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ABSTRACT

This paper proposes a new approach to strategic asset allocation for central banks' management of foreign reserves. This eclectic approach combines the behavioural portfolio management in the framework of mean-variance mental accounting (MVMA) with the improvements on asset return forecast offered by the Black-Litterman (B-L) model, proving particularly suitable for the reserve management policy with multiple objectives. The B-L model is embedded into the MVMA framework to obtain both the equilibrium and the B-L returns as our improved forecasts, formulating forward-looking investment strategies. The approach is applied to the case of China to derive optimal asset allocation for the Chinese central bank.

Key Words: Reserve Management, Strategic Asset Allocation, Mental Accounting, Black-Litterman model, China's Foreign Reserves.

JEL Codes: G11, G02, E58, C11, C61.

1. Introduction

In recent decades, foreign reserves held by central banks have surged to a record level. According to the IMF, the world's total foreign reserves had reached 10.2 trillion US dollars at the end of 2011, of which 66.69 %, or 6.8 trillion US dollars, are held by emerging and developing economies. The rapid accumulation of foreign reserves posts a vexing challenge to central banks, particularly those large reserve holders. For countries like China, the colossal reserve stocks means a slight management glitch would easily incur the country a huge loss of national wealth. As the largest emerging economy, China has accumulated a colossal amount of foreign reserves; by the end of March 2012, these stood at 3,305 billion US dollars (People's Bank of China). Hence, how to optimally allocate such enormous reserves is vital for China's central bank.

Central banks hold foreign reserves for a multiplicity of purposes, such as to back up a country's domestic currency, manage the exchange rate through market intervention, and therefore support and maintain confidence in the monetary and exchange rate policies. Reserves can also be employed to protect a country from external vulnerability by maintaining sufficient liquidity to absorb shocks during financial crisis (IMF, 2004).

This set of purposes conditions central banks' management of reserve investment into a multi-facet process. Central to this process is the multiple objectives of reserve management featuring "safety, liquidity and profitability". Central banks are highly risk-averse investors in the first place. This psychological profile predetermines that preservation of the capital value of reserves is central banks' utmost priority, leading to their investment concentrating on programs that can ensure the safety of their assets, although the returns on such investment are inescapably low (Fisher and Lie, 2004). Second, to align with the missions that central banks hold reserves for, reserve management is tasked to assign an appropriate portion of assets which have a high degree of liquidity to smooth impacts of negative shocks in the world economy. Third, given the massive stocks of foreign assets, it would be desirable that the reserves can bring home reasonable returns from prudent investment of the foreign assets while reserve management is conducted in an international environment characterised by uncertainty and volatile capital movements. With the rapid accumulation of foreign reserves in many emerging and developing countries, it also become possible for central banks in these countries to allocate certain portion of their external assets to higher investment without compromising the comprehensive health of the country's reserve holdings. In the circumstances, Berkelaar et al. (2010b) and Borio et al, (2008) report that, with the amount of reserves being in excess of what is needed, many central banks are seeking higher returns on their reserve assets. As a result, 'safety, liquidity and profitability (returns)' are generally accepted as the objectives of reserve management (Nugee, 2000, and IMF 2001).With these objectives, central banks have more than one goal for their reserve management. In other words, not only do central banks desire to fulfil their responsibility of preserving capital and maintaining adequate liquidity, but also they would pursue relatively high returns to accomplish efficient management of massive reserves.

Traditional strategic asset allocation for foreign reserves relies mainly on the mean-variance (MV) approach to portfolio management originally proposed by Markowitz (1952). In this approach, mean-variance investors view their portfolio as a whole and derive optimal asset allocation based on the overall expected returns and risk. For central banks' management of foreign reserves, the approach does not explicitly inform how central banks may invest their external assets in a multiple-goal way.

Behavioural finance has emerged as a complement of the conventional finance including the portfolio management. Shefrin and Statman (2000) develop the behavioural portfolio theory (BPT), which is a goal-based theory in which investors divide their wealth into a variety of mental accounts of a set of portfolios corresponding to various goals [1]. A central feature of BPT is that investors take their portfolios not as a whole, but as distinct mental accounts in a set of assets, where mental accounts are connected with particular goals and where attitudes toward risk vary across mental accounts (Statman, 2008).

Das, Markowitz, Scheid and Statman (2010) present a further development in the field of asset allocation by offering a framework that incorporates the mean-variance theory of portfolio management (MVT) and the behavioural portfolio theory (BPT). In this framework, investors view their portfolios as collections of mental accounting (MA) sub-portfolios, where each sub-portfolio is connected with a goal and each goal has a threshold level. Risk in each sub-portfolio can be measured by the probability of failing to reach the threshold level by means of VaR (Value at Risk). Known as the mean-variance mental accounting (MVMA) approach, Das et al. (2010) demonstrate that their framework is mathematically equivalent to the mean-variance solution. MVMA investors seek to choose the portfolio with maximum expected returns subject to the VaR constraint capturing the account's motive. Consistent with Markowitz (1952), optimal

[1] For an introduction to mental accounting, see for example Thaler (1999) and Nofsinger (2011, Chapters 6 and 7).

portfolios within various accounts are on the mean-variance frontier. Finally, as a combination of sub-portfolios, the aggregate portfolio is also on the efficient frontier.[2]

With the rapid accumulation of foreign reserves in many emerging markets, recent years has witnessed a growing interest in the literature on strategic allocation of a country's external assets. For example, Cardon and Coche (2004) propose a blueprint for the management of the strategic asset allocation for central banks, where asset allocation decisions can be carried out by a three-tier governance structure consisting of an oversight committee, investment committee and portfolio management. Fisher and Lie (2004) provide a framework for reserves' strategic asset allocation that considers various assets (e.g. government bonds, non-government bonds, equities and currency) and guaranteeing sufficient liquidity for trade and intervention requirements. In this framework, they show that relaxing various constraints can obtain better returns for the same level of risks De Cacella et al. (2010) develop a multi-objective evolutionary optimisation algorithm to obtain a set of viable portfolios using a variable time horizon. Volumes edited by Berkelaar, Coche and Nyholm (2010a), and Coche, Nyholm and Petre (2011) contain a number of studies which contribute to strategic asset allocation for central banks.

In the field of asset allocation, a very important new advance in recent years is the Black-Litterman (B-L) model, which can overcome the error-maximising of the mean-variance optimisation, i.e. the high sensitivity of the optimal portfolio weights to the expected-return inputs often results in extreme solutions (Michaud, 1989, and Best and Grauer, 1991). Black and Litterman (1992) extend the mean-variance analysis by incorporating the Bayesian estimation

[2] This result assumes that short sales are allowed. In the case where short sales are not allowed, Das et al. (2010) suggest that the aggregate portfolio lies close to the mean-variance frontier.

into their model. Lucid discussion of the Bayesian analysis and the B-L model can be found in Lee (2000) and Christodoulakis (2002). Walters (2011) and Meucci (2010) survey the original B-L model and its various extensions.

The intuitive appeal of the B-L model is the use of the equilibrium excess returns as prior for the distribution of asset returns derived from the CAPM model. This implies that the expected excess returns in this model are obtained from the assumption that the market portfolio is the optimal portfolio of risky assets. Based on these equilibrium returns, the specific views of each investor (which can be regarded as additional information or further insights) are combined with the prior to generate the posterior distribution of asset returns. Barros Fernandes et al. (2012) combine the B-L approach with the resampling approach of Michaud and Michaud (2008) to generate a portfolio optimisation for central banks' strategic asset allocation. Petrovic (2010) applies the Black-Litterman model to central banking practice. León and Vela (2011) present a long-term-dependence-adjusted and non-loss-constrained version of the Black-Litterman model to derive the efficient frontier based on their estimation of the Board of Directors' risk aversion.

Other than the applications of the B-L model, some studies concentrate on its various extensions. For example, Qian and Gorman (2001) present a method to integrate views on the covariance matrix as well as views on the returns. Fusai and Meucci (2003) propose a way to measure how consistent a posterior estimate of the mean is with regards to the prior, or some other estimate. Krishnan and Mains (2005) present a method to incorporate additional factors into the B-L model. Giacometti et al (2007) investigate the improvement of the original B-L model by both applying more realistic approaches to asset returns, e.g. the normal, the t-student, and the stable distributions, and alternative risk measures such as dispersion-based risk measures, VaR, and CVaR (conditional value at risk).

On the practical front, the recent trend of reserve management can be summarised in two aspects. On the one hand, some studies on strategic asset allocation use the so-called ‘investment tranche’ as their way of deriving optimal asset allocation when the central banks accumulating huge reserve stocks. For example, Berkelaar et al (2010b) suggest that some central banks have notionally divided their reserve assets into separate tranches, which includes a tranche of investment in broader asset classes that shows the low risk appetite to seek higher returns. On the other hand, with the unfolding of the global financial crisis that started around 2007, central banks have changed their risk preference, shifting their investable wealth towards favouring safe assets. In the circumstances, reserve managers now have a strong tendency to hold safe assets in view of both their value preservation policy and their need for ready liquidity.

Inspired by the central banks’ practice of investment tranching and by the recent advances of behavioural portfolio models featuring mental accounting, in this paper we propose a new method for central banks’ strategic asset allocation by combining the behavioural approach to asset allocation with improvements on return forecast offered by the Black-Litterman model. This eclectic approach takes into consideration of behavioural factors that may influence reserve managers’ risk-return profile. Underscoring the practical importance of mean variance mental accounting (MVMA), we assume that central banks have two mental accounts (sub-portfolios) or tranches. One is a ‘precautionary sub-portfolio’ showing higher risk aversion. Governed by the precautionary motive, this sub-portfolio is tasked to play safe hence earn lower returns. If successful, this sub-portfolio fulfils both safety and liquidity objectives. The other account is presented by an ‘investment sub-portfolio’ showing lower risk aversion. This sub-portfolio is investment oriented, which seeks for relatively high returns and thus to fulfil the return or profitability objective. An aggregate portfolio is then constructed by combining the two sub-portfolios. In each account, risk is measured by the probability of not reaching the threshold return level. For robust tests, we also design alternative aggregate portfolios by making different allocations of the total investable reserves into combinations of the two sub-portfolios. This will allow capturing the distinct risk attitudes of reserve managers.

With the MVMA framework, we use the Black-Litterman model to obtain both the equilibrium returns and the B-L returns as our improved forecasts, and therefore to derive the two sets of optimal asset allocation for foreign reserve. In the case of China, taking into consideration of the recent trends of both China’s holdings of US Treasury securities and the investable universe of reserve managers, we combine the MVMA analysis and the B-L model to derive optimal asset allocation for China’s central bank.

Our study contributes to the literature in this area in several ways: (1) We extend Das et al.’s (2010) work to reserve management; (2) conventional models are extended to the case where the investor has multiple goals; and (3) the Black-Litterman model is extended to take into consideration of influences of behavioural factors. For the first time we offer an approach that is designed to help central banks’ manage their strategic asset allocation of foreign reserves, which takes into consideration of behavioural factors that affects reserve management and take advantage of improved return forecast provided by the Black-Litterman model.

The rest of the paper is organised as follows. In section 2, we present the behavioural reserve management framework by combining the MVMA framework with the B-L model, based on which we propose a multiple-goal strategic asset allocation for central banks, with explicit consideration of behavioural influences. In section 3, we apply the approach to the Chinese case of reserve management. Conclusions are presented in section 4.

2. The Behavioural Reserve Management Framework

In this section, we first depict the mean-variance mental accounting (MVMA) framework by Das et al. (2010) to underpin the theoretical consideration in relation to reserve management. We then use the Black-Litterman (B-L) model as a means to improve return forecasts. By combining the MVMA framework and the B-L approach, we finally derive strategic asset allocation for central banks, and thus propose a multiple-goal reserve management policy.

2.1 The Mean Variance Mental Accounting (MVMA) Problem

In our model setting, the reserve manager selects portfolio weights $\mathbf{w} = \left[w_1, ..., w_n\right]'$ for n assets, where the assets have an expected return vector $\boldsymbol{\mu} \in R^n$ and a return covariance matrix $\boldsymbol{\Sigma} \in R^{n \times n}$. The MV problem is expressed by:

$$\max_{w} w'\mu - \frac{\gamma}{2} w'\boldsymbol{\Sigma} w \qquad (1)$$

subject to the fully invested constraint

$$\mathbf{w}'\mathbf{1} = 1 \qquad (2)$$

where $\mathbf{1} = [1,1,...,1]' \in R^n$, and γ is the risk aversion coefficient, which balances the trade-offs in the mean-variance space.

Using the Lagrange-multiplier method, the solution to optimal asset allocation in closed form is[3]

$$\mathbf{w} = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}\left[\boldsymbol{\mu} - \left(\frac{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \gamma}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\right)\mathbf{1}\right] \in R^n \qquad (3)$$

Unlike the standard MV problem, the reserve managers specify γ, which means they choose different values for $\gamma > 0$, and then solve problem (1) in terms of solution (3). With a collection of different risk-aversion values in hand, they can maximise mean-variance utility to find corresponding points on the efficient frontier.

Based on equations (1) to (3), we introduce the Mental Accounting (MA) factor. The MVMA reserve manager takes her portfolios as collections of MA sub-portfolios, in which each sub-portfolio is mapped onto a goal. Following Das et al. (2010), we assume that the reserve manager has difficulties in stating her precise risk-aversion coefficient (γ), but is comfortable to announce her threshold levels for each goal and the maximum probabilities of failing to reach them. Ultimately, the MVMA reserve manager acts as if she has different risk preferences in each of the mental accounts. Thus, solving the MA problem is equivalent to solving a standard MV problem with a specific 'implied' risk-aversion coefficient.

[3] The detailed derivation of this solution can be found in the Appendix of Das et al. (2010).

For a certain mental account, the reserve manager considers a threshold level of return H for portfolio p, and regards the maximum probability of the portfolio failing to reach portfolio return $r(p)$ as α. Thus, she has

$$\text{Prob}\left[r(p) \le H\right] \le \alpha \qquad (4)$$

We assume that portfolio returns are normally distributed. In terms of VaR, inequality (4) implies the following inequality:

$$H \le \mathbf{w}'\boldsymbol{\mu} + \Phi^{-1}(\alpha)\left[\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\right]^{1/2} \qquad (5)$$

where $\Phi(\bullet)$ is the cumulative standard normal distribution function.

The reserve manager's aim is to derive at optimal asset allocation from equation (3) subject to the constraint (5). Optimisation cannot be achieved unless the constraint (5) is an equality. As a result, the solution to the reserve manager's implied risk aversion γ and the optimal weights $\mathbf{w}(\gamma)$ is implied by the following equations:

$$H = \mathbf{w}(\gamma)'\boldsymbol{\mu} + \Phi^{-1}(\alpha)\left[\mathbf{w}'(\gamma)\boldsymbol{\Sigma}\mathbf{w}(\gamma)\right]^{1/2} \qquad (6)$$

where

$$\mathbf{w}(\gamma) = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}\left[\boldsymbol{\mu} - \left(\frac{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \gamma}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\right)\mathbf{1}\right] \qquad (7)$$

Plugging equation (7) into equation (6), it is straightforward to find the solution to equation (6) based on which one can obtain different values of the risk preference γ.

Thus, the portfolio optimisation problem for the MVMA reserve manager is specified by a threshold level of return H and a probability value α. When the reserve manager specifies her

MA preferences for each sub-portfolio through the parameter pair (H, α), they implicitly denote what their risk preferences (γ) are over the given portfolio choice set $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. With the risk aversion coefficient (γ), the reserve manager can derive their optimal asset allocation.

However, for reserve management, short selling is not allowed. Thus, we must call for quadratic programming (QP) optimisers. Following Das et al. (2010), the full problem with short-selling constraints is as follows:

$$\text{Solve}_{\gamma}\ \mathbf{w}(\gamma)'\boldsymbol{\mu} + \Phi^{-1}(\alpha)\sqrt{\mathbf{w}(\gamma)'\boldsymbol{\Sigma}\mathbf{w}(\gamma)} = H \qquad (8)$$

where $\mathbf{w}(\gamma)$ is the solution to the following MV problem:

$$\max_{w}\ w'\boldsymbol{\mu} - \frac{\gamma}{2}w'\boldsymbol{\Sigma}w \qquad (9)$$

subject to the full invested constraint and short-selling constraints

$$\mathbf{w}'\mathbf{1} = 1\ ,\ \mathbf{w} \geq 0 \text{ and } \mathbf{w} \leq 1 \qquad (10)$$

Thus, the reserve manager solves the nonlinear equation (8) based on the variable γ, which determines the portfolio weights $\mathbf{w}(\gamma)$ derived by solving the QP in equations (9) and (10). For a specified γ, the reserve manager needs to check whether the solution $\mathbf{w}(\gamma)$ can make equation (8) hold. If not, she must move γ appropriately and then solves the QP until equation (8) holds.

2.2. The Black-Litterman Model

We use the Black-Litterman model to improve our input forecast, i.e. the expected returns. This model employs the equilibrium returns as the starting point for its estimation. Equilibrium returns are inferred from the market capitalisation weights, using a ‘reverse optimisation process’.

Black and Litterman (1992) argue that this process, based on market capitalisation weights, can derive at consensus excess returns, which are consistent with the tangency portfolio of the Capital Asset Pricing Model. With the market forces of supply and demand in equilibrium, the weight allocation across the investment universe is expected to be optimal and the optimal weight can therefore act as the basis for asset allocation.

We follow Satchell and Scowcroft (2000) and Idzorek (2005) to state the Black-Litterman model. In this model, given the risk aversion coefficient δ that indicates the level of risk against returns of the market portfolio, the historical variance covariance matrix $\mathbf{\Sigma}$, and the vector of market capitalisation weights $\mathbf{w}_M$, the reverse optimisation process can provide the vector of implied equilibrium returns $\boldsymbol{\mu}_M$ in excess of the risk-free rate as

$$\boldsymbol{\mu}_M = \delta \mathbf{\Sigma} \mathbf{w}_M \qquad (11)$$

If the reserve manager does not agree with the implied equilibrium excess returns, she can introduce her own views. Specifically, she may take the implied equilibrium returns as the prior distribution and regards the corresponding forecasted returns as forward-looking views-based returns, to form the posterior Black-Litterman returns. For example, assume there are k views, which can be either relative or absolute and are represented in the $k \times 1$ vector $\mathbf{Q}$. The $k \times n$ matrix $\mathbf{P}$ then is used to define these views: $\mathbf{Q} = \mathbf{P} \cdot \mathbf{r}_a$. The first view is represented as a linear combination of expected returns denoted by the first row of $\mathbf{P}$. A confidence level is associated with each of the views implied by $\mathbf{Q}$. Therefore, the investor's beliefs can be described by a normal view distribution: $\mathbf{P} \cdot \mathbf{r}_a \sim N(\mathbf{Q}, \mathbf{\Omega})$, where $\mathbf{\Omega}$ is a $k \times k$ diagonal covariance matrix. In the same vein, the confidence in the equilibrium model and the derived implied returns can be defined. Consequently, we obtain the prior equilibrium distribution: $\mathbf{r}_a \sim N\left(\boldsymbol{\mu}_M, \tau\boldsymbol{\Sigma}\right)$, where τ is a known quantity indicating the uncertainty level to scale the historical covariance matrix $\boldsymbol{\Sigma}$.

Following the Bayesian estimation method, the reserve manager can generate the posterior Black-Litterman returns as follows:

$$E\left(\mathbf{r}_{BL}\right) = \left[\left(\tau\boldsymbol{\Sigma}\right)^{-1} + \mathbf{P}'\boldsymbol{\Omega}\mathbf{P}\right]^{-1} \times \left[\left(\tau\boldsymbol{\Sigma}\right)^{-1}\boldsymbol{\mu}_M + \mathbf{P}'\boldsymbol{\Omega}\mathbf{Q}\right] \qquad (12)$$

2.3. Strategic Asset Allocation with Multiple Goals

Combining the MVMA framework and the Black-Litterman model, we propose a multiple-goal reserve management policy to generate strategic asset allocation for central bankers.

The strategic asset allocation in this setting can be achieved through following steps. First, to comply with the objectives of safety, liquidity and profitability, the reserve manager regards her portfolios as a collection of two MA sub-portfolios. The first is a ‘precautionary sub-portfolio’, where the manager specifies higher risk-aversion parameters and expects this portfolio to earn lower returns with lower risks. This sub-portfolio can preserve capital and maintain liquidity, which makes it easier to intervene in the market during periods of crisis and thus to fulfil the central bank’s mission of stabilising the economy. The second portfolio is an ‘investment sub-portfolio’, where the reserve manager assigns lower risk-aversion parameters to pursue relatively higher returns with higher risks. This can satisfy the manager’s desire to seek higher returns given her enormous reserve positions. Then, an aggregate portfolio is constructed by combining these two sub-portfolios in a certain proportion. The different allocations of the total investable reserves across the two sub-portfolios imply distinct overall risk attitudes.

Next, before entering into her MA sub-portfolios, the reserve manager first selects her investment classes from the investment universe available to her. With that, she obtains the implied equilibrium returns $\boldsymbol{\mu}_M$ based on market capitalisation weights, and the B-L returns $E\left(\mathbf{r}_{BL}\right)$ based on their forecasts according to the updated information in a Bayesian approach.

Finally, with the equilibrium excess returns $\boldsymbol{\mu}_M$ and the B-L excess returns $E\left(\mathbf{r}_{BL}\right)$ already derived, the reserve manager works out the two sub-groups of optimal asset allocation by solving equations (8) to (10), and compares and analyses the results to make her final asset allocation decision.

3. Strategic Asset Allocation for China's Foreign Reserves

In this section, we illustrate application of our approach by way of the Chinese case. Before doing so, we first briefly introduce the structure of China's holdings of foreign reserves in terms of US Treasury securities. To put the case in comparative perspectives, we also discuss the investable universe for central banks around the global. Against this background, we show our selection of asset classes and analyse data characteristics.

3.1. Recent Trends of China's Holdings of US and Other Foreign Assets

The Chinese central bank, i.e. the People's Bank of China, has never publicly announced the allocation of its reserve investment. But one may obtain hints from the data of China's holdings of US Treasuries as published by US Department of the Treasury. Figure 1 below shows China's total amount of reserves and the breakdown of its holding of US Treasuries from January 2008 to May 2012.

<Figure 1 about here>

In Figure 1, the upper line shows the path of China's total amount of foreign reserves, while the filled line depicts China's holdings of US Treasuries. As of July 2011, China's holdings of US assets had reached the peak, i.e. 1314.90 billion US dollars. Since then, China's holdings of US assets have shown a moderate down trend. By May 2012, the latest figure available to this study shows that China holds 1169.60 billion US assets, accounting for 36.48% of China's total foreign reserves (excluding gold). Thus, the Figure indicates that despite the rapid accumulation of foreign reserves, the share of American assets that China holds has been stable, implying that is making diversification efforts, largely of those new stock of foreign reserves.

Table 1 illustrates the composition of China's holdings of US assets as of June 30, 2011. According to the table, 75.39% of them, or 1727 billion dollars in amount, is made up of long-term Treasury securities, while 14.19% are long-term agency securities. Others asset classes account for 10.42% of the total, including equity, long-term corporate securities, and short-term debt.

<Table 1 about here>

China's diversification efforts are palpable. On March 12th 2012, Reuters News Agency reports that Yi Gang, head of China's State Administration of Foreign Exchange (SAFE), states that the portion of China's $3.2 trillion of reserves invested in Eurozone assets has increased in value, making returns above the rate of inflation, and that more would go into yen-denominated assets when the time is right. In particular, he said that 'we will carry out investments in the Japanese government bond (JGB) market or other fixed-income products'.

3.2. Investment Universe for China and Selection of Asset Classes

It is vital for official reserve managers to first define their investment universe, i.e. the set of asset classes from assets will be selected to construct the portfolio. This is a crucial preliminary step towards forming the basis for the reserve investment policy. Several recent studies have investigated the trend of reserve management, which reveal important insights into central banks' investment universe,. According to the IMF (2011), government bonds have been the dominant asset class for central banks' reserve investment. Borio et al. (2008), however, suggest that central banks that hold a huge amount of foreign reserves have broadened the set of asset classes, which means more investable asset classes and instruments become available to them. With this new created possibility, these large reserve holders can invest their foreign reserves not only in traditional assets, i.e. lower risk assets for both liquidity and safety purposes such as Treasury Bills, bank deposits, government and supranational bonds, but also in higher risk assets, such as corporate bonds and equities, to satisfy the return objective. As a result, investment tranches have been established by some central banks other than liquidity tranches, seeking a higher return in the long run. While IMF (2011) claims that these investment tranches account for a small portion of total investable reserves, however, for large reserve holders such as China, the absolute total size of these investment tranche still could be formidable.

One critical development in central banks' reserve management is the fact that the global financial crisis has changed the risk preference of reserve managers, shifting more investable reserves towards safe assets, i.e. the assets with low credit and market risks and high market liquidity (IMF, 2012). McCauley and Rigaudy (2011) investigate the foreign reserve management in the crisis and after, suggesting that official reserve managers have shifted their reserves towards Treasury and Agency bills. IMF (2012) indicates that investors' shift to quality assets have given rise to an upsurge in demand for safe assets by various types of investors, such as banks, official reserve managers, and Sovereign Wealth Funds (SWF). IMF (2012) also investigates the universe of potentially safe assets, finding that global investors' universe of safe assets has broadened to include highly rated OECD government securities, relatively lower rated OECD government securities, supranational debts, US agency debt, and corporate debt (of investment grade).

Based on China's past investment practice and the recent development of global reserve management under the impact of the financial crises, we select sixteen asset classes as our investment opportunity set. Most of these are safe assets, including nine advanced countries' government bonds, US Treasury Bills (bank deposits), US corporate bonds, US agency securities, and supranational bonds. The rest are relatively higher risk assets, including some US, Eurozone, and UK equities.

3.3. Data and Implementation

Our empirical application is based on 16 indices of bonds and equities. For bonds, we employ nine advanced countries' government bond indices, one 3-month US Treasury Bill index, one US corporate bond index, one US agency index from Bank of America Merrill Lynch, and one supranational bond index from Citi Group Bond Index USBIG. For equities, S&P 100, S&P EURO, and S&P UK are used as the proxies for US, Eurozone, and UK equities, respectively. Monthly total return indices are used over the sample period from January 1995 to December 2011, with a total of 204 observations. All total return indices are calculated in a log-return style based on a US-dollar denomination and the 3-month US T-Bill is taken as the risk-free rate. Table 2 reports the descriptive statistics for all asset classes considered.

<Table 2 about here>

For bonds, US agency has the lowest standard deviation except for the 3-month T-Bill. For all government bonds, the Australia government bond has the best performance with the highest standard deviation, while the US government bond has the lowest standard deviation. All three equity assets have very high volatilities but relatively lower annual returns than during normal times, due to the fact that the sample period covers the recent financial crises.

For the sake of analytical exercise, we consider two different classifications of all selected asset classes. First, following IMF (2012), the selected asset classes are divided into two asset types, whereby all three equity asset classes are classified as risky assets, while other asset classes are classified as safe assets. Second, the selected asset classes are divided into their own currencies based on the location of their markets, although all are calculated on the basis of US dollars. Table 3 shows the asset classes and their corresponding currencies.

<Table 3 about here>

In Table 3, we consider six main categories of currency in terms of the COFER database published by the IMF. Supranational bonds are classified under US Dollars because of their US-dollar denomination, while Canada and Australia government bonds fall under Other Currencies.

Before entering into the MVMA framework, we use the Black-Litterman approach to improve our return forecasts. First, based on the market capitalisations of all asset classes considered, the reverse optimisation process by way of equation (11) is evoked to provide us with the equilibrium excess returns on all these asset classes. Second, taking the equilibrium excess returns as prior, reserve managers introduce their forward-looking investment views. Given the impact of the recent global financial crises, we assume that reserve managers favour more conservative investment strategies, under which government bonds are deemed the best way of flight to safety, hence government bonds are favourable than equities. Using equilibrium excess returns as a reference, reserve managers formulate their three conservative investment views as follows: (1) US equity will outperform US government bonds only by 2.60%; (2) Euro equity will outperform German government bonds only by 3.80%; and (3) UK equity will outperform UK government bonds only by 3.10%. The confidence levels of all three investment views are equal to 50%.

Table 4 presents market weights and two estimates of expected excess returns on all selected asset classes. According to Table 4, the equilibrium excess returns on US government bonds and US equity are 0.12% and 7.73%, respectively, a difference of 7.61%. However, the 2.60% in the first investment view is less than 7.61% by which the returns on US equity exceeds the returns on US government bonds, indicating that reserve managers expect the B-L approach to tilt the portfolio away from US equity in favour of US government bonds. Similarly, the other two investment views also imply their expectation of shifting the portfolio towards government bonds. Thus, all three investment views display that reserve managers have a tendency to invest in government bonds rather than in equities. As a result, comparing the third column and the last column in Table 4, the B-L excess returns on all three equities are less than the equilibrium excess returns of those.

<Table 4 about here>

With equilibrium and the B-L returns, we are now in a position to investigate the strategic reserve asset allocation using the MVMA framework. By solving equations (8) to (10), we derive at two sets of optimal weights for all asset classes. Based on each of these optimal weight sets, we construct our two MA sub-portfolios, i.e. the precautionary and investment sub-portfolios, through specifying different risk-aversion coefficients and considering the risk-return profiles. We also construct different aggregate portfolios by dividing the total investable reserves into two MA sub-portfolios in various proportions. Specifically, we construct three distinct aggregate portfolios: the first is based on an 80:20 division across the two sub-portfolios (80% of the total investable reserves from the precautionary sub-portfolio and 20% from the investment portfolio); the second is based on a 50:50 division, and the third on a 30:70 division. The three aggregate portfolios correspondingly indicate high, moderate, and low risk-aversion attitudes of the reserve managers, respectively. Finally, we probe into the mental accounting problem for all portfolios according to equation (5). In each portfolio, various threshold levels of returns correspond to the maximum probabilities of that portfolio failing to reach those threshold return levels.

3.4. Main Results

3.4.1. The results based on equilibrium returns

Table 5 provides information on holdings of MV efficient portfolios for the two MA sub-portfolios and three aggregate portfolios under the equilibrium return estimate. As suggested by Mehra and Prescott (1985), the range of risk aversion coefficient should be within the interval from 0 to 10. Thus, it is important to control this coefficient to ensure that it lies in the suggested interval. Classifying reserve managers as more conservative investors, we specify the risk aversion coefficient as between 5 and 10. Consequently, reserve managers do not care about seeking very high returns but emphasis on lower market risks.

<Table 5 about here>

According to Table 5, for the precautionary sub-portfolio, when the risk-aversion coefficient set at a higher value, i.e. $\gamma = 9.938$, the largest holdings in the portfolio would American 3-month T-Bill. For the investment portfolio, when the value of risk-aversion parameter is relatively low, i.e. $\gamma = 5.261$, largest holdings would be Japanese government bonds. For the three aggregate portfolios, holding the aggregate portfolio 1 displays the highest risk aversion. Here, the allocation of 43.14% of the total investable reserves to the US 3-month T-Bill implies that reserve managers focus more on their need to maintain ready liquidity in order for the government to intervene in the market if necessary. In contrast, holding the aggregate portfolio 3 indicates that reserve managers have relatively low risk aversion and therefore desire more to seek higher return.

Table 6 presents the optimal asset allocation between safe assets and risky assets based on the results from Table 5. For the precautionary sub-portfolio, nearly 90% of the holdings are in safe assets. For the investment sub-portfolio, the allocation between safe and risky assets is nearly 75:25. For the three aggregate portfolios, although the allocation to safe assets exceeds 75% in all of the cases the proportion of risky assets increases steadily.

<Table 6 about here>

Table 7 shows the resulting composition of currencies for the two sub-portfolios and three aggregate portfolios in terms of the classification of currencies from Table 3 and the results from Table 5. In all portfolios, the largest holdings are US assets. For the sub-portfolios, the greater the risk-aversion coefficient, the larger are the holdings of US assets. This is consistent with our findings about the asset structure in all the aggregate portfolios.

Furthermore, comparing the category of ‘US Dollars’ in Table 7 with the category of ‘Safe Assets’ in Table 6, in each portfolio, all the ratios of US assets to safe assets exceed 70%, which indicates that, for reserve managers favouring the conservative and value-preservation policies, US assets are viewed as the best for flying to safety. Other than US-dollar assets, Table 7 shows that the assets based on Japanese yen, the euro, and pounds-sterling denominations are the main channels for optimal portfolio diversification.

<Table 7 about here>

Table 8 investigates the MA problem by solving equation (5) and presents the combinations of threshold return levels and corresponding maximum probabilities of not reaching them for the two sub-portfolios and three aggregate portfolios. We can see that the maximum probabilities that the reserve managers would have negative returns are 11% and 20% for the precautionary and investment sub-portfolios, respectively, and for aggregate portfolios 1, 2 and 3 they are 13%, 16%, and 18%, respectively. These results correspond to decreasing risk aversions in the two sub-portfolios and three aggregate portfolios. It is convenient and efficient for using this VaR constraint to capture the risk perception of reserve managers in each portfolio. Equation (7) tells us that the portfolio weights are not linearly proportional to the risk-aversion parameter γ, which indicates that the risk-aversion parameter implied in all three aggregate portfolios is distinct from the weighted average of the risk-aversion parameters of the two sub-portfolios.

<Table 8 about here>

3.4.2 Results based on the B-L returns

In this part, all the results are based on the B-L return estimates. After reserve managers state their conservative investment views by shifting the portfolio towards government bonds, Table 9 to Table 12 correspond to and convey the same information as Tables 5 to 8, respectively.

Compared with the results in Table 5, Table 9 shows that in each portfolio, holdings of the US 3-month T-Bill increase while holdings of US equity decrease, which is the result of the more conservative investment strategy of reserve managers in response to the upsurge in demand for safe assets under the influence of the global financial crisis. Also, all the resulting expected portfolio returns and standard deviations in each portfolio after adding reserve manager's views are less than those under estimates with the equilibrium excess returns, indicating that reserve managers care more about managing the market risks than about seeking higher returns.

<Table 9 about here>

For proportion of the holdings between safe and risky assets, Table 10 displays that in each portfolio, holdings of safe assets under the conservative views are greater than those under equilibrium returns, compared with the results in Table 6.

<Table 10 about here>

In comparison with Table 7, Table 11 shows that, with the B-L returns, for each portfolio, holdings of both US and UK assets out of all investable asset classes increase, while holdings of both Euro and Japanese assets decrease.

<Table 11 about here>

For the MA problem, Table 12 indicates that in each portfolio, adding reserve managers' conservative views does decrease the maximum probabilities of failing to reach threshold return levels, due to the fact that in each portfolio the performances, i.e. the expected portfolio returns and standard deviations, decrease to some extent, compared with the results in Table 8.

<Table 12 about here>

4. Conclusions

In this paper, we propose a behavioural strategic asset allocation for foreign reserves to derive a multiple-goal reserve management policy for central banks. We design two sub-portfolios (mental accounts): the 'precautionary sub-portfolio' and the "investment sub-portfolio". The precautionary sub-portfolio exhibits higher risk aversion and favours safe and liquid assets. Such a sub-portfolio is therefore capable of fulfilling both safety and liquidity objectives of the reserve management. The 'investment sub-portfolio' exhibits lower risk aversion and can satisfy reserve managers' need to seek higher returns and thus fulfil the return objective. We also design different aggregate portfolios to display the distinct overall risk attitudes of reserve managers. Under this investment policy, we use the Black-Litterman approach to improve our return forecasts and therefore to overcome the maximisation problem of mean-variance optimisation.

Using equilibrium returns as a starting point, the B-L returns are obtained by adding the reserve managers' views.

Taking China as an example, we apply this behavioural reserve management framework to practical use. In line with the current trend of central banks' reserve management and their evolving investment universe, our approach sheds critical lights on optimal asset allocation in a volatile world under the impacts of the global financial crisis. Against conventional method, our approach shows several advantages. First, the creation of a sub-portfolio associated with a certain goal allows reserve managers to make investment decisions in a stratforward manner. Fail this, reserve managers with multiple investment objectives must choose portfolios based on the overall expected returns and their standard deviations. Second, risk can be measured by the maximum probabilities of not reaching the threshold of each goal as illustrated in Tables 7 and 11, rather than by the standard deviation of the returns of the overall portfolio. This measurement ensures that reserve managers can measure risks directly and efficiently. Using these advantages, not only can reserve managers specify different degrees of risk aversion to formulate their desired sub-portfolios, but also they can adjust the allocations of their total investable reserves across sub-portfolios to construct different aggregate portfolios, and can establish their desirable aggregate portfolio based on this risk measurement.

For future research, one may consider broadening the set of asset classes to expand the investable universe of official reserve managers. This may even include the real assets such precious metals and stones, real estate assets and commodities. Of particular interest would be gold as an asset class. Gold has a long history as a reserve asset. In the current volatile world where safe assets are in great demand, gold's attribute as a safe asset takes on a growing interests by global reserve managers. According to the IMF (2012), gold, as a potentially safe asset, has global market capitalisation of 8.4 trillion US dollars, accounting for 11% of total global safe assets. Thus, taking into consideration of investment opportunity in gold would be an important avenue for reserve managers to expand the diversification possibility.

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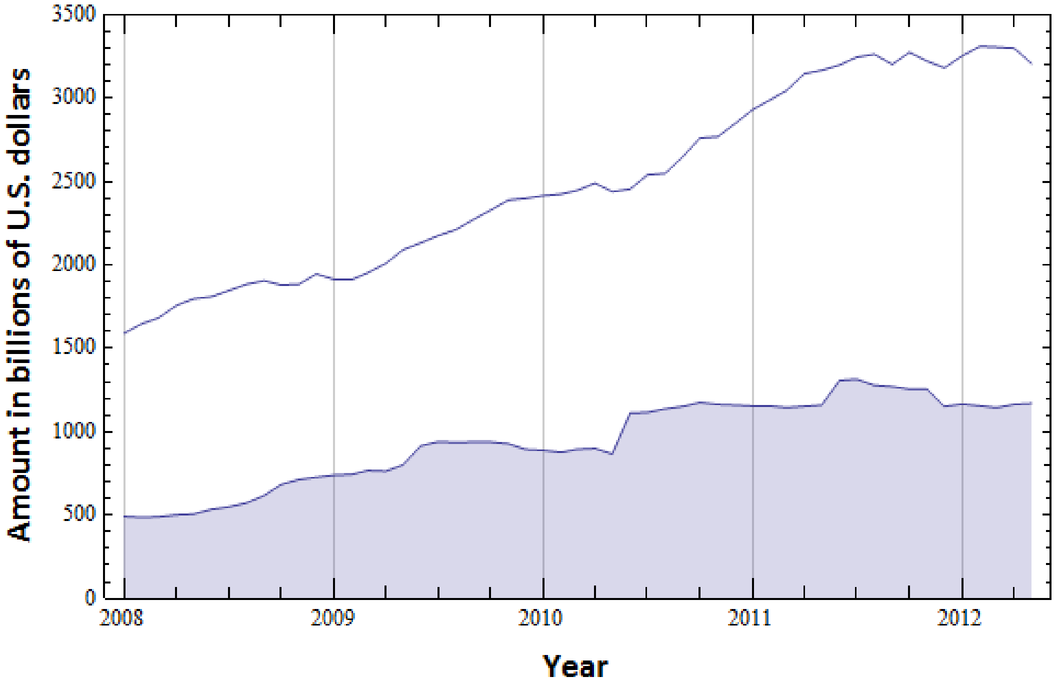

Figure 1 China's Total Amount of Reserves and its Holdings of US Treasury Securities
Sources: The People's Bank of China and US Department of The Treasury

Table 1 Composition of China's Holdings of Total US Assets

Type of Security	Long-Term Treasury	Long-Term Agency	Equity	Long-Term Corporate	Short-Term Debt	Total
Amount (billions of US dollars)	1302	245	159	16	5	1727
Percentage	75.39%	14.19%	9.21%	0.93%	0.29%	100%

Sources: US Department of the Treasury, Report on Foreign Portfolio Holdings of US Securities as of June 30, 2011.

Table 2 Descriptive Statistics

Name	Market	Instrument Type	Mean	Standard Deviation
US GOVT	USA	Long-term Bonds	5.67%	3.89%
CANADA GOVT	Canada	Long-term Bonds	8.58%	9.03%
AUSTRALIA GOVT	Australia	Long-term Bonds	8.86%	13.11%
UK GOVT	UK	Long-term Bonds	6.71%	10.10%
SWISS GOVT	Switzerland	Long-term Bonds	5.97%	11.65%
GERMANY GOVT	Germany	Long-term Bonds	5.81%	11.93%
FRENCH GOVT	France	Long-term Bonds	6.38%	12.06%
ITALIAN GOVT	Italy	Long-term Bonds	7.53%	12.13%
JAPAN GOVT	Japan	Long-term Bonds	4.09%	9.65%
US CORP	USA	Corporate	6.89%	6.49%
US AGENCY	USA	Agency	5.85%	3.61%
TBILL 3M	USA	Bank Deposit or Short-term Bonds	3.44%	2.02%
SUPRANATIONAL	Supranational	Supranational	6.36%	5.00%
US EQUITY	USA	Equity	8.13%	22.71%
EURO EQUITY	Euro Zone	Equity	8.82%	27.26%
UK EQUITY	UK	Equity	8.18%	25.42%

Notes: This table shows descriptive statistics of all selected asset classes. Our calculations use monthly data. The mean and standard deviations are annualised.

Table 3 Asset Classes and their Currencies

Currency	Market	Asset Class
US Dollars	USA & Supranational	US GOVT, US CORP, US AGENCY, TBILL 3M, US EQUITY, & SUPRANATIONAL
Pounds Sterling	UK	UK GOVT & UK EQUITY
Euros	Germany, France, Italy & Euro Zone	GERMANY GOVT, FRENCH GOVT, ITALIAN GOVT, & EURO EQUITY
Japanese Yen	Japan	JAPAN GOVT
Swiss Francs	Switzerland	SWISS GOVT
Other Currencies	Canada & Australia	CANADA GOVT & AUSTRALIA GOVT

Notes: All selected asset classes are classified into their own currencies based on the location of their markets. The currency categories are according to the database of Composition of Official Foreign Exchange Reserves (COFER) published by the IMF.

Table 4 Market Weights and Return Estimates

Name	Market Weights	Equilibrium Excess Returns	The B-L Excess Returns
US GOVT	22.90%	0.12%	0.14%
CANADA GOVT	1.96%	2.38%	1.80%
AUSTRALIA GOVT	0.79%	3.70%	2.59%
UK GOVT	2.63%	2.86%	1.96%
SWISS GOVT	0.24%	1.64%	1.15%
GERMANY GOVT	3.17%	2.41%	1.62%
FRENCH GOVT	3.24%	2.59%	1.71%
ITALIAN GOVT	3.55%	2.84%	1.80%
JAPAN GOVT	20.97%	0.72%	0.26%
US CORP	5.92%	1.65%	1.24%
US AGENCY	4.15%	0.16%	0.17%
TBILL 3M	2.89%	0.09%	0.09%
SUPRANATIONAL	1.73%	0.38%	0.37%
US EQUITY	15.27%	7.73%	5.25%
EURO EQUITY	6.34%	9.24%	6.93%
UK EQUITY	4.24%	7.98%	6.07%

Notes: Market weights are obtained by using market capitalisation data of all asset classes. Equilibrium excess returns are derived by the reverse optimisation process, i.e. equation (11). The B-L excess returns are gained via equation (12).

Sources: Market capitalisation data of all safe assets are from BIS Securities Statistics on the BIS official website. The data of all risky assets are from Standard & Poor's official website.

Table 5 (Equilibrium Returns)
Holdings of Efficient Portfolios for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Asset Classes	**Precautionary Sub-portfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
US GOVT	0.07%	12.34%	2.53%	6.21%	8.66%
CANADA GOVT	0.36%	2.33%	0.76%	1.35%	1.74%
AUSTRALIA GOVT	3.65%	1.29%	3.18%	2.47%	2.00%
UK GOVT	0.00%	3.23%	0.65%	1.62%	2.26%
SWISS GOVT	2.58%	1.48%	2.36%	2.03%	1.81%
GERMANY GOVT	3.39%	2.70%	3.25%	3.04%	2.90%
FRENCH GOVT	0.27%	2.71%	0.76%	1.49%	1.98%
ITALIAN GOVT	0.02%	3.51%	0.72%	1.76%	2.46%
JAPAN GOVT	15.73%	21.29%	16.84%	18.51%	19.62%
US CORP	7.93%	3.05%	6.95%	5.49%	4.51%
US AGENCY	0.06%	5.11%	1.07%	2.58%	3.59%
TBILL 3M	53.28%	2.57%	43.14%	27.93%	17.79%
SUPRANATIONAL	0.07%	11.49%	2.35%	5.78%	8.06%
US EQUITY	6.49%	16.57%	8.50%	11.53%	13.54%
EURO EQUITY	0.29%	7.00%	1.63%	3.64%	4.98%
UK EQUITY	5.82%	3.33%	5.32%	4.57%	4.08%
Total Weights	100%	100%	100%	100%	100%
Expected Return	5.00%	6.36%	5.27%	5.68%	5.95%
Std. Dev.	4.00%	7.45%	4.69%	5.72%	6.41%

Notes: The portfolio weights for all portfolios are obtained using the solutions in equations (6) to (10). The expected returns and standard deviations of all portfolios are presented at the bottom of the table.

Table 6 (Equilibrium Returns)
Holdings of Safe and Risky Assets for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	**γ = 9.938**	**γ = 5.261**	**80:20 Mix**	**50:50 Mix**	**30:70 Mix**
Asset Types	**Precautionary Su-bportfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
Safe assets	87.41%	73.11%	84.55%	80.26%	77.40%
Risky assets	12.59%	26.89%	15.45%	19.74%	22.60%
Total Weights	100%	100%	100%	100%	100%
Expected Return	5.00%	6.36%	5.27%	5.68%	5.95%
Std. Dev.	4.00%	7.45%	4.69%	5.72%	6.41%

Notes: The results are summarised based on the results from Table 5 and the classification of asset types is by the IMF (2012).

Table 7 (Equilibrium Returns)
Composition of Currencies for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	**γ = 9.938**	**γ = 5.261**	**80:20 Mix**	**50:50 Mix**	**30:70 Mix**
Currencies	**Precautionary Sub-portfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
US Dollars	67.89%	51.13%	64.54%	59.51%	56.16%
Pounds Sterling	5.82%	6.56%	5.97%	6.19%	6.34%
Euros	3.97%	15.91%	6.36%	9.94%	12.33%
Japanese Yen	15.73%	21.29%	16.84%	18.51%	19.62%
Swiss Francs	2.58%	1.48%	2.36%	2.03%	1.81%
Other Currencies	4.01%	3.63%	3.94%	3.82%	3.74%
Total Weights	100%	100%	100%	100%	100%
Expected Return	5.00%	6.36%	5.27%	5.68%	5.95%
Std. Dev.	4.00%	7.45%	4.69%	5.72%	6.41%

Notes: The results are obtained based on the results from Table 5 and the classification of currencies is from Table 3.

Table 8 (Equilibrium Returns)
Threshold Return Levels and Corresponding Maximum Probabilities of Not Reaching Them

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Threshold (H)	Precautionary Sub-portfolio Prob[r<H]	Investment Sub-portfolio Prob[r<H]	Aggregate Portfolio 1 Prob[r<H]	Aggregate Portfolio 2 Prob[r<H]	Aggregate Portfolio 3 Prob[r<H]
-15.00%	0.00	0.00	0.00	0.00	0.00
-10.00%	0.00	0.01	0.00	0.00	0.01
-5.00%	0.01	0.06	0.01	0.03	0.04
0.00%	0.11	0.20	0.13	0.16	0.18
5.00%	0.50	0.43	0.48	0.45	0.44
10.00%	0.89	0.69	0.84	0.78	0.74
15.00%	0.99	0.88	0.98	0.95	0.92
Expected Return	5.00%	6.36%	5.27%	5.68%	5.95%
Std. Dev.	4.00%	7.45%	4.69%	5.72%	6.41%

Notes: The results are computed using equation (5) after obtaining portfolio returns and the standard deviations for each portfolio.

Table 9 (The B-L Returns)
Holdings of Efficient Portfolios for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Asset Classes	**Precautionary Sub-portfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
US GOVT	0.60%	17.73%	4.03%	9.17%	12.59%
CANADA GOVT	3.52%	4.80%	3.77%	4.16%	4.42%
AUSTRALIA GOVT	0.42%	0.09%	0.35%	0.25%	0.19%
UK GOVT	0.03%	0.02%	0.03%	0.02%	0.02%
SWISS GOVT	3.92%	0.79%	3.29%	2.35%	1.73%
GERMANY GOVT	0.07%	0.05%	0.07%	0.06%	0.06%
FRENCH GOVT	0.05%	0.03%	0.04%	0.04%	0.03%
ITALIAN GOVT	0.02%	0.01%	0.02%	0.02%	0.02%
JAPAN GOVT	13.11%	17.49%	13.99%	15.30%	16.18%
US CORP	5.54%	0.07%	4.44%	2.81%	1.71%
US AGENCY	0.63%	2.17%	0.94%	1.40%	1.71%
TBILL 3M	62.43%	15.25%	52.99%	38.84%	29.41%
SUPRANATIONAL	0.72%	21.46%	4.86%	11.09%	15.23%
US EQUITY	0.02%	0.00%	0.01%	0.01%	0.01%
EURO EQUITY	0.46%	10.84%	2.54%	5.65%	7.73%
UK EQUITY	8.47%	9.20%	8.62%	8.84%	8.98%
Total Weights	100%	100%	100%	100%	100%
Expected Return	4.27%	5.02%	4.42%	4.65%	4.79%
Std. Dev.	2.99%	5.51%	3.49%	4.25%	4.75%

Notes: The portfolio weights of all portfolios are obtained using the solutions in equations (6) to (10). The expected returns and standard deviations of all portfolios are presented at the bottom of the table.

Table 10 (The B-L Returns)
Holdings of Safe and Risky Assets for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Asset Types	**Precautionary Sub-portfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
Safe assets	91.05%	79.96%	88.83%	85.50%	83.29%
Risky assets	8.95%	20.04%	11.17%	14.50%	16.71%
Total Weights	100%	100%	100%	100%	100%
Expected Return	4.27%	5.02%	4.42%	4.65%	4.79%
Std. Dev.	2.99%	5.51%	3.49%	4.25%	4.75%

Notes: The results are summarised based on the results in Table 9 and the classification of asset Types is by the IMF (2012).

Table 11 (The B-L Returns)
Composition of Currencies for Two Sub-Portfolios and Three Aggregate Portfolios

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Currencies	**Precautionary Sub-portfolio**	**Investment Sub-portfolio**	**Aggregate Portfolio 1**	**Aggregate Portfolio 2**	**Aggregate Portfolio 3**
US Dollars	69.92%	56.69%	67.28%	63.31%	60.66%
Pounds Sterling	8.50%	9.22%	8.65%	8.86%	9.00%
Euros	0.60%	10.93%	2.67%	5.77%	7.83%
Japanese Yen	13.11%	17.49%	13.99%	15.30%	16.18%
Swiss Francs	3.92%	0.79%	3.29%	2.35%	1.73%
Other Currencies	3.93%	4.89%	4.13%	4.41%	4.60%
Total Weights	100%	100%	100%	100%	100%
Expected Return	4.27%	5.02%	4.42%	4.65%	4.79%
Std. Dev.	2.99%	5.51%	3.49%	4.25%	4.75%

Note: The results are obtained based on the results in Table 9 and the classification of currencies in Table 3.

Table 12 (The B-L Returns)
Threshold Return Levels and Corresponding Maximum Probabilities of Not Reaching Them

Risk Aversion:	γ = 9.938	γ = 5.261	80:20 Mix	50:50 Mix	30:70 Mix
Threshold (H)	Precautionary Sub-portfolio Prob[r<H]	Investment Sub-portfolio Prob[r<H]	Aggregate Portfolio 1 Prob[r<H]	Aggregate Portfolio 2 Prob[r<H]	Aggregate Portfolio 3 Prob[r<H]
-15.00%	0.00	0.00	0.00	0.00	0.00
-10.00%	0.00	0.00	0.00	0.00	0.00
-5.00%	0.00	0.03	0.00	0.01	0.02
0.00%	0.08	0.18	0.10	0.14	0.16
5.00%	0.60	0.50	0.57	0.53	0.52
10.00%	0.97	0.82	0.94	0.90	0.86
15.00%	1.00	0.96	1.00	0.99	0.98
Expected Return	4.27%	5.02%	4.42%	4.65%	4.79%
Std. Dev.	2.99%	5.51%	3.49%	4.25%	4.75%

Notes: The results are computed using equation (5) after obtaining portfolio returns and standard deviations for each portfolio.