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November 2011

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First draft: November 2011

This version: May 2012

Abstract

I develop a theory of financial intermediation to explore how the availability of trading opportunities affects the link between the liquidity of financial institutions and their default decisions. In it, banks hedge against liquidity shocks either in the interbank market or by using a costly bankruptcy procedure, and depositors trade in the asset market without being observed. In this environment, the competitive pressure from the asset markets makes intermediaries choose an illiquid asset portfolio. I prove three results. First, illiquid banks default in equilibrium only when there is systemic risk and an unpredicted crisis hits the economy. Second, in contrast to the previous literature, the allocation at default is not socially optimal. Third, the constrained efficient allocation can be decentralized with the introduction of countercyclical liquidity requirements.

Keywords: Financial intermediation, bankruptcy, liquidity, hidden trades, insurance, optimal regulation

JEL Classification: E44, G21, G28

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1 Introduction

The connection between the illiquidity of financial institutions and their default is far from obvious: in principle, we would expect banks to be more prone to crises the more illiquid they are. However, as shown in figure 1, the liquidity ratios of U.S. banks were way lower in 2001 than in 2007-2009, and still the U.S. economy endured a much deeper financial crisis in the second than in the first case.

The aim of the present work is to show that the relationship between the liquidity of financial institutions and their default decisions crucially depends on the availability of trading opportunities, for both banks and individual investors. The claim that markets play a key role in the financial system stems from two well-known facts: first, wholesale interbank funding has become the main channel for the circulation of liquidity in the U.S. economy; second, in the last thirty years, financial liberalization has made available a whole new series of instruments – off-shore tax havens, international stock markets, hedge funds, and so on – that allow investors to by-pass the banking system.

To formally assess the microfoundations of default and its connection with markets and illiquidity, I develop a model of financial intermediation with both idiosyncratic and aggregate uncertainty, inspired by the work of Diamond and Dybvig (1983). The economy is populated by risk-averse depositors and risk-neutral intermediaries or banks. The first are hit by privately observed idiosyncratic shocks affecting the point in time (early or late) at which they are willing to consume, which makes them either "impatient" or "patient". The second provide insurance against these shocks by investing the total deposits in short-term (liquid) and long-term (illiquid) assets. After banks have chosen this initial portfolio strategy, an aggregate state of the world is revealed: banks are hit by asymmetric liquidity shocks, all happening with an ex ante known probability. These shocks affect the total fraction of depositors who turn out to be impatient, and might create an ex post budget imbalance, when the liquidity chosen ex ante is inadequate or excessive with respect to the actual liquidity demand from customers. Thus, in order to cover for these imbalances, banks trade among themselves in an economy-wide interbank market that opens after the aggregate state of the world has been

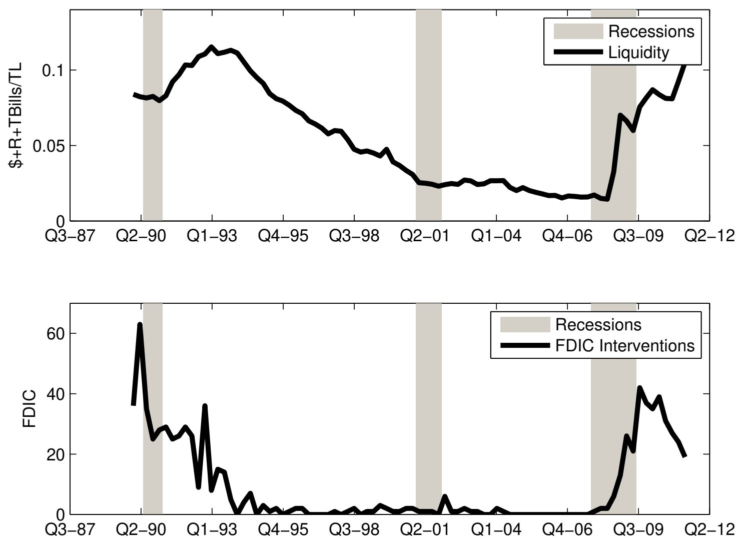

Figure 1: Liquidity ratios of U.S. chartered commercial banks versus the number of interventions by the FDIC. The liquidity ratio is defined as the sum of vault cash, reserves and Treasury securities as a percentage of total liabilities. Source: *Board of Governors of the Federal Reserve System*, Flow of Funds Accounts of the United States, and FDIC.

revealed.

I make two extensions to this basic environment. First, banks have two alternative strategies to interbank funding to transfer resources across states: when the liquidity is higher than expected, they can store it by using the short-term asset. When liquidity is instead insufficient, and the bank is unable to service its debt with the depositors, it can file for bankruptcy: in this case, the bank uses a costly liquidation technology to throw away the long-term assets in their portfolios and generate the extra cash they need. This can be seen as an expensive insolvency procedure through courts (similar to Chapter 11 in the U.S.) or a clearance scheme in which part of the capital gets lost, and is a way of i) clearly embedding bankruptcy costs in banks' budgets; ii) distinguishing between partial and full default; and iii) explicitly showing how illiquidity issues can lead to insolvency (when the bank is forced to fire sell all its assets).

The second feature that I introduce is instead an informational friction: depositors can engage in trades in the asset markets, without being observed by their banks. I model asset markets as institutions where individuals issue or buy uncontingent bonds, whose return is determined in equilibrium. The unobservability of these exchanges implies that the terms of the banking contract must satisfy an incentive compatibility constraint in equilibrium: the present value of the consumption bundle that each depositor is entitled to receive by the banks must be constant across types when evaluated at the return on the "hidden" bond.

With these hypotheses in hand, I characterize both the planner solution and the decentralized banking equilibrium. The competitive pressure from asset markets makes the banking system under-invest in liquidity as in Farhi et al. (2009): on one side, the planner provides insurance to individuals against the risk of being impatient by offering a higher present value of consumption (evaluated at the return on the long-term asset) to impatient depositors than to patient ones. Then, to lower the incentives for patient depositors to misreport their liquidity needs, the planner implicitly imposes a wedge between the return in the asset market and the return on the banks' long-term asset. On the other side, market clearing considerations imply that these two returns must be equal in the competitive equilibrium; hence, by incentive compatibility, intermediaries are forced to equalize early and late consumption (when evaluated at the return on the long-term asset). Put differently, the banking system is illiquid from an efficiency perspective since the competitive pressure from asset markets makes it hold relatively less liquid assets than the planner.

In this environment, I prove my main result: lower liquidity buffers do not lead to default when banks trade in the interbank markets, because they co-insure against liquidity shocks, but this conclusion dramatically changes when these shocks are positively correlated and interbank markets cease to function (but asset markets are still open): intermediaries must now use ex post storage or default to transfer resources across states of the world and ensure that incentive compatibility is satisfied. In this case, intermediaries choose their initial portfolio strategy such that the expected marginal benefit of having one more unit of liquidity (in terms of avoiding future default) is equal to the expected marginal cost of that one unit (in terms of the opportunity cost of future storage). As shown in a numerical example, corner solutions can also emerge. If the probability of having a high fraction of impatient depositors in the future is low enough, banks choose a completely illiquid portfolio strategy: they default to provide consumption to early depositors, and choose full bankruptcy when an unexpected financial crisis hits the economy. If instead the probability of having a high fraction of impatient depositors is high enough, banks do the opposite: they "fly to liquidity", i.e. invest all their capital in short-term assets to completely avoid default.

This result shows that default emerges as an equilibrium phenomenon only when interbank markets are shut down and, at the same time, depositors can trade in the asset markets: in fact, without private trades, intermediaries would indeed be able to offer a consumption plan contingent on their available liquidity, and avoid bankruptcy. Moreover, contrary to some key results in the literature (Allen and Gale, 2004), the allocation at default is not constrained efficient: even when liquidity shocks are positively correlated, the planner is able to tilt incentives by affecting the interest rate paid in the asset markets and provide higher welfare. By introducing private trades, I therefore provide a rationale for government intervention to mitigate the negative effects of a financial crisis when markets are not well-functioning.

In my second result, I move into normative analysis and characterize the optimal government intervention to solve the inefficiency of the competitive equilibrium. Despite the fact that the main distortion on the system stems from *asset markets* (the return on bonds is higher than its socially optimal level), in presence of fully functioning markets the planner's solution can be decentralized with an intervention on *banks*. Such a rule takes the form of a liquidity floor, i.e. a weighted average of future liquidity needs, weighted with both economy-wide and bank-specific factors. Thus, this result yields a theoretical background for the "Liquidity Coverage Ratio", introduced as part of the new architecture for macroprudential regulation in the Basel III Accord. On the contrary, when interbank markets are not functioning the Liquidity Coverage Ratio is not enough to provide welfare improvements to the decentralized outcome: that is because banks tend to hoard liquidity when the probability of a future crisis is relatively high, hence the constraint is binding only in those states in which there is a low probability of crisis (i.e. a high probability of storing liquidity). This result calls for further tailoring financial regulations to periods of aggregate uncertainty, through the introduction of countercyclical liquidity requirements: a liquidity floor when the probability of a future crisis is low, and a liquidity ceiling when the probability of a future crisis is high.

The rest of the paper is organized as follows. In section 2, I summarize part of the literature related to my work. In section 3, I define the environment of the model. I characterize the decentralized equilibrium with negatively correlated liquidity shocks in section 4. The competitive equilibrium with positively correlated liquidity shocks and the correspondent socially optimal allocation are analyzed in section 5. Finally, section 6 concludes with some open issues for future research.

2 Related Literature

The present paper finds inspiration in a recent and growing microeconomic literature on financial crises. Although a consensus exists that one of the main reasons for the current period of financial distress lies in excessive risk-taking by financial intermediaries, many different explanations have been proposed for why this behavior emerges. Farhi and Tirole (2011) focus their attention on strategic complementarities among banks that all expect to be bailed out ex post. In that sense, a crisis occurs because of an external ex post (and inefficient) government intervention. Diamond and Rajan (2010) provide a formal microfoundation of banks' behavior by assuming risk neutrality: financial institutions know that, with some probability, there will be a crisis, and they can insure against that by building up a buffer of liquidity ex ante. On the other hand, they also know that when a crisis hits, with some probability they will go bankrupt, but with some other probability they will survive and make profits, because asset values revamp precisely in those states. Risk neutrality then implies that banks will not create buffers, with disastrous consequences for the whole system. In that sense, risk neutrality is clearly key for their results, but if we think that financial markets ultimately exist because investors are willing to hedge risk (i.e., because they are risk averse), then we might ask why intermediaries do not insure against shocks at all.

This last question has been the center of analysis of a long-lasting line of research on financial intermediaries and markets that finds its cornerstone in the work of Diamond and Dybvig (1983). The authors develop an environment in which banks provide insurance to their depositors against unexpected liquidity shocks via demand deposits. Following their lead, Bhattacharya and Gale (1987) were the first to highlight how banks hit by shocks might avoid an unnecessary liquidation of long-term investments by exchanging resources in the interbank market. In particular, the authors account for the case where financial institutions have private information about the liquidity of their portfolios, and show how this leads them to over-invest in illiquid assets. The role of financial imperfections affecting the allocation of resources in the banking system has also been the center of more recent contributions. Freixas and Holthausen (2005) develop an environment with noisy cross-country information among intermediaries to show how "peer monitoring" helps improve the decentralized equilibrium outcome, and how the quality of information critically matters for the existence of an integrated interbank market. Freixas and Jorge (2008) address the role of asymmetric information in explaining the transmission of monetary policy in the economy. Heider et al. (2010) focus on counterparty risk and its effect on the pricing of liquidity.[1]

In a Diamond-Dybvig environment with a neoclassical definition of financial markets as trades in state-contingent claims, Allen and Gale (2004) prove some interesting results, in particular regarding the efficiency of the intermediated equilibrium. Their key conclusion is that when banks are *exogenously* constrained to offer incomplete (uncontingent) contracts to their customers (like the standard deposit contracts that we observe in reality), they might choose in equilibrium to use a bankruptcy procedure, because in this way they improve the contingency of the consumption allocation. In addition, such an equilibrium is constrained efficient: no government intervention can improve the market outcome. The present paper builds on this analysis, but delivers a completely opposite result: when banks are *endogenously* constrained to offer incomplete contracts, they might choose to default, but the resulting allocation is *not* constrained efficient.

In trying to endogenize the emergence of illiquidity in a banking equilibrium, my main reference has been a "folk theorem" in the theory of financial intermediation: the possibility that depositors might invest directly in the asset markets undermines banks' ability to implement the first best contract via demand deposits. This point, already made in some seminal papers (Jacklin, 1987; Diamond, 1997; von Thadden, 1999), has recently been restated by Farhi et al. (2009). The authors develop a version of the Diamond-Dybvig model without aggregate uncertainty, in which agents can engage in unobservable trades. This complex game of asymmetric information is then solved with mechanism design tools to show that private

[1]For an extensive review of the literature on financial intermediation, see Allen and Gale (2007) and Allen et al. (2011).

trades restrain banks from offering the efficient incentive compatible contract. To sum up, my work can then be seen as an extension of Allen and Gale (2004) to include private trades in the asset markets, or as a version of Farhi et al. (2009) with sectoral uncertainty, interbank markets and default.

3 Environment

The basic structure of the model is a Diamond-Dybvig model of financial intermediation with idiosyncratic and aggregate shocks. The economy lasts for three periods, labeled $t = 0, 1, 2$, and is divided into n groups or sectors of an equal unitary dimension populated by a continuum of individuals.[2] These are all ex ante identical, and at date 0 receive as endowment an homogeneous consumption good $e = 1$. In every group, there is also a continuum of Bertrand-competitive risk-neutral financial institutions or banks which operate in a market with free entry and offer real contracts to individuals. The relationship between customers and banks is *exclusive*, in the sense that agents can only deposit their endowments into a bank in their own group.

Intermediaries in the economy have access to two technologies to transfer resources across time: the first is a "short asset", which is essentially a way of storing the consumption good for one period. The second is a "long asset" delivering $\hat{R} > 1$ units of consumption in period $t = 2$ for each unit invested in $t = 0$, and can be seen as the marginal rate of transformation of firms producing the consumption good.[3] This long asset is partially illiquid, as there exists a liquidation technology through which banks can throw away part or all of its holdings before its natural maturity. That comes at a cost, as for each unit of the long asset only a fraction $r < 1$ can be recovered.[4]

[2]In this environment, groups can also be seen as regions of the same country, or countries in the world economy.

[3]In order to keep the focus on liquidity provision, here I do not model the supply side of the economy. The fact that the return is constant across sectors might be seen as an implicit consequence of integrated product markets. I analyze the case in which technologies in different sectors yield different returns in Panetti (2011).

[4] For simplicity, I adopt a linear technology. All results of the following sections would hold with an increasing and concave default function. See note 25 for details.

3.1 Uncertainty

The economy is affected by two types of uncertainty. An aggregate shock is defined over a finite set of states of the world, labeled by $s = 1, \dots, S$. Each state is realized with probability $\nu(s) > 0$ and $\sum_s \nu(s) = 1$. Aggregate uncertainty is resolved at the beginning of date 1, and affects the sectoral distribution of a preference shock. This shock is an idiosyncrasy affecting all individuals. Being ex ante equal, in $t = 1$ every consumer draws a type $\theta \in \{0, 1\}$ which is private information to herself.[5] Types affect the point in time at which individuals enjoy consumption according to the utility function $U(c_1, c_2, \theta) = (1-\theta)u(c_1) + \beta\theta u(c_2)$. Clearly, if $\theta = 0$ the agent is willing to consume only at date 1, while if $\theta = 1$ she will consume only at date 2. As is customary in this line of research, I then refer to type-0 and type-1 individuals as early (or impatient) and late (or patient) consumers, respectively. The felicity function $u(c)$ is increasing, twice continuously differentiable, strictly concave, and satisfies Inada conditions. Moreover, I restrict myself to the class of functions with relative risk aversion larger than or equal to unity. The discount factor β is such that $\beta\hat{R} > 1$.

The probability of being of type θ in group i and state s is labeled $\pi^i(\theta, s)$. Preference shocks are independent across agents so, by the law of large numbers, the cross-sectional distribution of types is equivalent to the probability distribution. Hence, $\pi^i(\theta, s)$ is equal to the fraction of agents that turn out to be of type θ in state s, and $\sum_\theta \pi^i(\theta, s) = 1$ in every group. Importantly, $\sum_i \pi^i(\theta, s) = \Pi(\theta, s) = \Pi(\theta, s') \equiv \Pi(\theta)$ for any s, s': the total fraction of agents in liquidity need *in the whole economy* is constant. Therefore, each state of the world is different from the others only with respect to the distribution of consumers' types across sectors, i.e. there is no systemic risk.

3.2 The Banking Contract

At the beginning of date 0, agents deposit their endowment into banks in their own group, and sign a banking contract. This indicates the amount of consumption goods $\{w_t^i(\theta, s)\}$ that each depositor is entitled to withdraw at dates 1 and 2, depending on the reported type and

[5] The model can easily be extended to include continuous types, but that would change the main results proposed in the next sections.

the realization of the aggregate state. In order to finance the contract and allocate resources across time, banks buy short and long assets in amounts X^i and Y^i, respectively.

Banks have three instruments to transfer resources across states of the world at date 1, after the aggregate state of the economy has been revealed. First, they can trade in an intersectoral interbank market. This is modeled as a market for a bond $Z^i(s)$ yielding a return $\tilde{R}(s)$ to be determined in equilibrium in each state.[6]

As an alternative to market trades, banks can use two other channels. If they have too much liquidity, because total demand from their depositors is unexpectedly low, they can move it forward to date 2 by using the short asset for an amount $M^i(s) \geq 0$. If instead their liquidity turns out to be inadequate, they can file for bankruptcy. In this case, the bank can employ the liquidation technology to get rid of an amount $D^i(s) \geq 0$ of the long assets (by giving up on $\hat{R}$ units of consumption at date 2 to get an amount r at date 1). Notice that since the probability distribution of the aggregate state is known at date 0, the intermediaries choose these strategies ex ante, with full commitment. If the liquid resources cumulated at date 0 are enough to cover for the consumption needs of the depositors who report to be impatient, then the banks are "liquid". If instead the available cash is inadequate, but banks are able to borrow in the market, they are "illiquid but solvent". Finally, if they choose the default procedure, they are in financial distress, i.e. "insolvent".

I summarize the set of policy decisions and consumption allocations in a compact vectorial definition:

Definition 1. *A banking contract is a vector* $C^i(\theta, s) = \{w_t^i(\theta, s), X^i, Y^i, Z^i(s), D^i(s), M^i(s)\}$ *for any type* $\theta \in \{0, 1\}$ *and state of the world* $s = 1, \ldots, S$.

3.3 Hidden Trades

At date 1, after the state of the world has been revealed to everyone, agents can withdraw the amount of consumption good stated in the contract from their banks and eventually engage in private trades in an asset market. I model this feature of the economy as unobservable

[6] The uncontingency of the securities traded is a direct consequence of the fact that such a market opens after the aggregate state of the world has been revealed, but this is not a restrictive hypothesis. All results here would go through even in an environment where intermediaries trade state-contingent claims at date 0.

exchanges across groups, through which individuals can freely borrow and lend via an uncontingent bond, yielding a "hidden" return $R(s)$ to be determined in equilibrium. Notice three things. First, the fact that agents trade only uncontingent bonds is not an a priori restriction on the completeness of the market, but an endogenous feature of the environment (see appendix A). Second, I follow Farhi et al. (2009) and assume that asset markets only open ex post and work as a secondary borrowing/lending channel for individuals. Third, the results proposed here hinge neither on the fact that banks cannot access this market themselves nor on the date that the market opens, but only on the fact that the depositors can borrow and lend without being observed.

More formally, the investor's problem in the asset market reads:

$$\begin{aligned} V(C^i(\theta,s),R(s),\theta,s) = &\max_{c_1^i,c_2^i,b^i,\theta'} U(c_1^i,c_2^i,\theta), \\ s.t. \qquad & c_1^i + b^i = w_1^i(\theta',s), \\ & c_2^i - R(s)b^i = w_2^i(\theta',s). \end{aligned} \tag{1}$$

Given the terms of the banking contract $C^i(\theta,s)$, the return on the hidden investment $R(s)$, and the realizations of the idiosyncratic and aggregate states, each agent decides which type θ' to report, how much to consume in the two periods (c_1^i and c_2^i) and how much to borrow or lend (b^i) in order to maximize her welfare, subject to her budget constraint.[7] At date 1, after reporting type θ', the depositor receives consumption $w_1^i(\theta',s)$ from her bank. She can then borrow or lend an amount b^i and consume the remaining part c_1^i. At date 2, the depositor then gets $w_2^i(\theta',s)$, pays back the bond (or earns the proceedings on the amount lent at date 1) and consumes what is left.

The environment so far describes a complex game of asymmetric information between the banks and their customers. Nevertheless, by the Revelation Principle, I can focus on direct mechanisms in which depositors truthfully report their types.The incentive compatibility constraint can then be defined in the following way:

[7]Notice that here I choose to simplify the notation: the control variables are all explicit functions of $(C^i(\theta,s),R(s),\theta,s)$.

Definition 2. *A banking contract $C^i(\theta, s)$ is incentive compatible if:*

$$V(C^i(\theta, s), R(s), \theta, s) \geq V(C^i(\theta', s), R(s), \theta, s),$$

for any $\theta, \theta' \in \{0, 1\}$ and any realization of the aggregate state $s = 1, \ldots, S$.

Incentive compatibility states that each agent should find it optimal to truthfully report her type, but given the presence of only two types, this can be simplified:

Lemma 1. *A banking contract $C^i(\theta, s)$ is incentive compatible if:*

$$w_1^i(0, s) + \frac{w_2^i(0, s)}{R(s)} = w_1^i(1, s) + \frac{w_2^i(1, s)}{R(s)}, \tag{2}$$

for any realization of the aggregate state $s = 1, \ldots, S$.

Proof. In Appendix C. ■

Truth-telling implies that the banking contract should give the same present value of consumption to each type, evaluated at the return on the hidden investment. In this way, agents have no incentive to retrade in the asset market. An obvious consequence of the lemma is that, in this environment, individuals only care about the present value of their consumption. This feature will be crucial in what follows.

3.4 Timing

In the rest of the paper, I will focus on pure strategy symmetric equilibria, where intermediaries in the same group make the same investment choices. Therefore, without loss of generality, I can restrict myself to the analysis of a representative bank for each sector.

The timing of actions and events is the following: at date 0, agents deposit their endowments; hence, the size of each representative intermediary is 1. Banks then set up fully state-contingent incentive-compatible contracts $C^i(\theta, s)$. At date 1, the aggregate state is revealed to everyone, and agents get to know their private types. Banks then trade among themselves across sectors, store or declare (partial) bankruptcy, and pay consumption to those depositors who report being impatient. After that, asset markets open and agents can engage in unobservable trades across sectors. Finally, at date 2, agents are paid the amount stated in the banking contract and, eventually, the return on their hidden investment.

Table 1: Timeline of actions and events

t = 0	t = 1	t=2
Agents make deposits	Short asset matures	Long asset matures
Banks set up banking contract	Uncertainty is realized	Interbank market clears
Banks buy short and long assets	Interbank market opens	Late withdrawals
	Storage/default	Asset market clears
	Agents report their types	Late consumption
	Early withdrawals	
	Asset market opens	
	Early consumption	

3.5 Planner's Problem

As a benchmark for the decentralized environment of the next sections, I start my analysis by characterizing the constrained efficient allocation. The availability of private trading opportunities for individuals imposes a further constraint on the social planner. That is, the efficient contract must provide utility in such an amount that agents have incentives to truthfully report their own private types, and not operate at all in the asset markets. It is important to highlight that without unobservable trades (but with private individual types) the planner would be able to ensure perfect risk sharing both within and between groups, as shown in Appendix B. In that environment, it can also be shown that a version of the First Welfare Theorem holds; hence, the decentralized competitive equilibrium is efficient and equivalent to the first best (Allen and Gale, 2004).

The planning problem reads:

$$\max_{\substack{\{w_1^i(\theta,s),w_2^i(\theta,s)\}_{s=1,\dots,S}\\ \theta\in\{0,1\}\\ i=1,\dots,n}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta,s) V(C^i(\theta,s), R(s), \theta, s), \tag{3}$$

subject to the incentive compatibility constraint (2) and the intertemporal resource constraint:

$$\sum_i \sum_\theta \pi^i(\theta,s) \left[w_1^i(\theta,s) + \frac{w_2^i(\theta,s)}{\hat{R}} \right] \leq n, \qquad \forall s = 1,\dots,S.$$

The planner chooses a consumption profile to maximize the expected total welfare of the economy. In order to do that, she employs all resources available (equal to n) to finance a consumption bundle whose present value is evaluated at the marginal rate of transformation. Notice that neither bankruptcy nor storage emerges in equilibrium, because the planner knows that the total fraction of agents in early liquidity need is constant and equal to $\Pi(0)$ in any state. The following proposition characterizes the efficient allocation with private trades:

Lemma 2. *In any state $s = 1, \ldots, S$ and group $i = 1, \ldots, n$, the constrained efficient allocation reads:*

$$\begin{aligned} &w_1^i(1, s) = w_2^i(0, s) = 0, \\ &w_1^i(0, s) = \mathcal{I}^P, \\ &w_2^i(1, s) = R^P \mathcal{I}^P, \end{aligned}$$

where $\{R^P, \mathcal{I}^P\}$ is the solution to:

$$\begin{aligned} &\beta \hat{R} u'(R^P \mathcal{I}^P) = u'(\mathcal{I}^P), \\ &\mathcal{I}^P = \frac{n}{\Pi(0) + \frac{R^P}{\hat{R}} \Pi(1)}. \end{aligned}$$

and $b^i = 0$.

Proof. In the appendix C. ■

The planner optimally chooses to provide no consumption to late consumers in the first period and to early consumers in the second period, so that they do not have any resource to borrow or lend in the secondary market. The remaining part of the contract is set according to an Euler equation, hence there is perfect risk sharing within each group. At the same time, the planner also ensures perfect cross-sectoral risk sharing: agents of the same types are entitled to the same amount of consumption, regardless of the sector to which they belong. Hence, the equilibrium characterized here is equivalent to the one that emerges in the constrained problem with private types only which, in turn, is equivalent to the unconstrained optimum. This is the multi-sectoral version of the main proposition in Farhi et al. (2009), and states that the planner can tilt incentives and (implicitly) prices so as to implement the first best.

More importantly for the results of the next sections, the planner chooses the efficient allocation by taking into account the spread between the hidden and the official return on assets $R^P/\hat{R}$. By rearranging the Euler equation, such a spread turns out to be strictly less than unity, as:

$$1 < R^P \leq \beta\hat{R} < \hat{R},$$

in every state of the world.[8] The intuition for this result is straightforward. As previously mentioned, the planner knows that without hidden savings the first best is achievable. Therefore, she finds it optimal to close down the private market by imposing a wedge between the return on bank assets and the return on the private technology. The planner is then able to efficiently allocate resources and provide optimal insurance. Because early consumers are valued more than late consumers ($\beta\hat{R} > 1$), the planner compresses the ex post income profile by transferring resources from patient to impatient agents. Although that would not be incentive compatible (the consumption bundle of the impatient depositors $c_1(0, s) > 1$ is more valuable than that of the patient ones $c_2(1, s)/\hat{R} < 1$ in present value), the imposition of a wedge between the two returns ensures that patient depositors do not mis-report their types and retrade.

4 Banking Equilibrium with Interbank Markets

4.1 Competitive Equilibrium

In this section, I define and characterize the equilibrium of a decentralized environment that I call "Banking Equilibrium". Here, banks only care about the expected welfare of their own customers,[9] and allocate resources across time and states of the world by buying assets (short and long technologies), trading claims in the interbank market, and by defaulting/storing ex

[8]Notice that R^P must also be uncontingent.

[9]Recall that banks are Bertrand-competitive, hence in equilibrium they have zero profits.

post. More formally, the representative bank of each sector solves the dual problem:

$$\max_{\substack{\{C^i(\theta,s)\}_{\theta\in\{0,1\}} \\ s=1,\dots,S}} \sum_s \nu(s) \sum_\theta \pi^i(\theta,s) V(C^i(\theta,s), R(s), \theta, s), \tag{4}$$

subject to the incentive compatibility constraint (2), the date-0 budget constraint:

$$X^i + Y^i \leq 1, \tag{5}$$

and the budget constraints at date 1 and 2, which must hold for any state:

$$X^i + rD^i(s) \geq \sum_\theta \pi^i(\theta,s) w_1^i(\theta,s) + Z^i(s) + M^i(s), \tag{6}$$

$$\hat{R}(Y^i - D^i(s)) + \tilde{R}(s)Z^i(s) + M^i(s) \geq \sum_\theta \pi^i(\theta,s) w_2^i(\theta,s), \tag{7}$$

$$0 \leq D^i(s) \leq Y^i, \tag{8}$$

$$0 \leq M^i(s) \leq X^i + rD^i(s) - Z^i(s). \tag{9}$$

A bank maximizes the total expected welfare of its depositors by choosing the best possible banking contract. From the definition of the hidden problem in (1), it is easily seen that total welfare $\sum_\theta \pi^i(\theta,s)V(C^i(\theta,s), R(s), \theta, s)$ is equal to:

$$\pi^i(0,s) u \left(w_1^i(0,s) + \frac{w_2^i(0,s)}{R(s)} \right) + \beta \pi^i(1,s) u(R(s) w_1^i(1,s) + w_2^i(1,s)). \tag{10}$$

In each state, a fraction $\pi^i(0,s)$ of depositors will be early consumers, and enjoy utility from what they receive at date 1 ($w_1^i(0,s)$) and from whatever amount they can borrow in the asset market against the consumption good $w_2^i(0,s)$ that they will receive at date 2. Similarly, there will be $\pi^i(1,s)$ late consumers who lend $w_1^i(1,s)$ at the rate $R(s)$ and therefore consume $R(s)w_1^i(1,s) + w_2^i(1,s)$.

The budget constraints also deserve an accurate explanation. At date 0, banks allocate the total deposits among short and long assets. At date 1, they then receive the return on the storage technology X^i and, if they file for bankruptcy, the return on the liquidation technology

$rD^i(s)$. They use these resources to pay early consumption, borrow or lend an amount $Z^i(s)$ in the interbank market and possibly store an amount $M^i(s)$ for the next period. Finally, at date 2, banks receive the net return on the long assets still in the portfolio $(\hat{R}(Y^i - D^i(s)))$, clear their trades in the interbank market $(\tilde{R}(s)Z^i(s))$, and use the storage from the previous period to finance late consumption.

The last two constraints need some more thoughts. The expression in (8) states that intermediaries cannot liquidate a negative amount of assets, nor throw away more than the long assets they hold. Similarly, they cannot store a negative amount from date 1 to date 2, nor store more than the maximum available resources: total liquidity plus the amount defaulted minus what they lent to other banks in the wholesale market.

I use (5)-(7), and the fact that bonds traded in the interbank market must be in zero net supply in equilibrium $\left(\sum_i Z^i(s) = 0\right)$, to derive the following key definition:

Definition 3. *In any state of the world, an allocation is feasible if it satisfies the resource constraint:*

$$\sum_i \sum_\theta \pi^i(\theta, s) \left[w_1^i(\theta, s) + \frac{w_2^i(\theta, s)}{\tilde{R}(s)} \right] =$$
$$\sum_i \left[X^i + \frac{\hat{R}}{\tilde{R}(s)} Y^i - \left(1 - \frac{1}{\tilde{R}(s)}\right) M^i(s) - \left(\frac{\hat{R}}{\tilde{R}(s)} - r\right) D^i(s) \right]. \tag{11}$$

The sum of all consumption expenditures (in present value), evaluated at market prices, must be equal to the market value of asset portfolios minus the deadweight losses from storage and bankruptcy, also evaluated at market prices.

The definition of the banking equilibrium is then straightforward:

Definition 4. *Given an endowment $e = 1$ for each agent and a probability distribution $\{\nu(s)\}$ for the aggregate states, a banking equilibrium with interbank markets is a return on the hidden bonds $R^B(s)$, a return on the interbank bonds $\tilde{R}^B(s)$, a set of feasible banking contracts $\{C^i(\theta, s)\}$, and bonds $b^i(C^i(\theta, s), R(s), \theta, s)$ traded in the asset market by individuals, for any state $s = 1, \ldots, S$, group $i = 1, \ldots, n$ and type $\theta = \{0, 1\}$, such that:*

- *Given prices the allocation solves the banking problem;*
- *Given prices the allocation solves the asset market problem for each agent;*

- *Markets clear:*

$$\sum_i \sum_\theta \pi^i(\theta, s) b^i(C^i(\theta, s), R(s), \theta, s) = 0,$$
$$\sum_i Z^i(s) = 0.$$

The characterization of the equilibrium starts from the price system. In particular, it must be the case that $R^B(s) = \hat{R}$ in any state. The intuition for such a result comes from a market-clearing consideration. Assume that $R^B < \hat{R}$. Then, the investment in long assets would be more profitable ex ante. Every bank would only invest in long assets, and give consumption to early consumers at time 2. These would accept the offer, because they only care about the present discounted value of their consumption bundle. But then there would only be borrowers and no lenders in the asset market; hence, that cannot be an equilibrium because it would violate zero net supply. Similar lines of reasoning lead us to exclude the possibility that $R^B > \hat{R}$, hence it must be the case that the two are actually equal.

With this result in hand, I can complete the characterization of the solution. The simultaneous presence of interbank markets and asset markets in turn has a dramatic effect on the equilibrium:

Proposition 1. *In the banking equilibrium with interbank markets, intermediaries promise their customers the following consumption bundle:*

$$w_1^i(1, s) = w_2^i(0, s) = 0,$$
$$w_1^i(0, s) = 1,$$
$$w_2^i(1, s) = \hat{R},$$

in any state $s = 1, \ldots, S$ *and group* $i = 1, \ldots, n$. *The return on the bonds exchanged in the interbank market is* $\tilde{R}(s) = \hat{R}$ *in any state. The ex ante investment strategy is:*

$$X^i = \sum_k \nu(k) \pi^i(0, k),$$
$$Y^i = \sum_k \nu(k) \pi^i(1, k).$$

The amount of bonds traded in the interbank market is:

$$Z^i(s) = \sum_k \nu(k)\pi^i(0,k) - \pi^i(0,s).$$

The equilibrium ex post strategies are $D^i(s) = M^i(s) = 0$*, and the investment in the asset market is* $b^i = 0$*.*

Proof. In Appendix C. ■

The intuition for the result is the following. Given all the possible investment strategies available to banks that I plot in figure 2, in equilibrium it must be the case that they all yield the same return, hence $R^B(s) = \tilde{R}^B(s) = \hat{R}$. This implies that the intermediaries are indifferent between investing in long and short assets. The equilibrium is then derived in the following intuitive way. As a consequence of unobservability, banks should offer a consumption bundle whose present value is constant across types when evaluated at *market returns*, so that individuals have no incentive to retrade. But the fact that the equilibrium return on

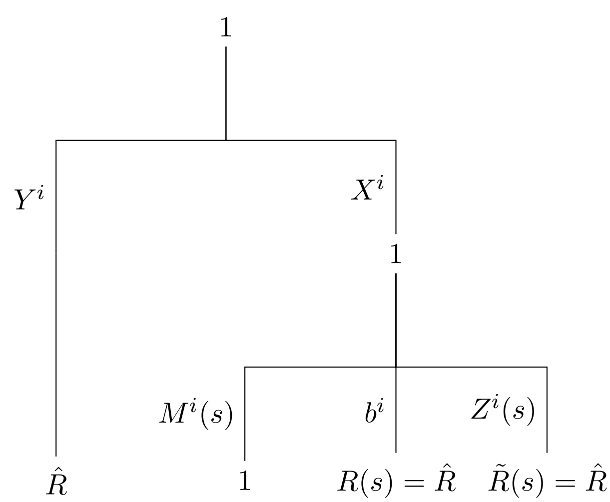

Figure 2: How to finance $w_2^i(\theta, s)$ with 1 unit of consumption at date $t = 0$ in the presence of interbank markets.

hidden assets is equal to the return on the long technology implies that intermediaries offer a contract whose present value is also constant across types when evaluated at *banking returns*. Moreover, trade in the interbank market ensures that banks provide perfect cross-sectoral risk sharing, as agents of the same types are entitled to consume exactly the same amount of goods, irrespective of the group they are born into. Therefore, the fact that in equilibrium there is neither ex post storage nor default implies that banks offer a contract whose present value of consumption is uncontingent, constant across types and exactly equal to the initial endowment, i.e. a “deposit account”.[10] Put differently, at equilibrium prices, the objective of providing incentives to truth-telling collides with the insurance motivation, and banks are not able to offer the efficient amount of within-country risk sharing.[11]

The ex ante investment in liquid and illiquid assets of intermediaries is such that they are equal to the average fractions of impatient and patient agents among their depositors, respectively. This means that banks might turn out to be illiquid, if the realized sectoral shock (the total number of depositors in early liquidity need) is higher than expected. Nevertheless, banks will not cover for any ex post excess or lack of liquidity by using the default or storage technologies, but by borrowing and lending in the interbank market, because that is always a cheaper option. The hypotheses that the total number of individuals of each type in the whole economy is constant (remember $\sum_i \pi^i(\theta, s) = \Pi(\theta, s) = \Pi(\theta, s')$ for any state s, s') and that the interbank market is completely frictionless are in that sense crucial: there will always be enough demand or supply of bonds in the interbank market. To sum up, when interbank markets are available, banks might turn out to be illiquid, but will never be insolvent.

In order to conclude this section, I now compare the banking equilibrium with the benchmark allocation chosen by the social planner, derived in section 3. For this purpose, I introduce an index of liquidity $\mathcal{L}$ as the ratio between total short-term assets and total long-term assets. As far as the planner is concerned, such a measure reads:

$$\mathcal{L}^P \equiv \frac{X^P}{Y^P} = \frac{\Pi(0)\mathcal{I}^P}{\Pi(1)\frac{R^P\mathcal{I}^P}{\hat{R}}} = \frac{\Pi(0)}{\Pi(1)}\frac{\hat{R}}{R^P},$$

[10]In the real world, a deposit contract yields a fixed return in the future, and the possibility for depositors to withdraw from (or even close) the account upon demand and receive exactly the amount originally deposited.

[11]Total welfare turns out to be equivalent to the one agents would get if no banks were in place. This “reversion to autarky” is a well-known result in the literature on hidden savings (see Ales and Maziero, 2010), and can be further seen as a way of rationalizing the co-existence of direct access and intermediated access to asset markets.

while in the banking equilibrium it is:

$$\mathcal{L}^B \equiv \frac{\sum_i X^i}{\sum_i Y^i} = \frac{\sum_i \sum_s \nu(s)\pi^i(0,s)}{\sum_i \sum_s \nu(s)\pi^i(1,s)} = \frac{\Pi(0)}{\Pi(1)}.$$

Clearly, the latter is less than $\mathcal{L}^P$ because I proved that $R^P < \hat{R}$ in every state of the world. Put differently, the banking system is always more illiquid than it should be from an efficiency perspective. The intuition for this result lies in the fact that in the decentralized equilibrium, the returns on hidden assets are too high; hence, in trying to keep up with the asset markets, banks are forced to invest relatively more in long-term securities and relatively less in safe liquid ones.

The main lesson of this section is that inefficient investment strategies are not enough to explain why banks might be in distress. Negatively correlated shocks and the availability of interbank markets in which banks hedge against them are in that sense crucial. When allowed to trade among themselves across sectors, banks might be illiquid, but never bankrupt: there will always be enough demand and supply of bonds in the market, so banks are always able to smooth consumption across states of the world.

4.2 Optimal Regulation

How can we affect the illiquidity of financial intermediaries' portfolios and decentralize the efficient allocation? The present set up is extremely suitable for providing an answer to this question, because the inefficiency of the equilibrium is endogenous and can be clearly identified by comparing the banking equilibrium to the solution to the planner's problem. Hence, we can think of some regulatory intervention to affect it at its very source.

As mentioned above, the banking equilibrium is inefficient because the returns in the asset markets are too high. The obvious consequence of such an observation would then be to directly regulate markets, for example through the imposition of taxes.[12] Unfortunately, this is impossible in theory because trades are observable to neither intermediaries nor regulators. Moreover, that might also be impossible in reality: financial transactions (for example in the

[12] Some of the proposals about the so-called "Tobin Tax" go in that direction: exogenously lowering the returns on the asset markets to achieve a better economic outcome.

stock markets) are difficult to track, and even if governments regulate some securities, capital might fly away to the "shadow banking system", or financial innovation would ensure that new unregulated instruments would be issued exactly to avoid such limitations. Therefore, what I propose here is an indirect approach: regulate markets by regulating banks.

Specifically, the regulatory intervention such that banks autonomously implement the constrained efficient allocation is a sector-specific liquidity floor imposed on their initial portfolios:

$$X^i \geq F^i. \tag{12}$$

The justification of such a rule is the following. In the new regulated equilibrium, the returns in the asset market will be lower than those in the unregulated equilibrium. This means that the short asset would be dominated by the long asset, and no intermediary would hold liquidity at all. This cannot be an equilibrium since clearing in the unobservable market would be violated: impatient consumers would like to borrow, but no one would lend to them. Thus, the only way the banking system can support an equilibrium in which the hidden return is lower than the return on the long-term asset is via the introduction of a minimum liquidity requirement, so that banks are forced to hold enough resources to finance early consumption. By picking the right floor, the regulator can then manipulate asset prices *indirectly* and intermediaries' portfolios *directly*, and let them implement the efficient allocation.

Assume that interbank markets are open and well-functioning. The banking problem in (4)-(9) is modified with the additional constraint (12). The following proposition characterizes the optimal regulatory intervention:

Proposition 2. *The minimum liquidity requirement F^i that implements the planner's solution is:*

$$F^i = \sum_{k=1}^{S} \nu(k)\pi^i(0,k)\mathcal{I}^P, \tag{13}$$

where $\mathcal{I}^P$ comes from the solution to the planner's problem.

Proof. In Appendix C. ■

This is the multi-sectoral version of the optimal regulatory intervention proposed by Farhi et al. (2009). The liquidity floor is a weighted average of all sector-specific expenses that banks face at date 1 if impatient depositors are entitled to receive the efficient amount of consumption ($\pi^i(0,s)\mathcal{I}^P$), weighted by a factor $\nu(s)$, i.e. the *economy-wide* probability of each state to be realized.

This result is interesting because it provides a theoretical rationale for the so-called "Liquidity Coverage Ratio", which is a key part of the liquidity regulation proposed by the Basel Committee on Banking Supervision (2010).[13] Moreover, the theoretical liquidity floor features two of the main characteristics of the rules in the "Basel III Accord", as (i) it dampens the cyclicality of budget requirements, by creating an ex-ante uncontingent rule, and (ii) promotes forward-looking provisions, by weighing all possible future states of the economy with common and sector-specific factors. In addition to those, a global regulatory standard is also supposed to affect systemic risk, and more generally tame moral hazard in the financial system. The conclusion here is that we have a further reason to introduce requirements on banks: to affect asset markets. This is an interesting yet novel way of rationalizing financial regulation.

5 Banking Equilibrium without Interbank Markets

To study the role of markets and liquidity shocks for the emergence of default as an equilibrium phenomenon, in this section I relax the hypothesis of no systemic risk, and analyze the opposite case, in which the liquidity shocks affecting the banks are instead positively correlated. This means that interbank markets do not clear, because all banks wants to either borrow or lend. As a consequence, default and storage are now the only instruments that banks can use to transfer resources across states of the world and ensure that the incentive compatibility constraint is satisfied ex post.[14]

[13] The Liquidity Coverage Ratio is the ratio between total liquid assets and the estimated net cash outflow of each bank, and it is supposed to be larger than 1 at any point in time.

[14] Recall that, if no hidden trades are possible, banks would be able to offer an equilibrium contract contingent on the realized per capita available liquidity, and never store or default. In that sense, this environment can be seen as an extension of Farhi et al. (2009) with aggregate uncertainty and storage/default.

5.1 Competitive Equilibrium

The objective function of the banks is the same as before. Moreover, early consumers do not receive any consumption at date 2, and similarly late consumers at date 1. This means that, by incentive compatibility, $w_2^i(1,s) = R(s)w_1^i(0,s)$. I rearrange the budget constraints in (5)-(7) (with $Z^i(s) = 0$) and make use of the incentive compatibility constraint to derive:

$$w_1^i(0,s) = 1 - \left(1 - \frac{1}{\hat{R}}\right) M^i(s) - (1-r)D^i(s), \tag{14}$$

$$D^i(s) = \frac{\pi^i(0,s) - X^i}{\pi^i(0,s) + r\pi^i(1,s)}, \tag{15}$$

$$M^i(s) = \hat{R}\frac{X^i - \pi^i(0,s)}{\pi^i(0,s) + \hat{R}\pi^i(1,s)}, \tag{16}$$

where I also make use of the fact that $R(s) = \hat{R}$ as before. The total present value of the bank incentive compatible expenditure[15] must be equal to the total available resources, lowered by the deadweight losses from either storing or defaulting. These two must be non-negative by assumption, hence intermediaries either store liquidity, if the realized fraction of early consumers is lower that the amount of short assets in the portfolio, or default in the opposite case.

The banking problem then boils down to the decision of how much liquidity to keep at date 0, before the uncertainty is realized, and how much to default upon or store in the future. From the first order conditions of the problem, I can derive the following expressions, governing the policy decision:

$$\sum_s \lambda(s)\left[\frac{1-r}{\pi^i(0,s) + r\pi^i(1,s)} - \frac{1-\frac{1}{\hat{R}}}{\pi^i(0,s) + \hat{R}\pi^i(1,s)}\right] = 0,$$

$$\lambda(s) = \pi^i(0,s)u'(w_1^i(0,s)) + \pi^i(1,s)\hat{R}u'(\hat{R}w_1^i(0,s)).$$

The bank chooses liquidity so as to equalize a weighted average of the deadweight losses from default and storage in every state of the world, using as weights the shadow values of consumption. As it is evident, the fact that the deadweight losses from storage and default are

[15] The left hand side of (14) comes from the simplification of $w_1^i(0,s)\left[\pi^i(0,s) + \pi^i(1,s)\frac{\hat{R}}{\hat{R}}\right]$.

unavoidable (for given liquidity, banks ex post will either store or default) and asymmetric implies that the former expression is highly non-linear, and an analytical solution does not exist. Thus, here I provide a numerical characterization. There is also an alternative reason to do so: by characterizing the solution in a calibrated exercise, I can also evaluate whether the model is good at replicating what happens during a financial crisis, by comparing it to the U.S. financial system in 2007-2009.

As far as the functional form of the felicity function is concerned, recall that relative risk aversion is restricted to be larger than or equal to 1 by assumption. Hence, I choose a standard logarithmic formulation. I also assume there are only two possible states of the world: with probability γ the fraction of impatient depositors is .99, and with probability $1-\gamma$ it is .01. These are the two most extreme cases such that it is still meaningful to talk about hidden trades, and I label them "crisis" and "no crisis", respectively.

The three parameters that I match with the data are the return on the long asset $\hat{R}$, the recovery rate of the liquidation technology r and the intertemporal discount factor β. I back up the first one from the average prime rate imposed by U.S. chartered commercial banks on short-term loans to business from Q2-2007 to Q4-2009.[16] For the same time period, I also choose r to be equal to the mean recovery rate on banks' loans according to Moody's (2009),[17] and β from the average market yield on 1-year U.S. Treasury bonds.[18] The calibration is summarized in table 2. Notice that the implied deadweight losses from storage $(1-r)$ turn out to be considerably lower than the ones from liquidation $(1-1/\hat{R})$: .0476 versus .55.

Table 2: Calibrated values of the parameters of the model

Parameter	Parameter Name	Value
$\hat{R}$	Long asset yield	1.05
β	Discount factor	0.9852
r	Recovery rate	0.45

In figures 3 and 4, I plot the equilibrium allocation for given probability of a future crisis. There exists a relevant area of the state space where banks choose an interior solution: liquidity

[16]Source: *Board of Governors of the Federal Reserve System.*

[17]This number is not far from the structural estimate (0.41) proposed by Chen (2010).

[18]Source: *Board of Governors of the Federal Reserve System.* The average yield is $\rho = 1.5$ per cent, and $\beta = 1/(1+\rho)$.

is such that the expected marginal benefit of having one more unit of it (in terms of avoiding liquidation in case of bankruptcy) is equal to its expected marginal costs (in terms of storage if it turns out to be excessive). Given that the deadweight losses from default are more than ten times as large as those from storage, the transition between the two corner solution is steep and happens for relatively low values of γ.

For extreme values of the probability of a future crisis, the intermediaries instead choose equally extreme portfolio strategies: if the probability of a crisis is low enough, they invest only in long assets and nothing in liquidity, and then choose to go bankrupt if a crisis is actually realized (the top left panel of figure 4). In contrast, when the probability of a run is high, the banks "fly to liquidity". Such an acute form of precautionary liquidity savings implies that intermediaries never default ex post, but store liquidity if no crisis happens at date 1[19] (the bottom middle panel of figure 4). In both cases, there will be welfare losses due to storage or liquidation, that are clear from the drop in promised consumption (the right panels of figure 4).

Before going into policy analysis, we need to check whether the model is a good representation of the reality. To shed light on this point, I check whether it is able to replicate some feature of the U.S. economy that is not targeted in the calibration. In particular, I focus my attention on the implied probability of a financial crisis. I take the liquidity ratios of U.S. chartered commercial banks, plotted in figure 1 at quarterly frequencies for the relevant period (from Q4-2007 to Q2-2009). I plug them into the model to back up the probability of a crisis consistent with the theory. Then, I calculate the probability of a financial crisis implied by a credit default swap index of U.S. banks,[20] and compare the two. The results are shown in table 3.

The model almost perfectly matches the average probability of a crisis according to the data (3.73 per cent versus 3.87 per cent). In contrast, it only accounts for about 9 per cent of

[19] The lower and upper bounds on the policy function for liquidity are consequences of the assumption on the probability distribution. Changing this would just modify the bounds accordingly, without affecting the qualitative pattern of the optimal decision.

[20] I use the quarterly averages of the "North American Banks 5-year CDS Index", built by Thompson-Reuters and published by Datastream. I assume that the probability of default until maturity is constant over time, and that the recovery rate in case of insolvency is 40 per cent. Thus, the probability of a financial crisis implied by the data is derived by rearranging the pricing formula of the credit default swap (it is priced at zero profits) and is $CDS/(1-REC)$.

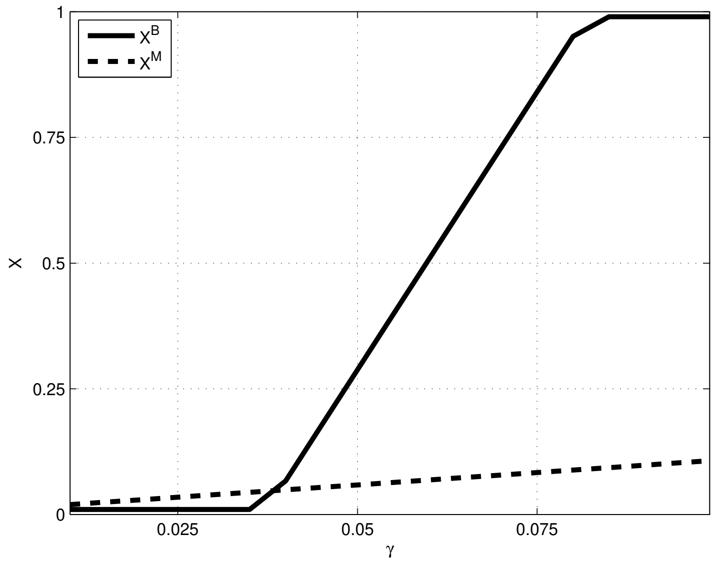

Figure 3: Equilibrium liquidity with negatively correlated liquidity shocks $\left(X^{M}\right)$ and with positively correlated liquidity shocks $\left(X^{B}\right)$, for different probabilities of a financial crisis γ (on the x-axis).

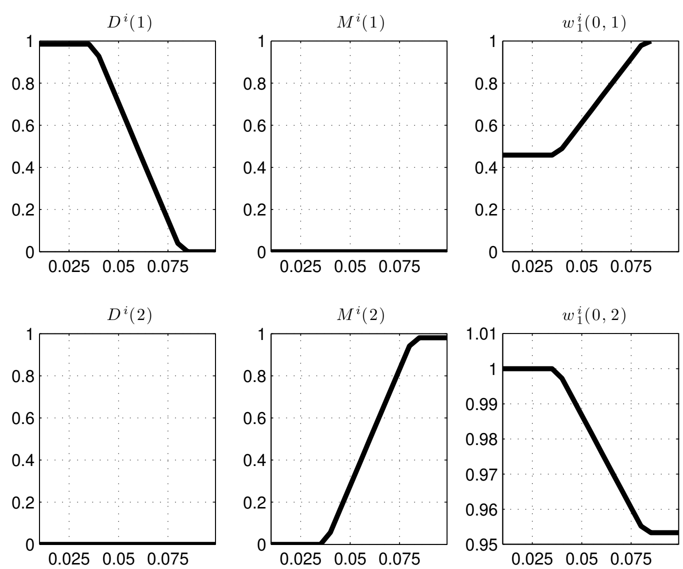

Figure 4: Equilibrium policy functions (default, storage and consumption) chosen by the banks for different probabilities of a financial crisis (on the x-axis) in the two states of the world. In state 1 $\pi^i(0,1) = .99$, and in state 2 $\pi^i(0,2) = .01$.

the total volatility, but that is expected given the model's lack of dynamic features. Moreover, the calibrated series of probabilities exhibits the same increasing trend of the data, even if at a lower magnitude. This is confirmed by the fact that the two series are highly correlated (0.8154).

To check the robustness of these outcomes, I also repeat the analysis using an alternative calibration drawn from He and Xiong (2011).[21] In this case, the model overestimates the probability of a financial crisis, and gets slightly closer to actual volatility. The fact that the correlation is still high suggests that the structure of the theory works well in replicating the data, at least in qualitative terms.

Table 3: Calibration results

	Data	Baseline Calibration	Alternative Calibration
Avg Pr(crisis) (%)	3.73	3.87	7.54
Std Pr(crisis) (%)	1.3588	0.1196	0.1523
Correlation with data	1	0.8154	0.8046

5.2 Policy Experiment

The results of the previous section allow me to use this calibrated model of financial intermediation to run a policy experiment. In particular, I want to study whether the liquidity floor that decentralizes the socially optimal allocation when interbank markets are available improves welfare also when banks cannot exchange resources among themselves. This is an important question because, in section 4, I showed that such a regulation is actually equivalent to the "Liquidity Coverage Ratio" imposed as part of the Basel III Accord. Therefore, we can check if the regulation in times of no systemic risk is also good in periods of market freeze.

To answer this question, I solve again the calibrated example with the additional constraint:

$$X^i \geq \gamma \pi^i(0,1) w_1^i(0,1) + (1-\gamma)\pi^i(0,2) w_1^i(0,2),$$

which states that the amount of liquidity must be at least as large as banks' expected cash

[21] In their dynamic model of bank runs, He and Xiong choose the return on banks' assets to be equal to 1.07 (the average mortgage rate between 2005 and 2008) and a recovery rate of 0.55.

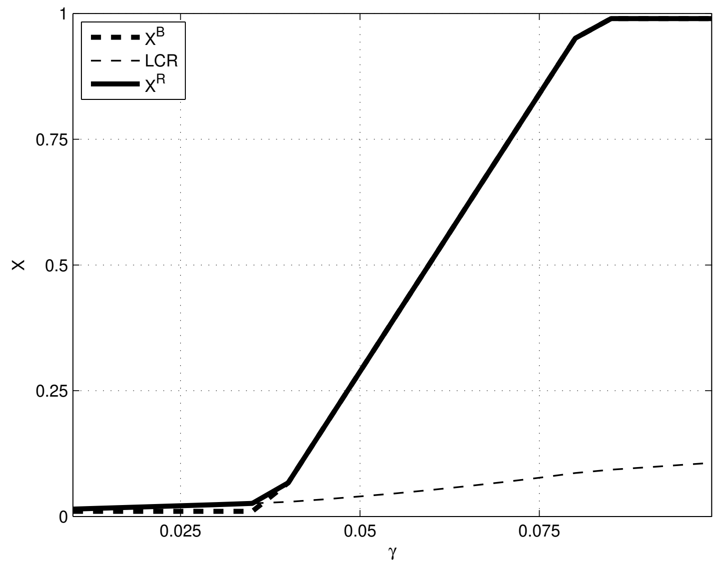

Figure 5: Equilibrium liquidity in the unregulated $\left(X^B\right)$ versus regulated equilibrium $\left(X^R\right)$ for different probabilities of a financial crisis γ (on the x-axis). In state 1 $\pi^i(0,1) = .99$, and in state 2 $\pi^i(0,2) = .01$.

outflow. Notice that the allocation chosen by the financial intermediaries in equilibrium affects the tightness of the constraint via both liquidity and promised consumption. The portfolio strategy is plotted in figure 5.

The liquidity coverage ratio is binding only for relatively low probability of a future crisis, as banks still hoard liquidity to avoid default. This means that regulated banks liquidate a lower amount of assets in case of bankruptcy, but with a higher probability they store more in case of no crisis. Thus, the expected welfare gains from the introduction of the liquidity floor (in consumption terms) are actually negative (between 0 and -.01 per cent).[22]

5.3 Planner's Solution

This last result points out that the liquidity coverage ratio is not a sufficient policy intervention when shocks are positively correlated and markets do not work, and that public authorities

[22] The values reported here are the the welfare gains in permanent consumption equivalent units, and they are equal to the weighted average of the utility gains in the two states of the world:

$$\kappa = \frac{\gamma[\pi^i(0,1) + \beta\pi^i(1,1)]ln\left(\frac{w_1^R(0,1)}{w_1^i(0,1)}\right) + (1-\gamma)[\pi^i(0,2) + \beta\pi^i(1,2)]ln\left(\frac{w_1^R(0,2)}{w_1^i(0,2)}\right)}{\gamma[\pi^i(0,1) + \beta\pi^i(1,1)] + (1-\gamma)[\pi^i(0,2) + \beta\pi^i(1,2)]}.$$

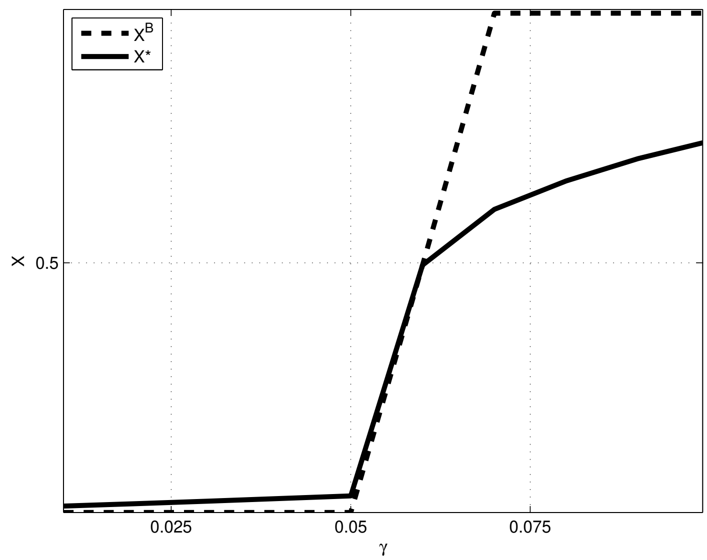

Figure 6: Equilibrium liquidity with positively correlated liquidity shocks $\left(X^B\right)$ versus planner solution (X^*) for different probabilities of a financial crisis γ (on the x-axis). In state 1 $\pi^i(0,1) = .70$, and in state 2 $\pi^i(0,2) = .30$.

should further tailor liquidity regulation to these cases. To this end, I evaluate whether there exists a feasible allocation that Pareto-dominates the competitive equilibrium. I do so by characterizing the socially optimal allocation, that I report in figures 6 and 7 (solid line), together with the competitive equilibrium (dashed line).[23]

The intuition for these results is the following. Remember that, as showed in section 3, the planner can choose the allocation *and* the return on the hidden trades $R^*(s)$, as long as they satisfy the incentive compatibility constraint. Therefore, in all effects she holds more instruments than the banks. Assume that the probability of a crisis (γ) is low. On one side, the banks choose low liquidity ex ante, because they are afraid of the deadweight losses from storage if a crisis is not realized ex post. On the other side, in states of no crisis the planner does not rebalance her budget ex post by storing liquidity, but by lowering $R^*(s)$: in this way, the early consumption increases (i.e., the impatient individuals borrow at lower rates) and the late consumption decreases (i.e., the patient individuals lend at lower rates). This means

[23] I assume that in state 1 (happening with probability γ) the fraction of impatient depositors is .70, and in state 2 (happening with probability $1-\gamma$) the fraction is .30. This change is exclusively done for simplicity of exposition, and does not qualitatively affect the results in any way.

that the planner is free to choose ex ante a higher level of liquidity than the banks, when the probability of a future crisis is low. Similarly, if the probability of a future crisis is high, the banks engage in precautionary liquidity savings, while the planner can actually increase $R^*(s)$, and in this way lower the early consumption (and increase the late consumption). Hence, the planner is free to choose lower liquidity than the banks, when the probability of a future crisis is high.

Put differently, the planner may use the interest rate on hidden bonds as an alternative instrument to avoid storage and default. Therefore, the first order conditions of the problem pin down some bounds for $R^*(s)$, that with log-utility read:

$$\beta \leq R^*(s) \leq \beta \frac{\hat{R}}{r}.$$

The intuition for this is straightforward. On one side, when $R^*(s)$ is equal to the upper bound, the planner, in periods of crisis, is indifferent between covering liquidity imbalances through higher costs of borrowing or through the default procedure. On the other side, when $R^*(s)$ is equal to the lower bound, the planner, when there is no crisis, is indifferent between lower returns to lending and storage. In between those bounds, where the planner use neither storage nor the default procedure, $R^*(s)$ is decreasing in the probability of a crisis. To see this, notice that, for given amount of liquidity, the incentive compatible return on hidden assets is:

$$R^*(s) = \frac{w_2^*(1,s)}{w_1^*(0,s)} = \hat{R} \frac{1-X^*}{X^*} \frac{\Pi(1,s)}{\Pi(0,s)}.$$

This is a decreasing function of the amount of liquidity, which in turn is increasing in the probability of a future crisis.

The early consumption $w_1^i(0,s)$ is clearly decreasing in $R^*(s)$, because it is the rate at which the impatient depositors borrow from the patient ones. Hence, as the probability of a crisis increases, the interest rate decreases and consumption of the impatient depositors increases. When the lower bound of $R^*(s)$ is hit (as in the bottom right panel of figure 7), the planner actually starts storing liquidity, and that comes at the cost of decreasing consumption. In a similar way, when the upper bound is hit (as in the top right panel of figure 7), the planner

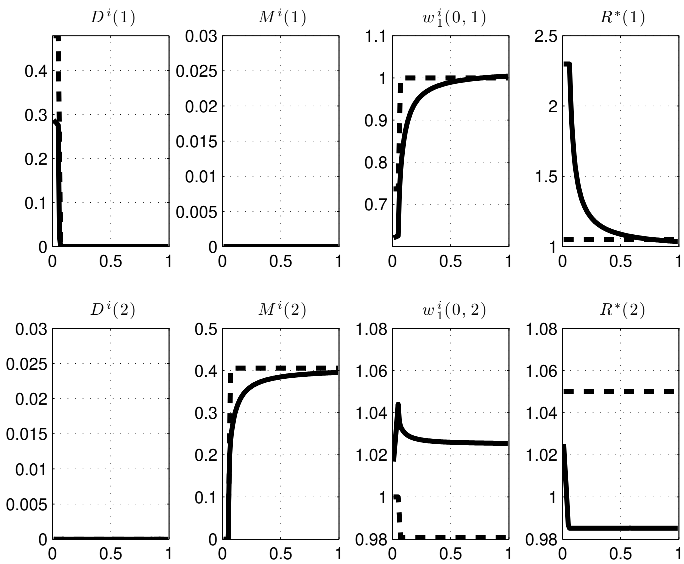

Figure 7: Equilibrium policy functions (default, storage and consumption) chosen by the banks (dashed line) and by the planner (solid line) for different probabilities of a financial crisis (on the x-axis) in the two states of the world. In the right panels, I plot the interest rate $R^*(s)$ chosen by the planner versus the return on the banking technology $\hat{R}$. In state 1 $\pi^i(0,1) = .70$, and in state 2 $\pi^i(0,2) = .30$.

finds more convenient to enter bankruptcy rather than further increase the interest rate.

From this analysis we can draw two conclusions regarding optimal regulation in presence of positively correlated shocks. First, there is space for government intervention to improve the allocation of liquidity, and as a consequence the default and storage decisions of the banks. This is a critical result, because it disproves the constrained efficiency of bankruptcy (proved in Allen and Gale, 2004) by imposing an informational constraint on intermediaries, and showing that a social planner has an advantage in dealing with it. The second conclusion is that, depending on the probability of a future crisis, there is a need for different types of regulations: when the probability of crisis is low, the banks are cumulating an inefficiently low amount of liquidity, hence a liquidity floor is necessary. In contrast, when the probability of crisis is high, the banks are afraid of default and choose an extremely safe strategy, that should be counteracted with the introduction of a liquidity ceiling. This result provides a rationale for government intervention through countercyclical liquidity requirements.

6 Concluding Remarks

Financial markets play a key role in linking illiquidity and distress in the financial system. In a stylized model of financial intermediation, it is possible to show that the emergence of default is a consequence of the simultaneous presence of trading opportunities for depositors and the absence of markets for banks. Moreover, in contrast to the previous literature, the corresponding allocation is not constrained efficient: there is the space for a public authority to improve welfare by imposing a countercyclical regulation.

This means that, in general, economists and policy makers need not only study how illiquidity emerges and how to solve it ex ante with macroprudential regulations, but also how investors (both banks and individuals) interact among themselves in the financial markets. In the present environment, interbank markets are a stabilizing force, while asset markets are a distortion to the economy. It is not difficult to argue that, in reality, these roles can switch: interbank markets can become channels for contagion, and asset markets can operate as substitutes for unavailable wholesale funding. I analyze this case in a companion paper (Panetti, 2011).

In a similar way, it can be argued that the present work only focuses on two extremes of the spectrum of market availability: complete or absent interbank markets. In between the two, there is, in all likelihood, a continuum of cases that can be analyzed with the introduction of some formal frictions, such as asymmetric information or counterparty risk. This can be done either by assuming a short cut (like the imposition of debit limits), or by fully characterizing equilibria when banks exchange funding without full commitment. A second crucial hypothesis regards the functioning of asset markets, as individuals in the present environment can borrow and lend freely. It might be interesting to characterize environments where there is instead limited commitment, as in the work of Antinolfi and Prasad (2008), or limited participation, as in Diamond (1997). These, in turn, would affect the equilibrium return on the hidden bonds, possibly in crucial ways. I leave these issues to future research.

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Appendices

A Why Only Bonds in the Asset Markets?

For this proof, I follow Golosov and Tsyvinski (2007). Remember that, when borrowing and lending in the market, individual types are still private information. In order to complete the set of traded securities, we may then add claims paying 1 unit of consumption conditional on reporting type θ in state s. Define the price of such securities as $Q(\theta, s)$. I can prove the following:

Lemma 3. *$Q(\theta, s) \geq \frac{1}{R(s)}$ for every type $\theta \in \{0, 1\}$ and $s = 1, \ldots, S$.*

Proof. $1/R(s)$ is the price of a risk-free bond delivering one unit of consumption in the following period for each unit invested. I prove the lemma by contradiction. Assume that $Q(\theta, s) < \frac{1}{R(s)}$ for some θ and s. That would give rise to arbitrage opportunities: agents would issue an infinite amount of uncontingent bonds, buy the same amount of those state-contingent securities, then report exactly type θ in state s, and enjoy infinite profits. That cannot be an equilibrium. ■

Given that $Q(\theta, s) \geq \frac{1}{R(s)}$, no type-contingent claim will be traded: individuals will never exchange securities which yield one unit of consumption if a specific type is reported, when they have the opportunity to trade a cheaper bond which yields one unit of consumption whatever type is reported.

B Planner Problem without Hidden Trades

The social planner chooses the optimal contract and the efficient portfolio allocation in order to maximize the total ex-ante welfare of the economy. In doing so, she is subject to the constraint that the portfolio allocation must provide enough resources to pay consumption in both periods to any agent of any type. In addition, individuals still have private information about their individual types. Then, I can apply the Revelation Principle and restrict the social planner problem to truth-telling mechanisms in which every agent correctly reports her type.

Formally, the planner's problem is:

$$\max_{\substack{X,Y,\{w_t^i(\theta,s)\}_{s=1,\dots,S} \\ \theta\in\{0,1\} \\ i=1,\dots,n \\ t=1,2}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta,s) U(C^i(\theta,s),\theta),$$

subject to:

$$X + Y \leq n, \tag{17}$$

$$\sum_i \sum_\theta \pi^i(\theta,s) w_1^i(\theta,s) \leq X, \tag{18}$$

$$\sum_i \sum_\theta \pi^i(\theta,s) w_2^i(\theta,s) \leq \hat{R}Y, \tag{19}$$

for each $s = 1,\dots,S$, and:

$$U(C^i(0,s),0) \geq U(C^i(1,s),0), \tag{20}$$

$$U(C^i(1,s),1) \geq U(C^i(0,s),1), \tag{21}$$

for each $i = 1,\dots,n$ and $s = 1,\dots,S$. I report the solution to this problem in the next lemma, which fully characterizes the equilibrium:

Lemma 4. *The planner chooses the optimal allocation such that in every state $s = 1,\dots,S$, and for every $i,j = 1,\dots,n$:*

$$\begin{gathered} w_1^i(1,s) = w_2^i(0,s) = 0, \\ \frac{u'(w_1^i(0,s))}{u'(w_1^j(0,s))} = 1 = \frac{u'(w_2^i(1,s))}{u'(w_2^j(1,s))}, \\ \beta\hat{R}u'(w_2^i(1,s)) = u'(w_1^i(0,s)), \end{gathered} \tag{22}$$

and

$$\sum_i \left[\pi^i(0,s) w_1^i(0,s) + \pi^i(1,s)\frac{w_2^i(1,s)}{\hat{R}}\right] = n. \tag{23}$$

The constrained efficient allocation is equivalent to the unconstrained optimum.

Proof. Guess (20) and (21) are slack. Re-write (17), (18) and (19) as:

$$\sum_i \sum_\theta \pi^i(\theta, s) \left[w_1^i(\theta, s) + \frac{w_2^i(\theta, s)}{\hat{R}} \right] \leq n, \qquad \forall s = 1, \ldots, S. \tag{24}$$

Assign multipliers $\lambda(s)$ to each constraint. Clearly, $w_1^i(1, s)$ and $w_2^i(0, s)$ are optimally set to zero, since they would be only costs for the planner and provide no utility to individuals. The first-order conditions with respect to $w_1^i(0, s)$ and $w_2^i(1, s)$ read:

$$u'(w_1^i(0, s)) = \lambda(s), \tag{25}$$

$$\beta u'(w_2^i(1, s)) = \lambda(s) \frac{1}{\hat{R}}, \tag{26}$$

for each $s = 1, \ldots, S$. Then we easily derive (22). A simplification of (24) leads to (23).

Finally, I need to verify that the incentive compatibility constraints are actually slack. The expressions in (20) and (21) now become:

$$u(w_1^i(0, s)) \geq u(w_1^i(1, s)) = 0, \tag{27}$$

$$u(w_2^i(1, s)) \geq u(w_2^i(0, s)) = 0. \tag{28}$$

I need to prove that in equilibrium $w_1^i(0, s) > 0$ and $w_2^i(1, s) > 0$ for each i and s. More specifically, given that the Euler equation holds in each country, I need just one of the two. Assume that $w_1^i(0, s) = 0$ for some i and s. Then, by Inada conditions $u'(w_1^i(0, s)) = +\infty$ and by (22) also $u'(w_1^j(0, s)) = +\infty$ for any $j \neq i$ and s. This implies $w_1^i(0, s) = 0$ for any i, and by the Euler equation $w_2^i(1, s) = 0$ for any i. But then (23) clearly gives $0 = n$, which is a contradiction. ∎

C Proofs

Proof of lemma 1. Rewrite the problem of the agent in the asset market as:

$$V(C^i(\theta, s), R(s), \theta, s) = \max_{c_1^i, c_2^i, b^i, \theta'} U(c_1^i, c_2^i, \theta),$$

$$s.t. \qquad c_1^i + \frac{c_2}{R(s)} = w_1^i(\theta', s) + \frac{w_2^i(\theta', s)}{R(s)}.$$

For type 1 and 2, the incentive compatibility then reads, respectively:

$$V(C^i(0,s), R(s), 0, s) \geq V(C^i(1,s), R(s), 0, s),$$
$$V(C^i(1,s), R(s), 1, s) \geq V(C^i(0,s), R(s), 1, s),$$

which can be rewritten as:

$$u\left(w_1^i(0,s) + \frac{w_2^i(0,s)}{R(s)}\right) \geq u\left(w_1^i(1,s) + \frac{w_2^i(1,s)}{R(s)}\right),$$
$$u(R(s)w_1^i(1,s) + w_2^i(1,s)) \geq u(R(s)w_1^i(0,s) + w_2^i(0,s)).$$

The result then follows. ■

Proof of lemma 2. I follow Farhi et al. (2009) and reduce the program to a simple one:

Lemma 5. *The planner's problem in* (3) *is equivalent to:*

$$\max_{\substack{\{R(s),\mathcal{I}^i(s)\}_{s=1,\ldots,S} \\ i=1,\ldots,n}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta,s) \tilde{V}(\mathcal{I}^i(s), R(s), \theta, s), \tag{29}$$

subject to:

$$\sum_i \sum_\theta \pi^i(\theta, s)\left[x_1^i + \frac{x_2^i}{\hat{R}}\right] \leq n,$$

where:

$$\tilde{V}(\mathcal{I}^i(s), R(s), \theta) = \max_{x_1^i, x_2^i} U(x_1^i, x_2^i, \theta), \tag{30}$$
$$s.t. \qquad x_1^i + \frac{x_2^i}{R(s)} \leq \mathcal{I}^i(s).$$

Proof. I want to prove that the allocation that solves (3) can be implemented for some $\{R(s), \mathcal{I}^i(s)\}$ satisfying (29), and that given $\{R(s), \mathcal{I}^i(s)\}$ solution to (29) we can set $w_t^i(\theta, s)$ $= x_t^i(\mathcal{I}^i(s), R(s), \theta, s)$ and check that it is feasible in (3).

Start from the first part. Take any allocation $w_t^i(\theta, s)$ solution to (3). We know that it satisfies incentive compatibility, hence:

$$w_1^i(0, s) + \frac{w_2^i(0, s)}{R(s)} = w_1^i(1, s) + \frac{w_2^i(1, s)}{R(s)}. \tag{31}$$

Call this present discounted value $\mathcal{I}^i(s)$. Notice that $w_t^i(\theta, s)$ from (3) is the solution to the "hidden problem" delivering $V(C^i(\theta, s), R(s), \theta, s)$ with equilibrium return $R(s)$, hence it also solves (29) provided that $\theta' = \theta$, i.e. the true type is reported. That means that any solution to (3) can be implemented with the right choice of $\mathcal{I}^i(s)$ and $R(s)$. This ends the first part of the proof.

Now for the second part. Assume that $\{R^P(s), \mathcal{I}^P(s)\}$ is the solution to (29). Pick $w_t^i(\theta, s) = x_t^i\left(\mathcal{I}^P(s), R^P(s), \theta, s\right)$. We want to see whether this allocation is feasible in (3). It clearly satisfies the resource constraint. Notice that:

$$\mathcal{I}^P(s) = w_1^i(0, s) + \frac{w_2^i(0, s)}{R^P(s)} = w_1^i(1, s) + \frac{w_2^i(1, s)}{R^P(s)}, \tag{32}$$

hence the incentive compatibility constraint is satisfied, too.This ends the proof. ■

The lemma ensures that instead of solving for the whole consumption contract, the planner can choose a present value of consumption $\mathcal{I}^i(s)$ for every agent, and a return on the hidden bonds $R(s)$ through which they might rearrange consumption across time. Therefore, I can solve the program (29) while having definitely less unknowns ($(1 + n)S$ vs. $2nS$). For type-0 agents, the problem (30) reads:

$$\max_{x_1^i, x_2^i} \quad u(x_1^i) \qquad s.t. \qquad x_1^i + \frac{x_2^i}{R(s)} = \mathcal{I}^i(s).$$

Clearly, $x_1^i = \mathcal{I}^i(s)$, because impatient agents only care about consuming at date 1. In the same way, I can solve type-1's maximization, and derive $x_2^i = R(s)\mathcal{I}^i(s)$. The planner's problem then simplifies to:

$$\max_{\substack{\{R(s),\mathcal{I}^i(s)\}_{s=1,\dots,S}\\ i=1,\dots,n}} \sum_i \sum_s \nu(s)\big[\pi^i(0,s)u(\mathcal{I}^i(s)) + \beta\pi^i(1,s)u(R(S)\mathcal{I}^i(s))\big],$$

subject to the resource constraint:

$$\sum_i \mathcal{I}^i(s)\left[\pi^i(0,s) + \pi^i(1,s)\frac{R(s)}{\hat{R}}\right] \leq n,$$

which must hold in any state.

Attach multiplier $\lambda(s)$ to the resource constraint. The first-order conditions are:

$$\mathcal{I}^i(s): \quad \nu(s)[\pi^i(0,s)u'(\mathcal{I}^i(s)) + \beta R(s)\pi^i(1,s)u'(R(s)\mathcal{I}^i(s))] = \lambda(s)\left[\pi^i(0,s) + \pi^i(1,s)\frac{R(s)}{\hat{R}}\right], \tag{33}$$

$$R(s): \quad \nu(s)\beta\sum_i \pi^i(1,s)u'(R(s)\mathcal{I}^i(s))\mathcal{I}^i(s) = \frac{\lambda(s)}{\hat{R}}\sum_i \pi^i(1,s)\mathcal{I}^i(s). \tag{34}$$

Multiply both sides of (33) by $\mathcal{I}^i(s)$ and sum across i. Making use of (34), we can then express $\lambda(s)$ as:

$$\lambda(s) = \frac{\nu(s)\sum_i \pi^i(0,s)u'(\mathcal{I}^i(s))\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)\mathcal{I}^i(s)}. \tag{35}$$

Use (35) back into (34) to derive the following condition:

$$\frac{\sum_i \pi^i(1,s)\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)\mathcal{I}^i(s)} = \frac{\beta\hat{R}\sum_i \pi^i(1,s)u'(R(s)\mathcal{I}^i(s))\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)u'(\mathcal{I}^i(s))\mathcal{I}^i(s)}. \tag{36}$$

The unconstrained optimum in (22) is the solution to the constrained efficient problem. To see that, plug it into (36) and check that it is satisfied. The resources are also exhausted. From the Euler equation notice that:

$$\hat{R} > \beta\hat{R} = \frac{u'(\mathcal{I}^P)}{u'(R^P\mathcal{I}^P)} \geq R^P, \tag{37}$$

where we used the fact that $\beta < 1$ and the hypothesis on relative risk aversion.[24] Moreover,

[24]The assumption about relative risk aversion is crucial to show this result. Rewrite $-\frac{u''(c)c}{u'(c)} \geq 1$ as $-\frac{u''(c)}{u'(c)} \geq$

rewrite the Euler equation as:

$$f(R) = \frac{u'(\mathcal{I}^P)}{u'(R\mathcal{I}^P)} - \beta \hat{R}. \tag{38}$$

Then, $f(1) = 1 - \beta \hat{R} < 1$ together with the fact that $f(R)$ is increasing gives the result that $R^P > 1$. This ends the proof. ■

Proof of proposition 1. Attach multipliers λ^i, $\xi^i(s)$ and $\chi^i(s)$ to the constraints (5), (6) and (7), respectively. Split the constraints (8) and (9) into two parts. Then assign multipliers $\zeta_D^i(s)$ and $\zeta_M^i(s)$ to the non-negativity constraints of $D^i(s)$ and $M^i(s)$, and multipliers $\eta_D^i(s)$ and $\eta_M^i(s)$ to the upper bounds. The first-order conditions of the program then read:

$$w_1^i(0,s): \quad \nu(s) \left[\pi^i(0,s)u'(w_1^i(0,s)) + \beta\pi^i(1,s)u'(w_2^i(1,s))\hat{R}\right] = \pi^i(0,s)\xi^i(s) + \pi^i(1,s)\hat{R}\chi^i(s), \tag{39}$$

$$Z^i(s): \quad \xi^i(s) = \tilde{R}(s)\chi^i(s) - \eta_M^i(s), \tag{40}$$

$$X^i: \quad \lambda^i = \xi^i(s) + \eta_M^i(s), \tag{41}$$

$$Y^i: \quad \lambda^i = \hat{R}\chi^i(s) + \eta_D^i(s), \tag{42}$$

$$D^i(s): \quad r\xi^i(s) + \zeta_D^i(s) + r\eta_M^i(s) = \hat{R}\chi^i(s) + \eta_D^i(s), \tag{43}$$

$$M^i(s): \quad \xi^i(s) + \eta_M^i(s) = \chi^i(s) + \zeta_M^i(s). \tag{44}$$

Concavity of $u(c)$, Inada conditions, equations (39), (41), (42) and non-negative multipliers ($\eta_D^i(s), \eta_M^i(s) \geq 0$) give that $\lambda^i > 0$. Equations (41), (42) and (43) in equilibrium give $\zeta_D^i(s) = (1-r)\lambda^i > 0$, hence $D^i(s) = 0$ for any state s and country i and $\eta_D^i(s) = 0$ by complementary slackness..[25] Similarly, equations (41), (42) and (44) give $\zeta_M^i(s) = (1 - 1/\hat{R})\lambda^i > 0$, hence $M^i(s) = 0$ and $\eta_M^i(s) = 0$. As a consequence, both budget constraints at date 1 and 2 are

$\frac{1}{c}$. This in turn means that $-(log[u'(c)])' \geq (log[c])'$. Integrate between z_1 and $z_2 > z_1$ so as to obtain $log[u'(z_1)] - log[u'(z_2)] \geq log[z_2] - log[z_1]$. Once taken the exponent, the last expression gives $\frac{u'(z_1)}{u'(z_2)} \geq \frac{z_2}{z_1}$. If $z_1 > z_2$, the inequality is reversed.

[25] As mentioned in footnote 4, this result would go through in the presence of more general increasing and concave default functions. For the sake of the argument, assume that for each unit $D^i(s)$ sold off the bank gets in liquidity an amount $l(D^i(s)) = log(1 + D^i(s))$. Then, in equilibrium, the multiplier on the non-negativity constraint of $D^i(s)$ would be:

$$\zeta_D^i(s) = (1 - l')\lambda^i = \left(1 - \frac{1}{1 + D^i(s)}\right)\lambda^i.$$

Clearly, $D^i(s) > 0$ would imply $\zeta_D^i(s) > 0$, and complementary slackness would be violated. Therefore, the only possible equilibrium is still the one with $D^i(s) = 0$.

binding, because $\xi^i(s)$ and $\chi^i(s)$ are strictly positive. Moreover, equations (40), (41) and (42) give $\tilde{R}(s) = \hat{R}$. The resource constraint in (11) then gives $w_1^i(0,s) = 1$ since $R(s) = \hat{R}$ for every s. The initial portfolio allocation $X^i = \sum_k \nu(k)\pi^i(0,k)$ and $Y^i = \sum_k \nu(k)\pi^i(1,k)$, together with the amount of bonds issued/purchased in the interbank market $Z^i(s) = \sum_k \nu(k)\pi^i(0,k) - \pi^i(0,s)$, clears the market and balances banks' budgets in every state of the world. ■

Proof of proposition 2. Given that I am looking for the capital requirement that implements the efficient allocation, I can impose $D^i(s) = M^i(s) = 0$ right away. The date-0 banking problem reads:

$$\max \sum_s \nu(s) \left[\pi^i(0,s) u \left(w_1^i(0,s) + \frac{w_2^i(0,s)}{R(s)} \right) + \beta \pi^i(1,s) u(R(s) w_1^i(1,s) + w_2^i(1,s)) \right],$$

subject to:

$$X^i + Y^i \leq 1, \tag{45}$$

$$X^i \geq \sum_\theta \pi^i(\theta,s) w_1^i(\theta,s) + Z^i(s), \tag{46}$$

$$\hat{R} Y^i + \tilde{R} Z^i(s) \geq \sum_\theta \pi^i(\theta,s) w_2^i(\theta,s), \tag{47}$$

$$X^i \geq F^i, \tag{48}$$

and the incentive compatibility constraint in (2). Use (45)-(47) to derive the intertemporal resource constraint:

$$\sum_\theta \pi^i(\theta,s) \left[w_1^i(\theta,s) + \frac{w_2^i(\theta,s)}{\hat{R}} \right] + \left(1 - \frac{\tilde{R}(s)}{\hat{R}} \right) Z^i(s) \leq 1. \tag{49}$$

Similarly, constraint (48) becomes:

$$\sum_\theta \pi^i(\theta,s) w_1^i(\theta,s) + Z^i(s) \geq F^i. \tag{50}$$

Apply the following change of variables:

$$\mathbf{I}^i(s) = w_1^i(0,s) + \frac{w_2^i(0,s)}{R(s)},$$
$$\mathbf{H}^i(s) = \sum_\theta \pi^i(\theta,s) w_2^i(\theta,s).$$

First, I express the constraints of the program in terms of $\mathbf{I}^i(s)$ and $\mathbf{H}^i(s)$. The capital requirement in (50) becomes:

$$\begin{aligned}
&\pi^i(0,s)w_1^i(0,s) + \pi^i(1,s)w_1^i(1,s) + w_1^i(0,s) - w_1^i(0,s) + Z^i(s) = \\
&= w_1^i(0,s) + \pi^i(1,s)(w_1^i(1,s) - w_1^i(0,s)) + Z^i(s) = \\
&= w_1^i(0,s) + \pi^i(1,s)\frac{w_2^i(0,s) - w_2^i(1,s)}{R(s)} + Z^i(s) = \\
&= \left(w_1^i(0,s) + \frac{w_2^i(0,s)}{R(s)}\right) - \frac{\pi^i(0,s)w_2^i(0,s) + \pi^i(1,s)w_2^i(1,s)}{R(s)} + Z^i(s) \\
&= \mathbf{I}^i(s) - \frac{\mathbf{H}^i(s)}{R(s)} + Z^i(s) \geq F^i. \qquad (51)
\end{aligned}$$

Similarly, the intertemporal budget constraint now reads:

$$\mathbf{I}^i(s) - \mathbf{H}^i(s)\left(\frac{1}{R(s)} - \frac{1}{\hat{R}}\right) + \left(1 - \frac{\tilde{R}(s)}{\hat{R}}\right) Z^i(s) \leq 1. \qquad (52)$$

The problem then is to choose $\{\mathbf{I}^i(s), \mathbf{H}^i(s), Z^i(s)\}$ to maximize the objective function, subject to (51) and (52). Attach multipliers η^i and ξ^i, respectively. Then, the first-order conditions with respect to $\mathbf{I}^i(s)$ and $\mathbf{H}^i(s)$ are:

$$\xi^i - \eta^i = \nu(s)[\pi^i(0,s)u'(\mathbf{I}^i(s)) + \beta R(s)\pi^i(1,s)u'(R(s)\mathbf{I}^i(s))], \qquad (53)$$
$$\frac{\eta^i}{R(s)} = \xi^i\left[\frac{1}{R(s)} - \frac{1}{\hat{R}}\right], \qquad (54)$$
$$\eta^i = \xi^i\left(1 - \frac{\tilde{R}(s)}{\hat{R}}\right). \qquad (55)$$

Plug the constrained efficient allocation into the program:

$$\mathbf{I}^i(s) = \mathcal{I}^P,$$

$$\mathbf{H}^i(s) = \pi^i(1,s) R^P \mathcal{I}^P,$$
$$R(s) = R^P.$$

I need to prove that, at the constrained efficient allocation, the multipliers are positive, FOCs are satisfied and markets clear for some positive prices. Notice that, at the constrained efficient allocation, the RHS of (53) can be simplified using the Euler equation. Merge the FOCs into:

$$\xi^i \frac{R^P}{\hat{R}} = \nu(s) u'(I^P) \left[\pi^i(0,s) + \pi^i(1,s) \frac{R^P}{\hat{R}} \right],$$

which is positive by concavity of the utility function. The multiplier η^i is positive by (54), since ξ^i is positive and $R^P < \hat{R}$. Therefore, the minimum capital requirement is a binding constraint and it must be so for any representative bank in all sectors.

Since we are decentralizing the efficient allocation, it must then be the case that $\sum_i F^i = \Pi(0)\mathcal{I}^P$; hence I derive (13). In equilibrium, the amount of bonds traded in the interbank markets is:

$$Z^i(s) = \left[\sum_k \nu(k) \pi^i(0,k) - \pi^i(0,s) \right] \mathcal{I}^P, \tag{56}$$

so that $\sum_i Z^i(s) = 0$ and the market clears. Finally, from (54) and (55), we obtain $\tilde{R}(s) = R^*$ in each state s. ∎