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ABSTRACT

The framework of mathematical dynamics of economic systems is applied to the development of financial crisis. A view is proposed that the severity of financial crises can be explained by means of superposition of the fluctuations on connected markets exhibited in the form of a resonance phenomenon. The practical actions of the central banks are criticized as contradicting to theoretical implications of the model.

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1 Introduction

Current financial crisis has produced a theoretical challenge to the thoughtful researchers. How can it happen that relatively small disturbances on the financial markets have produced so heavy consequences in the economic life? I was contemplating about this too. I tried to find a rational answer in the framework of mathematical dynamics of economic systems (Krouglov, 2006; 2009). And I think that I have found a plausible hypothesis. The very nature of financial crisis implies a superposition of the forces acting on two connected economic markets – financial market and real market. And in some circumstances these fluctuations are amplifying each other, which is a dynamic process known in mathematics as resonance phenomenon.

Below I describe a mathematical model of economic system consisting of two connected markets. The first one is a market for single product and the second is a financial market. The economic forces acting on the markets are expressed through the system of ordinary differential equations.

I also try to discuss the historical allusions related to the financial crises and some practical implications following from the proposed theoretical model.

2 Basic Mathematical Model of Single-Product Economy

In this section I repeat some results from the framework of mathematical dynamics of economic systems taken from Krouglov, 2006; 2009, which are helpful to the development of the model of financial crisis.

2.1. Real Product Market

In mathematical parlance, when there are no disturbing economic forces, the market is in equilibrium position, i.e., the product supply and demand are equal, and they are developing with a constant rate and the product price is fixed.

When the balance between the product supply and demand is broken, the product market is experiencing an economic force, which acts to bring the market to a new equilibrium position.

These economic forces are described by the following ordinary differential equations regarding to the product supply $V_S(t)$, demand $V_D(t)$, and price $P_R(t)$ (see Krouglov, 2006; 2009),

$$\frac{dP_R(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t)\right) \tag{1}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP_R(t)}{dt} \tag{2}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P_R(t)}{dt^2} \tag{3}$$

In Equations (1 – 3) above the values λ_P, λ_S, $\lambda_D \geq 0$ are constants.

I assume that the market had been in an equilibrium position until time $t = t_0$, volumes of product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with constant rate r_D^0.

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{4}$$

$$V_S(t) = V_D(t) \tag{5}$$

where $V_D(t_0) = V_D^0$.

2.2. Financial Market

Analogously to the aforementioned real product market, I assume that money supply and demand were equal initially, and the financial market was in an equilibrium position. It means that the money supply and demand were both developing with a constant rate and the price of money was fixed (where the price of money is represented by an interest rate).

When such balance between the money supply and demand is broken, financial market is experiencing the economic forces, which act to bring the market to a new equilibrium position.

These market forces are described by the following ordinary differential equations regarding to the money supply $M_S(t)$, demand $M_D(t)$, and price $P_M(t)$,

$$\frac{dP_M(t)}{dt} = -\mu_P \cdot (M_S(t) - M_D(t)) \tag{6}$$

$$\frac{d^2 M_S(t)}{dt^2} = \mu_S \cdot \frac{dP_M(t)}{dt} \tag{7}$$

$$\frac{d^2 M_D(t)}{dt^2} = -\mu_D \cdot \frac{d^2 P_M(t)}{dt^2} \tag{8}$$

In Equations (6 – 8) above the values $\mu_P,\ \mu_S,\ \mu_D \geq 0$ are constants.

Initially until time $t = t_0$, it has taken place that volumes of the money supply $M_S(t)$ and demand $M_D(t)$ on the market were equal, and they both were developing with constant rate m_D^0.

$$M_D(t) = m_D^0 (t - t_0) + M_D^0 \tag{9}$$

$$M_S(t) = M_D(t) \tag{10}$$

where $M_D(t_0) = M_D^0$.

2.3. *Joined Market of Real Product and Money*

In this section I describe a joined market of the real product and of money. First, let me combine Equations (1 – 3) and (6 – 8) into one integrated system of ordinary differential equations using the linking condition $M_S(t) = V_D(t)$,

$$\frac{dP_R(t)}{dt} = -\lambda_P \left(V_S(t) - M_S(t) \right) \tag{11}$$

$$\frac{dP_M(t)}{dt} = -\mu_P \left(M_S(t) - M_D(t) \right) \tag{12}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP_R(t)}{dt} \tag{13}$$

$$\frac{d^2 M_S(t)}{dt^2} = \mu_S \frac{dP_M(t)}{dt} - \lambda_D \frac{d^2 P_R(t)}{dt^2} \tag{14}$$

$$\frac{d^2 M_D(t)}{dt^2} = -\mu_D \frac{d^2 P_M(t)}{dt^2} \tag{15}$$

I assume that both the market of real product and the market of money had been in an equilibrium position until time $t = t_0$. It means that volumes of the money supply $M_S(t)$ and demand $M_D(t)$ and volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on markets were equal, and they were developing with constant rate m_D^0 and constant rate r_D^0.

$$M_D(t) = m_D^0 (t - t_0) + M_D^0 \tag{16}$$

$$M_S(t) = M_D(t) \tag{17}$$

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{18}$$

$$V_S(t) = V_D(t) \tag{19}$$

where $M_D(t_0) = M_D^0$, and $V_D(t_0) = V_D^0$; and two linking conditions $M_S(t) = V_D(t)$, and $m_D^0 = r_D^0$.

3 Disturbances on the Market and Fluctuations

Here I will try to apply the abovementioned mathematical framework to the problem of financial crises.

3.1. Product Supply Plunge due to Financial Crisis

First, here is a qualitative description of the processes we are interested in on the joined market of real product and money. I assume that demand on the financial market plunges. It causes supply of a financial product to drop on the financial market. Since supply of financial product is linked to the demand of a real product in the model, the drop in the supply of financial product causes the demand for a real product to plunge. And eventually, the latter causes the supply of real product to decrease.

Now, I describe the abovementioned processes mathematically. I also simplify some equations to eliminate the obscure details and make the results more prominent.

At some point in time $t = t_0$ the equilibrium situation was broken, and the demand on financial market was reduced by the amount $\Delta_0 > 0$,

$$M_S(t) = \begin{cases} M_D(t), & t < t_0 \\ M_D(t) + \Delta_0, & t = t_0 \end{cases} \qquad (20)$$

where $0 < \Delta_0 \le M_D^0$.

Let me show what happened on the joined market of a real product and money using a simplified version of Equations (11 – 15).

First, I simplify equations by eliminating the condition of damped oscillations on the financial market made by Equation (15) and assume $\mu_D = 0$. Therefore, the oscillations on financial market will be undamped. Second, I also simplify equations by eliminating the condition of damped oscillations on the market of a real product made by Equation (14) and assume $\lambda_D = 0$ as well. Therefore, the oscillations on market of the real product market will be undamped too.

Correspondingly, we may obtain the following solutions for time $t \geq t_0$ (Piskunov, 1965; Petrovski, 1966).

$$M_D(t) = m_D^0 (t - t_0) + M_D^0 - \Delta_0 \quad (21)$$

$$M_S(t) = m_D^0 (t - t_0) + M_D^0 - \Delta_0 + \Delta_0 \cos\left(\sqrt{\mu_P \mu_S}(t - t_0)\right) \quad (22)$$

$$M_S(t) = V_D(t) \quad (23)$$

$$\begin{aligned} V_S(t) &= m_D^0 (t - t_0) + M_D^0 \\ &- \Delta_0 \left[\frac{\mu_P \mu_S}{\lambda_P \lambda_S - \mu_P \mu_S} \cos\left(\sqrt{\lambda_P \lambda_S}(t - t_0)\right) - \frac{\lambda_P \lambda_S}{\lambda_P \lambda_S - \mu_P \mu_S} \cos\left(\sqrt{\mu_P \mu_S}(t - t_0)\right) + 1 \right] \end{aligned} \quad (24)$$

When value $(\lambda_P \lambda_S - \mu_P \mu_S)$ is small then $V_S(t)$ starts experiencing very large oscillations (so-called resonance phenomenon[1]), which may far exceed the initial drop of demand Δ_0 on the financial market.

3.2. *Historical Developments of Financial Crises*

2.3.1 Cause of the Great Depression

Caution, I am not a scholar of the Great Depression. However, based on the aforementioned theoretical framework of financial crisis I see the following economic developments. After the stock market crash of 1929 there was weaker demand for the financial products. Population started withdrawing money from the banks, which caused many banks to fail. Consecutively, there was smaller supply of the financial products (including credit) on the market. Therefore, there was weaker demand for real products on the market. And as a result, there was smaller supply of the real products on the market.

[1] Strictly speaking, resonance condition is named when $\lambda_P \lambda_S = \mu_P \mu_S$.

Thus, the main cause of the Great Depression was a “hardware failure” of the banking system to maintain demand for financial products on the markets manifested itself in the form of bank bankruptcies.

2.3.2 Cause of the Great Recession

The Great Recession has been developing in front of our eyes. Here is how I see its causes. It had been revealed in 2008 that collateralized debt obligations, which represented significant part of the financial products, were priced previously according to the defective formula. That fact caused a drop in the demand for these financial products, which caused their prices to plummet. Investment banks were destined to fail since their assets significantly lost value. To prevent worse, the US authorities inserted a large amount of liquidity into the banking system to prevent it from the complete failure. Simultaneously, there was smaller supply of financial products (including credit) on the market. In turn, smaller supply of financial products provoked weaker demand for real products on the market. Successively, weaker demand for real products caused smaller supply of the real products on the market.

Hence, the main cause of the Great Recession was a “software failure” of the banking system to reinstate demand for financial products on the markets despite of progress the world’s economies managed to avoid repeating a “hardware failure” of the banking system thanks to the coordinated government actions.

3.3. Practical Reflection s on the Great Recession

As I have mentioned above, the Federal Reserve under Ben Bernanke, in concert with other authorities, helped to prevent the banking system from bankruptcy by inserting a large amount of liquidity into the latter. In the model framework, this procedure was implemented by increasing the supply of financial products on the market.

However, a continual increase of the supply of financial products has not brought expected significant economic improvement for the markets.

Why hasn't it happened? The answer can be found in the aforesaid model. As one can see from Equations (21 – 24), the cause of the crisis is a decrease of demand for the financial products shown by Equation (21). However, repetitive actions of the Federal Reserve were trying to increase supply of the financial products shown by Equation (22). Thus, excessive supply of financial products orchestrated by the Federal Reserve was creating a surplus of financial products on financial market, where the surplus has to be absorbed by other markets such as, for example, commodity and stock markets (where in turn that surplus was driving the asset prices to fundamentally unsustainable levels and thereby creating not substantiated distortions), and was not solving the origin of the financial crisis.

Additionally, if the hypothesis of a resonance phenomenon proposed earlier has been correct, then in order to even partially overcome the consequences of financial crisis it would be required for the Federal Reserve to perform continuous government interventions on a very massive scale, and correspondingly it possibly could create massive long-lasting distortions on the markets.

It looks to me that right course of actions for the US and other authorities would be to work on the demand expansion requirements for financial products on the markets.

4 Conclusions

Thus, presented is a simplified mathematical model, which tries to explain some obscure aspects intrinsic to the financial crises. The model also gives us opportunities to theoretically evaluate what kind of actions would be more appropriate for the practical resolution of consequences of the financial crises.

One may also ask if it is possible that adding complexities to the model could expose more characteristics of the financial crises that are still hidden now with current simplified mathematical model. I really don't know the answer yet. Let me think about this question and possibly experiment with the model further.

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