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## Home-seekers in the Housing Market

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#### **Abstract**

This housing market matching model considers two types of home seekers: people who search for a house both in the rental and in the homeownership market, and people who only search in the homeownership market. The house-search process leads to several types of matching and in turn this implies different prices of equilibrium. Also, the house-search process connects the rental market with the homeownership market. This model is thus able to explain both the relationship between the rental price and the selling price and the price dispersion which exists in the housing market. Furthermore, this theoretical model can be used to study the impact of taxation in the two markets. Precisely, it is straightforward to show the effects of two different taxes: the tax on property sale and the tax on rental income.

### **Keywords:**

Rental market, homeownership market, house price dispersion, taxation

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## 1. Introduction

Although recent, housing market studies that adopt search and matching models are not new in the economic literature (notably, Wheaton, 1990; Krainer, 2001; Albrecht *et al.*, 2007; Caplin and Leahy, 2008; Novy-Marx, 2009; Ngai and Tenreyro, 2009; Diaz and Jerez, 2009; Albrecht *et al.*, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012). Precisely, two goals are usually pursued: analysing the formation process of house price in a decentralised market with search and matching frictions; explaining the behaviour of the housing market, in particular the price dispersion and the relationship among prices, time-on-the-market and sales.

The empirical "anomaly" known as 'price dispersion' is probably the most important distinctive feature of housing markets (see e.g. Leung, Leong and Wong, 2006). It refers to the phenomenon of selling two houses with very similar attributes and in near locations at the same time but at very different prices. The literature has mainly responded to the price dispersion puzzle by introducing the heterogeneity of economic agents. In Leung and Zhang (2011), in fact, a necessary condition for explaining the housing price dispersion, as well as the relationship among prices, time-on-the-market and sales, is the heterogeneity on the seller's and/or the buyer's side which generates corresponding submarkets.

Nevertheless, price dispersion may arise from the different states of home seekers in the search process. The basic idea behind the paper is the following: when a household or person needs to change its home (for business reasons or family needs), the goal is to buy a new or better house. However, the tenant state is often a satisfactory temporary situation, an intermediate step before buying in the homeownership market. In short, in the model the tenant state is modelled as a staging post for searching in the homeownership market. Nevertheless, some home-seekers can immediately find a home in the homeownership market. As a result, in this model there are two types of home seekers: the tenants who are waiting to become owners of a dwelling, thus searching only in the homeownership market, and people who search for a dwelling both in the rental and in the homeownership market (we simply refer to the latter as "seekers"). Hence, the search process leads to several types of matching; in turn, this implies different prices of equilibrium. Also, the search process connects the rental market with the homeownership market. Indeed, this paper analyses the

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<sup>&</sup>lt;sup>1</sup> Obviously, house price dispersion may also be due to missing housing characteristics (not observable or difficult to measure), the so-called unobserved good heterogeneity.

situation when both the homeownership market and the rental market are subject to search and matching frictions. As far as we are aware, this topic has been overlooked by housing market studies which adopt search and matching models. Indeed, papers in this literature omit the rental housing market from consideration (Diaz and Jerez, 2009) or rely on the standard asset-market equilibrium condition (Ngai and Tenreyro, 2009),<sup>2</sup> thus assuming a rental market without frictions (Kashiwagi, 2011).<sup>3</sup>

Therefore, the main aim of this paper is to develop a search and matching model of the housing market which is able to explain both the price dispersion and the relationship between rental and selling prices, relying only on the different states of home seekers in the search process. Furthermore, the proposed theoretical model can be used to study the impact of taxation in the housing market. Precisely, we consider the effects of two different taxes: the tax on property sale and the tax on rental income. We find that the tax on property sale increases the selling price and reduces the rental price; whereas, the tax on rental income increases both the rental price and the selling price, thus also increasing the time-on-the-market in both markets. Thus, a property sale tax may be better than a rental income tax. However, in the model there is the distinction between sellers and landlords, and thus further and potential effects of taxation on house prices are not considered.

The rest of the paper is organised as follows: section 2 presents the housing market matching model; section 3 shows the existence of price dispersion and describes the equilibrium of the model where the relationship between selling price and rental price play a key role; while section 4 discusses some effects of taxation on house prices and time-on-themarket; finally, section 5 concludes the work.

## 2. The housing market matching model

The housing market consists of the rental market and the homeownership market. In the homeownership market, the home-seeker who finds a dwelling and pays the selling price becomes the (new) owner of the house; whereas, this does not happen in the rental market, where the rental price only ensures the use of the house for a certain period of time. We

<sup>&</sup>lt;sup>2</sup> Assuming perfectly competitive housing markets, in equilibrium the risk-adjusted returns for homeowners and landlords should be equated across investments. This yields the usual user cost formula  $\dot{a}$  la Poterba (1984) where the rental price covers the user cost of housing, which is equal to the house price multiplied by the user cost, i.e. the sum of the real after-tax interest rate, the combined depreciation and maintenance rate, and the expected future house price appreciation.

<sup>&</sup>lt;sup>3</sup> Well-functioning rental markets can smooth out fluctuations in housing market liquidity (Krainer, 2001).

distinguish these two (sub-)markets by the subscript  $i = \{R, S\}$ , where R = rental market and S = homeownership or sale market. Hence,  $p_{R}$  is the rental price and  $p_{S}$  is the selling price. There are two main categories of home seekers in this housing market matching model: people who search for a dwelling both in the rental and in the homeownership market, simply named "seekers" (h), and people who pay a rent, thus searching only in the homeownership market, named "tenants" ( $h_s$ ). We assume that the mass of households/persons who need to change their home (for business reasons or family needs) increases over time and all the "new" home-seekers  $\lambda$  (where  $\lambda$  is a positive and exogenous number) initially search on both markets, i.e. they enter the seekers pool (h). As regards the supply side, i.e. the housing offer, there is free entry into the market. Hence, it is the free entry condition which allows the equilibrium value of vacant houses to be determined. In short, new vacant houses will be posted until the value of a further vacancy becomes equal to zero. In equilibrium, in fact, all the profit opportunities derived from opening new vacancies have been exploited, therefore the value of an additional vacancy is equal to zero (see Pissarides, 2000). Precisely, in this model, sellers post vacancies in the homeownership market and landlords open vacancies in the rental market.<sup>5</sup> Hence, landlords only meet with the seekers (h).

In order to formalise the housing market, we adopt a standard matching framework  $\dot{a}$  la Mortensen-Pissarides (see e.g. Pissarides, 2000) with random search and prices determined by Nash bargaining. The housing market is a "matching market" like the labour market, that clears not only through price, but also through time and money that the parties spend on the market. Thus, the search and matching approach is arguably more appropriate also for this type of market. As is usual in matching-type models (see Pissarides, 2000; Petrongolo and Pissarides, 2001), the meeting of vacant houses and home seekers is regulated by an aggregate matching function, m:

$$m_R = m(v_R, h);$$
  $m_S = m(v_S, (h+h_S))$ 

where  $v_R$  and  $v_s$  are the number of vacancies in the rental market and in the homeownership market, respectively. Precisely, the matching function gives the number of

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<sup>&</sup>lt;sup>4</sup> The zero-profit (or free-entry) condition makes sense in the housing market if houses (in both sub-markets) are supplied by competitive house builders, in addition to being supplied by owners who no longer need them for occupation.

<sup>&</sup>lt;sup>5</sup> The distinction between sellers and landlords is obviously a simplification of the model, since the sellers can rent their house and landlords can sell their house. Matters thus become simpler without loss of generality.

matches (i.e. contracts) formed per unit of time, given the number of vacant houses and the share of home seekers in the market. Recall that both the seekers (h) and the tenants ( $h_s$ ) search in the homeownership market. The matching function is non-negative, increasing and concave in both arguments and performs constant returns to scale. In order to clarify the properties of the matching function, one can consider the functional form commonly used in matching models, i.e. the Cobb-Douglas function:  ${}^6 M_R = m(v_R, h) = v_R^{1-a} h^a$ , where 0 < a < 1 is the (constant) elasticity of the matching function with respect to the share of seekers. The two instantaneous probabilities that characterise the matching process can thus be obtained:  $g(\vartheta_R) = \frac{m_R}{h} = \frac{v_R^{1-a} h^a}{h} = v_R^{1-a} h^{a-1} = \left(\frac{v_R}{h}\right)^{1-a} = \vartheta_R^{1-a}$  is the instantaneous probability of finding a home, and  $g(\vartheta_R) = \frac{m_R}{v_R} = \frac{v_R^{1-a} h^a}{v_R} = v_R^{-a} h^a = \left(\frac{v_R}{h}\right)^{-a} = \vartheta_R^{-a}$  is the instantaneous probability of filling a vacant house. It follows that the key variable of the model, the so-called  $market \ tightness$ ,  $\vartheta$ , with  $i = \{R, S\}$ , can be introduced:

$$\vartheta_R \equiv \frac{v_R}{h}; \qquad \vartheta_S \equiv \frac{v_S}{(h+h_S)}$$

the ratio between vacancies and home seekers identifies the market frictions which prevent (or delay) the matching between the parties. Note that  $\vartheta_i$ , with  $i = \{R, S\}$ , is the housing market tightness from the standpoint of sellers and landlords. Hence, an increase in market tightness (vacant houses) causes a positive (negative) effect on the demand (supply) side due to the *congestion externalities* effect on the sellers/landlords' side. Accordingly, the home-finding rate, i.e. the ratio between the matching function and the share of home seekers:

$$g(\vartheta_R) = \frac{m(v_R, h)}{h} = m(\vartheta_R, 1); \qquad g(\vartheta_S) = \frac{m(v_S, (h + h_S))}{(h + h_S)} = m(\vartheta_S, 1)$$

is positive, increasing and concave in market tightness, while the vacancy-filling rate, i.e. the ratio between the matching function and the number of vacancies

$$q(\vartheta_R) = \frac{m(v_R, h)}{v_R} = m(1, \vartheta_R^{-1}); \qquad q(\vartheta_S) = \frac{m(v_S, (h + h_S))}{v_S} = m(1, \vartheta_S^{-1})$$

<sup>6</sup> In this instance we take into account only the rental market but the same pattern applies to the homeownership market.

<sup>&</sup>lt;sup>7</sup> In the matching literature (see Pissarides, 2000), in fact, market tightness is usually calculated from the firm's standpoint.

is a positive, decreasing and convex function in market tightness.<sup>8</sup> Intuitively, this is straightforward to understand since if market tightness increases (decreases), the probability of filling a vacant house is lower (higher), while the probability of finding a home is higher (lower).

In order to study the matching between the parties in the two markets, it is necessary to introduce the value functions of the model. The value functions describe the expected marginal values (from which the positive and exogenous interest rate r has been deducted) associated with the differing conditions of housing market participants, basically comparing them to financial securities:  $^9$ 

$$rH = -e + g(\vartheta_R) \cdot [T - H] + g(\vartheta_S) \cdot [x - p_S - H]$$
[1]

$$rT = -e_s - p_R + g(\vartheta_s) \cdot [x - p_s - T] + \delta \cdot [H - T]$$
 [2]

$$rV_{R} = -c + q(\vartheta_{R}) \cdot [D - V_{R}]$$
[3]

$$rD = p_R + \delta \cdot [V_R - D] \tag{4}$$

$$rV_{s} = -c + q(\vartheta_{s}) \cdot \beta \cdot \left[ p_{s}^{h_{s}} - V_{s} \right] + q(\vartheta_{s}) \cdot (1 - \beta) \cdot \left[ p_{s}^{h} - V_{s} \right]$$
 [5]

where H is the discounted present value of an infinite life of a seeker (h); T is the discounted present value of an infinite life of a tenant ( $h_s$ );  $V_R$  is the discounted present value of a vacant house in the rental market; D is the discounted present value of an infinite life of a landlord, and  $V_s$  is the discounted present value of a vacant house in the homeownership market. In the rental market, existing leases are cancelled at the exogenous rate  $\delta$ , and thus at the rate  $\delta$  a tenant ( $h_s$ ) becomes a seeker (h). Instead, in the homeownership market if a contract is legally binding (as hypothesised) it is no longer possible to return to the circumstances preceding the bill of sale (unless a new and distinct contractual relationship is set up); hence, the discounted present value of an infinite life of a seller is simply given by the selling price ( $p_s$ ). In short, the destruction rate in the rental market is  $\delta > 0$  (lease destruction rate), while it is zero for the sale market. The terms on the right hand side of the value functions are, respectively, the "dividends" associated with the different conditions and the "capital gains". As regards the "dividends", e is the effort (in

<sup>&</sup>lt;sup>8</sup> Also, standard technical assumptions are usually assumed:  $\lim_{\vartheta_i \to 0} q(\vartheta_i) = \lim_{\vartheta_i \to \infty} g(\vartheta_i) = \infty$ , and  $\lim_{\vartheta_i \to 0} g(\vartheta_i) = \lim_{\vartheta_i \to \infty} q(\vartheta_i) = 0$ ,  $\forall i$ .

<sup>&</sup>lt;sup>9</sup> Time is continuous and individuals are risk neutral, live infinitely and discount the future at the exogenous interest rate *r*. It is common practice in the literature to make use of linear utility functions. Assuming that individuals are risk neutral not only simplifies the analysis, but also allows to focus on the consequences of the search and matching process rather than on the deficiencies of the insurance markets.

monetary terms) made by the home seekers to find and visit the largest possible number of houses: obviously,  $e > e_s$ , since the seeker (h) search in both markets; c is the cost of opening a vacant house and in this case it also includes the cost of building new homes; finally, x is the buyer's benefit which coincides with the value of the house and depends on the housing characteristics. As will become clear later, x can differ from the market price because of the matching frictions and bargaining power. The "capital gain", instead, is the transition from one condition to the other, influenced by the probability of finding a home  $g(\vartheta_i)$ , of filling a vacancy  $q(\vartheta_i)$ , with  $i = \{R, S\}$ , and by the lease destruction rate  $\delta$ . Consider equation [1], for example (the same reasoning applies for the other value functions): a seeker (h) bears cost flows ( $e_h$ ) during the search ( $negative\ dividends$ ); whereas, s/he becomes a tenant at the rate  $g(\vartheta_s)$ , thus obtaining the value T, and gets the house and pays the selling price at the rate  $g(\vartheta_s)$ . Hence, at the rates  $g(\vartheta_R)$  and  $g(\vartheta_s)$ , s/he finds a home as tenant or as homeowner ( $capital\ gains$ ).

Because potential buyers are different, the selling prices are also different: in fact, the seller may be matched with either a tenant  $(h_s)$  or a seeker (h). Hence,  $\beta = h_s / (h_s + h)$  and  $(1-\beta) = h / (h_s + h)$  in equation [5] are, respectively, the share of tenants  $(h_s)$  and seekers (h). In this model, however, the home-seekers differ only with respect to their state in the search process. Furthermore, they can change their condition in the house-search process: in fact, a seeker (h) can become a tenant  $(h_s)$  and vice versa. Therefore, we assume that sellers are not able to distinguish between different states of buyers in the search process, i.e. the buyers always appear identical to sellers ex ante. Hence, also the selling prices appear identical to sellers ex ante, namely  $p_s^{h_s} = p_s^h = p_s$ , and thus equation [5] collapses to:

$$rV_{s} = -c + q(\vartheta_{s}) \cdot [p_{s} - V_{s}]$$
 [6]

However, when the parties meet each other, the seller will observe the state of buyer ex post. Nevertheless, s/he always decides to sell since the search is costly in terms of time and money. In a nutshell, if the search is costly and random, it is not convenient for the seller to wait for a new match. Hence, sellers accept offers as long as the selling price is higher than the value of a vacant house.

<sup>&</sup>lt;sup>10</sup> According to the hedonic price theory, the value of the house, and thus the buyer's benefit, can be higher or lower according to the mix of desired and undesidered housing characteristics.

Finally, the value of being a tenant T is modelled as a staging post for searching in the homeownership market. Hence, a necessary condition for a non trivial equilibrium requires that:

$$(T-H) = \frac{(e-e_s)-p_R}{r+\delta+g(\vartheta_R)+g(\vartheta_S)} > 0$$

which is true if  $(e-e_s)>p_R$ , namely if the cost of being a seeker (h) in both markets is higher than the cost of being a tenant ( $h_s$ ). In this case, the tenant state is a satisfactory temporary situation.

To summarise, in the value functions [1] – [6], we introduce four endogenous variables ( $p_s$ ,  $p_R$ ,  $\vartheta_s$  and  $\vartheta_R$ ), while all the other variables are exogenous. In other words, as is usual in matching-type models, the variables that characterise the model are market prices and matching frictions. Hence, once the equilibrium values of  $p_s$ ,  $p_R$ ,  $\vartheta_s$  and  $\vartheta_R$  are obtained, the value functions are determined. Precisely, the "zero profit" equilibrium condition or free-entry equilibrium condition, normally used by matching models (see Pissarides, 2000), gives the key relationship of the model between price and market tightness. Indeed, by using the condition  $V_i = 0$ , with  $i = \{R, S\}$ , in equation [3] – [4] and [6], we get:

$$\frac{1}{q(\vartheta_R)} = D \Rightarrow \frac{1}{q(\vartheta_R)} = \frac{p_R}{c \cdot (r + \delta)} \Rightarrow \underbrace{q(\vartheta_R)^{-1}}_{l.h.s.} = \underbrace{\frac{p_R}{c \cdot (r + \delta)}}_{l.h.s.}$$
 [7]

$$\frac{1}{q(\vartheta_s)} = \frac{p_s}{c} \Rightarrow q(\vartheta_i)^{-1} = \frac{p_s}{c}$$
 [8]

unlike the labour market matching model (which describes a negative relationship between market tightness and wage), in this case the free-entry condition yields a positive relationship between market tightness and price: in fact,  $q(\vartheta_i)^{-1}$  is increasing in  $\vartheta_i$ , with  $i = \{R, S\}$ . This positive relationship is very intuitive: in fact, if the price increases, more vacancies will be on the market. However, equations [7] and [8] define a system of two equations in four unknowns. Thus, we need to introduce the two price equations.

## 3. Price equations and housing market equilibrium

We assume that market tensions are exogenous at the microeconomic level, in the sense that each individual takes  $\vartheta_{R}$  and  $\vartheta_{S}$  as given in the price bargaining.

The generalised Nash bargaining solution, usually used for decentralised markets, allows the price to be obtained through the optimal subdivision of surplus deriving from a successful match. The surplus is defined as the sum of the seller/landlord's and home-seeker's value when the trade takes place, net of the respective external options (the value of continuing to search). Hence, a trade takes place between the parties at a price determined by Nash bargaining if the surplus is positive. Precisely, the price (both rental and selling) solves the following optimisation condition:

price = 
$$argmax\{(net \ gain \ of \ seller/landlord)^{\gamma} \cdot (net \ gain \ of \ homeseeker)^{1-\gamma}\}$$
 [9] where  $\gamma \in (0,1)$  is the bargaining power of the seller/landlord. The bargained price crucially depends on the surplus deriving from the matching. Precisely, in this model three kinds of matching can occur, thus leading to different surpluses:

- 1) The seeker (h) finds a home in the homeownership market. This matching produces an equilibrium selling price of  $p_s^1 = argmax\{(p_s V_s)^{\gamma} \cdot (x p_s H)^{1-\gamma}\};$
- The tenant  $(h_s)$  finds a home in the homeownership market. Hence, the equilibrium selling price is  $p_s^2 = argmax\{(p_s V_s)^{\gamma} \cdot (x p_s T)^{1-\gamma}\};$
- 3) The seeker (h) finds a home in the rental market. This matching produces an equilibrium rental price of  $p_R = argmax\{(D_R V_R)^{\gamma} \cdot (T H)^{1-\gamma}\}$ .

Therefore, the existence of price dispersion can be straightforwardly shown. In fact, in the homeownership market the net gain of seeker (h) is different from that of tenant ( $h_s$ ), and this produces two different surpluses. Eventually, from equation [9] two different selling prices ( $p_s^1$  and  $p_s^2$ ) are obtained. It follows that the origin of price dispersion is due to the different states of home seekers in the search process. Indeed, this result holds true even in the presence of an identical bargaining power, identical search costs and also when the same house (namely, the same the buyer's benefit, x) is considered.

As regards the selling prices, i.e. the matching 1) and 2) in the homeownership market, solving the optimisation conditions yields (recall that in equilibrium  $V_i = 0$ ,  $\forall i$ ):

$$(x - p_s^1 - H) = \frac{1 - \gamma}{\gamma} \cdot p_s^1 \Rightarrow p_s^1 = \gamma \cdot (x - H)$$

$$(x-p_s^2-T)=\frac{1-\gamma}{\gamma}\cdot p_s^2 \Rightarrow p_s^2=\gamma\cdot (x-T)$$

Given the properties of equations [1] and [2], both  $p_s^1$  and  $p_s^2$  depend positively on  $p_R$  (yet remaining different since  $T \neq H$ ): in fact, an increase in the rental price reduces both T (directly) and H (indirectly through T). Therefore, without loss of generality, we can express this relationship in a broader form as follows: <sup>11</sup>

$$\rho_{s} = \rho_{s}(\rho_{R}) \tag{10}$$

with  $\partial p_s/\partial p_R > 0$ . Furthermore, if the rental price tends to zero, no one will have convenience to buy a house and the value of being a tenant will be at the maximum. As a result, the selling price will also tend to zero, since it cannot be negative or null (since the surplus is positive).

Instead, as regards the matching 3) in the rental market, we obtain:

$$(T-H) = [(1-\gamma)/\gamma] \cdot (D_R - V_R)$$

$$\Rightarrow (T - H) = \frac{1 - \gamma}{\gamma} \cdot \frac{p_R + c_R}{r + \delta + q(\vartheta_R)} \Rightarrow \frac{\gamma \cdot (r + \delta + q(\vartheta_R))}{1 - \gamma} \cdot (T - H) - c_R = p_R$$

We know that an increase in selling price reduces both T and H, since both home-seekers search in the homeownership market. Nevertheless, as long as the tenant state is an appealing perspective, i.e. as long as  $g(\vartheta_{\scriptscriptstyle R}) > \delta$ , the decrease in T is stronger than the decrease in H, i.e.  $\left| \frac{\partial T}{\partial p_{\scriptscriptstyle S}} \right| > \left| \frac{\partial H}{\partial p_{\scriptscriptstyle S}} \right|$ . Indeed, buying a home is the only future perspective for a

tenant. Hence, in this case we obtain a negative relationship between rental price and selling price:

$$p_{R} = p_{R}(p_{S})$$
 with  $\partial p_{R}/\partial p_{S} < 0$ .

Therefore, the relationship between selling and rental prices can be represented in the diagram with axes [ $p_s$ ,  $p_R$ ], where only a steady-state equilibrium exists in the housing market with positive prices (see *Figure* 1a).

Eventually, given  $p_{\scriptscriptstyle R}^*$  and  $p_{\scriptscriptstyle S}^*$ , we obtain a unique value of tightness for each market  $(\vartheta_{\scriptscriptstyle R}^*$  and  $\vartheta_{\scriptscriptstyle S}^*)$  at the macroeconomic level. This testable proposition is made possible by a

<sup>&</sup>lt;sup>11</sup> Alternatively, one could see  $p_s$  as a function of the two selling prices  $(p_s^1, p_s^2)$  and set up a system of four equations in four unknowns  $(p_s, p_s^1, p_s^2, p_R)$ . However, this solution would add complexity but no further insight.

downward sloping price function which forms the right hand side (*r.h.s.*) of the free-entry conditions (see equations [7]-[8] and *Figure* 1b). In fact, *ceteris paribus*,  $\partial p_R/\partial \vartheta_R < 0$  and  $\partial p_S/\partial \vartheta_S < 0$ , since an increase in market tightness increases T and H and reduces  $q(\vartheta_R)$ .

Finally, we close the model by describing the evolution of h and  $h_{\rm S}$  in the course of time t: $^{12}$ 

$$\dot{h} \equiv \frac{\partial h}{\partial t} = \delta \cdot h_s + \lambda - [g(\vartheta_R) + g(\vartheta_S)] \cdot h$$
 [12]

$$\dot{h}_{s} \equiv \frac{\partial h_{s}}{\partial t} = g(\vartheta_{R}) \cdot h - g(\vartheta_{s}) \cdot h_{s}$$
 [13]

where  $\delta \cdot h_s$  represents seekers inflows, i.e. existing leases cancelled at rate  $\delta$ ;  $h \cdot [g(\vartheta_R) + g(\vartheta_S)]$  describes the seekers outflows, i.e. the seekers (h) that find a home as tenant or as homeowner, and  $\lambda$  are the "new" home seekers. Likewise,  $g(\vartheta_R) \cdot h$  and  $g(\vartheta_S) \cdot h_S$  describe, respectively, the inflows and outflows in/from the tenant state.

In steady state equilibrium, where  $\it h$  and  $\it h_{\it s}$  are constant over time, it follows that:

$$\dot{h} = 0 \Longrightarrow \delta \cdot h_{s} + \lambda = [g(\vartheta_{R}) + g(\vartheta_{s})] \cdot h$$

$$\dot{h}_{s} = 0 \Rightarrow g(\vartheta_{R}) \cdot h = g(\vartheta_{s}) \cdot h_{s}$$

sufficiently low:

therefore, given the value of search frictions in both markets, we get a system of two equations in two unknowns: h and  $h_{\scriptscriptstyle S}$ . Sufficient condition for the existence of an interior equilibrium is that  $g(\vartheta_{\scriptscriptstyle S})\cdot\frac{\left(g(\vartheta_{\scriptscriptstyle R})+g(\vartheta_{\scriptscriptstyle S})\right)}{g(\vartheta_{\scriptscriptstyle R})}\!>\!\delta$ , namely  $g(\vartheta_{\scriptscriptstyle S})$  is sufficiently high or  $\delta$  is

$$\lambda = \left\{ \left[ g(\vartheta_R) + g(\vartheta_S) \right] \cdot \frac{g(\vartheta_S)}{g(\vartheta_R)} - \delta \right\} \cdot h_S$$
 [14]

$$h = \frac{g(\vartheta_s)}{g(\vartheta_R)} \cdot h_s \tag{15}$$

In words, if the probability of finding a home in the sale market is sufficiently high and/or the lease destruction rate is sufficiently low, the perspective of finding a home in both markets is very attractive. This is consistent with the story told in this model, where the goal of each home seeker is to buy a house and the tenant state is a satisfactory temporary situation.

<sup>&</sup>lt;sup>12</sup> The equilibrium usually characterised by these models is in fact the stationary state, in which the values of the variables are not subject to further changes over time.

## 4. Effects of taxation on house prices

By considering rental and homeownership market together in a matching framework, one can study how changes in the relative tax treatment of owner and rental housing influence the two markets. Indeed, the proposed theoretical model can be used to show the effects of both property sale tax and rental income tax.

Basically, from a microeconomic point of view, the taxation ( $\tau$ ) increases the house prices, since the sellers/landlords with a sufficient bargaining power react by increasing the price charged to the home-seekers. This can be straightforwardly shown by introducing the term  $-\tau_i$ , with  $i = \{R, S\}$ , in the value of an occupied home, viz.:<sup>13</sup>

$$rD = p_{R} - \tau_{R} + \delta \cdot [V_{R} - D]$$
 [16]

$$rV_{s} = -c + q(\vartheta_{s}) \cdot [p_{s} - \tau_{s} - V_{s}]$$
[17]

Precisely, by using equations [10] and [11] together, it is possible to show that a tax on property sale ( $\tau_s$ ) leads to an increase in selling price and a decrease in rental price (see also figure 2a); whereas, a tax on rental income ( $\tau_R$ ) leads to an increase in both selling and rental prices (see also figure 2b).

====== Figure 2 about here now at the end =======

The change in house prices, in turn, affects the time it takes to sell (to rent) a property, the so-called time-on-the-market (TOM), which measures the degree of illiquidity of the real estate market. By using the free-entry conditions, it is straightforward to show that the house with a higher price has a longer time-on-the-market. In fact, with a probability of filling a vacant house of  $q(\vartheta_i)$ , the (expected) time-on-the-market is  $q(\vartheta_i)^{-1}$  which is increasing in  $\vartheta_i$ , with  $i = \{R, S\}$ . As a result, with a tax on rental income the time-on-the-market increases for both markets (since both prices are higher); whereas, with a tax on property sale the time-on-the-market increases in the homeownership market but decreases in the rental market. Thus, a property sale tax may be better than a rental income tax. The explanation is that the tax on property sale is a lump-sum cost for sellers, while the tax on rental income is a cost flow for landlords.

Nevertheless, the model's prediction that levying rental income tax will increase house price might seem very counter-intuitive. In particular, it is inconsistent with the classical *four-quadrant model* (see DiPasquale and Wheaton, 1992, 1996). However, in this

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<sup>&</sup>lt;sup>13</sup> Recall that the value of an occupied home for a seller is simply given by the selling price.

simple model there is the distinction between sellers and landlords. By introducing the possibility that the sellers can rent their house and landlords can sell their house, the rental income tax  $\tau_R$  introduces a further effect into the model developed here. Precisely, an increase in  $\tau_R$  reduces the value of being a landlord (D). Hence, many landlords may choose to sell their house rather than to offer rental units, thus increasing vacant houses and market frictions in the sale market. This, in turn, has a negative effect on the house price, since  $\rho_S$  depends negatively on  $\vartheta_S$  (due to the congestion externalities effect on the sellers' side). An analogous reasoning applies to the tax on property sale ( $\tau_S$ ). Therefore, it can be useful to develop in the future an extended version of the model in order to investigate the net effect of taxation on house prices.

## 5. Conclusions

In this paper we develop a matching theoretic-model that is able to capture the main characteristic of the housing market, namely the house price dispersion, and considers the rental and homeownership market together. Precisely, this housing market matching model considers two types of home seekers: people who search for a house both in the rental and in the homeownership market, and people who only search in the homeownership market. The house-search process leads to several types of matching and in turn this implies different prices of equilibrium. Also, the house-search process connects the rental market with the homeownership market. This paper is thus able to explain both the price dispersion and the relationship between rental and selling prices, relying only on the different states of home-seekers in the search and matching process. Also, this theoretical model can be useful to study the effects of taxation in the housing market.

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## **Figures**

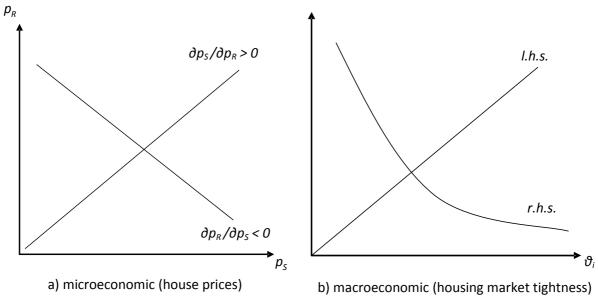


Figure 1. Equilibrium

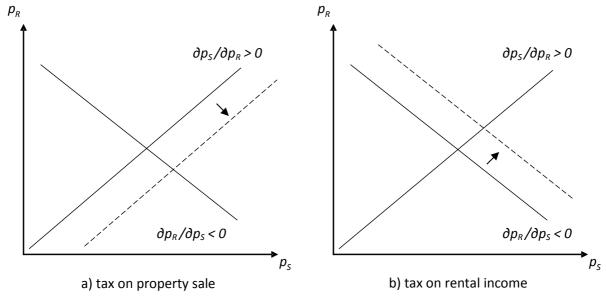


Figure 2. Effects of taxation