Macroeconomic framework for financial stability for Morocco

firano, zakaria and Benbachir, Saad and Abouch, Mohammed

University Mohammed V Rabat Agdal

2012

Macroeconomic framework for financial stability for morocco

Dr. Firano Zakaria
Department of Management - University of Mohamed 5, Agdal
e-mail:firanou@yahoo.fr

Dr. Abouch Mohammed
Department of Management - University of Mohamed 5, Agdal
e-mail: m_abouch @yahoo.fr

Dr. Benbachir Saâd
e-mail: benbachirsaad@gmail.com
Department of Management - University of Mohamed 5, Agdal

Summary

In this paper, a macroeconomic model with financial frictions was estimated with a view to evaluate the interactions between the Moroccan banking system and the evolutions of the macroeconomic framework. Indeed, two heterogeneous commercial banks were introduced into an objective to highlight the frictions and financial shocks which affect the evolutions of the macroeconomic economic conditions. The results obtained made it possible to confirm that this model reproduced, to a certain extent, the various fundamental characteristics of the Moroccan economic system. In addition, some shocks were planned in order to measure their impacts on the equilibrium of the system. The results were convincing and the introduction of the banking system into this model proved to be persuasive.

Key words: Macroeconomics, financial frictions, banking system, financial stability

JEL classification: E44, G21

1. **Introduction**

The economic and empirical literature treating macroeconomic models with microeconomic foundation neglected, during a certain time, the existence of frictions in the activity of financing and financial intermediation. In the traditional models of monetary policy, the financial intermediation was often ensured by the means of the central bank or through the emissions of the obligations of companies whose the owners are the households. Moreover, the majority of these models do not take into account the banking system, and when it is taken into consideration, these models omit the existence of an interbank market and informational imperfections resulting from the market of the credit.

Several assumptions were adopted during the implementation of these models, namely the perfect rationality of the agents, efficiency of the various markets and the neutrality of finance vis-à-vis the real economy. These assumptions have created a wide gap between the results of different theoretical models and the economic and financial reality.

It is only after the advent of the international financial crisis of 2008, which confirmed the interconnection between the financial sphere and the persistence of the business cycle, that empirical and theoretical work started to question the utility to actively integrate the financial intermediation in the dynamic stochastic general equilibrium models (DSGE). In the same way, the costs which were assumed by the various agents (households and firms), accelerated the process of realization of this type of models which integrate frictions and financial shocks generated by the existence of a system of financial intermediation. In addition, the taking into account of the financial stability function by the central banks aroused the interest to integrate the macroeconomic models with banking system in the analytical device making it possible to reinforce the macro prudential regulation and to provide models able to produce inputs for different stress test exercises.

Actually, several recent works concentrated on the problems of integrating the financial system in the macroeconomic models. The objective is to highlight the importance of the various frictions and financial shocks that may endanger the real economy. The introduction of the financial system, and in particular the banking system, will make it possible to describe at the same time the cyclic behaviors of the economy and to give microeconomic explanations to the evolutions of some financial sizes ignored until now. Indeed, the taking into account of the banking system will make it possible to understand the interactions between the deposits, the loans, the banks' profits, the effects of contagion and the impacts of the evolutions of the default rates and other financial variables on macro-economic and monetary stability. This will allow, among other things, to highlight the situation of the banking system in terms of financial stability. For this purpose, the establishment of a macroeconomic model incorporating financial system will help to measure the effects of inside and outside shocks that may constrain the activity of the optimal allocation of the financial system and its ability to maintain its resilience to extreme situations.

The introduction of the macroeconomic models with financial frictions does not call into question the utility of the other tools intended to evaluate financial stability (FSI1, models of macro stress test and early warning systems). However, the existence of great models integrating the financial system, makes it possible to answer the global objectives of a model of financial stability. Although the reduced models (panel data, time series, structural models) make it possible to quantify the responses of the financial system to the shocks affecting some factors of vulnerability (credit risk, liquidity risk and others), they remain less effective for analyzing more general questions. The introduction of an evaluation system, using a macroeconomic model incorporating the financial system, will make it possible to have a more complete view by introducing the majority of the shocks affecting the financial system and in addition to quantify their impact on the real economy. The pioneering work of English theorists have confirmed that the DSGE models are best suited to respond effectively to this type of requirement (see Goodhart et al. (2006)).

The macroeconomic modeling exercise integrating the banking system is very recent. Works relating to the question remain limited and meet sometimes heterogeneous aims. Model BGG of Bernanke and Al (1999) made it possible to include the procyclical behavior of the loans via frictions related to the existence of a financial accelerator. This model continues to show its success, by reproducing the evolutions of the business cycle. However, there remains imperfect in the direction where it considers only one function for the banking system which is the granting of the credits. Aspects as the default and the existence of the interbank market were unfortunately

---

[1] Financials soundness indicators (IMF (2006))

neglected. Agustin and Al (2010) were based on work of Goodhart et al. (2006) to elaborate a model DSGE for Colombia. Their work made it possible to obtain a model with acceptable quality of prediction in the short run. Their model envisages the existence of three banks on the interbank market, the households, companies and also the central bank as a monetary authority and banking supervisory. Dib (2010) succeeded to develop a model DSGE on the basis of BGG model which include at the same time, the financial accelerator and the financial frictions on the interbank market, through the existence of an informational asymmetry between the borrowers and the lenders. The work of Walque and Al (2010) for the bank of Luxemburg made it possible to produce a DSGE with two commercial banks and a banking supervision authority distinct from the central bank. Their work included, moreover, specificities of Basle II and permit to have answers in term of capital requirement and in term of injection of liquidity in financial crisis situations. Goodhart and al (2010) succeded to elaborate a DSGE model with two commercial banks and some financial frictions which are the default rates and the liquidity injections. Other works, treating these question gave satisfying results permitting to confirm the utility to include a banking system in macroeconomic models, for example the works of Goodfriend and Al (2007), Gertler and Al (2009) and Christiano and Al (2009).

DSGE models incorporating the banking system, put in production during these two last years, tried to confirm their relevance in term of economic and financial analysis. The results obtained remain contrasted, but the fact of integrating the financial system gave more rigor to the economic dynamic. Moreover, factors such as the default rates, regulatory capital and the solvency rations can henceforth be handled and be the subject of several tests of macro stress testing.

In this paper, we will try to develop a dynamic stochastic general equilibrium (DSGE) model adapted to the Moroccan economy including the banking system. This work is mainly inspired from the work of Goodhart et al. (2006 and 2010). The model we propose aims to measure the financial stability of the Moroccan banking system through its integration into a macroeconomic model. Two types of heterogeneous representative banks were indeed introduced, with the possibility of a deal on the interbank market. In addition, we consider including the frictions related to the credit market through the existence of different default rates.

However, before beginning the presentation of the microeconomic foundations of the model, we intend to briefly present the current context of the Moroccan economy to emphasize the utility of this type of model for the Moroccan economy.

The decision to maintain a fixed exchange rate regime, with a partial closure of the capital account, allowed the country to remain immune from the first negative fluctuations of international financial markets. However, and through the real economy, the economic crisis experienced by the developed countries has affected the evolution of several macroeconomic aggregates in close relationship with changes in foreign economies (European). Indeed, the foreign demand in Morocco, the remittances from Moroccans living abroad and the tourism revenues shrank, reducing the non-agricultural added value and the net foreign assets of the country. The regression of the Moroccan economic activity quickly spread to the banks activity, combined with lower growth rates of credit to the economy and of customer deposits. Therefore, the needs for increased liquidity and injections made by the Central Bank have increased, reflecting the birth of a period of lack of liquidity in the banking system. Moreover, and beyond the negative effects of the financial crisis, these period highlighted the interconnection between the banking activity and the evolution of the macroeconomic framework of the country. The Moroccan banking remains in fact dominated by the traditional activities of transformation of maturities, and the majority of bank assets consist of loans to the economy. This dominance of the intermediation activity makes the balance sheet of the Moroccan banking system increasingly exposed to the fluctuations of the habits of economic agents in terms of savings and investments as well as to the changes in the added value of the country. In addition, the importance of short-term deposits as liabilities of the various banks, continuously expose them to a risk of deformation of the economic conditions of households and firms in terms of consumption, savings and investment. A simple change in consumer preferences of economic agents may limit the strength of bank balance sheets and especially when the sources of financing emanating from the financial market are limited. This structure of the Moroccan banking system allows confirming that the conception of a macroeconomic model with banking system can reproduce the different existing microeconomic relationships.

2. Model

In order to evaluate the interactions between the real economy and the Moroccan banking system, a stochastic dynamic model of general stability integrating the banking system is elaborate. This model includes five actors, namely: households, firms of production, the central bank and two trade banks heterogeneous. The first bank has the role the collection of the deposits near the households and second is intended to satisfy the needs for liquidity of the companies and this through the granting of the appropriations and the financing of the requirements in working capital.

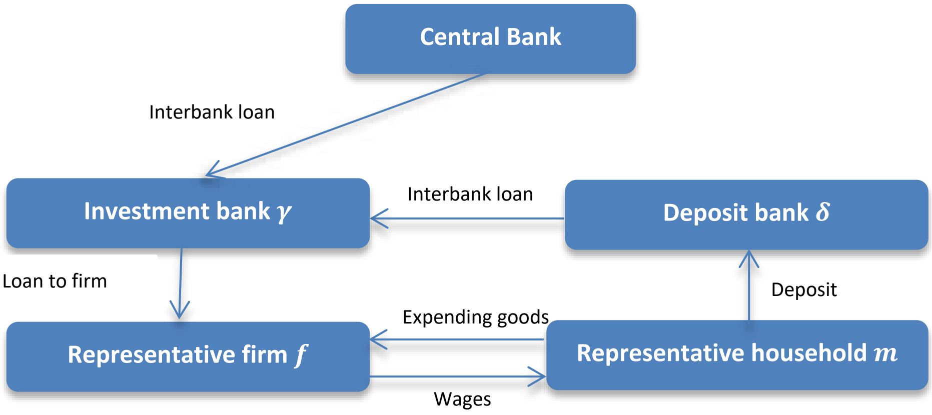


Figure 1: Financial flows between the economic agents

The representative household is considered to be the owner of both banks and his savings generate a rate of return which allows to increase his future consumption. His second source of income is the salary from his activity within the productive system. The business sector is considered to be monopolistic with manufacturing production. The outputs of the representative firm of the production system are intended to meet their needs and those of households. To achieve its production purposes, the firm uses capital originating from the investment bank in return for a remuneration rate.

The relationship between the two banks of the system is realized through the existence of an interbank market. The bank noted δ uses the funds of depositors (households) to grant credits to the bank noted γ which ensures intermediation with the productive sector. The bank γ also receives credits from the central bank and provides loans to the productive sector. The profit of the two banks, respectively, for the bank δ, the difference between the remuneration rate to the bank γ and the interest rate on deposits and, for the bank γ, is the difference between the interest rate on the producing sector and the interest rate earned by the bank δ and the central bank. The competition between the two banks has been neglected which leaves prevailing a monopolistic structure in the supply and demand market. The interbank market is considered complete in the sense that transactions between the two banks are continuous and uninterrupted.

The central bank intervenes in this structure of the economy through its ability to inject money into the money market. He was admitted in the model that the central bank intervenes through open market operations. In this context, it can increase its intervention to satisfy a massive need for liquidity and to meet the additional demand in the money market. In this perspective, the model neglects the existence of a Taylor rule and the central bank claims just his role of liquidity provider.

The five economic agents of the model are regarded as being rational agents, able to maximize their expected intertemporal utility at the date $t = 0$ by taking into account all the constraints involved. We will be interested by the set of discrete times $T = \{0, \cdots, t-1, t, t+1, \cdots\}$. In this paragraph, we briefly present the various problems of optimization in order to lead to the equilibrium conditions which will make it possible to obtain the optimal solution of the model. In the appendices, we give the necessary conditions of first order thus obtained and the various steps of optimization.

2.1. Representative household m

In the exercise of modeling, we chose to represent the whole of the households by a representative household m which maximizes its expected intertemporal utility at the date $t = 0$:

$$\max_{bm_t\,,\, Lm_t,\, dm_t:\, \forall t \in T} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(U\left(\frac{bm_t}{p_t}\right) + \lambda U(Nm - Lm_t) \right) \right\} \qquad (1)$$

under the constraint:

$$bm_t + dm_t = mm_t + \omega_t . Lm_t + Rm_t \cdot dm_{t-1}(1 + r\delta_{t-1}) + prd_t + prc_t, \quad \forall t \in T \qquad (2)$$

(i.e expenditure in goods + deposits = monetary endowment for m + labour income + refunding received on the deposits + profits on the two banks)

with:

m_t: Monetary endowment for the representative household m at the date T
bm_t: Amount of money allocated by m for the purchase of the consumer goods at the date T
dm_t: Deposits of m in the deposit bank δ at the date T
p_t: Price of the consumer goods at the date T
Nm: Available time of m
Lm_t: Time devoted by m to work at the period T

ω_t: Wage rate per unit of time worked in period t
Rm_t: Expected refunding rate of the deposit bank δ with m during the time $t-1$
$r\delta_t$: Expected repayment rate of deposit bank δ to m in period t-1
prd_t : Profit of the deposit bank δ at the period T
prc_t: Profit of the investment bank γ at the period T
β: Actualization factor
λ: Leisure relative preference parameter

The representative household income consists of wages, interest earned on deposits, dividends obtained from the two banks and the money at its disposal. These revenues are either consumed or saved in the bank deposits..

2.2. Productive sector represented by the firm f

We will represent the whole of the manufacturing production companies by a representative firm noted f which maximizes its expected intertemporal utility at the date $t = 0$:

$$\max_{lf_t,\, qf_t\,,\, cf_t,\, af_t:\, \forall t \in T} E_0\{\sum_{t=0}^{\infty} \beta^t \big(U(Yf_t - qf_t)\big)\} \quad (3)$$

under the constraints:

$$af_t \cdot cf_{t-1} = mf_t + p_{t-1} \cdot qf_{t-1}, \quad \forall t \in T \qquad (4)$$

(i.e. Credits reimbursement of f = Monetary endowment for f + Income from sales of f at the previous period)

$$lf_t = \frac{cf_t}{1+rf_t} - \Delta f_t(1 - af_t)\, cf_{t-1}, \quad \forall t \in T \qquad (5)$$

(i.e. Expenditure of f devoted to wages= Credits of f– Credits costs of the previous period)

The function of production is modeled by;

$$Yf_t = A_t \left(\frac{lf_t}{\omega_t}\right)^{\alpha}, \quad \forall t \in T \qquad (6)$$

with:
Yf_t: Production of consumer goods by the representative firm f at the period t
qf_t: Quantity of good offered by f to the sale at the period t
cf_t: Debt of f at the period t
af_t: Repayment rate on debt of f at the investment bank γ at the period t
rf_t: Interest rate on loans granted by the investment bank γ to f at the period t
lf_t: Amount of money allocated by f to labor at the date t
A_t: Technological factor at the period t
ω_t: Wage rate per unit of time worked in the period t
mf_t: Monetary endowment for f at the period t
Δf_t: Penalty of default of f on its debt at the period t
α: Returns to scale parameter

The representative firm maximizes its expected intertemporal profit expected under two types of constraints. The first relates to the repayment obligations of the bank debt and the second relates to the remuneration of different inputs which contribute to the formation of added value.

The modifications of the equilibrium conditions are supposed to intervene following a fluctuation in the technological factor of the firm. To simulate the shocks on the technological factor of the firm, we suppose that this factor A_t follows an autoregressive process of order 1:

$$ln(A_t) = \rho_A ln(\bar{A}) + (1 - \rho_A) ln(A_{t-1}) + \varepsilon_{A,t} \qquad (7)$$

Where $\bar{A}$ is the value of A_t in equilibrium and $(\varepsilon_{A,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{A,t} \backsim N(0, \sigma_A^2) \qquad (8)$$

Another type of shock can occur threatening the activity of the firm, it is the one related to credit risk when the firm is facing solvency problems towards the investment bank γ. To model this shock, we assume that the default penalty To model this shock, we suppose that the penalty of default Δf_t follows an autoregressive process of order 1:

$$ln(\Delta f_t) = \rho_f ln(\overline{\Delta f}) + (1 - \rho_f) ln(\Delta f_{t-1}) + \varepsilon_{f,t} \quad (9)$$

Where $\overline{\Delta f}$ is the value of Δf_t in the equilibrium and $(\varepsilon_{f,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{f,t} \backsim N(0, \sigma_f^2) \qquad (10)$$

2.3. Deposit bank δ

The first type of bank considered in this model is a deposit bank, whose the heart of its business is the collection of savings from households. However, and in order to fructify funds it is associated, in the interbank market, with the second bank to finance investment and contribute to economic growth. In this perspective, it maximizes the expected intertemporal utility at the following $t = 0$ date:

$$\max_{c\delta_t,\, d\delta_t,\, a\delta_t:\, \forall t \in T} E_0\{\textstyle\sum_{t=0}^{\infty} \hat{\beta}^t \cdot U(prd_t)\} \ (11)$$

under the constraint:

$$d\delta_t = m\delta_t + \frac{c\delta_t}{1+r\delta_t} - \Delta\delta_t\,(1 - a\delta_t)\, c\delta_{t-1} \qquad \forall t \in T \qquad (12)$$

(i.e. interbank loan of the deposit bank δ towards the investment bank γ = Monetary endowment of the bank δ + Deposit of the households in the bank δ − Cost on the credit of δ towards the households)

with the profit of the deposit bank δ defined by:

$$prd_t = R\delta_t(1 + r\gamma_{t-1})\, d\delta_{t-1} - a\delta_t \cdot c\delta_{t-1},\ \forall t \in T \ (13)$$

The variables and the parameters are defined by:

$m\delta_t$: Monetary endowment of the bank of deposit δ at the beginning of the period t
$c\delta_t$: Debt of the bank of deposit δ vis-à-vis the representative household m
$R\delta_t$: Expected refunding rate of the investment bank γ towards the deposit bank δ during the period $t - 1$
ri_t: Interbank interest rate during the time t
$d\delta_t$: Deposits of the deposit bank δ in the investment bank γ at the period t
$r\delta_t$: Remuneration rate of the deposits of m at the deposit bank δ at the period t
$a\delta_t$: Repayment rate of the debt of the deposit bank δ towards the representative household m at the date t
$\Delta\delta_t$: Penalty of default of the deposit bank δ on its debt towards the representative household m at the period t

prd_t : Profit of the deposit bank δ at the period t
$\hat{\beta}$: Actualization factor of the banks

The profit of the deposit bank depends on the evolution of the interbank interest rate and the conditions of saving of the representative household. The evolution of the deposits of the representative household at the deposit bank is conditioned by the fluctuations of the refunding rate of the deposit bank towards the representative household, influencing thus the penalty of default. In order to emphasize the effects of a contraction of the deposits, we suppose that the penalty of default $\Delta\delta_t$ follows an autoregressive process of order 1:

$$ln(\Delta\delta_t) = \rho_\delta ln(\overline{\Delta\delta}) + (1-\rho_\delta)ln(\Delta\delta_{t-1}) + \varepsilon_{\delta,t} \qquad (14)$$

Where $\overline{\Delta\delta}$ is the value of $\Delta\delta_t$ in equilibrium and $(\varepsilon_{\Delta\delta,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{\delta,t} \sim N(0, \sigma_\delta^2) \qquad (15)$$

2.4. Investment Bank γ

The second type of bank considered in this model is an investment bank noted γ. While profiting from the resources emanating from the deposit bank, the investment bank γ contributes directly to the economic growth via the financing the productive firm. In this respect, the bank γ maximizes its expected intertemporal utility at the date $t = 0$:

$$\max_{d\gamma_t,\, c\gamma_t,\, a\gamma_t:\, \forall t\in T} E_0\{\textstyle\sum_{t=0}^{\infty} \hat{\beta}^t \cdot U(prc_t)\} \qquad (16)$$

under the constraint:

$$d\gamma_t = m\gamma_t + \frac{c\gamma_t}{1+ri_t} - \Delta\gamma_t(1 - a\gamma_t)\, c\gamma_{t-1} \qquad \forall t \in T \qquad (17)$$

(i.e. Loans of the bank γ granted the firm f = Monetary endowment of γ + Loans of γ in the interbank market − Interbank credit cost of γ)

with :

$$prc_t = R\gamma_t(1 + rf_{t-1})\, d\gamma_{t-1} - a\gamma_t \cdot c\gamma_{t-1} \qquad \forall t \in T \qquad (18)$$

The variables and the parameters are defined by:
$m\gamma_t$: Monetary endowment of the investment bank γ at the beginning of the period t
$c\gamma_t$: Debt of the investment bank γ in the interbank market during the period t
$R\gamma_t$: Refunding rate of the representative firm f hoped by the commercial bank γ during the time $t-1$
rf_t: Interest rate of the credit granted by the investment bank γ to f during the period t
ri_t: Interbank interest rate during the period t
$d\gamma_t$: Deposits of the investment bank γ at the representative firm f in period
$a\gamma_t$: Depreciation rate of the debt of the investment bank γ in the interbank market during the period t
$\Delta\gamma_t$: Penalty of default of the investment bank γ on its debt in the interbank market during the period t
prc_t: Profit of the investment bank γ at the period t

We suppose that the penalty of default $\Delta\gamma_t$ of the investment bank in the interbank market follows an autoregressive process of order 1:

$$ln(\Delta\gamma_t) = \rho_\gamma ln(\overline{\Delta\gamma}) + (1-\rho_\gamma) ln(\Delta\gamma_{t-1}) + \varepsilon_{\gamma,t} \qquad (19)$$

Where $\overline{\Delta\delta}$ is the value of $\Delta\delta_t$ in equilibrium and $(\varepsilon_{\delta,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{\delta,t} \leadsto N(0, \sigma_\delta^2) \qquad (20)$$

2.5 Fiscal policy and outside money

We define the outside money as an injection of liquidity in the economy either by the government or by foreign transfers. The outside money enters the system free and clear of any obligation and accumulates in the economy.

If G_t is the outside money entering the economy at the period t, i.e. entering the system in the form of aggregate monetary endowment, then we choose to model the shock of the outside money by a perturbation of his growth rate which is defined by:

$$g_t = \frac{G_t}{G_{t-1}} \qquad (21)$$

The aggregate monetary endowment is distributed across all the economic agents:

$$G_t = mm_t + mf_t + m\delta_t + m\gamma_t \qquad (22)$$

where

mm_t : monetary endowment for the representative household
mf_t : monetary endowment for the representative firm
$m\delta_t$: monetary endowment for the deposit bank
$m\gamma_t$: monetary endowment for the investment bank

We suppose that, in the model, the agents receive their endowment with the same proportions:

$$\begin{cases} mm_t = \omega_1 \cdot G_t \\ mf_t = \omega_2 \cdot G_t \\ m\delta_t = \omega_3 \cdot G_t \\ m\gamma_t = \omega_4 \cdot G_t \end{cases} \qquad (23)$$

Where the weights respect:

$$\omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \qquad (24)$$

In order to stationnarize the variable G_t which is in constant growth, we will divide in the model all the monetary variables by G_{t-1} . So, if X_t is a monetary variable we will replace it by:

$$\hat{X}_t = \frac{X_t}{G_{t-1}} \qquad (25)$$

To simulate the fiscal shock we suppose that the growth rate g_t follows an autoregressive process of order 1:

$$ln(g_t) = \rho_g ln(\bar{g}) + (1-\rho_g) ln(g_{t-1}) + \varepsilon_{g,t} \qquad (26)$$

Where $\bar{g}$ is the value of g_t in equilibrium and $(\varepsilon_{g,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{g,t} \leadsto N(0, \sigma_g^2) \qquad (27)$$

The fiscal shock leads to an increase of money available to economic agents, causing mutations in the behavior of savings, of consumption and of production. These agents will have their ability to meet their various commitments improved following the abundant liquidity in their portfolios. Companies in finding that the money is fairly present among the households will be encouraged to reduce their quantities of production to fight against a likely decline in prices. This behavior of firms will reduce the output and the added value created and the level of wages and employment. Facing this situation, households will respond by reducing the allocation for savings and deposits with the aim to smooth their consumption and to stabilize their baskets of consumer goods. Banks on their part will be encouraged to reduce the amount of funds allocated to the economy and promote a policy of credit rationing. The decline in deposits and then in loans will have the effect to increase the interest rates in the deposit and credit market as well as in the interbank market. This will result in the decline of the asset prices and in the triggering of a process of deleveraging and deflation which will not be beneficial for economic agents, in a situation of credit rationing.

2.6. ***Monetary policy and inside money***

The inside money represents the interventions of the Central bank on the interbank market. The liquidities injected leave the system when the banks of investment repay their obligations. We model the shocks of the inside money by the changes on the level of the open market operations.

$$\widehat{M}_t = \xi_{M,t}\widehat{\bar{M}}_t \qquad (28)$$

where

$$\widehat{M}_t = \frac{M_t}{G_{t-1}} \quad \text{and} \quad \widehat{\bar{M}}_t = \frac{\bar{M}_t}{G_{t-1}} \qquad (29)$$

M_t being the inside money and $\bar{M}_t$ the inside money in the equilibrium state.

The parameter $\xi_{M,t}$ is supposed to follow an autoregressive process of order 1:

$$ln(\xi_{M,t}) = \rho_M ln(\bar{\xi}_M) + (1-\rho_M)ln(\xi_{M,t-1}) + \varepsilon_{M,t} \qquad (30)$$

Where $(\varepsilon_{M,t})$ is a Gaussian white noise, i.e:

$$\varepsilon_{M,t} \sim N(0,\sigma_M^2) \qquad (31)$$

3. Calibration of the model

Based on the Moroccan economic and financial data, spanning from a period 2000 to 2010 with quarterly frequency, we identified the parameters and initial values that will be used for simulation purposes. The data are taken from the official sites of various Moroccan institutions responsible for the publication of public data relating to financial and economic conditions of Morocco.

Initially, the choice of initial values of the model was carried out according to two options: the first is based on the average of the macroeconomic series for which available data are deep enough, while the second is based on the taking into consideration of a base-year, our choice was established on the base-year of 2009.

The endogenous variables which were initialized through the average of their evolutions during the years 2000 and 2010 are the credit to economy, the deposits of customers, the inflation rate, the GDP, the money and interbank interest rate.

Variables	Averages
Interbank interest rate	3.366%
Growth rate of m3	1.027
Inflation rate	2.2%
Deposits of the customers (log)	5.560

Loan of the customers (log)	5.4578
GDP (log)	5.095

Table 1: Inside variables initialized by their average

According to the evolutions recorded during the year 2009, the interbank debt, the bank's profitability, the proportion to be consumed, the interest rates presented the following values:

Variables	**Value in 2009**
Interbank loan (log)	7.78
Banking profitability	0.14
Proportion to be consumed	0.58
Deposit interest rate	0.06
Credit interest rate	0.03
Quantity of goods offered by f (qf)	0.38
Monetary endowment for f (mf)	2

Table 2: Inside variables initialized by their value in 2009

In the second time, the calibration of the model was elaborates by an analysis of the evolution of the macroeconomic aggregates of the country. This stage of work initially consisted in determining the evolutions of elasticities of some equations of the model, then in taking into account the various elasticities used in the empirical work applied to other countries. The discount coefficients for the households and the banks were evaluated hypothetically to the unit. The various repayments rates of the bilateral debts are considered close to 1, we consider thus that there does not exist any phenomenon of insolvency on behalf of the various economic agents in equilibrium situation. The technological factor was approximated according to the theory by a value equal to the unit. Concerning the rates of profit of the companies, it was allowed that these rates are in the neighborhood of 20% which is equivalent to an internal rate of return higher than the cost of the capital on the Moroccan financial market. The other parameters that track the inertia in the system were evaluated by using time series models. The table below shows the different values thus selected [2].

Parameter	**Value**	**Parameter**	**Value**
α	1	$\bar{g}$	1.008
β	1	$\bar{A}$	1
Δf	0.94	ρ_g	0.96
$\Delta\delta$	0.97	ρ_A	0.8
$\Delta\gamma$	0.99	ρ_f	0.8
Nm	1	ρ_δ	0.8
ω_1	0.25	ρ_γ	0.8
ω_2	0.25		
ω_3	0.25		
ρ_M	0.7		
$\bar{\xi}_M$	0.5		

Table 3: Parameters initializing the model

---

[2] The share of the money held initially all agents is leveling and is estimated at 25% of the total of the money in circulation.

3.1. Results of the calibrated model

The elaboration of a macroeconomic model with financial frictions aims to reproduce the various interactions noted within the Moroccan economy. Moreover, the introduction of the banking system into this type of model will make it possible to refine the various economic results and will contribute to formulate more plausible economic interpretations. The relevance of the model is thus related to its capacity to generate the same behaviors of the macro-economic variables as those noted at the empirical level. From this point of view, we intend in this section to evaluate this model through, on the one hand, an analysis of the theoretical and empirical moments, and on the other hand via some statistics obtained after calibration.

The analysis of the moments generated by the macroeconomic model makes it possible to validate its relevance. Its capacity to reproduce the various behaviors of the Moroccan economy is measured through the similarity between the empirical moments resulting from the available data and the theoretical moments which were produced by the model. The variables of interest which were analyzed are GDP, interbank interest rate, credit to economy, deposit interest rate, credit interest rate, money and deposits of the customers. The relative standard deviations calculated make it possible thus to corroborate the agreement between the empirical and theoretical series. Indeed, all the results were convincing and confirm that the model is able to reproduce the business cycles in Morocco as well as the interactions between the real sphere and the financial sphere. In this respect, it would be appropriate to use the model for simulation and also for testing the financial soundness of the banking system faced to different possible shocks. The results obtained are presented in the table below:

Relative standard deviation		
Variables	**Theoretical moments**	**Empirical moments**
Interbank credits	42%	26%
GDP (Yf)	1%	2%
Growth rate M3 (g)	1%	2%
Deposits of the households (dm)	5%	2%
Credit interest rates (rf)	12%	22%

Table 4: Theoretical and empirical relative standard deviation

Concerning the analysis of the autocorrelations between the various endogenous variables, they made it possible to reproduce the inertia noted within the macroeconomic series. The table below summarizes the results:

Simulated variables	**AR (1)**
Interbank rate	0.25
Credit interest rates	0.22
Deposits of the households	0.43
Interbank loan	0.42
Credit to firm	0.35
GDP (log)	0.26

Table 5: Autocorrelations of order 1

These various results of the model confirm its robustness and its capacity to reproduce the various economic interactions. Through this model, calibrated to date, we have the ability to imagine a number of shocks that may affect the Moroccan banking system and to evaluate the economic costs of failure of some economic agents.

3.2. Simulations of the shocks

The exercise of simulation consists in imagining various macroeconomic shocks and analyzing their impact on the various economic variables. During the elaboration of the theoretical model, we highlighted five shocks allowing to judge the relevance of the economic policies. Indeed, we highlighted a fiscal shock, a monetary shock, a technological shock and default shocks.

The fiscal shock made it possible to increase the fraction of liquidity held by the various agents, which was combined by a reduction of the manufacturing output and consequently by a rise of the prices. Moreover, this situation led the households to reduce their deposits at the deposit banks to maintain their levels of consumption. This resulted in a fall of the loans intended for the firms as well as a significant rise of the credit interest rate.

The productivity shock realized through a technological shock was reflected positively on the quantity of production of the firm, thus reducing the prices of the consumer goods and incenting the agents to consume even more. This improvement on the conditions of production will induce a fall in the quantity of work as well as in the wages paid by the company. Thus, the deposits of the customers will decrease with a regression of the profits of the deposit bank. While the investment bank will profit from the situation of the firm and their profits will increase. The improvement of the economic conditions, and especially those of the firm, and the increase of the demand of the households will lead to a significant decrease of interest rates. Thus, the interbank interest rate and the interest rate in the credit market will respond to this fall and will encourage the companies to have recourse more to the credit market.

The monetary shock has a direct impact on the objective of the interest rate. Indeed, the injections of the central bank induced a fall of interest rates on the interbank market, which results by a fall of the profits' deposit bank and conversely of those of the investment bank. This decline of the interbank rate will decrease the credit interest rates, which lets prevail an expansion in the granting of the credits becoming cheaper. Thus, the increase in the loans towards companies with a reduction of the cost of the capital will generate a fall in the general level of the prices which will stimulate the household consumption.

Finally, the shock of defaults considered in the model resulted in a fall of the capacity of the firm to refund its engagements towards the investment bank. This involved a rise in the credit interest rates which resulted in increasing the income of the investment bank. Information about the quality of the companies will lead the households to reduce their savings following a decline of the wages and the time of work. This situation will worsen consequently to the fall of the household consumption and of the amount of money allocated to savings. Consequently, the general level of the prices will post an important fall, when we know that the firm continues to maintain its production intended to the sale.

In order to show the relevance of the integration of the banking system in a DSGE model, we consider that it is useful to make the comparison between two types of models, the first taking account of the financial frictions and the second not taking account. The results which we obtained during various simulations made it possible to describe the behaviors of the economic agents with respect to particular economic situations. The comparison between the results obtained by the two models confirms the robustness of the model integrating the financial frictions.

4. Estimation of the model by the Bayesian method

4.1. Estimation method

The estimation of the DSGE model with financial frictions will help to validate the calibrated version already elaborated and whose the results were conclusive. The usual estimation techniques used and often rely on statistical inference remain unsuitable for this type of modeling, hence the use of Bayesian technique (Fernández-Villaverde (2009)).

The latter is commonly used in order to estimate the DSGE models for reasons of convenience and effectiveness. More precisely, the recourse to this econometric technique is justified by the existence of several local minimum and maximum in the stochastic dynamic models and especially those incorporating the imperfections of the markets (nominal and real rigidities and financial frictions) as well as the flat likelihoods in the majority of the parameters. These incompleteness is due primarily to the scarcity of the data and the flexibility of the DSGE models which can generate a similar behavior with different relatively combinations from the values of the parameters.

The environment of DSGE models is characterized by restricted data, which generate difficulties as to the use of real information. Indeed, the recourse to the maximum of likelihood requires complete information on the parameters in order to have effective and convincing estimates. Because of the scarcity of the data, only the Bayesian technique guaranteed a better approximation of the reality.

The Bayesian technique indeed uses the idea of the theorem of Bayes where it is enough to carry out a judgment on the distributions of the parameters and to confront them directly with the real or simulated data. Indeed, this technique is an optimal rule in information processing as indicated by Zellner (1988), it uses all available information on the data from a sample large or small, and whose reliability is proved.

The relation of Bayes admits that the knowledge of some information available contributes to increase information produced a posteriori. Precisely, the Bayes' formula applies to the prior density parameters and returns the posterior density of parameters conditional on the observations.The formula of Bayes applied to densities of probability is written:

$$\pi(\theta/y) = \frac{f(y/\theta) \times \pi(\theta)}{m(y)} \qquad (32)$$

where

$\pi(\theta)$: marginal density a priori of θ

$f(y/\theta)$: likelihood of the data y

$\pi(\theta/y)$: density a posteriori of θ

$m(y)$:marginal density of the data y

The a posteriori distribution of the parameters is based on two a priori information available, that is the marginal distribution of θ as well as the information on y. This combination of information can generate more reliable estimates by minimizing the variance of the parameters determined a priori and a posteriori. Indeed, Gelman et al. (2004) argued that one might expect that the posterior variance of the parameters is smaller than the prior variance because it takes into account information on the parameters through the data.

Indeed, the two authors managed to affirm that the variance of the estimated parameters is of minimal variance:

$$V(\theta) = E\big(V(\theta/y)\big) + V\big(E(\theta/y)\big) \qquad (33)$$

This implies that the variance a posteriori $V(\theta/y)$ is on average smaller than the variance a priori $V(\theta)$. The difference $V\big(E(\theta/y)\big)$ depends on the variability of the average a posteriori of the distribution of the available data. Gelman deduced that more this variability is large more the a posteriori variability of θ is small.

More generally, we don't refer to expectations, the posterior mean and posterior variance can be approximated as a compromise between a priori theory and data. The weights associated with these two sources of information depend respectively on the variability of the observed data and the variability of the a priori information. The larger the sample size increases more the relative weight of the prior decreases. So, the Bayesian approach provides greater efficiency on the estimation of the density of the estimated parameters, with the combined use of both available information (distribution and data).

Thus through the Bayes relation we can obtain the estimates of the various parameters of interest. However, the use of the method requires methods of simulation of the a posteriori distribution. The methods of simulation which we chose to use are the Markov Chain Monte Carlo[3] (MCMC).

4.2. Estimation results

In the estimation phase of the model with financial frictions we used two types of estimations namely: the estimation of a small-scale model (NKM[4]) as well as the estimation by the recourse to the DSGE-VAR[5].

The estimated parameters of the various models were obtained using the available data on the Moroccan economy. The series used are quarterly frequency from 1985 to 2010. The use of the Bayesian method could reduce the problems of scarcity of the data for some series non available during the estimation of the DSGE model.

The following table summarizes the results obtained after the estimation of the DSGE-VAR:

Parameters	**DSGE-VAR**	
	Value	**Distributions**
β	0.89	Beta
α	0.9905	Normal
λ	0.8902	Beta
ρ_M	0.9	Normal
ρ_A	0.7001	Normal
ρ_f	0.7	Normal
ρ_δ	0.6999	Normal
ρ_Y	0.7003	Normal
Δf	0.7892	Beta
$\Delta\delta$	0.8014	Beta

---

[3] The methods known as MCMC rest on the construction of chains of Markov having the property to converge towards the distribution which one seeks to simulate.

[4] New Keynesian Model

[5] For more details to see Del Negro, Mr., and F. Schorfheide (2004).

$\Delta\gamma$	0.8116	Beta

Table 6: Parameters estimated by the Bayesian method

4.3. *Comparison with the BVAR*[6]

In order to validate the estimates resulting from the DSGE model with financial frictions, we deduced from this last a small-scale model (NKM[7]) that can be compared with the BVAR model whose power was proven in the literature.

Parameters	**BVAR**		**NKM**	
	Value	**Distributions**	**Value**	**Distributions**
β	1.06	Gamma	1.04	Gamma
Persistence IS	1.16	Inv-gamma	1.01	Inv-gamma
Persistence LM	0.78	Inv-gamma	0.69	Inv-gamma
Philips curve	0.17	Normal	0.17	Normal

Table 7: Parameters estimated by the two small-scale models

The results obtained are almost identical, thus we can confirm that the introduction of the financial frictions into the DSGE model could contribute to improve the dynamics of the model.

We also compared the predictive capacity of the three models. In this perspective, we can affirm that the estimates of the three models have a satisfactory predictive quality. The latter claims that the DSGE model with financial frictions largely traces the trends in Moroccan macroeconomic aggregates. Thus, we chose to compare the three models in terms of forecasting inflation.

---

[6] Bayesian Vectorial Autoregressive (See Villemot (2007)).

[7] After the log-linearization of the model

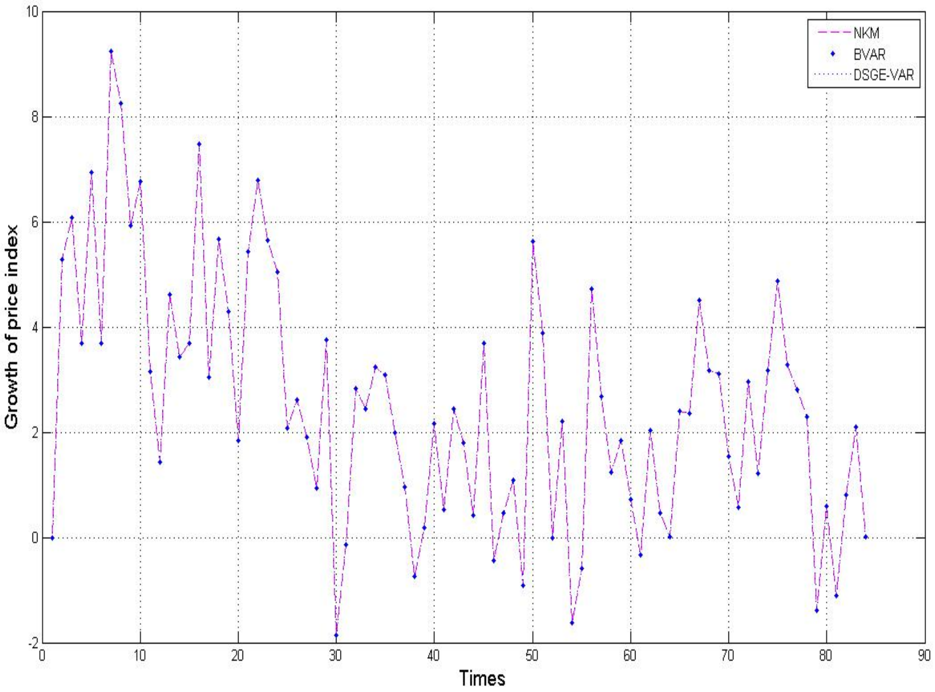


Figure 2: Inflation rate estimated by the three models

4.3. *Interpretation of shocks simulation*

The results obtained made it possible to affirm that the introduction of the financial frictions into the macroeconomic model improved the impulse responses as to the various economic policies.

Indeed, we considered, in similarity with the responses produced by the calibrated model, to work out several shocks of economic policies to have an idea on the responses that estimated DSGE model with financial friction can produce. Thus, the three economic shocks are: a fiscal shock, a monetary shock and a technological shock.

Fiscal shock:

The reinforcement of the liquidity of the economic agents by the means of an increase in their consumption in time causes positive reactions compared to the behaviors of the responses of the agents. The shock thus added, gave responses in conformity with the economic theory. Thus, we see, in the horizon of six quarters, that the fiscal shock leads to an increase in bank profitability, in the price level, in the interbank rate and in the debt to finance growth. Concerning the economic growth, the effects of expansionary fiscal policy are positive but small. However, this policy which aims to strengthen the capacity of economic agents in terms of consumption, will cause the opposite effect with a lower rate of consumption which can be explained through the higher prices and interest rate markets (Ricardian equivalence).

Monetary shock:

The responses obtained for a monetary shock which affects the various components of the real and financial sectors were in conformity with the economic theory. Thus, a rise of the central bank money positively affects the economic growth, the prices and the household consumption. By opposition, the cost of the debt of the firms, the outputs of the deposit banks and the interbank rate showed a fall in response of the rise of the central bank money.

Technological shock:

The impact of a shock on the productivity of the firms involves necessarily various effects on the Moroccan macroeconomic aggregates. Indeed, the responses obtained remain in conformity with economic perception. In the facts, rises of the efforts of productivity result in a fall of the prices, the debt, the interbank rate as well as the capacities of the households to make deposits at the finance firms. On the other hand, this economic policy involves a significant rise of the economic growth, the consumption and the profits of the commercial banks.

Figure 3: Technological shock

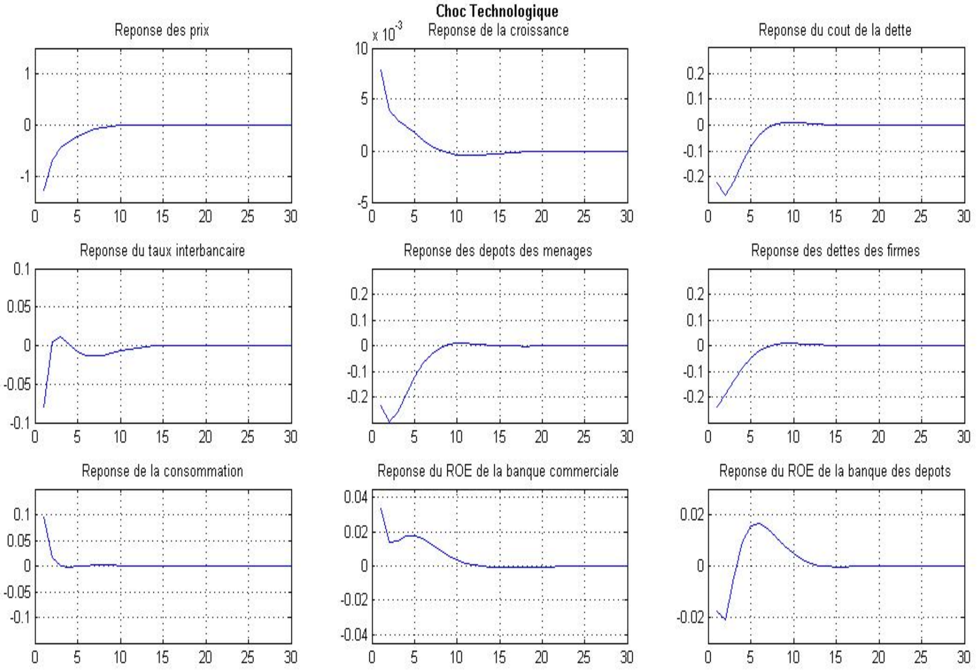


Figure 4: Monetary shock

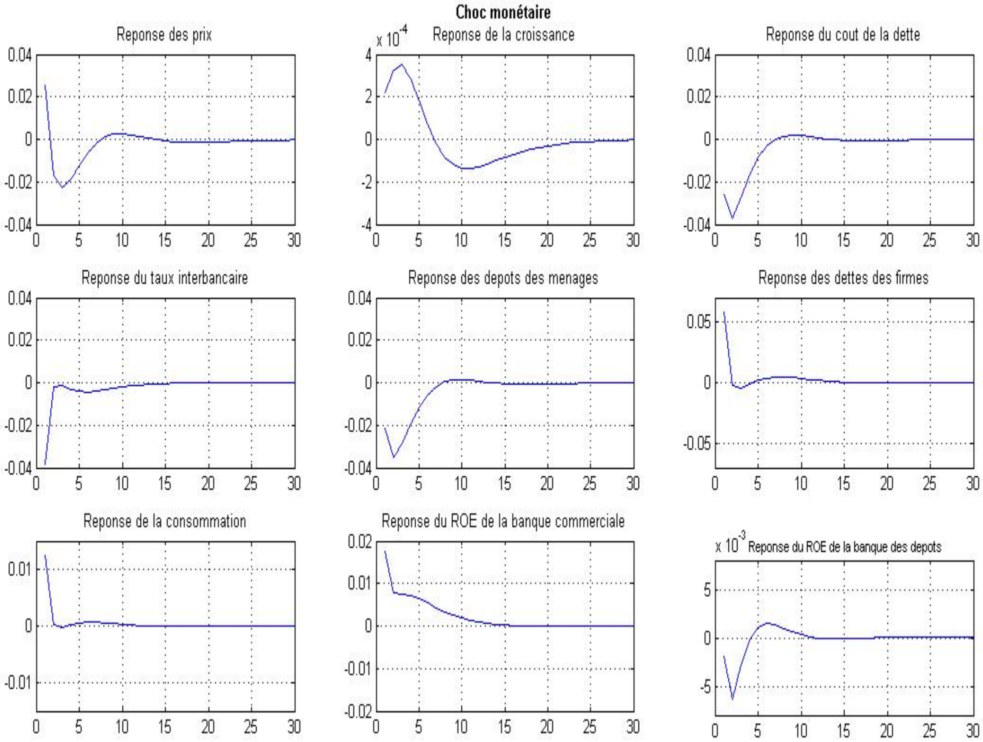
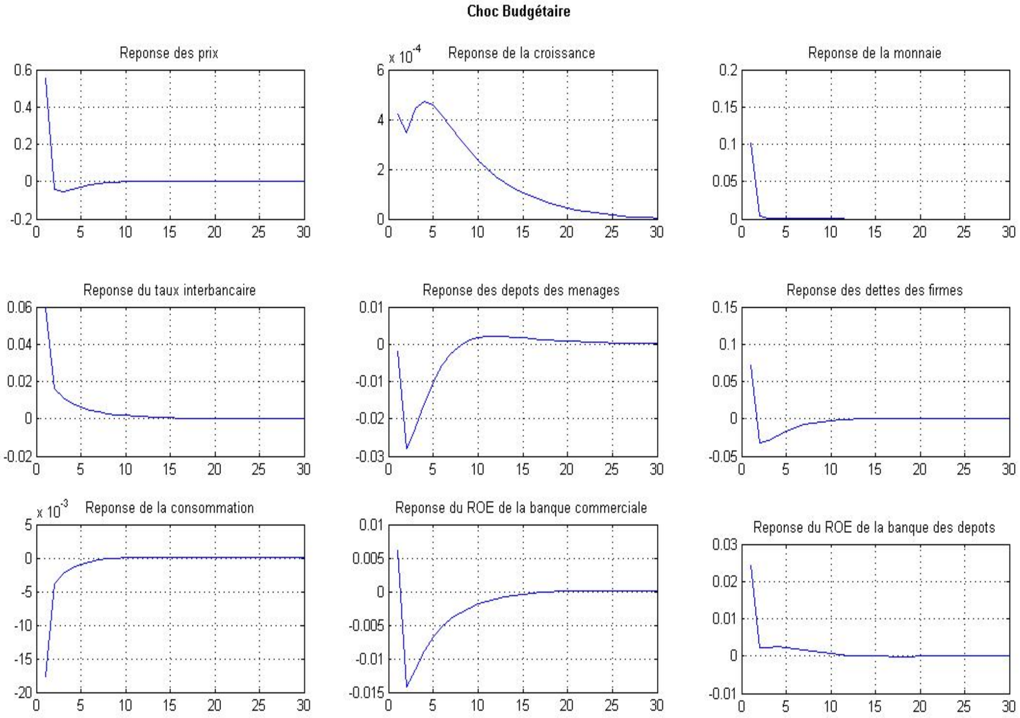


Figure 4: Fiscal shock



4.Conclusion

The implementation of the DSGE model with financial frictions aims to achieve macroeconomic modeling incorporating the Moroccan banking system. This will allow us to measure the different interactions between the real and financial spheres and combine exercises incorporating the dynamic simulations and financial cycles that characterize the activity of intermediation.

Based on the Moroccan data, one could obtain a calibrated model and found to describe the various changes in economic conditions. The integration of the banking system has quantified the economic costs of a shock to the various banks. In this preliminary version of the model, only a few shocks have been identified, namely those affecting the technology, liquidity, fiscal policy and the defaults of the firms.

The comparison between the model with and without financial frictions showed us that the integration of the mechanisms of financial frictions has a positive effect on the responses of the model. The impulses generated through this model pertinently describe the rational behaviors of the various economic agents. In addition, the incorporation of the banking system in modeling contributes to describe the responses of the economic and financial variables as to various possible shocks, which can help to evaluate the financial stability of the Moroccan banking system.

This model will be able to contribute, in addition, to generate plausible macroeconomic shocks in order to work out macro stress tests aiming to evaluate the robustness of the Moroccan banking system. In the same wake, the financial design of the model will make it possible to measure the articulation between some stabilization policies such as: monetary, fiscal and macro prudential policies.

For now, this model does not include prudential variables, to judge the solvency of banking system, but in other works witch we will later realize we will take into consideration all these improvements. The incorporation of a prudential variable can be a reference for decisions concerning the financial stability.

References

Bernanke, B., Mr. Gertler, and S. Gilchrist (1999), The Financial Accelerator in has Quantitative Business Cycles Framework in J. Taylor and M.Woodford (eds) off Handbook Macroeconomics, Volume 1C, North Holland.

Blanchard, O.J and C.M. Kahn. 1980. The Solution off Linear Difference Models under Rational Expectations. Econometrica 48:1305 - 11.

Blanchard, O.J and NR. Kiyotaki. 1987. Monopolistic Competition and the Effects off Aggregate Demand. The American Economic Review 77:647 - 66.

Buiter, W.H. (1999), The Fallacy off the Tax Theory off the Price Level, NBER Working Paper No 7302. 16

Christiano, L., R. Motto, and Mr. Rostagno. 2010. Financial Factors in Economic Fluctuations. "Central European Bank Working Paper No 1192.

Curdia, V. and Woodford, Mr. 2009, Credit Spreads and Monetary Policy, NBER Working Paper No 15289.

De Walque, G., O. Pierrard there A. Rouabah, (2008), Financial (in) stability, supervision, and liquidity injections: has dynamic general equilibrium approach, Working Paper Research 148, National Bank off Belgium.

Dib A. 2010. Year Estimated Canadian DSGE Nominal Model with and Real Rigidities Working Paper, Bank off Canada

Dubey, P., and J. Geanakoplos, (2003), Real Determinacy with Nominal Assets, Cowles Foundation Discussion Paper No 1427.

Gali, J. 1999. Technology, Employment, and the Business Cycle: C Technology Shocks Explain Aggregate Fluctuations? The American Economic Review 89:249 - 71.

Goodfriend, Mr. and R. King. 1997. The New Neoclassical Synthesis and the Role off Monetary Policy. NBER Macroeconomics Annual 231-83.

Goodhart CAE, D. Tsomocos and A. Vardoulakis, (2009), Modelling has Housing and Mortgage Crisis, in Financial Stability, Monetary Policy and Central Banking. Edited by C. Echavarria, R. Fuentes and D. Gray, Series one Central Banking, Analysis and Economics Policies, Central Bank off Chile, forthcoming, 2009.

Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2004), has Model to Analyse Financial Fragility: Applications, Newspaper off Financial Stability, 1, pp. 1-30.

Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2006), has Model to Analyse Financial Fragility, Economic Theory (27), pp. 107-142.

Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2008), Optimal The Monetary Instrument for Prudential Purposes Newspaper off Financial Stability, forthcoming.

Iacoviello, Mr. and S. Neri, (2007), Housing Markets Spillovers: Obviousness from year Estimated DSGE Model. Working Paper 659, Boston College Department off Economics.

Kiyotaki NR. and J. Moore, (2007), Credit Cycles., Newspaper off Political Economy, 105, pp. 211248.

Leao E., and P. Leao, (2007), central Modelling the bank repo spleen in has dynamic general equilibrium framework, Economic Modelling, 24, pp. 571-610.

Mendoza, G.E. 1991. Real Business Cycles in has Small Open Economy. The American Economic Review 81:797 - 818.

Notarpietro, A., (2007), Credit Frictions and Household Debt in the US Business Cycles: In Bayesian Approach,. Working Paper Universit Bocconi.

Tsomocos, D.P., (2003), Equilibrium Analysis, Banking and Financial Instability, Newspaper off Mathe- matical Economics, 39 (5), pp. 619-655.

Tsomocos, D.P., (2008), Generic Determinacy and Money Non-neutrality off International Monetary Equilibria, Newspaper off Mathematical Economics, 44, pp. 866-887.

Woodford, Mr. 2003. Interest and Prices: Foundations off off has Theory Monetary Policy, Princeton, NJ, Princeton University Press.

Appendices

I. Resolution of the first order conditions for DSGE model with financial frictions

Problem of optimization of the representative household$\boldsymbol{m}$:

$$\max_{bm_t\,,\,Lm_t,\,dm_t:\,\forall t\in T} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(U\left(\frac{bm_t}{p_t}\right)+\lambda U(Nm-Lm_t)\right)\right\} \tag{34}$$

under the constraint:

$$bm_t+\ dm_t=mm_t+\omega_t.Lm_t+Rm_t\cdot dm_{t-1}(1+r\delta_{t-1})+prd_t+prc_t,\quad \forall t\in T \tag{35}$$

The function of Lagrange is given by:

$$L=E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(U\left(\frac{bm_t}{p_t}\right)+\lambda U(Nm-Lm_t)+\boldsymbol{\gamma_t}(mm_t+\omega_t.Lm_t+Rm_t\cdot dm_{t-1}(1+r\delta_{t-1})+prd_t+prc_t-bm_t-\ dm_t)\right)\right\} \tag{36}$$

First order conditions:

1) $\frac{\partial\mathcal{L}}{\partial dm_t}=0\Leftrightarrow\beta^t\left(-\gamma_t\right)+\beta^{t+1}E_t\{\gamma_{t+1}Rm_t(1+r\delta_t)\}=0$

$$\Leftrightarrow\gamma_t=\beta.E_t\{\gamma_{t+1}Rm_{t+1}(1+r\delta_t)\}$$

2) $\frac{\partial\mathcal{L}}{\partial Lm_t}=0\Leftrightarrow\beta^t\{-\lambda U'(Nm-Lm_t)+\gamma_t\omega_t\}=0$

$$\Leftrightarrow\gamma_t=\lambda\frac{U'(Nm-Lm_t)}{\omega_t}$$

3) $\frac{\partial\mathcal{L}}{\partial bm_t}=0\Leftrightarrow\beta^t\left\{\frac{1}{p_t}U'\left(\frac{bm_t}{p_t}\right)-\gamma_t\right\}=0$

$$\Leftrightarrow\gamma_t=\frac{1}{p_t}U'\left(\frac{bm_t}{p_t}\right)$$

1st formula:

$$\lambda\frac{U'(Nm-Lm_t)}{\omega_t}=\frac{1}{p_t}U'\left(\frac{bm_t}{p_t}\right)$$

$$\Leftrightarrow\frac{\omega_t}{p_t}=\lambda\frac{U'(Nm-Lm_t)}{U'\left(\frac{b_t^{\theta}}{p_t}\right)}$$

$$\Leftrightarrow\frac{\widehat{\omega}_t}{\hat{p}_t}=\lambda\frac{U'(Nm-Lm_t)}{U'\left(\frac{bm_t}{p_t}\right)} \tag{37}$$

2nd formula:

$$\frac{1}{p_t}U'\left(\frac{bm_t}{p_t}\right)=\ \beta.E_t\left\{\frac{1}{p_{t+1}}U'\left(\frac{bm_{t+1}}{p_{t+1}}\right)Rm_{t+1}(1+r\delta_t)\right\}$$

We set: $cm_t=\frac{bm_t}{p_t}$

Thus:

$$\Leftrightarrow U'(cm_t) = p_t.\beta.E_t\left\{\frac{1}{p_{t+1}}U'(cm_{t+1})Rm_{t+1}(1+r\delta_t)\right\}$$

$$\Leftrightarrow U'(cm_t) = \hat{p}_t.G_{t-1}.\beta.(1+r\delta_t).E_t\left\{\frac{1}{\hat{p}_{t+1}.G_t}U'(cm_{t+1})Rm_{t+1}\right\}$$

$$\Leftrightarrow \frac{G_t}{G_{t-1}}U'(cm_t) = \beta.(1+r\delta_t).E_t\left\{\frac{U'(cm_{t+1})Rm_{t+1}}{\frac{\hat{p}_{t+1}}{\hat{p}_t}}\right\}$$

$$\Leftrightarrow g_t.U'(c) = \beta.(1+r\delta_t).E_t\left\{\frac{U'(cm_{t+1})Rm_{t+1}}{\frac{\hat{p}_{t+1}}{\hat{p}_t}}\right\} \qquad (38)$$

Problem of optimization of the representative firm f:

$$\max_{lf_t,\, qf_t\,,\, cf_t,\, af_t:\, \forall t\in T} E_0\left\{\sum_{t=0}^{\infty}\beta^t\big(U(Yf_t - qf_t)\big)\right\}$$

under the budgetary constraints:

$$af_t \cdot cf_{t-1} = mf_t + p_{t-1}\cdot qf_{t-1}, \quad \forall t\in T$$

$$lf_t = \frac{cf_t}{1+rf_t} - \Delta f_t(1 - af_t)\, cf_{t-1}, \quad \forall t\in T$$

and

$$Yf_t = A_t\left(\frac{lf_t}{\omega_t}\right)^{\alpha}, \quad \forall t\in T$$

The function of Lagrange:

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{\infty}\beta^t\big(U(Yf_t - qf_t)\big) + \alpha_t\big(mf_t + p_{t-1}\cdot qf_{t-1} - af_t\cdot cf_{t-1}\big) + \gamma_t\left(\frac{cf_t}{1+rf_t} - \Delta f_t(1 - af_t)\, cf_{t-1} - lf_t\right)\right\}$$

1) $\frac{\partial\mathcal{L}}{\partial\, lf_t} = 0 \Leftrightarrow \beta^t\left(U'(Yf_t - qf_t).\alpha.\frac{Yf_t}{lf_t} - \gamma_t\right) = 0$

$$\Leftrightarrow \gamma_t = \alpha.\frac{Yf_t}{lf_t}U'(Yf_t - qf_t)$$

2) $\frac{\partial\mathcal{L}}{\partial af_t} = 0 \Leftrightarrow \beta^t(-\alpha_t\cdot cf_{t-1} + \gamma_t\cdot\Delta f_t\cdot cf_{t-1}) = 0$

$$\Leftrightarrow \alpha_t = \gamma_t\cdot\Delta f_t$$

3) $\frac{\partial\mathcal{L}}{\partial qf_t} = 0 \Leftrightarrow \beta^t\big(-U'(Yf_t - qf_t)\big) + \beta^{t+1}E_t(p_t.\alpha_{t+1}) = 0$
$\Leftrightarrow U'(Yf_t - qf_t) = \beta.p_t.E_t(\alpha_{t+1})$

4) $\frac{\partial\mathcal{L}}{\partial cf_t} = 0 \Leftrightarrow \beta^t\left(\frac{\gamma_t}{1+rf_t}\right) + \beta^{t+1}E_t\big(-\alpha_{t+1}\, af_{t+1} - \gamma_{t+1}\cdot\Delta f_{t+1}(1 - af_{t+1})\big)$

$$\Leftrightarrow \frac{\gamma_t}{1+rf_t} = \beta E_t(\gamma_{t+1}\cdot\Delta f_{t+1})$$

1st formula:

$$\alpha.\frac{Yf_t}{lf_t}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\alpha.\frac{Yf_{t+1}}{lf_{t+1}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

$$\Leftrightarrow \frac{Yf_t}{lf_t}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\frac{Yf_{t+1}}{lf_{t+1}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

Where : $Yf_t = A_t\left(\frac{lf_t}{\omega_t}\right)^{\alpha} = A_t(\,Lm_t)^{\alpha}$

Thus:

$$\frac{A_t(\,Lm_t)^{\alpha}}{lf_t}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\frac{A_{t+1}(\,Lm_{t+1})^{\alpha}}{lf_{t+1}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

$$\Leftrightarrow \frac{A_t(\,Lm_t)^{\alpha-1}}{\omega_t}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\frac{A_{t+1}(\,Lm_{t+1})^{\alpha-1}}{\omega_{t+1}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

$$\Leftrightarrow A_t(\,Lm_t)^{\alpha-1}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\frac{A_{t+1}(\,Lm_{t+1})^{\alpha-1}}{\frac{\omega_{t+1}}{\omega_t}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

$$\Leftrightarrow A_t(\,Lm_t)^{\alpha-1}U'(Yf_t - qf_t) = \beta(1 + rf_t)E_t\left(\frac{A_{t+1}(\,Lm_{t+1})^{\alpha-1}}{\frac{G_t\widehat{\omega}_{t+1}}{G_{t-1}\widehat{\omega}_t}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right)$$

$$\Leftrightarrow g_t.A_t(\,Lm_t)^{\alpha-1}U'(Yf_t - qf_t)$$

$$= \beta(1 + rf_t).E_t\left(\frac{A_{t+1}(\,Lm_{t+1})^{\alpha-1}}{\frac{\widehat{\omega}_{t+1}}{\widehat{\omega}_t}}\cdot \Delta f_{t+1}.U'(Yf_{t+1} - qf_{t+1})\right) \tag{39}$$

2nd formula:

$$\frac{U'(Yf_t - qf_t)}{\beta.p_t} = \frac{\alpha.\frac{Yf_t}{lf_t}U'(Yf_t - qf_t)}{\beta.(1 + rf_t)}$$

$$\Leftrightarrow \frac{1 + rf_t}{p_t} == \alpha.\frac{Yf_t}{lf_t} \Leftrightarrow \frac{1 + rf_t}{p_t}\ \alpha.\left(\frac{A_t}{lf_t}\right)\left(\frac{lf_t}{\omega_t}\right)^{\alpha} \Leftrightarrow \frac{\omega_t(1 + rf_t)}{p_t} = \alpha A_t(\,Lm_t)^{\alpha-1} \Leftrightarrow \frac{\omega_t(1 + rf_t)}{p_t} = \alpha\frac{A_t(\,Lm_t)^{\alpha}}{Lm_t}$$

$$\Leftrightarrow \frac{\omega_t(1 + rf_t)}{p_t} = \alpha\frac{Yf_t}{lf_t}$$

$$\Leftrightarrow \frac{\widehat{\omega}_t}{\hat{p}_t}(1 + rf_t) = \alpha\frac{Yf_t}{lf_t} \tag{40}$$

Problem of optimization of the bank δ:

$$\max_{c\delta_t,\, d\delta_t,\, a\delta_t:\ \forall t\in T} E_0\left\{\sum_{t=0}^{\infty}\hat{\beta}^t\cdot U(\,prd_t)\right\}$$

under constraint:

$$d\delta_t = m\delta_t + \frac{c\delta_t}{1 + r\delta_t} - \Delta\delta_t\,(1 - a\delta_t)\,c\delta_{t-1} \qquad \forall t\in T$$

With $prd_t = R\delta_t(1 + r\gamma_{t-1})\,d\delta_{t-1} - a\delta_t\cdot c\delta_{t-1},\ \forall t\in T$

The function of Lagrange:

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{\infty} \hat{\beta}^t [\, prd_t + \lambda_t(\, R\delta_t \cdot (1 +\ r\gamma_{t-1}) \cdot\ d\delta_{t-1} -\ a\delta_t \cdot\ c\delta_{t-1} -\ d\delta_t)]\right\}$$

First order conditions:

1) $$\frac{\partial \mathcal{L}}{\partial\, d\delta_t} = 0 \Leftrightarrow \hat{\beta}^t(-\lambda_t) + \hat{\beta}^{t+1} E_t\{U'(\, prd_{t+1})\, R\delta_{t+1}(1 +\ r\gamma_t)\} = 0$$
$$\Leftrightarrow \lambda_t = \hat{\beta}.(1 +\ r\gamma_t).E_t\{U'(\, prd_{t+1})\, R\delta_{t+1}\}$$

2) $$\frac{\partial \mathcal{L}}{\partial\, a\delta_t} = 0 \Leftrightarrow \hat{\beta}^t(-U'(\, prd_t)\, c\delta_{t-1} + \lambda_t \Delta\delta_{t-1}\, c\delta_{t-1}) = 0$$
$$\Leftrightarrow U'(\, prd_t) = \lambda_t \Delta\delta_t$$

3) $$\frac{\partial \mathcal{L}}{\partial\, c\delta_t} = 0 \Leftrightarrow \hat{\beta}^t\left(\lambda_t\left(\frac{1}{1 + r\delta_t}\right)\right) + \hat{\beta}^{t+1} E_t\{-U'(\, prd_{t+1})\, a\delta_{t+1} - \lambda_{t+1}\Delta\delta_{t+1}(1 -\ a\delta_{t+1})\}$$
$$\Leftrightarrow \frac{\lambda_t}{1 +\ r\delta_t} = \hat{\beta}.E_t\{U'(\, prd_{t+1})\, a\delta_{t+1} + \lambda_{t+1}\Delta\delta_{t+1} - \lambda_{t+1}\Delta\delta_{t+1}\, a\delta_{t+1}\}$$
$$\Leftrightarrow \frac{\lambda_t}{1 +\ r\delta_t} = \hat{\beta}.E_t\{U'(\, prd_{t+1})\, a\delta_{t+1} + \lambda_{t+1}\Delta\delta_{t+1} - U'(\, prd_{t+1})\, a\delta_{t+1}\}$$
$$\Leftrightarrow \frac{\lambda_t}{1 +\ r\delta_t} = \hat{\beta}.E_t\{\lambda_{t+1}\Delta\delta_{t+1}\}$$

1st formula:

$$\frac{U'(\, prd_t)}{\Delta\delta_t} = \hat{\beta}.(1 +\ r\gamma_t).E_t\{U'(\, prd_{t+1})\, R\delta_{t+1}\}$$
$$\Leftrightarrow U'(\, prd_t) = \hat{\beta}.(1 +\ r\gamma_t).E_t\{U'(\, prd_{t+1})\, R\delta_{t+1}\Delta\delta_t\} \qquad (41)$$

2nd formula:

$$\frac{U'(\, prd_t)}{\Delta\delta_t(1 +\ r\delta_t)} = \hat{\beta}.E_t\left\{\Delta\delta_{t+1}\frac{U'(\, prd_{t+1})}{\Delta\delta_{t+1}}\right\}$$
$$\Leftrightarrow U'(\, prd_t) = \hat{\beta}.(1 +\ r\delta_t).E_t\{\Delta\delta_t \cdot U'(\, prd_{t+1})\} \qquad (42)$$

Problem of optimization of the bank γ:

$$\max_{d\gamma_t,\, c\gamma_t,\, a\gamma_t:\ \forall t \in T} E_0\left\{\sum_{t=0}^{\infty} \hat{\beta}^t \cdot U(\, prc_t)\right\}$$

Under the constraint:

$$d\gamma_t =\ m\gamma_t + \frac{c\gamma_t}{1 +\ ri_t} -\ \Delta\gamma_t(1 -\ a\gamma_t)\, c\gamma_{t-1} \qquad \forall t \in T$$

with $prc_t =\ R\gamma_t(1 +\ rf_{t-1})\, d\gamma_{t-1} -\ a\gamma_t \cdot\ c\gamma_{t-1} \qquad \forall t \in T$

The function of Lagrange:

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{\infty} \hat{\beta}^t\left(U(\, prc_t) + \lambda_t\left(\, m\gamma_t + \frac{c\gamma_t}{1 +\ ri_t} -\ \Delta\gamma_t(1 -\ a\gamma_t)\, c\gamma_{t-1} -\ d\gamma_t\right)\right)\right\}$$

First order conditions:

1) $\frac{\partial \mathcal{L}}{\partial d\gamma_t} = 0 \Leftrightarrow \hat{\beta}^t(-\lambda_t) + \hat{\beta}^{t+1}E_t\{U'(\,prc_{t+1})\, R\gamma_{t+1}(1+\, rf_t)\} = 0$

$$\Leftrightarrow \lambda_t = \hat{\beta}.(1+\, rf_t).E_t\{U'(\,prc_{t+1})\, R\gamma_{t+1}\}$$

2) $\frac{\partial \mathcal{L}}{\partial a\gamma_t} = 0 \Leftrightarrow \hat{\beta}^t(-U'(\,prc_t)\, c\gamma_{t-1} + \lambda_t\, \Delta\gamma_t\, c\gamma_{t-1}) = 0$

$$\Leftrightarrow U'(\,prc_t) = \lambda_t\, \Delta\gamma_t$$

3) $\frac{\partial \mathcal{L}}{\partial\, c\gamma_t} = 0 \Leftrightarrow \hat{\beta}^t\left(\lambda_t\left(\frac{1}{1+ ri_t}\right)\right) + \hat{\beta}^{t+1}E_t\{-U'(\,prc_{t+1})\, a\gamma_{t+1} - \lambda_{t+1}\, \Delta\gamma_{t+1}(1-\, a\gamma_{t+1})\}$

$$\Leftrightarrow \frac{\lambda_t}{1+\, ri_t} = \hat{\beta}.E_t\{U'(\,prc_{t+1})\, a\gamma_{t+1} + \lambda_{t+1}\, \Delta\gamma_{t+1} - \lambda_{t+1}\, \Delta\gamma_{t+1}\, a\gamma_{t+1}\}$$

$$\Leftrightarrow \frac{\lambda_t}{1+\, ri_t} = \hat{\beta}.E_t\{U'(\,prc_{t+1})\, a\gamma_{t+1} + \lambda_{t+1}\, \Delta\gamma_{t+1} - U'(\,prc_{t+1})\, a\gamma_{t+1}\}$$

$$\Leftrightarrow \frac{\lambda_t}{1+\, ri_t} = \hat{\beta}.E_t\{\lambda_{t+1}\, \Delta\gamma_{t+1}\}$$

1st formula:

$$\frac{U'(\,prc_t)}{\Delta\gamma_t} = \hat{\beta}.(1+\, rf_t).E_t\{U'(\,prc_{t+1})\, R\gamma_{t+1}\}$$

$$\Leftrightarrow U'(\,prc_t) = \hat{\beta}.(1+\, rf_t).E_t\{U'(\,prc_{t+1})\, R\gamma_{t+1}\, \Delta\gamma_t\} \qquad (43)$$

2nd formula:

$$\frac{U'(\,prc_t)}{\Delta\gamma_t(1+\, ri_t)} = \hat{\beta}.E_t\left\{\Delta\gamma_{t+1}\frac{U'(\,prc_{t+1})}{\Delta\gamma_{t+1}}\right\}$$

$$\Leftrightarrow U'(\,prc_t) = \hat{\beta}.(1+\, ri_t).E_t\{\,\Delta\gamma_t U'(\,prc_{t+1})\} \qquad (44)$$

Note: It is supposed that all the economic agents have a utility logarithmic function in the form

$$U(x) = ln(x) \qquad (45)$$

Thus :

$$U'(x) = \frac{1}{x} \qquad (46)$$

II. Distributions of the estimated parameters

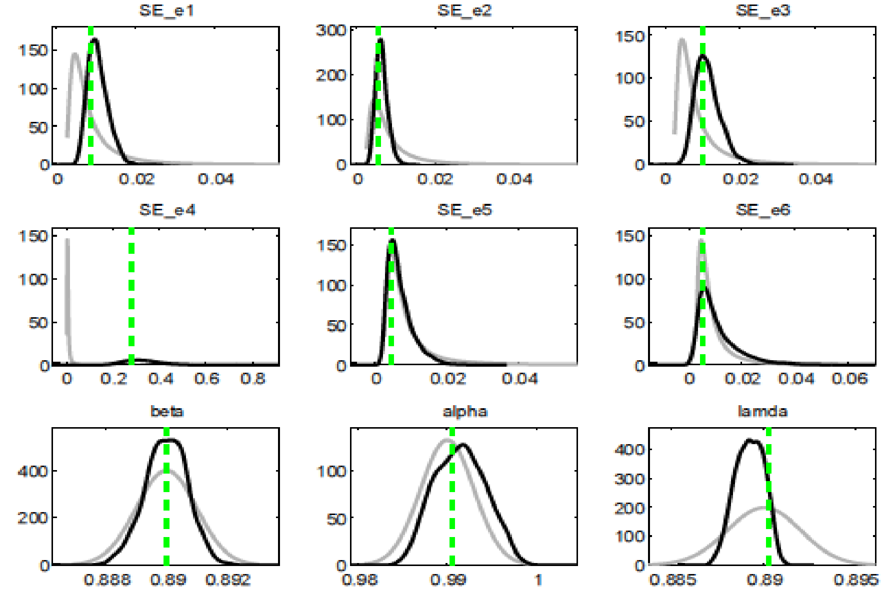


Figure 6: Distributions of the parameters of DSGE model with financial frictions
SE_ei : Standard deviations of the various shocks $\varepsilon_{A,t}$,$\varepsilon_{g,t}$,$\varepsilon_{M,t}$,$\varepsilon_{f,t}$,$\varepsilon_{\delta,t}$ and $\varepsilon_{\gamma,t}$.
beta=β, alpha=α, lamda=λ

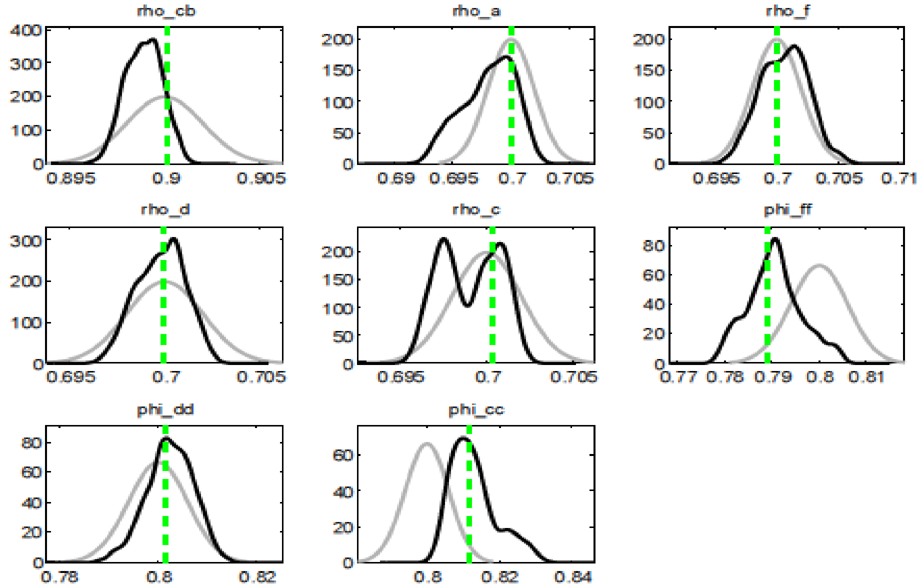


Figure 7: Distributions of the parameters of DSGE model with financial frictions
$phi\_dd=\Delta\delta$,$phi\_cc=\Delta\gamma$, $phi\_ff=\Delta f$
Rho_a=ρ_A, *Rho_c*=ρ_γ, *Rho_d*=ρ_δ, *Rho_bc*=ρ_M, *Rho_f*=ρ_f

III. Cross correlation between the variables of interest of DSGE model with financial frictions

Variables	p	c	r_d	v_d	w	d_m	pr_c	pr_d	l	r_f	d_f	v_f	v_i	r_i	d_i	M	g	A	phi_cb	phi_f	phi_d	phi_c	y
p	1	-0.91	0.39	0.58	0.88	0.9	-0.71	0.6	0.83	0.86	0.97	0.37	-0.1	0.88	0.92	0.06	0.3	-0.9	0.06	-0.12	-0.21	-0.03	-0.95
c	-0.91	1	-0.64	-0.55	-0.61	-0.69	0.71	-0.37	-0.97	-0.92	-0.82	-0.11	-0.19	-0.91	-0.71	0	-0.11	0.98	0	0.1	0.19	0.01	0.9
r_d	0.39	-0.64	1	0	0.01	-0.02	-0.23	0.15	0.6	0.73	0.22	-0.18	0.74	0.66	0.02	-0.27	0.33	-0.52	-0.27	-0.06	-0.19	0.06	-0.29
v_d	0.58	-0.55	0	1	0.5	0.56	-0.49	0.27	0.55	0.45	0.58	0.22	-0.16	0.46	0.61	-0.07	0.06	-0.57	-0.07	-0.2	0.24	0.18	-0.54
w	0.88	-0.61	0.01	0.5	1	0.94	-0.55	0.7	0.51	0.58	0.94	0.55	-0.4	0.62	0.95	0.17	0.42	-0.62	0.17	-0.12	-0.18	-0.03	-0.8
d_m	0.9	-0.69	-0.02	0.56	0.94	1	-0.7	0.59	0.6	0.62	0.93	0.49	-0.47	0.67	1	0.06	0.15	-0.71	0.06	-0.09	-0.22	-0.09	-0.91
pr_c	-0.71	0.71	-0.23	-0.49	-0.55	-0.7	1	-0.28	-0.64	-0.68	-0.62	-0.48	0.34	-0.76	-0.71	0.11	0.01	0.69	0.11	0.13	0.12	-0.26	0.74
pr_d	0.6	-0.37	0.15	0.27	0.7	0.59	-0.28	1	0.23	0.49	0.6	0.43	-0.19	0.48	0.61	-0.1	0.59	-0.31	-0.1	-0.08	-0.64	-0.04	-0.46
l	0.83	-0.97	0.6	0.55	0.51	0.6	-0.64	0.23	1	0.8	0.77	-0.02	0.27	0.79	0.62	0.15	-0.05	-0.99	0.15	-0.13	-0.14	0.05	-0.85
r_f	0.86	-0.92	0.73	0.45	0.58	0.62	-0.68	0.49	0.8	1	0.72	0.26	0.18	0.98	0.65	-0.27	0.38	-0.83	-0.27	-0.09	-0.21	-0.06	-0.79
d_f	0.97	-0.82	0.22	0.58	0.94	0.93	-0.62	0.6	0.77	0.72	1	0.32	-0.17	0.74	0.94	0.18	0.25	-0.85	0.18	-0.11	-0.21	-0.01	-0.93
v_f	0.37	-0.11	-0.18	0.22	0.55	0.49	-0.48	0.43	-0.02	0.26	0.32	1	-0.7	0.39	0.5	-0.02	0.48	-0.06	-0.02	-0.1	0.05	-0.04	-0.23
v_i	-0.1	-0.19	0.74	-0.16	-0.4	-0.47	0.34	-0.19	0.27	0.18	-0.17	-0.7	1	0.04	-0.44	-0.16	0.03	-0.14	-0.16	-0.01	-0.07	0.18	0.16
r_i	0.88	-0.91	0.66	0.46	0.62	0.67	-0.76	0.48	0.79	0.98	0.74	0.39	0.04	1	0.7	-0.21	0.36	-0.83	-0.21	-0.12	-0.19	-0.08	-0.81
d_i	0.92	-0.71	0.02	0.61	0.95	1	-0.71	0.61	0.62	0.65	0.94	0.5	-0.44	0.7	1	0.04	0.2	-0.73	0.04	-0.1	-0.19	-0.07	-0.91
M	0.06	0	-0.27	-0.07	0.17	0.06	0.11	-0.1	0.15	-0.27	0.18	-0.02	-0.16	-0.21	0.04	1	-0.14	-0.13	1	0.05	0.1	-0.02	-0.08
g	0.3	-0.11	0.33	0.06	0.42	0.15	0.01	0.59	-0.05	0.38	0.25	0.48	0.03	0.36	0.2	-0.14	1	0.03	-0.14	-0.08	0.03	0.04	-0.02
A	-0.9	0.98	-0.52	-0.57	-0.62	-0.71	0.69	-0.31	-0.99	-0.83	-0.85	-0.06	-0.14	-0.83	-0.73	-0.13	0.03	1	-0.13	0.09	0.18	0	0.92
phi_cb	0.06	0	-0.27	-0.07	0.17	0.06	0.11	-0.1	0.15	-0.27	0.18	-0.02	-0.16	-0.21	0.04	1	-0.14	-0.13	1	0.05	0.1	-0.02	-0.08
phi_f	-0.12	0.1	-0.06	-0.2	-0.12	-0.09	0.13	-0.08	-0.13	-0.09	-0.11	-0.1	-0.01	-0.12	-0.1	0.05	-0.08	0.09	0.05	1	-0.06	-0.18	-0.01
phi_d	-0.21	0.19	-0.19	0.24	-0.18	-0.22	0.12	-0.64	-0.14	-0.21	-0.21	0.05	-0.07	-0.19	-0.19	0.1	0.03	0.18	0.1	-0.06	1	0.14	0.27
phi_c	-0.03	0.01	0.06	0.18	-0.03	-0.09	-0.26	-0.04	0.05	-0.06	-0.01	-0.04	0.18	-0.08	-0.07	-0.02	0.04	0	-0.02	-0.18	0.14	1	0.12
y	-0.95	0.9	-0.29	-0.54	-0.8	-0.91	0.74	-0.46	-0.85	-0.79	-0.93	-0.23	0.16	-0.81	-0.91	-0.08	-0.02	0.92	-0.08	-0.01	0.27	0.12	1

Table 8: Cross correlation between the variables of interest of DSGE model with financial frictions