Trade, productivity, income, and profit: the comparative advantage of structural axiomatic analysis

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Trade, Productivity, Income, and Profit: The Comparative Advantage of Structural Axiomatic Analysis

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Abstract

The classical case of comparative advantage is put into a new formal framework, that is, the behavioral axioms of standard economics are replaced by a set of structural axioms. This enables a comprehensive analysis that takes the effects on income and profit explicitly into account. The axioms in combination with the conditions of market clearing, budget balancing and initial zero profit determine all measurable variables objectively. It is the purpose of the present paper to formally restate the notion of comparative advantage and to ascertain whether this leads to a well-grounded new perspective on this time-honored doctrine.

JEL F10, F16, F31, F41

Keywords new framework of concepts; structure-centric; axiom set; consistency; comparative advantage; terms of trade; exchange rate; division of labor; factor immobility

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‘Notwithstanding its limitations, the theory of comparative advantage is one of the deepest truths in all of economics.’ (Samuelson and Nordhaus, 1998, p. 696). The point, however, is that this truth derives more evidence from the common sense of a simple barter model and less from a theory in the proper sense:

> The undefined and defined terms, the axioms, and all the theorems that can be derived from them constitute a theory. (Stigum, 1991, p. 36)

Seen from the methodological perspective the theory of comparative advantage cannot stand alone but must be part of a comprehensive framework that combines the interaction of real and nominal variables for the economy as a whole. Exemplifying his case with a convincing example that compares the respective labor inputs of two countries caused some trouble for Ricardo because the theory of international value ‘really rips up the entire fabric of Ricardo’s theory of value’ (Schumpeter, 1994, p. 612). If one takes the formal definition of theory literally Ricardo advanced a rudimentary piece. The task would have been to derive the theorem of comparative advantage from a consistent set of axioms for the economy as a whole. To proceed in this way was clearly not the first priority of Political Economics.

> Since advocacy of free-trade policy was the main practical purpose the ‘classical’ writers had in mind when they developed their theory of international values, they were naturally more interested in displaying the ‘gains’ that accrue to a nation from foreign trade. (Schumpeter, 1994, p. 609)

General equilibrium theory, in marked contrast to Ricardo’s approach, explicitly complies to the formal definition of theory (Debreu, 1959, p. x). Its first drawback, though, is that it rests on a set of *behavioral* axioms (Arrow and Hahn, 1991, p. v). Its second drawback is the irresistible bias to explain the fact of long running positive profits away:

> Wherever entrepreneurs make profits (beyond the market return on their own land, labor, and capital) they expand production; wherever they incur losses, production is contracted. In equilibrium therefore, there are neither profits nor losses. Walras thus created the abstraction of the zero-profit entrepreneur under perfect competition. (Niehans, 1994, p. 214)

This bias prevents deeper insights into the nature and function of profit. The zero-profit situation is a convenient analytical limiting case but evidently no feature of the real world. Hence, the long term existence of positive profits for the economy as a whole has to be explained first. In the second step profit has to be made an integral part of the theory of international trade.

Ricardo’s line of thought ran in real terms. His principle of comparative advantage is, after a long hibernation, ‘back to center stage’ (Eaton and Kortum, 2012, p. 66). The new approaches expand Ricardo’s simple model by introducing concepts drawn from general equilibrium theory, thereby implicitly carrying over its basic its assumptions. Taking the widespread critique of GET seriously this is hardly a promising line of inquiry. In the aforementioned reference the term profit – the pivot of the market economy – does not appear once.

The present paper starts from a entirely new foundation. Its general thesis says that human behavior does not yield to the axiomatic method, yet the axiomatization of the money economy's fundamental structure is feasible. The methodological case for structural axiomatization has been made at length elsewhere (2012).

It is the specific purpose of this paper to reformulate the notion of comparative advantage consistently in structural axiomatic terms and to see whether this yields answers to some unsettled questions of theoretical economics.

The formal ground is prepared in Section 1. In Section 2 profit is defined in terms of the axiomatic variables. In Sections 3 to 6 three elementary cases are compared with regard to the required adaptations of prices and wage rates in consequence of the merger of two regions: (A) two regions, two products, (B) two regions, equal productivities, different wage rates and prices, and (C) two regions, equal wage rates, different productivities and prices. The criterion for necessary adaptations is that the pre- and post-merger situations are indifferent in real terms. In Section 7 the elementary cases are formally combined to the classical case of comparative advantage. In Section 8 it is shown that the merger of the two regions ultimately leaves business and workers unaffected in all real aspects. In Section 9 the consequences of factor immobility are considered. Section 10 summarizes.

1 Axioms and definitions

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production conditions. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other state activity.

> The simplicity of our theory summarized in axioms and describing all possible configurations is therefore dearly bought. Except in case of a miniature theory ..., the implications are difficult to see by merely examining the axioms. ... Via the axioms information is by a large provided implicitly. A theory is a logical filing system. That which is delivered enveloped in axioms must be carefully retrieved by means of deduction. (Klant, 1984, p. 10)

Albeit quite obvious, it is worth to re-emphasize that all axiomatic variables are measurable in principle. No nonempirical concepts like equilibrium, rationality or perfect competition are put into the premises.

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms (Boylan and O'Gorman, 2007, p. 431). With (4) wage income Y_W and distributed profit income Y_D is defined as:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

With (5) the expenditure ratio ρ_E and the sales ratio ρ_X is defined as:

$$\rho_E \equiv \frac{C}{Y} \qquad \rho_X \equiv \frac{X}{O} \quad |t. \tag{5}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures are equal to income, or, in other words, that the household sector's budget is balanced. A value of $\rho_X = 1$ of the sales ratio means that the quantities produced and sold are equal in period t or, in other words, that the product market is cleared.

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit have to be thoroughly kept apart.

2 Profit

The business sector's profit in period t is defined with (6) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures C – and costs – here identical with wage income Y_W:[1]

$$\Delta Q_{fi} \equiv C - Y_W \quad |t. \tag{6}$$

In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

$$\Delta Q_{fi} \equiv PX - WL \quad |t. \tag{7}$$

Using the first axiom (1) and the definitions (4) one gets

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad |t. \tag{8}$$

The three definitions are formally equivalent. If distributed profit Y_D is set to zero then profit or loss of the business sector is determined solely by expenditures and income. For the business sector as a whole to make a profit consumption expenditures C have in the simplest case to be greater than wage income Y_W. So that profit comes into existence in the pure consumption economy the household sector must run a deficit at least in one period. This in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy (cf. Keynes, 1973, p. 85). Mention should be made that neither classicals, nor neoclassicals, nor Keynesians ever came to grips with profit (Desai, 2008, p. 10), (Tómasson and Bezemer, 2010).[2]

3 Case A: two regions, two products

We have at first two regions M and N, respectively. Each region consists of one firm A and B, respectively. The inhabitants of region M buy at first only the product of firm A; the inhabitants of region N buy only the product of firm B.

Total income in each region follows from (1) and is given by:

$$\begin{aligned} Y_{\mathbf{M}} &\equiv W_A L_A + \underbrace{D_A N_A}_{0} \\ Y_{\mathbf{N}} &\equiv W_B L_B + \underbrace{D_B N_B}_{0} \qquad |t. \\ Y &= Y_{\mathbf{M}} + Y_{\mathbf{N}} \end{aligned} \tag{9}$$

[1] Profits from changes in the value of nonfinancial assets are neglected here, i.e. the condition of market clearing $O = X$ holds throughout. For details about changes of inventory see (2011c, p. 5).

[2] For the full implications of the difference between profit and distributed profit in (8) see (2011a) and (2011d).

To simplify matters, distributed profits are set to zero.

Consumption expenditures in each region follow from (3) and are given by:

$$\begin{aligned} C_{\mathbf{M}} &\equiv P_A X_A \\ C_{\mathbf{N}} &\equiv P_B X_B \qquad |t. \\ C &= C_{\mathbf{M}} + C_{\mathbf{N}} \end{aligned} \tag{10}$$

Each firm's financial profit follows from (7) and is given by:

$$\begin{aligned} \Delta Q_{fiA} &\equiv P_A X_A - W_A L_A \\ \Delta Q_{fiB} &\equiv P_B X_B - W_B L_B \end{aligned} \quad |t. \tag{11}$$

This can, under the condition that both product markets are cleared, i.e. $\rho_X = 1$, be rewritten as:

$$\begin{aligned} \Delta Q_{fiA} &= P_A R_A L_A \left(1 - \frac{W_A}{P_A R_A}\right) \quad \text{if} \quad \rho_{XA} = 1 \\ \Delta Q_{fiB} &= P_B R_B L_B \left(1 - \frac{W_B}{P_B R_B}\right) \quad \text{if} \quad \rho_{XB} = 1 \end{aligned} \quad |t. \tag{12}$$

In the initial period profits of both firms are set to zero, i.e. the factor cost ratio

$$\rho_F \equiv \frac{W}{PR} \quad |t \tag{13}$$

is unity for each firm. Under the zero profit condition follows from (11) that wage income is equal to consumption expenditures. With the zero profit condition the market clearing prices for both firms are determined as:

$$P_A = \frac{W_A}{R_A} \qquad P_B = \frac{W_B}{R_B} \quad |t. \tag{14}$$

The prices are, in the simplest case, equal to the respective unit wage costs. Note that no subjective element enters the price determination. The three *objective* conditions: market clearing, budget balancing, and zero profit are sufficient. Any additional behavioral condition, e.g. utility or profit maximization, amounts to formal over-determination. There is no spare room for the marginal principle.

We now merge the two regions. The inhabitants of M may also buy product B, and the inhabitants of N may also buy product A. The opportunity set expands. For the households in M that give up $X_{A\Theta}$ and buy $X_{B\Theta}$ there must be households in N that give up $X_{B\Theta}$ and buy $X_{A\Theta}$. Any complementary combination is feasible. The quantities traded are determined by that region that is not prepared to give up as much as the other would like to exchange. Total consumption expenditures (10) for both regions taken together remain unaltered:

$$C = C_{\mathbf{M}} + C_{\mathbf{N}} \Rightarrow C = \underbrace{P_A\left(X_A - X_{A\Theta}\right) + P_B X_{B\Theta}}_{\mathbf{M}} + \underbrace{P_B\left(X_B - X_{B\Theta}\right) + P_A X_{A\Theta}}_{\mathbf{N}} \quad |t. \tag{15}$$

Therefore it holds:

$$P_A X_{A\Theta} = P_B X_{B\Theta} \quad |t. \tag{16}$$

Both regions' current accounts are balanced. Under the given conditions it is of no consequence whether the traded quantities are large or small; the necessary condition is that they are complementary.

From (14) follow the terms of trade:

$$\rho_\Theta \equiv \frac{P_A}{P_B} \quad \Rightarrow \quad \rho_\Theta \equiv \frac{\dfrac{W_A}{R_A}}{\dfrac{W_B}{R_B}} \equiv \frac{W_A}{W_B}\,\frac{R_B}{R_A} \quad |t. \tag{17}$$

The terms of trade are determined by relative wage rates and inverse productivities. For each unit of product A that the inhabitants of M give up they can buy more of X_B if the wage rate W_A is higher relative to W_B and if the productivity R_B is higher relative to R_A. Vice versa for N. Only if the wage rates happen to be equal, the terms of trade do not depend on the ratio of nominal magnitudes but solely on the ratio of productivities. Alternatively, the region with the higher wage rate has better terms of trade relative to the benchmark of real magnitudes. By introducing an exchange rate the effect of different nominal wage rates can be neutralized.

For the firms the merging of regions does not change much; only the composition of customers is now more mixed. Profits are zero before and after the merger. The households enjoy a greater variety of choice and are, on this score, clearly better off. The real wages that follow from (14), i.e. $W_A/P_A = R_A$ respectively $W_B/P_B = R_B$, are different (to be precise: numerically and qualitatively different) and remain unaltered. This difference is of no behavioral consequence if it is outweighed by the possibly greater noneconomic benefits of staying in the home region.

Equation (17) looks rather Ricardian as it makes no explicit reference to demand. What happens if the inhabitants of the merged regions develop a stronger taste for product A?

Total consumption expenditures are partitioned in the relation:

$$\frac{C_{\mathbf{M}}}{C_{\mathbf{N}}} = \frac{\dfrac{C_{\mathbf{M}}}{Y}}{\dfrac{C_{\mathbf{N}}}{Y}} = \frac{\rho_{E\mathbf{M}}}{\rho_{E\mathbf{N}}} \tag{18}$$

$$\text{with} \quad C = C_{\mathbf{M}} + C_{\mathbf{N}} \quad \Rightarrow \quad \rho_E \equiv \rho_{E\mathbf{M}} + \rho_{E\mathbf{N}} \quad |t.$$

From this in combination with (10) and (11) then follows under the zero profit condition:

$$\frac{L_A}{L_B} = \frac{\dfrac{W_B}{W_A}}{\dfrac{\rho_E}{\rho_{E\mathbf{M}}} - 1} \tag{19}$$

$$\text{with} \quad L \equiv L_A + L_B \ \text{ constant} \quad \text{and} \quad \rho_E = 1 \quad |t.$$

The condition of overall budget balancing translates into $\rho_E = 1$. With the wage rates given, a relative increase of consumption expenditures in favor of product A amounts to an increase of $\rho_{E\mathbf{M}}$. And this in turn necessitates according to (19) a reallocation of total labor input L. An increase of L_A in firm A and a complementary decrease of L_B entails a migration of labor form region N to M. Thereby output adapts to the change of preferences which expresses itself as an increase of the expenditure ratio $\rho_{E\mathbf{M}}$ and a complementary decrease of $\rho_{E\mathbf{N}}$. The stronger demand for one product is adequately satisfied by a shift of production.

This change of tastes and the consequent adaptation of production, though, does not alter the terms of trade (17), which depend on unvaried productivities and unvaried wage rates. Hence demand shifts are, contrary to J. S. Mill and Marshall (Niehans, 1994, pp. 131, 242), (Schumpeter, 1994, p. 608), neutral with regard to the terms of trade. The partitioning of total nominal demand determines the structure of production. This follows without regress to any behavioral assumptions from the axiom set and the rules of algebra. The usual demand–supply analysis is inadequate because it ignores the interconnections between consumption expenditures and income. These interconnections are determined by the objective conditions of market clearing, budget balancing and zero profits. Therefore there is no room left for independent demand and supply schedules.

4 Case B: equal productivities, different wage rates and prices

The set-up that is given with equations (9) to (12) remains the same with one exception. It is assumed now that the productivities are equal in both regions. The wage rates differ. For the qualitatively identical output we then have, compared to (14), the following new market clearing prices:

$$R_A = R_B = R \quad \Rightarrow \quad P_A = \frac{W_A}{R} \qquad P_B = \frac{W_B}{R} \qquad \text{if} \quad \rho_{XA} = 1,\ \rho_{XB} = 1 \quad |t. \tag{20}$$

If the wage rate is higher in region M, i.e. $W_A > W_B$, then the market clearing price is higher, i.e. $P_A > P_B$, and M is the high wage–high price region. Real wages W_A/P_A and W_B/P_B are equal to productivity R.

There is an obvious incentive for the inhabitants of M to buy the qualitatively identical product at a lower price in region N. This one-sidedness would derange the initial autonomy of the two regions. To fix this behavioral bias, different currencies are introduced. With an exchange rate of 2 M/N the incentive vanishes, if, for example, wage rate and price in M are double of that in N. At this exchange rate there occurs neither supply nor demand and by consequence no transaction in the foreign exchange market. Each region is supposed to buy its own product.

If the exchange rate is not fixed at 2 M/N but at, for example, 0.49 N/M then there arises a small incentive for the inhabitants of region N to buy in M. It is assumed that those and all other remaining small incentives are swamped by transport costs, thus no trade takes place in the clear-cut initial state. The high wage–high price and the low wage–low price regions at first simply coexist. A lower wage rate and a lower price do not *per se* set up an advantage. The nominal differentials are equalized by the exchange rate, which in this case does not ‘clear’ the market but prevents any trade between the regions. If wage rate and price double in region M then the exchange rate doubles without any transfer of goods between the regions or any transaction on the foreign exchange market. The specie flow mechanism remains inactive. The exchange rate operates as an inhibitor of trade.

We now merge the two regions. This calls for the equalization of the product prices. From (2), (10) and (3) follows:

$$P^{\star} = \frac{C}{X} = P_A \frac{L_A}{L} + P_B \frac{L_B}{L}$$
$$\text{if } \rho_X = 1, \text{ with } C = C_{\mathbf{M}} + C_{\mathbf{N}},\ X \equiv X_A + X_B,\ L \equiv L_A + L_B \quad |t. \tag{21}$$

If both regions are of equal size, measured in labor input, then the new market clearing price is located exactly in the middle between the formerly higher price of region M and the lower price of region N. In the general case, the new price is a weighted average that depends on the relative size of the regions. The price in M decreases, i.e. $P^{*} < P_A$, the price in N increases, i.e. $P^{*} > P_B$. The inhabitants of N are momentarily worse off, and the inhabitants of M are momentarily better off.

Profit was hitherto zero for both firms. Equations (11) change to:

$$\begin{aligned} \Delta Q_{fiA} &\equiv P^{\star} X_A - W_A L_A \\ \Delta Q_{fiB} &\equiv P^{\star} X_B - W_B L_B \end{aligned} \quad |t. \tag{22}$$

With a lower selling price $P^{\star}$ firm A now posts a loss; firm B on the other hand makes a profit. Firm A is momentarily worse off, and firm B is better off. The situation of households and firms is inverted in each region. In order to restore the zero profit state it follows from (20) that the wage rates have to change according to:

$$W_A^{\star} = P^{\star} R = W_B^{\star} \quad \text{if} \quad \rho_{XA} = 1,\ \rho_{XB} = 1 \quad |t. \tag{23}$$

After the product price equalization the wage rates in both firms cannot remain different. This entails that the wage rate falls in firm A, i.e. $W_A^{\star} < W_A$, and rises in firm B, i.e. $W_B^{\star} > W_B$. The real wages remain unvaried. Total income Y, too, remains unchanged. This can be shown with the help of (23), (21) and (20):

$$Y^{\star} = W_A^{\star} L_A + W_B^{\star} L_B = Y \quad |t. \tag{24}$$

At the end of the day nothing has changed in real terms. How could the necessary, but ultimately indifferent, price and wage rate changes be brought about?

5 Auctioneer vs. invisible hand

For the Walrasian auctioneer the task is simple. There is no need for a tâtonnement, the auctioneer can calculate the new price vector under the given objective conditions from the given data. All that is necessary is a simultaneous implementation that keeps the agents at the *same* position on their indifference curves. Since agents are supposed to reckon in real terms, firm A accepts a price reduction while its workers accept a wage rate reduction. Firm B accepts a price increase and its workers a wage rate increase. It is *not* the case that firm B accepts the price increase but rejects the wage increase. And it is *not* the case that firm A accepts the wage rate reduction but rejects the price reduction. There is no cherry picking. All this is implied in the assumption of economic rationality which in turn implies the *simultaneous* execution of all nominal changes. Simultaneity presupposes a coordinating central agency (Nadal, 2004, p. 108). Implementation presupposes no extra-market authority provided the agents rationally accept the indifferent nominal changes as determined by the auctioneer.

For the invisible hand the task is more demanding because it has to rely on market forces that effect the price and wage rate changes by means of quantitative changes of supply and demand. The invisible hand is not allowed to touch prices.

After the regions' merger in Case B it initially pays for firm B to transport part of its output to region M and sell it there. According to the logic of market forces, this will drive down the price in M and drive up the price in N. As long as this process continues firm B makes a profit and firm A faces accumulating inventories and losses. It is implicitly assumed that the price adaptation process ends before firm A goes bankrupt. Firm A's skill to stay afloat therefore specifies the notion of the long run. Bankruptcy of firm A is not an indicator that the markets work properly and in good Darwinist fashion merely weed out the weak and sick firms, just the contrary; in the case under consideration it is the very epitome of inefficiency.

Since firm A's situation at first deteriorates after the price equalization it has a strong motive and cogent arguments to effect wage cuts. In order to save jobs workers can be expected to acquiesce more or less rapidly. In the meantime, losses accumulate. Firm B, on the other hand, will be slow to raise its wage rate and rather prefer to expand its profitable business by hiring more workers. This subjectively rational procrastination, though, is not conductive to satisfying results for the economy as a whole. Since financial profit for the merged economy is zero by construction the situation is behaviorally unstable as long profit in one firm is positive, because there is a complementary loss in the other firm. What is needed, then, is full downward *and* upward flexibility of product prices *and* wage rates. This perfect behavioral symmetry is no feature of the real world. In the limiting case of symmetry and simultaneity no resources consuming reallocation of goods and labor input would be necessary in order to arrive at an economically indifferent end state.

The invisible hand's clumsy and wasteful quantitative adaptations are an inferior substitute for the purely nominal adaptations of the auctioneer. All the more so, since it is by no means clear whether and how the insufficiently specified market forces converge to the algebraically derived correct values of prices and wage rates before unwarranted casualties happen. The move from an initial state to the perfectly indifferent end state implies a Walrasian adaptation which is fundamentally different from a market-forces adaptation.[3] To bring about nominal changes by moving around real quantities and people is comparatively less efficient. The present inquiry is simplified, for the moment, by assuming Walrasian adaptations.

6 Case C: equal wage rates, different productivities and prices

The formal frame that is given with equations (9) to (12) remains almost the same. The exception concerns the wage rates that are now assumed to be equal in both regions. The quality of the products is identical, which is here indicated by a dot. From (14) follow the market clearing prices as:

$$W_A = W_B = W \quad \Rightarrow \quad P_A = \frac{W}{R_A^{\bullet}} \qquad P_B = \frac{W}{R_B^{\bullet}} \quad |t. \tag{25}$$

If the productivity is higher in region M, i.e. $R_A^{\bullet} > R_B^{\bullet}$, then the price is lower, i.e. $P_A < P_B$, and M is the high productivity–low price region. Accordingly, the real wage in M is higher than in region N, i.e. $W/P_A > W/P_B$. Let us assume for simplicity a relation of 2 to 1. Taken the real wages without regard to any other considerations there exists an incentive for the workers to migrate from region N to M. Put into a greater context, though, it is rational to stay in N if the differences in working and living conditions outweigh the real wage differential. This is assumed to be the case. Then, however, there remains the incentive of the inhabitants of N to buy the qualitatively identical product in M at half price. When different currencies are introduced and the exchange rate is set at 2 N/M this incentive vanishes. At this exchange rate no transactions occur in the foreign exchange market.

We now merge the two regions. This calls for the equalization of the product prices. From (2), (10) and (3) follows:

$$P^{\star} = \frac{C}{X} = \frac{P_A + P_B \Upsilon}{1+\Upsilon}$$

$$\text{if} \quad \rho_X = 1, \quad \text{with} \quad C = C_N + C_M,\ X \equiv X_A + X_B,\ \Upsilon \equiv \frac{R_B^{\bullet} L_B}{R_A^{\bullet} L_A} \equiv \frac{O_B}{O_A} \qquad |t. \tag{26}$$

If outputs are numerically equal in both regions, i.e. $\Upsilon = 1$, then the new market clearing price is located exactly in the middle between the lower price of region M

[3] "One of the most important aspects of a specific market format is the temporal pattern of price setting; that market clearing takes time inherently violates the presumption that the entire target stock will be sold at a unique price .... Far from being a fusty antiquarian issue, the controversy is still relevant to the modern neoclassical fascination with the "law of one price" .... (Mirowski, 2004, p. 347). Niehans, for one, maintains to the contrary that both adjustment processes are complementary instead of mutually exclusive (1994, p. 245).

and the higher price of region N. In the general case, the new price is a weighted average that depends on the relative outputs of the regions. The outputs can only be equal if the smaller region, measured in labor input, has the higher productivity. Then both regions have the same economic weight.

The price in M increases, i.e. $P^{\star} > P_A$, the price in N decreases, i.e. $P^{\star} < P_B$. Real wages are no longer equal, they decrease in M, and increase in N. The inhabitants of M are momentarily worse off, and the inhabitants of N are momentarily better off. Region N benefits from the higher productivity and the lower price in M.

Profit was hitherto zero for both firms. Equations (11) change to:

$$\begin{aligned} \Delta Q_{fiA} &\equiv P^{\star} X_A - W L_A \\ \Delta Q_{fiB} &\equiv P^{\star} X_B - W L_B \end{aligned} \quad |t. \tag{27}$$

With a higher selling price firm A now makes a profit; firm B posts a loss because its former selling price was higher. The business sector's situation is inverse to that of the household sector in each region. To restore the zero profit situation it follows from (25) that the wage rates must change to:

$$W_A^{\star} = P^{\star} R_A^{\bullet} \quad W_B^{\star} = P^{\star} R_B^{\bullet} \quad \text{if} \quad \rho_{XA} = 1;\ \rho_{XB} = 1 \quad |t. \tag{28}$$

The new uniform market clearing price in combination with different productivities demands a wage rate differentiation. The wage rate rises in region M and falls in N, i.e. $W_A^{\star} > W$ and $W_B^{\star} < W$. Thereafter the real wage in M is again higher than in N, just as in the pre-merger situation. Total income remains unchanged. This can be shown with the help of (28), (26) and (25):

$$Y^{\star} = W_A^{\star} L_A + W_B^{\star} L_B = Y. \tag{29}$$

At the end of the day nothing has changed in real terms. Firms and households alike are, without any economic blessings, at the same point on their respective indifference curves, provided all nominal changes happen simultaneously.

7 Case D: comparative advantage

Each region consists now of both, a corn producer A and a cloth producer B. Region M has a higher productivity in corn production, region N in the production of cloth. The respective productivities are given by:

$$\begin{array}{lllll} R_{\mathbf{M}A} & R_{\mathbf{N}A} = \alpha R_{\mathbf{M}A} & \alpha < 1 & \alpha = \frac{1}{2} & \text{corn} \\ R_{\mathbf{M}B} & R_{\mathbf{N}B} = \beta R_{\mathbf{M}B} & \beta > 1 & \beta = 2 & \text{cloth} \end{array} \quad |t. \tag{30}$$

The productivity of corn production in region N is assumed to be half of that in region M, yet the productivity of cloth production in region N is double of that in region M. This concrete values facilitate the exposition, for the general case the factors α and β may assume arbitrary values. What matters is that region M has a relative advantage in agrarian production and region N in industrial production. The product qualities of corn and cloth are identical.

With wage rates initially set equal for all firms, profits (7) are differentiated as follows:

$$\left.\begin{aligned} \Delta Q_{fi\mathbf{M}A} &= P_{\mathbf{M}A} R_{\mathbf{M}A} L_{\mathbf{M}A} \left(1 - \frac{W}{P_{\mathbf{M}A} R_{\mathbf{M}A}}\right) \quad \text{if} \quad \rho_{X\mathbf{M}A} = 1 \\ \Delta Q_{fi\mathbf{M}B} &= P_{\mathbf{M}B} R_{\mathbf{M}B} L_{\mathbf{M}B} \left(1 - \frac{W}{P_{\mathbf{M}B} R_{\mathbf{M}B}}\right) \quad \text{if} \quad \rho_{X\mathbf{M}B} = 1 \\ \Delta Q_{fi\mathbf{N}A} &= P_{\mathbf{N}A} R_{\mathbf{N}A} L_{\mathbf{N}A} \left(1 - \frac{W}{P_{\mathbf{N}A} R_{\mathbf{N}A}}\right) \quad \text{if} \quad \rho_{X\mathbf{N}A} = 1 \\ \Delta Q_{fi\mathbf{N}B} &= P_{\mathbf{N}B} R_{\mathbf{N}B} L_{\mathbf{N}B} \left(1 - \frac{W}{P_{\mathbf{N}B} R_{\mathbf{N}B}}\right) \quad \text{if} \quad \rho_{X\mathbf{N}B} = 1 \end{aligned}\right| t. \qquad (31)$$

From this follow the market clearing prices for corn in region M and N, respectively:

$$P_{\mathbf{M}A} = \frac{W}{R_{\mathbf{M}A}} \qquad P_{\mathbf{N}A} = \frac{W}{\alpha R_{\mathbf{M}A}} \qquad P_{\mathbf{M}A} < P_{\mathbf{N}A} \qquad \text{corn} \quad |t. \qquad (32)$$

The price of corn $P_{\mathbf{M}A}$ in region M is half of that in region N.

By the same token the market clearing prices of cloth in both regions are given by:

$$P_{\mathbf{M}B} = \frac{W}{R_{\mathbf{M}B}} \qquad P_{\mathbf{N}B} = \frac{W}{\beta R_{\mathbf{M}B}} \qquad P_{\mathbf{M}B} > P_{\mathbf{N}B} \qquad \text{cloth} \quad |t. \qquad (33)$$

The price of cloth $P_{\mathbf{M}B}$ in region M is double of that in region N. Corn is cheaper in M, cloth is cheaper in N. Hence it is advantageous for the households of both regions to buy *all* corn in M and *all* cloth in N. The situation is not behaviorally stable unless the two regions are kept apart in one way or another. Let us suppose that the exchange rate has been set at 2 N/M, then it is no longer advantageous for the households in region N to buy corn in region M. The lower corn price in M is compensated for by the exchange rate. However, the households in region M now face an exchange rate of 0.5 M/N *and* a lower cloth price in region N. Their incentive to buy cloth in region N quadruples. There is not one single exchange rate that can level the price differentials between the regions. To keep them definitively apart for the moment we therefore have to introduce transportation costs or tariffs that compensate for the price differentials, such that trade is prevented and the households keep on buying in their respective home regions.

Consumption expenditures (3) in both regions are given by:

$$C_{\mathbf{M}} \equiv P_{\mathbf{M}A} X_{\mathbf{M}A} + P_{\mathbf{M}B} X_{\mathbf{M}B} \qquad |t. \tag{34}$$
$$C_{\mathbf{N}} \equiv P_{\mathbf{N}A} X_{\mathbf{N}A} + P_{\mathbf{N}B} X_{\mathbf{N}B}$$

Wage incomes (1) in both regions are given by:

$$Y_{\mathbf{M}} \equiv W\left(L_{\mathbf{M}A} + L_{\mathbf{M}B}\right) \qquad |t. \tag{35}$$
$$Y_{\mathbf{N}} \equiv W\left(L_{\mathbf{N}A} + L_{\mathbf{N}B}\right)$$

The household sectors' budgets in both regions are balanced, i.e. $\rho_{E\mathbf{M}} = 1$ and $\rho_{E\mathbf{N}} = 1$, the product markets are cleared and profits of all firms are zero.

We now merge the two regions. Inhabitants of *M* may freely enter the markets of *N,* and vice versa. From (34) follows the uniform price in the corn market under the aforementioned conditions as:

$$P_A^\star = \frac{P_{\mathbf{M}A} X_{\mathbf{M}A} + P_{\mathbf{N}A} X_{\mathbf{N}A}}{X_{\mathbf{M}A} + X_{\mathbf{N}A}} = \frac{P_{\mathbf{M}A} + P_{\mathbf{N}A} \Upsilon_A}{1 + \Upsilon_A} \qquad |t. \tag{36}$$
$$\text{if} \quad \rho_{XA} = 1 \quad \text{with} \quad \Upsilon_A \equiv \alpha \frac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}}$$

If $\Upsilon_A = 1$ then $P_A^\star$ is a simple average, the new price is located exactly halfway between the hitherto lower corn price in *M* and the higher price in *N*; otherwise (36) yields a somewhat more sophisticated weighted average of the initial prices that depends on the relative size of the firms and the productivity factor α.

Seen from the corn producer in *M* the new market clearing price $P_A^\star$ is above the former regional price, i.e. $P_A^\star > P_{\mathbf{M}A}$, that is, the corn producer in *M* makes a profit. Seen from the corn producer in *N* the new market price is below the former regional price, i.e. $P_A^\star < P_{\mathbf{N}A}$, that is, the corn producer in *N* makes a loss.

From (34) follows the market clearing price in the cloth market as:

$$P_B^\star = \frac{P_{\mathbf{M}B} X_{\mathbf{M}B} + P_{\mathbf{N}B} X_{\mathbf{N}B}}{X_{\mathbf{M}B} + X_{\mathbf{N}B}} = \frac{P_{\mathbf{M}B} + P_{\mathbf{N}B} \Upsilon_B}{1 + \Upsilon_B} \qquad |t. \tag{37}$$
$$\text{if} \quad \rho_{XB} = 1 \quad \text{with} \quad \Upsilon_B \equiv \beta \frac{L_{NB}}{L_{MB}}$$

If $\Upsilon_B = 1$ then $P_B^\star$ is a simple average, otherwise it is a somewhat more sophisticated weighted average of the initial prices that depends on the relative size of the firms and the productivity factor β.

Seen from the cloth producer in *M* the new market price is below the former regional price, i.e. $P_B^\star < P_{\mathbf{M}B}$, that is, the cloth producer in *M* makes a loss. Seen from the cloth producer in *N* the new market price is above the former regional price, i.e. $P_B^\star > P_{\mathbf{N}B}$, that is, the cloth producer in *N* makes a profit.

The market price equalization disrupts the zero profit situation. The firms with the relatively high productivity now make a profit; the firms with the relatively low productivity make a loss. Seen under the regional perspective one firm makes a profit and the other a loss. It is evident that this distribution of profits and losses cannot last for long.

To restore the zero profit configuration the respective wage rates have to be adapted. Analogous to (28) one gets for the corn producers

$$W^{\star}_{\mathbf{M}A} = P^{*}_{A} R_{\mathbf{M}A} \qquad W^{\star}_{\mathbf{N}A} = P^{*}_{A} R_{\mathbf{N}A} \qquad W^{\star}_{\mathbf{M}A} \uparrow \quad W^{\star}_{\mathbf{N}A} \downarrow \tag{38}$$

and for the cloth producers

$$W^{\star}_{\mathbf{M}B} = P^{\star}_{B} R_{\mathbf{M}B} \qquad W^{\star}_{\mathbf{N}B} = P^{\star}_{B} R_{\mathbf{N}B} \qquad W^{\star}_{\mathbf{M}B} \downarrow \quad W^{\star}_{\mathbf{N}B} \uparrow \quad |t. \tag{39}$$

The wage rate changes follow the price changes in each firm. The corn producing workers in M earn more in nominal terms after the merger of the regional markets, in N they earn less, and vice versa for the cloth producing workers.

The consumption expenditures in both regions, as compared to (34), are now given by:

$$\begin{aligned} C^{\star}_{\mathbf{M}} &\equiv P^{\star}_{A} X_{\mathbf{M}A} + P^{\star}_{B} X_{\mathbf{M}B} \\ C^{\star}_{\mathbf{N}} &\equiv P^{\star}_{A} X_{\mathbf{N}A} + P^{\star}_{B} X_{\mathbf{N}B} \end{aligned} \quad |t. \tag{40}$$

The wage incomes in both regions, as compared to (35), are now given by:

$$\begin{aligned} Y^{*}_{\mathbf{M}} &\equiv W^{\star}_{\mathbf{M}A} L_{\mathbf{M}A} + W^{\star}_{\mathbf{M}B} L_{\mathbf{M}B} \\ Y^{\star}_{\mathbf{N}} &\equiv W^{\star}_{\mathbf{N}A} L_{\mathbf{N}A} + W^{\star}_{\mathbf{N}B} L_{\mathbf{N}B} \end{aligned} \quad |t. \tag{41}$$

Due to the zero profit condition the respective consumption expenditures and wage costs are equal for each firm. From the zero profit condition in combination with (31) follow the real wages of the workers in the two firms of region M:

$$\frac{W^{\star}_{\mathbf{M}A}}{P^{*}_{A}} = R_{\mathbf{M}A} \quad \text{if} \quad \rho_{X\mathbf{M}A} = 1 \qquad \frac{W^{\star}_{\mathbf{M}B}}{P^{*}_{B}} = R_{\mathbf{M}B} \quad \text{if} \quad \rho_{X\mathbf{M}B} = 1 \quad |t. \tag{42}$$

In the same manner follow the real wages of the workers in the two firms of region N:

$$\frac{W^{\star}_{\mathbf{N}A}}{P^{*}_{A}} = R_{\mathbf{N}A} \quad \text{if} \quad \rho_{X\mathbf{N}A} = 1 \qquad \frac{W^{\star}_{\mathbf{N}B}}{P^{*}_{B}} = R_{\mathbf{N}B} \quad \text{if} \quad \rho_{X\mathbf{N}B} = 1 \quad |t. \tag{43}$$

Since all productivities remain unchanged, the real wages in all firms remain unchanged, too. The merger of the two regions ultimately leaves the workers unaffected in real terms. And since the profits in all firms are zero before and after the merger all agents stay put at the same position on their respective indifference curves. Nobody gains or looses because of the integration of product markets – provided wage rates are adapted in *both* directions. Wage cuts alone are inadequate.

In order to achieve real improvements, overall productivity increases are required. With *given* productivities in each firm, this in turn calls for more specialization and a reallocation of the given total labor input among firms. The integration of product markets that is accompanied by appropriate wage rate adaptations in both directions by itself has no real effect whatsoever, only a more profound division of labor has.

There are, however, changes of nominal incomes. From (41) and (42) follows:

$$Y^{\star}_{\mathbf{M}} \equiv W_{\mathbf{M}A}\, L_{\mathbf{M}A} \underbrace{\frac{P^{\star}_{A}}{P_{\mathbf{M}A}}}_{>1} + W_{\mathbf{M}B}\, L_{\mathbf{M}B} \underbrace{\frac{P^{\star}_{B}}{P_{\mathbf{M}B}}}_{<1} \quad |t. \tag{44}$$

The previous wage incomes in region M are now increased or decreased by a factor that depends on the relation of the new to the former market clearing prices. Hence the new wage income of the corn producing workers is higher than before and that of the cloth producing workers shrinks

The numerical value of the first price relation in (44) follows from (36)

$$\frac{P^{\star}_{A}}{P_{\mathbf{M}A}} = \frac{1 + \dfrac{P_{\mathbf{N}A}}{P_{\mathbf{M}A}} \alpha \dfrac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}}}{1 + \alpha \dfrac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}}} \quad \text{if} \quad \rho_{XA} = 1 \quad |t. \tag{45}$$

Likewise for the second price relation. The countervailing income changes in both firms as given by (44) do not cancel out exactly. By consequence, total nominal income in region M changes. Likewise for region N. These income changes, though, do not affect the real wages which stay put, as we have seen earlier. Since the overall expenditure ratio is unity consumption expenditures move in step with income.

The new terms of trade follow from (36) and (37) as:

$$\rho^{\star}_{\Theta} \equiv \frac{P^{\star}_{A}}{P^{*}_{B}} \quad \Rightarrow \quad \rho^{\star}_{\Theta} = \frac{P_{\mathbf{M}A} + P_{\mathbf{N}A}\Upsilon_{A}}{P_{\mathbf{M}B} + P_{\mathbf{N}B}\Upsilon_{B}} \, \frac{1+\Upsilon_{B}}{1+\Upsilon_{A}}$$
$$\text{if} \quad \rho_{XA} = 1,\ \rho_{XB} = 1 \qquad \Upsilon_{A} \equiv \alpha \frac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}},\ \Upsilon_{B} \equiv \beta \frac{L_{\mathbf{N}B}}{L_{\mathbf{M}B}} \quad |t. \tag{46}$$

After the substitution of prices by (32) and (33) this reduces to:

$$\rho^{\star}_{\Theta} = \frac{R_{\mathbf{M}B}}{R_{\mathbf{M}A}} \, \frac{\left(1 + \dfrac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}}\right)}{\left(1 + \alpha \dfrac{L_{\mathbf{N}A}}{L_{\mathbf{M}A}}\right)} \, \frac{\left(1 + \beta \dfrac{L_{\mathbf{N}B}}{L_{\mathbf{M}B}}\right)}{\left(1 + \dfrac{L_{\mathbf{N}B}}{L_{\mathbf{M}B}}\right)}$$
$$\text{if} \quad \rho_{XA} = 1,\ \rho_{XB} = 1 \quad |t. \tag{47}$$

The new terms of trade depend on the inverse productivities $R_{\mathbf{M}A}$ and $R_{\mathbf{M}B}$, the relative size of firms measured in labor inputs, and the productivity factors which in turn define the comparative advantages.[4] If the comparative advantages vanish, i.e. $\alpha = 1$; $\beta = 1$, the corn producers A, as well as the cloth producers B, are identical and the terms of trade are equal to the inverse productivities in the two lines of production. It has to be emphasized that the terms of trade are objectively given and in no way dependent on subjective idiosyncrasies. All structural axiomatic variables are measurable in principle.

The real wages are the same as in the initial situation but, to recall, they were different among firms then. This leaves us with a possible incentive for the workers to move from low-real-wage firms to high-real-wage firms. This incentive, though, is of no consequence if it is compensated for by countervailing differentials in working and living conditions.

8 Full specialization

It is assumed now that both regions focus on their comparative advantages, that is, region M allocates all available labor input to corn production and region N to cloth production. This entails that the workers in region M move voluntarily from firm B to firm A. This move is motivated by the higher real wage in firm A and accompanied by a net increase of total wage income in region M. Firm B vanishes in the process, which may entail second round effects like the devaluation of former investments or of real estate. Likewise for region N.

The financial profits of the two remaining firms are then given by:

$$\begin{aligned} \Delta Q_{fi\mathbf{M}A} &\equiv P^{\circ}_{\mathbf{M}A} X_{\mathbf{M}A} - W^{\star}_{\mathbf{M}A} L_{\mathbf{M}A} \\ \Delta Q_{fi\mathbf{N}B} &\equiv P^{\circ}_{\mathbf{N}B} X_{\mathbf{N}B} - W^{\star}_{\mathbf{N}B} L_{\mathbf{N}B} \end{aligned} \quad |t. \tag{48}$$

From the zero profit condition follow the market clearing prices as:

$$P^{\circ}_{\mathbf{M}A} = \frac{W^{\star}_{\mathbf{M}A}}{R_{\mathbf{M}A}} \qquad P^{\circ}_{NB} = \frac{W^{\star}_{\mathbf{N}B}}{R_{\mathbf{N}B}} \quad |t. \tag{49}$$

From this in turn follow the terms of trade:

$$\rho^{\circ}_{\Theta} \equiv \frac{P^{\circ}_{\mathbf{M}A}}{P^{\circ}_{\mathbf{N}B}} \quad \Rightarrow \quad \rho^{\circ}_{\Theta} \equiv \frac{W^{\star}_{\mathbf{M}A}}{W^{\star}_{\mathbf{N}B}} \frac{R_{\mathbf{N}B}}{R_{\mathbf{M}A}} \quad |t. \tag{50}$$

This is the same relation as (17) in Case A. Both cases are structurally identical. By substituting (42) and (43) we finally arrive at:

$$\frac{P^{\circ}_{\mathbf{M}A}}{P^{\circ}_{\mathbf{N}B}} = \frac{P^{\star}_{A}}{P^{\star}_{B}} \quad |t. \tag{51}$$

In the limiting case of full specialization the terms of trade are equal to the terms of trade in the general Case D. Under full specialization larger quantities are traded

[4] This is a variant of the structural value theorem. For details see (2011b, pp. 5-7).

but they exchange in the same relation, that is, region M gives up larger quantities of corn and obtains larger quantities of cloth. The higher real wage in both regions translates into larger quantities of both products. Full specialization, though, does not lead to better terms of trade for one region or the other. This results holds under the condition that the productivities remain constant.

If full specialization is, in *addition*, conductive to increasing returns, then it must occur in both firms in the same proportion, otherwise the terms of trade improve for the region with the comparatively smaller productivity increases.

It is therefore not precisely to the point to speak of the gains from trade as Ricardo (1981, p. 128) and J. S. Mill (2004, p. 8) already noted. The real gain originates in the sphere of production. The overall quantitative increases and the higher real wages in both regions are due to the intensification of the division of labor, which in turn presupposes deep going structural changes. The interlocked effects of: (a) specialization on the production line with a relatively high productivity, (b) larger quantitative trade volumes that come along with more resource consumption in transportation, (c) structural monoculture, and (d) intensified mutual dependencies have to be weighted against each other in order to determine the overall net gain. An obvious alternative to full specialization is to boost productivities in the comparatively weak production lines. This amounts to setting the productivity factors in (29) to unity.

By positing an expenditure ratio of unity and a distributed profit ratio of zero we have confined the analysis to the elementary zero profit case. Under the condition of zero profit business is entirely indifferent to the higher degree of specialization. The gains from specialization and trade therefore take the form of higher real wages in both regions. The structural axiom set of course contains also the the more complex general case of *overall* positive profits. All that is necessary is to allow for an expenditure ratio greater one and a distributed profit ratio greater zero. This analysis is left for another occasion.

9 Factor immobility between regions

Case A is once more the point of departure. It is assumed now that the partitioning of consumption expenditures shifts in favor of product A. This led in case A to the reallocation of labor input from the region that produces the less preferred output to the region that produces the more preferred output. Now the migration of labor input from region N to region M is excluded. Two possible ways of adaptation to the new situation are considered: (a) the price mechanism and (b) the reallocation of labor input within region N.

9.1 The exchange rate mechanism

The initial partitioning of the given total consumption expenditures C_0 changes as follows:

$$\begin{array}{rcl} C_0 = C_{\mathbf{M}0} + C_{\mathbf{N}0} & \rightarrow & C_{\mathbf{N}1} = C_0 - C_{\mathbf{M}1} \\ C_{\mathbf{M}1} > C_{\mathbf{M}0} & \rightarrow & C_{\mathbf{N}1} < C_{\mathbf{N}0}. \end{array} \tag{52}$$

Consumption expenditures increase in region M and decrease by the same amount in region N, that is, the inhabitants of N spend part of their incomes in region M. Total income Y remains unchanged and is equal to (9). The partitioning of labor input between the two regions remains unchanged. With given productivities the respective output quantities remain unchanged, too.

Given this conditions, the market clearing price of product A must rise according to (3)

$$P^{\star}_{A1} = \frac{C_{\mathbf{M}1}}{X_{A0}} \quad \text{if} \quad \rho_{XA1} = 1. \tag{53}$$

and analogous for region N

$$P^{\star}_{B1} = \frac{C_{\mathbf{N}1}}{X_{B0}} \quad \text{if} \quad \rho_{XB1} = 1 \tag{54}$$

where the market clearing price must fall.

Profit was initially zero in both firms. Due to the price changes we have now:

$$\begin{aligned} \Delta Q_{fiA1} &\equiv P^{\star}_{A1} X_{A0} - W_{A0} L_{A0} \\ \Delta Q_{fiB1} &\equiv P^{\star}_{B1} X_{B0} - W_{B0} L_{B0}. \end{aligned} \tag{55}$$

Firm A now makes a profit and firm B a loss. We can rewrite the new consumption expenditures $C_{\mathbf{M}1}$ as the sum of initial consumption expenditures of region M and additional expenditures of region N

$$\begin{aligned} C_{\mathbf{M}1} &\equiv C_{\mathbf{M}0} + C_{A\mathbf{N}1} \\ C_{\mathbf{N}1} &\equiv C_{\mathbf{N}0} - C_{A\mathbf{N}1} \end{aligned} \tag{56}$$

and analogous for the new consumption expenditures of region N. From (55) then follows:

$$\begin{gathered} \Delta Q_{fiA1} \equiv C_{A\mathbf{N}1} \\ \Delta Q_{fiB1} \equiv -C_{A\mathbf{N}1} \\ \text{with} \quad C_{\mathbf{M}0} = W_{A0} L_{A0},\ C_{\mathbf{N}0} = W_{B0} L_{B0.} \end{gathered} \tag{57}$$

The profit of region M is equal to its exports. The loss of region N is equal to its imports. This configuration is clearly not reproducible over a longer time span. What has to be done is to counteract the demand shift by an appropriate price increase. This works if the consumers react to a price increase with a cutback of consumption expenditures $C_{\mathbf{M}}$.

The price elasticity of consumption expenditures is defined as:

$$\varepsilon_{CP} \equiv \frac{\dddot{C}_{\mathbf{M}}}{\dddot{P}_A} \equiv \frac{\dfrac{C_{\mathbf{M}} - C_{\mathbf{M}1}}{C_{\mathbf{M}1}}}{\dfrac{P_A - P^{\star}_{A1}}{P^{\star}_{A1}}}. \tag{58}$$

This can be rewritten with the help of (53) as:

$$C_{\mathbf{M}} \equiv C_{\mathbf{M}1} - \varepsilon_{CP}\,(P_A - P^\star_{A1})\,X_{A0}. \tag{59}$$

If the price P_A is above the actual market clearing price $P^\star_{A1}$ then consumption expenditures $C_{\mathbf{M}}$ are below the current consumption expenditures $C_{\mathbf{M}1}$ provided the elasticity ε_{CP} is > 0. We are looking for the price P°_A that reduces $C_{\mathbf{M}}$ again to $C_{\mathbf{M}0}$. From (59) follows:

$$P^\circ_A = P^\star_{A1} + \frac{1}{\varepsilon_{CP} X_{A0}}\,(C_{\mathbf{M}1} - C_{\mathbf{M}0}) \qquad \text{if} \quad C_{\mathbf{M}} = C_{\mathbf{M}0}. \tag{60}$$

From this limiting price follows the appropriate exchange rate as:

$$\chi^\circ_{\mathrm{N/M}} \equiv \frac{P^\circ_A}{P_{A0}}. \tag{61}$$

By substituting (60) this finally gives:

$$\chi^\circ_{\mathrm{N/M}} \equiv \frac{P^\star_{A1}}{P_{A0}}\left(1 + \frac{1}{\varepsilon_{CP}}\right) - \frac{1}{\varepsilon_{CP}}. \tag{62}$$

The exchange rate that neutralizes the demand shift depends on the relation of the market clearing prices after and before the demand shift and the price elasticity of consumption expenditures. If the elasticity is infinite the exchange rate is equal to the relation of the market clearing prices. At this exchange rate there occurs neither supply nor demand and by consequence no transaction in the foreign exchange market.[5] Each region buys again its own product and profits are again zero. This configuration is in principle reproducible for an indefinite time span. The exchange rate, however, indicates a latent real demand for product A.

9.2 Domestic reallocation of labor input

It is now assumed that firm A reacts to the additional demand from region N with an expansion of output and employment. Conversely, firm B cuts back employment. The respective prices remain unaltered. The new labor inputs follow from (3), (2) and the market clearing condition as:

$$L_{A1} = \frac{C_{\mathbf{M}1}}{P_{A0} R_{A0}} \quad \text{if} \quad \rho_{XA} = 1$$
$$L_{B1} = \frac{C_{\mathbf{N}1}}{P_{B0} R_{B0}} \quad \text{if} \quad \rho_{BA} = 1. \tag{63}$$

[5] In the familiar terminology one could say that the demand and supply schedules should intersect exactly on the y-axis.

If there was full employment in each region in the initial period then we have now overemployment in region M since from $C_{\mathbf{M}1} > C_{\mathbf{M}0}$ follows $L_{A1} > L_{A0}$. Conversely, region N now experiences underemployment and has a strong motive to change the situation.

Since factor movements between the regions are ruled out by assumption region N cannot export unemployment. Hence the only possibility that is left under the given conditions is to build up a firm C in region N with the same productivity as firm A. This firm absorbs the unemployed and produces the output that is now more preferred than product B. The increased demand for product A is thereby satisfied and full employment is restored in both regions. This solution, although economically perfectly satisfactory, is not exactly in the spirit of the classical free trade doctrine. After the reallocation of labor input in region N there is no trade between the regions. No trade, though, is only a special case of balanced trade. And balanced trade is the condition for longer term reproducibility. An export-surplus in one region redistributes the profit of the world economy as a whole according to (57). Only in the ideal case of a balanced current account foreign trade makes no difference with regard to reproducibility.

10 Summary

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The present paper suggests three nonbehavioral axioms as groundwork for the consistent real *and* nominal analysis of foreign trade.

Four cases with increasing complexity are considered under the objective conditions of market clearing, budget balancing and initial zero profit of all firms in a pure consumption economy. Two regions are at first taken in isolation and then merged. Depending on the initial configuration of regional productivities, wage rates, and prices the merger leads to a structural adaptation of product prices and wage rates. The main results for the classical case of comparative advantages are:

- Since all productivities remain unaffected by the merger the real wages in all firms remain unchanged. And since the profits in all firms are zero before and after the merger all agents stay put at the same position on their respective indifference curves. Nobody gains or looses because of the integration of product markets – provided wage rates are adapted in *both* directions.

- The firms with a relatively low productivity must accept price reductions while its workers must accept simultaneous wage rate reductions. The firms with a relatively high productivity must accept simultaneous price and wage rate increases. Partial wage rate increases are as indispensable as partial wage rate reductions.

- The real gains from full specialization and trade accrue, in the zero profit consumption economy, exclusively to the workers.
- In the case of a demand shift for the two products of two regions three options are available: (a) the reallocation of labor input between the regions, (b) the implementation of an exchange rate that perfectly neutralizes the demand shift, (c) the build-up of a new firm that absorbs the demand shift within the region that is adversely affected by it. Measures (b) and (c) maintain full employment in both regions and prevent a profit redistribution between the regions.

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