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Eliezer Borenstein and David Elkayam

Abstract

In this paper we attempt to reproduce both the business cycle facts and the equity premium of the Israeli economy—an economy which is "typical" in the sense that investment is much more volatile than output (and consumption). We show that GHH preferences, which are quite common in RBC models of small open economies, are not suited for reproducing both the business cycle and the equity premium facts of a "typical" small open economy. We found that a way to progress is to "correct" the GHH preferences by adding some degree of wealth effect on labor supply. That is, by switching to the Jaimovich-Rebelo (2006) type of preferences. However, in this case we also need to add to the model some kind of limitations on labor supply (we used both real wage rigidity and habits in labor). Our main finding is that the use of Jaimovich-Rebelo preferences considerably improves the results relative to that achieved by GHH preferences. The reason for this is that the GHH preferences are characterized by a relatively high degree of substitutability between consumption and leisure and this moderates the volatility of the stochastic discount factor (SDF). By adding some degree of wealth effect we can achieve a significant increase in the volatility of the SDF, and hence an increase in the equity premium and in the volatility of investment. Following the relevant literature we used three shocks: to productivity, to government expenditure and to the world interest rate. Our analysis suggests that by adding one or more of two kinds of shocks: shocks to wealth and shocks to the real exchange rate – one can achieve a significant progress in reproducing both the business cycle facts and the equity premium.

פרמיית המניות במשק קטן ופתוח, ויישום למשק הישראלי

דוד אלקיים ואליעזר בורנשטיין

תקציר

בנייר זה אנו מנסים לשחזר את מאפייני מחזור העסקים הריאלי בד בבד עם פרמיית המניות במשק הישראלי, משק שהנו "טיפוסי" במובן זה שההשקעות בו תנודתיות בהרבה מן התוצר (ומן הצריכה). אנו מראים כי פונקצית התועלת מסוג GHH, אשר הינה מקובלת למדי בניתוח מחזורי עסקים ריאליים (RBC) למשקים קטנים ופתוחים, אינה מתאימה לשחזור של מאפייני מחזור העסקים יחד עם פרמיית המניות במשק קטן ופתוח סטנדרטי. מצאנו כי ניתן לשפר את התוצאות באמצעות "תיקון" פונקצית התועלת בכיוון הוספת מידה מסוימת של השפעת רכוש על היצע העבודה, זאת באמצעות מעבר לפונקצית התועלת של Jaimovich-Rebelo (2006). עם זאת, במקרה זה עלינו להוסיף למודל גם מגבלות מסוימות על היצע העבודה (הוספנו הנחה של קשיחות של השכר הריאלי וקיום הרגלים בהיצע העבודה). הממצא העיקרי שלנו הוא כי השימוש בפונקצית התועלת של Jaimovich-Rebelo משפר באופן משמעותי את התוצאות ביחס לאלו המושגות תחת העדפות GHH. הסיבה לכך היא שהעדפות GHH מאופיינות במידה גבוהה יחסית של תחליפיות בין הצריכה לפנאי ולכך יש השפעה ממתנת על התנודתיות של מקדם ההיוון הסטוכסטי (SDF). על ידי הוספת מידה מסוימת של השפעת רכוש על היצע העבודה (ובתוספת עם מגבלות על יכולת ההתאמה של היצע העבודה) מתקבלת עלייה ניכרת בתנודתיות של ה – SDF, ובעקבותיה עלייה בפרמיית המניות ובתנודתיות של ההשקעות. בעקבות הספרות הרלוונטית השתמשנו בשלושה זעזועים: לפריון, להוצאה הממשלתית, ולשער הריבית העולמי. הניתוח שלנו מצביע על כך שעל ידי הוספת אחד או יותר משני סוגי זעזועים: זעזועי רכוש וזעזוע לשער החליפין הריאלי, ניתן להשיג שיפור ניכר ביכולת לשחזר את מאפייני מחזור העסקים יחד עם פרמיית המניות.

1. Introduction

A well known result in the business cycle–asset pricing literature is that the standard[1] real business cycle (RBC) model is unable to reproduce the equity premium observed in the data.[2] In order to be able to produce a reasonable equity premium, there must be some real frictions in the real business cycle model which make it difficult for the consumer to fully and freely smooth marginal utility of consumption in response to external shocks.[3]

Abel (1990) and Constantinides (1990) have shown that high risk aversion and high degree of habit formation in consumption can generate an equity premium in an endowment economy. Jermann (1998) extended the results to a production economy with endogenous capital but with constant labor input. The ability of the consumer-producer to freely adjust investment-savings in response to external shocks enables the consumer to smooth the marginal utility of consumption. In order to produce an equity premium in his set up, Jermann added capital adjustment costs to the production process of capital. The higher those adjustment costs are, the harder it is for the representative consumer-producer to adjust investment-savings in response to external (productivity) shocks. When one allows labor input to become endogenous in Jermann's standard RBC model, the equity premium usually disappears. The reason is that the consumer can adjust labor supply, in response to a productivity shock, and so to continue smoothing marginal utility of consumption.

Endogenous labor supply and the existence of a relatively high wealth effect on labor supply, in the commonly used preferences in real business cycle models, often cause a countercyclical behavior of hours worked in the model, while in the data labor input tend to be pro-cyclical. This outcome, often found in the models, is the result of a positive wealth effect on labor supply. A negative productivity shock reduces income (and consumption). The consumer would like to compensate for this (to smooth marginal utility of consumption) by working more hours (increase labor supply). The result is often a small (or even negative) correlation between output (and consumption) and hours worked, while in the data this correlation is usually highly positive.

Uhlig (2006, 2007) suggested overcoming the above two problems (countercyclical labor input and small equity premium) by adding the assumption of real wage rigidity to the model.

[1] By "standard" we mean a frictionless, one agent (homogenous) model such as the one presented by King, Plosser and Rebelo (2000).
[2] For a survey see, for example, Cochrane (2001).
[3] As is already known (for example Lettau and Uhlig (2002)) and will be discussed below, the shape of the utility function also has an important role in producing an equity premium.

In his set up, the labor market is always in a situation of excess supply and the quantity of labor is determined by the demand of firms. A positive productivity shock increases the demand for labor and if the shift (to the left) in the labor supply (wealth effect) is not too strong, then labor will continue to grow with output. That means that in such a situation the consumer does not have an effect on the quantity of labor (it is determined by demand only). In this set up, Uhlig showed that a sizeable equity premium can emerge in an otherwise standard RBC model.[4]

The aforementioned literature dealt with closed economy models. When one moves to an open economy (such as the model of Mendoza (1991) and Schmitt-Grohe and Uribe (2003)) another degree of freedom is added: consumers can adjust their external borrowing position. In order to produce an equity premium in such a set up, Parvar et al. (2012) added to the model adjustment costs for adjusting the external debt position of the consumer. They applied the model to the data of three South American countries (Brazil, Argentina and Chile) and succeeded in matching both the business cycle facts and the equity premium of those countries. (To the best of our—and their—knowledge, this is the only paper that tries to match business cycle moments and equity premium in a small open economy).

Parvar et al. (2012) used a model with GHH[5] preferences that is quite common in models of small open economies. An important characteristic of these preferences is the absence of a wealth affect on labor supply. This helps to reproduce the business cycle moments of an open economy without the need to "add" real wage rigidity. However, the business cycle facts of the above mentioned countries differ at least with one respect from the data of a typical small open economy. As can be seen in Table 3 of Parvar et al. (2012), in the above mentioned South American countries (and in the specific time period of their research[6]) the volatility of investment is quite similar to that of consumption.[7] On the other hand, in our data (as well as in most small open economies[8]) the standard deviation of investment is significantly larger

[4] We can name at least two other ways to overcome the above two problems caused by endogenous labor supply (in a representative agent model). Boldrin, Christiano and Fisher (2001) added friction to the labor market by specifying limited sectoral mobility. Jaccard (2010) added to the model internal habit formation in labor.

[5] This function was first proposed by Greenwood, Hercowitz, Huffman, (1988).

[6] Their research covers the period 1993 to 2007.

[7] The standard deviation of investment and consumption is 5.56 and 4.73 percent respectively in Argentina. In Brazil, the numbers are 19.9 and 18.66, respectively, and in Chile 10.8 and 6.8. In Israel, the figures are 12.3 and 2.2, respectively.

[8] For example: Canada, Portugal, Finland, Norway, Portugal and Belgium. In these countries the volatility of investments is more than 3 times the volatility of consumption.

than that of consumption[9]. As will be discussed later in this paper, it seems that the GHH utility used (by Parvar et al.) is not suited to reproduce both the business cycle and the equity premium facts of a "typical" small open economy.

In this paper we are trying to reproduce both the business cycle facts and the equity premium of the Israeli economy, which is typical in the sense that investment is much more volatile than output (and consumption). Following Parvar et al. (2012), we started with the GHH utility. Using this function we found it difficult to reproduce both the business cycle facts and the equity premium of our data, which are quite similar to a typical small open economy. We found that a way to progress is to "correct" the GHH preferences by adding some degree of wealth effect on labor supply. We do this (as will be explained later) by switching to the Jaimovich-Rebelo (2006) type of preferences. However, in this case we also need to add to the model some kind of limitations on labor supply (we used both real wage rigidity, of the kind proposed by Uhlig (2006, 2007), and habits in labor). As we shall see later, the addition of some degree of wealth effect (and wage rigidity) can improve the fit even when we look only on the business cycle moments (i.e., ignoring the equity premium).

Our main finding in this paper is that the use of Jaimovich-Rebelo (henceforth JR) preferences considerably improves the results relative to that achieved by GHH preferences. The reason for this, as will be detailed later, is that the GHH preferences are characterized by a relatively high degree of substitutability between consumption and leisure, and this moderates the volatility of the stochastic discount factor (SDF). By adding some degree of wealth effect we can get a significant increase in the volatility of the SDF, and hence an increase in the equity premium and in the volatility of investment.

An intuitive explanation is as follows. Assume a negative technology shock which reduces consumption (and thus increases the marginal utility of consumption) and reduces the demand for labor. In both GHH and JR preferences, consumption and leisure are substitutable. In the GHH case the wealth effect on labor supply is zero—that is, the decline in labor moderates, to a large degree, the increase in the marginal utility of consumption, such that the consumer is not interested in increasing labor supply. In the case of JR preferences the wealth effect is positive—that is, the consumer wishes to increase labor supply in order to moderate the increase in the marginal utility of consumption. But, because of the existence of wage rigidity, he is prevented from doing so freely and this generates increased volatility of the marginal utility of consumption relative to the case of GHH.

[9] In fact, even for the above mentioned South American countries, if we look at a longer period then that used by Parvar et al. we see much more regular behavior of the business cycle moments.

In the model we used three shocks: to productivity, to government expenditure and to the world interest rate. An interesting finding of this paper is that additional shocks might make large contributions to the equity premium. As will be detailed later, this may include shocks to wealth and shocks to the real exchange rate.

In the next section we present the model. In section 3 we present and discuss the data and in section 4 we discuss the calibration of the parameters. In section 5 we review in brief the relevant asset pricing equations. In section 6 we present and discuss the results and section 7 concludes. Most of the technical aspects of the paper are left to the appendix.

2. The model

2.1 Households

Following the relevant literature[10] we assume a small open economy with infinite number of identical households. The representative household has the following momentary utility function:

(1) $U_t = U(C_t - \chi^c \tilde{C}_{t-1}, H_t - \chi^h \tilde{H}_{t-1})$

Where: C_t and H_t represent consumption and labor input of the representative household. We assume the existence of external habit formation both in consumption and in labor input. $\tilde{C}_{t-1}$ and $\tilde{H}_{t-1}$ represent aggregate consumption and aggregate labor input and 0< χ^c <1 and 1< χ^h <1 are parameters representing the degree of habit in consumption and in labor input.

In each period the (representative) household faces a budget constraint that is represented by the following two equations:

(2) $W_t H_t + V_t^k K_{t-1} = C_t + [I_t + \Phi(K_t - K_{t-1})] + \Gamma_t + [TB_t + \Theta(D_t - D_o)]$

(3) $D_t = R_{t-1}^f D_{t-1} - TB_t$

The left hand side of equation (2) represents household current income, which is the sum of labor income and capital income, where W_t and V_t^k represent the wage rate and the rental rate of capital. The right hand side of the equation represents the uses of that income: consumption (C_t), investment in physical capital (I_t), lump sum taxes (Γ_t), investment abroad (the trade balance) (TB_t) and two special components: a cost of adjusting capital, $\Phi(K_t - K_{t-1})$, and a cost of adjusting foreign assets ($\Theta(D_t - D_o)$). K_t and D_t are the capital

[10] In the specification of the model we follow Uribe and Schmitt-Grohe (2003).

stock and the foreign debt at the end of period t (the beginning of period $t+1$), and $\Phi(.)$ and $\Theta(.)$ are concave cost functions. Equation (3) represents the evolution of foreign debt, where R_{t-1}^{f} is the world (gross) interest rate at period t, which is determined at the end of $t-1$. We assume that R_{t-1}^{f} is exogenous and stochastic.

Equation (4) describes the evolution of the capital stock, where δ is depreciation rate.

(4) $$K_t = (1-\delta)K_{t-1} + I_t$$

We substitute TB_t and I_t from (3) and (4) into (2). Households choose a process $\{C_t, H_t, Y_t, I_t, K_t, D_t\}_{t=0}^{\infty}$ that maximizes lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1})$$

Subject to equation (2), where $0<\beta<1$ is the rate of time preference.[11]

Let Λ_t be the Lagrange multiplier on equation (2). The first order conditions of the maximization problem are equations (2) to (4) above and (5) to (8) ahead:

(5) $$\Lambda_t = \frac{\beta R_t^{f} E_t(\Lambda_{t+1})}{(1-\Theta_D(D_t - D_0))}$$

(6) $$\Lambda_t = U_c(C_t - \chi^c \tilde{C}_{t-1}, H_t - \chi^h \tilde{H}_{t-1})$$

(7) $$-U_h(C_t - \chi^c \tilde{C}_{t-1}, H_t - \chi^h \tilde{H}_{t-1}) = \Lambda_t W_t$$

(8) $$\Lambda_t = \beta E_t[R_{t+1}^{eq} \Lambda_{t+1}]$$

Where: R_{t+1}^{eq} is the gross return on investing in capital stock and is given by:

(9) $$R_{t+1}^{eq} = \frac{V_{t+1}{}^{k} + (1-\delta) + \Phi_k(K_{t+1} - K_t)}{1 + \Phi_k(K_t - K_{t-1})}$$

The above first order conditions (equations (5) to (8)) are quite standard. Equation (6), the first order condition with respect to consumption, defines the shadow value of wealth (Λ_t) in terms of consumption.[12] Equation (7) is the first order condition with respect to supply of labor. Equation (5) and (8) are the pricing equations for foreign bonds and stocks.

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[11] The maximization is also subjected to a no-Ponzi constraint with respect to D_t.

[12] For ease of exposition we treat the utility function as time separable. In this case the derivative of lifetime utility is equal to that of the momentary utility. This is true for GHH or KPR preferences (see next section). For time non-separable preferences one should replace $U_c()$ in (6) by the derivative of lifetime utility with respect to c_t.

That is, the inter-temporal Euler equations that describe the conditions that the returns on bonds and stocks need to satisfy.

2.2 Firms

We assume an infinite number of identical competitive firms. They are owned by households and produce a final good that is a perfect substitute to the foreign produced final good. The (representative) firm hires labor services and rents capital stock from households, to produce output Y_t, according to a Cobb-Douglas production function:

(10) $$Y_t = A_t F(K_{t-1}, H_t) = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)},$$

Where A_t is the technology level. In each period the firm chooses H_t and K_{t-1} to maximize its profits: $A_t K_{t-1} H_t^{(1-\alpha)} - V_t^k K_{t-1} - W_t H_t$

The first order conditions are:

(11) $$W_t = A_t F_h(K_{t-1}, H_t) = A_t (1-\alpha)(\frac{K_{t-1}}{H_t})^{\alpha} = (1-\alpha)\frac{Y_t}{H_t}$$

(12) $$V_t^k = A_t F_k(K_{t-1}, H_t) = A_t \alpha (\frac{K_{t-1}}{H_t})^{(\alpha-1)} = \alpha(\frac{Y_t}{K_{t-1}})$$

2.3 The driving forces

Government consumption (G_t) is assumed exogenous, stochastic, nonproductive and financed by lump-sum taxes. That is:

(13) $$G_t = \Gamma_t$$

For the exogenous variables we assume the following AR(1) process:

(14) $$\ln G_t = (1-\rho^g)\ln G_0 + \rho^g \ln G_{t-1} + \varepsilon_t^{\,g}$$

(15) $$\ln R_t^{\,f} = (1-\rho^f)\ln R_0^{\,f} + \rho^f \ln R_{t-1}^{\,f} + \varepsilon_t^{\,r}$$

(16) $$\ln A_t = \rho^a \ln A_{t-1} + \varepsilon_t$$

Where G_0 and $R_0^{\,f}$ are steady state values. The innovations ($\varepsilon_t^{\,g}, \varepsilon_t^{\,r}, \varepsilon_t$) are assumed to be i.i.d variables, with variances $\sigma^2{}_g, \sigma^2{}_r, \sigma^2$, and are also not correlated with each other.

For the cost functions $\Phi(K_t - K_{t-1})$ and $\Theta(D_t - D_o)$ we assume the following specification:

(17) $$\Phi(K_t - K_{t-1}) = 0.5\phi^k (K_t - K_{t-1})^2$$

(18) $\Theta(D_t - D_o) = 0.5\phi^d (D_t - D_{t-1})^2$

To solve the model we need to specify an explicit utility function. The solution is a set of stochastic processes of the endogenous variables $\{C_t, H_t, Y_t, I_t, K_t, D_t, W_t, V_t^k, \Gamma_t\}_{t=0}^{\infty}$ and of the driving forces $\{G_t, R_t^f, A_t\}_{t=0}^{\infty}$ satisfying equations (2)-(16) given equations (17)-(18) and the initial conditions for K_t, D_t and for the shocks $\varepsilon_t^g, \varepsilon_t^r, \varepsilon_t$.

2.4 The utility function

For the utility function we shall use the specification that was suggested by **Jaimovich-Rebelo** (2006):

(20) $$U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \frac{\{(C_t - \chi^c \tilde{C}_{t-1}) - \psi[(H_t - \chi^h \tilde{H}_{t-1})]^{1+\gamma^h} X_t\}^{1-\gamma^c} - 1}{1-\gamma^c}$$

Where

(21) $$X_t = (C_t - \chi^c \tilde{C}_{t-1})^{\gamma} X_{t-1}^{1-\gamma}$$

γ^c and γ^h are curvature parameters[13] and ψ is a scale parameter.

The parameter $0 \le \gamma \le 1$ governs the magnitude of the wealth elasticity of labor supply. As γ declines the (negative) income effect on labor supply declines (in absolute value). In the polar case of γ=0, the function gets the form of GHH utility which has the form:

(20a) $$U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \frac{[(C_t - \chi^c \tilde{C}_{t-1}) - \psi[(H_t - \chi^h \tilde{H}_{t-1})^{1+\gamma^h}]^{1-\gamma^c} - 1}{1-\gamma^c}$$

One of the characteristics of that function is that labor supply depends only on the real wage and is independent of the marginal utility of income (the income elasticity of labor supply is null). In the other polar case, when γ=1, the function gets the form of KPR[14] utility:

(20b) $$U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \frac{\{(C_t - \chi^c \tilde{C}_{t-1})[1 - \psi(H_t - \chi^h \tilde{H}_{t-1})^{1+\gamma^h}]\}^{1-\gamma^c} - 1}{1-\gamma^c}$$

Several papers have shown that a GHH utility function is more suited than the KPR function to reproduce the business cycles facts of a small open economy.[15] As we shall see

[13] In the absence of consumption habits γ^c is the coefficient of relative risk aversion and in the absence of labor habits γ^h is the inverse of the Frisch elasticity of labor supply.

[14] After King, Plosser, Rebelo,(1988).

[15] Correia et al. (1995), Uribe and Grohe (2003).

later, by increasing γ from 0 to about 0.05 we can do better (relative to GHH and KPR utilities) in reproducing the business cycle facts of the Israeli economy.

Recently, Parvar et al. (2012) used a model with GHH preferences and successfully reproduced both the business cycles facts and the equity premium of several South American countries. However, the business cycle properties of those countries differ in several respects from our data.[16] As can be seen in Table 3 of Parvar et al. (2012), in the South American countries (to which the paper refers), and during the specific period under study, the volatility of investment is similar to that of output.[17] On the other hand, in our data (as well as in many small open economies) the standard deviation of investment is much larger than that of output and consumption. We shall refer to this issue later on and claim that it is not possible to reproduce both the business cycle facts and the equity premium of our data with a GHH utility. For that purpose we shall have to add some wealth effect by using the Jaimovich-Rebelo type of utility (that is, increasing γ above zero).

2.5 Adding real wage rigidity

Preliminary simulations with Jaimovich-Rebelo preferences with a positive wealth effect (positive γ) produced a much lower correlation between output (y_t) and labor (h_t) than in the data. This is a well known problem with this kind of utility function. To overcome this problem (and to be able to produce an equity premium, as will be detailed later) we followed Uhlig (2006, 2009) and added the assumption that the real wage is rigid. More specifically, we replaced the labor supply equation (7) with the following two equations:

(22) (7′) $-U_h(C_t - \chi^c \tilde{C}_{t-1}, H_t - \chi^h \tilde{H}_{t-1}) = \Lambda_t W_t^{\,f}$

(23) (7″) $W_t = (W_t^{\,f})^{1-\mu} W_{t-1}^{\mu}$

Where $W_t^{\,f}$ stands for the frictionless real wage and $0 \leq \mu \leq 1$ is the degree of real wage rigidity. As has been shown by Uhlig (2006, 2009) and will be also demonstrated below, real wage rigidity can be an important source of the equity premium.

[16] Parvar et al (2010) used quarterly data for the years 1996 to 2007. We used yearly data for the period 1960 to 2008.

[17] See footnotes 7 and 8.

3. Asset pricing

3.1 The equity premium and the Sharpe ratio

In this section we present the asset pricing equations that were used to calculate the equity premium and the Sharpe ratio[18].

Let $x_t = \ln(X_t)$ be normally distributed, then:

(24) $\ln E(X_t) = E\ln(X_t) + 0.5\,\text{var}(\ln(X_t)) = E(x_t) + 0.5\,\text{var}(x_t)$

The F.O.C for capital, equation (8) above can be written as:

(25) $1 = E_t(M_{t+1}R_{t+1}^{eq})$

Where: R_{t+1}^{eq} is the gross return on equity (capital) and M_{t+1} is the S.D.F, that is: $M_{t+1} = \dfrac{\beta\Lambda_{t+1}}{\Lambda_t}$. Taking logarithms and assuming that R_{t+1}^{eq} and M_{t+1} are log-normally distributed, we obtain, after a little bit of algebra (using equations (24) and (25)):

(27) $\ln E_t(R_{t+1}^{eq}) + \ln E_t(M_{t+1}) = -Cov_t(r_{t+1}^{eq}, m_{t+1})$

where: $Var_t(m_{t+1}) = E_t[m_{t+1} - E_t(m_{t+1})]^2$ etc.

For a risk free asset, the return at time t+1, R_{t+1}^{f}, is known at time t. Using this in the pricing equation (27), and using the fact that

$Var_t(r_{t+1}^{f}) = 0$ And $Cov_t(r_{t+1}^{f}, m_{t+1}) = 0$, we get the pricing equation for a risk free asset:

(28) $r_{t+1}^{f} = -\ln E_t(M_{t+1})$

From (27) and (28) we get the following condition for the equity risk premium:

(29) $EP_t = \ln E_t(R_{t+1}^{eq}) - r_{t+1}^{f} = -Cov_t(r_{t+1}^{eq}, m_{t+1})$

Now, observe that: $m_{t+1} = \ln(\dfrac{\beta\Lambda_{t+1}}{\Lambda_t}) = \ln\beta + \lambda_{t+1} - \lambda_t = \ln\beta + \Delta\lambda_{t+1}$

Using this in (28) we have:

(30) $r_{t+1}^{f} = -\ln\beta - E_t(\Delta\lambda_{t+1}) - 0.5Var_t(\Delta\lambda_{t+1})$

Note also that:

$$Var_t(\Delta\lambda_{t+1}) = E_t(\Delta\lambda_{t+1} - E_t(\Delta\lambda_{t+1}))^2 = E_t(\lambda_{t+1} - \lambda_t - E_t(\lambda_{t+1} - \lambda))^2 =$$
$$E_t(\lambda_{t+1} - E_t(\lambda_{t+1}))^2 = Var_t(\lambda_{t+1})$$

[18] The material of this section is based on Uhlig (2006). For more detailed derivation see the appendix.

Using this in (29) we get:

$$(31)\quad \begin{aligned} EP_t &= \ln E_t(R_{t+1}^{eq}) - r_t^f = -Cov_t(r_{t+1}^{eq}, \Delta\lambda_{t+1}) = -Cov_t(r_{t+1}^{eq}, \lambda_{t+1}) \\ &= -\rho_t(r_{t+1}^{eq}, \lambda_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(\lambda_{t+1}) \end{aligned}$$

and the Sharpe ratio is

$$(32)\quad SR_t = \frac{EP_t}{\sigma_t(R_{t+1}^{eq})} = \frac{-Cov_t(r_{t+1}^{eq}, \lambda_{t+1})}{\sigma_t(R_{t+1}^{eq})} = -\rho_t(r_{t+1}^{eq}, \lambda_{t+1})\sigma_t(\lambda_{t+1})$$

To ease the calculation of the EP and SR from the outcomes of a log-linearized DSGE model, note that for each variable X_{t+1}:

$Var_t(\ln X_{t+1}) = Var_t(\ln X_{t+1} - \ln X_0)$ etc., where: X_0 is the steady state value of X .

3.2 The equity premium and the Sharpe ratio in terms of the elasticities of the marginal utility

Log linearization of the F.O.C (6) above yields:

$$(33)(6')\quad \lambda_t = -\eta_{cc}(c_t - \chi^c \tilde{c}_{t-1}) + \eta_{ch,h}(h_t - \chi^h \tilde{h}_{t-1})$$

where: $\eta_{cc} = -\dfrac{C_0 U_{cc|0}}{U_{c|0}}$ and $\eta_{ch,h} = \dfrac{H_0 U_{ch|0}}{U_{c|0}}$

Substitute λ_t from (33) in (31) we get:

$$(35)(31')\quad \begin{aligned} EP_t &= \eta_{cc} Cov_t(r_{t+1}^{eq}, c_{t+1}) - \eta_{ch,h} Cov_t(r_{t+1}^{eq}, h_{t+1}) \\ &= \eta_{cc}\rho_t(r_{t+1}^{eq}, c_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(c_{t+1}) - \eta_{ch,h}\rho_t(r_{t+1}^{eq}, h_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(h_{t+1}) \end{aligned}$$

For the equity premium;

And for the Sharpe ratio we have

$$(36)(32')\quad SR_t = \eta_{cc}\rho_t(r_{t+1}^{eq}, c_{t+1})\sigma_t(c_{t+1}) - \eta_{ch,h}\rho_t(r_{t+1}^{eq}, h_{t+1})\sigma_t(h_{t+1}) .$$

In appendix A.4 we present the detailed calculation of η_{cc} and $\eta_{ch,h}$ for the case of JR, GHH and KPR preferences.

4. The data

For the calibration and for the empirical moments we used the following data:

National Accounts

We use yearly data for the period 1960 to 2008 on gross domestic product (Y), private consumption excluding durables (C),[19] gross domestic investments (I), government consumption (excluding direct defense imports) (G) and trade balance (TB) to output ratio (TBY)[20]. All series are in terms of fixed prices and per capita. For the analysis we expressed the variables in terms of percentage deviations from HP trend.

Wages and Labor

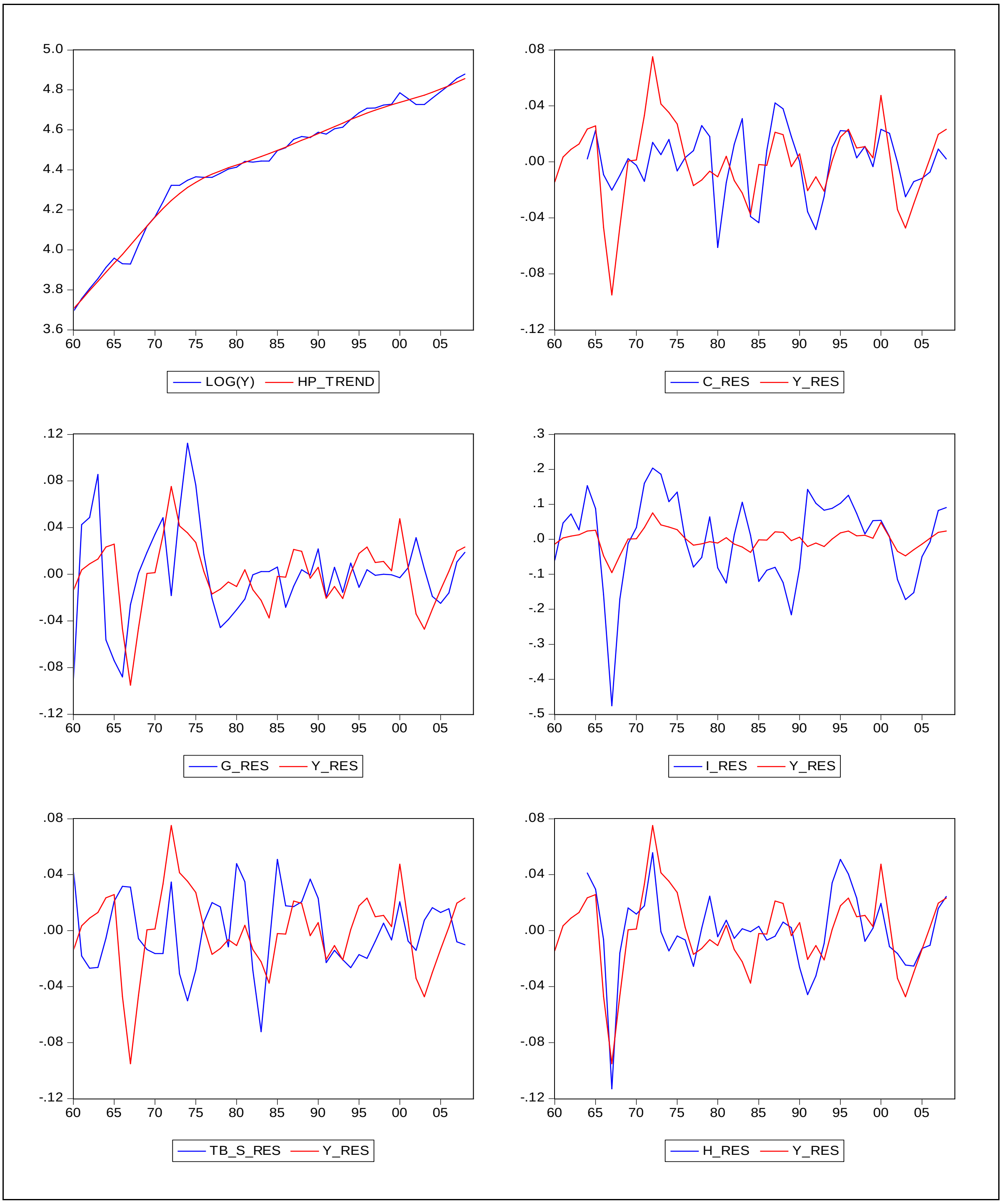
For labor we used the number of total employees (per capita) multiplied by the average working hours per employee (i.e., total hours worked per capita). For wage we used the averaged monthly wage per employee post, deflated by the CPI. Both series are expressed in terms of percentage deviations from an HP trend.

Graph 1 presents the behavior of the main components of the national accounts variables during the period 1960 to 2008. As can be seen, consumption's volatility is quite similar to GDP's volatility, whereas Investment's volatility is much higher. Another noticeable fact is the high positive correlation between Labor and GDP (that is, labor is pro-cyclical).

[19] We also checked the series of total private consumption and report its statistics in the relevant tables.

[20] The trade balance (TB) was calculated as the difference between the GDP and the sum of the three uses: private consumption, government consumption and gross domestic investments,

Graph 1: Behavior of main National Accounts data during 1960 to 2008



Financial Data

For the foreign risk free rate, we used monthly data (for 1954–2009) on the nominal yield to maturity on one-year US Treasury bills. We subtracted actual annual inflation (i.e., in the last twelve months) from the nominal yield and then the data were yearly averaged. For the domestic risk free asset, we used data (for 1966–2009) on the yield to maturity on one-year Israeli government CPI indexed bonds.[21]

For the domestic stocks we used monthly data (for 1971–2010) on the real total return on stocks. As another alternative to the holding of a risk free asset, we also used data on the real total return on the whole portfolio of government indexed bonds (for 1976–2010).

Table 1 summarizes the main asset pricing facts for the Israeli economy during the years 1971 to 2010. During that period, the average yearly real return on stocks was 12.73 percent, while the real return on one year governments bonds was 3.34 percent. That is, we observe an average equity premium of 9.39 percent per year, which is quite in line with what is observed in other countries. We should also note the very high volatility of the yearly rate of return on stocks, 35.06 percent, which is by far larger than that observed in other countries. Note that the equity premium is quite stable during most of the period (in 2000-2010 it seemed to decline due to a decline in stocks return). Due to the very large volatility of stocks' return the Sharpe ratio (SR) is relatively low, about 0.26 compared to a figure of about 0.50 in US.

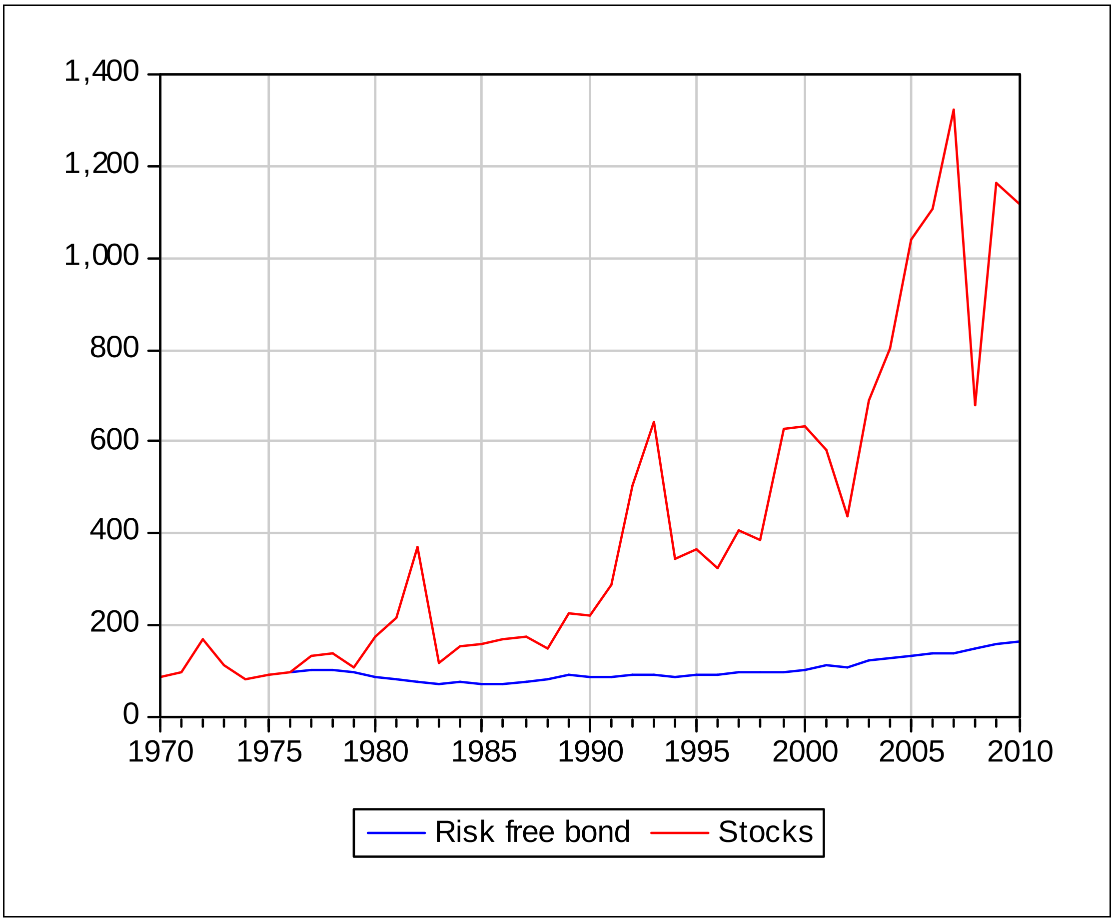
Graph 2 presents the value of the two following portfolios: The first, represented by the lower line, is composed of government indexed bonds. The second, represented by the upper line, is composed of stocks. Both portfolios were scaled to 100 in 1976 (the beginning of our risk free bond data). As is clear from the graph, the value of the stock portfolio is much more volatile than the bond portfolio, and its growth was much higher—while the "risk free" portfolio's value grew by 60%, the value of the stock portfolio grew by more than 1000%. This graph offers a good visual presentation of the basic risk-return tradeoff: for higher returns one has to bear higher risks.

[21] Constructed from various sources.

Table 1 – Real return on stocks and bonds in the Israeli economy, 1971–2010

	Equity total return		Yield to maturity of a 1-year indexed government bond		Total real return on a portfolio of indexed bonds	
Period	**Mean**	**Standard deviation**	**Mean**	**Standard deviation**	**Mean**	**Standard deviation**
1971–2010	12.73	35.06	3.34	2.69		
1977–2010	13.63	35.07	2.88	2.51	1.62	5.98
1987–2010	13.27	33.39	3.17	2.27	3.54	4.99
1992–2010	13.12	35.74	3.68	2.07	3.36	4.59
2000–2010	10.60	34.61	3.22	2.35	4.84	4.98

Graph 2: Asset pricing facts of the Israeli economy – 1970–2010



5. Calibration

We first refer to the calibration of the model under the GHH preferences. Later we will present the changes made under Jaimovich-Rebelo preferences.

We need to calibrate the following list of parameters:

$$\{ \gamma^c, \gamma^h, \psi, \chi^c, \chi^h, \beta, \alpha, \delta, R_0, \rho^r, \rho^g, \sigma_r, \sigma_g, s_g, s_{tb}, \rho, \sigma, \phi^k, \theta^d, \mu \}$$

In doing so we followed a similar strategy to that of Parvar et al. (2012). We divided the parameters of the model into three groups.

The first group contains the parameters of the utility function: $\{ \gamma^c, \gamma^h, \psi, \chi^c, \chi^h, \beta \}$. We do not have a strong a priori knowledge of the values of the curvatures γ^c and γ^h and on the habits parameters χ^c and χ^h. We started with values that are common in the relevant literature and performed sensitivity checks. Thus, for the curvature parameter on consumption, γ^c, we used three values: 1, 2 and 5, and eventually we chose 5. For the curvature on employment, γ^h, we checked several values in the range of 0.5-2, and we chose 0.9 eventually[22] in order to get the volatility of labor close to the data. For the habit in consumption and in labor we tried 0, 0.4 and 0.6 and eventually we chose 0.6 for both. The labor parameter ψ was chosen such that the steady state value of labor is the same as in Schmitt-Grohe and Uribe (2003)[23] (note that ψ depends also on the habits coefficients). β is set to fulfill the steady state relation: $\beta = 1/R_0$.

The second group includes parameters that we calibrated using historical data. This group contains: $\{ \alpha, \delta, R_0, \rho^r, \rho^g, \sigma_r, \sigma_g, s_g, s_{tb}, \mu \}$.

$1-\alpha$, the share of labor in national income, is set at 0.67, based on National Accounts data. To estimate parameters of the world interest rate (ρ^r, σ_r) we used yearly data (for the years 1954 to 2009) on the real yield to maturity on one-year US Treasury bills (details in appendix). The estimates are 0.695 and 0.0137 for ρ^r and σ_r respectively. For the steady

[22] In the absence of habit in labor this means a Frisch elasticity of 1.1.

[23] In their paper they have a steady state value of 1.00742 for labor. Correia et al. (1995) applied the following methodology to calibrate the steady state value of h: they assumed that there are 7×14=98 potential working hours in a week and that average work week is 40 hours. Multiplying this by the employment rate yields a value for the steady state of H. In Israel, the sample average of the employment rate is 0.56. So applying this methodology to Israel yields: $H_0 = 0.56 * 40 / 98 = 0.23$. However, using this value caused us problems in the solution of the model when the risk aversion and habit parameters took high values. Therefore, we decided to stay with the Schmitt-Grohe and Uribe calibration.

state risk free rate $(R_0 - 1)$ we used the sample mean of real yield to maturity on a one-year Israeli government bond (0.033[24]). We set: $\delta = 0.1$, which is quite close to what is derived from the sample mean of the ratio of investment to output.[25]

The parameters ρ^g and σ_g were estimated using an auto-regression of the H.P. filtered deviations of government consumption, for the period 1980 to 2008.[26] To estimate s_g, the government consumption share in output, we used the sample for that period.[27]

For s_{tb}, the trade balance share of output, we used the sample mean in the period 1960 to 2008, and got value of -0.048.[28]

As for the parameter μ (wage rigidity), we first tried to estimate it by a regression based on data of real wage, consumption and employment (all in terms of deviations from H.P trend). The log linearized version of equations (20) and (21) is[29]:

(39) (7''') $\quad w_t^f = (\eta_{hh} - \eta_{ch,h})(h_t - \chi^h h_{t-1}) + (\eta_{cc} + \eta_{hc,c})(c_t - \chi^c c_{t-1})$

(40)(7''') $\quad w_t = (1-\mu) w_t^f + \mu w_{t-1}$

Plugging w_t^f from (39) into (40) we get:

(41) $\quad w_t = (1-\mu)(\eta_{hh} - \eta_{ch,h})(h_t - \chi^h h_{t-1}) + (1-\mu)(\eta_{cc} + \eta_{hc,c})(c_t - \chi^c c_{t-1}) + \mu w_{t-1}$

---

24 Another possibility is to use the mean of the sample in terms of U.S data (which happens to be 0.017). We prefer to base the estimate of $R_0 - 1$ on Israeli data. The difference (0.033-0.017) can be interpreted as a "constant" risk premium.

25 In steady state (ignoring growth) we have: $\frac{I_0}{Y_0} = \frac{\alpha\delta}{R_0 - 1 + \delta} = \frac{0.33 * 0.1}{0.03 + 1} = 0.254$.

In the data, if investment includes durables consumption goods, the average investment output ratio is 0.246 (without durables goods the ratio is 0.215).

26 We shortened the period because of major change that took place in the process for government consumption.

27 The value in the data was 0.31. We used a value a bit lower, 0.28, since it performed better in the model solution, due to numerical issues.

28 Since we also wanted to calibrate the debt to GDP ratio to be similar to the data (around 0.2), we slightly modified eq. 3 such that the debt evolvement is determined by the current account and not the trade balance by itself. The ratio of current account to GDP was calibrated such that the steady state value of debt to GDP will equal 0.2.

29 For the GHH (39) takes the form: $\frac{\gamma^h}{(1-\chi^h)} h_t = w_t - 0 \cdot c_t$ and for the KPR we have:

$\frac{\gamma^h + Z_0}{(1-\chi^h)(1-Z_0)} h_t = w_t - 1 \cdot c_t$ where $Z_0 = \psi[(1-\chi^h)H_0]^{1+\gamma^h}$ for details see appendix A4.

Using the calibrated values for χ^c, χ^h in (41) we can estimate μ in (41). We estimated (41) by OLS and by 2SLS[30]; in both cases the estimates were close to 0.76. For the calibration we used a slightly higher value, 0.85.

Table 2: Calibrated and estimated parameters

Parameter	γ^c	γ^h	χ^c	χ^h	β
Final value	5	0.5	0.6	0.6	$1/R_0$
Sensitivity	1,2,5	2,1,0.5	0,0.4,0.6	0,0.4, 0.6	

Parameter	α	δ	R_0	ρ^r	ρ^g	σ_r	σ_g	s_g	s_{tb}	μ
Value	0.33	0.1	1.033	0.695	0.248	0.0137	0.0147	0.28	-0.048	0.85

The third group contains five parameters $\{\rho, \sigma, \phi^k, \theta^d, \gamma\}$. To find the value of those parameters, we conducted a grid search on the first four, for various values of γ (as will be detailed in the next section). In the grid search we tried to match the following ten moments: standard deviation of output (σ_y), standard deviation of consumption (σ_c), standard deviation of investments (σ_i), standard deviation of labor input (σ_h), standard deviation of trade-balance to output ratio (σ_{tby}), first order auto-correlation of output ($\rho_{y,y_{t-1}}$), labor-output correlation ($\rho_{h,y}$), investment-output correlation ($\rho_{i,y}$), trade-balance to output ratio-output correlation ($\rho_{tby,y}$) and mean equity premium (EP). Specifically, we searched for the parameters that minimize a loss function composed of the square distance between the simulated moments and the moments in the data.[31]

We conducted the grid search under two alternatives. In the first, alternative 1, we looked only at the business cycle aspect of the model. That is, we excluded the equity premium from the loss function. In the second alternative, alternative 2, we added the equity premium to the loss with a weight of 0.5 (i.e., we assigned a 50 percent weight to the real business cycle moments and a 50 percent weight to the equity premium). The calibrated values of the first and the second group of parameters are listed in Table 2.

[30] We used the following instruments: $w_{t-2}, w_{t-3}, c_{t-2}, c_{t-3}, h_{t-2}, h_{t-3}$, for the period 1986-2008.

[31] For details see appendix A.6.

6. Results

In the first subsection we will present the main results. In the second we shall expand on the explanations. In the third subsection we shall refer to the role of the government expenditure shock and to the potential role of other wealth shocks. In the fourth subsection we shall refer to the potential role of shock to the real exchange rate.

6.1 Main results

In Table 3 we present the results obtained under various values of γ, between 0 and 1. In the first column of Table 3 we present the relevant moments of the Israeli data. Apart from the first column, the table contains four additional **blocks** and each block contains two **columns**. In the first column of each block (labeled "No EP") we present the results obtained when we consider only the business cycle aspect of the data (i.e., ignoring the risk premium in the loss function; we also refer to that alternative as "alternative 1"). In the second column of each block (labeled "EP") we present the results when the equity premium is added to the loss function (we also refer to that alternative as "alternative 2"). The last two rows of each column in blocks 1 to 4 in Table 3 contain the value of the loss function calculated with the relevant parameters. The first row, labeled "No EP" presents the loss when the equity premium is absent from the loss function. The second row, labeled "EP", presents the loss when the equity premium is included in the loss function. In each column we present also the optimum values of the four "searched by grid" parameters (ϕ^k , θ^d , ρ , σ).

In block 1 we present the results under GHH preferences (that is, under $\gamma = 0$). Looking at the first column of block 1 we can see that most of the moments are quite close to the data. This is not surprising since GHH preferences are known to provide successful replication of the business cycle moments of a "typical" small open economy.[32] The equity premium under this alternative is 0. When we move to alternative 2—that is, we add a 9 percent equity premium to the loss function with a weight of 50 percent—we achieve a large, close to data, equity premium (8.87) but the business cycle "fit" is worsened considerably.[33] The value of the loss function in this case is 0.1035. As can be seen, in order to achieve a large equity premium, ϕ^k and ϕ^d must increase considerably. The "price" is a large reduction in the

[32] Correia et al. (1995) and Uribe-Grohe (2003).
[33] The value of the loss function of alternative 1, with the searched by grid parameters of alternative 2 is 0.2069 compared to a value of 0.0510 achieved with the parameters of alternative 1 (that is it is 4 times bigger) .

standard deviations of i and of tb/y. The standard deviation of investment reduced to 3.6, from a value of 10.6 under alternative 1 (where in the data the relevant number is 12.3) and the standard deviation of the trade balance to output ratio declines to 0.4 from a value of 2.4 under alternative 1 (where in the data the relevant number is 2.6). This is not a surprising result. In order to reproduce an equity premium in a standard RBC model we need to "add" significant real rigidities. A larger value of ϕ^k means a larger cost of adjusting the capital stock and this makes it difficult for the producer-consumer to smooth the marginal utility of consumption in response to external shocks. Similarly, a large value of ϕ^d means a large cost of adjusting the external borrowing position for the producer-consumer when he tries to smooth the marginal utility of consumption in response to shocks.

As will be detailed below, it seems that this result (the large reduction of the standard deviation of investment) is general in the sense that GHH preferences are not so suitable for replicating both the business cycle and the equity premium of a typical small open economy. We found that adding some wealth effect to labor supply, (that is, to increase γ above zero) helps substantially in matching the data.

Block 2 presents the results when $\gamma = 0.05$. This value yielded the lowest loss (0.0733 under alternative 2). Comparing the results of alternative 2 in blocks (2) and (1) we see that increasing γ (from 0 to 0.05) enables us to achieve a similar equity premium but with a much better fit of the business cycle moments. We see an improvement in the standard deviations of y, c, i and h and in the ratio of the standard deviation of c, i, and h relative to that of y. Notice also that the ratio of the standard deviation of i to c increased from about 1.13 (3.6/3.2 in block 1, to about 3.1 (8.0/2.6) in block 2 (which is still, however, lower than the ratio of 5.6 (12.3/2.2) in the data). In the next subsection we shall expand on the causes of this improvement.

When we compare the results of alternative 1 in blocks (2) and (1) we see that the results under $\gamma = 0.05$ are better than those achieved using the GHH preferences (the loss reduced from 0.0510 to 0.0175). That is, at least in our calibration, even when we concentrate only on the business cycle moments we can substantially improve the fit by increasing γ. We can see an improvement in the standard deviation of output, consumption and investments and in the ratio between them (that is, in the ratio of the standard deviation of consumption and investment relative to that of income). Note also the improvement of the correlation of i and h with y. In blocks 3 and 4 we present the results when we get closer to KPR preferences

($\gamma = 0.50$ and $\gamma = 0.99$). As can be seen, under both alternatives there is an increase in the loss (i.e., the fit worsens) the closer we are to the KPR preferences. Comparing the results to the GHH preferences we see that when $\gamma = 0.99$, that is, we are very close to KPR preferences, the loss under alternative 1 is lower than in the GHH case, whereas under alternative 2 it is higher.

It should be noted that in all of the cases, when we search for an equity premium, σ_{r^f}, the standard deviation of the risk free interest rate is very high compared to its standard deviation in the data (2.7%). This is known as the risk free rate volatility puzzle and is a feature of many DSGE models. Parvar et al. (2012) claim that their model does not present this problem since the risk free rate according to them is pinned down by the world's interest rate which is not very volatile, and yet the equity premium is high enough because the debt adjustment costs cause the IMRS to become volatile enough. In our view, the correct interest rate that should be considered as the risk free rate, and the one that should be compared to the data, is the one taking into account the adjustment costs of borrowing and lending from abroad (i.e., the effective risk free interest rate). This effective risk free interest rate is the one we present in our tables, and it is calculated as follows: $r_t^{ef} = \dfrac{R_t^{\,f}}{(1 - \Theta_D(D_t - D_0))}$.

When deciding whether to invest in a local bond or in a foreign bond, the investor will take into account the adjustment costs incurred by changing the foreign debt position. Therefore, a no-arbitrage condition will equate the expected return on the local bond with r_t^{ef}. In the data, the standard deviation of the local bond is 2.7%. The standard deviation of r_t^{ef} when $\gamma = 0.05$ is much higher, 24%. Thus, as can be seen, we are not able to get over the risk free rate volatility puzzle in any of the cases. This problem remains in the next sections as well.

Table 3: Business cycle moments and the equity premium, Israeli data and the model under the Jaimovich-Rebelo utility function

		(1)		(2)		(3)		(4)	
γ		0.0001		0.05		0.5		0.99	
	Data Israel	No EP	EP	No EP	EP	No EP	EP	No EP	EP
Standard deviations									
Y	2.9	2.8	2.4	3.2	3.3	3.3	3.1	3.4	3.1
C	2.2	2.9	3.2	2.5	2.6	2.4	2.9	2.2	3.6
I	12.3	10.6	3.6	11.9	8.0	12.5	6.8	13.4	5.4
H	2.8	1.9	1.7	2.3	2.2	1.9	1.7	1.3	1.1
Tb/y	2.6	2.4	0.4	2.4	0.4	2.4	0.4	2.4	0.4
Serial correlations									
Y	0.61	0.6	0.46	0.60	0.39	0.53	0.29	0.54	0.23
Correlations with output									
C	0.51	1.00	0.98	1.00	0.97	0.99	0.98	0.95	0.98
I	0.71	0.42	0.91	0.57	0.95	0.61	0.96	0.67	0.96
H	0.71	0.98	0.98	0.86	0.51	0.87	0.61	0.83	0.82
tb/y	-0.08	0.14	0.26	0.14	0.42	0.13	0.38	0.12	0.37
Equity premium									
EP	9.00	0.00	8.87	0.01	8.95	0.01	8.89	0.01	8.59
$\sigma_{r^{ef}}$		0.24	29.96	0.39	23.93	0.35	29.37	0.40	32.74
SR		0.01	0.21	0.01	0.24	0.01	0.21	0.01	0.19
Parameter Values									
θ^d		0.36	6.8E+05	0.363	6.8E+05	0.357	2.0E+07	0.354	1.1E+11
ϕ^k		6.7E-06	26.790	2.1E-11	10.908	1.1E-10	11.874	1.9E-11	14.588
ρ		0.22	0.29	0.44	0.50	0.44	0.40	0.38	0.24
σ		0.0101	0.0103	0.0114	0.0195	0.0152	0.0213	0.0196	0.0235
Loss function value									
No EP		0.0510	0.2069	0.0175	0.1466	0.0251	0.1747	0.0425	0.2478
EP		0.5250	0.1035	0.5081	0.0733	0.5118	0.0874	0.5203	0.1249

Table 3a presents a decomposition of the equity premium to its components. The upper part of Table 3a ("Total") presents the decomposition of the equity premium under each alternative, according to equation 35 in the text. It should be noted that η_{cc} and $\eta_{ch,h}$ are not a function of the parameters that we searched over, but $Cov_t(r_{t+1}^{eq},c_{t+1})$ and $Cov_t(r_{t+1}^{eq},h_{t+1})$ do depend on those parameters.

The table indicates several points. First, when we compare alternatives 1 and 2 (in each block) we see that there is a difference in the covariance of consumption and labor with the return on equity. The higher equity premium under the "EP" alternatives is achieved through an increase in the covariance between consumption and the return on equity. For example, under alternative 2 in the first block, the equity premium is achieved by increasing the covariance between consumption and the return on equity from 0.01 (in alternative 1) to 0.76 (in alternative 2).

Second, the increase in the covariance between consumption and the return on equity is accompanied by a large increase also in the covariance between labor and the return on equity, which has a negative contribution to the equity premium, and this component moderates the positive contribution of the former component.

Third, when moving from γ = 0.001 ("GHH") to γ = 0.05, both η_{cc} and $\eta_{ch,h}$ fall. However, $\eta_{ch,h}$ falls relatively much more than η_{cc} and this helps to achieve the equity premium. We shall return to this point soon when we shall review the sources of the better results obtained under JR preferences.

The rest of the table presents the contribution of each shock to the total equity premium and to the covariances. It is clear that the biggest source of the equity premium is the technology shock ("eps"), accounting for an equity premium of 7.67 percent, out of a total value of 8.87, in the GHH case, for example.

It is also clear that the shock to the world interest rate ("epsrw"), does not contribute at all to the equity premium and to the covariances. This happens because the change in the world interest rate is not sufficient to generate a strong response of r^{eq} [34], which thus stays almost stable.

The government shock ("epsg") contributes a small part of the equity premium (1.2 percent in the GHH case). However, note that this contribution (1.2 percent) is associated with a very low covariance of c and r^{eq} (in the GHH case: 0.03 percent, as opposed to 0.76 of the technology shock) and also with a very low covariance of h and r^{eq}. When we increase γ this covariance even becomes negative, which means a positive contribution to the equity premium. This suggests that large government shocks, and in general, expenditure or wealth shocks, may have a relatively strong effect on the equity premium. This point will be elaborated in subsection 6.3.

[34] This outcome depends on the value of ϕ^d. If it is small enough, the reaction of r^{eq} could be stronger, thus having a bigger influence on the equity premium and the covariance.

Table 3a: The equity premium and its sources under Jaimovich-Rebelo preferences

$$EP_t = \eta_{cc} Cov_t(r^{eq}_{t+1}, c_{t+1}) - \eta_{ch,h} Cov_t(r^{eq}_{t+1}, h_{t+1})$$

	(1)		(2)		(3)		(4)	
γ	0.0001		0.05		0.5		0.99	
	No EP	EP	No EP	EP	No EP	EP	No EP	EP
Total								
eta_cc	38.75	38.75	29.17	29.17	20.48	20.48	12.63	12.63
eta_ch,h	49.85	49.85	26.78	26.78	20.11	20.11	13.06	13.06
cov(c,r)	0.01	0.76	0.01	0.58	0.01	0.76	0.01	0.95
cov(h,r)	0.00	0.41	0.01	0.31	0.01	0.34	0.00	0.27
Equity premium %	0.00	8.87	0.01	8.95	0.01	8.89	0.01	8.59
eps								
cov(c,r)	0.01	0.59	0.01	0.77	0.01	0.95	0.01	0.59
cov(h,r)	0.01	0.33	0.01	0.35	0.00	0.27	0.01	0.33
Equity premium %	0.00	7.67	0.01	8.40	0.01	8.67	0.01	8.38
epsrw								
cov(c,r)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
cov(h,r)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity premium %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
epsg								
cov(c,r)	0.00	0.03	0.00	-0.01	0.00	0.00	0.00	0.01
cov(h,r)	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00
Equity premium %	0.00	1.20	0.00	0.21	0.00	0.23	0.00	0.21

* The shocks' contributions to the general equity premium don't always add up exactly to 100% due to approximations.

6.2 Moving from GHH to JR preferences: the sources of improvement in the loss function

Table 4 helps in isolating the sources of the improvement in the loss function when moving from GHH preferences to JR preferences. Column 1 of Table 4 presents the results of the optimization in the case of GHH when looking for an equity premium (it is the same as the EP alternative in the first block of Table 3). As noted earlier, the biggest drawback of the results under the GHH preferences is the very low (absolute and relative) standard deviation of investments (3.6% as opposed to 12.3% in the data, and a ratio of 1.125 between the standard deviations of investments and consumption, as opposed to 5.6 in the data). When moving to the JR preferences, under γ = 0.05 (column 4 in Table 4) we get an improvement in the loss function value - from a loss of 0.1035 under GHH preferences, to a value of 0.0733 under γ = 0.05. It seems that the most noticeable difference in the γ = 0.05 case is the standard deviation of investments which is higher both in absolute value, 8.0 percent, and relative to the standard deviation of consumption, 3.1 (also due to a small reduction in the standard deviation of consumption).

As we see, the move to JR preferences helps in raising the absolute and relative standard deviation of investments without having to compromise on the equity premium. Note that one of the causes of the low standard deviation of investments under the GHH case is the high level of capital adjustment costs (ϕ^k), 26.79. Under γ = 0.05 ϕ^k is much lower, 10.908, which may explain part of the larger volatility of investments.

To assess the contribution of the reduction in ϕ^k we performed a simulation under the GHH preferences, using the same parameter values of column 1 except of ϕ^k, for which we used a value of 10.908 (The value found optimal under γ = 0.05). The results which are presented in block 2 of Table 4 show that the standard deviation of investments goes up a little, to 4.5 percent, while mildly lowering the standard deviation of consumption, resulting in a small improvement of the relative volatility of investments to consumption's volatility, 1.5. Hence the lower value of ϕ^k, explains only a small part of the improvement in the loss function value. The equity premium in this case declines sharply to 2.39 percent. Block 3 presents the results of running the simulation with the same parameters of block 2, while changing γ to 0.05. It can be seen that now we observe a much better relative volatility of investments, 3.1.[35] Block 4 presents (again) the results of the optimal parameter values for γ = 0.05, where the optimal ρ and σ were found to be higher. The relative volatility of investments stays the same, but the higher standard deviation and persistence of the technology shock raises the absolute standard deviations of all of the variables, thus getting the moments much closer to those in the data.

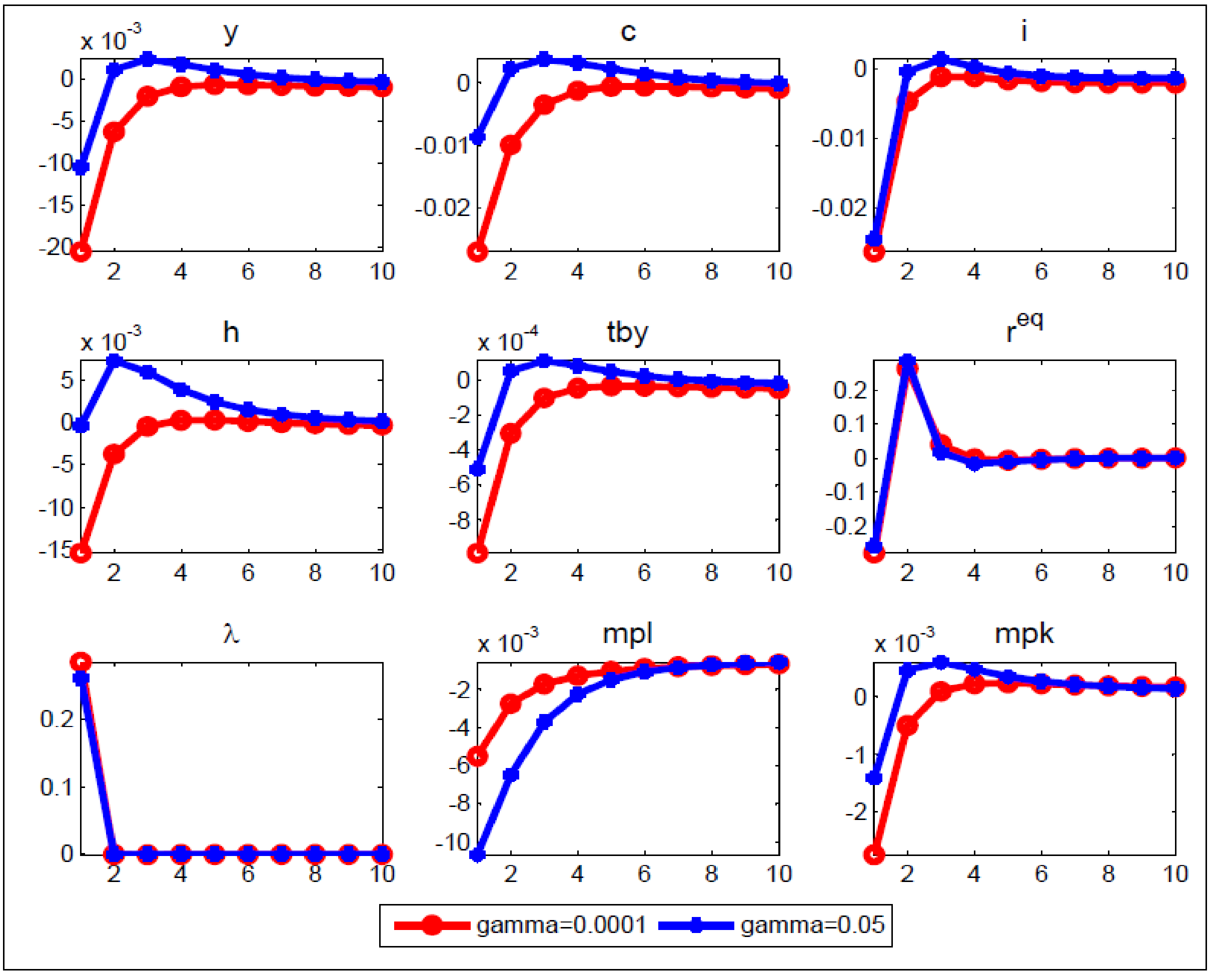
Table 4: Moving from GHH to the Jaimovich-Rebelo utility function: The sources of improvement in the loss function

		(1)	(2)	(3)	(4)
γ	Data Israel	0.0001	0.0001	0.05	0.05
Standard deviations					
y	2.9	**2.4**	**2.5**	**1.6**	**3.3**
c	2.2	**3.2**	**3.0**	**1.3**	**2.6**
i	12.3	**3.6**	**4.5**	**4.0**	**8.0**
h	2.8	**1.7**	**1.7**	**1.2**	**2.2**
tb/y	2.6	**0.4**	**0.4**	**0.4**	**0.4**
Serial correlations					
y	0.61	**0.5**	**0.54**	**0.21**	**0.39**
Correlations with output					
c	0.51	**0.98**	**0.99**	**0.98**	**0.97**
i	0.71	**0.91**	**0.92**	**0.84**	**0.95**
h	0.71	**0.98**	**0.98**	**0.65**	**0.51**
tb/y	-0.08	**0.26**	**0.26**	**0.31**	**0.42**
Equity premium					
EP	9.00	**8.87**	**2.39**	**2.49**	**8.95**
$\sigma_{r^{ef}}$		29.96	13.74	15.73	23.93
SR		0.21	0.12	0.11	0.24
Parameter Values					
θ^{d}		**680801**	**680801**	**680801**	**680801**
ϕ^{k}		**26.790**	**10.908**	**10.908**	**10.908**
ρ		**0.29**	**0.29**	**0.29**	**0.50**
σ		**0.01**	**0.01**	**0.01**	**0.0195**
Loss function value					
No EP		0.2069	0.1825	0.2458	0.1466
EP		**0.1035**	**0.3610**	**0.3845**	**0.0733**

[35] This ratio is very close to the ratio in most of the small open economies we've examined (see footnote 8 for details).

To understand how the move to JR preferences helps in raising the relative volatility of investments it is useful to examine the IRF of the model. Figure 1 presents the IRF of a negative technology shock (with a size of a one standard deviation) for selected variables, under the GHH preferences (in the red line) and under the JR preferences (when γ = 0.05, in the blue line). The parameter values used to calculate the presented IRF are the same as those that were used to construct the optimal results for the GHH utility function (Alternative 2 in block 1 of Table 3). That is, changes in the results for the γ = 0.05 case are only due to the change in γ .

Figure 1 – IRF: A negative technology shock



Let us look first at the GHH case (the red line). A negative technology shock lowers the firms' demand for labor. If the labor supply of households doesn't change (as in the GHH case), the lower demand results in a decline in the amount of labor (h), which, together with the decline in productivity, causes a fall in output (y). The lower output causes households to reduce both consumption (c) and investment (i).[36] The fall in the demand for investments together with the fall of productivity lowers the return on capital (r^{eq}). At the same time, the decline in consumption causes the marginal utility of consumption (λ) to go up, thus causing a negative correlation between λ and r^{eq} which generates a positive equity premium.

In the case of JR preferences (the blue line), in response to the negative productivity shock, households increase their labor supply (a wealth effect) and this moderates the decline in labor and thus also in output (relative to the GHH case). This results in a weaker fall of consumption, relative to the GHH case. Investments, however, fall roughly the same as in the GHH case[37], resulting in a quite similar fall of the return on capital.

In regard to the equity premium, note that the marginal utility of consumption (λ) increases in the JR case the same as in the GHH, even though the level of consumption falls less. This happens because of the higher level of labor[38], but also because of the lower value of $\eta_{ch,h}$[39], that, as was noted earlier, falls relatively stronger than η_{cc}. Since λ and r^{eq} react about the same as in the GHH case, the equity premium under both cases is similar. Hence, a small increase of γ, to 0.05, generates a similar equity premium, with a much higher volatility of investments relative to consumption.

6.2.1 The role of wage rigidity

In the following we shall demonstrate that the role of the wage rigidity in reproducing the equity premium in the GHH case is negligible. On the other hand in the JR preferences it has a central role. More specifically, what matters is the interaction of the wealth effect on labor supply and the limitation on free adjustment of labor (caused by wage rigidity) which generates the equity premium.

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[36] The demand for investments falls because households don't want consumption to fall too much, and also because the productivity of capital makes investment less profitable.

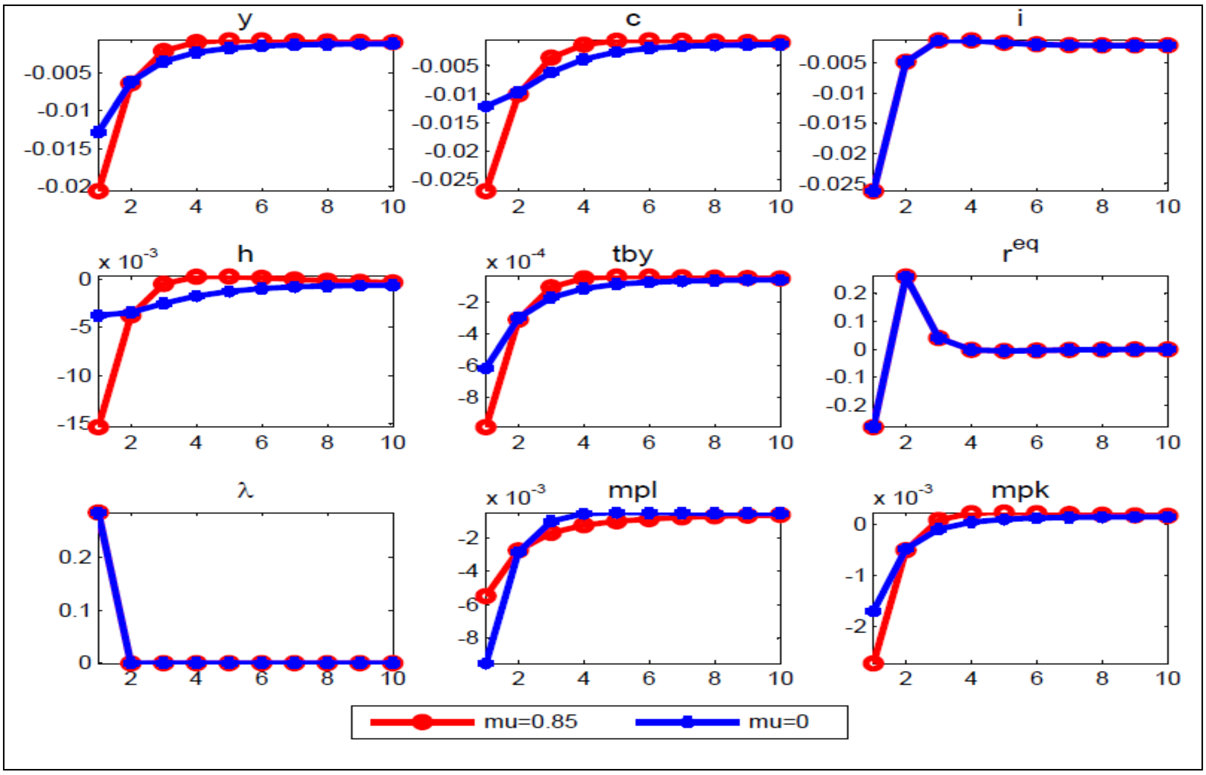
[37] This is because of two offsetting effects: Households want to consume more in the JR case, and this should make investment fall even further than in the GHH case. Yet at the same time, since the amount of labor is higher in the JR case, the expected marginal productivity of capital (mpk) is higher, thus making investments more desirable. As mentioned, the net outcome of these two channels is close to zero (under the specific parameters used in this case),

[38] Which is by itself also a result of the lower value of $\eta_{ch,h}$.

[39] Recall that $\lambda_t = -\eta_{cc}(c_t - \chi^c \tilde{c}_{t-1}) + \eta_{ch,h}(h_t - \chi^h \tilde{h}_{t-1})$, so that a lower value of $\eta_{ch,h}$ makes λ higher.

To illuminate the role of the wage rigidity, we plot the IRF of a negative technology shock for both of the cases presented in Figure 1, compared to the same cases without wage rigidity ($\mu = 0$). The IRFs are presented in Figures 2 and 3.

Figure 2 – IRF: A negative technology shock, GHH preferences

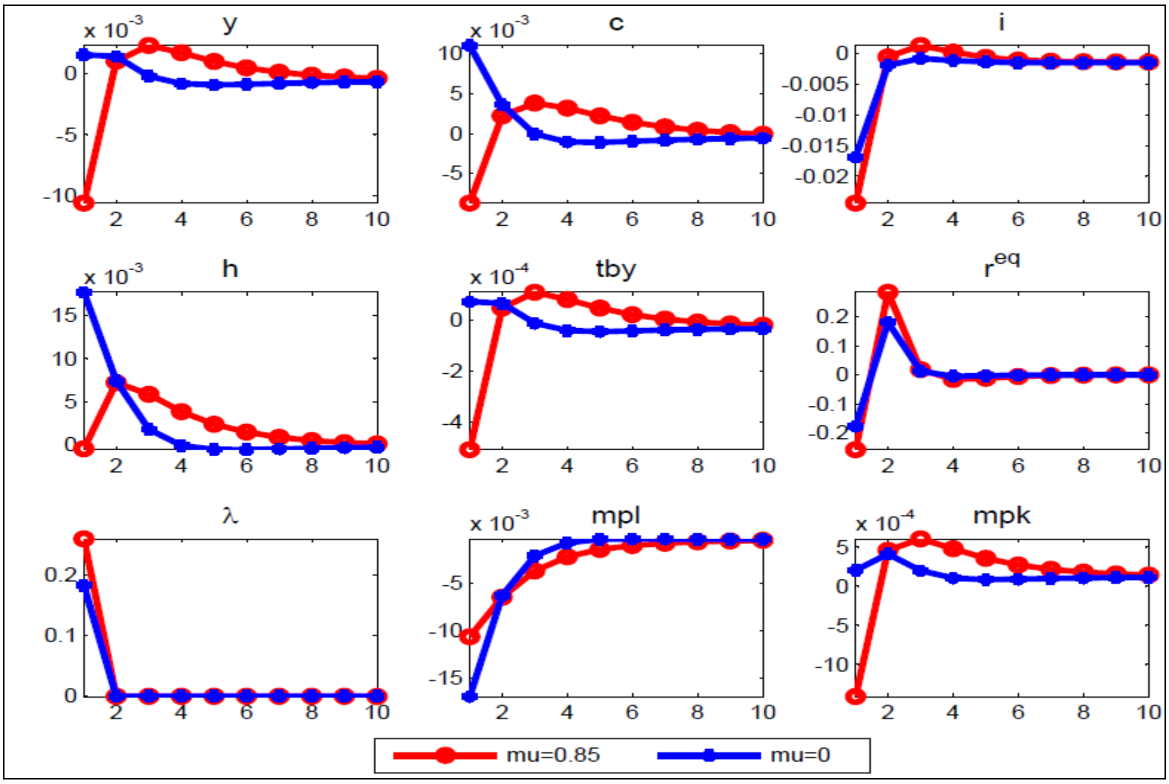


Looking at Figure 2, a negative technology shock induces a reduction in the demand for labor. Under GHH preferences, there is no shift of the labor supply curve[40], so the amount of labor falls. When wages are rigid, they go down very slowly. Thus, the fall in the amount of labor is stronger in the case of the wage rigidity, as is the fall in consumption. However, despite the stronger fall of consumption, the marginal utility of consumption (λ) reacts the same in the case of the wage rigidity. This is the result of the fall in labor (increase in leisure) and the strong substitutability of leisure and consumption in the GHH utility function (that is, households are "compensated" for their lower consumption, with more leisure). Since the

[40] Note that this happens because of two offsetting forces: Households would want to work more in order to prevent consumption from falling. Yet at the same time, since they consume less, their utility of leisure is higher, and therefore they would like to work less. In GHH preferences, these two forces exactly offset each other, resulting in the labor supply staying unchanged.

marginal utility of consumption (λ) responds the same[41] the equity premium stays here the same.

Figure 3 – IRF: A negative technology shock, JR preferences ($\gamma = 0.05$)



Looking at Figure 3, the negative technology shock induces a reduction in the demand for labor, as in the GHH case. However, Under JR preferences, the substitution effect between consumption and leisure is not strong enough to make households choose to consume less and work less, as in the GHH case. Here, there is a wealth effect on labor supply—households would like to increase their labor supply in order to stabilize their marginal utility of consumption. For this reason, when $\mu = 0$, labor goes up. In the specific parameter values used here, consumption actually rises initially. When $\mu = 0.85$, labor does not go up, and hence consumption falls. Notice that demand for investments falls more when wages are rigid, to prevent consumption from falling further. This causes the return on equity to fall stronger. The fall in consumption resulting from the wage rigidity causes the marginal utility of consumption to react stronger in this case, and together with the stronger fall in the return on equity, the result is a higher equity premium. In contrast with the GHH case, where

[41] Together with the fact that return on equity responds the same, which is also mainly a result of the similar response of λ .

households' labor supply did not shift to the right in response to the shock, in this case it does. Adding the friction to the adjustment of wages prevents households from increasing their labor effort as they would like to. That is, it is the interaction of the wealth effect of labor supply and the limitation on the free adjustment of labor, caused by the wage rigidity, which generates the increase in the equity premium.

6.3 The role of government demand shocks in reproducing an equity premium

In the following subsection we shall demonstrate the relative efficiency of the government expenditure shock in reproducing the equity premium. This leads us to conclude that other shocks, with similar attributes to the government demand shock, that are currently absent from our model, might have an important role in reproducing the equity premium. These features are likely to be a result of expenditure or wealth shocks, both of which have a dominant wealth effect.

Table 3a presents the contribution of each of the shocks to the total equity premium and to the covariances. As was noted, the government shock contributes a relatively small part of the equity premium (1.2 percent in the GHH case). However, the equity premium generated by this shock is accompanied by a very low covariance of c and r^{eq} (in the GHH case: 0.03 percent, as opposed to a contribution of 0.76 percent of the technology shock). In a sense, this means that the government shock is more "efficient" than the technology shock, since it is able to produce a significant equity premium with only a small volatility of consumption.[42] This result comes from the fact that the covariance of h and r^{eq} is much lower in the government shock than in the technology shock, so that the second term in equation 35 does not reduce the equity premium as much as in the technology shock, and it may even have a positive contribution to the equity premium if the covariance of h and r^{eq} is negative.

To see the difference in the effects of the government shock and the technology shock we present in Figure 4 the IRF of a one standard deviation of a positive government shock (which reduces consumption) in comparison to a one standard deviation of a negative technology shock (which reduces consumption as well). The parameters used for the IRF calculation are the ones from the GHH results, with a slight difference – for expositional purposes, the standard deviation and the persistence of the technology shock were adjusted

[42] Note that $Cov_t(r_{t+1}^{eq}, c_{t+1}) = \rho_t(r_{t+1}^{eq}, c_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(c_{t+1})$ and from the impulses we can see that the low covariance between c and r is primarily due to low variance of c.

such that both shocks would yield a similar IRF for the consumption. This calibration yielded a standard deviation of 0.0011 for the technology shock, as opposed to a standard deviation of 0.01 for the government shock.

From Figure 4 it is clear that while both shocks have the same effect on consumption, the government shock has a much larger effect on the marginal utility of consumption (λ). This is the result of the difference in the reaction of h: in the case of technology shock the amount of labor decreases because of the reduction of firms' labor demand, while in the case of government shock there is no change in the demand for labor, and hence its level does not fall on impact. Since h does not fall, there is no mitigation of the effect of the fall in consumption on λ. Since λ rises a lot in the government shock case, households choose to reduce investments strongly in order to stabilize the marginal utility of consumption, so the demand for investment falls harder and with it also the return on equity. So, for a given response of consumption, the government shock causes a much bigger response of λ and of r^{eq} which enlarges the equity premium.

Although the contribution of the government shock in our simulations (presented in Table 3a) is quite low, due to the relatively small standard deviation of government shocks in our data, the results show that in the presence of larger government demand shocks, their role may be important in explaining the equity premium and the business cycle moments observed in the data.[43] To demonstrate this, we conduct a grid search as in section 6.1, with the difference that we assume a much higher standard deviation of the government shock – 10 times higher than in the basic calibration. The results are presented in Tables A.5 and A.5a in Appendix A.5. Table A.5 shows that the larger volatility of the government demand has a negligible effect on the loss function value in the GHH case. In the JR case, however, the effect is substantial. Looking at the results with the optimal value of γ (which is now higher, 0.5) it can be seen that the loss function value decreased significantly to 0.0287 as opposed to 0.0733 in the basic case. Note that one of the causes of the improvement is the better fit of the volatility of the trade balance to output. This is the outcome of the lower value of ϕ^{d}, which means that the economy is much more open. In essence, once government demand shocks are dominant[44], there is no need to close the economy so much in order to

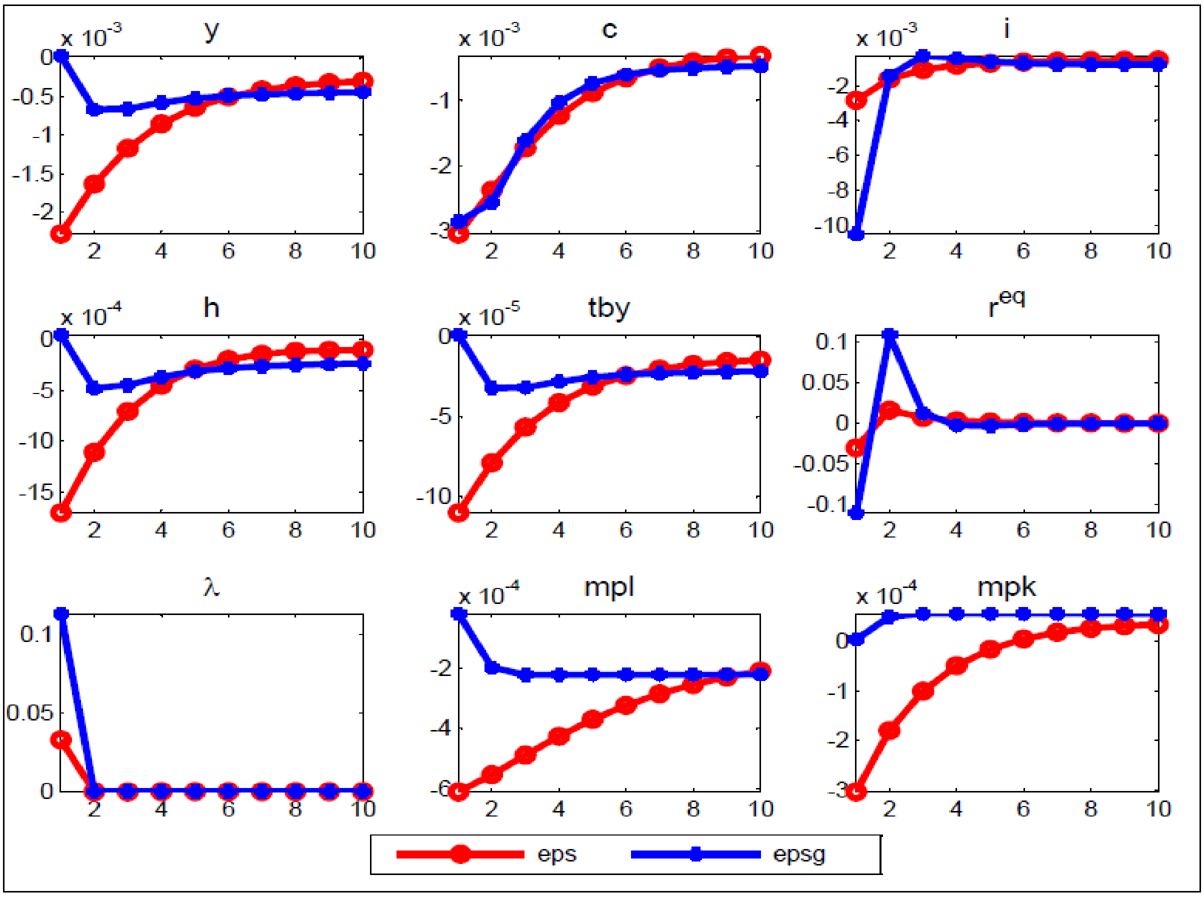
[43] For example, Parvar et al. 2012 report a very large standard deviation of the government expenditure in Argentina and Brazil (0.113 and 0.123, respectively).

[44] To see the dominance of the government shock, note in table 5a their high contribution to the equity premium.

match the equity premium, and hence the dynamics of the trade balance are much closer to the data.

The much improved fit of the model when the standard deviation of the government expenditure shock is higher might imply that other shocks, with similar attributes to the government demand shock, that are currently absent from our model, have an important role in reality. More specifically, shocks whose dominant effect is a reduction in consumption and in the return on equity, accompanied by an increase in labor supply, are good candidates. These features are likely to be a result of expenditure or wealth shocks that have a dominant wealth effect.

Figure 4 – IRF: A positive government shock vs. a negative technology shock, GHH preferences



6.4 The role of shocks to the international interest rate in reproducing an equity premium

As can be seen in Table 3a the contribution of the international interest rate to the equity premium is low. In this subsection we shall explain why.[45] Also note that a basic assumption in our model (as well as in other standard RBC models of a small open economy) is that the real exchange rate is constant. This assumption is quite at odds with reality. In reality, the actual return on foreign bonds in domestic terms is influenced by fluctuations in the real exchange rate. In subsection 6.4.1 we shall also show that the presence of large real exchange rate fluctuations may have an important contribution to the equity premium.

To assess the low contribution to the equity premium of shocks to the international interest rate, we present in Figure 5[46] the IRF of this shock. A positive shock to the world's interest rate has two effects. The first effect of the shock is that it makes lending abroad more attractive than investing in domestic capital and hence should reduce the demand for investment. However, in the presence of high adjustment costs in changing the debt position, as is the case here, households will be averse to changing their debt position in response to the shock. Therefore, on impact, the demand for investments does not change, and the return on equity stays stable as well.

A second effect of the shock has to do with the effect on households' income: households in the economy are net borrowers from the rest of the world[47]; thus, a rise in the world's interest rate means that households have to pay more on their debt, and thus they reduce consumption. In sum, although in response to the shock the marginal utility of consumption goes up, the return on equity stays stable (on impact), and this results in a low contribution of this shock to the equity premium.

Note that although investments don't respond to the shock on impact, after the initial period, when consumption starts to decline strongly, household have to reduce investments in order to prevent consumption from falling even further. The fall in investment is much larger than the fall in consumption and output, meaning that the investment volatility induced by this shock is much higher than consumption's or output's volatility.

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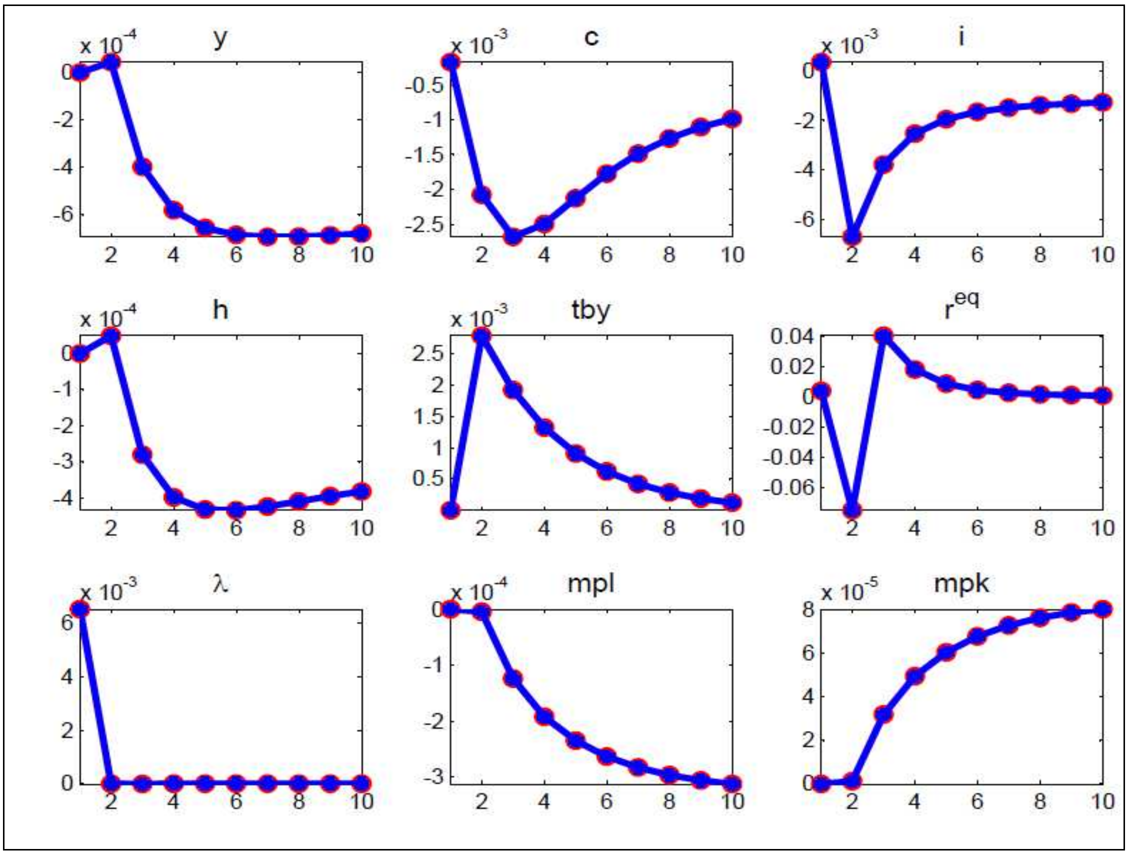
[45] Note, however, that its contribution to business cycle properties of the model are quit significant, as can be seen by comparing the IRF in figures 5 and 2.

[46] The IRF was constructed with the parameters of the GHH case.

[47] This is a result of our calibration, not a general feature of the model. This calibration was chosen since in our sample period, the Israeli economy was, on average, a net borrower from abroad (see footnote 28).

Figure 5 – IRF: A positive shock to the international interest rate, GHH preferences



6.4.1 An alternative calibration of the world's interest rate dynamics

In the formulation of our model, we assumed that the domestic good is a perfect substitute for the foreign good, so that their prices must be equal. In the following we shall relax this assumption but only with regard to capital movements. Our aim is to to show that uncertainty with regard to changes in the real exchange rate, might have a large positive affect on the equity premium in a small open economy.

Till now we assumed that investing in a foreign bond yields R^f units of the foreign good which is a perfect substitute to the domestic good. We shall now assume that in order to use the foreign good in the local economy we need to transfer it to local goods. The yield in terms of the domestic good is affected by the change in the relative prices of the domestic and foreign goods—i.e., by the change in the real exchange rate. Let Δe_t denote the change in the (log of the) exchange rate at time t. The yield at period t in terms of domestic good is $R^f_{t-1}(1+\Delta e_t)$. Note that R^f_{t-1}, the return in terms of foreign good, is known in advance, but the return in terms of the local good is unknown in advance. The yield on foreign bonds in terms of the domestic good is known only when Δe is realized. This means that the volatility of the realized return (in domestic terms) is larger than the volatility of the ex ante return.

In order to take account of the volatility of the realized returns in terms of the domestic good, we first estimate an equation for the process of the real exchange rate of the following form[48]:

43) $\Delta e_t = \varepsilon_t^e$.

The estimate of the standard deviation of ε_t^e (σ^e) is 0.129%, much higher than the standard deviation of ε_t^r (0.0137%), reflecting a very high volatility of the real exchange rate.

In order to incorporate the above equation in the model we slightly modified the basic model, to account, in an ad-hoc manner[49], for the high volatility of the realized returns of the foreign bonds in local terms. Note that equation (43) implies that $E_{t-1}(\Delta e_t) = 0$, so that the ex ante return in domestic terms is equal to the ex ante return in foreign terms. Hence, the process for the ex ante world interest rate stays the same (evolves according to equation (15)), but equation (3), describing the evolvement of the economy's debt is slightly changed:

(3)' $D_t = R_{t-1}^f(1+\Delta e_t)D_{t-1} - TB_t = R_{t-1}^f(1+\varepsilon_t^e)D_{t-1} - TB_t$

That is, the debt is affected by the shocks to the real exchange rate as well.

Table 5 and 5a present the results with the new shock. As can be seen, the results with the optimal value of γ (which is now 0.4) in terms of the loss function value are much better: 0.0218 as opposed to 0.0733 in the basic case. One source for this improvement is the standard deviation of the trade balance to output ratio, which inclined from 0.4 in the basic case, to 2.7, while in the data the standard deviation is 2.6. A second noticeable source for the improvement is higher, closer to data, volatility of consumption.

In Table 5a it can be seen that the shock to the realized return on bonds has a significant contribution to the equity premium. In the optimal case, where γ = 0.4, it is responsible for more than half of the equity premium.

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48 A constant term and an autoregressive term were found to be not significantly different from zero.

49 It is an ad-hoc manner since in reality, it is likely that Δe is correlated with the shocks that we have in our model. Modeling explicitly the channels that affect the real exchange rate is beyond the scope of this paper. Hence, we simply assume here that Δe evolves according to some exogenous process, and check its effect on the world's interest rate in the data.

Figure 6 presents the IRF for this shock. Since households are net borrowers from abroad[50], the rise in the interest rate raises the cost of the debt, thus inducing a negative wealth effect on consumption and on labor: consumption actually rises in the first periods, but this rise is temporary, and right afterward it falls and stays lower than usual for a prolonged period of time. To prevent consumption from falling even further, households choose to work more, and labor goes up. Thus, the marginal utility of consumption rises. The return on equity goes down because of the fall in investment induced by households' desire to prevent consumption from falling further. Notice the similarity of this shock to the government demand shock. Both shocks affect directly households' disposable income, thus causing consumption and investment to decrease (when the shock is positive), and labor to increase. The result of this process is an increase in the marginal utility of consumption accompanied by a fall of the return on equity, which generates a positive correlation between the two and causes households to demand a positive equity premium.

To conclude, the presence of large real exchange rate fluctuations may be a potential major driver of the business cycle, as well as an important source of the equity premium.

[50] It should be noted that although in this specific model shocks to the real exchange rate have a contribution to the risk premium only when the foreign debt differs from zero, in reality this mechanism does not require the debt to be different than zero: in our model, there is no difference between the domestic good and the foreign good, so shocks to the real exchange rate had to be accounted for through their effect on the volatility of the interest rates, which has a direct income effect on households only because the debt is not zero. In reality, changes in the exchange rate affect households' wealth even in the absence of debt. For example, a depreciation of the domestic good means that households can't consume as much as before, since the foreign goods are now more expensive. Thus, even when there is no debt, real exchange rate fluctuations should be taken into account as a (possibly) important contributor to the equity premium.

Table 5: Business cycle moments and the equity premium,

Israeli data and the model under Jaimovich-Rebelo utility function, σ^e =0.129

		(1)		(2)	
γ		0.0001		0. 4	
	Data Israel	No EP	EP	No EP	EP
Standard deviations					
y	2.9	3.0	2.9	3.2	3.3
c	2.2	3.5	3.5	2.5	2.7
i	12.3	10.1	9.9	12.7	10.5
h	2.8	1.7	1.7	1.9	2.2
tb/y	2.6	2.3	2.6	2.6	2.7
Serial correlations					
y	0.61	0.78	0.71	0.53	0.40
Correlations with output					
c	0.51	1.00	0.96	0.98	0.97
i	0.71	0.47	0.47	0.58	0.49
h	0.71	0.98	0.98	0.82	0.64
tb/y	-0.08	0.09	0.05	0.07	0.30
Equity premium					
EP	9.00	0.43	8.88	0.01	8.92
$\sigma_{r^{ef}}$		5.06	32.39	0.48	31.18
SR		0.06	0.20	0.01	0.21
Parameter Values					
θ^d		2.19	307	2.35	299435
ϕ^k		1.5E+00	8.348	3.5E-02	7.539
ρ		-0.67	-0.32	0.36	0.46
σ		0.0038	0.0063	0.0137	0.0185
Loss function value					
No EP		0.0781	0.0759	0.0224	0.0435
EP		0.4919	0.0380	0.5101	0.0218

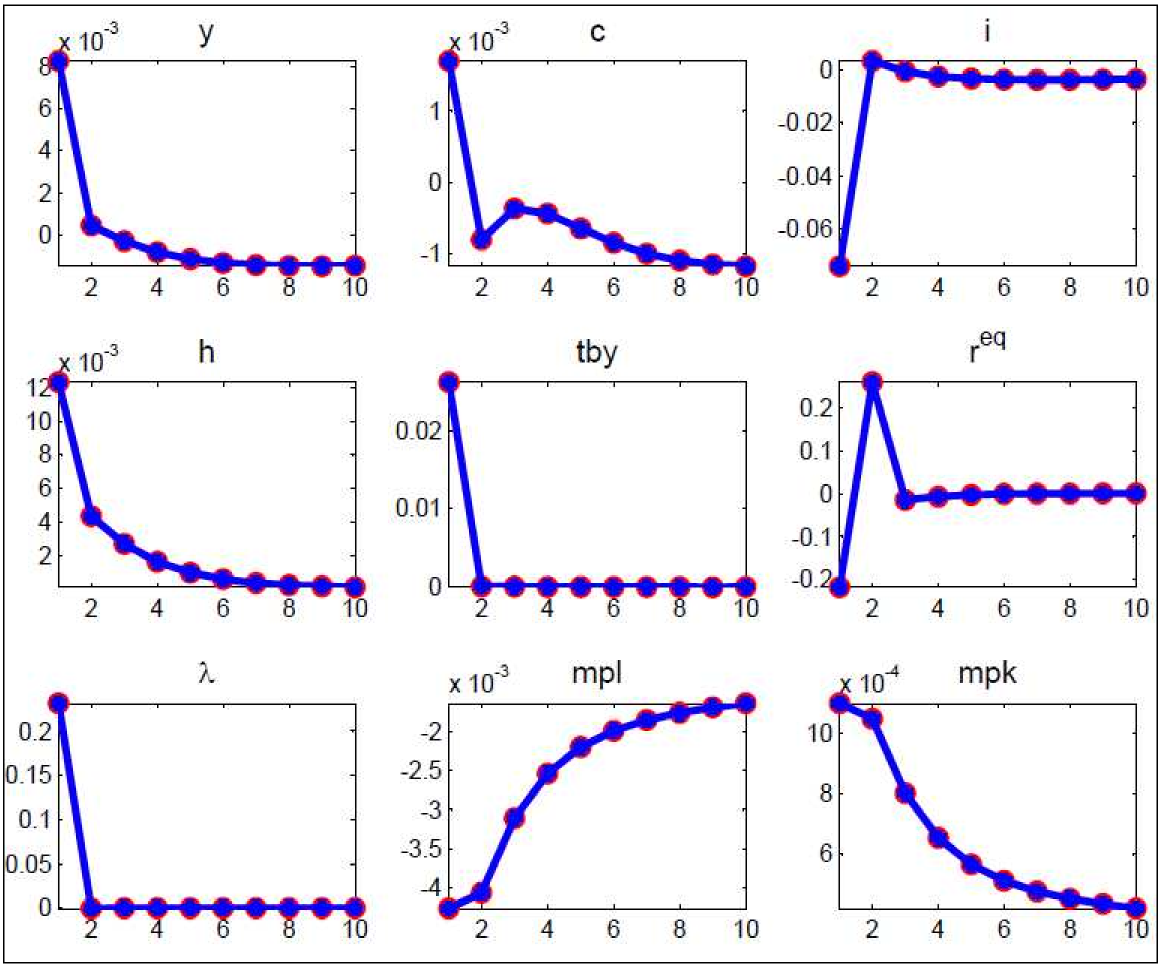
Table 5a: The equity premium and its sources under

Jaimovich-Rebelo preferences, σ^{e}=0.129

$$EP_t = \eta_{cc} Cov_t(r_{t+1}^{eq}, c_{t+1}) - \eta_{ch,h} Cov_t(r_{t+1}^{eq}, h_{t+1})$$

	(1)	(2)
γ	0.0001	0.4
	EP	EP
Total		
eta_cc	38.75	22.37
eta_ch,h	49.85	21.39
cov(c,r)	0.31	0.39
cov(h,r)	0.06	0.00
Equity premium %	8.88	8.92
eps		
cov(c,r)	0.09	0.44
cov(h,r)	0.06	0.27
Equity premium %	0.48	3.92
epsrw		
cov(c,r)	0.0000	-0.0001
cov(h,r)	0.0000	-0.0001
Equity premium %	-0.0007	-0.0005
epsg		
cov(c,r)	0.01	0.00
cov(h,r)	0.00	-0.01
Equity premium %	0.22	0.13
Epsrw_lag		
cov(c,r)	0.21	-0.04
cov(h,r)	0.00	-0.26
Equity premium %	8.19	4.88

Figure 6 – IRF: A positive shock to the realized international interest rate, Jaimovich-Rebelo preferences, γ =0.4, σ^e =0.129



7. Conclusions

In this paper we tried to reproduce both the business cycle facts and the equity premium of the Israeli economy. For this purpose we used data on the main real business cycle moments of the Israeli economy during the period 1960 to 2008, as well as data on the equity returns and the risk free bonds returns. We formulated an RBC model for a small open economy which included three driving forces of the economy's dynamics: a productivity shock, a government expenditure shock, and a world interest rate shock.

By defining a loss function which consists of the sum of the differences between the moments in the data to simulated moments, and trying to minimize it, we checked what structure of the economy describes the Israeli economy in the best way. Specifically, we checked what utility function is the most suitable for this purpose and what are the quantitative properties of the unobserved features of the economy such as capital and foreign debt adjustment costs and the technological shock process.

Our first finding is that GHH preferences, which are quite common in RBC models of small open economies, are not suited to reproduce both the business cycle and the equity premium facts of a "typical" small open economy. A typical small open economy, such as Israel's, is characterized by a relatively high volatility of investments (compared to output and consumption). With GHH preferences the model is not able to yield an equity premium close to the data together with a large enough volatility of investments. A main finding of this paper is that the use of Jaimovich-Rebelo preferences considerably improves the results relative to that achieved by GHH preferences. The reason for this is that the GHH preferences are characterized by a relatively high degree of substitutability between consumption and leisure and this moderates the volatility of the stochastic discount factor (SDF). By adding some degree of wealth effect we can get a significant increase in the volatility of the SDF, and hence an increase in the equity premium and in the volatility of investment. We also found, that in order to prevent households from freely adjusting their labor supply (thus smoothing their marginal utility of consumption over time), we need to add to the model some kind of limitations on labor supply (we used both real wage rigidity and habits in labor). We further showed that it is the interaction between the limitations on labor supply and the wealth effect in the Jaimovich-Rebelo preferences that increases the equity premium.

Another finding in the paper is that shocks that have a large effect on households' expenditure or wealth might play an important role in the presence of the equity premium. These kind of shocks cause households to consume less and work more, thus the marginal utility of consumption in the Jaimovich-Rebelo preferences (where consumption and leisure are substitutable) changes a lot in response to these shocks, thus yielding a high equity premium. One shock of this type is the government demand shock which directly affects households' disposable income and hence seemed to have the ability of explaining a significant part of the equity premium. However, the magnitude of this shock in the Israeli data is not very large, and hence it explains only a small part of the equity premium. A second shock that might have similar properties is a shock to the realized return on foreign assets. This kind of shock could arise from a shock to the exchange rate, which changes the return on foreign assets in terms of the local consumption basket. We have shown that the volatility in households' returns due to exchange rate fluctuations is considerable, and it significantly helps in fitting the data.

With the few shocks in our model we are able to get a good fit of the data. Nonetheless, it is clear that in reality there may be other shocks, that are absent from our model and have similar properties to the aforementioned type of shocks, and they have a contribution to the equity premium as well.

One shortcoming of our results is the excess volatility of the risk free interest rate, relative to its volatility in the data. That is, our model does not solve the risk free rate volatility puzzle.

References

Boldrin M., Christiano L, and Fisher J. (2001). "Habit Persistence, Assets Returns and the Business Cycle", *American Economic Review 91,* 149-196.

Campbell, John Y. (2003). "Consumption-Based Asset Pricing". In: *Handbook of the Economics of Finance,* edited by G.M. Constantinides, M. Harris, and R. M. Stulz, Volume 1, Chapter 13, pages 803-887 (Elsevier).

Correia, Neves, Rebelo (1995). "Business Cycles in a Small Open Economy", *European Economic Review 39,* 1089-1113.

De Graeve F., et al. (2010). "Risk Premium and Macroeconomic Dynamics in a Heterogeneous Agent Model" *Journal of Economic Dynamics and Control (in press).*

De Paoli B., et al. (2009). "Asset Pricing Implications of a New Keynesian Model", Working Paper 326, *(Bank of England).*

Greenwood, J., Hercowitz, Z., Huffman, G. (1988). "Investment, Capacity Utilization and the Real Business Cycle", *American Economic Review 78,* 1054-1087.

Jaccard, I. (2010). "Asset Pricing, Habit Memory, and the Labor Market", Working Paper Series 1163 (ECB).

Jaimovich, N., Rebelo, S. (2009). "Can News about the Future Drive the Business Cycle?", *American Economic Review 99,* 1097-1118.

Jermann, U., (1998). "Asset Pricing in Production Economies", *Journal of Monetary Economics 41,* 257-275.

King, R., Plosser, C., Rebelo S. (1988). "Production, Growth and the Business Cycle I: The Basic Neoclassical Growth Model", *Journal of Monetary Economics 21,* 195-232.

Mendoza, E. (1991). "Real Business Cycles in a Small Open Economy", *American Economic Review 81,* 797-818.

Parvar, J., et al. (2012). "Equity Returns and Business Cycles in Small Open Economies", MPRA

Schmitt-Grohe, S., Uribe M., (2003). "Closing Small Open Economy Models", *Journal of International Economics 61,* 163-185.

Uhlig, H. (2006). "Macroeconomics and Asset Markets: Some Mutual Implications", Humboldt University Berlin (draft).

Uhlig, H. (2007). "Explaining Asset Prices with External Habits and Wage Rigidity in a DSGE Model", *American Economic Review 97,* 239-243.

Appendix

A1. Log linearization of the F.O.C

(5) $\Lambda_t = \dfrac{\beta R_t^{\ f} E_t(\Lambda_{t+1})}{(1-\Theta_D(D_t - D_0))}$

(5') $\lambda_t = \dfrac{1}{(1+r_o)} r_t + E_t(\lambda_{t+1})$ 5'') $\lambda_t = (r_t - r_0) + E_t(\lambda_{t+1})$

(6) $\Lambda_t = U_c(C_t - \chi^c \widetilde{C}_{t-1}, H_t - \chi^h \widetilde{H}_{t-1})$

(6') $\lambda_t = -\eta_{cc}(c_t - \chi^c c_{t-1}) + \eta_{ch,h}(h_t - \chi^h h_{t-1})$

Where: $\eta_{cc} = -\dfrac{C_0 U_{cc/0}}{U_{c/0}}$ and $\eta_{ch,h} = \dfrac{C_0 U_{ch/0}}{U_{h/0}}$

(7) $-U_h(C_t - \chi^c \widetilde{C}_{t-1}, H_t - \chi^h \widetilde{H}_{t-1}) = \Lambda_t W_t = \Lambda_t A_t F_h(K_{t-1}, H_t)$

(7') $\eta_{hh}(h_t - \chi^h h_{t-1}) + \eta_{hc.c}(c_t - \chi^c c_{t-1}) = \lambda_t + w_t = \lambda_t + a_t + \alpha(k_{t-1} - h_t)$

Where: $\eta_{hh} = -\dfrac{H_0 U_{hh/0}}{U_{h/0}}$ and $\eta_{hc,c} = \dfrac{C_0 U_{ch/0}}{U_{h/0}}$

From 6') and 7') we get:

(7'') $\eta_{hh}(h_t - \chi^h h_{t-1}) + \eta_{hc.c}(c_t - \chi^c c_{t-1}) = -\eta_{cc}(c_t - \chi^c c_{t-1}) + \eta_{ch,h}(h_t - \chi^h h_{t-1}) + w_t$

Where $w_t = a_t + \alpha(k_{t-1} - h_t)$

Or:

(7''') $(\eta_{hh} - \eta_{ch,h})(h_t - \chi^h h_{t-1}) = -(\eta_{cc} + \eta_{hc,c})(c_t - \chi^c c_{t-1}) + w_t$

(8) $\Lambda_t = \beta E_t[R_{t+1}^{eq} \Lambda_{t+1}]$

Where: $R_{t+`1}^{\ eq}$ is the gross return in investing in capital stock and is given by:

(9) $R_{t+1}^{eq} = \dfrac{V_{t+1}^{eq} + (1-\delta) + \Phi_k(K_{t+1} - K_t)}{1 + \Phi_k(K_t - K_{t-1})}$

(8') $\lambda_t = E_t(r_{t+1}^{eq}) + E_t(\lambda_{t+1})$

(9') $r_{t+1}^{eq} = A - B$

Where:

$A = \beta(\beta - 1 + \delta)[E_t(a_{t+1}) + (1-\alpha)E_t(h_{t+1}) - (1-\alpha)k_t + \beta E_t(k_{t+1}) - \beta\phi^k k_0 k_t]$

$B = \phi^k k_0 k_t - \phi^k k_0 k_{t-1}$

A.2 Asset pricing

Let $x_t = \ln(X_t)$ be normally distributed, then:

(1) $\ln E(X_t) = E\ln(X_t) + 0.5\,\mathrm{var}(\ln(X_t)) = E(x_t) + 0.5\,\mathrm{var}(x_t)$

The F.O.C for capital, equation (8) above can be written as:

(2) (8') $1 = E_t(M_{t+1}R^{eq}_{t+1})$

Where: $R^{eq}{}_{t+1}$ is the gross return on equity (capital) and M_{t+1} is the S.D.F, that is: $M_{t+1} = \dfrac{\beta\Lambda_{t+1}}{\Lambda_t}$. Taking logarithms and assuming that R_{t+1} and M_{t+1} are log-normally distributed we obtain:

$$
\begin{aligned}
&0 = \ln E_t(M_{t+1}R^{eq}_{t+1}) = E_t\ln(M_{t+1}R^{eq}_{t+1}) + 0.5Var_t\ln(M_{t+1}R^{eq}_{t+1}) = E_t\ln(M_{t+1}) + E_t\ln(R^{eq}_{t+1})\\
(3)\quad &+0.5[Var_t\ln(M_{t+1}) + Var_t\ln(R^{eq}_{t+1}) + 2Cov_t(\ln R^{eq}_{t+1}, \ln M_{t+1})]\\
&= E_t(m_{t+1}) + E_t(r^{eq}_{t+1}) + 0.5[Var_t(m_{t+1}) + Var_t(r^{eq}_{t+1}) + 2Cov_t(r^{eq}_{t+1}, m_{t+1})]
\end{aligned}
$$

The above equation can also be written as follows:

(3') $[E_t(r^{eq}_{t+1}) + 0.5Var_t(r^{eq}_{t+1})] + [E_t(m_{t+1}) + 0.5Var_t(m_{t+1})] = -Cov_t(r^{eq}_{t+1}, m_{t+1})$

Or also as:

(3'') $\ln E_t(R^{eq}_{t+1}) + \ln(M_{t+1}) = -Cov_t(r^{eq}_{t+1}, m_{t+1})$

Where: $Var_t(m_{t+1}) = E_t[m_{t+1} - E_t(m_{t+1})]^2$ etc.

For a risk free asset the return (r^f_{t+1}) is known at time t. Using this in the asset pricing equation 3'') and using the fact that $Var_t(r^f_{t+1}) = 0$ and $Cov_t(r^f_{t+1}, m_{t+1}) = 0$, we have that the pricing equation for the risk free rate is:

(4) $r^f_t = -\ln E_t(M_{t+1}) = -E_t(m_{t+1}) - 0.5Var_t(m_{t+1})$

Using 4) and 3'') we get the following expression for the equity premium:

(5) $EP = [E_t(r^{eq}_{t+1}) + 0.5Var_t(r^{eq}_{t+1})] - r^f_t = \ln E_t(R^{eq}_{t+1}) - r^f_t = -Cov_t(r^{eq}_{t+1}, m_{t+1})$

Now, notice that: $m_{t+1} = \ln(\dfrac{\beta\Lambda_{t+1}}{\Lambda_t}) = \ln\beta + \lambda_{t+1} - \lambda_t = \ln\beta + \Delta\lambda_{t+1}$

Using this in 4) we have:

(30) $r^f_{t+1} = -\ln\beta - E_t(\Delta\lambda_{t+1}) - 0.5Var_t(\Delta\lambda_{t+1})$

Note also that:

$Var_t(\Delta\lambda_{t+1}) = E_t(\Delta\lambda_{t+1} - E_t(\Delta\lambda_{t+1}))^2 = E_t(\lambda_{t+1} - \lambda_t - E_t(\lambda_{t+1} - \lambda))^2 = E_t(\lambda_{t+1} - E_t(\lambda_{t+1}))^2 = Var_t(\lambda_{t+1})$

Using this in 5) we get:

(6)

$$EP_t = \ln E_t(R_{t+1}^{eq}) - r_t^f = -Cov_t(r_{t+1}^{eq}, \Delta\lambda_{t+1}) = -Cov_t(r_{t+1}^{eq}, \lambda_{t+1}) = -\rho_t(r_{t+1}^{eq}, \lambda_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(\lambda_{t+1})$$

And the sharpe ratio is

$$(7) \qquad SR_t = \frac{EP_t}{\sigma_t(R_{t+1}^{eq})} = \frac{-Cov_t(r_{t+1}^{eq}, \lambda_{t+1})}{\sigma_t(R_{t+1}^{eq})} = -\rho_t(r_{t+1}^{eq}, \lambda_{t+1})\sigma_t(\lambda_{t+1})$$

To ease the calculation of the EP and SR from the outcomes of log-linearized DSGE model, note that for each variable X_{t+1}:

$Var_t(\ln X_{t+1}) = Var_t(\ln X_{t+1} - \ln X_0)$ etc. , where: X_0 is the steady value of X .

A3. The equity premium and Sharpe ratio in terms of the elasticity's of the marginal utility:

Log linearization of the F.O.C (6) and (7) above yields:

$$(33)(6') \qquad \lambda_t = -\eta_{cc}(c_t - \chi^c c_{t-1}) + \eta_{ch,h}(h_t - \chi^h h_{t-1})$$

Where: $\eta_{cc} = -\dfrac{C_0 U_{cc|0}}{U_{c|0}}$ and $\eta_{ch,h} = \dfrac{H_0 U_{ch|0}}{U_{c|0}}$

$$(34)(7') \qquad \eta_{hc,c}(c_t - \chi^c c_{t-1}) - \eta_{hh}(h_t - \chi^h h_{t-1}) = \lambda_t + a_t + \alpha(k_{t-1} - h_t)$$

Where: $\eta_{hc,c} = \dfrac{C_0 U_{ch|0}}{U_{h|0}}$ and $\eta_{hh} = -\dfrac{H_0 U_{hh|0}}{U_{h|0}}$.

Substitute λ_t from (33) in (31) we get:

$$(35)(31') \qquad \begin{aligned} EP_t &= \eta_{cc} Cov_t(r_{t+1}^{eq}, c_{t+1}) - \eta_{ch,h} Cov_t(r_{t+1}^{eq}, h_{t+1}) \\ &= \eta_{cc}\rho_t(r_{t+1}^{eq}, c_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(c_{t+1}) - \eta_{ch,h}\rho_t(r_{t+1}^{eq}, h_{t+1})\sigma_t(r_{t+1}^{eq})\sigma_t(h_{t+1}) \end{aligned}$$

For the equity premium, and for the sharpe ratio we get:

$$(36)(32') \qquad SR_t = \eta_{cc}\rho_t(r_{t+1}^{eq}, c_{t+1})\sigma_t(c_{t+1}) - \eta_{ch,h}\rho_t(r_{t+1}^{eq}, h_{t+1})\sigma_t(h_{t+1})$$

A4. Utility specification and its derivatives

GHH:

(2) $$U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \frac{[(C_t - \chi^c \widetilde{C}_{t-1}) - \psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{1-\gamma^c} - 1}{1-\gamma^c}$$

(2.1) $$U_c = [(C_t - \chi^c \widetilde{C}_{t-1}) - \psi[(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-\gamma^c}$$

(2.2) $$U_h = -\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h} U_c$$

(2.3) $$\begin{aligned} U_{cc} &= -\gamma^c[(C_t - \chi^c \widetilde{C}_{t-1}) - \psi[(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-\gamma^c - 1} \\ &= -\gamma^c[(C_t - \chi^c \widetilde{C}_{t-1}) - \psi[(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-1} U_c \le 0 \end{aligned}$$

(2.4) $$U_{hh} = -\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}[\gamma^h (H_t - \chi^h \widetilde{H}_{t-1})^{-1} U_c + U_{ch}] \le 0$$

(2.5) $$\begin{aligned} U_{ch} &= \gamma^c \psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}[(C_t - \chi^c \widetilde{C}_{t-1}) - \psi[(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-\gamma^c - 1} \\ &= \gamma^c \psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}[(C_t - \chi^c \widetilde{C}_{t-1}) - \psi[(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-1} U_c \\ &= -\gamma^c \psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h} U_{cc} \end{aligned}$$

(2.6) $$\eta_{cc} = -\frac{C_0 U_{cc/0}}{U_{c/0}} = \gamma^c[(1-\chi^c)C_0 - \psi(1-\chi^h)^{1+\gamma^h} H_0^{1+\gamma^h}]^{-1} C_0$$

(2.7) $$\eta_{hh} = -\frac{H_0 U_{hh/0}}{U_{h/0}} = (1-\chi^h)^{-1}[\gamma^h + \psi(1+\gamma^h)(1-\chi^h)^{1+\gamma^h} H_0^{1+\gamma^h} \eta_{cc} C_0^{-1}]$$

(2.8) $$\eta_{ch,h} = -\frac{C_0 U_{ch/0}}{U_{c/0}} = \psi(1+\gamma^h)(1-\chi^h)^{\gamma^h} H_0^{1+\gamma^h} \eta_{cc} C_0^{-1}$$

(2.9) $$\eta_{ch,c} = -\frac{C_0 U_{ch/0}}{U_{h/0}} = -\eta_{cc}$$

From (2.9): $\eta_{cc} + \eta_{ch,c} = 0$

From (2.7) and (2.8) and assuming $\chi^c = \chi^h = 0$ we get:

$$\eta_{hh} - \eta_{ch,h} = (1-\chi^h)^{-1}\gamma^h$$

KPR:

(3) $$U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \frac{\{(C_t - \chi^c \widetilde{C}_{t-1})[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})]^{1+\gamma^h}\}^{1-\gamma^c} - 1}{1-\gamma^c}$$

(3.1) $$U_c = \{(C_t - \chi^c \widetilde{C}_{t-1})[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]\}^{-\gamma^c}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]$$

(3.2) $$U_h = (-\psi)(1+\gamma^h)(C_t - \chi^c \widetilde{C}_{t-1})(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}]^{-1})U_c$$

(3.3) $$\begin{aligned} &U_{cc} = -\gamma^c\{(C_t - \chi^c \widetilde{C}_{t-1})[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]\}^{-\gamma^c-1}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^2 \\ &= -\gamma^c(C_t - \chi^c \widetilde{C}_{t-1})^{-1}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-1}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]U_c = \\ &-\gamma^c(C_t - \chi^c \widetilde{C}_{t-1})^{-1}U_c \le 0 \end{aligned}$$

(3.4)

$$\begin{aligned} &U_{ch} = \gamma^c[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}](C_t - \chi^c \widetilde{C}_{t-1})\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}\{(C_t - \chi^c \widetilde{C}_{t-1}) \\ &[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]\}^{-\gamma^c-1} - \{(C_t - \chi^c \widetilde{C}_{t-1})[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]\}^{-1}\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h} \\ &= \{(C_t - \chi^c \widetilde{C}_{t-1})[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]\}^{-\gamma^c}\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}(\gamma^c - 1) = \\ &(\gamma^c - 1)\psi(1+\gamma^h)(H_t - \chi^h \widetilde{H}_{t-1})^{\gamma^h}[1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}]^{-1}U_c \end{aligned}$$

(3.5) $$U_{hh} = U_h h_t^{-1}[\gamma^h + \frac{\gamma^c(1+\gamma^h)\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}}{1-\psi(H_t - \chi^h \widetilde{H}_{t-1})^{1+\gamma^h}}] \le 0$$

(3.6) $$\eta_{cc} = -\frac{C_0 U_{cc/0}}{U_{c/0}} = \gamma^c[(1-\chi^c)C_0 - \psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}]^{-1}C_0 = \frac{\gamma^c}{(1-\chi^c)}$$

(3.7)

$$\eta_{ch,h} = \frac{H_0 U_{ch/0}}{U_{c/0}} = \frac{(\gamma^c - 1)H_0\psi(1+\gamma^h)[(1-\chi^h)^{\gamma^h}H_0^{\gamma^h}]}{1-\psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}} = \frac{(\gamma^c - 1)(1+\gamma^h)\psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}}{(1-\chi^h)[1-\psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}]}$$

(3.8) $$\eta_{ch,c} = \frac{C_0 U_{ch/0}}{U_{h/0}} = \frac{1-\gamma^c}{1-\chi^c}$$

(3.9) $$\eta_{hh} = \frac{H_0 U_{hh/0}}{U_{h/0}} = (1-\chi^h)^{-1}[\gamma^h + \frac{\gamma^c(1+\gamma^h)\psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}}{[1-\psi(1-\chi^h)^{1+\gamma^h}H_0^{1+\gamma^h}]}]$$

From (3.6) and (3.8) we have: (3.10) $\eta_{cc} + \eta_{ch,c} = \dfrac{1}{1-\chi^c}$

From (3.7) and (3.9) and assuming we have:

(3.11) $\eta_{hh} - \eta_{ch,h} = \dfrac{\gamma^h + \psi(1-\chi^h)^{1+\gamma^h} H_0^{1+\gamma^h}}{(1-\chi^h)[1-\psi(1-\chi^h)^{1+\gamma^h} H_0^{1+\gamma^h}]}$

Jaimovich-Rebelo:

(4) $U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1}) = \dfrac{[(C_t - \chi^c \tilde{C}_{t-1}) - \psi(H_t - \chi^h \tilde{H}_{t-1})^{1+\gamma^h} X_t]^{1-\gamma^c} - 1}{1-\gamma^c}$

Where: (5) $X_t = (C_t - \chi^c \tilde{C}_{t-1})^{\gamma} X_{t-1}^{1-\gamma}$ And $X_0 = (1-\chi^c)C_0$ or $X_0 = 1$ if $\gamma = 0$

When γ=0 the function gets the form of GHH utility. When γ=1 the function gets the form of KPR utility.

Let also $U_t = U(C_t - \chi^c C_{t-1}, H_t - \chi^h H_{t-1})$ and $V_t = (C_t - \chi^c \tilde{C}_{t-1}) - \psi(H_t - \chi^h \tilde{H}_{t-1})^{1+\gamma^h} X_t$

For ease of calculations, in the following we shall relate to the case of $\chi^c = 0$.

Let us start with the derivatives of X_t:

(5.1) $X'_{c,t} = \gamma C_t^{\gamma-1} X_{t-1}^{1-\gamma} = \gamma C_t^{-1} X_t$ And $X'_{c,0} = \gamma(1-\chi^c) C_0^{-1} X_0$

(5.2) $X''_{c,t} = \gamma(\gamma-1) C_t^{-2} X_t$ And $X''_{c,0} = \gamma(\gamma-1) C_0^{-2} X_0$

(5.3) $X_{t+i} = C_{t+i}^{\gamma} C_{t+i-1}^{\gamma(1-\gamma)^1} C_{t+i-2}^{\gamma(1-\gamma)^2} .... C_t^{\gamma(1-\gamma)^i} X_{t-1}^{(1-\gamma)^{i+1}}$

(5.4) $X'_{c,t+i} = \gamma(1-\gamma)^i C_t^{-1} X_{t+i}$

(5.5) $X''_{c,t+i} = \gamma(1-\gamma)^i C_t^{-2} X_{t+i} [\gamma(1-\gamma)^i - 1]$

Lifetime utility is:

(5.6) $\tilde{U}_t = U_t + \beta U_{t+1} + \beta^2 U_{t+2} + ...$

Let us compute $\tilde{U}'_{c,t}$.

(5.7) $\tilde{U}'_{c,t} = V_t^{-\gamma^c} V'_{c,t} + \beta V_{t+1}^{-\gamma^c} V'_{c,t+1} + \beta^2 V_{t+2}^{-\gamma^c} V'_{c,t+2} + ...$

(5.8) $V'_{c,t} = 1 - \psi H_t^{1+\gamma^h} X'_{c,t}$

(5.8) $V'_{c,t+i} = -\psi H_{t+i}^{1+\gamma^h} X'_{c,t+i}$

so

(5.7.1) $\tilde{U}'_{c,t} = V_t^{-\gamma^c}(1-\psi H_t^{1+\gamma^h} X'_{c,t}) - \psi \sum_{i=1}^{\infty} \beta^i H_{t+i}^{1+\gamma^h} V_{t+i}^{-\gamma^c} X'_{c,t+i}$

And

(5.7.2) $\tilde{U}'_{c,0} = V_t^{-\gamma^c}(1-\psi H_0^{1+\gamma^h} C_0^{-1} X_0) - \psi H_0^{1+\gamma^h} V_0^{-\gamma^c} C_0^{-1} X_0 \sum_{i=1}^{\infty} \beta^i \gamma (1-\gamma)^i$

$$= V_t^{-\gamma^c}\left(1-\frac{\psi\gamma H_0^{1+\gamma^h} C_0^{-1} X_0}{1-\beta(1-\gamma)}\right) = (C_0 - \psi H_0^{1+\gamma^h} X_0)^{-\gamma^c}\left(1-\frac{\psi\gamma H_0^{1+\gamma^h} C_0^{-1} X_0}{1-\beta(1-\gamma)}\right)$$

A similar calculations leads for the following expression for the second derivative (with respect to consumption) of lifetime utility:

(5.8)

$$\tilde{U}''_{c,t} = (V_t^{-\gamma^c})'_c - \psi H_0^{1+\gamma^h}[(V_t^{-\gamma^c})'_c X'_{c,t} + V_t^{-\gamma^c} X''_{c,t}] - \psi \sum_{i=1}^{\infty} \beta^i H_{t+i}^{1+\gamma^h}[(V_t^{-\gamma^c})'_c X'_{c,t+i} + V_{t+i}^{-\gamma^c} X''_{c,t+i}]$$

$$= A_t + B_t + C_t$$

Where:

$$A_t = (V_t^{-\gamma^c})'_c = -\gamma^c V_t^{-\gamma^c} V_t^{-1}(1-\psi H_t^{1+\gamma^h} \gamma^c c_t^{-1} X_t)$$

$$B_t = -\psi H_t^{1+\gamma^h}[(V_t^{-\gamma^c})'_c X'_{c,t} + V_t^{-\gamma^c} X''_{c,t}] = -\psi H_t^{1+\gamma^h} \gamma^c X_t C_t^{-1}[A_t + V_t^{-\gamma}(\gamma^c - 1) C_t^{-1}]$$

$$C_t = -\psi \sum_{i=1}^{\infty} \beta^i H_{t+i}^{1+\gamma^h}[(V_t^{-\gamma^c})'_c X'_{c,t+i} + V_{t+i}^{-\gamma^c} X''_{c,t+i}]$$

$$= -\psi \sum_{i=1}^{\infty} \beta^i H_{t+i}^{1+\gamma^h}[\gamma^c \gamma^2 \psi H_{t+i}^{1+\gamma^h} V_{t+i}^{-\gamma^c} V_{t+i}^{-1}(1-\gamma)^{2i} C_t^{-2} X_{t+i}^{-2} + V_{t+i}^{-\gamma^c} \gamma (1-\gamma)^i C_t^{-2} X_{t+i}(\gamma(1-\gamma)^i - 1)]$$

And:

$$A_0 = -\gamma^c V_0^{-\gamma^c} V_0^{-1}(1-\psi H_0^{1+\gamma^h} \gamma^c c_0^{-1} X_0)$$

$$B_0 = -\psi H_t^{1+\gamma^h} \gamma^c X_0 C_0^{-1} V_0^{-\gamma^c}[-\gamma^c V_0^{-1}(1-\psi H_0^{1+\gamma^h} \gamma^c c_0^{-1} X_0) + (\gamma^c - 1) C_0^{-1}]$$

$$C_0 = -\psi \sum_{i=1}^{\infty} \beta^i H_0^{1+\gamma^h}[\gamma^c \gamma^2 \psi H_0^{1+\gamma^h} V_0^{-\gamma^c} V_0^{-1}(1-\gamma)^{2i} C_0^{-2} X_0^{-2} + V_0^{-\gamma^c} \gamma (1-\gamma)^i C_0^{-2} X_0(\gamma(1-\gamma)^i - 1)]$$

$$= -\psi H_0^{1+\gamma^h} V_0^{-\gamma^c} \gamma C_0^{-2} X_0[\gamma^c \gamma \psi H_0^{1+\gamma^h} V_0^{-1} X_0 \frac{\beta(1-\gamma)^2}{1-\beta(1-\gamma)^2} + \gamma \frac{\beta(1-\gamma)^2}{1-\beta(1-\gamma)^2} - \frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}]$$

Using the above we have:

$$\eta_{cc} = -\frac{C_0 \tilde{U}''_{c,0_t}}{\tilde{U}'_{c,0_t}}$$

(5.9)
$$= \gamma^c C_0 V_0^{-1}(1-G_0)[1-\frac{G_0}{1-\beta(1-\gamma)}]^{-1}$$
$$+ C_0 G_0[-\gamma^c V_0^{-1}(1-G_0) + (\gamma-1)c_0^{-1}][1-\frac{G_0}{1-\beta(1-\gamma)}]^{-1} =$$
$$+ G_0[\gamma^c c_0 V_0^{-1} G_0 \frac{\beta(1-\gamma)^2}{1-\beta(1-\gamma)^2} + \gamma\frac{\beta(1-\gamma)^2}{1-\beta(1-\gamma)^2} - \frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}][1-\frac{G_0}{1-\beta(1-\gamma)}]^{-1}$$

Where: $G_0 = \psi H_0^{1+\gamma^h}\gamma^c c_0^{-1} X_0$

It can be easily seen that for the case $\gamma = 0$ we get the relevant elasticity of the GHH utility (equation (2.6) above) and for the case $\gamma = 1$ we get the relevant elasticity of the KPR utility (equation (3.6)).

Now we continue to calculate the cross derivative $\tilde{U}''_{ch,t}$. Differentiating equation (5.7.1) with respect to H_t, we get:

(5.10) $$\tilde{U}''_{ch,t} = \psi\gamma^{1+h} H_t^{1+\gamma^h} H_t^{-1} V_t^{-\gamma^c}[\gamma^c V_t^{-1} - \psi H_t^{1+\gamma^h} C_t^{-1} X_t \gamma^c V_t^{-1} - \gamma C_t^{-1}]$$

Using this we get, after some algebra:

(5.11) $$\eta_{ch,h} = \frac{H_0 \tilde{U}''_{ch,0}}{\tilde{U}'_{c,0}} = \gamma^{-1} C_0 \psi G_0[\gamma^c V_0^{-1}(1-G_0) - \gamma^c C_0^{-1}][1-\frac{G_0}{1-\beta(1-\gamma)}]^{-1}$$

A similar calculation yields:

(5.12) $$\eta_{ch,c} = \frac{C_0 \tilde{U}''_{ch,0}}{\tilde{U}'_{h,0}} = \gamma - \gamma^c V_0^{-1} C_0 (1-G_0)$$

Note that for $\gamma = 0$ we get $\eta_{ch,c} = -\eta_{cc}$ and for $\gamma = 1$ we get $\eta_{ch,c} = 1-\gamma^c$

The second derivative with respect to H_t is:

(5.13) $$\tilde{U}''_{hh,t} = U''_{hh,t} = -\psi(1+\gamma^{1+h}) H_t^{\gamma^h} H_t^{-1} X_t V_t^{-\gamma^c}[\gamma^c V_t^{-1}\psi(1+\gamma^{1+h}) H_t^{1+\gamma^h} C_t^{-1} X_t + \gamma^{1+h}]$$

And the relevant elasticity is:

(5.14) $$\eta_{hh,0} = \frac{H_0 U''_{hh,0}}{U'_{h,0}} = (1+\gamma^{1+h})[\gamma^c V_0^{-1}\psi(1+\gamma^{1+h}) H_0^{1+\gamma^h} X_0 + 1] - 1$$

A.5 Illustrating the potential role of Government expenditure shock

Table A5: Business cycle moments and the equity premium,
Israeli data and the model under Jaimovich-Rebelo utility function, with σ^{G} *10

		(1)		(2)	
γ		0.0001		0. 5	
	Data Israel	No EP	EP	No EP	EP
Standard deviations					
y	2.9	2.3	2.5	3.0	3.0
c	2.2	3.8	4.2	2.9	2.9
i	12.3	3.7	4.4	10.8	8.8
h	2.8	1.4	1.5	1.6	2.3
tb/y	2.6	4.2	3.8	2.5	2.0
Serial correlations					
y	0.61	0.6	0.64	0.6	0.56
Correlations with output					
c	0.51	0.73	0.70	0.9	0.76
i	0.71	0.78	0.83	0.5	0.28
h	0.71	0.97	0.98	0.6	0.56
tb/y	-0.08	0.07	0.04	0.1	0.15
Equity premium					
EP	9.00	2.17	8.91	1.1	8.92
$\sigma_{r^{ef}}$		3.99	10.80	5.3	20.35
SR		0.16	0.30	0.1	0.25
Parameter Values					
θ^{d}		0.16	0.83	1.0	5.99
ϕ^{k}		1.3E+02	92.16	2.9	10.68
ρ		0.08	-0.38	0.4	0.58
σ		0.0071	0.0056	0.0	0.0140
Loss function value					
No EP		0.1951	0.2054	0.048	0.0574
EP		0.3855	0.1027	0.414	0.0287

Table A5a: The equity premium and its sources under Jaimovich-Rebelo preferences, σ^G *10

$$EP_t = \eta_{cc} Cov_t(r^{eq}_{t+1}, c_{t+1}) - \eta_{ch,h} Cov_t(r^{eq}_{t+1}, h_{t+1})$$

	(1)	(2)
γ	**0.0001**	**0. 5**
	EP	**EP**
Total		
eta_cc	**38.75**	**18.99**
eta_ch,h	**49.85**	**18.68**
cov(c,r)	**0.25**	**0.37**
cov(h,r)	**0.02**	**-0.09**
Equity premium %	**8.91**	**8.92**
eps		
cov(c,r)	**0.03**	**0.22**
cov(h,r)	**0.02**	**0.15**
Equity premium %	**0.06**	**1.52**
epsrw		
cov(c,r)	**0.0068**	**0.0011**
cov(h,r)	**-0.0001**	**-0.0003**
Equity premium %	**0.2703**	**0.0240**
epsg		
cov(c,r)	**0.22**	**0.15**
cov(h,r)	**0.00**	**-0.24**
Equity premium %	**8.58**	**7.38**

A.6 The grid search for optimal parameter values

In order to find the optimal parameter values of { ρ , σ , ϕ^k , θ^d } we defined the following loss function :[51]

$$L = [(\frac{\sigma_{y\_\text{model}} - \sigma_{y\_data}}{\sigma_{y\_data}})^2 + (\frac{\sigma_{c\_\text{model}} - \sigma_{c\_data}}{\sigma_{c\_data}})^2 + (\frac{\sigma_{i\_\text{model}} - \sigma_{i\_data}}{\sigma_{i\_data}})^2 + (\frac{\sigma_{h\_\text{model}} - \sigma_{h\_data}}{\sigma_{h\_data}})^2$$

$$+ (\frac{\sigma_{tby\_\text{model}} - \sigma_{tby\_data}}{\sigma_{tby\_data}})^2 + (\rho_{y\_\text{model}} - \rho_{y\_data})^2 + (\rho_{i,y\_\text{model}} - \rho_{i,y\_data})^2 + (\rho_{h,y\_\text{model}} - \rho_{h,y\_data})^2$$

$$+ (\rho_{tby,y\_\text{model}} - \rho_{tby,y\_data})^2 + z(EP_{\text{model}} - EP_{data})^2]/(9+z)$$

[51] A somewhat similar methodology was used by Jermann 1998, Uhlig 2006 and Parver et al. 2012.

Where z is a binary variable getting the value 0, when we consider only the business cycle moments, and 9 when we want to match the equity premium as well. The value 9 was chosen in order to give equal weights to the business cycle side and the asset pricing side of the model. As can be seen, the loss function is normalized by the factor (9+z).

Once we have defined the loss function, we want to find the parameter values that minimize it. For this, we use the "fminsearch" procedure of MATLAB, which gets as an input an initial value and returns a value that is a local minimum of the function. A potential problem with this procedure is that in many cases the local minimum is not the global one. To deal with this potential problem we proceeded in the following way: we conducted a grid search on the four parameters. For each parameter we chose a range of values to search over, and the size of the jump between the different values. This defines for each parameter a list of values to be searched over. Then we constructed a set comprised of all of the different combinations of these parameter values. i.e., we constructed a large list of grid points, each of them defined by the value of the four parameters. After constructing a list of grid points, we used each of them as an initial value for the "fminsearch" procedure. For each grid point we ran the minimization procedure and saved the value it returned. At the end of the process we found the parameter values that returned the lowest value of the loss function and used them as the optimal ones.

In theory, we would like to check a very large amount of grid points in order to thoroughly cover the whole array of possibilities. In practice, however, this process is very time consuming. Therefore, we chose the parameter range that seemed to give the best results in terms of this trade off between accuracy and efficiency. It seems that the combination of using the minimization procedure of MATLAB together with the grid search is an improvement in this aspect, relative to using each of these schemes by its own mean.