Two-moment decision model for location-scale family with background asset

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Abstract: This paper studies the impact of background risk on the indifference curve. We first study the shape of the indifference curves for the investment with background risk for risk averters, risk seekers, and risk-neutral investors. Thereafter, we study the comparative statics of the change in the shapes of the indifference curves when the means and the standard deviations of the returns of the financial asset and/or the background asset change. In addition, we draw inference on risk vulnerability and investment decisions in financial crises and bull and bear markets.

Key Words : Mean-variance model, indifference curve, location-scale family, background risk, utility function, risk aversion, risk seeking.

JEL Classification : C0, D81, G11

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1 Introduction

The mean-variance (MV) analysis from the pioneer work of Markowitz (1952) has been widely used in economics and finance to analyze how people make choices among risky assets. Using the means and variances of prospects' returns as the criteria for portfolio investment, Markowitz (1959) presents the critical line algorithm for computing the efficient frontier of portfolios to obtain the highest possible expected return, given their level of standard deviation or risk.[1] He also demonstrates that if the ordering of alternatives is to satisfy the von Neumann-Morgenstern (1944) (NM) axioms of rational behavior, a quadratic (NM) utility function is consistent with an ordinal expected utility function that depends solely on the mean and variance of the return.

There are three major types of people: risk averters, risk neutrals and risk seekers. Their corresponding utility functions are concave, linear, and convex; all are increasing functions. Tobin (1958) develops the MV selection rules to state properties of the indifference curves for risk averters, risk neutrals and risk seekers. Thereafter, Feldstein (1969), Hanoch and Levy (1969), and others comment that the MV criterion is applicable when the decision maker's utility function is quadratic and the probability distribution of return is normal. Meyer (1987) and others extend the MV theory to include general utility functions and a comparison between distributions that differ by location and scale parameters, while Wong and Ma (2008) generalize the results to a multivariate setting. The advantage of applying the mean-variance analysis is that it is simple and easy to interpret. For example, the mean-variance analysis could represent preferences on investment as functions of the mean and the variance or standard deviation of final wealth. This model setup could be used in portfolio selection (Fishburn and Porter, 1976), firm behavior (Sandmo, 1971), insurance demand (Meyer, 1992), hiring under uncertainty (Feder, 1977), linear risk tolerance (Wagener, 2005) and many others.

Another important area of work is to study the impact of a background risk. For example, Kihlstrom et al. (1981) study whether the conditions for more risk aversion are preserved under random background wealth. Eeckhoudt et al. (1996) examine conditions on the expected utility function under which some changes in the distribution of the background risk lead to more risk-averse behavior towards endogenous risk. Caballe and Pomansky (1997) have found conditions under which the introduction of an additional

[1] Recently, Bai et al. (2009) have developed new bootstrap-corrected estimators of the optimal returns for the Markowitz mean-variance optimization.

independent background risk induces more (or less) risk aversion when preferences display mixed risk aversion. Recently, Alghalith et al. (2012) develop a stochastic factor model with an additive background risk and thereafter developed a dynamic model of simultaneous multiplicative background risk and additive background risk.

Some studies link the mean-variance model with background risk. For example, Eichner and Wagener (2003a) employ the notion of variance vulnerability to characterize the effects of changing an independent background risk in a generic decision problem. Eichner and Wagener (2003b) elucidate the equivalence of the equivalence of decreasing absolute prudence and the concavity of utility as a function of mean and variance. Eichner and Wagener (2009) analyze the comparative static effects under uncertainty when a decision maker has mean-variance preferences and faces a generic, quasi-linear decision problem with both an endogenous risk and a background risk. Analyzing risk taking in the presence of a dependent background risk, Eichner and Wagener (2012) characterize the comparative statics of changes in the distribution and dependence structure of the background risk and present the necessary and sufficient restrictions on preferences.

This paper follows Eichner and Wagener (2003a, 2003b, 2009, 2012) and others in linking the mean-variance model with background risk by developing some properties of indifference curves for risk averters and risk seekers on their investment with background risk and examining the impact of background risk on the indifference curve. In this paper, we consider the background risk to be unpleasant as well as pleasant. Many studies support this consideration. For example, Guiso et al. (1996) find that households facing uninsurable income risks reduce their holdings of risky assets, while Arrondel et al. (2010) document a negative correlation between earnings risks and households' willingness to hold risky financial assets. These findings support our contention that the background risk can be unpleasant as well as pleasant. For example, the findings from Guiso et al. (1996) could imply that households might hold more risky assets when they have insurable income, whereas the findings from Arrondel et al. (2010) could imply that households are willing to hold more risky financial assets when earnings risks are smaller.

In this paper we study the impact of background risk on the indifference curve. We find that similar to the shapes of the indifference curves on investment without background risk, as shown in Tobin (1958) and others, the indifference curves on the investment with background risk is convex upward for risk averters, concave downward for risk seekers, and horizontal for risk-neutral investors. We then find that when an agent displays decreasing (constant, increasing) absolute risk aversion, an increase in the mean of either the financial asset or the background risk can yield a decrease (no change, increase) in the slope of the indifference curve. In addition, we find that when we do not take the restriction of constant expected utility into account, the necessary and sufficient condition for an increase in the slope upon the increase of the standard deviations of either the financial asset or background risk is that the agent displays decreasing (constant, increasing) relative risk aversion. When we impose the condition of constant expected utility, we conclude that an agent will increase the slope of the indifference curve upon an increase in the variance of returns from the financial asset and/or background risk if the agent is risk averse. Otherwise, the slope will decrease upon an increase in the variance of returns from the financial asset and/or background risk.

In addition, we demonstrate the applicability of the theory developed in this paper by drawing some inferences on risk vulnerability and investment decisions in financial crises and bull and bear markets. We find that in order to maintain the same mean return, an increase (decrease) in the variance of an exogenous, independent background risk induces the agent to choose a lower (higher) level of risky activities, regardless of whether the agent is risk averse or risk seeking. We also find that in order to keep the same mean return, investors will invest in less-risky assets during financial crises and invest in more-risky assets during bull markets, regardless of whether they are risk averters or risk seekers.

The remainder of the paper is organized as follows. In the next section we develop the theory to study the shapes of the indifference curves on investment with background risk for risk averters, risk seekers, and risk neutrals. Section 3 applies the theory developed in Section 2 to draw inference on risk vulnerability and investment decisions in financial crises and bull and bear markets. The last section wraps up the paper by providing some discussions and some suggestions for further research.

2 The Theory

Before we develop the theory for the indifference curve, we first state the definition of utility functions for risk averters and risk seekers as follows:

Definition 2.1 *U_j^A and U_j^D are the sets of twice differentiable utility functions u such* *that*

$$
\begin{aligned}
U_2^A &= \{u : (-1)^i u^{(i)} \leq\ 0\ ,\ i = 1, 2\} \quad \textit{and} \\
U_2^D &= \{u : u^{(i)} \geq (>)\, 0\ ,\ i = 1, 2\}\ ,
\end{aligned}
$$

where $u^{(i)}$ *is the* i^{th} *derivative of* u*.*

We note that investors in U_2^A are risk averse, while investors in U_2^D are risk seeking. If investors with utility u belong to both U_2^A and U_2^D, then they are risk neutral. In addition, we note that choosing between F and G in accordance with a consistent set of preferences will satisfy the von Neumann-Morgenstern (1944) consistency properties. Accordingly, F is (strictly) preferred to G, or equivalently, Y is (strictly) preferred to Z if $\Delta Eu \equiv E[u(F)] - E[u(G)] \equiv E[u(Y)] - E[u(Z)] \geq (>) 0$, where $E[u(F)] \equiv E[u(Y)] \equiv \int_a^b u(x) dF(x)$ and $E[u(G)] \equiv E[u(Z)] \equiv \int_a^b u(x) dG(x)$.

Let X_0 be a seed random variable with zero mean and unit variance and the location-scale family $\mathcal{D}_{X_0}$ generated by X_0 is

$$
\mathcal{D}_{X_0} = \{ X \mid X = \mu_X + \sigma_X X_0 \quad , \quad -\infty < \mu_X < \infty \quad , \quad \sigma_X > 0 \} . \tag{2.1}
$$

Considering $X = \mu_X + \sigma_X X_0$ to be the return on an asset, such as a financial asset, with mean μ_X and standard deviation σ_X, academics and practitioners are interested in studying the shapes of the indifference curves for risk averters, risk seekers, and risk neutrals possessing utility u who invest in an asset with return X when μ_X and σ_X vary. To do so, Meyer (1987) and others study the expectation of utility u on the random variable X such that

$$
U(\sigma_X, \mu_X) = E[u(X)] = \int_a^b u(\mu_X + \sigma_X t)\, d\, F_{X_0}(t)\ ,
$$

where F_{X_0} is the distribution function of X_0. $U(\sigma_X, \mu_X)$ is used to represent the expected utility $E[u(X)]$ to state the set of alternatives for different combinations of σ_X and μ_X.

Since it is well-known that there could exist background risk in one's investment as discussed in the Introduction, in this paper we are interested in studying the shapes of indifference curves for risk averters, risk seekers, and risk neutrals possessing utility u who invest in an asset with return X and there exists a background risk with return B such that $Y = X + B$. Before we develop the theory, we first make the following assumption:

Assumption 2.1 *Let $Y = X + B$ in which $X = \mu_X + \sigma_X X_0$ be a random variable generated by the location-scale family $\mathcal{D}_{X_0}$ stated in (2.1), let $B = \mu_B + \sigma_B B_0$ with σ_B, B_0 to be a seed random variable of mean zero and unit variance, and let X_0 and B_0 be independent. Then, there exists a location-scale family, $\mathcal{D}_{Y_0}$, generated by a seed variable, say, Y_0 such that $Y = \mu + \sigma Y_0$, the mean and the standard deviation of Y_0 are 0 and 1, respectively, and $\sigma^2 = \sigma_X^2 + \sigma_B^2$ and $\mu = \mu_X + \mu_B$ are, respectively, the variance and mean of Y. In addition, we suppose that F_{X_0} is the distribution function of X_0, F_{B_0} is the distribution function of B_0, and let F be the distribution function of Y with support $[a, b]$.*

We note that some studies, for example, Eichner and Wagener (2003a,b, 2009, 2012), treat a background risk as an unpleasant exogenous risk, and thus, they let the mean of the background risk equal zero. However, as discussed in the Introduction, we believe that the background risk can be unpleasant (as in the bear market) as well as neutral or pleasant (as in the bull market), and thus, in this paper we assume the mean of the background risk could be positive, zero, or negative. It is trivial that $\mathcal{D}_{Y_0}$ in Assumption 2.1 is not an empty set. For example, if both X_0 and $B_0 \sim N(0, 1)$ or both X_0 and B_0 are distributed as gamma distributions, then there will exist a location-scale family $\mathcal{D}_{Y_0}$ such that $Y = X + B$ belongs to $\mathcal{D}_{Y_0}$. In this model framework, the expected utility $U(\sigma, \mu)$ of the utility u on the random variable $Y = X + B$ could be represented as

$$U(\sigma, \mu) = E[u(X + B)] = \int_a^b u(\mu + \sigma s)\, dF(s) \ , \tag{2.2}$$

in which all the terms are defined in Assumption 2.1. The expected utility $U(\sigma, \mu)$ of the utility u stated in (2.2) represents a two-parameter family of random variables parameterized by their mean μ and standard deviation σ.

For any constant α, the indifference curve drawn on the (σ, μ) plane such that $U(\sigma, \mu)$ is a constant can be expressed as:

$$C_\alpha = \{(\sigma, \mu) | U(\sigma, \mu) \equiv \alpha\} \ . \tag{2.3}$$

We note that some academics, for example, Meyer (1987), study the shapes of indifference curves without imposing the condition stated in (2.3), while, on the other hand, some academics, for example, Wong and Ma (2008), impose such a condition in their study. In this paper, we will include both situations. Assuming that the utility function u is twice continuously differentiable, we follow the approach used in Meyer (1987) and others to obtain the following equation for the expected utility $U(\sigma, \mu)$ stated in (2.2) for risk averters and risk seekers:

$$U_\mu(\sigma, \mu)\, d\mu + U_\sigma(\sigma, \mu)\, d\sigma = 0$$

or

$$U_\mu(\sigma, \mu)\, \frac{d\mu}{d\sigma} + U_\sigma(\sigma, \mu) = 0$$

where

$$U_\mu(\sigma, \mu) = \frac{\partial\, U(\sigma, \mu)}{\partial\, \mu} = \int_a^b u'(\mu + \sigma s)\, d\, F(s), \tag{2.4}$$

$$U_\sigma(\sigma, \mu) = \frac{\partial\, U(\sigma, \mu)}{\partial\, \sigma} = \int_a^b u'(\mu + \sigma s) s\, d\, F(s). \tag{2.5}$$

We first state the shapes of the indifference curves for risk averters and risk seekers with the expected utility $U(\sigma, \mu)$ for the utility u on the random variable Y as stated in the following proposition:

Proposition 2.1 *If $Y = X + B$ satisfies Assumption 2.1 with mean μ and variance σ^2 belongs to a location-scale family, and, for any utility function u, if $u' > 0$, the indifference curve C_α can be parameterized as $\mu = \mu(\sigma)$ with slope*

$$S(\sigma, \mu) = -\frac{U_\sigma(\sigma, \mu)}{U_\mu(\sigma, \mu)}\ ,$$

in which $U_\mu(\sigma, \mu)$ and $U_\sigma(\sigma, \mu)$ are defined in (2.4) and (2.5), respectively. In addition,

1. *if $u'' \le 0$, the indifference curve $\mu = \mu(\sigma)$ is a convex upward function of σ; and*

2. *if $u'' \ge 0$, the indifference curve $\mu = \mu(\sigma)$ is a concave downward function of σ.*

Proposition 2.1 implies that, similar to the shapes of the indifference curves on investment without background risk as shown in Tobin (1958) and others, the indifference curves on the investment with background risk is convex upward for risk averters, concave downward for risk seekers, and horizontal for risk-neutral investors, to include the general conditions stated by Meyer (1987). We note that the slope $S(\sigma, \mu)$ of the investor's indifference curve in (σ, μ)-space at (σ, μ) is the marginal rate of substitution between risk, σ, and return, μ. We also note that because comparisons of risk aversion are determined only from the family of risks in (2.2), risk aversion can be measured in terms of standard deviation and mean, and thus, it can be measured by the slope $S(\sigma, \mu)$.

Now, we turn to studying the comparative statics of the shapes of indifference curves for risk averters and risk seekers with respect to the means and the standard deviations of the returns from a financial asset and/or a background asset. We first examine the change in the shapes of indifference curves with respect to the change in the means of the financial asset and background risk as stated in the following proposition:

Proposition 2.2 *Under the conditions stated in Proposition 2.1, for any utility function u with $u' > 0$, $\partial S(\sigma,\mu)/\partial \mu_X$ and $\partial S(\sigma,\mu)/\partial \mu_B \leq (=,\geq) 0$ if and only if $u(\mu+\sigma s)$ displays decreasing (constant, increasing) absolute risk aversion for any $\mu+\sigma s$. Furthermore, we have $\partial S(\sigma,\mu)/\partial \mu_X = \partial S(\sigma,\mu)/\partial \mu_B$.*

We next investigate the change in the shapes of indifference curves for risk averters and risk seekers with respect to the change in the standard deviations of the financial asset and background risk as stated in the following proposition:

Proposition 2.3 *Under the conditions stated in Proposition 2.1 and for any utility function u with $u' > 0$, we have*

1. *without the restriction of $U(\sigma,\mu) \equiv \alpha$, $\partial S(\sigma,\mu)/\partial \sigma_X$ and $\partial S(\sigma,\mu)/\partial \sigma_B \leq (=,\geq) 0$ if and only if $u(\mu+\sigma s)$ displays decreasing (constant, increasing) relative risk aversion for any $\mu+\sigma s$, and*

2. *with the restriction of $U(\sigma,\mu) \equiv \alpha$, an agent will increase the slopes of the indifference curves upon an increase in σ_X or σ_B if $u'' < 0$. Otherwise, she will decrease the slope upon an increase in σ_X or σ_B.*

3. *Furthermore, for both situations as stated in (1) and (2), $\partial S(\sigma,\mu)/\partial \sigma_X \propto \sigma_X$, $\partial S(\sigma,\mu)/\partial \sigma_B \propto \sigma_B$ and $\partial S(\sigma,\mu)/\partial \sigma_X = \frac{\sigma_X}{\sigma_B}\partial S(\sigma,\mu)/\partial \sigma_B$.*

3 Applications

In this section we demonstrate the applicability of the theory developed in Section 2 by drawing some inferences on risk vulnerability and investment decisions in financial crises and bull and bear markets. We first obtain the following proposition from Proposition 2.3:

Proposition 3.1 *Under the conditions stated in Proposition 2.3, in order to keep the same mean return, we have that*

1. *an increase in the variance of an exogenous, independent background risk induces the agent to choose a lower level of risky activities, and*

2. *a decrease in the variance of an exogenous, independent background risk induces the agent to choose a higher level of risky activities,*

regardless of whether the agent is risk averse or risk seeking.

Gollier and Pratt (1996) examine the restriction on utility functions by adding an unfair background risk to wealth, which makes risk-averse individuals behave in a more risk-averse way with respect to any other independent risk. They call this concept *risk vulnerability.* Eichner and Wagener (2003a) further propose and characterize the concept of variance vulnerability to formally capture the idea that an agent reduces her risky activities when confronted with the increase in the variance of an independent background risk. They impose a zero-mean assumption in their model setting and show that both EU- and two-parameter approaches are compatible in settings with independent background risk if and only if the distributions are Gaussian. In this paper, we extend their findings as shown in Proposition 3.1, and we show that the property of the variance vulnerability holds not only for risk averters but also for risk seekers. In addition, we establish the situation in which both risk averters and risk seekers could take more risk. Moreover, in our results we relax both the zero-mean assumption on the background risk and the Gaussian assumption.

Morever, Proposition 3.1 enables us to get the following proposition:

Proposition 3.2 *Under the conditions stated in Proposition 2.3 and assuming that the variance of the background risk during bull markets is smaller than that during bear markets, in order to keep the same mean return, investors will*

1. *invest in less-risky assets during bear markets, and*

2. *invest in more-risky assets during bull markets,*

regardless of whether they are risk averse or risk seeking.

We note that the main reason why investors like to take more risk during bull markets is that during bull markets, the economy is doing well, unemployment is low, consumers are willing to spend more money, and thus, a company will be doing well and stock prices will go up. The reverse argument holds during bear markets. Nonetheless, it is well known that the risk is bigger in bear markets, especially during financial crisis, than in bull markets. Thus, the results stated in Proposition 3.2 are still true and reflect investors' actual behaviors during bull and bear markets. We also note that the result in Proposition 3.2 implies that investors will invest in less-risky assets during financial crises because the market is an extremely bear market during any financial crisis.

4 Concluding Remarks

In this paper, by considering that a background risk could be pleasant as well as unpleasant, we study the impact of background risk on the indifference curves. Thereafter, we study the comparative statics of the shapes of the indifference curves when the means and the standard deviations of the returns on the financial asset and/or the background asset change.

We note that there could be many applications of the theory developed in this paper. In this paper we only demonstrate the applicability of our theory by drawing some inferences on risk vulnerability and investment decisions in financial crises and bull and bear markets. There could be many other applications. We note that the theory developed in this paper could provide more information to academics and investors if one could incorporate our theory with other theories. For example, one could incorporate our theory with the mean-variance (MV) rules for risk averters (Markowitz, 1952) and risk seekers (Wong, 2007). In addition, recently, Bai, et al. (2012a) develop the mean-variance ratio (MVR) tests that are uniformly most powerful and unbiased, while Bai, et al. (2012b) apply the tests to compare the performance of commodity trading advisors. The MV rules and the MVR tests assist risk averters and risk seekers to draw preferences among different assets, whereas the theory developed in our paper helps risk averters and risk seekers to know the shapes of their indifference curves and the change in their indifference curves when the means and the standard deviations of the returns on the financial asset and/or the background asset change. Thus, the theory developed in this paper provides more information to investors in their investment decision making.

Another example of incorporating the theory developed in our paper with other theories is to work with the stochastic dominance theory for risk averters (Feldstein, 1969;

Hanoch and Levy, 1969) and for risk seekers (Li and Wong, 1999; Wong and Li, 1999). For example, based on the findings from Fong et al. (2005), Sriboonchitta, et al. (2009) conclude that risk averters prefer to invest in "winners," whereas risk seekers prefer to invest in "losers" in momentum portfolios. Broll, et al. (2006) analyze export production in the presence of exchange rate uncertainty under MV preferences. Qiao et al. (2013) find that risk averters prefer to invest in spot market while risk seekers prefer to invest in futures market. Investors could include the theory developed in our paper to provide more information on the shapes of the indifference curves and the comparative statics for decision makers investing in spot, futures, momentum portfolios, and export production.

There could be many extensions of the theory developed in our paper. One important area of extension is to relax the independent assumption between the asset return and the background risk imposed in this paper. Some studies in the literature (see, for example, Eichner and Wagener (2012)) develop results using the mean-variance model with background risk in which the asset return and the background risk are dependent. Academics could use their approach to extend the theory developed in our paper to relax the independence assumption between the asset return and the background risk. In addition, Levy and Wiener (1998), Levy and Levy (2002, 2004), and Wong and Chan (2008) have developed the stochastic dominance theory for investors with S-shaped and reverse S-shaped utility functions. Thereafter, Broll, et al. (2010), Egozcue, et al. (2011) and others have developed some properties for the indifference curves of investors with reverse S-shaped utility functions but they have not included the background risk in their studies. Thus, it would be interesting to extend the theory developed in this paper to include investors with S-shaped and reverse S-shaped utility functions.

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