Bank Liquidity, Market Participation, and Economic Growth

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Abstract

We report evidence that bank liquidity ratios (liquid assets as a percentage of total assets) decrease during the process of economic development. To reconcile this observation with (i) the increasing importance of financial markets and (ii) the increasing direct participation of individual investors in them, we build a neoclassical growth model with banks and markets. In this environment, banks engage in cross-subsidization of the impatient depositors to keep up with the competitive pressure from the markets. Moreover, as the economy grows, it becomes easier for the individuals to access the market, and the banks react to this by lowering their liquidity ratios. In a panel of 45 countries, we find evidence that such a mechanism is in place: a one-unit increase in an index of securities market liberalization leads to a drop in the bank liquidity ratio between 15 and 22 per cent.

Keywords: Financial intermediation, liquidity, market participation, economic growth

JEL Classification: E44, D91, G21, O16
1 Introduction

An extensive empirical literature has showed how the “financial architecture” of an economy (i.e., the mix of financial intermediation and markets) evolves during the process of economic development, and might generate non-trivial feedback effects on the process itself. Despite this, most of the times the theoretical literature on finance and growth has thought of the financial sector just as a “black box” that simply allows individuals to pool risk among themselves. With the present paper, we want to open that black box.

Our aim is twofold. First, we report evidence that the liquidity ratio (liquid assets as a percentage of total assets) of financial intermediaries decreases during the process of economic development. Second, we reconcile this evidence with (i) economic growth and (ii) two other well-known facts about the evolution of the financial architecture during the process of development: the increasing importance of financial markets, at the expenses of more traditional financial intermediaries, and the increase of direct participation of individual investors into them.

To this end, we build upon the idea that financial intermediaries (or more commonly, “banks”) and markets compete in offering two types of services to the economy: insurance against idiosyncratic shocks, and investment opportunities. Here we use the word “markets” to identify all those investment opportunities that allow individuals to effectively by-pass the banks to make their investments. For example, we can think of them as tax havens, or as those institutions that provide banking services without being regulated as such, which have been labeled “new financial intermediaries” or more recently “shadow banking system”.

More formally, we embed into a general equilibrium growth model the most standard theory of financial intermediation and markets that the microeconomic literature gives us: the one that goes back to the seminal work of Diamond and Dybvig (1983). In their world, banks collect deposits and set up contracts providing insurance to their customers against some liquidity shocks that make them “impatient” to consume. For a given amount deposited $w_t$, it is a well-known result that, with logarithmic utility, the bank offers to pay back exactly $w_t$ to those customers who turn out to be impatient (i.e., banks never “break the buck”),
and $R_t w_t$ to those who turn out to be patient, where $R_t$ is the return on the capital that banks lend to the production sector. It is also well-known that, in this environment, the agents would get exactly the same consumption bundle \( \{w_t, R_t w_t\} \) (and, as a consequence, the same expected welfare) by independently accessing the capital market and making their own investment decision.\(^1\) Hence, the individuals are completely indifferent between these two investment opportunities.

To break up this indifference, we tweak the environment in two directions. First, banks pay an iceberg-type cost on the return on their capital investment. This cost can be seen as emerging from regulation, limiting the ways banks can invest their capital, or as a technology constraint. Moreover, by imposing it only onto banks, we replicate the preferential tax treatment enjoyed by capital gains with respect to interests from deposits, that is typical of many developing and developed countries (including the U.S.). Second, individuals who invest directly in the market must pay a fixed entry cost. This can be seen as a transaction cost or an institutional impediment that prevent individuals from accessing the market, and is a tool that has been extensively used in the macroeconomic literature on finance and growth (Acemoglu and Zilibotti, 1997; Townsend and Ueda, 2006).

The interplay between the bank iceberg cost and the individual fixed entry cost is at the cornerstone of our analysis. We assume that the individuals engage in a discrete investment decision, by choosing between investing in the bank or directly in the market, and thus show how the competitive pressure from this alternative investment opportunity affects the bank asset portfolio. Technically, we solve a banking problem, augmented with the imposition of a participation constraint: the banking contract must be such that the depositors are in expectation at least as well off as they would be by trading in the market.\(^2\) Depending on whether this constraint is binding or not, we will have very different results. At low levels of development, the salary of the individuals is not enough for them to afford the fixed cost to access the market. Hence, the economy is in a pure banking equilibrium, in which we demonstrate

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\(^1\)This is a consequence of the assumption of logarithmic utility. In their original paper, Diamond and Dybvig assume a strictly concave utility function with relative risk aversion larger than 1, and show that banks increase expected welfare with respect to the market equilibrium by cross-subsidizing the (marginally more valuable) impatient individuals.

\(^2\)This is similar in spirit to the “disintermediation constraint” analyzed in Allen and Gale (1997).
that the bank liquidity ratio is constant in the long run. This result dramatically changes as the economy grows, though. After a threshold, the salary that the individuals receive is high enough so that the participation constraint binds. After this point, banks know that, due to their technological/regulatory constraint, they cannot compete with the markets in providing services to the patient individuals. Hence, they ensure participation by focusing on “early” (impatient) consumption, i.e. they increase the amount of liquidity in their portfolios relative to capital. This is an interesting result, because we find that in equilibrium banks cross-subsidize the impatient depositors as a consequence not of their high risk aversion (as in the original Diamond and Dybvig’s paper), but of competition and technology/regulation.

As the economy keeps growing, this bank “comparative advantage” in providing early consumption disappears, as the fixed cost for the individuals to enter the market becomes less and less important. Therefore, banks are forced to invest relatively more and more into capital, i.e. the liquidity ratio goes down. This dynamic evolution gets to a point in which the individual fixed cost becomes negligible, and banks cannot enforce participation any more: the economy is in a pure market equilibrium from then onwards.

We are currently working on extending our analysis in two different directions, that we will further characterize in future drafts of this paper. First, we want to relax the hypothesis of discrete investment choice, and assume that individuals play a “financial lottery”. That is, we allow the agents in the economy to randomize between banks and markets by playing an individual-specific lottery, and insure against the resulting income risk in a perfectly competitive insurance market, in a fashion similar to the “employment lotteries” pioneered by Hansen (1985) and Rogerson (1988). In this way, we simultaneously characterize the investment decision of the banks, the behavior of the investors in the market, and the evolution of the market participation rate. Moreover, we provide the base for some future extensions of this environment, that can be possibly used to analyze issues like optimal taxation, or calibrated to study the evolution of the financial architecture at business cycle frequencies.

Second, we want to validate our intuition that banks lower their liquidity ratios because of the competitive pressure from the external investment opportunities available to their potential customers. To this end, we make use of a database of bank liquid reserves, constructed
by the World Bank for around 100 different countries for the period 1947-2003. We proxy the availability of external investment channels with an index of securities market policy, provided by the IMF, that increases as regulatory changes are imposed to advance the development of markets. Our results show that a one-unit increase in this index leads to an average drop in the bank liquidity ratio between 15 and 22 per cent. Moreover, we prove that this effect is stable when controlling for other types of financial liberalizations, and highly nonlinear: only an extensive process of markets liberalization has a significant effect on the behavior of banks.

The rest of the paper is organized as follows. In section 2 and 3, we first summarize the literature related to our work and the empirical evidence, respectively. In section 4, we describe the economic environment of our model, and in section 5 we characterize its equilibrium. In section 6, we show the results of our econometric analysis. In section 7, we go back to the theory, and introduce financial lotteries. Finally, section 8 concludes.

2 Related Literature

Our paper contributes to several lines of research. First, it extends the literature on the evolution of the roles of banks and financial markets, the so-called “financial architecture”. For example, Gurley and Shaw (1955) are the first to point out the increasing importance of markets during the process of economic development. Song and Thakor (2010) study a mechanism according to which the interaction between banks and markets is based not only on competition, but also on complementarity and co-evolution. These considerations lie on an extensive empirical literature. The work of Demirguc-Kunt et al. (2012) is only the last example of a series of papers that finds that, as a country develops, the size of both banks and securities markets increase. The authors also find that the association between economic growth and an increase in securities market development increases through time, thus reinforcing the view that the role of markets become more and more important.

Despite the extension of the empirical analysis, not many authors have focused on explaining the mechanism underlying the evolution of the financial architecture, Boyd and

\(^3\text{See Levine (2005) for a survey.}\)
Smith (1998) and Deidda and Fattouh (2008) being two notable exceptions. They both study environments in which asymmetric information about the profitability of an investment opportunity pushes for some costly state verification. Boyd and Smith (1998) use this assumption to analyze the evolution of debt and equity markets, while Deidda and Fattouh (2008) instead point their attention at the interactions between banks who gather information and disclosure laws in the stock markets. Our work differs from theirs in three respects. First, we do not explain the coexistence of banks and markets with the need for monitoring, but instead focus on the willingness of individual investors to insure themselves against idiosyncratic shocks, as in the seminal work by Diamond and Dybvig (1983). Second, we aim at jointly explaining the evolution of financial architecture and of the bank asset portfolios. Third, we use a different environment to answer our question, by embedding a standard microfounded model of banking into an equally standard neoclassical growth model. In that sense, our work is inspired by some dynamic models of banking used in different set-ups in the literature (Bencivenga and Smith, 1991; Qi, 1994; Ennis and Keister, 2003; Gaytan and Ranciere, 2006), which here are extended by considering commitment problems similar to those in Thomas and Worrall (1988).

Finally, by studying the evolution of the financial system during the process of economic development, the present paper bridges two important strands of macroeconomic research: finance and growth, and structural change. The literature on the first one finds its cornerstone in the work of Greenwood and Jovanovic (1990), and focuses on the role of increasing individual participation in the financial system as a mechanism to enhance growth (Khan, 2001) and risk diversification (Acemoglu and Zilibotti, 1997), and to affect income inequalities (Townsend and Ueda, 2006). The second one instead points its attention to some unbalanced features of the economy in the process of development, as highlighted by the seminal works of Kuznets (1966) and Baumol (1967), and in more recent paper by Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).
3 Empirical Evidence

In this section, we provide some evidence showing that a decreasing liquidity ratio in the financial system is a general characteristic of the process of economic growth. In figure 1, we plot a measure of relative liquidity of U.S. financial businesses: liquid assets as a percentage of total liabilities. Following a common practice in the literature, we define liquid assets as the sum of vault cash, excess and required reserves, and Treasury bonds. The series exhibits a significant downward trend: it started at around 40 percent in the 1950s, and fell to a level lower than 5 percent in the 2000s. The average growth rate is -3.34% while, even after the steep decline of the period 1950-1970, relative liquidity has decreased at an average rate of -2.40%. These observations are robust to an alternative specification of the liquidity ratio, that is also used in the literature: in figure 2, we substitute total financial assets at the denominator, and we find a pattern that is qualitatively identical to the original one.

It can be argued that the downward trend in the liquidity ratio is a consequence of an underlying downward trend in the relative price of liquidity. In order to rule out this argument, we proxy the relative price of liquidity with the inverse of the compounded return on a 10-year U.S. Treasury bond, relative to the return on the S&P500. From figure 3, it is evident that the series has decreased in the period 1970-2010. This pattern was expected, given that it reflects the fall of the equity risk premium, which is a well-known phenomenon and has been extensively analyzed in the past. What is interesting to notice here is that the fall in the relative price of liquidity has been quantitatively negligible: its average growth rate is just -0.06%. This allows us to rule out any considerations regarding prices, and focus on the behavior of economic agents in the following sections.

Finally, the decreasing liquidity in the financial system is not a phenomenon that exclusively regards the United States. In figure 4, we plot a measure of relative liquidity, calculated

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4According to the IMF, such a measure “provides an indication of the liquidity available to meet expected and unexpected demands for cash. The level of liquidity indicates the ability of the deposit-taking sector to withstand shocks to their balance sheet”.

5There exists also some evidence for the period 1880-1910, derived from Weber (2000). We find that the liquidity ratio of the U.S. commercial banks, after a steep decline in the first ten years of the sample (from 36 to 20 percent) has stayed almost constant for the following twenty years, fluctuating between 20 and 26 percent. The data are available on request.

6Jagannathan et al. (2000); Lettau et al. (2008)
by the Bank of England for the United Kingdom, and show that even this series exhibits a negative average growth rate (-6.64%), with a sharp decline in the Seventies, and a continuing downward trend in the following years. There exists some strong evidence at a cross-sectional level, too. In a sample of 100 countries in the period 1970-2010, low-income countries have on average an amount of bank liquid reserves (as a percentage of total assets) of around 26.3 percent, versus 18.2 percent for middle income countries, and 8.6 percent for high-income countries.\textsuperscript{7} This pattern has been consistent throughout our sample. In figure 5, we plot the bank liquid reserves as a percentage of total assets in the available years, and show that in every period the higher the income is, the lower the amount of relative liquidity.

4 The Model

4.1 Preferences, Endowments and Technology

Time is infinite and discrete. The economy is populated by a series of 2-period-living overlapping generations. At each point in time, a new cohort is born, represented by a unitary continuum of identical individuals. Each individuals is endowed with one unit of labor when born, and zero units in the second period of the lifetime.

Individual preferences take the following form:

\[
U(c_t^1, c_{t+1}^1, \theta) = (1 - \theta)\log(c_t^1) + \theta\log(c_{t+1}^1),
\]

where the superscript indicates the birth date, and the subscript the period in which the individuals consume. The parameter \(\theta\) represents a preference shock, that takes values 0 with probability \(\pi\), and 1 with probability \((1 - \pi)\). Shocks are independent across individuals, idiosyncratic, and publicly revealed at the end of period \(t\).\textsuperscript{8} Their role is to affect the point in time at which individuals are willing to consume. Thus, in line with the literature, we define type-0 individuals as “impatient”, and type-1 individuals are “patient”. The presence of a continuum of individuals implies that the law of large numbers holds, so the probability

\textsuperscript{7}Source: World Development Indicators, World Bank; International Financial Statistics, IMF.

\textsuperscript{8}By assuming that the preference shocks are publicly revealed, we implicitly rule out bank run equilibria, whose effects on growth are analyzed by Ennis and Keister (2003) and Gaytan and Ranciere (2006).
distribution of the preference shock is equivalent to its cross-sectional distribution.

In order to finance their consumption in $t$ and $t+1$, two different technologies are available. The first one is employed by a perfectly competitive firm, and is represented by a neoclassical production function $y_t = f(K_t, A_tL_t)$, with constant returns to scale, $f' > 0$ and $f'' < 0$. Labor, inelastically provided\(^9\) by the newborn cohort at each point in time, is augmented by the exogenous technological process $A_t$, and yields an immediate return (i.e., a salary) $w_t = A_t f_2(K_t, A_tL_t)$. Capital instead needs “time to build”: the amount invested in $t-1$ matures only in $t$, yields a return $r_t = f_1(K_t, A_tL_t)$, and then fully depreciates. Importantly, we rule out intergenerational transfers, i.e. there exists no mechanism to transfer the return on capital received by the current old cohort to the current impatient individuals. This hypothesis, together with time to build, ensures that capital will only be held ex post by patient individuals. Therefore, there exists a role for a second technology to finance the consumption of those who turn out to be impatient: we call it “liquidity”. This is simply a storage technology, that yields one unit of the consumption good for each unit invested at the beginning of the period.

4.2 Investment Opportunities

The preference structure and the available technologies imply that the individuals in this economy must make a non-trivial investment decision: once they receive their salaries, they decide their holdings of liquidity and capital, before knowing if they turn out to be patient or impatient (because the idiosyncratic shock is revealed at the end of period $t$). In order to do this, there are two investment channels that can be exploited: the individuals can directly trade in the markets, or they can negotiate a deposit contract with a bank, before the markets open. This investment decision is discrete, i.e. individuals either deposit their salary in the banks or invest it all in the market, and is taken without commitment: individuals can always renege on the banking contract and choose the market, instead. More formally, the investment choice is governed by the dummy variable $\phi_t$, which takes value 1 if the individuals invest in the market, or 0 if they go to the banks. We relax the hypothesis of discrete investment

\(^9\)Because individuals do not enjoy utility from leisure.
choice in section 7, where we introduce financial lotteries.

When the individuals invest in the market, they take their own investment decisions, after the payment of a fixed cost $\xi$. The problem in the market then reads as follows:

$$V^M(w_t, r_{t+1}, p_t) = \max_{\{x_t^t, x_{t+1}^t, z_t^M, k_{t+1}^M\}} \pi \log(x_t^t) + (1 - \pi) \log(x_{t+1}^t),$$

subject to:

$$z_t^M + k_{t+1}^M = w_t - \xi,$$  \hspace{1cm} (2)

$$z_t^M + p_t k_{t+1}^M = x_t^t,$$  \hspace{1cm} (3)

$$R r_{t+1} \left( k_{t+1}^M + \frac{z_t^M}{p_t} \right) = x_{t+1}^t.$$  \hspace{1cm} (4)

Individuals choose how much to consume in $t$ and $t+1$ and their holding of liquidity and capital ($z_t^M$ and $k_{t+1}^M$, respectively) to maximize their expected welfare, subject to the budget constraints. At time $t$, they get a salary $w_t$, pay the fixed entry cost $\xi$, and invest what remains in liquidity and capital. At the end of $t$, they all get to know their individual types, and a secondary market opens, in which individuals can exchange liquidity and capital at price $p_t$. This means that the consumption of the impatient individuals (at $t$) will be equal to the liquidity held ($z_t^M$), plus the proceedings that they earn from selling their holdings of capital ($p_t k_{t+1}^M$). Similarly, the consumption of the patient individuals (at $t+1$) will be equal to the return on the capital invested in $t$ ($R r_{t+1} k_{t+1}^M$), plus the return on the capital bought in the secondary market with their holding of liquidity ($R r_{t+1} z_t^M / p_t$). $R$ is a scaling parameter, whose importance will be explained soon. Individuals are price-takers, so the total welfare $V^M$ that they enjoy from directly trading in the market is a function of the salary $w_t$, of the return on capital $r_{t+1}$, and of the price $p_t$ in the secondary market, which are determined in equilibrium.

The second channel that individuals can employ to make their investments is through financial intermediation. That is, we assume that the economy is populated by a banking sector, in which a large number of 2-period-lived banks compete in order to maximize their profits. To this end, they collect deposits at zero costs, and invest them in liquidity and
capital.

In order for this portfolio problem to be interesting, we need three assumptions. First, we impose that the banks pay an iceberg cost \( \tau \) on the return on the invested capital \( Rr_t \). Second, the parameter \( R \) is such that \( (1 - \tau)Rr_t > 1 \); if that was not the case, the banks would invest all their capital in liquidity, and roll it over for two periods to finance late consumption.\(^{10}\) Third, there exists a technology through which the banks can liquidate their capital investment before its due maturity, but its return is lower than unity: in this way, the banks are forced to hold liquidity to finance early consumption.\(^{11}\) Finally, we assume free entry, so that bank profits in equilibrium are zero, and we can focus on a representative bank solving the dual problem:

\[
\begin{align*}
\max_{\{c^t_t, c^t_{t+1}, z^B_t, k^B_{t+1}\}} \pi \log(c^t_t) + (1 - \pi)\log(c^t_{t+1}), \quad (5)
\end{align*}
\]

subject to the budget constraints:

\[
\begin{align*}
z^B_t + k^B_{t+1} &= w_t, \quad (6) \\
z^B_t &= \pi c^t_t, \quad (7) \\
(1 - \tau)Rr_{t+1}k^B_{t+1} &= (1 - \pi)c^t_{t+1}, \quad (8)
\end{align*}
\]

and to the participation constraint:

\[
\pi \log(c^t_t) + (1 - \pi)\log(c^t_{t+1}) \geq V^M(w_t, r_{t+1}, p_t). \quad (9)
\]

The representative bank exploits the law of large numbers to choose how to optimally invest the total deposits in liquidity and capital, so as to maximize the depositors’ welfare.\(^{12}\) The total amount of liquidity \( z^B_t \) is employed to pay consumption \( c^t_t \) to \( \pi \) impatient individuals. Similarly, the return on the total capital invested \( (1 - \tau)Rr_{t+1}k^B_{t+1} \) is employed to pay con-

\(^{10}\) Incidentally, we can choose \( R \) to be high enough to ensure that \( c^t_{t+1} \geq c^t_t \), so that our results do not change even when the realization of the idiosyncratic types is private information.

\(^{11}\) Notice that we rule out the possibility for banks to trade claims to capital in the secondary market, as the individuals do. Bencivenga and Smith (1991) extensively comment on the rationale for such a restriction.

\(^{12}\) We choose a logarithmic felicity function for simplicity. All the results presented here would go through if we assumed a standard CRRA functional form.
sumption $c_{t+1}^t$ to $(1 - \pi)$ patient individuals. The assumption made above about the absence of a system of intergenerational transfers calls for the assumption of 2-period-lived banks.\footnote{Qi (1994) develops an overlapping generation model with financial intermediation similar to ours, but in which an infinitely lived bank operates as a social planner and does allow intergenerational transfers. The author shows that in this environment the bank optimally chooses to keep no liquidity in its portfolio. Since our focus is also on the evolution of the bank asset portfolios, we rule out such a case.}

Finally, the deposit contract $\{c_t^t, c_{t+1}^t\}$ must be such that the individuals have no incentive to renege and invest in the market. This is ensured with the imposition of the participation constraint in (9).

With these descriptions in hand, we are ready to define the object of our analysis, that we call “banking equilibrium”:

**Definition 1.** Given an initial value $K_0$, a banking equilibrium is a price vector $\{r_t, w_t, p_t\}$, a bank portfolio strategy $\{z_{t}^B, k_{t+1}^B\}$, and a deposit contract $\{c_t^t, c_{t+1}^t\}$, an individual portfolio strategy $\{z_t^M, k_{t+1}^M\}$, a consumption allocation $\{x_t^t, x_{t+1}^t\}$, a participation choice $\phi_t \in \{0, 1\}$, and production inputs $\{K_t, L_t\}$ for every $t = 0, 1, \ldots$ such that:

- For given prices, the deposit contract and the bank portfolio strategy solve the banking problem in (5);
- For given prices, the consumption allocation and the individual portfolio strategy solve the problem in the market in (1);
- For given prices, the production inputs maximize firm profits;
- Markets clear:

$$K_t = (1 - \phi_t)k_t^B + \phi_t k_t^M,$$

$$L_t = 1.$$
4.3 Timing

To conclude this section, we sum up the timing of actions. In each period $t$, (i) the representative bank negotiates a deposit contract with the newborn generation, stating the amount of consumption goods that it will receive at the end of period $t$ and at $t+1$; (ii) production takes place, with the capital cumulated by the patient individuals born at $t-1$ (which characterizes the state of the economy) and the labor provided by the newborn cohort; (iii) the patient individuals from period $t-1$ consume the return on capital $r_t$, and the young individuals receive the salary $w_t$ and decide whether to renege on the banking contract; (iv) types are publicly revealed; (v) the secondary market for capital opens; (vi) impatient individuals consume.

4.4 Unconstrained Banking Equilibrium

As a benchmark to the main problem, we start our analysis with the characterization of the unconstrained banking equilibrium. That is, we solve the problem in (5), subject only to the budget constraints (6)-(8): the representative bank collects the wages, and invests them in liquidity and capital on behalf of their customers, so as to have the right amount of consumption good at $t$ and $t+1$. Call the expected welfare from this problem $V^B_U$. From the date-$t$ budget constraint, it is easy to see that in the long run it must be the case that liquidity, capital investment and salaries (and as a consequence, production) grow at the same rate, and the liquidity ratio $L_t \equiv z_t^B/w_t$ is constant. We summarize this result in the following:

**Lemma 1.** In the unconstrained banking equilibrium, the liquidity ratio is constant in the long run.

**Proof.** In the text above. ■

The equilibrium of this environment is characterized by the resource constraint of the economy (equivalent to the budget constraint of the bank) and by the Euler equation:

$$u'(w_t(K_t) - K_{t+1}) = (1-\tau)Rr_{t+1}(K_{t+1})u'\left(\frac{(1-\tau)Rr_{t+1}(K_{t+1}K_{t+1})}{1-\pi}\right).$$

This is an implicit difference equation that regulates the evolution of the total capital in the
economy. Notice that in this environment we only have intermediated capital, so the market clearing condition implies that $K_t = k_t^B$. With a logarithmic felicity function, a closed-form solution can be found, in which the liquidity ratio $L_t$ is constant at every point in time (also during the transition), and equal to $\pi$. This is a consequence of the fact that the income and substitution effects cancel out, thus any change in the return on capital does not affect the relative allocation of the initial portfolio.

With relative risk aversion larger than 1, as in the original Diamond and Dybvig’s paper, a closed-form solution of the Euler equation in (10) is instead not possible. Nevertheless, an interesting result emerges from the numerical analysis: the income effect dominates the substitution effect. This means that, as the economy grows and capital increases, $r_t$ decreases and the bank reallocates its portfolio by relatively increasing its investment in capital. That is, the liquidity ratio decreases in the transition, as showed in figure 6. \footnote{We assume a Cobb-Douglas production function $y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$, with $A_t = (1 + \gamma)^t$, and felicity function of the form $u(c) = c^{1-\sigma}/(1 - \sigma)$. The parameters of the model are as follows: $\alpha = .4$, $\gamma = .0193$, $\tau = .1$, $\pi = .15$, $R = 2.5$, $\sigma = 3$.}

5 The Banking Equilibrium

The first step in the characterization of the banking equilibrium defined above is the solution to the problem in the market. We enunciate a key result, that holds true in any case analyzed here, in the following lemma:

**Lemma 2.** In every banking equilibrium, $p_t = 1$.

*Proof.* In Appendix B.

The intuition for this result is simple: given that the individuals are all ex ante equal, the only possible equilibrium is one in which they are also ex ante indifferent between holding liquidity and capital. If that was not the case, everyone would invest only in one of the two assets, and the secondary market would not clear. Despite this indeterminacy, we use the budget constraints in (2)-(4) and still characterize the consumption allocation that individuals gets in the market, which is $x_t^i = w_t - \xi$ and $x_{t+1}^i = R r_{t+1} (w_t - \xi)$. \footnote{We assume a Cobb-Douglas production function $y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$, with $A_t = (1 + \gamma)^t$, and felicity function of the form $u(c) = c^{1-\sigma}/(1 - \sigma)$. The parameters of the model are as follows: $\alpha = .4$, $\gamma = .0193$, $\tau = .1$, $\pi = .15$, $R = 2.5$, $\sigma = 3$.}
This result formally shows where the inefficiency of the market solution comes from: prices do not reflect the willingness to smooth consumption across time, as a consequence of the fact that the economic agents do not have access to a complete set of state-contingent claims.\textsuperscript{15} In the literature on financial intermediation, these considerations provide a rationale for the emergence of a banking equilibrium, equivalent to our unconstrained equilibrium, which dominates the market solution (Diamond and Dybvig, 1983). Here instead the presence of the bank iceberg-type cost and of the market entry cost allows us to rule out this dominance, because it makes the bank compete with an institution (i.e., the market) that in all effects employs a different technology. For the banks to react to the presence of markets, we then need to impose that $V^M > V^B$, i.e. that individuals would rather invest in the market unless the banks change their behavior. These considerations lead us to impose the following constraint:

\textbf{Assumption 1.} $w_t > \frac{\xi}{1 - (1 - \tau)^{1 - \pi}} \equiv w^*$. 

To find the solution to the constrained problem, we plug the budget constraints (6)-(8) and the market allocation \{$x_t^t, x_t^{t+1}$\} derived above into the participation constraint, and assume that, if the individuals are indifferent between staying with their banks and going to the market, they will choose the first option (hence $\phi_t = 0$), so that by market clearing $K_t = k_t^B$. Therefore, we derive the following:

\textbf{Proposition 1.} \textit{In the constrained banking equilibrium, the law of motion of capital is given by:}

$$[w_t(K_t) - K_{t+1}]^\pi K_{t+1}^{1-\pi} = \Psi [w_t(K_t) - \xi],$$

\textsuperscript{15}As highlighted by Wallace (1988), the restriction on the individual access to state-contingent claims naturally arises in every model in which financial intermediation plays a role. Assume that such a market exists: individuals pay $q_1$ and $q_2$ to buy two assets that pay 1 unit of consumption at the end of period $t$ if they turn out to be impatient and patient, respectively (if they turn out to be patient, individuals store that unit of consumption to the following period). The maximization problem reads:

$$\max \pi u(c_t) + (1 - \pi) u(c_{t+1}),$$

subject to:

$$q_1 \pi c_t^t + q_2 (1 - \pi) c_{t+1} = w_t - \xi$$

By a no arbitrage condition, it must be the case that $R r_{t+1}/p_t = 1/q_2$, and that $q_1 = 1$. Hence, from the optimality conditions it comes out that the price in the secondary market should be:

$$p_t = R r_{t+1} q_2 = R r_{t+1} \frac{u'(c_{t+2})}{u'(c_{t+1})}$$
where:

\[ \Psi \equiv \frac{\pi^\tau (1 - \pi)^{1-\pi}}{(1 - \tau)^{1-\pi}}. \]

Proof. In the text above.

This is an implicit difference equation characterizing the amount of capital in \( t + 1 \), as a function of the current state of the economy (the amount of capital \( K_t \)), of the idiosyncratic demand for liquidity \( \pi \), and of the technological parameters \( \tau \) and \( \xi \). In figure 7, we show a typical pattern for the evolution of capital and liquidity ratio in this environment.\(^{16}\)

From this proposition, we can derive a series of important corollaries that characterize the evolution of the bank portfolio allocation. We enunciate and comment them all together:

**Corollary 1.** In the constrained banking equilibrium:

1. The liquidity ratio \( L_t \) is strictly larger than \( \pi \);
2. The liquidity ratio \( L_t \) is decreasing;
3. Asymptotically, there exists no constrained banking equilibrium.

Proof. In Appendix B.

The intuition for these results is the following. Once the market channel is available, the bank knows that it cannot compete with it in offering services to the patient individuals, due to the technological/regulatory constraint that prevents the bank from fully enjoying the yields on its capital investment. This means that the bank optimally chooses to ensure participation by focusing on the impatient individuals, and invests relatively more than necessary in liquidity. Incidentally, notice that here the cross-subsidization of the early consumption emerges in equilibrium, but not as a consequence of high risk aversion as in the original Diamond and Dybvig’s paper, but as a consequence of the interplay between technological constraints and competition from the market.

As the economy grows, the bank “comparative advantage” in the provision of liquidity becomes less and less important, because the fixed cost to enter the market becomes more

\(^{16}\)We make the same assumptions as in figure 6 regarding the parameterization of the numerical simulation. See note 14 for the details.
and more negligible. Hence, in order to ensure participation, the bank is forced to increase its investment in capital relative to liquidity, i.e. to lower its liquidity ratio. At infinity, individuals in the market do not “break the buck”, i.e. the return on their portfolio is such that they get at least as much as their initial investment (in present value), regardless of the realization of their idiosyncratic type. To ensure participation, the bank then needs to provide the same type of service, so it invests an amount $\pi w_t$ of deposits in liquidity, and pay $c_t = w_t$ to the impatient depositors. In this way, it is left with an amount of capital equal to $(1 - \pi)w_t$, which is not enough to avoid breaking the buck with the patient depositors, since $c_{t+1}/R_{t+1} = (1 - \tau)$ is less than 1.

6 Econometric Analysis

The main result of the previous sections highlights a mechanism to explain the decreasing trend in the bank liquidity ratios that we observe in the data: banks lower the relative amount of liquid assets in their portfolios to keep up with the competitive pressure coming from other channels that their potential customers can employ to hedge against idiosyncratic uncertainty. The aim of this section is to go back to the data, and test the validity of this rationale by estimating the following equation:

$$\log(\text{liqratio}_{it}) = \beta_0 + \beta_1 \text{SMP}_{it} + \beta_2 \log(\text{RGDP}_{it}) + \beta_3 X_{it} + \mu_i + \mu_t + \epsilon_{it}.$$  

We want to exploit the panel dimension of the database of bank relative liquidity that we briefly analyzed in section 3. Thus, our dependent variable $\text{liqratio}$ is defined as the ratio of domestic currency holdings and deposits with the monetary authorities to claims on other governments, nonfinancial public enterprises, the private sector, and other banking institutions. As we mentioned earlier, this is a commonly-used indicator of the liquidity available to the banks to meet expected and unexpected demands for cash.\(^{17}\)

As independent variable, we instead need a real-world equivalent of the exogenous fixed cost $\xi$, which again we see as an intrinsic transaction cost or institutional impediment that

\(^{17}\)Source: World Bank. More details are available in Appendix ??.
affects the possibility of accessing the alternative market channel. We find one in the database of financial reforms produced by the IMF (Abiad et al., 2008), that covers various regulatory changes affecting the financial system (credit controls, interest rate controls, entry barriers, state ownership, etc.) in around 90 countries for the period 1970-2008. In particular, we use the variable labeled “Securities market policy” ($SMP$), which is an index summarizing all the regulatory interventions that make the access to security markets easier.$^{18}$ The index takes value 0 if a securities market does not exist, and increases in discrete steps up to 3, as the system becomes fully liberalized. The prediction from our theory says that, as the access to the market becomes easier (i.e., the index of securities market policy increases) the bank liquidity ratios should decrease.

We also add to the analysis a series of controls. First, since the theory predicts the liquidity ratios to decrease with economic development, we add as regressor the real GDP per capita. Second, since the process of financial liberalization almost never pertained only one part of the financial system, but generally came as a wave of complex regulation (and deregulation) of various aspects of the system itself, we want to test whether the evolution of our securities market policy index does not reflect some broader regulatory changes. Hence, we include as controls other measures of financial liberalization: the indices of credit controls and banking sector supervision drawn from the same IMF database, and the Chinn-Ito measure of openness in capital account transactions (Chinn and Ito, 2008). Finally, to take care of any other country-specific characteristic that might influence the bank liquidity ratio, we estimate a fixed effect model, and add time dummies:

We report our results in table 1. The coefficient on securities market policy is significant and has the expected sign (column 1): a one-unit increase in the index leads to a drop in the bank liquidity ratio of around 21.8 per cent. The inclusion of real GDP as a regressor affects neither the significance nor the magnitude of the coefficient in a notable way (column 2). Instead, once we control for other forms of financial liberalization (columns 3 to 5), the coefficient slightly falls, while still being highly significant: at its lowest estimate, a one-unit

---

$^{18}$According to the authors, these include “[...] the auctioning of government securities, establishment of debt and equity markets, and policies to encourage development of these markets, such as tax incentives or developments of depository and settlement systems”.
increase in the index leads to a drop in the bank liquidity ratio of around 14.9 per cent.

In order to further analyze the connection between securities market liberalization and the bank liquidity ratio, we split our indicator into four dummy variables, corresponding to the four values that it takes in the data. Then, we study the nonlinear effects of financial liberalization both directly onto the bank liquidity ratio and indirectly, through the interaction of the dummies with real GDP. The results are reported in table 2. The first two columns show that the negative average effect found in the previous table kicks in only when an extensive process of securities market liberalization is implemented: moving the index from 0 to 2 would lead to a drop in the bank liquidity ratio of almost 24 per cent, while moving from 0 to 3 would lead to a drop of around 35 per cent. The introduction of very mild reforms (i.e. moving the index from 0 to 1) has instead an insignificant effect, and tends to increase the bank liquidity ratio. These results find confirmation when we interact the dummies with real GDP: moving the index from 0 to 2 would increase the negative effect of GDP on the liquidity ratio by around 3.8 percentage points, while moving from 0 to 3 would increase the negative effect by more than 5.7 per cent. Moving from 0 to 1 would instead have an effect not statistically distinguishable from nihil.

7 Financial Lotteries

In this section, we go back to our theory, and relax the hypothesis of discrete investment choice. Specifically, we introduce the possibility for the individuals to play a financial lottery, i.e. we assume that each newborn agent, after receiving her salary but before investing it, plays an individual-specific lottery, in which with probability \( \phi_t \in [0, 1] \) she goes to the market, and with probability \( 1 - \phi_t \) she makes a deposit into the bank. Thus, in all effects the individuals are randomizing between the bank and the market, as they do between working and not working in the seminal paper by Rogerson (1988). The law of large number holds, so \( \phi_t \) also represents the participation rate in the market.

In order to insure against the income risk generated by the lottery, the individuals purchase an insurance in a perfectly competitive insurance sector, in which the relative price of the market outcome versus the bank outcome is \( \phi_t / (1-\phi_t) \). More formally, the investment problem
reads:

\[
\max_{\{X_t^t, X_{t+1}^{t+1}\} \cap \{C_t^t, C_{t+1}^{t+1}, \phi_t\}} \phi_t [\pi u(X_t^t) + (1 - \pi)u(X_{t+1}^{t+1})] + (1 - \phi_t) [\pi u(C_t^t) + (1 - \pi)u(C_{t+1}^{t+1})],
\]

subject to:

\[
\phi_t X_t^t + (1 - \phi_t) C_t^t \leq \phi_t (w_t - \xi) + (1 - \phi_t) c_t^t,
\]

\[
\phi_t X_{t+1}^{t+1} + (1 - \phi_t) C_{t+1}^{t+1} \leq \phi_t Rr_{t+1} w_t - \xi + (1 - \phi_t) c_{t+1}^{t+1},
\]

and \(0 \leq \phi_t \leq 1\). Individuals choose the probability of investing in the market and in the bank, and how much to consume in the two following periods in the two cases: \(\{X_t^t, X_{t+1}^{t+1}\}\) if they go to the market, and \(\{C_t^t, C_{t+1}^{t+1}\}\) if they go to the bank. The expected liquid resources available at the end of period \(t\) are on the right hand side of (13): with probability \(\phi_t\) the individuals go to the market and get \((w_t - \xi)\), and with probability \((1 - \phi_t)\) they go to the bank and get \(c_t^t\) (fixed by the banking contract). Similarly, the right hand side of (14) gives the expected resources available in period \(t+1\). These are used to purchase the bundle \(\{X_t^t, X_{t+1}^{t+1}\}\) at price \(\phi_t\), and the bundle \(\{C_t^t, C_{t+1}^{t+1}\}\) at price \((1 - \phi_t)\), on the left hand sides of (13) and (14).

The hypothesis of perfect competition in the insurance market allows perfect insurance against the income fluctuations generated by the lottery, hence:

**Lemma 3.** In each equilibrium with financial lotteries, \(X_t^t = C_t^t\) and \(X_{t+1}^{t+1} = C_{t+1}^{t+1}\).

**Proof.** In Appendix B.

This result greatly simplifies the problem, that now reads:

\[
\max \pi u(C_t^t) + (1 - \pi)u(C_{t+1}^{t+1}),
\]

subject to:

\[
C_t^t \leq \phi_t (w_t - \xi) + (1 - \phi_t) c_t^t,
\]

\[
C_{t+1}^{t+1} \leq \phi_t Rr_{t+1} (w_t - \xi) + (1 - \phi_t) c_{t+1}^{t+1},
\]
and $0 \leq \phi_t \leq 1$. We focus on an interior solution. We provide the characterization of the equilibrium of this economy in the following proposition:

**Proposition 2.** The allocation of the banking equilibrium with financial lotteries satisfies the optimality conditions:

\[
\pi u'(C^t_t) \left[ c^t_t - (w^t_t - \xi) \right] = (1 - \pi) u'(C^t_{t+1}) \left[ Rr^t_{t+1}(w^t_t - \xi) - c^t_{t+1} \right],
\]

\[
u'(c^t_t) = (1 - \tau) Rr^t_{t+1} u'(c^t_{t+1}).
\]

**Proof.** In Appendix B.

These two expressions, together with the budget constraints and the price equations, fully characterize the banking equilibrium with financial lotteries. The first equation regulates the choice between investing in the market or in the bank. Each individual optimally chooses her own participation rate $\phi_t$ so as to equalize the expected marginal costs and benefits of her decision. The expected marginal cost is on the left hand side of (18): with probability $\pi$, the individual turns out impatient and, if she invests one marginal unit in the market, she loses the extra utility coming from the cross-subsidization offered by the bank ($[c^t_t - (w^t_t - \xi)]$).

The expected marginal benefit of participating in the market is instead on the right hand side of (18): with probability $(1 - \pi)$, the individual turns out patient and, if she invests one marginal unit in the market, she gets the extra return $[Rr^t_{t+1}(w^t_t - \xi) - c^t_{t+1}]$. The second equation instead regulates the portfolio allocation of the banks: they choose the contract $\{c^t_t, c^t_{t+1}\}$ so as to satisfy an Euler equation, as in the unconstrained banking equilibrium. This also means that, having depositors gained a further degree of freedom through the possibility to randomize, the bank can allocate its capital by equalizing early and late consumers at the margin. As a consequence, the evolution of its portfolio of assets will again depend on the interplay of income and substitution effects.

The equilibrium dynamics of this economy is driven by the fact that the participation rate affects the prices (specifically, the return on capital), which affect the bank portfolio, which in turn affects the participation rate back. This means that, in general, a closed form solution for the system cannot be found. Thus, we characterize the equilibrium in a numerical
example. We impose a CRRA functional form for the felicity function $u(\cdot)$, with relative risk aversion equal to 3. The probability of being impatient is $\pi = 0.15$, roughly equivalent to the average bank liquidity ratio among the U.S. financial businesses in our sample. The production function is assumed to be a standard Cobb-Douglas $y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$, with the exogenous labor-augmenting technology being the deterministic process $A_t = A_0 (1 + \gamma)^t$. We choose $\alpha = 0.4$, $A_0 = 0.1$ and $\gamma = 0.0193$, equal to the average growth rate of U.S. GDP in the period 1970-2010. Finally, as far as the institutional parameters are concerned, we pick $R = 2.5$, and $\tau = \xi = 0.1$.

The results are reported in figure 8. As the economy grows, the fixed cost $\xi$ to access the market becomes more and more negligible, hence the participation rate increases (panel A). The increasing capital accumulation leads to a decreasing return on capital, that affects the bank portfolio in the expected way: the income effect dominates the substitution effect (because agents are more risk averse than log), hence a decreasing return on capital pushes banks to lower their liquidity ratios, i.e. the per capita amount of capital invested by the banks gets relatively bigger than the amount of liquidity (panel B).

In the last three panels of figure 8, we analyze the evolution of private credit and equity capital, defined as the total amount of capital provided by the banks and by the market, respectively. By definition, the first one is equal to $K_t^B = (1 - \phi_t)k_t^B$, and the second is equal to $K_t^M = \phi_t (1 - \pi) (w_t - \xi)$. Clearly, the per capita bank capital $k_t^B$, the salary $w_t$ and the total amount of capital in the economy $K_t$ must grow at the same rate of GDP. Therefore, equity capital grows faster than total capital and GDP because of the increasing participation rate (panel C and E). On the contrary, private credit grows slower than GDP, and as a consequence private credit as a percentage of GDP falls (panel D).\footnote{This last result is at odds with some empirical observations, finding that private credit and equity capital are often complementary (Beck et al., 2000), but is an obvious consequence of the increasing participation rate and of the fact that the two channels (banks and markets) here compete for the collection of individual savings.} We can conclude that the model is able to replicate, at least qualitatively, the three stylized facts highlighted at the beginning of the paper: increasing market participation, decreasing bank liquidity ratio, and increasing importance of equity capital.
8 Concluding Remarks

In the present paper, we develop a theory of finance and growth to jointly explain (i) the decreasing liquidity ratio of financial institutions and (ii) some well-known stylized facts on the evolution of the architecture of the financial system during the process of economic development. We characterize the equilibrium of a growth model, in which banks and markets play a key role, by competing for the provision of insurance services and fruitful investment opportunities to the savers, and show that the competitive pressure from the markets pushes banks to lower their liquidity ratios. We also find evidence of such a mechanism being into place in the real world: a one-unit increase in the index of securities market liberalization (that we take as a proxy for the market entry costs) leads to a drop in the bank liquidity ratio between 15 and 22 percentage points.

To conclude, we believe that our work is interesting because it allows us to have a first look at a class of models that, despite its simplicity, can be used in the future to answer very different questions. For example, we think of studying optimal Ramsey taxation, and find the optimal mix of taxes on capital gains and on interests on deposits. We can also extend this environment by formally modelling financial innovation: for example, we can endogenize the bank technological/regulatory constraint $\tau$ and the market entry cost $\xi$ by introducing search in the capital markets. We can also run a growth accounting exercise, to quantitatively evaluate the contribution of intermediaries and markets on the observed growth rate of GDP, or feed the model with high frequency shocks and study its business cycle properties. These two last extensions would require the abandonment of the current OLG structure, that is not well-suited for a quantitative analysis, in favor of a representative agents set-up. We leave all these considerations for future research.
References


Appendices

A Data Appendix

- **Bank Supervision**: Index of banking sector supervision. It is the sum of three indices, normalized on a 0-3 scale. It includes: a dummy indicating with 1 when a country adopted a capital adequacy ratio based on the Basel standard; an index of independence of the banking supervisory agency, on a 0-2 scale; an index of effectiveness of banking supervision, on a 0-2 scale. Source: IMF (Abiad et al., 2008).

- **Credit Controls**: Index of credit controls and reserve requirements. It is the sum of three indices, normalized on a 0-3 scale. It includes: an index of restrictiveness of reserve requirements, on a 0-2 scale; a dummy indicating with 1 if mandatory credit allocations to certain sectors are eliminated or do not exist; a dummy indicating with 1 when the mandatory requirement of credit allocation at subsidized rates is eliminated or banks do not have to supply credits at subsidized rates. Source: IMF (Abiad et al., 2008).

- **Financial Openness**: The Chinn-Ito index of financial openness. It is based on the binary dummy variables that codify the tabulation of restrictions on cross-border financial transactions reported in the IMF’s Annual Report on Exchange Arrangements and Exchange Restrictions. The data are available for the period 1970 to 2010, for 182 countries. Source: Chinn and Ito (2008).

- **liqratio**: Bank liquid reserves to bank assets ratio (%). It is the ratio of domestic currency holdings and deposits with the monetary authorities to claims on other governments, nonfinancial public enterprises, the private sector, and other banking institutions. Data are available for the period 1947 to 2003, for 106 countries. Source: World Development Indicators, World Bank, and International Financial Statistics, IMF.

- **Real GDP**: PPP Converted GDP Per Capita (Chain Series), at 2005 constant prices. Data are available for the period 1950 to 2009, for 189 countries. Source: Penn World Tables (Heston et al., 2011).
- **Securities Market Policy**: Index of securities market liberalization. It takes value 0 if a securities market does not exist; 1 when a securities market is starting to form with the introduction of auctioning of T-bills or the establishment of a security commission; 2 when further measures have been taken to develop securities markets (tax exemptions, introduction of median and long-term government bonds in order to build the benchmark of a yield curve, policies to develop corporate bond and equity markets, or the introduction of a primary dealer system to develop government security markets); 3 when further policy measures have been taken to develop derivative markets or to broaden the institutional investor base by deregulating portfolio investments and pension funds, or completing the full deregulation of stock exchanges. Source: IMF (Abiad et al., 2008).
B Proofs

Proof of Lemma 2. We prove the lemma by contradiction. Assume that $p_t > 1$. Then, liquidity is dominated by capital. This means that impatient individuals would like to sell their holdings of capital in the secondary market, but there would be nobody there to buy, and the price would go to zero. This is evidently a contradiction. Assume now that $p_t < 1$, so that, contrary to the previous case, capital is dominated by liquidity. This means that patient individuals would like to use their holdings of liquidity to buy capital, but there would be no seller in the secondary market, and the price would go to infinity. This is again a contradiction.

Proof of Corollary 1. We prove the first part of the corollary by contradiction. Assume that the liquidity ratio $\leq \pi$. Divide both sides of (11) by $w_t$ and derive:

$$\mathcal{L}_t^\pi (1 - \mathcal{L}_t)^{1-\pi} = \Psi \left[ 1 - \frac{\xi}{w_t} \right]. \quad (20)$$

Define:

$$\Phi \equiv \frac{\mathcal{L}_t^\pi (1 - \mathcal{L}_t)^{1-\pi}}{\pi^\pi (1 - \pi)^{1-\pi}} \leq 1,$$

because $\mathcal{L}_t \leq \pi$. Rearranging (20), we obtain:

$$w_t = \frac{\xi}{1 - \Phi (1 - \pi)^{1-\pi}} \leq w^*,$$

where we also used the definition of $\Psi$. This contradicts assumption 1, thus it cannot be true.

Now for the second part of the corollary. By definition, the liquidity ratio is decreasing if $K_{t+1}$ grows at a higher rate than $w_t$. Take the total differential of $K_{t+1}$, i.e. $\frac{dK_{t+1}}{dt} = \frac{\partial K_{t+1}}{\partial w_t} \frac{dw_t}{dt}$. Then, the instantaneous growth rate of capital is:

$$g^K \equiv \frac{dK_{t+1}}{dt} \frac{K_{t+1}}{K_{t+1}} = \frac{\partial K_{t+1}}{\partial w_t} \frac{dw_t}{dt} \frac{w_t}{K_{t+1}} \frac{1}{\frac{\partial K_{t+1}}{\partial w_t}} \frac{w_t}{K_{t+1}} g^w = \frac{\partial K_{t+1}}{\partial w_t} \frac{w_t}{K_{t+1}} g^w.$$
Hence, $g^K > g^w$ if $\frac{\partial K_{t+1}}{\partial w_t} \frac{w_t}{K_{t+1}} > 1$. Using the implicit function theorem, this becomes:

$$
\Psi \frac{w_t}{K_{t+1}} > (1 - \pi) \Psi \frac{w_t - \xi}{K_{t+1}} + \pi \left( \frac{K_{t+1}}{w_t - K_{t+1}} \right)^{1-\pi} \frac{w_t}{K_{t+1}} - 1
$$

$$
0 > -\Psi \frac{\xi}{K_{t+1}^{\pi}} - \pi \Psi \frac{w_t - \xi}{K_{t+1}} + \pi \left( \frac{K_{t+1}}{w_t - K_{t+1}} \right)^{1-\pi}
$$

$$
0 > -\Psi \frac{\xi}{K_{t+1}}
$$

which is always true.

Finally, we move to the third part of the corollary. At infinity, the right hand side of (20) is equal to $\Psi$. Define:

$$
f(L_t) \equiv L_t^\pi (1 - L_t)^{1-\pi} - \Psi.
$$

Notice that $f(L_t)$ is strictly concave, $f(0) = -\Psi = f(1)$, $L_t^\ast \equiv \max f(L_t) = \pi$ and:

$$
f(L_t^\ast) = \pi^\pi (1 - \pi)^{1-\pi} \left[ 1 - \left( \frac{1}{1 - \tau} \right)^{1-\pi} \right] < 0
$$

because $\tau < 1$. Thus, there is no solution. ■

**Proof of Lemma 3.** Attach multipliers $\lambda_{1t}$ and $\lambda_{2t}$ to (13) and (14), respectively. The first order conditions give $u'(X_t^t) = \lambda_{1t} = u'(C_t^t)$ and $u'(X_{t+1}^t) = \lambda_{2t} = u'(C_{t+1}^t)$, which simplify to the result. ■

**Proof of Proposition 2.** Attach multipliers $\lambda_{1t}$ and $\lambda_{2t}$ to (16) and (17), respectively. The first order conditions read:

$$
C_t^t : \quad \pi u'(C_t^t) = \lambda_{1t},
$$

$$
C_{t+1}^t : \quad (1 - \pi) u'(C_{t+1}^t) = \lambda_{2t},
$$

$$
\phi_t : \quad \lambda_{1t} \left( c_t^t - (w_t - \xi) \right) = \lambda_{2t} \left[ Rr_{t+1}(w_t - \xi) - c_{t+1}^t \right].
$$

Plugging (21) and (22) into (23), we derive (18). For the characterization of the banking equilibrium, see section 4.4. ■
### Table 1: Bank Liquidity Ratio and Market Participation: Direct Effect

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t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Country and year effects included.
Table 2: Bank Liquidity Ratio and Market Participation: Nonlinear and Interacted Effects

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</tr>
<tr>
<td>SMP==3</td>
<td>-0.430**</td>
<td>-0.429**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.238)</td>
<td>(-2.252)</td>
<td></td>
</tr>
<tr>
<td>(Log of) Real GDP</td>
<td>-0.693***</td>
<td>-0.661***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.271)</td>
<td>(-3.129)</td>
<td></td>
</tr>
<tr>
<td>(Log of) Real GDP * SMP==1</td>
<td>0.0160</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.394)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Log of) Real GDP * SMP==2</td>
<td>-0.0382**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.346)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Log of) Real GDP * SMP==3</td>
<td>-0.0571**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.517)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.533***</td>
<td>3.983**</td>
<td>3.743**</td>
</tr>
<tr>
<td></td>
<td>(-8.918)</td>
<td>(2.350)</td>
<td>(2.141)</td>
</tr>
</tbody>
</table>

Observations | 716 | 716 | 716 |
Number of countries | 45 | 45 | 45 |
R-squared | 0.193 | 0.206 | 0.212 |

T-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1.
Country and year effects included as controls.
Other financial liberalization variables included as controls.
Figure 1: Liquid assets of the U.S. financial businesses as a percentage of total liabilities. Liquid assets are defined as the sum of vault cash, reserves and Treasury securities. Source: Flow of Funds Accounts of the United States.

Figure 2: Liquid assets of the U.S. financial businesses as a percentage of total financial assets. Liquid assets are defined as the sum of vault cash, reserves and Treasury securities. Source: Flow of Funds Accounts of the United States.
Figure 3: Relative price of liquidity with respect market prices. The relative price of liquidity is calculated as the ratio between the compounded returns (base date: 1940) on the S&P500 and on a 10-year U.S. Treasury bond. Source: Aswath Damodaran, available at: http://pages.stern.nyu.edu/~adamodar/.

Figure 4: Sterling liquid assets relative to total asset holdings of UK banking sector. Source: Bank of England.
Figure 5: Bank liquid reserves to bank assets ratio (%).

Figure 6: The evolution of the total capital and of the liquidity ratio in the Unconstrained Banking Equilibrium, with CRRA felicity function and $RRA > 1$. 
Figure 7: The evolution of the total capital and of the liquidity ratio in the Constrained Banking Equilibrium.
Figure 8: Equilibrium with Financial Lotteries