$ but the value of the extracted resource $\dot{S}$ estimated in the marginal resource productivity F_R, which, with no distortion, coincides with the market price. Therefore, G corresponds to a combination of investment and resource policies.

In the general case, GI is defined as $G(t) := \Sigma_i p_i \dot{K}_i(t)$ where K_i are various forms of man-made (including human and intangible) and natural capital, and p_i are the shadow prices equal to the marginal change in social welfare V resulting from a change in K_i: $p_i := \partial V / \partial K_i$ (Arrow et al. 2003).[12] By construction, G must coincide with the change in social welfare resulting from the combination of investment and resource policies at the

[12] For example, under the utilitarian criterion, $V(t) = \int_t^\infty U[K_1(s), K_2(s), \ldots]\pi(s)ds$, where π is a discount factor, and, under maximin, $V(t) = U^*[K_1(t), K_2(t), \ldots]$, where U^* is the maximum level of utility that can be maintained forever given the current assets $K_1(t), K_2(t), \ldots$.

current moment: $\dot{V}(K_1,K_2,\ldots) = \Sigma_i\left(\partial V / \partial K_i\right)\dot{K}_i$ (if V does not depend on time explicitly).[13] Then, if V reflects the ability of the economy to maintain non-declining utility, the indicator GI shows the change in sustainability of the economy.

As was mentioned above, sustainability of real economies is always evaluated under imperfections including imperfections in knowledge and in models that are used for this evaluation. Therefore, in practical sustainability evaluation, a model welfare function and the correspondent indicator GI never include all the factors that will affect the long-term path of utility in the real economy. In order to examine the effect of this discrepancy, this study considers indicator GI in the form of (7), which corresponds to the "undistorted" economy.

Since utility may be distorted by D_2, the dependence of utility on consumption may be nonmonotonic; therefore, consumption cannot substitute utility as a measure of well-being (see, for example, Section 4.2). Hence, the proposition below establishes the link between G and U, which includes the link between G and C as a special case.

Proposition 1. *Current utility change (CUC) in distorted economy (1)-(4) is*

$$\dot{U} = (v - \dot{G}^0 / G^0)G^0 U_C + \Psi, \tag{8}$$

where G^0 is the GI indicator for the undistorted economy, and

$$\Psi := -U_C[D_5 F_R R + (\dot{D}_3 - \dot{D}_1 F_{D_1} - \dot{D}_2 U_{D_2}/U_C - \dot{D}_4 F_R)(1 + \partial X / \partial S) + \dot{D}_4 F_R]$$

is the influence of dynamic inefficiency.

Proof is in Appendix 2.

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[13] If social welfare depends on endogenous processes such as population growth or technical change, the time derivative is $\dot{V}(t,K_1,K_2,\ldots) = \partial V / \partial t + \Sigma_i\left(\partial V / \partial K_i\right)\dot{K}_i$.

In a particular case, when the economy is dynamically efficient ($D_5 = \Psi = 0$), formula (8) can be obtained from the results of Dixit et al. (1980) and Hamilton and Withagen (2007), expressed in present prices (see the proof). With no distortion, Eq. (8) coincides with the result of Hamilton and Hartwick (2005). In the general case, Eq. (8) shows that investment (7) can indeed determine U if the influence of Ψ is relatively small. However, $\dot{U}$ can be also completely determined by Ψ when the term $(v - \dot{G}/G)GU_C$ is close to zero.

Of course, sharp changes in G can determine an instant sign of U despite the large values of Ψ. Formula (8) shows that if there is a $t = \bar{t}$, such that $\Psi(t)$ has a large positive (negative) value, $\dot{U}(t)$ can be negative (positive) if $G(\bar{t})$ is negative (positive) and $\dot{G}(\bar{t})/G(\bar{t})$ has a large positive (negative) value. However, these cases are not relevant to sustainability due to the boundedness of investments, whereas distortions in general are less restricted. The boundedness of investment implies that the larger is Ψ, the shorter is the period of time when these cases are possible. Therefore, neglecting the short-run oscillations, it can be assumed, for determinateness, that $\left|\dot{G}/G\right| < v < \infty$,[14] where $v > 0$, and the current investment $\dot{K}$ is bounded by the current output Q. Then a feasible investment can be defined as follows.

Definition 1. Investment $\dot{K}(t) = w(t)Q(t)$ is *feasible* if $w(t) \in$ (0,1) and $\left|\dot{G}/G\right| < v$ for any $t \geq 0$.

Definition 1 results in the following Corollary.

Corollary 1. *If $D_4 \equiv 0$, Eq. (8) implies that*

[14] The analysis can be easily complemented with the case where $\left|\dot{G}/G\right| \geq v$.

(I) for a feasible investment,

$$\dot{U} \gtrless 0 \text{ if and only if}$$

$$G \gtrless RF_R D_5 / \left(v - \dot{G}/G\right) \text{ or } w \gtrless \left[RF_R / Q\right]\left[1 + D_5 / \left(v - \dot{G}/G\right)\right],[15] \text{ or} \tag{9}$$

$$\Psi \gtrless -\left(v - \dot{G}/G\right)GU_C \text{ or}$$

$$D_5 \lessgtr \left(v - \dot{G}/G\right)\left(\dot{K}/\left(RF_R\right) - 1\right). \tag{10}$$

(II) a feasible investment can change the sign of $\dot{U}$ if and only if

$$-\left(v - \dot{G}/G\right) < D_5 < \left(v - \dot{G}/G\right)\left(Q/\left(RF_R\right) - 1\right). \tag{11}$$

The following examples show that the impact of dynamic inefficiency on the efficacy of investment depends on the level of output and the share of the resource rent in output. Assume that $D_4 = 0$, $v = 0.06$, and $\dot{G}/G = 0.03$ at $\bar{t} \geq 0$.

(a) Large resource-poor economy. Let $Q(\bar{t}) = 101$ and $R(\bar{t})F_R(\bar{t}) = 1$. Then (Corollary 1) an investment policy ($\dot{K}$) can change the sign of $\dot{U}(\bar{t})$ if and only if

$$-0.03 < D_5 < 3.$$

(b) Small resource-rich economy. For $Q(\bar{t}) = 11$ and $\dot{S}(\bar{t})F_R(\bar{t}) = -10$, an investment policy can affect the sign of $\dot{U}(\bar{t})$ if and only if

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[15] When $Q = F(K,R) = K^{\alpha}R^{\beta}$ and $D_5 = \xi^*(N-1)/K$, where N is the number of the owners of the common resource and ξ^* is the coefficient of violation of property rights, condition (9) takes the form: $w \gtrless \beta\left[1 + \xi^*(N-1)/(\alpha Q - \delta K - \dot{Q}K/Q)\right]$, which, for $\delta = 0$ and under requirement of constant per capita consumption, becomes $w = \beta\left[1 + \xi^*(N-1)/(\alpha Q)\right]$, coinciding with the result of van der Ploeg (2011). In more detail, this example is considered in Section 4.3 with a more general production function.

$-0.03 < D_5 < 0.003.$

It is intuitive that a large economy has more opportunities in investment than a small one, and so the range for D_5, in which investment is able to affect the sign of utility change, is larger in case (a) than in case (b). Another difference between these two cases is that investment in a large resource-poor economy can change the sign of $\dot{U}$ mostly when D_5 affects $\dot{U}$ negatively (positive D_5 reduces $\dot{U}$). This asymmetry is inverted in case (b).

The boundedness of investments implies that the current states of economies along the planner's paths belong to the one of the four following types determined by different roles of resource (institutional) and investment policies in the current change of utility depending on the level of inefficiency D_5.

(A) $D_5 \geq (v - \dot{G}/G)[Q/(RF_R) - 1]$: utility declines regardless of investment; non-negative values of $\dot{U}$ can be obtained only by reduction of the inefficiency if it is still possible.[16]

(B) $0 < D_5 < (v - \dot{G}/G)[Q/(RF_R) - 1]$: utility growth can be achieved by investment policy alone; the optimal saving rate is higher than under $\Psi = 0$ (see the second inequality in condition (9)) in order to compensate not only for the shrinking natural capital but for the negative effect of inefficiency. Without a policy reducing D_5, the level of utility may be lower than under $\Psi = 0$.

[16] Possibility of reduction of inefficiency depends on the state of the economy with respect to tipping points. This problem is not considered in this study.

(C) $-\left(v - \dot{G}/G\right) < D_5 < 0$: utility growth can be achieved by investment policy alone; the optimal saving rate may be lower than under $\Psi = 0$ due to the positive effect from D_5; decline in utility is still possible when $G < RF_R D_5 / \left(v - \dot{G}/G\right) < 0$.

(D) $D_5 < -\left(v - \dot{G}/G\right)$: utility grows regardless of investments; investment policy is important as a determinant of the level of utility along the growing path (Bazhanov 2008).

Types C and D may correspond to an economy where the planner underestimates future production possibilities (Section 4.1).

Condition (9) shows that, for $\Psi < 0$, the minimum investment G that provides non-declining utility may be essentially higher than zero. The next section illustrates that such a value of G, guaranteeing $\dot{U} \geq 0$, may not exist.

4. Dynamic inefficiency and sustainability: examples

In the examples below, D_5^0 denotes a deviation of the ratio $\dot{F}_R / F_R$ along the planner's optimal path from a first-best path and D_5^* – a deviation of this ratio along the first-best path from the planner's path ($D_5^* = -D_5^0$). For succinctness, the planner's optimal paths and the models that do not take into account some of the distortions are called below "undistorted."

4.1. Resource-augmenting technical change

Assume that D_4 is the only distortion in a real economy: $\dot{S} = -R + D_4(X, L_R/L)$, where L_R/L is the share of the exploration sector and $D_4(\bullet) \geq 0$ is the increment of the resource stock due to research (Takayama 1980).

If a planner does not use the information about D_4 and works with the undistorted model, the planner's decisions are inefficient with

$$D_5 = (\partial D_4 / \partial X)(\partial X / \partial S) = \dot{D}_4(\partial X / \partial S)/R.$$

However, the claim of Proposition 1 in this case is[17]

$$\dot{C} = (v - \dot{G}^0 / G^0)G^0,$$

which formally coincides with the efficient case. Indeed, if a sustainable path is feasible, the planner can implement it with the same policy G^0 even without additional opportunities. Inefficiency of the path $C(t)$ can be shown, for example, when the planner follows a constant-consumption criterion. As is known from theory, a higher resource stock results in a higher optimal level of constant consumption. Therefore, the efficient planner recalculates the optimal path $C^*(t)$ with the updates in the stock, which results in a piecewise-constant path with growing levels. Hence, $C^*(t)$ will be Pareto-superior to the continuous path $C(t) \equiv C(0)$.

In this example, the discrepancy between theory and real life may result in a sustainable but inefficient path. Dynamic inefficiency can be reduced only by adjustment of

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[17] Utility is not distorted here ($U_{D_2} = 0$); hence, formula (8) becomes $\dot{C} = (v - \dot{G}^0 / G^0)G^0$ since $\dot{U} = U_C\dot{C} + U_{D_2}\dot{D}_2$ and $\Psi / U_C = -D_5F_RR + \dot{D}_4F_R(\partial X / \partial S) = 0.$

the resource policy when the planner updates the information about reserve estimates. This adjustment can result in sustainable and asymptotically efficient economy.

4.2. Inadequate accounting system

Assume that production and social utility are negatively affected in a real economy by the damage $D = D_1 = D_2$ caused by a stock externality[18] ($D_X > 0$, $U_D < 0$, $F_D < 0$). If a planner uses an accounting system that disregards the damage, then, according to Lemma 1, the planner's paths are dynamically inefficient with $D_5^0 = -(F_D + U_D / U_C) D_X / F_R > 0$.[19] The planner's problem reduces in this case to the one of Solow (1974) - Hartwick (1977), where, under the maximin criterion, the path of extraction starts from a higher level than in the efficient case,[20] and the economy follows a constant-consumption path (due to $G = 0$) with a higher level than the initial level of the efficient path, which is measured in utility units. Since the planner assumes that $U_D = F_D = 0$, formula (8) becomes

$$\dot{C} = (v - \dot{G}^0 / G^0) G^0.$$

In reality, however, the change in well-being is[21]

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[18] For example, D may result from irreversible climate change (Stollery 1998).

[19] As usual, $U_C > 0$.

[20] When damage affects only production, Stollery (1998, p. 735) showed that the optimal extraction starts from a lower initial level and declines slower than in the case with no damage. The same result for damage in utility was obtained in Bazhanov (2012, formula (33), Fig. 4).

[21] Eq. (12) can be obtained from Eq. (8) using the expression for D_5^* and the fact that $\dot{D} = R D_X$.

$$\dot{U} = (v - \dot{G}^0 / G^0) G^0 U_C + (F_D U_C + U_D)\dot{D}, \qquad (12)$$

which is negative for the planner's paths of investment and extraction ($G^0 = 0$) since $(F_D U_C + U_D)\dot{D} < 0$. The investment G that provides $\dot{U} > 0$ does not exist here when the damage is large, namely, when

$$(F_D U_C + U_D)\dot{D} < -[v(Q - RF_R) - \dot{G}]U_C < 0.$$

Hence, the undistorted policies result in inefficiency and unsustainability of this economy.

A non-declining path of the true quality of life (utility) can be achieved in this example only when the planner recognizes the damages and reconsiders the measure of progress in the society.[22] This done, the planner, situated in the Solow-Hartwick case, can obtain a sustainable and first-best optimal path by changing the resource policy alone, namely, by reducing extraction, while the investment rule remains the same.[23]

4.3. Insecure property rights

Following Arrow et al. (2003, p. 664), assume that the owner i ($i = 1 \ldots N; N \geq 2$) extracts a liquid resource from the pool with the stock S_i. All N owners are identical, non-

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[22] A review on development of the theory of social accounting is in Aronsson and Löfgren (2010). A practical illustration of the changes in the measure of social progress is the development of the Integrated Environmental and Economic Accounting, which was first offered as a handbook on environmental accounting in UN (1993) and eventually became a legal base in EU (2011).

[23] Stollery (1998) showed that the Hartwick rule ($G = 0$) is still optimal in this economy.

cooperative, and the pools are separated by porous barriers. The resource diffuses from larger pools to smaller ones with the same rate $\lambda > 0$.[24] Then the depletion equations are

$$\dot{S}_i = \lambda \sum_{j \neq i} \left(S_j - S_i \right) - R_i,\ i = 1 \ldots N,$$

where $R_i = R_i(t)$ is the rate of extraction of the owner i at the moment t. The necessary conditions for PV-maximization of the each owner's utility yield equation (5) with $\tau = D_5 = (N-1)\lambda > 0$ (socially efficient paths require N=1). This inefficiency results in the distorted path of extraction

$$\tilde{R}(t) = \sum_{i=1}^{N} R_i(t) = [(\rho + D_5)/\eta] S_0 e^{-(\rho + D_5)t/\eta}$$

with the higher initial rate $\tilde{R}(0)$ and faster decline $\dot{\tilde{R}}(0)$ than for the efficient path

$$R(t) = [\rho/\eta] S_0 e^{-\rho t/\eta}.$$

In these formulas, $\rho > 0$ is the social discount rate, and $\eta > 1$ – the elasticity of marginal utility. Hence, the distorted equation for the whole reserve is $\dot{S} = -R + D_4$, where $D_4 = R - \tilde{R}$. In this example, D_4 does not depend directly on the extracted resource X, and deviation D_5 does not depend on D_4; both these distortions result from imperfect institutions, expressed in $N \geq 2$ and $\lambda > 0$.[25]

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[24] No barriers corresponds to $\lambda \rightarrow \infty$.

[25] Formally, the link $D_5(D_4)$ is $D_5 = -\rho - (\eta/t) W\left[D_4 t / S_0 - (\rho t/\eta) e^{-(\rho t/\eta)} \right]$, where $W[\bullet]$ is the Lambert W function. Numerically, using computational software (for example, Maple), this formula gives $D_5 = (N-1)\lambda$ for any t when $\arg\{W[\bullet]\} > -1/e$. Also, formally, D_4 changes with X since both are changing in time. However, it can be shown that D_4 cannot be represented as a function of X only.

For illustration, assume that $v = 0.06$, and $\dot{G}/G = -0.04$. Since $D_5 \geq 0$ for any $t \geq 0$, a feasible investment $\dot{K}$ can change the sign of $\dot{U}$ if and only if (Corollary 1)

$$(N-1)\lambda < 0.1\left[Q/\left(RF_R\right)-1\right] > 0.$$

When this condition is not satisfied, only institutional changes and resource policies can prevent decline in utility. An investment provides non-declining utility here if and only if

$$G \geq 10(N-1)\lambda RF_R > 0 \quad \text{or} \quad w \geq \left[10(N-1)\lambda + 1\right]\left(RF_R/Q\right),$$

which may be very restrictive for $N > 1$.

It is illustrative to consider two cases.

(a) Large resource-poor economy ($Q = 101; RF_R = 1$). In this case, $\dot{K}$ can change the sign of $\dot{U}$ if and only if $(N-1)\lambda < 10$, which means, for example, that, for $\lambda = 1$, utility declines for any $\dot{K}$ (type A) if $N > 10$. Let $N = 5$. Then the saving rate, compensating for the shrinking resource and inefficiency, should be no less than $w_{\min} = 41/101$ (or $G/Q \geq 40/101$), whereas with no distortion ($N = 1$), utility grows for any $w > 1/101$ (or $G > 0$).

(b) Small resource-rich economy ($Q = 11; RF_R = 10$). $\dot{K}$ can change the sign of $\dot{U}$ if and only if $(N-1)\lambda < 0.01$; for example, for $\lambda \geq 0.01$ and $N \geq 2$, utility declines regardless of any feasible investment. Let $\lambda = 0.009$ and $N = 2$. Then not declining utility is possible when

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Namely, the assumption $D_4 = D_4(X)$, given τ, the expression $D_4 = R - \tilde{R}$, and using Lemma 1, results in $D_4 = (N-1)\lambda X + D_4(0)$, since $D_1 = D_2 = D_3 = 0$. Here, $D_4(0) = -(N-1)\lambda S_0/\eta$. However, since the reserve S_0 is fixed, D_4 must result only in intertemporal redistribution of the resource, namely, the condition $\int_0^{\infty} D_4(t)dt = 0$ must hold, which is not true for D_4 derived in this way.

almost all output is being invested, namely, $w \geq 10.9/11$, although, for this resource-dependent economy, even with no distortion ($N = 1$), the saving rate yielding at least constant utility must be very high, namely, $w_{\min} = RF_R / Q \approx 0.91$.

The use of the policies for undistorted model results in this example in inefficiency and unsustainability; moreover, a feasible investment compensating for the inefficiency and providing non-declining utility may not exist. Therefore, the policy for increasing sustainability by increasing investments, which is recommended by the undistorted model, may cause a sharp decline in current utility below a subsistence minimum without reaching the goals of sustainability. This outcome implies that institutions must be corrected first in order to reduce inefficiency and only secondly the policies for the undistorted model can be applied.

5. Model dependency of sustainability evaluation

As is known, the gap between sustainability evaluation and actual sustainability depends not only on disregarded inefficiencies. The example provided in this section shows how sustainability evaluation may depend on the form of the indicator and on the specification of the same effects in the model.

5.1. Indicators GI based on CUC and on the change in intergenerational welfare

Indicator GI that shows CUC may not reflect the change in the ability of an economy to maintain non-declining utility during a long period of time. For example, in the DHS economy ($Q = F(K, R) = K^{\alpha} R^{\beta}$), indicator (7) expressed as a share of GDP takes the form $G/Q = w - \beta$, where w is the rate of investment ($\dot{K} = wQ$). Assume, for simplicity, that utility monotonically depends on consumption (no damages). In this economy with $\alpha > \beta$, an economic program with non-declining consumption exists at $t = t_0$ if and only if[26]

$$G_\infty / Q\big|_{t=t_0} := w - KR\beta / [S_0 Q(\alpha - \beta)]\big|_{t=t_0} \geq 0. \tag{13}$$

In this important particular case of the indicator GI, the resource price shows the change in the maximum level of consumption that can be maintained forever, while the resource stock is changed by one unit. Note that G_∞, as it is expressed in (13), is always non-negative for $\alpha < \beta$, i. e., when a non-declining path of consumption does not exist (Solow 1974). Therefore, if α and β are to be determined by calibration (for example, Bazhanov 2011, Section 3), a more convenient form of this indicator is $G_\infty = \alpha - \beta - KR\beta / (S_0 \dot{k})\big|_{t=t_0}$. This expression is always negative when a non-declining path of consumption does not exist.

Hence, this economy is *potentially sustainable* at $t = t_0$ (a program with non-declining utility exists)[27] if and only if $R(t_0) / S_0 \leqslant wQ(\alpha - \beta) / [\beta K]\big|_{t=t_0}$, which means that potential

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[26] Derivation of G_∞, which is called the *level of potential sustainability*, is provided in Bazhanov (2011).

[27] This definition, offered in Bazhanov (2011), is partly equivalent to the following definition of Pezzey (2004): an economy is *sustainable at time* t_0, if $U(t_0) \leq U_{\max}$ (the economy is not overconsuming at t_0), where $U_{\max}$ is the maximum sustainable level of utility that can be maintained forever, given the stocks of man-made and natural capital at t_0. The equivalence is only partial because, as Pezzey noted, his definition works only for efficient economies. An overextracting inefficient economy may be underconsuming due to inefficient use of the resource. For such an economy, a sustainable program may not exist; therefore, for sustainability evaluation of real economies, it is preferable to use the tools that can work under inefficiencies since real economies are, as a rule, inefficient.

sustainability can be achieved by a change in resource policy only, regardless of the value of $G/Q|_{t=t_0}$ and the sign of $\dot{C}(t_0)$. The indicators G and G_∞ coincide if and only if the state of the economy satisfies a "perfection" condition ($R/S_0 = Q(\alpha - \beta)/K]$) with respect to a constant-consumption criterion (Bazhanov 2010).

On the other hand, the indicator G_∞ does not reflect CUC and current level of utility. As a result, a positive value of this indicator may "approve" a sharp decline in the current rate of the resource extraction, which may lead to a drop in the level of utility below a subsistence minimum, violating intergenerational justice. Of course, unsustainability of an economy ($G_\infty < 0$) calls for changes in investment and resource policies to increase G_∞, which requires 1) to define the maximum value of G_∞ for an optimal growth; and 2) the optimal path of acceptable sacrifices in current utility for future sustainable development.

5.2. The models of Russian oil extraction and sustainability evaluation

For any sustainability indicator, the result of sustainability evaluation may be predetermined by a specification of the model. For example, Bazhanov (2011) provides a comparison of sustainability policies for the Russian economy, resulting from three different models that describe the role of oil in the economy. The assumptions of these models and the corresponding policy recommendations are briefly discussed below.

(a) Oil as an equivalent to a financial asset. The studies of IMF (International Monetary Fund) offer a model for recommending sustainable budget policy to oil-exporting countries including Russia (Barnett, Ossowski 2003; Jafarov et al. 2006). Oil enters the model only as

an export good but not as a production factor. The budget constraint in this model can be written as follows:

$$C(t)+\dot{K}(t)\leq F_I[A(t),K(t),L(t)]+p(t)R(t)+iK_W(t),$$

where $C(t)$ – the aggregate government and private consumption at the time t; $\dot{K}(t)$ – investment into government and private "non-oil" capital $K(t)$; $R(t)$ – the rate of oil extraction (all oil is being exported); $p(t)$ – the export price of oil; K_W – the government holdings in the world's financial assets; i – the rate of return on K_W (constant); $F_I[A(t),K(t),L(t)]$ – domestic "non-oil" production function, where F_I – Cobb-Douglas function; $A(t)$ – the level of technology, exogenously growing at a constant rate; $L(t)$ – labour.

Indicators GI are not relevant to this model since the resource does not enter the production function. This model assumes that oil and financial assets are perfect substitutes (the elasticity of substitution between them is infinity). Therefore, the model can maintain non-declining consumption during an infinite period of time relying only on the interest from financial assets (iK_W) when oil is depleted. As a result, the IMF studies offer prospects of infinitely non-declining per capita consumption under the assumption that oil reserves will be exhausted by the middle of the 21st century.

(b) Closed DHS model. Evaluation of sustainability requires that the model must allow both for sustainable and unsustainable outcomes depending on the paths of extraction and investment. The DHS model, which is based on the Cobb-Douglas production function with a resource as a factor, is the simplest model that satisfies this requirement:

$$Q(t)=F(t,K(t),R(t))=A(t)K(t)^{\alpha}R(t)^{\beta}L(t)^{\gamma},$$

where $\alpha,\beta,\gamma \in (0,1); \alpha+\beta+\gamma=1$, Q – GDP, A – the scale multiplier (TFP), K – the stock of capital, R – the rate of resource extraction, L – labor, which is constant and equal to population. In the DHS model, the resource is a necessary[28] factor of production, and the elasticity of substitution between the resource and man-made capital is unity.

Calibration of this model based on the data of the Russian economy[29] showed that $G>0$ and $G_\infty>0$ (the model is potentially sustainable). However, this model recommends a more cautious resource policy than in case (a) since the existence of a program with non-declining per capita consumption requires that the rates of oil extraction only asymptotically approach zero, always remaining positive. If oil is depleted in finite time, the model collapses. In practice, this means that a sustainable policy can be implemented, for example, by development of renewable energy and the use of oil only for production of recyclable materials with gradual reduction of oil input due to decreasing dissipation of these materials as a result of technical progress.

(c) Open model (Bazhanov 2011, Section 5.2). In comparison with case (b), the model below specifies the production of GNP in more detail, which presumably should result in a more accurate sustainability evaluation; the incomes from oil export and foreign assets are considered here (as in case (a)) as separate parts of production:

$$y = c + \dot{k}_T = f_I(k_I, r_I) + pr_E + ik_W,$$

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[28] A resource is *necessary* for production if output is zero in the absence of the resource and positive in the presence of any positive amounts of the resource and other necessary factors.

[29] In more detail, see Bazhanov (2011), Sections 3 and 5.1.

where y – GNP, $f_I = Ak_I^{\alpha_I} r_I^{\beta_I}$ – domestic production except oil export, r_I, r_E – the rates of domestic use and export of oil, p – the price of oil, k_W – holdings in the world's financial assets: $k_W = k_T - k_I$, k_T – total capital (domestic k_I plus the assets abroad), i – interest rate on k_W. The variables y, f_I, c r_I, r_E, r_W, k_T, k_W, and k_I are in per capita units. The main difference between this model and the model in case (a) is that the flow of oil r_I, which is used in domestic production, is included in production function as a factor.

Unlike the closed case (b), the open model, calibrated in the same way and using the same data (plus the data on incomes from foreign assets and oil exports), proved to be not only unsustainable ($G_\infty < 0$ and $G < 0$) but *unsurvivable* (consumption declined to zero along any feasible program).[30] Some hypothetical changes in the data, which may correspond to a more thrifty resource policy and capital-labour augmenting advances in technical progress, may result in potential survivability ($\alpha > \beta$) with $G_\infty < 0$ and $G > 0$, or in potential sustainability with $G_\infty > 0$ and $G > 0$.

Hence, a model that is used for sustainability evaluation may predetermine a too optimistic forecast[31] if this model 1) does not reflect the dependence of the production on a resource, as in case (a); or 2) does not include trade effects, as in case (b). In fact, the model

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[30] The model was unsurvivable because calibration yielded $\alpha < \beta$, violating the Solow (1974) condition ($\alpha > \beta$) that allows to stretch out a limited reserve over an infinite period of time.

[31] A model, of course, may predetermine a too pessimistic forecast if, for example, in a real economy, oil can be substituted by renewable sources of energy but a model assumes that the elasticity of substitution between oil and man-made capital is zero: $f = A\min\{\alpha k, \beta r\}$. In this case, output declines to zero with the exhaustion of the resource despite any efforts in investment.

in case (c), which is the most "pessimistic" in this example, may also be too optimistic because of the following unrealistic assumptions: 1) the resource is always productive; 2) the economy is always non-wasteful; 3) the model does not include all the resources that are necessary in production; and 4) there are no damages to utility and production, resulting from the resource use. Therefore, a resource policy based on this model can be considered only as an upper bound for the real path of extraction.

6. Conclusions

This paper examined the effects of unaccounted inefficiencies on credibility of sustainability indicators and efficacy of investment. The study assumed that a social planner constructed optimal paths using a model that is not sufficiently adequate to the problem, for example, due to imperfections in knowledge or institutions. In order to imitate this natural discrepancy between theory and real life, this study extended the result of Hamilton and Hartwick (2005) regarding the role of genuine investment (GI) in current utility change (CUC) by assuming that 1) there are distortions that affect utility, production, balance equation, and the dynamics of the resource stock; and 2) the planner ignores some of the distortions. As a result, the planner's paths are dynamically inefficient. Proposition 1 established the link between GI and CUC depending on the influence of this inefficiency.

Proposition 1 and Corollary 1 showed that CUC can be determined by GI only when the influence of inefficiency is close to zero. These results entail a classification of resource economies by the importance of institutional, investment, and resource policies for CUC.

The examples of inefficient economies demonstrated that 1) in the presence of inefficiencies, a feasible level of investment that provides a positive CUC may not exist; 2) an economy may be sustainable, when the planner underestimates future production possibilities, or unsustainable, when the planner uses an inadequate accounting system or ignores institutional imperfections.

The results of this study imply that, for sustainability of real economies, 1) institutional and resource policies may be more important than investment when the level of inefficiency is high; and 2) it is preferable that a resource policy is more conservative than is prescribed by a theory. In the former conclusion, investment policy, of course, is still important as a determinant of the level of utility along a growing or declining path and as a determinant of growth when the level of inefficiency is low. In the latter conclusion, an overly conservative resource policy may result in dynamic inefficiency, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

Besides the influence of disregarded inefficiencies, an indicator that is linked to CUC may not reflect sustainability, because, by construction, this indicator does not show the change in the long-term ability of the economy to maintain non-declining utility. In order to evaluate current sustainability change, the change in the resource stock should be measured in prices that show, for example, the change in the maximum level of utility that can be maintained forever. However, an indicator based on these prices may not reflect CUC. As a result, an increase in the long-term welfare may be obtained at the cost of an unacceptable decline in current utility, violating the principles of sustainability and intergenerational justice. The properties of various forms of sustainability indicators imply that they may complement each other in sustainability evaluation. Alternatively, another indicator that is consistent with a criterion of intergenerational justice at any moment in time can be constructed.

Despite all the difficulties with the practical use of indicator GI, the algorithm of its calculation undoubtedly provides useful information for policymakers by showing changes in factors that influence social welfare and sustainability. It is apparent that the development of knowledge will further improve the processes of gathering this information and its use for sustainability policies.

7. Appendix 1. Proof of Lemma 1

Since optimal paths are always efficient, a necessary condition of dynamic efficiency for economy (1)-(4) can be obtained from optimality conditions, for example, in the problem of PV-maximization[32] of $\int_0^{\infty} U(C,D_2)\pi(t)dt$ with a discount factor $\pi(t)$. The Hamiltonian of this problem is $H = U(C,D_2)\pi(t) + \mu_K(F - C - \delta K - D_3) + \mu_S(D_4 - R)$,[33] and the Pontryagin-type necessary conditions are

$$H_C = U_C\pi(t) - \mu_K = 0, \tag{14}$$

$$H_R = \mu_K F_R - \mu_S = 0, \tag{15}$$

$$\dot{\mu}_K = -\partial H/\partial K = -\mu_K(F_K - \delta), \tag{16}$$

$$\dot{\mu}_S = -\partial H/\partial S = -\{\pi(t)U_{D_2}\,\partial D_2/\partial X + \mu_K(F_{D_1}\,\partial D_1/\partial X - \partial D_3/\partial X)$$

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[32] The maximin, formulated as $\max\limits_{R,C} \int_0^{\infty} \overline{U}\delta e^{-\delta t}dt \equiv \overline{U} = const(R,C)$ with the additional constraint $U(C,D_2) = \overline{U}$, yields the same result (Leonard and Long (1992, 300–304)).

[33] Here and below, μ_K and μ_S are indexed dual variables for capital and resource stock unlike $H_C, H_R, U_C, U_{D_2}, F_K$, and F_R, which are the partial derivatives of H, U, and F.

$$+\mu_S \, \partial D_4 / \, \partial X\} \, \partial X / \, \partial S. \tag{17}$$

Eq. (17) with μ_K from (14) becomes

$$\dot{\mu}_S = -\{\pi(t)U_{D_2} \frac{\partial D_2}{\partial X} + U_C \pi(t)(F_{D_1} \frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X}) + \mu_S \frac{\partial D_4}{\partial X}\} \frac{\partial X}{\partial S}. \tag{18}$$

The time derivative of Eq. (15) is $\dot{\mu}_S = \dot{\mu}_K F_R + \mu_K \dot{F}_R$, which, combined with (18), results in

$$\dot{\mu}_K F_R + \mu_K \dot{F}_R = -\{\pi(t)[U_{D_2} \frac{\partial D_2}{\partial X} + U_C (F_{D_1} \frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X})] + \mu_S \frac{\partial D_4}{\partial X}\} \frac{\partial X}{\partial S}.$$

The last equation, after dividing through by F_R and substitutions for $\dot{\mu}_K$ from (16) and μ_S from (15), becomes

$$-\mu_K (F_K - \delta - \frac{\dot{F}_R}{F_R}) = -\{\frac{\pi(t)}{F_R}[U_{D_2} \frac{\partial D_2}{\partial X} + U_C (F_{D_1} \frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X})] + \mu_K \frac{\partial D_4}{\partial X}\} \frac{\partial X}{\partial S},$$

which, divided through by μ_K with substitution for μ_K from (14), yields $\dot{F}_R / F_R = F_K - \delta + \tau(\mathbf{D})$, where $\tau(\mathbf{D})$ is defined by Eq. (6)■

8. Appendix 2. Proof of Proposition 1

The proof follows the approach of Hamilton and Hartwick (2005, Proposition 1), which was first applied in Hartwick (1977). The differences are that the current proof uses: 1) utility as a measure of well-being (due to distortion D_2); 2) a modified HR to substitute for $\dot{F}_R$ instead of the standard HR. Namely, equations (1), (3), and (5) give

$$\begin{aligned}\dot{C} &= \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 = \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 + R\dot{F}_R - R\dot{F}_R \\ &= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R R - \left[\ddot{K} - d(RF_R)/dt\right] + \dot{D}_1 F_{D_1} - \dot{D}_3.\end{aligned}$$

The planner uses the indicator GI for the undistorted model: $G = \dot{K} - RF_R$. Then

$$\dot{C} = vG - \dot{G} + \dot{D}_1 F_{D_1} - \dot{D}_3 - \tau F_R R. \tag{19}$$

Assumptions 1 and 2 imply that $\dot{D}_i = \dot{X}\, \partial D_i / \partial X$. Then, using Lemma 1,

$$\tau F_R R = D_5 F_R R - \{\dot{D}_1 F_{D_1} + \dot{D}_2 U_{D_2} / U_C - \dot{D}_3 + \dot{D}_4 F_R\}\, \partial X / \partial S.$$

Substitution of this expression into (19) results in

$$\dot{C} = vG - \dot{G} - D_5 F_R R + (\dot{D}_1 F_{D_1} - \dot{D}_3)(1 + \partial X / \partial S) + (\dot{D}_2 U_{D_2} / U_C + \dot{D}_4 F_R)\, \partial X / \partial S,$$

which, after substitution into $\dot{U} = U_C \dot{C} + U_{D_2} \dot{D}_2$, yields

$$\dot{U} = U_C [vG - \dot{G} - D_5 F_R R - (\dot{D}_3 - \dot{D}_1 F_{D_1} - \dot{D}_4 F_R)(1 + \partial X / \partial S) - \dot{D}_4 F_R] + \dot{D}_2 U_{D_2} (1 + \partial X / \partial S).$$

The use of the definition of Ψ results in Eq. (8) of the proposition.

When economy (1)-(4) is dynamically efficient ($D_5 = 0$), Eq. (8) can be obtained from the result of Dixit et al. (1980, Theorem 1) or from a generalization of this result in Hamilton and Withagen (2007). Namely, in terms of the present value prices of utility, capital, and the resource, defined in Lemma 1 as $\pi(t), \mu_K,$ and $\mu_S,$ these results claim that

$$\pi(t)\dot{U} = -\frac{d}{dt}\left(\mu_K \dot{K} + \mu_S \dot{S}\right),$$

which, using Eqs. (14)-(16), can be rewritten as follows:

$$\begin{aligned}\pi(t)\dot{U} &= -\frac{d}{dt}\left[\mu_K\left(\dot{K} + \dot{S}\mu_S / \mu_K\right)\right] = -\frac{d}{dt}\left[\mu_K\left(\dot{K} + F_R \dot{S}\right)\right] \\ &= -\frac{d}{dt}\left[\mu_K G\right] = -\mu_K\left(G\dot{\mu}_K / \mu_K + \dot{G}\right) = \mu_K\left[\left(F_K - \delta\right)G - \dot{G}\right].\end{aligned}$$

Then, with the use of formula (14) and the notation $v := F_K - \delta,$ it becomes

$$\dot{U} = (v - \dot{G} / G) G U_C,$$

which is Eq. (8) for $D_5 = \Psi = 0$ ■

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