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Abstract

While much work in macroeconomics considers the formation of price expectations, there has been relatively little work analyzing wage expectations. This study develops models in which workers form expectations of average wages in choosing levels of effort and on-the-job search, under the assumption that information on lagged average wages is available at a low fixed cost, while acquiring other information requires an additional variable cost. Under reasonable conditions, workers’ expectations are at least partly adaptive. It is argued that wage expectations may be more important than price expectations in explaining unemployment fluctuations.

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1. Introduction

In recent years economists have done a good deal of research on the issue of price expectations. One line of inquiry involves testing whether survey measures of expected inflation satisfy the criteria for rational expectations. In a second line of research, Mankiw and Reis (2002) assume that some firms operate with out-of-date information about optimal prices, and they demonstrate that this “sticky information” model can explain output and inflation dynamics better than a model with sticky prices. However, while price expectations have received much attention, a related issue that has been overlooked is workers’ expectations about average wages.

This study models the formation of workers’ wage expectations and argues that expectations of average wages may be at least as important as expectations of the price level in explaining unemployment fluctuations. Workers’ wage expectations are analyzed in the context of their effort and on-the-job search decisions, since these decisions depend on the relationship between a worker’s current wage and his or her expectations of the mean of the aggregate wage distribution. Under reasonable conditions, it is demonstrated that workers’ wage expectations are partly adaptive and that they make systematic errors in their expectations.

The motivation for this work is that systematic errors in wage expectations may have significant macroeconomic consequences. If workers’ expectations about average wages are not completely rational, they may appear to exhibit money illusion in their effort and quit decisions. In Section 2 it is argued that the wage expectations of workers may be more important than the price expectations of either firms or workers in explaining business cycle fluctuations.

Section 3 develops a model of information acquisition for the effort decision of workers. It is assumed that workers can form expectations of average wages both by using an adaptive
expectations calculation based on lagged information that is available at a low fixed cost and by paying an additional variable cost to acquire further information that yields a more accurate estimate. A worker seeks to minimize the sum of information acquisition costs and the utility loss that results from making decisions with imperfect information (since the effort exerted by a worker who forms incorrect wage expectations will be non-optimal). Two methods for acquiring additional information about average wages are considered: gathering and analyzing macroeconomic data and sampling wages at a subset of firms. It is demonstrated that workers’ expectations are likely to be at least partly adaptive, and may be a mixture of rational and adaptive expectations. It is also demonstrated that demand shocks have persistent effects if expectations are adaptive, but not if expectations are based on macroeconomic data or sampling wages. The model is modified in Section 4 to analyze the job search decision, which affects workers’ quit behavior. Section 5 concludes and discusses implications of the model.

The main contributions of this study lie in identifying the benefits of information to workers and developing microfoundations to explain why utility-maximizing workers may form expectations that are partly adaptive.¹ In doing so, it provides theoretical justification for efficiency wage modeling in which effort and quits depend partly on lagged average wages. In addition, this study shows how microeconomic parameters affect the degree to which expectations are based on lagged average wages vs. additional costly information, and how these parameters determine the weights placed on various lags of wages.

2. Why Wage Expectations Matter

There is an extensive literature that involves testing the rationality of price expectations, and much of this work uses the survey of economists conducted by Joseph Livingston and/or the household survey conducted by the Michigan Institute of Social Research. Taken together, these
studies suggest that expectations are neither completely rational nor completely adaptive. On one hand, Evans and Gulamani (1984), Batchelor and Dua (1989), Roberts (1997), Thomas (1999), and Mankiw, Reis, and Wolfers (2003) find that expectations do not satisfy the criteria for rational expectations, as they show that forecast errors can be predicted by information available at the time of the forecast (e.g., money supply growth, unemployment, the budget deficit, interest rates, the output gap, and lagged inflation). On the other hand, the findings of Mullineaux (1980), Gramlich (1983), and Baghestani and Noori (1988) indicate that expectations are not purely adaptive. In addition, Fuhrer (1997) and Roberts (1998) demonstrate that expectations can be described as a mixture of rational and adaptive expectations. In a similar vein, Pfajfar and Santoro (2010) examine a cross section of individuals’ inflation forecasts, and they find that some individuals have rational expectations, some form their expectations adaptively, while the expectations of others are based on adaptive learning and sticky information.

The assumption that expectations are not purely rational is made in Mankiw and Reis’s (2002) sticky information model. Mankiw and Reis assume that some firms (chosen randomly) operate with current information about optimal prices, while the remaining firms operate with out-of-date information, and they demonstrate that their model outperforms the sticky price model in explaining output and inflation dynamics.

In the studies that test the rationality of inflationary expectations and in the sticky information model, the variable of interest is price expectations. In contrast, the present study assumes that individuals have imperfect information about average wages. While this study differs from the rest of the literature by considering wage expectations rather than price expectations, the macroeconomic consequences of imperfect information may be more important for wage expectations than for price expectations.
Labor market outcomes are determined from the interactions between firms and workers. From the perspective of firms, it is not clear why their knowledge of the price level would be inaccurate enough to cause large fluctuations in unemployment, since information on the price level is available on the internet from the Bureau of Labor Statistics on a monthly basis. Even if firms lack perfect information about the price level, there is no obvious mechanism through which imperfect information would translate into large changes in unemployment. For example, Mankiw and Reis consider firms’ pricing and output decisions but do not consider their wage and employment decisions, and there is no reason why firms in their model would not continually set wages at their market-clearing level.

The behavior of workers is likely to be influenced more by their expectations about average wages than by their expectations about the price level. Workers make several decisions that may affect the wages set by firms: at what value to set their reservation wage, whether to quit, how much effort to exert, and how much labor to supply. Theoretical considerations suggest that these decisions are likely to depend more on relative than on real wages. Reservation wages (controlling for labor supply) and quits should depend on relative wages, since workers make these decisions by comparing wages at a given firm with wages elsewhere. Two models, the shirking model and the gift-exchange model, emphasize the effect of wages on workers’ effort. In the shirking model the relative wage is the relevant variable, since wages elsewhere affect the cost of job loss. In the gift-exchange model, effort could depend on either relative or real wages, depending on whether workers are more concerned with how their pay compares with the market wage or with the prices of goods and services. The one decision that unambiguously depends on real wages is labor supply. However, most studies find a small elasticity of labor supply with respect to the real wage.\(^4\) In addition to these theoretical arguments, the dramatic rise in real
wages over the past 60 years, without a correspondingly significant decline in the quit rate or significant increase in effort, also suggests that quits and effort depend more on relative wages than on their real wages. Thus, it is reasonable to believe that workers’ behavior is influenced more by their expectations of average wages than by their expectations of the price level.

In addition, workers probably make larger errors in wage expectations than in price expectations. As previously discussed, the Consumer Price Index is published monthly and is available on the internet. On the other hand, the relevant wage for workers’ effort and quit decisions is the average wage for workers in the same occupation with similar qualifications (e.g., experience and education), and this information is not easily obtainable. In fact, employers in Bewley’s (1999) survey believed that their workers did not have a very precise idea about the wages at other firms.

If firms pay efficiency wages, workers’ imperfect information about average wages may have significant macroeconomic consequences, since firms take into account the reaction of workers in setting wages. In particular, an efficiency wage model with imperfect information about average wages can explain nominal wage rigidity. If workers’ expectations are partly adaptive, firms have an incentive to adjust nominal wages slowly in response to contractionary shocks, out of concern that adjusting wages too quickly would adversely affect effort and quits, and this sluggish adjustment would likely cause unemployment to rise.

3. Expectation Formation in Choosing Optimal Effort

3.1 General model

One explanation for a positive relationship between wages and effort is the shirking model of Shapiro and Stiglitz (1984), in which a higher wage raises the cost of job loss and induces workers to exert more effort. The cost of job loss depends negatively on the wages
offered by other firms, which means that workers’ effort depends on the relationship between their current wages and average wages in the rest of the economy.

In the shirking model, it is generally assumed that all firms pay the same wage, which means that workers know with certainty the average wage offered by other firms. In reality, however, this is not a reasonable assumption. Wages vary across employers, even for jobs that are similar, and workers generally do not have perfect information concerning the wages offered by other firms. Lacking perfect information, workers need to form expectations about wages elsewhere in order to provide the optimal amount of effort. The relevant comparison for workers is likely to be average wages in their occupation.

A worker who forms incorrect expectations of the average wage will exert a non-optimal level of effort and suffer a utility loss. A worker overestimating average wages will exert less than optimal effort, so that on average, the loss of future earnings resulting from the increased probability of dismissal will exceed the utility gain from lower effort. A worker who underestimates average wages will suffer the opposite type of utility loss.

This section models information acquisition by workers, who incur costs from acquiring information and from making decisions with imperfect information. It is assumed that wages vary across firms and that the average wage is unobserved. While the mean of the wage distribution is unknown, workers have two main ways to estimate it. First, they can observe past average wages at a low fixed cost and can predict this mean from an adaptive expectations procedure. This fixed cost is assumed to be low enough that everyone acquires this information. For example, as part of its Occupational Employment Statistics program, the Bureau of Labor Statistics publishes figures on average wages for more than 800 occupational groups, based on the May CPS. However, these statistics are not published until March of the following year, a lag
of ten months. The fixed cost can be viewed as the time spent looking up lagged average wages in one’s occupation on the BLS website.

Second, workers can incur an added variable cost to obtain additional information about average wages. They face the tradeoff that acquiring more information is costly, but it enables them to more accurately estimate the mean of the wage distribution, resulting in effort that is closer to its optimal level. There has been little previous work on how workers estimate average wages, so it is not clear how they form their estimates. In addition to observing past wages, possible ways to estimate average wages include gathering and analyzing macroeconomic data (e.g., the growth rate of money, fiscal policy, and unemployment) and obtaining information on wages at a subsample of firms (e.g., by contacting firms, talking to friends, reading help-wanted ads, and searching on websites that have information on employment opportunities).  

It is assumed that the amount of information acquired by workers is determined by the following framework. Suppose a worker chooses effort \( e \) to maximize

\[
E[U] = E \sum_{t=1}^{\infty} \frac{1}{(1 + \delta)^{t-1}} \left[ \ln(c_t) + u(e_t) \right]
\]

s.t. \( \sum_{t=1}^{\infty} \frac{1}{(1 + r)^{t-1}} c_t = \sum_{t=1}^{\infty} \frac{1}{(1 + r)^{t-1}} \left[ LI_t, Pr[Emp_t] + B(1 - Pr[Emp_t]) \right] \),

where \( \delta \) is the discount rate, \( c \) is consumption, \( u(e_t) \) is the utility or disutility of effort, \( r \) is the interest rate, \( LI \) is labor income, \( Pr[Emp_t] \) is the probability that a worker is employed in period \( t \), and \( B \) represents the income of an unemployed individual. The probability of dismissal is assumed to depend negatively on a worker’s effort, so that the probability of employment in period \( t \) depends on a worker’s effort in previous periods. Expected labor income in future periods is assumed to equal the worker’s current wage if he or she is still at the same firm and to equal the expected average wage if he or she is employed elsewhere.
Campbell (2006) demonstrates that the expected utility of a worker who maximizes the above utility function can be expressed as \( V(w_t, \bar{w}_t, e_t) \), where \( w_t \) is the worker’s current wage and \( \bar{w}_t \) is the average wage. This expression for \( V \) takes into account the disutility of effort and the utility of consumption, which depends on income in the current and future periods. Effort has two opposing effects on lifetime utility. An increase in effort reduces current utility, but it also reduces the probability of dismissal, which increases expected future income and thus raises expected lifetime consumption. Optimal effort is determined from the condition \( dV/de = 0 \).

Suppose that a worker does not know with certainty the true mean of the wage distribution (\( \bar{w}_t \)) and forms an estimate of this mean, denoted by \( \bar{w}^e_t \). Let \( e(\bar{w}_t) \) represent effort when a worker knows that the mean of the wage distribution is \( \bar{w}_t \) and \( e(\bar{w}^e_t) \) represent effort when the worker estimates that the mean is \( \bar{w}^e_t \). The utility loss (\( VL \)) resulting from incorrect expectations of average wages can be expressed as

\[
VL(\bar{w}^e_t - \bar{w}_t) = V(w_t, \bar{w}_t, e(\bar{w}_t)) - V(w_t, \bar{w}_t, e(\bar{w}^e_t)),
\]

with \( VL(0) = 0 \) and \( VL'' > 0 \).

The assumption that \( VL'' > 0 \) means that the utility loss rises at an increasing rate as the difference between \( \bar{w}^e_t \) and \( \bar{w}_t \) increases.

The total expected utility loss (\( TEUL \)) equals the cost of acquiring information plus the expected utility loss from imperfect information, so that

\[
TEUL = C(I) + E[VL(\bar{w}^e_t - \bar{w}_t)],
\]

where \( C(I) \) is the cost of acquiring information. Approximating (3) with a second-order Taylor expansion around the point where \( \bar{w}^e_t = \bar{w}_t \) yields
This expression can be simplified by making the substitutions \( VL(0) = 0 \) and \( VL'(0) = 0 \) and by using the fact that \( VL''(0) \) is a constant.\(^{10}\) Accordingly,

\[
TEUL \approx C(I) + E[VL(0)] + E[VL'(0)(\bar{w}_i^e - \bar{w}_i)] + \frac{1}{2} E[VL''(0)(\bar{w}_i^e - \bar{w}_i)^2].
\]

where \( E[(\bar{w}_i^e - \bar{w}_i)^2] \) depends negatively on the quantity of the worker’s information. A worker acquires the amount of information that minimizes this expected utility loss.

To obtain an expression for \( E[(\bar{w}_i^e - \bar{w}_i)^2] \) it is necessary to make assumptions about demand and wage setting. It is assumed that demand follows an autoregressive process,

\[
m_t = \rho m_{t-1} + \varepsilon_t,
\]

where \( m_t \) is the log of demand, which is assumed to be unobserved, and \( \varepsilon_t \) is a white noise error with variance \( \sigma^2 \). As for wage setting, Appendix A demonstrates that the efficiency wage model of Campbell (2010) implies that the log of the wage set by the \( i \)th firm is described by the relationship,

\[
w_{it} = \bar{w}_i^e + \frac{\lambda}{1 - \lambda} m_t - \frac{\lambda}{1 - \lambda} \bar{w}_i + \eta^*_t + \xi_{it},
\]

where \( \bar{w}_i^e \) is the average worker’s expectation of the average wage, \( \bar{w}_i \) is the actual average wage, \( \lambda \) depends on microeconomic parameters, \( \eta^*_t \) is a common white noise error, and \( \xi_{it} \) is an idiosyncratic white noise error that sums to 0 across firms. The presence of this idiosyncratic error term means that wages vary between firms. By aggregating across firms, the average wage is given by,
(6b) \[ \bar{w}_t = \lambda m_t + (1 - \lambda) \bar{w}_{t-1} + \eta_t, \]

where \( \eta_t = (1 - \lambda) \eta^*_t \) and \( \text{Var}(\eta_t) = \sigma_w^2 \). Taking expectations of both sides of (6b) yields

(7) \[ \bar{w}^{\varepsilon}_t = m^\varepsilon_t, \]

where \( m^\varepsilon_t \) is the average worker’s expectation of demand.

As previously discussed, all workers are assumed to incur the fixed cost to obtain information on past average wages. The next subsection develops a model in which workers form expectations of current average wages by supplementing this information by gathering and analyzing macroeconomic data. Then subsection 3.3 assumes that workers sample wages at other firms to update the information contained in lagged average wages. Both subsections show how microeconomic parameters affect the amount of additional information acquired and the degree to which expectations are based on lagged average wages relative to additional information.

3.2 Observing lagged average wages and gathering macroeconomic data

This subsection models the wage expectations of workers who use information on lagged average wages and macroeconomic data. Workers are likely to differ in their wage expectations, and it will be assumed that individual wage expectations are normally distributed around the average wage expectation. Since the wage expectation of the average worker is what determines the firm’s optimal wage, this subsection considers the average wage expectation. To estimate current average wages from past average wages, the average worker is assumed to use information on average wages in period \( t-1 \) (\( \bar{w}_{t-1} \)) and his or her own prior expectations of average wages in period \( t-1 \) (\( \bar{w}^{\varepsilon}_{t-1} \)), where \( \bar{w}^{\varepsilon}_{t-1} \) represents the expectations formed prior to \( t-1 \) of average wages in period \( t-1 \). Workers use this information to estimate demand in \( t-1 \) (\( m_{t-1} \)), which
is a predictor of current demand, based on the relationship in (5). From (7), workers’ expectations of average wages equal their expectations of demand. The average worker obtains information about $m_{t-1}$ from the relationship in (6b), which can be rewritten as

\begin{equation}
\bar{w}_{t-1} - (1 - \lambda)\bar{w}_{t-1}^e = \lambda m_{t-1} + \eta_{t-1}.
\end{equation}

In addition, individuals can incur a cost in period $t$ to obtain information on demand in period $t-1$ by gathering and analyzing macroeconomic variables such as GDP, unemployment, fiscal policy, and the growth rate of the money supply. This information ($m^l_{t-1}$) is assumed to provide an unbiased but noisy estimate of the true value of demand, so that

\begin{equation}
m^l_{t-1} = m_{t-1} + \nu_{t-1},
\end{equation}

where $\nu_t$ is a serially uncorrelated error term with variance $\sigma^2_t$. Let $I$ represent the amount of information acquired, over and above information on lagged average wages. It is assumed that $\sigma^2_I$ depends negatively on $I$ and that more information reduces this variance at a non-increasing rate, implying that $\sigma^2_I$ can be expressed as $\sigma^2_I(I)$ with $d\sigma^2_I / dI < 0$ and $d^2 \sigma^2_I / dI^2 \geq 0$. In addition to acquiring information about $m_{t-1}$ in the current period, it is also possible that workers acquire revised information about macroeconomic variables in periods prior to $t-1$ that enables them to update their expectations of demand in previous periods (i.e., $m^l_{t-2}$, $m^l_{t-3}$, ...), since macroeconomic data are often revised and since current information about demand may shed light on demand conditions in previous periods.

A worker’s expectation of demand (and thus his or her expectations of average wages) can be viewed as being determined from a Kalman filtering process. The relevant equations are (5) and the system of equations in (8) and (9),
Appendix B demonstrates that the average worker’s wage expectation is

\[ \bar{w}_t = K_1 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} \bar{w}_{t-j} + K_2 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} m_{t-j}^i, \]

where

\[ K_1 = \frac{\rho v^2 \lambda \sigma_i^2}{\lambda^2 v^2 \sigma_i^2 + \sigma_i^2 v^2 + \sigma_i^2 \sigma_i^2}, \]

and

\[ K_2 = \frac{\rho v^2 \sigma_w^2}{\lambda^2 v^2 \sigma_i^2 + \sigma_i^2 v^2 + \sigma_i^2 \sigma_i^2}. \]

In these expressions for \( K_1 \) and \( K_2 \), \( v^2 \) represents the variance of the difference between \( m_i^e \) and the true value of \( m_i \). Appendix B derives an expression for \( v^2 \) and demonstrates that

\[ \frac{\partial v^2}{\partial \sigma_i^2} > 0, \quad \frac{\partial^2 v^2}{\partial (\sigma_i^2)^2} < 0, \quad \frac{\partial v^2}{\partial \sigma_i^2} > 0, \quad \text{and} \quad \frac{\partial^2 v^2}{\partial \sigma_i^2 \partial \sigma_i^2} > 0. \]

The value of \( \sigma_i^2 \) (and thus of \( v^2 \)) depends on the amount of information acquired. To decide how much information to obtain, workers need to calculate the expected utility loss from imperfect information, which depends on \( E[\bar{w}_i - \bar{w}_i]^2 \). The difference between \( \bar{w}_i \) and \( \bar{w}_i^e \) is

\[ \bar{w}_i - \bar{w}_i^e = \lambda m_i + (1 - \lambda) \bar{w}_i^e + \eta_i - \bar{w}_i^e = \lambda (m_i - \bar{w}_i^e) + \eta_i, \]

so that,

\[ E[\bar{w}_i - \bar{w}_i]^2 = \lambda^2 v^2 + \sigma_w^2. \]
Let \( c \) represent the cost of each unit of information. Then the total expected utility loss can be expressed as

\[
\text{TEUL} \approx cI + \frac{1}{2} \nu^2(0) \{ \lambda^2 \nu^2(I, \sigma^2_e) + \sigma^2_w \}.^{13}
\]

Appendix B demonstrates that

\[
\frac{dI}{dc} < 0 \quad \text{and} \quad \frac{dI}{d\sigma^2_e} > 0,
\]

implying that workers acquire less information as the cost of acquiring information increases and acquire more information as demand becomes more variable.\(^{14}\) It should be noted that \( c \) may depend partly on the transparency of policymakers, since increased transparency reduces the cost of macroeconomic information.

If \( 0 < \sigma^2_t < \infty \), (11) demonstrates that workers’ wage expectations are a mixture of the expectations formed with lagged average wages and the expectations formed with macroeconomic data.\(^{15}\) The relative weight that workers place on lagged average wages, as compared to macroeconomic variables, depends on the sums of coefficients on these variables, which, in turn, depend on the model’s microeconomic parameters. The sums of coefficients are

\[
\sum \text{coeff's on } \bar{w}_{t-j} = \frac{K_1}{1 - \rho + K_1 + K_2}, \quad \text{and}
\]

\[
\sum \text{coeff's on } m'_{t-j} = \frac{K_2}{1 - \rho + K_1 + K_2}.^{16}
\]

Thus, the ratio between the sums of coefficients equals the ratio of \( K_1 \) to \( K_2 \), which is

\[
\frac{K_1}{K_2} = \frac{\lambda \sigma^2_t}{\sigma^2_w}. \quad (14)
\]
The effects of \( c \) and \( \sigma^2_\varepsilon \) on this ratio are

\[
\frac{d}{dc} \left( \frac{K_1}{K_2} \right) = \frac{\lambda}{\sigma_w^2} \frac{d\sigma^2_i}{dI} \frac{dI}{dc} > 0, \quad \text{and}
\]

\[
\frac{d}{d\sigma^2_\varepsilon} \left( \frac{K_1}{K_2} \right) = \frac{\lambda}{\sigma_w^2} \frac{d\sigma^2_i}{dI} \frac{dI}{d\sigma^2_\varepsilon} < 0.
\]

Thus, workers place relatively more weight on lagged average wages as the cost of information increases, so that expectations become relatively more adaptive. As demand variability rises, they place relatively more weight on macroeconomic information, implying that their expectations become less adaptive. In addition, (11) shows that the model’s microeconomic parameters also determine the coefficients on each value of lagged wages.

The difference between expectations formed with lagged wages and expectations formed with macroeconomic data is important because wages respond qualitatively differently to demand shocks in these two cases. In particular, demand shocks have persistent effects if expectations are formed with lagged wages, but not if they are formed by observing macroeconomic variables.

Suppose the economy has been in a steady-state equilibrium prior to period 1, with \( m_t = \bar{w}_t = 0 \) for \( t \leq 0 \). Then suppose there is a one-time demand shock of \( \varepsilon^* \) in period 1, so that \( \varepsilon_t = \varepsilon^* \) and \( \varepsilon_t = 0 \) for \( t \geq 2 \). As a result, \( m_t = \rho^{t-1} \varepsilon^* \) for \( t \geq 1 \). We first consider the case in which expectations are based solely on lagged average wages. (This will occur if \( \sigma^2_i = \infty \), meaning that macroeconomic data contribute no relevant information.) In this case, Appendix C demonstrates that wages in period \( t \) are
\[
\bar{w}_t = \rho^{t-1} \left[ 1 - (1 - \lambda) \left( \frac{\sigma_x^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^{t-1} \right] \varepsilon^t. \]

Thus, \(|\bar{w}_t| < |m_t|\) at each value of \(t\), although this difference decreases over time. From (A8), the deviation of the unemployment rate \((u_t)\) from the natural rate \((u^*)\) is \(u_t - u^* = \bar{w}_t - m_t\). As a result, demand shocks cause persistent changes in unemployment, with \(u_t\) gradually approaching \(u^*\) over time.

In contrast, suppose wage expectations are based solely on macroeconomic data. (This will occur if \(\sigma_i^2 = 0\).) In this case, \(K_1 = 0\) and \(K_2 = \rho\), so that \(\bar{w}_t^e = \rho m_{t-1}^I\). Therefore, \(\bar{w}_1^e = 0\) and \(\bar{w}_t^e = \rho^{t-1} \varepsilon^*\) for \(t \geq 2\). From (6b), average wages are \(\bar{w}_1 = \lambda \varepsilon^*\) and \(\bar{w}_t = \rho^{t-1} \varepsilon^*\) for \(t \geq 2\). As a result, \(\bar{w}_t - m_t = (1 - \lambda) \varepsilon^*\) and \(\bar{w}_t - m_t = 0\) for \(t \geq 2\), meaning that demand shocks affect unemployment in period 1, but have no effect thereafter.

The model developed in this subsection assumes that \(\varepsilon_t\) is unpredictable at the beginning of period \(t\). The model could be extended to make \(\varepsilon_t\) partly dependent on policymakers’ actions and to enable individuals to form expectations about these policies. If workers have complete information about policy shocks, these shocks will raise \(m_t\), \(\bar{w}_t^e\), and \(\bar{w}_t\) by the same amount, so unemployment would be unchanged in the current period, as well as in future periods.

A relevant issue is whether the expectations formed with macroeconomic information satisfy the criteria for rational expectations. Suppose rational expectations are defined as expectations that are unbiased and whose errors are serially uncorrelated and unpredictable to agents with superior information. Since \(m_{t-j}^{I_j} = m_{t-j} = \nu_{t-j}\) for all \(j\), the term \(K_2 \sum (\rho - K_1 - K_2)^{j-1} m_{t-j}^I\) is an unbiased predictor of the actual value of demand and is thus an
unbiased predictor of the wage that would prevail under perfect information. In addition, if agents with superior information do not know the subset of information used by individual workers, expectational errors will be unpredictable to these agents. However, this term may exhibit serial correlation, since it includes all past values of $m^I$. If workers do not update their old information about aggregate demand, past errors in demand expectations would be correlated with current errors. On the other hand, if workers acquire updated information that enables them to form new estimates of lagged values of demand and if the new errors are orthogonal to the old,$^{19}$ then wage expectation errors will be serially uncorrelated, so that overall expectations will be a mixture of rational and adaptive expectations.

3.3 Observing lagged average wages and sampling wages at other firms

A second way for workers to acquire additional information about the mean of the wage distribution is by randomly sampling wages at other firms. In this subsection, it is assumed that workers use information about lagged average wages (employing the Kalman filtering process described in the previous subsection) to form their prior beliefs about the mean of the wage distribution, but do not acquire information about macroeconomic variables. They then randomly sample other firms’ wages, which are assumed to be normally distributed (in terms of logs) with a variance of $\sigma^2_{wd}$, to update their prior beliefs. In this case, expectations are a mixture of rational and adaptive expectations, and closed-form solutions can be obtained for the amount of information acquired and for the degree to which expectations are rational vs. adaptive.

The assumption that workers do not acquire information about macroeconomic variables implies that $\sigma^2_I = \infty$. Given this assumption, workers’ prior beliefs about the mean of the wage distribution, based on lagged wages, are given by (11) with $K_2=0$ and
\[ K_i = \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2}. \]

As before, the variance of the difference between this estimate and the actual average wage is \( \lambda^2 v^2 + \sigma_w^2 \), and in this case \( v^2 \) is calculated at \( \sigma_i^2 = \infty \).

Let \( I \) represent the number of wages sampled, \( c \) represent the cost of each observation, and \( \bar{w}_i^{e,l} \) represent the average of the wages sampled. The value of \( \bar{w}_i^{e,l} \) should be an unbiased predictor of \( \bar{w}_i \), and the difference between these variables should be serially uncorrelated and unpredictable to other agents. Given the mean and variance of the prior distribution, the mean of the posterior distribution is

\[ \bar{w}_i^{e} = \omega \bar{w}_i^{e,l} + (1 - \omega) \bar{w}_i^{e, LW}, \]

where

\[ \omega = \frac{(\lambda^2 v^2 + \sigma_w^2)I}{\sigma_{WD}^2 + (\lambda^2 v^2 + \sigma_w^2)I}, \]

and the variance of the posterior distribution is

\[ E[\bar{w}_i^{e} - \bar{w}_i] = \frac{\sigma_{WD}^2 (\lambda^2 v^2 + \sigma_w^2)}{\sigma_{WD}^2 + (\lambda^2 v^2 + \sigma_w^2)I}. \]

Since the expectations formed by sampling wages have the characteristics of rational expectations, (15) implies that expectations are a weighted average of rational and adaptive expectations, with \( \omega \) measuring the degree to which expectations are rational. In this case, the total expected utility loss is

\[ TEUL \approx cI + \frac{1}{2} VL''(0) \frac{\sigma_{WD}^2 (\lambda^2 v^2 + \sigma_w^2)}{\sigma_{WD}^2 + (\lambda^2 v^2 + \sigma_w^2)I}. \]
Appendix B derives closed-form solutions for $I$ and $\omega$, and demonstrates that

$$\frac{dI}{dc} < 0, \quad \frac{dI}{d\sigma^2_x} > 0, \quad \frac{d\omega}{dc} < 0, \quad \text{and} \quad \frac{d\omega}{d\sigma^2_x} > 0.$$

Thus $\omega$, which measures the degree to which expectations are rational, depends negatively on the cost of sampling wages and depends positively on the variability of demand.

The positive dependence of $\omega$ on $\sigma^2_x$ may provide an explanation for Lucas’s (1973) finding that countries with greater variability in inflation experience smaller increases in output in response to nominal demand shocks. As demonstrated in this section, expectations become relatively more rational and relatively less adaptive as the variance of demand increases. As expectations become more rational and less adaptive, Campbell (2010) shows that the Phillips curve becomes steeper, and Campbell (2009) shows that the aggregate supply curve becomes steeper, which means that a given nominal demand shock has a smaller effect on real output.

Subsection 3.2 shows that when expectations are formed solely with macroeconomic information, unpredictable demand shocks do not change wage expectations in the current period and thus affect current unemployment (although these shocks have no effect on future unemployment). In contrast, if expectations are formed solely by sampling wages, then demand shocks affect current wage expectations by the same amount, on average, and thus do not systematically cause changes in current unemployment.

4. **Expectations Formation in Choosing Optimal Job Search Intensity**

Section 3 develops a model in which workers acquire information about average wages in choosing their level of effort. Another decision workers make is how much time to spend looking for a different job, and this decision also depends on the ratio of their own wages to average...
wages elsewhere. Workers need to estimate the mean of the wage distribution to determine how much time to devote to job search, and their process of information acquisition can be modeled in the same way as in Section 3. In fact, the job search decision is probably more important for macroeconomic outcomes than the effort decision described in Section 3 (based on the shirking model), since survey evidence suggests that theories involving turnover are much more relevant than theories involving shirking in explaining the rigidity of wages.20

In deciding how much job search to undertake, workers face the tradeoff that job search is costly, but that it may enable them to find a job that will yield higher income in the future. The present value of a worker’s expected lifetime utility can be denoted \( V(w_i, \overline{w}_i, s_i) \), where \( w_i \) and \( \overline{w}_i \) are defined as in Section 3, and \( s_i \) measures the worker’s job search intensity. If \( s(\overline{w}_i) \) represents the optimal amount of search when a worker knows that the mean of the wage distribution is \( \overline{w}_i \) and \( s(\overline{w}_i) \) represents the optimal amount of search when the worker estimates that the mean is \( \overline{w}_i \), the utility loss resulting from imperfect expectations of average wages is

\[
VL(\overline{w}_i - \overline{w}_i) = V(w_i, \overline{w}_i, s(\overline{w}_i)) - V(w_i, \overline{w}_i, s(\overline{w}_i^e)).
\]

Using a model similar to the one developed in Section 3, it can be demonstrated that, in choosing how much time to devote to job search, workers’ expectations of average wages will likely be at least partly adaptive.

Workers will quit their present jobs if they find a more attractive job at a different firm. The probability that a worker quits depends on two factors: 1) the difference between the worker’s current wage and the actual mean of the wage distribution, and 2) the worker’s search intensity. The worker’s search intensity matters because, controlling for the first factor, the
probability of finding one of the jobs that offers a higher wage depends on how hard he or she searches. Accordingly, the probability of a worker quitting can be expressed as

\[ q = f(w_i - \bar{w}_t, s(w_i - \bar{w}_t^e)) \].

Since quits depend on job search intensity, and since job search depends on \( \bar{w}_t^e \), workers’ quit propensities depend on their expectations of average wages, as well as on the actual average wage. The fact that quits are a function of average wage expectations means that quits will depend on lagged average wages if workers’ wage expectations are partly adaptive.

Campbell (1995) finds support for the hypothesis that quits depend partly on lagged average wages. In this study, quit rates in 2-digit SIC manufacturing industries are regressed on current and lagged industry wages and on current and lagged values of average manufacturing wages. Industry wages have a negative effect on quits, while average manufacturing wages have a positive effect. The hypothesis that industry wages and average manufacturing wages have an equal (but opposite) long-run effect on industry quits cannot be rejected. However, industry quit rates respond almost immediately to industry wages, but respond with a relatively long lag to average manufacturing wages, suggesting that expectations of average wages are partly adaptive.

5. Conclusion

This study develops a model in which workers obtain information on lagged average wages at a low fixed cost and can then supplement this with additional information for an added variable cost. This additional information enables them to more accurately predict the mean of the aggregate wage distribution, helping them to make better choices about their effort and job search intensity. It is demonstrated that workers’ expectations of average wages are likely to be at least partly adaptive. Their expectations are a mixture of rational and adaptive expectations if
they obtain additional information by sampling wages or if they acquire macroeconomic data and orthogonally update previous information. The degree to which expectations are rational vs. adaptive depends on the cost of acquiring information and on the variability of demand.

The main contributions of this work lie in highlighting the importance of workers’ wage expectations and in identifying the benefits of information about average wages. This information is valuable to workers because it enables them to more optimally choose their effort level and job search intensity. In addition, this study shows how microeconomic parameters affect the degree to which expectations are rational vs. adaptive and how these parameters affect the coefficients on the various lags of wages. The model developed here provides microfoundations for modeling in which expectations about average wages are partly adaptive.

The distinction between adaptive expectations and expectations formed with unbiased information is qualitatively significant. To the extent that expectations are adaptive, demand shocks have persistent real effects. In contrast, demand shocks have real effects for one period, at most, if expectations are formed by analyzing macroeconomic data or sampling wages.

This study also provides an alternative explanation for Lucas’s finding that nominal demand shocks have a smaller effect on real output in countries with greater inflation variability. It is demonstrated that workers’ expectations become relatively less adaptive as the variability of demand increases. As expectations become relatively less adaptive, the Phillips curve and the aggregate supply curve become steeper, so that nominal shocks have smaller real effects.

One implication of this study is that more work should be done on the issue of how workers estimate wages elsewhere. While there has been a great deal of research on price expectations, there has been relatively little (if any) research on the formation of wage expectations. This study discusses several methods that workers may use in estimating average
wages, such as analyzing macroeconomic data and sampling wages at other firms. In addition, workers may employ other methods to estimate average wages. Investigating the ways that workers predict average wages would yield insights into the response of effort and quits to macroeconomic shocks. Since firms take the response of workers into account in setting wages, such research may provide a better understanding of the reasons for sluggish nominal wage adjustment.
Appendix A

Equations (4) and (6) in Campbell (2010) are

(A1) \[ Q_t = A_t^\phi L_t^\phi K_0^{1-\phi} e[W_t / \bar{W}_t^e, u_t]^\phi, \]

and

(A2) \[ L_t = W_t^{\frac{\gamma}{(\gamma - 1) - \gamma}} \left( \frac{\phi(\gamma - 1)}{\gamma} \right)^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}} Y_t^{\frac{1-\gamma}{\phi(\gamma - 1) - \gamma}} A_t^{\frac{\phi(\gamma - 1)}{\phi(\gamma - 1) - \gamma}} K_0^{\frac{(1-\phi)(\gamma - 1)}{\phi(\gamma - 1) - \gamma}} \times e[W_t / \bar{W}_t^e, u_t]^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}} \bar{P}_t^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}}, \]

where \( Q \) is output, \( A \) represents technology, \( L \) is employment, \( K_0 \) is the capital stock (assumed to be fixed), \( e \) is the efficiency of the average worker, which depends on the ratio between a worker’s wage (\( W \)) and the worker’s expectation of the average wage (\( \bar{W}^e \)) and on the unemployment rate (\( u \)), \( \gamma \) is the price elasticity of demand, \( Y \) is real aggregate demand per firm, and \( \bar{P} \) is the aggregate price level. Aggregate demand is assumed to be determined from the constant velocity specification, \( Y_t = M_t / \bar{P}_t \). Solving the aggregate demand equation for \( \bar{P} \) and substituting it into (A2) yields

(A3) \[ L_t = W_t^{\frac{\gamma}{(\gamma - 1) - \gamma}} \left( \frac{\phi(\gamma - 1)}{\gamma} \right)^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}} Y_t^{\frac{1-\gamma}{\phi(\gamma - 1) - \gamma}} A_t^{\frac{\phi(\gamma - 1)}{\phi(\gamma - 1) - \gamma}} K_0^{\frac{(1-\phi)(\gamma - 1)}{\phi(\gamma - 1) - \gamma}} \times e[W_t / \bar{W}_t^e, u_t]^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}} M_t^{\frac{-\gamma}{\phi(\gamma - 1) - \gamma}}. \]

Since output equals aggregate demand in equilibrium, \( Q \) from (A1) can be substituted into \( Y \) in (A3). Making this substitution gives the following expression for \( L_t \):

(A4) \[ L_t = \frac{M_t}{W_t} \left( \frac{\phi(\gamma - 1)}{\gamma} \right). \]
The unemployment rate can be expressed as

\[(A5) \quad u_t = \frac{N - L_t}{N},\]

where \(N\) is labor supply per firm. This expression implies that \(L_t = N(1 - u_t)\). Substituting this value for employment into (A4) yields

\[(A6) \quad N(1 - u_t) = \frac{M_t}{W_t} \left( \frac{\phi(\gamma - 1)}{\gamma} \right).\]

By taking logs of both sides, aggregating over the entire economy, and using the approximation, \(\ln(1 - u_t) \approx -u_t\), unemployment is

\[(A7) \quad u_t = \bar{w}_t - m_t + n - \ln \left( \frac{\phi(\gamma - 1)}{\gamma} \right),\]

where \(\bar{w}_t\) is the average wage and lower case letters represent natural logs (except for \(u_t\)). If the equilibrium values of \(w\) and \(m\) are normalized to be equal to one another, the difference between the unemployment rate and the natural rate \((u^*)\) is

\[(A8) \quad u_t - u^* = \bar{w}_t - m_t.\]

Equation (10) in Campbell (2010) can be expressed as

\[(A9) \quad w_{it} = \bar{w}_t^e + \frac{e_u - e_{wa}}{e_{ww}} (u_t - u^*).\]

Substituting (A8) into (A9) yields

\[w_{it} = \bar{w}_t^e - \frac{e_u - e_{wa}}{e_{ww}} (m_t - \bar{w}_t).\]
where \( w_{it} \) is the wage of an individual firm and \( \bar{w}_t \) is the average wage. The above equation can be expressed as

\[
(A10) \quad w_{it} = \bar{w}_t^e + \frac{\lambda}{1-\lambda} m_t - \frac{\lambda}{1-\lambda} \bar{w}_t, \\
\text{where} \quad \lambda = \frac{e_u - e_{wu}}{e_{ww} - (e_u - e_{wu})}.
\]

In Campbell (2010), \( e_u > 0 \), \( e_{wu} < 0 \), and \( e_{ww} < 0 \), which implies that \( 0<\lambda<1 \).

In addition, there may be errors in wage setting. From (A9), a firm’s optimal wage depends on the values of \( e_u \), \( e_{wu} \), \( e_{ww} \), and \( \bar{w}_t^e \). Suppose that firms do not know these values with certainty and that there are both common and firm-specific errors in estimating these values. Then the actual wage set by firms will differ from its optimal value, as expressed in (A10), by both a common and firm-specific error term. Thus, the wage of the \( i \)th firm can be expressed as

\[
(A11) \quad w_{it} = \bar{w}_t^e + \frac{\lambda}{1-\lambda} m_t - \frac{\lambda}{1-\lambda} \bar{w}_t + \eta^*_t + \xi_t, \\
\text{where} \quad \eta^* \text{ is the common error and } \xi \text{ is the firm-specific error.}
\]
Appendix B

Derivation of (11), (12a), and (12b):

Given the system of equations in (10), a worker’s estimate of \( m_t \) can be expressed as

\[
(B1) \quad m_t^\epsilon = \left( \rho - K_1 \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \right) m_{t-1}^\epsilon + \left[ K_1 \begin{bmatrix} \bar{w}_{t-1} - (1-\lambda)\bar{w}_{t-1}^\epsilon \\ m_{t-1}^I \end{bmatrix} \right]
\]

\[
= (\rho - \lambda K_1 - K_2) m_{t-1}^\epsilon + K_1 \bar{w}_{t-1} - K_1 (1-\lambda)\bar{w}_{t-1}^\epsilon + K_2 m_{t-1}^I,
\]

where the values of \( K_1 \) and \( K_2 \) are

\[
[K_1 \quad K_2] = \rho v^2 [\lambda \quad 1] \begin{bmatrix} \lambda^2 & \sigma_w^2 \\ \sigma_i^2 & \sigma_w^2 \end{bmatrix}^{-1} + \begin{bmatrix} \sigma_w^2 \\ 0 \end{bmatrix}
\]

\[
= \rho v^2 [\lambda \quad 1] \begin{bmatrix} \lambda^2 v^2 + \sigma_w^2 & \lambda v^2 \\ \sigma_i^2 v^2 + \sigma_w^2 \sigma_i^2 & v^2 + \sigma_i^2 \end{bmatrix}^{-1}
\]

\[
= \rho v^2 [\lambda \quad 1] \frac{1}{\lambda^2 v^2 \sigma_i^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_i^2} \begin{bmatrix} v^2 + \sigma_i^2 & -\lambda v^2 \\ -\lambda v^2 & \lambda^2 v^2 + \sigma_w^2 \end{bmatrix}
\]

\[
(B2) \quad [K_1 \quad K_2] = \frac{\rho v^2 \lambda \sigma_i^2 \quad \rho v^2 \sigma_w^2}{\lambda^2 v^2 \sigma_i^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_i^2}.
\]

In (B2), \( v^2 \) represents the variance of the difference between \( m_t^\epsilon \) and the true value of \( m_t \).

Substituting (7) into (B1) yields the following relationships:

\[
(B3a) \quad m_t^\epsilon = (\rho - K_1 - K_2) m_{t-1}^\epsilon + K_1 \bar{w}_{t-1} + K_2 m_{t-1}^I, \quad \text{and}
\]

\[
(B3b) \quad \bar{w}_t^\epsilon = (\rho - K_1 - K_2) \bar{w}_{t-1} + K_1 \bar{w}_{t-1} + K_2 m_{t-1}^I.
\]
By solving (B3b) recursively, the average worker’s wage expectation is

\[ \bar{w}_t = K_1 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} \bar{w}_{t-j} + K_2 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} m_{t-j}. \]

Calculation of \( \nu^2 \) and its derivatives:

The value of \( \nu^2 \) can be calculated as follows:

\[
\nu^2 = \rho^2 \nu^2 + \sigma_v^2 - \rho^2 (\nu^2)^2 \left[ \lambda \begin{bmatrix} \lambda \\ \nu^2 \end{bmatrix} - \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_i^2 \end{bmatrix} \right]^{-1} \lambda
\]

\[
\nu^2 = \rho^2 \nu^2 + \sigma_v^2 - \rho^2 (\nu^2)^2 \left[ \lambda \begin{bmatrix} \nu^2 + \sigma_i^2 & -\lambda \nu^2 \\ -\lambda \nu^2 & \lambda^2 \nu^2 + \sigma_w^2 \end{bmatrix} \right] \lambda
\]

\[
\nu^2 = \rho^2 \nu^2 + \sigma_v^2 - \frac{\rho^2 (\nu^2)^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)}{\lambda^2 \nu^2 \sigma_i^2 + \sigma_v^2 \nu^2 + \sigma_w^2 \sigma_i^2}
\]

\[
\nu^2 \lambda^2 \nu^2 \sigma_i^2 + \nu^2 \sigma_v^2 \nu^2 + \nu^2 \sigma_w^2 \nu^2 = \rho^2 \nu^2 \sigma_w^2 \sigma_i^2 + \sigma_v^2 \lambda^2 \nu^2 \sigma_i^2 + \sigma_v^2 \sigma_w^2 \nu^2 + \sigma_v^2 \sigma_w^2 \sigma_i^2
\]

\[
[\lambda^2 \sigma_i^2 + \sigma_w^2] (\nu^2)^2 + [\sigma_v^2 \sigma_i^2 - \rho^2 \sigma_w^2 \sigma_i^2 - \lambda^2 \sigma_v^2 \sigma_i^2 - \sigma_v^2 \sigma_w^2] \nu^2 - \sigma_v^2 \sigma_w^2 \sigma_i^2 = 0
\]

\[
\nu^2 = \frac{1}{2} \sigma_v^2 + \frac{(1 - \rho^2) \sigma_w^2 \sigma_i^2 + \sqrt{B}}{2 (\lambda^2 \sigma_i^2 + \sigma_w^2)}
\]

where

\[
B = (1 - \rho^2) \sigma_w^2 \sigma_i^2 + 2 (1 + \rho^2) \sigma_w^2 \sigma_i^2 (\lambda^2 \sigma_i^2 + \sigma_w^2) \sigma_v^2 + (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 (\sigma_v^2)^2.
\]

The first derivative of \( \nu^2 \) with respect to \( \sigma_i^2 \) is

\[
\frac{\partial \nu^2}{\partial \sigma_i^2} = - \frac{2 (1 - \rho^2) (\sigma_w^2)^2 + B \frac{1}{2} (\lambda^2 \sigma_i^2 + \sigma_w^2) \frac{\partial B}{\partial \sigma_i^2} - 2 B \lambda^2}{4 (\lambda^2 \sigma_i^2 + \sigma_w^2)^2}
\]
\[
-2(1 - \rho^2)(\sigma_w^2)^2 + 2(\sigma_w^2)^2 \left[ \frac{(1 - \rho^2)^2 \sigma_w^2 \sigma_i^2 + (1 + \rho^2)(\lambda^2 \sigma_i^2 + \sigma_w^2)\sigma_e^2}{\sqrt{B}} \right]
= \frac{4(\lambda^2 \sigma_i^2 + \sigma_w^2)^2}{4(\lambda^2 \sigma_i^2 + \sigma_w^2)^2}
\]

\[
-(1 - \rho^2) + \frac{\sqrt{(1 - \rho^2)^2 (\sigma_w^2)^2 + 2(1 - \rho^2)^2 (1 + \rho^2)\sigma_w^2 \sigma_i^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)\sigma_e^2 + (1 + \rho^2)^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 (\sigma_e^2)^2}}{B}
= \frac{(\sigma_w^2)^2}{2(\lambda^2 \sigma_i^2 + \sigma_w^2)^2}
\]

\[
-(1 - \rho^2) + \frac{\sqrt{(1 - \rho^2)^2 B + 4 \rho^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 (\sigma_e^2)^2}}{B}
= \frac{(\sigma_w^2)^2}{2(\lambda^2 \sigma_i^2 + \sigma_w^2)^2}
\]

\[
\frac{\partial v^2}{\partial \sigma_i^2} = (\sigma_w^2)^2 (1 - \rho^2) - 1 + \frac{\sqrt{1 + \frac{4 \rho^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 (\sigma_e^2)^2}{B}}}{2(\lambda^2 \sigma_i^2 + \sigma_w^2)^2} > 0.
\]

The second derivative of \( v^2 \) with respect to \( \sigma_i^2 \) is

\[
\frac{\partial^2 v^2}{\partial (\sigma_i^2)^2} = (\sigma_w^2)^2 (1 - \rho^2) \left( \frac{(\lambda^2 \sigma_i^2 + \sigma_w^2)^2}{4(\lambda^2 \sigma_i^2 + \sigma_w^2)^4} \right) \frac{1}{2} \frac{\partial X}{\partial \sigma_i^2} + 4\lambda^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)(1 - \sqrt{X}) \right) \frac{\partial X}{\partial \sigma_i^2} + 4\lambda^2 (\lambda^2 \sigma_i^2 + \sigma_w^2)(1 - \sqrt{X})}
\]
where

\[
X = 1 + \frac{4 \left( \frac{\rho^2}{(1 - \rho^2)^2} (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 (\sigma^2) \right)}{B},
\]

and

\[
\frac{\partial X}{\partial \sigma_i^2} = \frac{4 \rho^2 (\sigma^2) (\lambda^2 \sigma_i^2 + \sigma_w^2) \left[ 2B \lambda^2 - (\lambda^2 \sigma_i^2 + \sigma_w^2) \frac{\partial \sigma_i^2}{\partial \sigma_i^2} \right]}{(1 - \rho^2)^2 B^2}
\]

\[
\frac{\partial X}{\partial \sigma_i^2} = \frac{8 \rho^2 (\sigma^2) (\lambda^2 \sigma_i^2 + \sigma_w^2) [(1 - \rho^2)^2 \sigma_i^2 + (1 + \rho^2)(\lambda^2 \sigma_i^2 + \sigma_w^2) \sigma_i^2]}{(1 - \rho^2)^2 B^2} < 0.
\]

Since \( \sqrt{X} > 1 \) and \( \partial X / \partial \sigma_i^2 < 0 \),

\[
\frac{\partial^2 v^2}{\partial (\sigma_i^2)^2} < 0.
\]

The derivative of \( v^2 \) with respect to \( \sigma_i^2 \) is

\[
\frac{\partial v^2}{\partial \sigma_i^2} = \frac{1}{2} + \frac{B}{2} \left[ \frac{(1 + \rho^2) \sigma_i^2 (\lambda^2 \sigma_i^2 + \sigma_w^2) + (\lambda^2 \sigma_i^2 + \sigma_w^2)^2 \sigma_i^2}{2(\lambda^2 \sigma_i^2 + \sigma_w^2)} \right] > 0.
\]

Finally, by taking the derivative of \( \partial v^2 / \partial \sigma_i^2 \) with respect to \( \sigma_i^2 \), the cross derivative of \( v^2 \) with respect to \( \sigma_i^2 \) and \( \sigma_i^2 \) is

\[
\frac{\partial^2 v^2}{\partial \sigma_i^2 \partial \sigma_i^2} = \frac{2X \left( \frac{1}{2} \rho^2 (\sigma_w^2)^3 \sigma_i^2 \sigma_i^2 \right) [(1 - \rho^2)^2 \sigma_i^2 + (1 + \rho^2)(\lambda^2 \sigma_i^2 + \sigma_w^2) \sigma_i^2]}{(1 - \rho^2) B^2} > 0.
\]
Calculation of $dl / dc$ and $dl / d\sigma^2_\varepsilon$ in Section 3.2:

Given the total expected utility loss in (13), the optimal amount of information is determined from the first-order condition,

\[
\frac{d\text{TEUL}}{dl} = 0 = c + \frac{1}{2} V L^*(0) \lambda^2 \frac{\partial v^2 (\sigma^2_i, \sigma^2_\varepsilon)}{\partial \sigma^2_i} d\sigma^2_i,
\]

where cost minimization requires that

\[
\frac{d^2\text{TEUL}}{dl^2} = \frac{1}{2} V L^*(0) \lambda^2 \left[ \frac{\partial v^2}{\partial \sigma^2_i} \frac{d^2 \sigma^2_i}{dl^2} + \frac{\partial^2 v^2}{\partial (\sigma^2_i)^2} \left( \frac{d\sigma^2_i}{dl} \right)^2 \right] > 0.
\]

Totally differentiating (B5) with respect to $I$, $c$, and $\sigma^2_\varepsilon$ yields the following comparative static results:

\[
\frac{dl}{dc} = -\frac{2}{VL^*(0) \lambda^2 \left[ \frac{\partial v^2}{\partial \sigma^2_i} \frac{d^2 \sigma^2_i}{dl^2} + \frac{\partial^2 v^2}{\partial (\sigma^2_i)^2} \left( \frac{d\sigma^2_i}{dl} \right)^2 \right]} < 0,
\]

\[
\frac{dl}{d\sigma^2_\varepsilon} = -\frac{\partial^2 v^2}{\partial \sigma^2_i \partial \sigma^2_\varepsilon} \frac{d\sigma^2_i}{dl} > 0.
\]

Calculation of $dl / dc$, $dl / d\sigma^2_\varepsilon$, $d\omega / dc$, and $d\omega / d\sigma^2_\varepsilon$ in Section 3.3:

From (18), the optimal amount of information is determined from the first-order condition,
Solving (B6) for $I$ yields

$$I = \sqrt{\frac{\sigma_{WD}^2 V L^*(0)}{2c}} - \frac{\sigma_{WD}^2}{\lambda^2 v^2 + \sigma_w^2}, \quad \text{with the boundary condition that } I \geq 0.$$  

Thus,

$$\frac{dI}{dc} = -\frac{1}{4} \left( \frac{\sigma_{WD}^2 V L^*(0)}{2c} \right)^{\frac{1}{2}} \frac{\sigma_{WD}^2 V L^*(0)}{c^2} < 0, \quad \text{and}$$

$$\frac{dI}{d\sigma_w^2} = \frac{\sigma_{WD}^2}{(\lambda^2 v^2 + \sigma_w^2)^2} \lambda^2 \frac{\partial v^2}{\partial \sigma_w^2} > 0.$$  

Substituting (B7) into (16) enables $\omega$ to be expressed as,

$$\omega = -\frac{1}{\lambda^2 v^2 + \sigma_w^2} \left( \frac{2\sigma_{WD}^2 c}{\sqrt{V L^*(0)}} \right)^{\frac{1}{2}}, \quad \text{with the boundary condition that } \omega \geq 0.$$  

By differentiating (B8), the following comparative statics results are obtained:

$$\frac{d\omega}{dc} = -\frac{1}{2} \frac{1}{\lambda^2 v^2 + \sigma_w^2} \left( \frac{2\sigma_{WD}^2 c}{V L^*(0)} \right)^{\frac{1}{2}} \frac{2\sigma_{WD}^2}{V L^*(0)} < 0, \quad \text{and}$$

$$\frac{d\omega}{d\sigma_w^2} = \frac{1}{(\lambda^2 v^2 + \sigma_w^2)^2} \left( \frac{2\sigma_{WD}^2 c}{\sqrt{V L^*(0)}} \lambda^2 \frac{\partial v^2}{\partial \sigma_w^2} > 0.\right.$$
Appendix C

If $\sigma_i^2 = \infty$, then (11) can be expressed as,

$$\bar{w}_i = z \sum_{j=1}^{\infty} x^{j-1} \bar{w}_{i-j}, \quad \text{where}$$

$$z = \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2}$$

and

$$x = \rho \frac{\lambda(\lambda - 1)v^2 + \sigma_w^2}{\lambda^2 v^2 + \sigma_w^2}.$$

From (6b),

(C1) $\bar{w}_i = \lambda m_i + (1 - \lambda) \bar{w}_i^e$

$$= \lambda m_i + (1 - \lambda) z \sum_{j=1}^{\infty} x^{j-1} \bar{w}_{i-j}$$

$$= \lambda m_i + (1 - \lambda) z \bar{w}_{i-1} + (1 - \lambda) zx \sum_{j=2}^{\infty} x^{j-2} \bar{w}_{i-j}.$$ 

Similarly,

(C2) $\bar{w}_{i-1} = \lambda m_{i-1} + (1 - \lambda) \bar{w}_{i-1}^e$

$$= \lambda m_{i-1} + (1 - \lambda) z \sum_{j=1}^{\infty} x^{j-2} \bar{w}_{i-j}.$$ 

Combining (C1) and (C2) yields

$$\bar{w}_i = \lambda m_i + (1 - \lambda) z \bar{w}_{i-1} + x(\bar{w}_{i-1} - \lambda m_{i-1})$$

$$= \lambda(m_i - xm_{i-1}) + [(1 - \lambda)z + x] \bar{w}_{i-1}.$$ 

By repeated substitution,

$$\bar{w}_i = \lambda(m_i - xm_{i-1}) + [(1 - \lambda)z + x] \lambda(m_{i-1} - xm_{i-2}) + [(1 - \lambda)z + x]^2 \lambda(m_{i-2} - xm_{i-3})$$

$$+ \cdots + [(1 - \lambda)z + x]^{i-2} \lambda(m_2 - xm_1) + [(1 - \lambda)z + x]^{i-1} [\lambda(m_1 - xm_0) + ((1 - \lambda)z + x)\bar{w}_0].$$
Since $m_0 = \bar{w}_0 = 0$ and since $m_t = \rho^{-1} \epsilon^*$ for $t \geq 1$, $\bar{w}_t$ can be expressed as

\[
\bar{w}_t = \lambda \left( \rho^{t-1} + [(1 - \lambda)z + x] \rho^{t-2} + [(1 - \lambda)z + x]^2 \rho^{t-3} + \cdots + [(1 - \lambda)z + x]^{t-1} \right) \epsilon^*
- \lambda x \left( \rho^{t-2} + [(1 - \lambda)z + x] \rho^{t-3} + [(1 - \lambda)z + x]^2 \rho^{t-4} + \cdots + [(1 - \lambda)z + x]^{t-2} \right) \epsilon^*
\]

\[
\bar{w}_t = \lambda \rho^{t-1} \left( 1 + [(1 - \lambda)z + x] \rho^{-1} + [(1 - \lambda)z + x]^2 \rho^{-2} + \cdots + [(1 - \lambda)z + x]^{t-1} \rho^{-(t-1)} \right) \epsilon^*
- \lambda x \rho^{-2} \left( 1 + [(1 - \lambda)z + x] \rho^{-1} + [(1 - \lambda)z + x]^2 \rho^{-2} + \cdots + [(1 - \lambda)z + x]^{t-2} \rho^{-(t-2)} \right) \epsilon^*.
\]

From the definitions of $z$ and $x$,

\[(1 - \lambda)z + x \rho^{-1} = \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2}.
\]

Thus,

\[
\bar{w}_t = \lambda \rho^{t-1} \left[ 1 - \left( \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^t \right] \epsilon^* - \lambda x \rho^{t-2} \left[ 1 - \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right] \epsilon^*
\]

\[
= \lambda \rho^{t-1} \left[ 1 - \left( \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^t \right] \frac{\lambda^2 \nu^2 + \sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \left[ 1 - \left( \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^{t-1} \right] \epsilon^*
\]

\[
= \lambda \rho^{t-1} \left[ \lambda^2 \nu^2 + \sigma_w^2 - \sigma_w^2 \left( \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^t \right] \frac{1}{\lambda^2 \nu^2} - \lambda^2 (1 - \lambda) \nu^2 + \sigma_w^2 \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \epsilon^*
\]

\[
\bar{w}_t = \rho^{t-1} \left[ 1 - (1 - \lambda) \left( \frac{\sigma_w^2}{\lambda^2 \nu^2 + \sigma_w^2} \right)^t \right] \epsilon^*.
\]
References


Footnotes

1 Conlisk (1988) discusses theoretical reasons for why rational behavior may result in expectations that are not unbiased. Under the assumption that it is costly to form accurate expectations of next period’s price, he demonstrates that optimal forecasts may be a weighted average of an unbiased estimate obtained from agents’ costly optimization activities and a “free estimator,” which may be determined from an adaptive expectations process. However, Conlisk does not identify the benefits of information.

2 Mullineaux (1980) and Gramlich (1983) find that, controlling for lagged inflation, other macroeconomic variables have significant effects on expectations, which suggests that economists and households use more than just past inflation to predict future inflation. Additional evidence that expectations are not purely adaptive comes from Baghestani and Noori (1988), who find that survey respondents predict inflation more accurately than an ARIMA model, implying that their expectations depend on more than just lagged inflation.

3 Fuhrer (1997) estimates Phillips curves in which expected inflation depends on a weighted average of lagged inflation and actual future inflation. He can reject the hypothesis that expectations are completely rational, but cannot reject the hypothesis that they are completely adaptive. However, he demonstrates that inflation dynamics are predicted more accurately by a model with mixed rational and adaptive expectations than by a model with completely adaptive expectations. Roberts (1998) shows that survey forecasts of inflation can be explained by a model in which part of the population has rational expectations and the rest has adaptive expectations.

4 Blundell and MaCurdy (1999) report on previous estimates of labor supply elasticities in Tables 1 and 2 of their study. The average uncompensated labor supply elasticity reported in these tables is 0.086 for men and 0.689 for married women. In addition, Card (1991, p. 22) reviews several previous studies of labor supply and concludes, “Taken together, the literature suggests that the elasticity of intertemporal substitution is surely no higher than 0.5, and probably no higher than 0.20.”

5 In conventional efficiency wage models, efficiency depends on the actual real or relative wage of workers, so these models are unable to explain nominal wage rigidity.

6 An additional way to form expectations of average wages is to use information on expected price inflation, since wage inflation and price inflation are highly correlated. As discussed in Carroll (2003), evidence suggests that some households use the forecasts of professional forecasters to form their expectations of price inflation.

7 The overall utility of effort may be either negative or positive, but the marginal utility of effort will be strictly negative for a utility-maximizing worker.

8 Campbell (2006) makes specific assumptions about workers’ utility, the probability of dismissal, and the probability of an exogenous separation. A more general version of the model is available from the author upon request.

9 For example, if the utility loss is represented by the quadratic equation, \( VL = \theta(\bar{w}_t - \bar{w})^2 \), then \( VL^* = 2\theta \). In the analysis in Section 3, \( VL(\bar{w}_t - \bar{w}) \) is treated as a general functional form. However, an equation representing this loss can be obtained if specific assumptions are made about the utility function of workers, the probability of dismissal, and the probability of an exogenous separation. In an unpublished appendix, which is available at http://www.niu.edu/econ/directory/faculty/campbell/CampbellApp1.pdf, the effort model of Campbell (2006) is used to derive a specific expression for \( VL(\bar{w}_t - \bar{w}) \).

10 The fact that \( VL'(0) = 0 \) is obtained from the relationship \( VL' = dVL / d(\bar{w}_t - \bar{w}) = (\partial VL / \partial e)(\partial e / \partial (\bar{w}_t - \bar{w})) \), where \( \partial (VL / \partial e) = 0 \) at the point \( \bar{w}_t = \bar{w} \).

11 The results are the same if we consider the wage expectation of an individual worker, except that the variance of the error would be higher by an additional error term representing the difference between the average wage expectation and the individual’s wage expectation.

12 It is assumed that data on macroeconomic variables for period \( t \) are not available until the end of the period.

13 This expression does not include the small fixed cost that workers incur to acquire information about lagged average wages.

14 Consistent with these results, Sethi and Franke (1995) develop a model in which agents can choose whether to use a costless adaptive expectations procedure or pay to acquire information that allows them to predict the true outcome with certainty. They find that agents are more likely to acquire this information when optimization costs are low or when the economy is characterized by a “high degree of exogenous variability.”

15 If \( \sigma^2 = 0 \) for a non-infinite value of \( I \) and if workers acquire this amount of information, then \( K_1 = 0 \). On the other hand, \( K_2 = 0 \) if macroeconomic variables contribute no relevant information (i.e., \( \sigma^2 = \infty \)).
If \( \rho = 1 \), the sum of coefficients on \( w_{t-j} \) plus the sum of coefficients on \( m_{t-j} \) equals 1. If \( \rho < 1 \), the sum of the sums of coefficients is less than 1 since, on average, \( m \) will be lower in period \( t \) than in period \( t-1 \) if \( \rho < 1 \).

This derivation assumes that \( \eta = 0 \) for all values of \( t \).

When \( K_1 = 0 \) and \( K_2 = \rho \), the coefficients on values of \( m' \) prior to period \( t-1 \) equal 0.

In particular, suppose that in period \( t-1 \) a worker obtained information about macroeconomic variables in period \( t-2 \) that enabled him or her to estimate demand in \( t-2 \) and let this estimate of demand be denoted \( m_{t-2} \). Then suppose in period \( t \) the worker receives revised information about macroeconomic variables in \( t-2 \) that leads the individual to estimate that demand in \( t-2 \) was \( m_{t-2} \). If \( m_{t-2} - m_{t-2} \) is uncorrelated with \( m_{t-2} \), then the new errors would be orthogonal to the old.

In Campbell and Kamlani’s (1997) and Bewley’s (1999) surveys of employers, respondents indicated that they viewed labor turnover as being much more important than shirking in explaining wage rigidity.

Equation (A9) in the present study differs from (10) in Campbell (2010) in that Campbell (2010) expresses wages and wage expectations as percentage deviations from steady-state values (rather than as natural logs), does not include firm subscripts on the wage (although these subscripts are implied since (10) is derived from the profit-maximizing behavior of individual firms), and expresses the difference between the unemployment rate and the natural rate as \( du_t \), rather than as \( u_t - u^* \). In Campbell (2010), \( e_u \), \( e_{uu} \), and \( e_{ww} \) represent, respectively, the derivative of efficiency with respect to the unemployment rate, the cross derivative of efficiency with respect to wages and unemployment, and the second derivative of efficiency with respect to wages.

In Ljungqvist and Sargent (2004, p. 123), see equation (5.6.2) for a statement of the equation for \( m' \) and equation (5.6.3a) for a statement of the equation for \( K_1 \) and \( K_2 \).

See Ljungqvist and Sargent (2004, p. 123, equation 5.6.3b) for a statement of this equation. In this study, \( \nu^2 \) is equivalent to \( \Sigma \) in Ljungfqvist and Sargent. The present study assumes that the economy is in a steady-state equilibrium, so that \( \Sigma_{n+1} = \Sigma \).