A Dynamic Optimization on Energy Efficiency in Developing Countries

Wang, Dong

24 November 2012

Working Paper

A Dynamic Optimization on Energy Efficiency in Developing Countries

Dong Wang

Crawford School of Public Policy, Australian National University, Canberra, Australia

Business School of Sichuan University, Chengdu, China

Email: wangdong.anu@gmail.com

ABSTRACT

This paper introduces a way for measuring the energy efficiency in economics besides the methods in physics. The linkage among energy efficiency, energy consumption and other macroeconomic variables is demonstrated primarily. Based on the methodology of dynamic optimization, a maximum problem of energy efficiency over time is subjected to the extended Solow growth model and instantaneous investment rate. In this model, energy consumption is set as control variable and investment is regarded as state variable. The analytic solutions can be derived and the diagrammatic analysis provides saddle-point equilibrium. With assigning values to parameters, a numerical simulation is presented; meanwhile the optimal paths of investment and energy consumption can be drawn. The discussion on modelling and implications is organized in the end. The dynamic optimization encourages governments in developing countries to pursue higher energy efficiency as it can reduce energy use without influencing the achievement of steady state in terms of Solow model.

Keywords: energy efficiency; dynamic optimization; develpment

1 Introduction

Social planners and policymakers have been attracted by dynamic optimization issues for many years. To some degree, ‘path choice problem’ always lies in the centre of policy debate not only in developed countries, but especially in developing countries. Although a large quantity of researches have discussed on the optimal path of economic growth, energy consumption or pollution reduction, seldom economists have set their feet in the energy efficiency issues under the view of dynamic. In fact, the improvement of energy efficiency is a dynamic procedure in the development and it is always related to growth, investment, technology change and many other economic variables.

Energy efficiency can be defined in three dimensions. The first definition stems from the laws of thermodynamics in physics. It is defined as a ratio of best practice energy input over energy input, *ceteris paribus*, which refers to technical efficiency[1] (Jin & Arons, 2009) and cannot be greater than one. The second definition is based on economic concepts and named energy intensity[2], which is the ratio of energy input over output (National Bureau of Statistics of China, 2010). However, this definition only considers energy as a unique input with ignoring the other factors in the production such as capital and labour. David Stern (2012) has developed this definition of economic energy efficiency under the multi-input framework. In his work, the economic technical efficiency is on the basis of Pareto principle and is associated with capital as another input. He stated that any economy has two inputs for production. The one is energy and the other is capital. People should utilize the input composition to attain the goal of output and growth. Thus, by this argument, there must be an optimal solution about the input combination in development, which can be seen as the economic energy efficiency. The things people need to do are to make the economy operating under the best energy efficiency condition. In this paper, I adapt the meaning of energy efficiency is based on the Stern's definition. In other words, I consider how to allocate and utilize energy and

---

[1] The thermodynamic definition of the energy efficiency η is $\eta = \frac{W_{in}^{min}}{W_{in}^{real}} \leq 1$

[2] energy intensity $= \frac{E}{Y}$ Where E is energy input and Y is total output of the society.

capital as two inputs efficiently for achieving the desired output and growth.

The attendant question is why the economic energy efficiency is crucial? In many developing countries, it is inevitable that the energy consumption is increasing with a rapid economic growth and the improvement of living standards. This may lead to energy security and environmental problems meanwhile. Probably, a continuous increase in energy efficiency is an appropriate solution for this dilemma, even though energy efficiency cannot be increased infinitely in terms of the second law of thermodynamics.

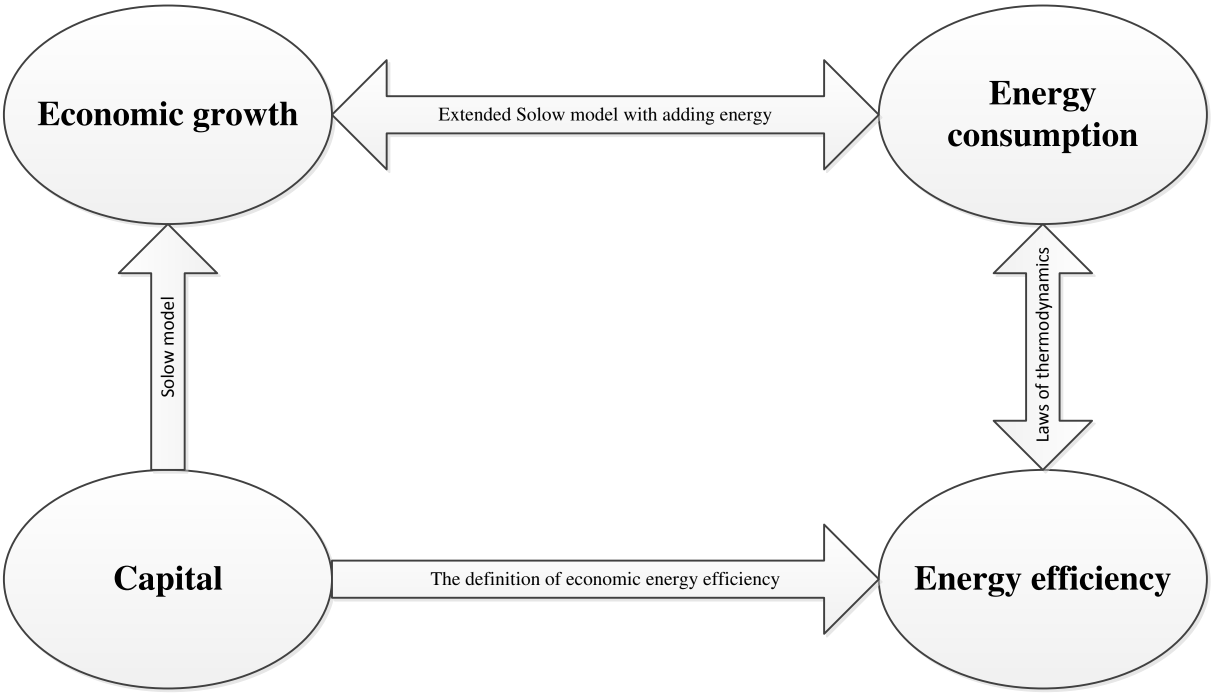
Apparently, advanced technology applications can increase energy efficiency. Besides that, the underlying drivers are capital, human resources and even energy itself. Firstly, investment and skilled labour can promote technology and energy utilization; secondly, different types of energy have different potential for energy efficiency promotion. The modern energy contains more exergy[3], meaning that higher energy efficiency could be achieved. However, the transition from conventional energy to modern energy in developing countries also needs sufficient capital accumulation and adequate economic growth. Hence, the improvement of energy efficiency is interlinked with investment capacity, labour force quality and economic growth stages.

Nonetheless, the labour force shifts spontaneously and cannot be controlled or planned easily. While the investment whether in its scale or speed, is controllable in most situations. Additionally, investment can influence the improvement of energy efficiency by means of technology and furthermore, influence the energy consumption and economic growth. That is to say, the pace of energy efficiency improvement should feature dynamic and be restricted by capital, growth and energy itself. Consequently, there may be a dynamic optimal path of energy efficiency improvement in development. The mechanism is illustrated as Figure 1.

---

[3] Exergy is available energy, which is the maximum useful work possible during a process that brings the system into equilibrium with a heat reservoir (Perrot, 1998).

Figure 1 The mechanism and linkage between energy efficiency and other economic variables



It is of importance and inspiration on both theory and practice. In theory, if we can model the dynamic path of energy efficiency linking with energy consumption and economic growth, we can introduce energy efficiency into traditional 'energy-economy-environment' (3E) analysis. Furthermore, the coming 3E analysis on technology transformation, factor allocation and energy transition could access to energy efficiency discussion. In practice, the modelling could reveal the best choice of controlled path for developing countries on how to enhance energy efficiency given the limited capital stock and investment.

The purpose of this paper is to model the dynamic optimal path of energy efficiency improvement given the investment shift and production function. Firstly, a function is established for measuring energy. Secondly, Solow growth is extended by adding energy and deriving the instantaneous state equation of investment. Lastly, the steady-state solution can be solved by dynamic optimization method. The paper is organised by six parts. The literature review follows the introduction, and then the methodology is introduced in section three. Results and a numerical simulation are presented in section four followed by some discussions in section five. The conclusion is arranged at the end of the paper.

2 Literature review

The reviewed literature includes three categories: the application of dynamic optimization in exhaustible resource economics; the recent work on the dynamic relationship among energy, economy and environment; and the literature on energy efficiency.

Dynamic optimization has been applied for resource exploration problems since 1970s (Pindyck, 1978, 1980). He established a basic model on the optimal exploration of non-renewable resources in 1978 and developed it in 1980 with adding uncertainty into exhaustible resource market analysis. Both of the two models are based on cost benefit analysis. Basically, they are general models and they only focus on the optimization in production. The shortcoming is that the energy depletion in the models has not been linked with economic growth and any other macroeconomic variables.

Some other economists (Stiglitz, 1974; Garg and Sweeney, 1978; Dasgupta and Heal, 1979) brought the optimal exploration problem into the framework of neoclassical model of growth. They have discussed well on the optimal sustainable growth path under the condition that the resources are scarcity and diminishing all the time. But the technology element was assumed exogenous in their models, which has aroused a wide controversy. Having the endogenous growth model been raised (Romer, 1990; Lucas, 1988), the technology change became endogenous so that the long-run analysis was feasible and reasonable. Nevertheless, the literature on endogenous growth model rarely includes the natural resources problems. The two recent papers for China's issues are written by Peng (2007) and Li et al. (2012). They developed the endogenous growth model with treating natural resources as a constraint and got an optimal path of development eventually. But they do not mention the issues about energy efficiency.

Another bulk of literature explores the relationship between energy and environmental issues from dynamic perspective. The most famous example is Forster model which was raised in 1980. The model constructed an instantaneous utility function which depends on the level of consumption and pollution by means of the principle of utility theory. Since the pollution can be associated with energy use in Forster model, we can establish a maximum problem with energy as a control variable. It is a regular model for considering the optimization problem in energy and environmental fields. In addition, Chiang provided a similar optimal example about anti-pollution policy (1992). Besides, Conrad (2001) linked energy with carbon emission and solved an optimal path for resource allocation. These researches provide a thinking direction on energy and environment, and inspire me somehow. But they do not discuss the energy efficiency either.

On the side of energy efficiency, the papers, usually, are case by case instead of providing a general and theoretical framework. Moreover, most of the discussions are on the technology level with using the technical definition not economic definition of energy efficiency. For instance, Jaffe and Stavins (1994) stated the five dimentions of optimal energy use for analying the energy-efficiency gap; two recent papers on energy efficiency are on the level of technology applications and management (Sanchez & Ruiz, 2009; Brennan, 2010). Although Stern (2012) models the international trends of energy efficiency detailedly, his work does not involve the dynamic improvement path issue. Thus, a generalised analysis on the optimal energy efficiency path from economics perspective is needed.

3 Methodology

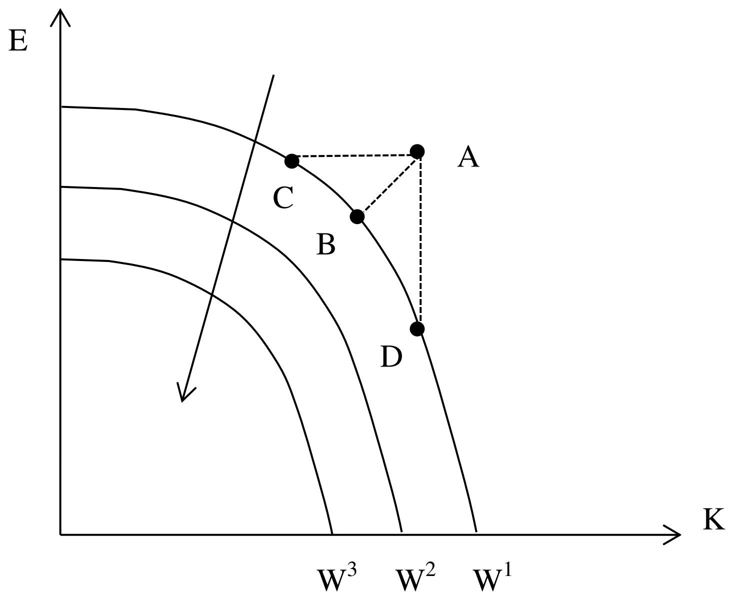
The optimal energy efficiency path is hard to discover mainly because the energy efficiency is hard to be defined apporprately. In this section, the modelling will start with a quantative definition which is given by mathmatical and geometrical method. And then, Solow model will be extended with taking energy consumption into account. After these preparations, the dynamic maximum problem with constraints will be modelled.

3.1 The meaning of economic energy efficiency

The economic energy efficiency can be defined as a function of capital stock and energy consumption under the multi-input framework. Obviously, energy efficiency depends on energy use. The reasons that we encompass capital in the model are from the views of stock and flow angles. For one thing, the level of capital stock determines the level of development and technology, which is the foundation in energy efficiency improvement. For another thing, the capital flow is directly associated with investment, which is the key driver of economic growth and technology progress in developing countries. Thus, capital is tightly related with the energy efficiency. In this model, technology is assumed exogenous and it is embodied by investment and capital stock level.

With the energy and capital as two inputs, to some extent, economic energy efficiency is similar to production function. On the other hand, some certain level of energy efficiency can be evaluated and compared with each other, similar to the methods used in the theory of ordinal utility. This definition is demonstrated geometrically in Figure 2.

Figure 2 The economic energy efficiency



In figure 2, the three curves represent three levels of energy efficiency under different technology, capital, energy consumption or output conditions. All points on the curves are efficient states. The direction of the arrow means energy efficiency increasing, implying that 'the less, the better' for inputs. Given the level of output, the less inputs use means the more efficient energy economic system is. W stands for energy efficiency, as a result, $W^3 > W^2 > W^1$.

The energy efficiency curve is concave rather than curving inward mainly because of diminishing marginal rate of substitution. That is how much capital input increase can substitute one unit of energy we reduce. It is similar to the production-possibility curve.

There are two ways for increasing energy efficiency, which are demonstrated by both arrow and dotted line. A is on the right of curve W^1, meaning that A use more energy and capital than the system indeed need given the best efficient curve W^1. So A is inefficient point. B, C and D on W^1 are all the efficient points. Now, we have three paths to haul the inefficient point A back to the efficient state W^1, which is called diminishing the energy distance. If we reduce both energy and capital, we can get B; if we only reduce energy with holding capital input unchanged, we can get D; if we only reduce capital with holding energy input unchanged, we can get C.

Another important thing is about other macroeconomic variables change including technology, investment and output. For any certain output, technology progress can improve the best level of efficiency (the curves). Put differently, the state shift from W^1 to W^3 may result from technology advance which is associated with the level of capital. In this situation, we call the economy climbing the energy efficiency curves. Modelling in this paper is indicating this situation instead of energy distance.

An equation for quantifying the level of energy efficiency W is

$$\mathrm{W(K, E)} = -E^2 - \alpha K\,, \qquad \alpha > 0 \qquad (1)$$

Where W is the level of energy efficiency; K is capital input and E is Energy input. α is a parameter. The projection of W on the E—K plane is a cluster of curves like figure 2.

3.2 The extended Solow model

For linking energy with growth and investment, we should extend neoclassical growth model. Holding technology as an exogenous variable, Stern (2011) developed the Solow model and discussed the role of energy in growth. However, his model is so complicated that it cannot be solved. In this paper, I provide a simple extended model as follow.

$$\mathrm{Y} = (1-\gamma)K^{\beta}L^{1-\beta} + \gamma E\,, \qquad 0<\gamma<1,\ \ 0<\beta<1 \qquad (2)$$

Equation (2) embeds Solow model, which is a Cobb-Douglas function of value added, with adding energy (E), which produces gross output Y. The term of $K^{\beta}L^{1-\beta}$ is the traditional component of Solow model with capital (K) and labour (L). β and γ are parameters. γ reflects the relative importance of energy and Solow value added.

Besides, we can get an equation on instantaneous investment state which reveals capital flow.

$$\dot{K} = s(Y-E) - \delta K\,, \qquad 0<s<1, 0<\delta<1 \qquad (3)$$

Here, $\dot{K}$ is the growth rate of the capital stock which refers to investment. s is the rate of saving. The capital depreciates at a constant rate δ. The term $(Y-E)$ is different from Solow model. It implies that, under the energy constraint, the true accumulated capital should be adjusted by energy consumption. Hence, the instantaneous increment of capital is the proportion of gross output with subtracting the depreciation of capital stock from the net accumulative capital.

3.3 The maximum dynamic problem

Modelling on the maximum dynamic problem of energy efficiency under the growth and energy constraints is expressed mathematically.

$$\text{Max} \int_0^T W(K,E)dt \qquad (4)$$

$$\text{s. t.} \quad \text{Y} = (1-\gamma)K^{\beta}L^{1-\beta} + \gamma E \qquad (2)$$

$$\dot{K} = s(Y-E) - \delta K \qquad (3)$$

$$K(0) = K_0 \qquad \text{K(T)} \ \ Free$$

The equation (2) and (3) are two constraints and equation (3) is state equation. Accordingly, capital (K) is the state variable and energy (E) is the control variable. The equation (4) does not include discounting rate in terms of the specific problem of energy efficiency. As mentioned above, the main meaning of the function (1) is for indicating the degree of energy efficiency and for comparing different levels of efficiency. Thus, it has little meaning in discounting.

Concerning about the boundary condition, the initial value of the state variable is given by E_0, and the terminal value is free. T is the ending time and it is flexible or $T \to \infty$. In this free ending point problem, the transversality condition is

$$\mu(0) = 0, \quad \mu(\text{T}) = 0 \qquad (5)$$

Substituting equation (2) into (3) and integrating them as one constraint, the Hamiltonian is given by

$$\text{H} = \text{U(K,E)} + \mu[\text{s(Y} - \text{E)} - \delta\text{k}]$$

$$= -E^2 - \alpha K + \mu\left[\text{s}\left((1-\gamma)K^{\beta}L^{1-\beta} + \gamma E - \text{E}\right) - \delta\text{K}\right] \qquad (6)$$

The first order condition is

$$\frac{\partial \text{H}}{\partial \text{E}} = 0 \qquad (7)$$

$$\dot{\mu} = -\frac{\partial H}{\partial K} \qquad (8)$$

$$\dot{K} = s(Y - E) - \delta K \qquad (9)$$

4 Results

4.1 Diagrammatic analysis

I use phrase-diagram to analyze the steady-state of this constrained problem. Suppose that labour is constant, denoting $\bar{L}$. That is, during the observed years, the gross number of the labour force in the country should not change. Solving for the steady-state point, the first order condition can be rewritten as

$$\frac{\partial H}{\partial E} = -2E + \mu\gamma - \mu = 0 \qquad (10)$$

$$\dot{\mu} = -\frac{\partial H}{\partial K} = \alpha - \mu\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1} + \mu\delta \qquad (11)$$

$$\dot{K} = s\big[(1-\gamma)K^{\beta}L^{1-\beta} + \gamma E - E\big] - \delta K = 0 \qquad (12)$$

From equation (10), we can get

$$\mathrm{E} = \frac{\mu}{2}(\gamma - 1) \qquad (13)$$

The derivative of E is

$$\dot{E} = \frac{(\gamma-1)}{2}\dot{\mu} \qquad (14)$$

Substitute equation (11) into (14),

$$\dot{E} = \frac{(\gamma-1)}{2}\dot{\mu} = \frac{(\gamma-1)}{2}[\alpha - \mu\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1} + \mu\delta\,] = 0 \qquad (15)$$

From equation (15), we can get the notation of K

$$\mathrm{K} = \left[\frac{\mu\beta s(1-\gamma)\bar{L}^{1-\beta}}{\alpha+\mu\delta}\right]^{\frac{1}{1-\beta}} \qquad (\dot{E}=0 \text{ curve}) \qquad (16)$$

Equation (16) is for $\dot{E}=0$ curve. It only depends on the parameters of the system and the scale of population. This is a constant in the E—K space as illustrated in Figure 3.

Solving equation (12), we can get the solution is

$$\mathrm{E} = \frac{\delta K}{(1-\gamma)} - \bar{L}^{1-\beta}K^{\beta} \qquad \left(\dot{K}=0 \text{ curve}\right) \qquad (17)$$

Equation (17) is for $\dot{K}=0$ curve as illustrated in Figure 3. Consequently, the differential system of E and K in E—K space is

$$\begin{cases} \dot{E} = \dfrac{(\gamma-1)}{2}(\alpha+\mu\delta) - \dfrac{(\gamma-1)}{2}\mu\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1} & (18) \\ \dot{K} = s(\gamma-1)E + s(1-\gamma)\bar{L}^{1-\beta}K^{\beta} - s\delta K & (19) \end{cases}$$

The direction of movement depends on the signs of the derivatives $\dot{E}$ and $\dot{K}$ at particular point in the E—K space. We can find by the differentiation from equation (18) and (19).

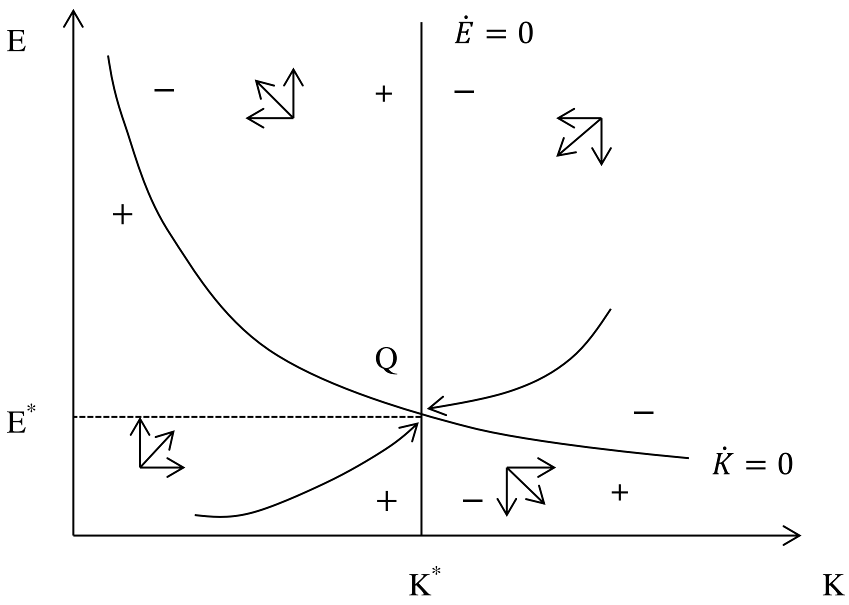
$$\frac{\partial\dot{E}}{\partial K} = (\beta-1)\frac{(\gamma-1)^2}{2}\mu\beta s\bar{L}^{1-\beta}K^{\beta-2} < 0 \qquad (20)$$

$$\frac{\partial\dot{K}}{\partial E} = s(\gamma-1) - \delta < 0 \qquad (21)$$

The negative sign of equation (20) implies that with K increasing, $\dot{E}$ should be decreasing. The negative sign of equation (21) implies that with E increasing, $\dot{K}$ should be decreasing. The directions are denoted in Figure 3 followed by drawing the possible path of their movement. The phrase-diagram analysis indicates that the equilibrium point Q is a saddle point. The mathematical solution is derived from equation (16) and (17). Thus, the steady-state is

$$
\begin{cases}
K^* = \left[\dfrac{\mu\beta s(1-\gamma)\bar{L}^{1-\beta}}{\alpha+\mu\delta}\right]^{\frac{1}{1-\beta}} \\[2em]
E^* = \dfrac{\delta}{(1-\gamma)}\left[\dfrac{\mu\beta s(1-\gamma)\bar{L}^{1-\beta}}{\alpha+\mu\delta}\right]^{\frac{1}{1-\beta}} - \bar{L}^{1-\beta}\left[\dfrac{\mu\beta s(1-\gamma)\bar{L}^{1-\beta}}{\alpha+\mu\delta}\right]^{\frac{\beta}{1-\beta}}
\end{cases}
$$

Figure 3 The phrase—diagram analysis



4.2 The solutions of dynamic optimal path

Firstly, we solve the co-state variable μ, from the equation (11) and calculate the integral of $\dot{\mu}$ for time t. The equation (11) can be rearranged as a regular linear differential equation of first order.

$$\dot{\mu} - \left[\delta - \beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}\right]\mu = \alpha$$

So, the analytic solution of μ is

$$\mu(t) = e^{\int[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]dt}\left[C+\int \alpha e^{\int -[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]dt}dt\right]$$

$$= Ce^{[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]t} - \frac{\alpha}{[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]}e^{-[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]t}$$

Combining with the transersality condition in equation (5), we can get the particular solution is

$$\mu(t) = \frac{\alpha}{[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]}e^{[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]t}\left[1-e^{-\{[\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1}]t+1\}}\right]$$

For simplicity, Let $\delta-\beta s(1-\gamma)\bar{L}^{1-\beta}K^{\beta-1} = A$, the solution of $\mu(t)$ can be simplified as

$$\mu(t) = \frac{\alpha}{A}e^{At}\left[1-e^{-(At+1)}\right] \qquad (22)$$

Secondly, the state variable K can be solved by substituting the equation (22) into (16). The optimal time path of the capital stock optimal capital path is

$$K = \left[\frac{\beta s e^{At}\left(1-e^{-(At+1)}\right)(1-\gamma)\bar{L}^{1-\beta}}{A+\delta e^{At}(1-e^{-(At+1)})}\right]^{\frac{1}{1-\beta}} \qquad (23)$$

Furthermore, we substitute equation (22) into (13) for solving the optimal path of control variable E.

$$E = \frac{\alpha(\gamma-1)}{2}e^{At}\left[1-e^{-(At+1)}\right] \qquad (24)$$

Note that equation (23) and (24), in equation (23), K does not depend on α, which is the parameter of capital in energy efficiency function (1). That is to say, whatever how important capital is in determining the energy efficiency, it cannot affect the optimal path of capital stock. While in equation (24), α is a crucial multiplier in determining optimal energy consumption. That is, if capital plays an important role in energy efficiency, it will have a great influence on the optimal energy consumption given the maximum energy efficiency.

4.3 Simulation: a numerical example

In this subsection, I will assign numbers on the parameters on the basis of common economic practice. And then, I plot the optimal path in the graphs for further discussion.

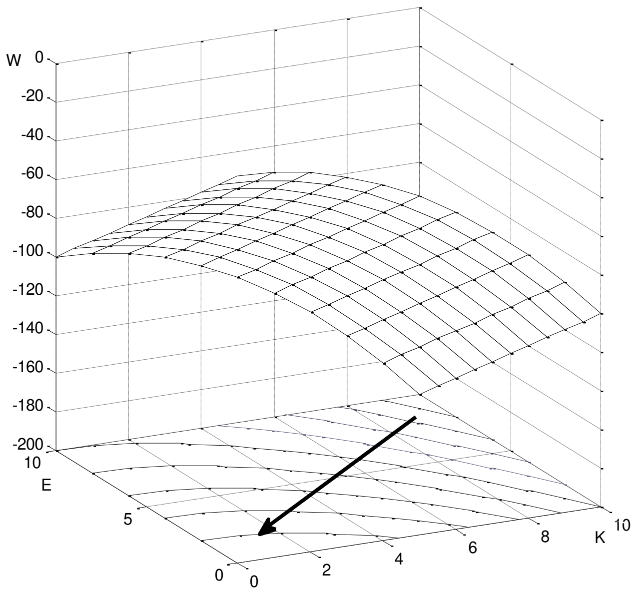
Suppose that α=10, β=0.3, γ=0.2, δ=0.1, s=0.6, $\bar{L} = 50$. This is reasonable for many developing countries.

Consequently, the economic energy efficiency function is

$$\mathrm{W} = -E^2 - 10K$$

This function can be visualized in Figure 4. We can see that the projection of W surface lies in the E—K plane and the energy efficiency increases following the direction of the arrow. Indeed, the curves in E—K plane are the ones have been illustrated in Figure 2. Every curve represents a state of efficiency. The process of increasing energy efficiency is the process of climbing the curves.

Figure 4 The simulation of economic energy efficiency function



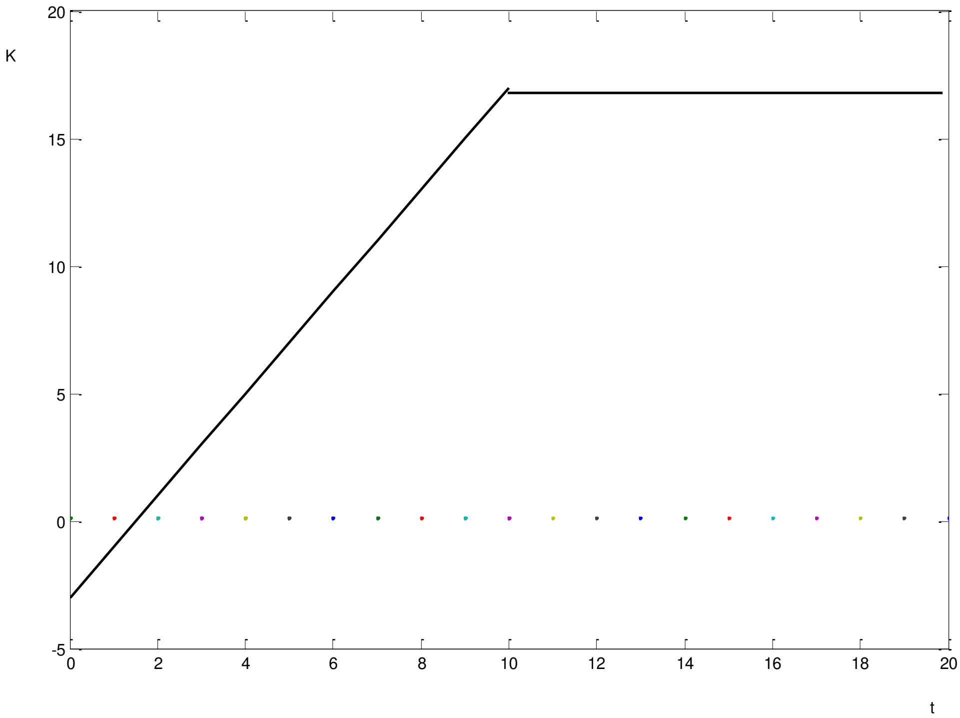
Next, I will simulate the optimal paths of K and E. Note that usually, the capital stock

K>>0. Given the numerical example, $A \approx \delta$. Hence, the function of K is

$$K = \left(\frac{2.23e^{0.1t} - 0.82}{0.1e^{0.1t} + 0.063}\right)^{-0.7}$$

The curve is drawn in Figure 5.

Figure 5 The optimal path of capital stock



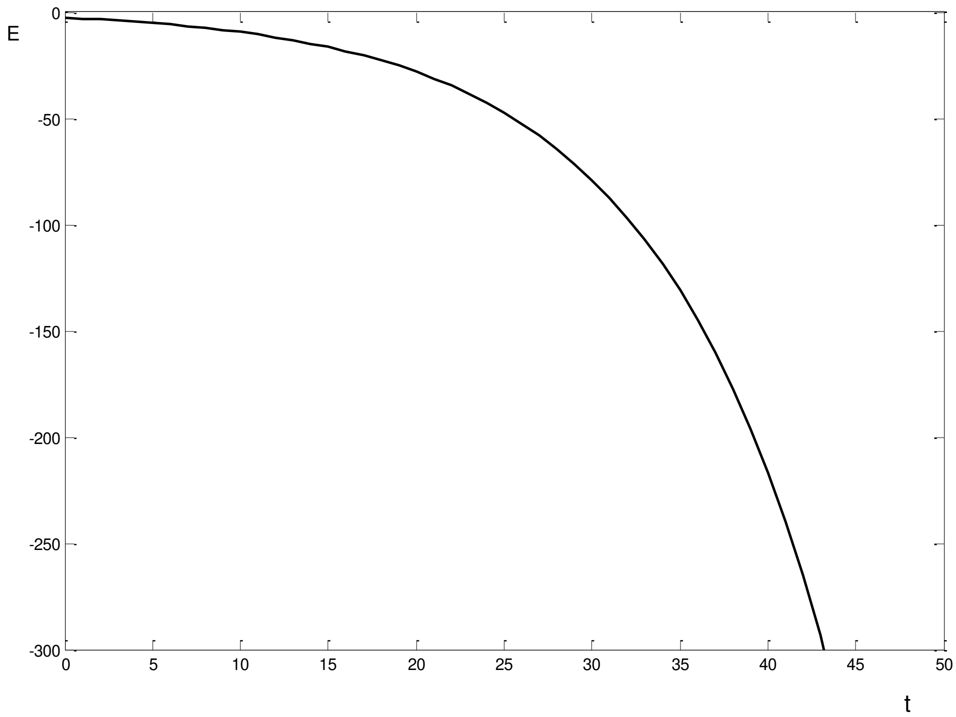
As Figure 5 illustrated, the path of capital stock increases at first, after ten periods, it will remain constant.

On the side of energy consumption, the simulated function is

$$E = -4e^{0.1t} + 1.47$$

The path can be investigated in Figure 6. As we can see, the energy consumption will always decline over time. Besides, its speed is slow at first but becomes more and more rapidly then.

Figure 6 The optimal path of energy consumption



5 Discussion

The modelling and results reveal some interesting policy implications.

Firstly, under the neoclassical growth mechanism and investment constraint, we can get the maximum of economic energy efficiency continuously. Even though the economic energy efficiency has two meanings including reducing the energy distance and climbing the best efficient curves, the optimization modelling mainly reflects the latter one and focus on long-term effect. In this paper, technology progress is assumed exogenous and it is an underlying reason for energy efficiency improvement in long-run. In other words, the effect of technology is not direct but embodied by investment in this model. For achieving the maximum efficiency, the economy should run in the optimal investment state by energy consumption being controlled. In reality, energy consumption is relative easy to control for many developing countries as investment is a fluctuant variable in many circumstances.

The second important thing is about the state of equilibrium. The saddle-point reveals that there is the only stable branch to reach the target point Q. If the economy gets onto the unstable branch unfortunately, it could never reach the optimization. Thus, the path choice is still vital for social planners. Additionally, the phrase—diagram indicates that the increase in investment is accompanying with an increase in the rate of energy consumption. This is proved by Figure 5 and 6 in the numerical simulation.

Next, we discuss the two transition paths of state variable and control variable. The result in Figure 5 is in accord with the statement of Solow model. After the steady state, the capital stock will not increase. In other words, the investment is only equal to the depreciation of capital. Hence, even though energy consumption is included in Solow model, it does not change the basic conclusion of neoclassical growth model. On the other hand, the curve in Figure 6 is also easy to understand. With the continuous improvement of energy efficiency, the amount of energy consumption is decreasing all the time. Initially, the rate of decline is small. But with the accumulation of technology and capital, the rate of decline becomes more faster. As capital will not change over ten periods, technology will be a key factor in long-run. Thus, the shift of slope can be explained by the scale effect of technology.

Lastly, the imposed restriction that the population is constant is somewhat unreasonable. It is mainly for getting an appropriate solution in modelling. In fact, population is always increasing in many developing countries. However, this restriction could also remind governments in those countries that population plan could be important for a better development.

6 Conclusion

The modelling on the dynamic optimization demonstrates some implications and inspirations. In development, energy efficiency is a key factor linking with other macroeconomic variables. On one hand, industrialization and modernization lead to the appetite for energy. The improvement of energy efficiency is essential for utilizing and conserving energy effectively. On the other hand, the improvement of energy efficiency largely depends upon the investment and technology, which are key drivers in development. Pursuing the maximum energy efficiency is not contradictory with investing and consuming energy. In contrast, they can be harmonized in growth and development. This fact could help developing countries crucial for developing countries realize a sustainable development.

Moreover, the modelling provides specific methods for approaching a better development. As the extended Solow model works well, governments can energy consumption effectively and make the optimal state of investment at any time. As a result, the levels of best energy efficiency could be attained in succession; the capital stock will increase until reaching the steady level of the golden rule which is illustrated in Solow model; while the energy consumption will decline continuously. Eventually, the sustainable development can be achieved automatically and optimally.

References

Brennan, TJ 2010, 'Optimal energy efficiency policies and regulatory demand-side management tests: How well do they match?', *Energy Policy,* Vol. 38, pp. 3874-3885.

Chiang, AC 1992, 'Chapter 7: optimal control', in *Elements of Dynamic Optimization,* Waveland Press, Illinois, pp. 200-204.

Conrad, K 2001, 'The optimal path of energy and CO_2 taxes for intertemporal resource allocation', CESifo Working paper, no.552, viewed 20 November 2012, <http://www.cesifo-group.de/portal/pls/portal/docs/1/1190566.PDF>

Dasgupta, PS & Heal, GM 1979, *Economic theory and exhaustible resources,* Oxford University Press, UK.

Forster, BA 1980, 'Optimal energy use in a polluted environment', *Journal of Environmental Economics and Management,* Vol.7, pp. 321-333.

Garg, PC & Sweeney, JL 1978, 'Optimal growth with depletable resources', *Resources and Energy,* Vol. 1, pp. 43-56.

Jaffe, AB & Stavins, RN 1994, 'The energy-efficiency gap', *Energy Policy,* vol.22, no.10, pp. 804-810.

Jin, Y & Arons, JS 2009, 'Chapter 3: the metabolic society', in *Resource, Energy, Environment, Society: Scientific and Engineering Principles for Circular Economy,* Chemical Industry Press, Beijing, pp. 92-93.

Li, H, Long, R & Lan, X 2012, 'Economic growth in resource-based cities: based on ecological constrains', *China Soft Scinece Magazine,* vol.26, no.6, pp. 53-59.

Lucas, R 1988, 'On the mechanics of economic development', *Journal of Monetary Economics,* vol.22, no.1, pp. 3-42.

National Bureau of Statistics of China, 2010, *Handbook of Energy Statistics,* China Statistics Press, Beijing.

Peng, S 2007, 'Natural resource depletion and sustainable economic growth based on a four-sector endogenous growth model', *Journal of Industrial Engineering and Engineering Management,* vol.21, no.4, pp. 119-124.

Perrot, P 1998, *A to Z of Thermodynamics,* Oxford University Press, England.

Pindyck, RS 1978, 'The optimal exploration and production of nonrenewable resources', *Journal of Political Economy,* vol.86, no.5, pp. 841-861.

Pindyck, RS 1980, 'Uncertainty and exhaustible resource markets', *Journal of Political Economy,* vol.88, no.6, pp. 1203-1225.

Romer, PM 1990, 'Endogenous technological change', *Journal of Political Economy,* Vol.98, pp. 71-102.

Sanchez, JA & Ruiz, PM 2009, 'Locally optimal source routing for energy-effiency geographic routing', *Wireless Netw,* Vol.15, pp. 513-523.

Stern, DI 2011, 'The role of energy in economic growth', *Annals of the New York Academy of Sciences*, pp. 26-51.

Stern, DI 2012, 'Modelling international trends in energy efficiency', *Energy Economics,* Vol. 34, pp. 2200-2208.

Stiglitz, J 1974, 'Growth with exhaustible natural resources: efficient and optimal growth paths', *Review of Economic Studies,* Vol.41, pp. 123-137.