Observability of information gathering in agency models

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Abstract

We consider an adverse selection model in which the agent can gather private information before the principal offers the contract. There are two scenarios. In scenario I, information gathering is a hidden action, while in scenario II, the principal observes the agent's information gathering decision. We study how the two scenarios differ with respect to the agent's expected rent, the principal's expected profit, and the expected total surplus. In particular, it turns out that the principal may be better off when the agent's information gathering decision is a hidden action.

Keywords: Hidden information; adverse selection; information gathering.

JEL Classification: D82; D86; C72

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1 Introduction

Agency models with precontractual private information play a central role in contract theory.[1] While in standard adverse selection models the information structure is exogenously given, more recently some authors have accounted for endogenous information structures.[2] The contributions to the literature on information gathering differ in several respects. In particular, some authors (e.g., Kessler 1998) assume that the information gathering decision is observable, while others (e.g., Crémer, Khalil, and Rochet, 1998) assume that it is a hidden action. Hence, it is interesting to investigate the effects of observability of information acquisition in a unified framework. Is the agent better off if information gathering is a hidden action? Is the principal better off if she can observe whether the agent has gathered private information?

In Section 2, we introduce a simple adverse selection model in which costly information gathering before the contract is offered may be pursed for rent seeking purposes only, since it is commonly known that it is always ex post efficient to trade.[3] We consider two scenarios. In the first scenario (Section 3), the principal cannot observe whether the agent has spent resources to gather information. In the second scenario (Section 4), the principal can observe the agent's information gathering decision. In Section 5, we analyze how the agent's expected rent, the principal's expected profit, and the expected total surplus differ between the two scenarios.

2 The model

Consider a principal and an agent, both of whom are risk-neutral. The principal wants the agent to produce the quantity $x \in [0,1]$ of a specific good. The principal's return is xR and the agent's production costs are xc.

At date 0, nature draws the cost parameter c. While both parties know that the distribution of $c \in \{c_l, c_h\}$ is given by $p = prob\{c = c_l\}$, at date 0 no one knows the realization of c. At date 1, the agent decides whether ($\lambda = 1$) or not ($\lambda = 0$) he wants

[1] See the seminal contributions by Myerson (1981), Baron and Myerson (1982), and Maskin and Riley (1984).

[2] For a survey, see Bergemann and Välimäki (2006).

[3] Information gathering is also a strategic rent-seeking activity in Crémer and Khalil (1992, 1994) and Crémer, Khalil, and Rochet (1998). While Crémer and Khalil (1994), Crémer, Khalil, and Rochet (1998), and Kessler (1998) also assume that information gathering can occur before the contract is offered, some authors have studied models in which information gathering can occur after the contract is offered but before it is accepted (see Crémer and Khalil, 1992 and Hoppe and Schmitz, 2010). Note that in the latter case observability of information gathering is irrelevant.

to incur information gathering costs $\gamma > 0$ to privately learn the realization of his production costs.[4] At date 2, the principal offers a contract to the agent. At date 3, the agent decides whether to reject the contract (so that the principal's payoff is 0 and the agent's payoff is $-\lambda\gamma$) or whether to accept it. If the agent accepts the contract, at date 4 production takes place and the principal makes the contractually specified transfer payment t to the agent. Then the principal's payoff is $xR - t$ and the agent's payoff is $t - xc - \lambda\gamma$.

We assume that $R > c_h > c_l$. Thus, it is common knowledge that $x = 1$ is the first-best trade level, regardless of the state of nature. This implies that costly information gathering is an unproductive rent-seeking activity only.

We will compare two scenarios. In scenario I, the principal cannot observe the agent's information gathering decision λ. In contrast, in scenario II the principal observes the agent's decision λ (while she can never observe the realization of c).

3 Scenario I

In scenario I, the principal cannot observe whether the agent has gathered information. Let $\pi \in [0, 1]$ denote the probability with which the agent gathers information at date 1.

Consider first the principal's contract offer. Suppose the principal believes the agent has gathered information with probability π. According to the revelation principle, the principal can confine her attention to direct mechanisms $[x_l, t_l, x_h, t_h, x_u, t_u]$ to maximize her expected payoff

$$\pi[p(x_l R - t_l) + (1-p)(x_h R - t_h)] + (1-\pi)(x_u R - t_u) \tag{1}$$

subject to the incentive compatibility constraints

$$\begin{aligned}
t_l - x_l c_l &\geq t_h - x_h c_l, && (IC_{lh})\\
t_l - x_l c_l &\geq t_u - x_u c_l, && (IC_{lu})\\
t_h - x_h c_h &\geq t_l - x_l c_h, && (IC_{hl})\\
t_h - x_h c_h &\geq t_u - x_u c_h, && (IC_{hu})\\
t_u - x_u E[c] &\geq t_h - x_h E[c], && (IC_{uh})\\
t_u - x_u E[c] &\geq t_l - x_l E[c], && (IC_{ul})
\end{aligned}$$

[4] We thus consider the same information gathering technology as Crémer and Khalil (1992, 1994) and Crémer, Khalil, and Rochet (1998). In contrast, Kessler (1998) studies a model in which the agent chooses information gathering expenditures that determine the probability with which he becomes informed.

the participation constraints

$$\begin{aligned} t_l - x_l c_l &\geq 0, & (PC_l) \\ t_h - x_h c_h &\geq 0, & (PC_h) \\ t_u - x_u E[c] &\geq 0, & (PC_u) \end{aligned}$$

and the feasibility constraints $x_l \in [0,1]$, $x_h \in [0,1]$, and $x_u \in [0,1]$.

Observe that the participation constraint (PC_l) of the low-cost type is redundant, as it is implied by (IC_{lh}) and (PC_h). Similarly, the participation constraint (PC_u) is redundant because it is implied by (IC_{uh}) and (PC_h). Moreover, note that the incentive compatibility constraints (IC_{lu}) and (IC_{ul}) imply the monotonicity constraint $x_l \geq x_u$, while similarly (IC_{hu}) and (IC_{uh}) imply $x_u \geq x_h$.

Ignore for a moment the incentive compatibility constraints (IC_{hl}), (IC_{hu}), and (IC_{ul}), which will turn out to be satisfied by our solution. It is then easy to see that (PC_h) must be binding; i.e., it is optimal for the principal to set

$$t_h = x_h c_h, \tag{2}$$

because otherwise we could increase the principal's expected profit by decreasing t_h without violating any of the remaining constraints. Furthermore, (IC_{uh}) must be binding, so that it is optimal for the principal to set

$$t_u = x_h(c_h - E[c]) + x_u E[c], \tag{3}$$

because otherwise she could decrease t_u without violating any of the remaining constraints. Observe that (2) and (3) together with the monotonicity constraint $x_u \geq x_h$ imply that the right-hand side of (IC_{lu}) is larger than the right-hand side of (IC_{lh}). Thus, it is optimal for the principal to set

$$t_l = x_h(c_h - E[c]) + x_u(E[c] - c_l) + x_l c_l, \tag{4}$$

so that (IC_{lu}) is binding. It is straightforward to check that the omitted constraints (IC_{hl}), (IC_{hu}), and (IC_{ul}) are indeed satisfied if (2), (3), (4), and $x_h \leq x_u \leq x_l$ hold.

Hence, the principal's problem can be simplified. She chooses $x_l \in [0,1]$, $x_h \in [0,1]$, and $x_u \in [0,1]$ in order to maximize her expected profit

$$\begin{aligned} & x_l[\pi p(R - c_l)] \\ & + x_u[R - E[c] - \pi(p(E[c] - c_l) + R - E[c])] \\ & + x_h[\pi(1-p)(R - E[c]) - (c_h - E[c])] \end{aligned} \tag{5}$$

subject to the monotonicity constraint

$$x_h \leq x_u \leq x_l. \tag{6}$$

The payments t_l, t_h, and t_u are given by (2), (3), and (4).

To solve the simplified problem, note that it is optimal for the principal to set $x_l = 1$, since $\pi p(R - c_l) \geq 0$. Moreover, the coefficient of x_u is strictly positive whenever

$$\pi < \hat{\pi}(R) := \frac{R - E[c]}{p(E[c] - c_l) + R - E[c]}. \tag{7}$$

Note that $0 < \hat{\pi}(R) < 1$. The coefficient of x_h is strictly positive whenever

$$\pi > \bar{\pi}(R) := \frac{c_h - E[c]}{(1-p)(R - E[c])}, \tag{8}$$

where $\bar{\pi}(R) > 0$.

Hence, it is easy to verify that in order to maximize her expected profit (5) subject to the monotonicity constraint (6), the principal sets x_h and x_u as displayed in Table 1.

	$\pi > \bar{\pi}(R)$	$\pi = \bar{\pi}(R)$	$\pi < \bar{\pi}(R)$
$\pi < \hat{\pi}(R)$	$x_h = x_u = 1$	$x_h \in [0,1]$, $x_u = 1$	$x_h = 0$, $x_u = 1$
$\pi = \hat{\pi}(R)$	$x_h = x_u = 1$	$x_h \in [0,1]$, $x_u \in [x_h, 1]$	$x_h = 0$, $x_u \in [0,1]$
$\pi > \hat{\pi}(R)$	$x_h = x_u = 1$ if $R - c_h > \pi p(R - c_l)$ $x_h = x_u \in [0,1]$ if $R - c_h = \pi p(R - c_l)$ $x_h = x_u = 0$ if $R - c_h < \pi p(R - c_l)$	$x_h = x_u = 0$	$x_h = x_u = 0$

Table 1. The principal's choice of x_h and x_u depending on π.

Consider now the agent's behavior. Suppose first that in equilibrium the agent always gathers information, so that $\pi = 1 > \hat{\pi}(R)$. Then according to Table 1, the principal would set $x_h = x_u =: x$. Yet, if the agent always gathers information, his expected payoff would then be given by $p(t_l - c_l) + (1-p)(t_h - xc_h) - \gamma = x(c_h - E[c]) - \gamma$, while his expected payoff would be $t_u - xE[c] = x(c_h - E[c])$ if he does not gather information. Hence, $\pi = 1$ cannot be part of an equilibrium.

Suppose next the agent never gathers information, so that $\pi = 0$. Then $\pi < \hat{\pi}(R)$ and $\pi < \bar{\pi}(R)$, so that according to Table 1 the principal would set $x_h = 0$, $x_u = 1$. If the agent gathers information, his expected payoff is $p(t_l - c_l) - \gamma = p(E[c] - c_l) - \gamma$, while his expected payoff if he does not gather information is $t_u - E[c] = 0$. Thus, if $\gamma \geq p(E[c] - c_l)$, then we have found the equilibrium. In contrast, if $\gamma < p(E[c] - c_l)$, then $\pi = 0$ cannot be part of an equilibrium.

Taken together, if $\gamma < p(E[c] - c_l)$, then in equilibrium the agent gathers information with probability $\pi \in (0, 1)$. This means that the agent must be indifferent between gathering and not gathering information, so that $p(t_l - c_l) + (1 - p)(t_h - x_h c_h) - \gamma = t_u - x_u E[c]$ must hold, which using (2), (3), and (4) simplifies to

$$x_h = x_u - \frac{\gamma}{p(E[c] - c_l)}. \tag{9}$$

Observe that (9) is equivalent to $x_u - x_h = \frac{\gamma}{p(E[c]-c_l)}$. Since $0 < \gamma < p(E[c] - c_l)$, this implies that in equilibrium $x_u - x_h \in (0, 1)$ must hold. Inspection of Table 1 shows that if $\pi > \bar{\pi}(R)$ or $\pi > \hat{\pi}(R)$, then $x_u - x_h = 0$. Moreover, if $\pi < \hat{\pi}(R)$ and $\pi < \bar{\pi}(R)$, then $x_u - x_h = 1$. Hence, in these cases the equilibrium conditions cannot be satisfied, so that there are only three cases left.

- If $\pi = \hat{\pi}(R) < \bar{\pi}(R)$, the principal sets $x_h = 0$ and $x_u = \frac{\gamma}{p(E[c]-c_l)}$.
- If $\pi = \bar{\pi}(R) < \hat{\pi}(R)$, the principal sets $x_h = 1 - \frac{\gamma}{p(E[c]-c_l)}$ and $x_u = 1$.
- If $\pi = \hat{\pi}(R) = \bar{\pi}(R)$, there are multiple equilibria. Specifically, the principal may set $x_h \in [0, 1 - \frac{\gamma}{p(E[c]-c_l)}]$ and $x_u = x_h + \frac{\gamma}{p(E[c]-c_l)}$. Since the principal makes the same expected profit in all these equilibria, we assume that she offers the contract that is the best one for the agent, which is $x_h = 1 - \frac{\gamma}{p(E[c]-c_l)}$ and $x_u = 1$.

Note that there exists a cut-off value $\tilde{R}$, which is defined by $\hat{\pi}(\tilde{R}) = \bar{\pi}(\tilde{R})$.[5]

Proposition 1 *Consider scenario I.*

(i) If $\gamma \geq p(E[c] - c_l)$, then in equilibrium the agent does not gather information, the principal's expected profit is $R - E[c]$, and the agent's expected rent is 0.

(ii) If $\gamma < p(E[c] - c_l)$, there are two cases. If $R < \tilde{R}$, the agent gathers information with probability $\hat{\pi}(R)$, the principal's expected profit is $\hat{\pi}(R)p(R - c_l)$, and the agent's expected rent is 0. If $R \geq \tilde{R}$, the agent gathers information with probability $\bar{\pi}(R)$, the principal's expected profit is $R - c_h$, and the agent's expected rent is $c_h - E[c] - \frac{\gamma}{1-p}$.

[5] It is straightforward to check that $\hat{\pi}(R)$ is strictly increasing in R, while $\bar{\pi}(R)$ is strictly decreasing. Moreover, $\hat{\pi}(c_h) < \bar{\pi}(c_h)$ and $\lim_{R\to\infty} \bar{\pi}(R) = 0$. Hence, there exists a unique $\tilde{R} > c_h$ such that $\hat{\pi}(R) < \bar{\pi}(R)$ whenever $R < \tilde{R}$, while $\hat{\pi}(R) > \bar{\pi}(R)$ whenever $R > \tilde{R}$.

4 Scenario II

In scenario II, the principal can observe whether at date 1 the agent has invested γ in information gathering. Suppose first the agent has not gathered information. Then at date 2 the principal offers the contract $x_u = 1$, $t_u = E[c]$, which the uninformed agent accepts since his expected production costs are covered. Thus, the principal obtains the expected first-best surplus $R - E[c]$, while the agent's expected payoff is zero.

Next, suppose that the agent has gathered information. When the principal offers the contract, she knows that the agent has private information about his costs, so that she faces a standard adverse selection problem. The principal offers the menu $[x_l, t_l, x_h, t_h]$ that maximizes her expected profit $p[x_l R - t_l] + (1-p)[x_h R - t_h]$ subject to the incentive compatibility constraints $t_l - x_l c_l \geq t_h - x_h c_l$ and $t_h - x_h c_h \geq t_l - x_l c_h$ and the participation constraints $t_l - x_l c_l \geq 0$ and $t_h - x_h c_h \geq 0$. As is well known (see e.g. Laffont and Martimort, 2002), only the incentive compatibility constraint of the low-cost type and the participation constraint of the high-cost type are binding, so that it is optimal for the principal to set $x_l = 1$, $x_h = I_{\{R - c_h \geq p(R - c_l)\}}$, $t_l = c_l + x_h(c_h - c_l)$, $t_h = x_h c_h$. Hence, if $R - c_h \geq p(R - c_l)$, the principal's expected payoff is $R - c_h$ and the agent's expected payoff is $c_h - E[c]$. If $R - c_h < p(R - c_l)$, the principal's expected payoff is $p[R - c_l]$ and the agent's expected payoff is 0.

Finally, consider the agent's decision whether to gather information. If $R - c_h \geq p(R - c_l)$, the agent gathers information whenever $\gamma \leq c_h - E[c]$. Otherwise, he never gathers information.

Proposition 2 *Consider scenario II. Let $\hat{R}$ be defined by $\hat{R} - c_h = p(\hat{R} - c_l)$. If $R \geq \hat{R}$ and $\gamma \leq c_h - E[c]$, the agent gathers information. The principal's expected profit is $R - c_h$ and the agent's expected rent is $c_h - E[c] - \gamma$. Otherwise, the agent does not gather information, the principal's expected profit is $R - E[c]$, and the agent's expected rent is* 0.

5 Comparison of the scenarios

We now analyze how the observability of information gathering affects the principal's expected profit, the agent's expected rent, and the expected total surplus. Propositions 1 and 2 imply the following result, which is illustrated in Figure 1.[6]

[6] One can check that $\hat{\pi}(\hat{R}) > \bar{\pi}(\hat{R})$, so that $\tilde{R} < \hat{R}$ must hold.

Proposition 3 *(i) Suppose $R < \tilde{R}$ and $\gamma < p(E[c] - c_l)$. Then the agent's expected rent is zero regardless of the prevailing scenario, but observability of information gathering increases the principal's expected profit and thus the expected total surplus.*

(ii) Suppose $\tilde{R} \leq R < \hat{R}$ and $\gamma < p(E[c] - c_l)$. Then observability of information gathering decreases the agent's expected rent and it increases the principal's expected profit as well as the expected total surplus.

(iii) Suppose $R \geq \hat{R}$. If $\gamma < p(E[c] - c_l)$, then observability of information gathering increases the agent's expected rent but it does not affect the principal's expected profit, so that it increases the expected total surplus. If $p(E[c] - c_l) \leq \gamma \leq c_h - E[c]$, observability of information gathering increases the agent's expected rent and it decreases the principal's expected profit as well as the expected total surplus.

(iv) Otherwise, the principal extracts the first-best expected total surplus in both scenarios.

As one might have expected, the principal is typically (weakly) better off if she can observe the agent's information gathering decision. Yet, there are also circumstances under which observability of information gathering reduces the principal's expected profit. This happens if the return R is sufficiently large and the information gathering costs γ are at an intermediate level. For large values of R, the principal has a strong interest to trade. Hence, when she observes that the agent is informed, she will make an offer that the agent accepts regardless of his type. Thus, if information gathering is observable, then from the agent's point of view information gathering has the additional advantage that it can influence the principal's offer, which is not the case if it is unobservable. Hence, for intermediate values of the information gathering costs γ, the agent still gathers information when it is observable, while he remains uninformed when information gathering is unobservable. As a consequence, when R is large and γ is at an intermediate level, the principal must leave an information rent to the agent when information gathering is observable, while she can extract the expected first-best surplus from the uninformed agent when information gathering is unobservable.[7]

[7] The result that observability of information gathering may reduce the principal's expected profit and increase the agent's expected rent may also hold in a model with more than two states. For example, suppose that $c \in \{c_l, c_m, c_h\}$. If $c_l = 30$, $prob\{c = c_l\} = 0.4$, $c_m = 50$, $prob\{c = c_m\} = 0.2$, $c_h = 60$, $R = 100$, and $\gamma = 10$, then one can show that the principal extracts the expected first-best surplus (54) in scenario I, while her expected payoff is only 40 in scenario II. The agent's expected rent is 4 in scenario II, while he obtains no rent in scenario I. (The calculation of the example is available from the author upon request.)

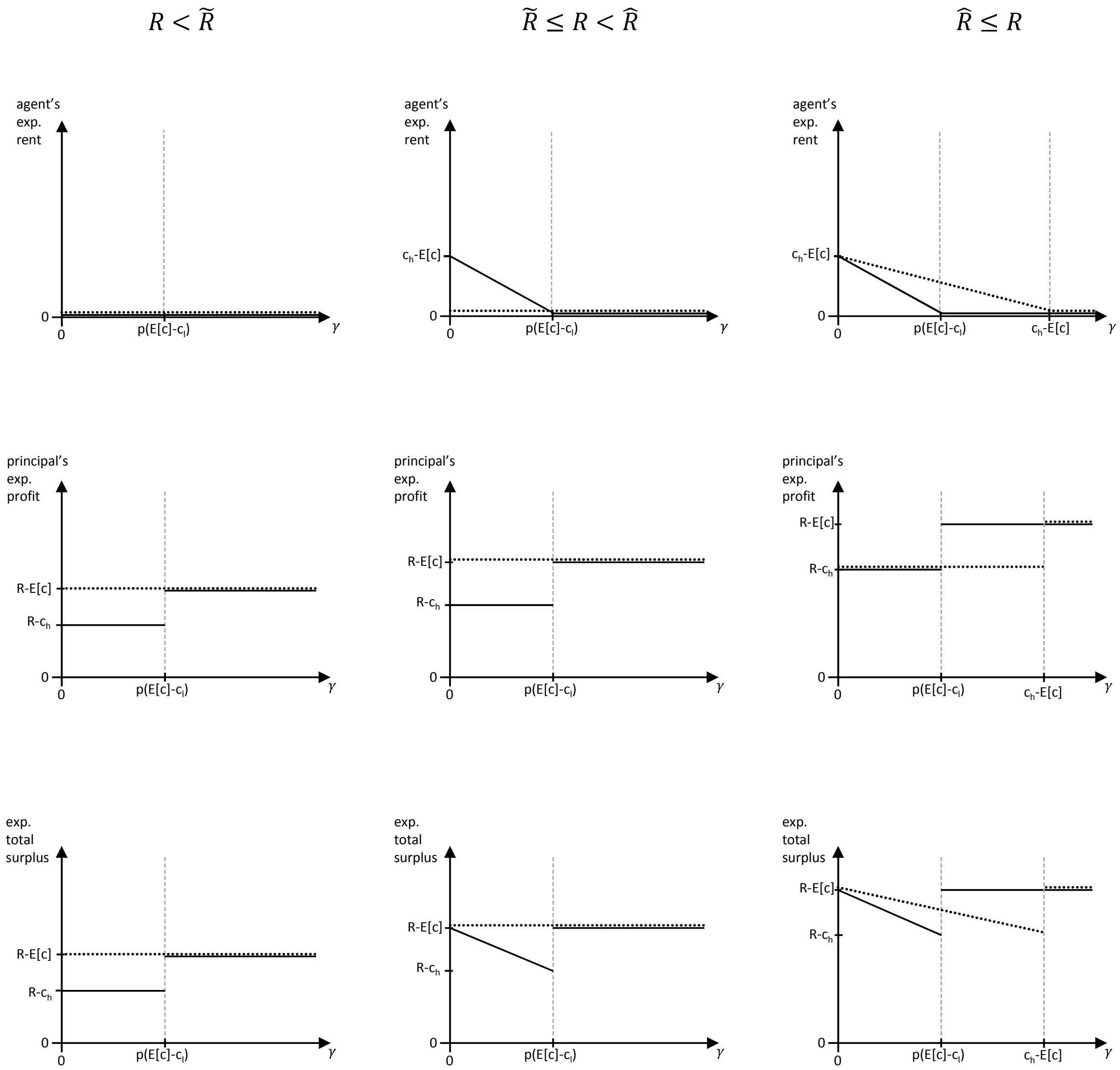

Figure 1. The solid curves correspond to scenario I, while the dotted curves depict scenario II.

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Supplementary material:

An example with three types (cf. footnote 7 of the paper)

1. The model

We consider the same model as in the paper except that now there are three different cost types. Specifically, we assume that the distribution of $c \in \{c_l, c_m, c_h\}$ is given by $p_l = prob\{c = c_l\}$ and $p_m = prob\{c = c_m\}$.

2. Scenario I: Unobservable information gathering

Suppose the principal believes that the agent has gathered information with probability π. Again we use direct mechanisms to solve the principal's problem; i.e. she maximizes her expected payoff

$$\pi[p_l(x_lR - t_l) + p_m(x_mR - t_m) + (1 - p_l - p_m)(x_hR - t_h)] + (1 - \pi)(x_uR - t_u) \quad (1)$$

subject to the incentive compatibility constraints

$$\begin{aligned}
t_l - x_lc_l &\geq t_h - x_hc_l, && (IC_{lh})\\
t_l - x_lc_l &\geq t_m - x_mc_l, && (IC_{lm})\\
t_l - x_lc_l &\geq t_u - x_uc_l, && (IC_{lu})\\
t_m - x_mc_m &\geq t_h - x_hc_m, && (IC_{mh})\\
t_m - x_mc_m &\geq t_l - x_lc_m, && (IC_{ml})\\
t_m - x_mc_m &\geq t_u - x_uc_m, && (IC_{mu})\\
t_h - x_hc_h &\geq t_m - x_mc_h, && (IC_{hm})\\
t_h - x_hc_h &\geq t_l - x_lc_h, && (IC_{hl})\\
t_h - x_hc_h &\geq t_u - x_uc_h, && (IC_{hu})\\
t_u - x_uE[c] &\geq t_h - x_hE[c], && (IC_{uh})\\
t_u - x_uE[c] &\geq t_m - x_mE[c], && (IC_{um})\\
t_u - x_uE[c] &\geq t_l - x_lE[c], && (IC_{ul})
\end{aligned}$$

the participation constraints

$$\begin{aligned}
t_l - x_lc_l &\geq 0, && (PC_l)\\
t_m - x_mc_m &\geq 0, && (PC_m)\\
t_h - x_hc_h &\geq 0, && (PC_h)\\
t_u - x_uE[c] &\geq 0, && (PC_u)
\end{aligned}$$

and the feasibility constraints $x_l \in [0, 1]$, $x_m \in [0, 1]$, $x_h \in [0, 1]$, and $x_u \in [0, 1]$.

Solving this optimization problem leads to tedious case distinctions. However, consider the following example: $c_l = 30$, $p_l = 0.4$, $c_m = 50$, $p_m = 0.2$, $c_h = 60$, $R = 100$, and $\gamma = 10$.

Observe that the participation constraint (PC_l) of the low-cost type is redundant, as it is implied by (IC_{lh}) and (PC_h). Similarly, the participation constraint (PC_m) is redundant because it is implied by (IC_{mh}) and (PC_h). The participation constraint (PC_u) which is implied by (IC_{uh}) and (PC_h) is redundant as well. Moreover, note that the incentive compatibility constraints (IC_{lu}) and (IC_{ul}) imply the monotonicity constraint $x_l \geq x_u$. Similarly, since $c_m > E[c]$ holds in the example, (IC_{um}) and (IC_{mu}) imply $x_u \geq x_m$. Moreover, (IC_{mh}) and (IC_{hm}) imply $x_m \geq x_h$.

Now ignore for a moment the incentive compatibility constraints (IC_{hl}), (IC_{hm}), (IC_{hu}), and (IC_{mu}), (IC_{ml}) as well as (IC_{ul}) which will turn out to be satisfied by our solution. It is then easy to see that (PC_h) must be binding; i.e., it is optimal for the principal to set

$$t_h = x_h c_h, \tag{2}$$

because otherwise she could increase her expected profit by decreasing t_h without violating any of the remaining constraints. Furthermore, (IC_{mh}) must be binding, so that it is optimal for the principal to set

$$t_m = x_h(c_h - c_m) + x_m c_m, \tag{3}$$

because otherwise she could decrease t_m without violating any of the remaining constraints. Observe that (2) and (3) together with the monotonicity constraint $x_m \geq x_h$ imply that in our example the right-hand side of (IC_{um}) is larger than the right-hand side of (IC_{uh}). Thus, it is optimal for the principal to set

$$t_u = x_h(c_h - c_m) + x_m(c_m - E[c]) + x_u E[c], \tag{4}$$

so that (IC_{um}) is binding. Similarly, (2), (3), and (4) together with the monotonicity constraint $x_u \geq x_m \geq x_h$ imply that the right-hand side of (IC_{lu}) is larger than both the right-hand side of (IC_{lh}) and the right-hand side of (IC_{lm}). Thus, it is optimal for the principal to set

$$t_l = x_h(c_h - c_m) + x_m(c_m - E[c]) + x_u(E[c] - c_l) + x_l c_l, \tag{5}$$

so that (IC_{lu}) is binding.

Hence, the principal's problem can be simplified. She chooses $x_l \in [0, 1]$, $x_m \in [0, 1]$, $x_h \in [0, 1]$, and $x_u \in [0, 1]$ in order to maximize her expected profit

$$
\begin{aligned}
& x_l[\pi p_l(R-c_l)] \\
& +x_u[(1-\pi)(R-E[c])-\pi p_l(E[c]-c_l)] \\
& +x_m[\pi p_m(R-c_m)+(1-\pi)(E[c]-c_m)-\pi p_l(c_m-E[c])] \\
& +x_h[\pi(1-p_l-p_m)(R-c_m)-(c_h-c_m)] \qquad (6)
\end{aligned}
$$

subject to the monotonicity constraint

$$x_h \leq x_m \leq x_u \leq x_l. \qquad (7)$$

The payments t_h, t_m, t_u, and t_l are given by (2), (3), (4), and (5).

To solve the simplified problem, note first that it is optimal for the principal to set $x_l = 1$ (since $\pi p_l(R - c_l) \geq 0$). Moreover, it is straightforward to show that the principal also sets $x_u = 1$.[1] Now consider the principal's choice of x_m. Note that the coefficient of x_m is strictly positive if

$$\pi > \bar{\pi}(R) := \frac{c_m - E[c]}{(1-p_l)(c_m - E[c]) + p_m(R - c_m)} \approx 0.32.$$

Thus, the princial sets $x_m = 1$ if $\pi > \bar{\pi}(R)$ and $x_m = 0$ if $\pi < \bar{\pi}(R)$.[2]

Finally consider the choice of x_h. The principal sets $x_h = 1$ if the coefficient of x_h is strictly positive; i.e. if

$$\pi > \tilde{\pi}(R) := \frac{c_h - c_m}{(1-p_l-p_m)(R-c_m)} = 0.5.$$

If the coefficient of x_h is strictly negative, she sets $x_h = 0$, while she sets $x_h \in [0,1]$ if the coefficient of x_h is equal to zero ($\pi = \tilde{\pi}(R)$).[3] Hence, the solution of the principal's maximization problem is given by $x_l = x_u = 1$, $t_l = t_u = x_h(c_h - c_m) + x_m(c_m - E[c]) + E[c]$, $t_m = x_h(c_h - c_m) + x_m c_m$, $t_h = x_h c_h$,

$$x_m \begin{cases} = 0 & \text{if } \pi < \bar{\pi}(R), \\ \in [0,1] & \text{if } \pi = \bar{\pi}(R), \quad \text{and} \\ = 1 & \text{if } \pi > \bar{\pi}(R), \end{cases}$$

[1] Specifically, she chooses $x_u = 1$ if the coefficient of x_u is positive. If the coefficient of x_u is negative, then due to the monotonicity constraint $x_m \leq x_u$ she sets $x_u = 1$ whenever the sum of the coefficient of x_u and the coefficient of x_m is positive; i.e. whenever $R - c_m > \pi\left[(1-p_m)(R-c_m) + p_l(c_m - c_l)\right]$. Given our example, this condition is always satisfied.

[2] If the coefficient of x_m is strictly negative, then due to the monotonicity constraint $x_h \leq x_m$ the principal would generally set $x_m = 1$ whenever the sum of the coefficient of x_m and the coefficient of x_h is positive; i.e. whenever $\pi > \hat{\pi}(R) := \frac{c_h - E[c]}{(R-E[c])(1-p_l)} \approx 0,43$. Obviously, in our example this condition is more restrictive than the condition $\pi > \bar{\pi}(R)$. Hence, in our example the principal sets $x_m = 0$ if the coefficient of x_m is strictly negative ($\pi < \bar{\pi}(R)$). Moreover, she sets $x_m \in [0,1]$ if the coefficient of x_m is equal to zero ($\pi = \bar{\pi}(R)$).

[3] Note that the monotonicity constraint $x_h \leq x_m$ is always satisfied since in the example $\tilde{\pi} > \bar{\pi}$.

$$x_h \begin{cases} = 0 & \text{if } \pi < \tilde{\pi}(R), \\ \in [0,1] & \text{if } \pi = \tilde{\pi}(R), \\ = 1 & \text{if } \pi > \tilde{\pi}(R). \end{cases}$$

It is straightforward to check that the omitted constraints (IC_{hl}), (IC_{hm}), (IC_{hu}), and (IC_{mu}), (IC_{ml}) as well as (IC_{ul}) are indeed satisfied by the solution.

Consider now the agent's behavior. Suppose first that in equilibrium the agent always gathers information, so that $\pi = 1 > \tilde{\pi}(R) > \bar{\pi}(R)$. Then the principal sets $x_m = x_h = 1$ such that $t_h = t_m = t_u = t_l = c_h = 60$. If the agent always gathers information, his expected payoff is given by $p_l(c_h - c_l) + p_m(c_h - c_m) - \gamma = 4$, while his expected payoff is $t_u - E[c] = c_h - E[c] = 14$ if he does not gather information. Hence, $\pi = 1$ cannot be part of an equilibrium.

Suppose next that the agent never gathers information; i.e. $\pi = 0$. Then $\pi < \bar{\pi}(R)$ and $\pi < \tilde{\pi}(R)$, so that the principal sets $x_m = x_h = 0$ such that $t_h = t_m = 0$ and $t_u = t_l = E[c] = 46$. If the agent gathers information, his expected payoff is $p_l(t_l - c_l) - \gamma = -3.6$, while it is $t_u - E[c] = 0$ if he does not gather information. Hence, we have found the equilibrium: In our example the agent does not gather information.[4]

Taken together, if information gathering is unobservable the principal offers the menu $[x_l,\, t_l,\, x_m,\, t_m,\, x_h,\, t_h,\, x_u,\, t_u] = [1,\, 46,\, 0,\, 0,\, 0,\, 0,\, 1,\, 46]$. Given that the principal will offer this menu, the agent does not gather information such that the principal extracts the expected first-best surplus, which is 54.

3. Scenario II: Observable information gathering

Suppose first that the principal observes that the agent has not gathered information. Then at date 2 she offers the contract $x_u = 1$, $t_u = E[c]$, which the uninformed agent accepts since his expected production costs are covered. Thus, the principal obtains the expected first-best surplus $R - E[c]$, while the agent's expected payoff is zero.

Next, suppose that the principal observes that the agent has gathered private information; i.e. she faces a standard adverse selection problem. Then the principal offers the menu $[x_l,\, t_l,\, x_m,\, t_m,\, x_h,\, t_h]$ that maximizes her expected profit $p_l[x_l R - t_l] + p_m[x_m R - t_m] + (1 - p_l - p_m)[x_h R - t_h]$ subject to the incentive

[4] It is immediate to show that there exists no equilibrium in which the agent gathers information with probability $\pi \in (0,1)$. The agent would do so if and only if his expected payoff if he gathered information $p_l\,(x_h(c_h - c_m) + x_m(c_m - E[c]) + E[c] - c_l) + p_m x_h(c_h - c_m) - \gamma$ was equal to his expected payoff if he did not gather information $x_h(c_h - c_m) + x_m(c_m - E[c])$. Given our example, this is equation cannot be satisfied.

compatibility constraints

$$\begin{aligned} t_l - x_l c_l &\geq t_h - x_h c_l, & (IC_{lh}) \\ t_l - x_l c_l &\geq t_m - x_m c_l, & (IC_{lm}) \\ t_m - x_m c_m &\geq t_h - x_h c_m, & (IC_{mh}) \\ t_m - x_m c_m &\geq t_l - x_l c_m, & (IC_{ml}) \\ t_h - x_h c_h &\geq t_m - x_m c_h, & (IC_{hm}) \\ t_h - x_h c_h &\geq t_l - x_l c_h, & (IC_{hl}) \end{aligned}$$

and the participation constraints

$$\begin{aligned} t_l - x_l c_l &\geq 0, & (PC_l) \\ t_m - x_m c_m &\geq 0, & (PC_m) \\ t_h - x_h c_h &\geq 0, & (PC_h) \end{aligned}$$

and the feasibility constraints $x_l \in [0,1]$, $x_m \in [0,1]$, and $x_h \in [0,1]$.

Again we consider our previous example. Observe that the participation constraint (PC_l) of the low-cost type is redundant, as it is implied by (IC_{lh}) and (PC_h). Similarly, the participation constraint (PC_m) is redundant because it is implied by (IC_{mh}) and (PC_h). Moreover, note that the incentive compatibility constraints (IC_{lm}) and (IC_{ml}) imply the monotonicity constraint $x_l \geq x_m$. Similarly, (IC_{mh}) and (IC_{hm}) imply $x_m \geq x_h$. Now ignore for a moment the incentive compatibility constraints (IC_{hl}), (IC_{hm}) and (IC_{ml}) which will turn out to be satisfied by our solution. It is then easy to see that (PC_h) must be binding; i.e., it is optimal for the principal to set

$$t_h = x_h c_h, \tag{8}$$

because otherwise the principal could increase her expected profit by decreasing t_h without violating any of the remaining constraints. Furthermore, (IC_{mh}) must be binding, so that it is optimal for the principal to set

$$t_m = x_h(c_h - c_m) + x_m c_m, \tag{9}$$

because otherwise she could decrease t_m without violating any of the remaining constraints. Observe that (8) and (9) together with the monotonicity constraint $x_m \geq x_h$ imply that the right-hand side of (IC_{lm}) is larger than the right-hand side of (IC_{lh}). Thus, it is optimal for the principal to set

$$t_l = x_h(c_h - c_m) + x_m(c_m - c_l) + x_l c_l, \tag{10}$$

so that (IC_{lm}) is binding. Hence, the principal's problem can be simplified. She chooses $x_l \in [0,1]$, $x_m \in [0,1]$, and $x_h \in [0,1]$ in order to maximize her expected profit

$$
\begin{aligned}
& x_l p_l(R-c_l) \\
& +x_m[p_m(R-c_m)-p_l(c_m-c_l)] \\
& +x_h[R-c_h-(p_l+p_m)(R-c_m)]
\end{aligned}
\tag{11}
$$

subject to the monotonicity constraint

$$x_h \leq x_m \leq x_l. \tag{12}$$

Note first that it is optimal for the principal to set $x_l = 1$, since $p_l(R-c_l) > 0$. Moreover, since $\frac{p_l c_l + p_m R}{p_l + p_m} > c_m$ holds in the example, the principal also sets $x_m = 1$. Furthermore, it is optimal for the principal to set $x_h = 1$ since $R-c_h > (p_l+p_m)(R-c_m)$ holds in our example. Hence, if the principal observes an informed agent, she offers the menu $[x_l,\, t_l,\, x_m,\, t_m,\, x_h,\, t_h] = [1,\, 60,\, 1,\, 60,\, 1,\, 60]$ such that her profit is $R - c_h = 40$. It is straightforward to check that the omitted constraints (IC_{hl}), (IC_{hm}) and (IC_{ml}) are indeed satisfied by the solution.

Finally, consider the agent's decision whether to gather information. If the agent gathers information, the principal offers $t_l = t_m = t_h = 60$ such that the agent's expected rent is $c_h - E[c] - \gamma = 4$. If the agent does not gather information, the principal offers $t_u = E[c]$ such that the agent obtains no rent. Hence, the agent gathers information.

Taken together, if information gathering is observable, the agent gathers information and the principal offers $[x_l,\, t_l,\, x_m,\, t_m,\, x_h,\, t_h] = [1,\, 60,\, 1,\, 60,\, 1,\, 60]$. The principal's profit thus is 40 and the agent's expected rent is 4.