Credit Rationing by Loan Size: A Synthesized Model

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Abstract

We construct a unified framework to study credit rationing by the loan size. Due to default risk, the loan offer curve is positive-sloping. At the equilibrium interest rate, increasing the loan size reduces the average cost of the loan, so the borrower always demands a larger loan than that the lender can offer even in a perfect credit market. We show that any agency cost may shift the loan offer curve upwards, enlarging the excess demand further. If agency costs are sufficiently high, the borrower is unable to obtain the loan that she needs at any interest rate. This is the common logic underlying the ex-post agency models of credit rationing.

Keywords: *agency cost, Jaffee and Rusell, loan size, collateral*

JEL Classifications: *D82, G21*

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1 Introduction

In the classical demand-supply economic theory, market clears through price. A buyer can get as many of the goods as she demands by offering the competitive market price. This is the case for any traditional goods, e.g. apples or desks. However, in credit markets, where the interest rate is referred as the loan price, the situation is different. Some borrowers may demand larger loans than lenders can offer at the equilibrium interest rate (e.g., Jaffee and Russell, 1976), and others cannot get the loans that they need at any interest rate (e.g., Tirole, 2006). These phenomena are called non-price credit rationing, in the sense that the interest rate is not the only used rationing device in the credit market and the lender would in many cases rather use non-price instruments, e.g. the loan size, to ration credit. Although we call both credit rationing by loan size, they are different in nature. In the former case, which we refer to as the *JR-type* rationing, changing the interest rate is sufficient for the borrower to get financed, but the borrower prefers a larger loan size at the ruling interest rate. In the latter case, as we call the *Tirole-type* rationing, the borrower cannot get financed at any interest rate.[1]

In the literature, the theories of the two types of credit rationing are relatively isolated. This paper constructs a unified framework to incorporate them. We start with a perfect credit market and illustrate the case of the *JR-type* credit rationing. This case is not associated with an efficiency loss. We then show that any agency problem may induce credit rationing with

[1] In the literature, there is no strict consensus concerning the definition and classification of credit rationing (Jaffee and Stiglitz, 1990). For example, De Meza and Webb (2006) classify credit rationing into two types. One is the *JR-type* rationing. The other is called random rationing proposed by Stiglitz and Weiss (1981). Random rationing is defined as the situation in which the lender randomly chooses borrowers to be granted credit or be rationed. In this case, none of the contract instruments is used to ration credit. The significance of random rationing has been questioned both theoretically (e.g. Arnold and Riley, 2009) and empirically (e.g. Berger and Udell, 1992).

a deadweight loss. When there is no severe agency problem, the borrower can be financed at equilibrium, but obtains a smaller loan than that she demands. For high enough agency costs, the borrower cannot be financed at any interest rate in the sense of the *Tirole-type* rationing.

In our baseline model, the borrower or the entrepreneur finances her investment project from a competitive credit market. Borrower's effort can reduce the initial cash investment with convex costs. Without any agency problem, the borrower chooses the optimal contract along the loan offer curve, which is the zero-profit curve of the lender. At equilibrium, the marginal cost of borrower effort is equal to the marginal cost of outside debt, one. However, due to default risk, the loan interest rate is not the *effective* price of the loan. A larger loan size requires a higher interest rate to compensate the lender, so the loan offer curve with respect to the interest rate is positive-sloping. Although the marginal cost of debt is constant along the loan offer curve, increasing the loan size for any given interest rate reduces the cost of the loan, so the borrower would prefer a larger and hence cheaper loan than the lender can offer. Such kind of rationing is not a proof of efficiency loss and hence is called efficient credit rationing by De Meza and Webb (1992).

Both our baseline model and the Jaffee and Russell (1976) model rely on the positive-slope loan offer curve to derive credit rationing, but there are key differences. Jaffee and Russell (1976) focus on the consumer credit market with unlimited liability and information imperfection. The loan supply curve is positive-sloping because, with a larger loan size, the proportion of *dishonest* borrowers is higher and so is the default rate. We instead focus on the commercial loan market, where borrowers have limited liability and the project's future payoff is uncertain. The positive slope of the loan offer curve stems from the higher default risk following a larger loan size, even if information is perfect. De Meza and Webb (1992) develop a divisible-investment model to illustrate the same form of rationing. Our model differs from the De Meza and Webb (1992) model mainly in that we assume a fixed initial investment, which simplifies the analysis and makes it possible to extend our model to allow for agency problems.

To extend our baseline mode, we then introduce ex-post agency problems. We show that agency costs raise the slope of the positive-sloping loan offer curve, reducing the borrowing capacity for any given interest rate. Again, the positive-slope loan offer curve induces the *JR-type* rationing at equilibrium, provided that the borrower is able to get financed when the agency cost is low. Because the marginal cost of debt is no longer constant along the offer curve due to agency costs, the equilibrium is associated with efficiency loss. For sufficiently high agency costs, the borrower cannot get financed at any interest rate, i.e., the *Tirole-type* rationing occurs. We illustrate the above idea through examples with various agency problems, including costly state versification (e.g. Gale and Hellwig, 1985; Williamson, 1987), money diversion (e.g. Hart and Moore, 1994), risk-shifting (e.g. Jensen and Meckling, 1976; Stiglitz and Weiss, 1981) and hidden shirking (e.g. Tirole, 2006). The paper hence constructs a unified framework to incorporates the two different rationing forms, on which theories in the literature are isolated. Our framework makes it easier to understand the underlying reasons for non-price credit rationing.

Our analysis also provides a couple of more insights concerning how agency costs induce credit rationing. First, agency costs may result in a first positive-sloping and then backward-bending loan offer curve. We show that such an offer curve corresponds to a hump-shaped expected return function of the lender, which is a necessary condition for the lender not to use the interest rate as the only rationing device, the logic shared by many agency models of credit rationing (e.g. Stiglitz and Weiss, 1981; Williamson, 1987). Second, agency costs reduce the pledgeable income and hence the borrowing capacity of the borrower. If the required debt amount is beyond the borrowing capacity, the borrower cannot get the loan at any interest rate. To expand the borrowing capacity, non-price instruments are necessary. We use collateral as an example to illustrate how non-price instruments can mitigate agency problems and hence ease debt finance. In our setting, collateral moves the loan offer curve downward by aligning the wedge of incentives between the borrower and the lender.

Our framework is built on the assumption of competitive credit markets. In a different setting where the lender is a monopolist, Schreft and Villamil (1992) illustrate that credit rationing by loan size may occur under imperfect information.

The rest of the paper is organized as follows. Section 2 constructs the baseline model with perfect information and derives the *JR-type* rationing without efficiency loss. Section 3 extends the baseline model and shows how various agency problems induce both the *JR-type* and *Tirole-type* credit rationing with efficiency loss. Section 3.2 discusses how the use of collateral expands the borrowing capacity. Section 4 concludes.

2 The Baseline Model

Throughout the paper, we assume that information is symmetric when the contract is signed. This distinguishes our model from the adverse selection models of credit rationing (e.g.

Stiglitz and Weiss, 1981; Wette, 1983).

2.1 Setup

Consider the borrowing-lending relationship between a borrower and her lender. The borrower is a firm with limited liability. The lender is a bank or some other financial intermediary. Both parties are risk neutral. The credit market is competitive in the sense that by expectation ex ante, the lender obtains zero-profit from any individual borrower. It follows that the loan offer curve is the zero-profit curve of the lender and that the borrower is free to choose any contract along this loan offer curve. Denote the cost of bank loanable funds as γ, which can be thought of as the deposit rate plus an intermediary fee.

There are two dates, date 0 and date 1. At date 0, the borrower starts her investment project that requires a fixed investment I and has stochastic total return x at date 1. The borrower cannot quit until the return is realized, so she only cares about her expected payoff at the end of the period. The distribution of x is exogenously given by the cumulative distribution function, $F(\cdot)$, or the density function, $f(\cdot)$, with support $[m, M]$ where $M > m \geq 0$. The project is profitable if fully financed by debt, i.e., $\int_m^M x\, dF(x) > \gamma I$.

For simplicity, we normalize the initial net worth of the borrower as zero. To start the project, the borrower exerts cash-equivalent effort E as well as raising debt D, where $D + E = I$. What we have in mind here is the case in which, although the initial investment of the project is fixed, the borrower is able to reduce debt borrowing by exerting more effort, for example, by more effectively organizing the project. This is the plausible case for SMEs in practice. For example, the owner of a small firm may use public transportation, instead of a fancy car, to do business. This reduces the initial investment, but costs more effort. In another case, the entrepreneur may work harder to reduce the number of employees and hence reduce the cash investment. Any other cost that can be reduced by the entrepreneur's effort, including time, patience and reputation, also fits our model.

Let effort E incur a cost $g(E)$, where g is continuous, twice-differentiable and convex, with support $[0, \overline{E}]$. That is, $g'(E) > 0$, $g''(E) > 0$ and $g'(\overline{E}) = +\infty$. We assume that debt is necessary to start the project, i.e. $\overline{E} < I$. As will be clear later, the only purpose for introducing the convex cost function of the borrower's cash-equivalent effort is to obtain a unique equilibrium in the model. If $\overline{E} = 0$, it is obvious that the loan demand is fixed at $D = I$ and there will be no credit rationing in the sense that the borrower prefers a larger loan size than the lender offers. To focus on the interesting case in which both debt and borrower effort are relevant, we further assume $g(0) = 0$ and $g'(0) = 0$ to ensure $D < I$ at optimum.

Denote the required payment per unit of loan as R and then the loan contract as (D, R). We only consider the standard debt contract. Namely, when the debt is due, either the firm pays the fixed amount RD, or the firm goes bankrupted and the bank receives the entire return of the project. There is no bankruptcy cost in the baseline model. Bankruptcy cost will be considered in section 3 when market imperfections are introduced.

2.2 The Loan Offer Curve

For the given project in our setting, the expected profit of the bank from the contract, (D, R), is

$$\pi_l(D, R) = \int_m^{DR} x \, dF(x) + DR \, [1 - F(DR)] - \gamma D \tag{1}$$

In a competitive credit market, the bank breaks even ex ante, so the loan offer curve is $\pi_l(D,R)=0$. The slope of this loan offer curve at point (D,R) is

$$\left.\frac{dR}{dD}\right|_{\pi_l=0}=\left.\frac{1}{D}\left[\frac{\gamma}{1-F(DR)}-R\right]\right|_{\pi_l=0}=\left.\frac{1}{D^2}\left[\frac{\int_m^{DR}x\,dF(x)}{1-F(DR)}\right]\right|_{\pi_l=0}\geq 0 \tag{2}$$

As illustrated in *Figure* 1, the loan offer curve has some special properties (see also Jaffee and Stiglitz, 1990). First, a small enough loan has zero default risk, so the offer curve is flat when $R=\gamma$ and $D\in[0,\underline{D}]$, where $\underline{D}=m/\gamma$. Second, above $\underline{D}$, there is default risk. A higher loan size increases default risk and has to be compensated by a higher interest rate. Hence, the loan offer curve is positive-sloping when $\gamma<R<\overline{R}$ and $D\in[\underline{D},\overline{D}]$ where $\overline{D}=\int_m^M x\,dF(x)/\gamma$ and $M=\overline{RD}$. Third, $\overline{D}$ is the maximum value of the loan size, beyond which the payoff to the bank is not increasing any more. When $R>\overline{R}$, the offer curve is vertical. Note $\overline{D}>I$ because the project is profitable when fully financed by debt.

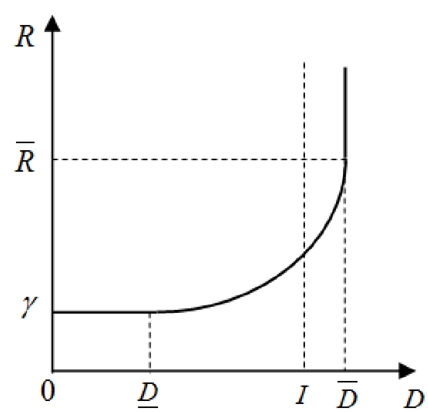

Figure 1: The Loan Offer Curve

In the presence of both uncertainty and limited liability of the borrower, the interest rate is not necessarily the effective loan price due to default risk. This is the key reason inducing the positive slope of the loan offer curve. In fact, the effective loan price is γ, at which the loan supply is inelastic, like any traditional goods (e.g. apple or eggs) in competitive markets.

2.3 The Equilibrium Debt Contract

Given contract (D, R) and borrower effort $E = I - D$, the expected profit of the borrower is

$$\pi_b(D, R) = \int_{DR}^{M} x\, dF(x) - DR\,[1 - F(DR)] - g(I - D). \tag{3}$$

or

$$\pi_b(D, R) = \int_{m}^{M} x\, dF(x) - \gamma D - g(I - D). \tag{4}$$

At equilibrium, the borrower maximizes her expected profit, $\pi_b(D, R)$, by choosing a debt contract along the loan offer curve, $\pi_l(D, R) = 0$. The equilibrium debt contract always exists because at least the borrower can choose full-debt financing. At equilibrium, the marginal cost of borrower effort equals the marginal cost of debt, i.e. $g'(E) - \gamma = 0$.

The equilibrium debt contract, (D^*, R^*) or $\mathcal{M}$, is illustrated in *Figure* 2. At $\mathcal{M}$, the loan offer curve and the indifference curve of the borrower are tangent. If the equilibrium debt is risky, the slope of the loan offer curve is positive at $\mathcal{M}$. If the equilibrium debt is risk free, the slope is zero. We only consider the interesting case with risky debt, in which $\mathcal{M}$ lies at the positive-sloping part of the loan offer curve. It is worth mentioning that, to solve for the optimal debt contract, we only use the indifference curve of the borrower and the loan offer curve, which is also the zero-profit indifference curve of the lender, but not the borrower's demand curve. Furthermore, the shape of the borrower's indifference curve is not necessarily concave everywhere but, form the first-order condition, $g'(E) - \gamma = 0$, we know that the tangent point is unique.

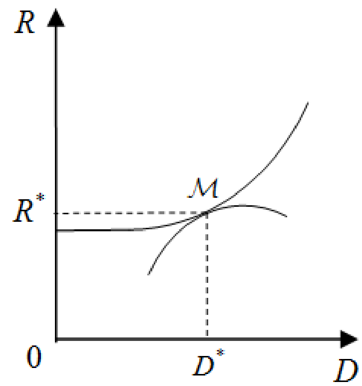

Figure 2: The Equilibrium Debt Contract

2.4 Credit Rationing in the Sense of Jaffee and Russell (1976)

In the classical demand-supply economic theory, any individual buyer faces an inelastic supply curve in the competitive market of a goods. The equilibrium contract for this good can be solved in the same way as we derive the equilibrium in the previous section. That is, the supply curve and the indifference curve of the borrower are tangent at equilibrium. This is illustrated in the left-hand side of *Figure* 3. The equilibrium contract, (Q^*, p^*) or $\mathcal{M}$, is the tangent point of the flat supply curve and the buyer's hump-shaped indifference curve.[2] By definition, the demand curve of the buyer gives the optimal demand given any price level, so it is the locus of the peaking points of the buyer's indifference curves. Because the supply curve is flat, $\mathcal{M}$ is also the peaking point of the indifference curve in the figure and hence lies on the demand curve of the buyer. That is, the three curves have the same intersection point, which is the Pareto-optimal equilibrium. At this equilibrium, demand equals supply, so market clears through the price and there is no non-price rationing.

[2] Again, the indifference curve of the buyer is not necessarily concave everywhere, but the tangent point with the supply curve is unique as long as the marginal rate of substitution between this goods and the others is decreasing. This is a widely accepted assumption in the classical theory.

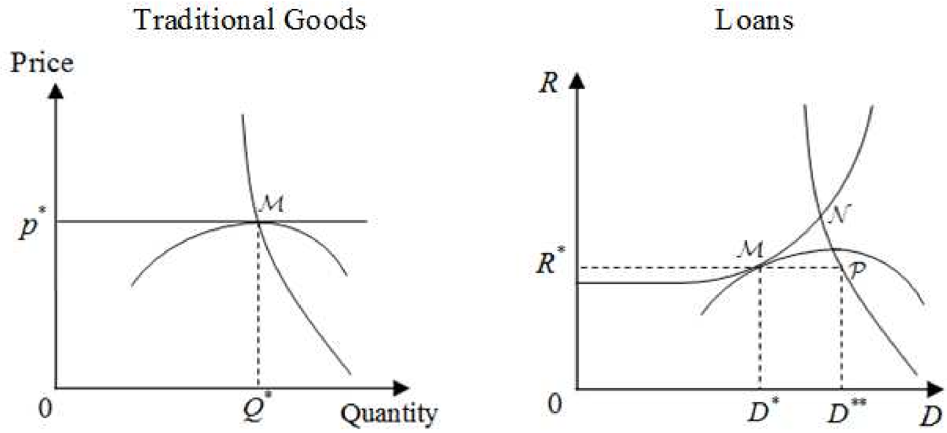

Figure 3: The Equilibrium Contract

However, this is not always the case in credit markets, shown in the right-hand side of *Figure* 3. For risk-free debt, default risk is irrelevant so that the loan supply curve is also flat and demand equals supply at equilibrium. In the presence of default risk, the loan offer curve is positively sloping, so at the equilibrium point, (D^*, R^*) or $\mathcal{M}$, the borrower's indifference curve also has a positive slope. This means that $\mathcal{M}$ does not lie on the demand curve and hence is no longer the intersection between the offer curve and the loan demand curve, $\mathcal{N}$. The borrower is better off by choosing $\mathcal{M}$ rather than $\mathcal{N}$ because $\mathcal{M}$ is Pareto-optimal. Note that at the equilibrium interest rate R^*, the demand of debt is D^{**} while the supply is D^*. Because $D^{**} > D^*$, the loan demand is larger than the loan supply. This is the case of the *JR-type* credit rationing.[3]

Jaffee and Russell (1976) derive credit rationing under ex-ante asymmetric information concerning the borrower's *honesty* or *dishonesty* and ex-post moral hazard. Their setting could be more suitable for consumer loans. Instead in our model, rationing is driven by uncertainty

[3] Please note that our reasoning here does not depend on the shape of the demand curve, which can be solved by deriving the optimal amount of debt for any given interest rate. Take the derivative of (5) with respect to D for a given R, we have $\frac{\partial \pi_b}{\partial D} = g'(I-D) - R[1-F(DR)]$. The demand curve is then $\partial \pi_b / \partial D = 0$

and the limited liability of the borrower, which could be more plausible for commercial loans. Like us, De Meza and Webb (1992) derive credit rationing under perfect information, but their model has a variable-investment setting. The fixed-investment setting in our model simplifies the analysis while keeps the key insights. Since the equilibrium with rationing in our baseline model as well as the De Meza and Webb (1992) model is Pareto-optimal, it is not a proof of efficiency loss.

It is worth emphasizing a couple of key points in the baseline model. First, due to default risk, the interest rate is no longer the effective loan price. A larger loan size is followed by higher default risk, resulting in a positive-sloping loan offer curve. Despite this positive slope, the marginal cost of debt is constant (i.e., γ) along the offer curve. Second, as long as the loan offer curve is positive-sloping and $D < I$ at equilibrium, there exists the *JR-type* credit rationing. For any (D, R) lying on the loan offer curve with $\underline{D} < D < I$, the contract (D', R), where D' is slightly larger than D, is preferred to (D, R) by the borrower. That is, for any risky debt contract, demand exceeds supply at the equilibrium interest rate. This conclusion does not depend on shape of the borrower's indifference curve and the demand curve. Third, the contract with rationing is Pareto-optimal and the borrower is able to increase the loan size by paying the effective price. For this reason, rationing here may only be a *pseudo image* because the borrower can always get the loan she needs at the effective loan price, similar to the case of traditional goods. Finally, in the model, we take the standard debt contract as exogenously given. If other forms of the contract can be chosen, there could be no rationing. For example, a linear claim by outside investors in the common equity contract is not associated with rationing under perfect information.

3 Credit Rationing due to Ex-post Agency Problems

In the baseline model, we show that any risky loan is characterized by credit rationing in the sense of Jaffee and Russell (1976), because the loan interest rate is not the effective loan price due to default risk. This kind of rationing occurs even under perfect information and hence is not a proof of efficiency loss. Jaffee and Stiglitz (1990) states "it is striking that a large part of the literature for monetary economics and corporate finance assumes that there exists a market in which people or firms can borrow as much as they like at a fixed rate of interest".[4] From this point of view, credit rationing in our baseline model is not an interesting phenomenon because, as long as the project has positive NPV, it will always get debt financed. Now we extend the baseline model to illustrate how different ex-post agency problems cause credit rationing with efficiency loss.

3.1 Agency Cost and Credit Rationing

We will show that the presence of agency costs shifts the loan offer curve upwards further, from the dashed one to the solid one (see *Figure* 4), and changes the borrowing capacity from $\overline{D}$ to $\widehat{D}$. In this case, if the equilibrium exists and lies in the new loan offer curve but not in the original one, the *JR-type* credit rationing occurs with efficiency loss. For sufficiently high agency cost, the equilibrium may not exist when no contract along the loan offer curve satisfies the participation constraint of the borrower. The project thus cannot get financed at any interest rate. This is the *Tirole-type* credit rationing. The necessary and sufficient condition for the *Tirole-type* credit rationing is that $\pi_b(D, R) < 0$ for any contract (D, R) such that $\pi_l(D, R) = 0$. In an extreme case when $\overline{E} + \widehat{D} < I$, this condition is satisfied.

[4]See page 847 in Jaffee and Stiglitz (1990).

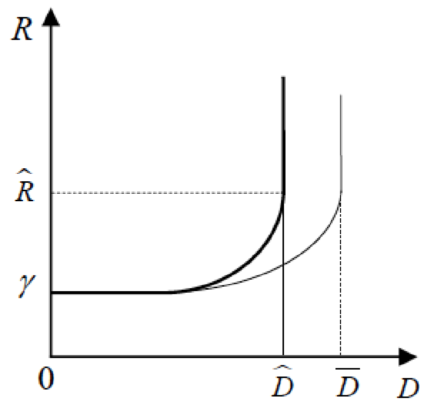

Figure 4: The Loan Offer Curve with Agency Cost

In the literature, it is common that a hump-shaped expected return function of the lender is derived as a necessary condition for credit rationing. For example, the Stiglitz and Weiss (1981) adverse selection model and the Williamson (1987) costly state verification model both rely on the humped shape of the lender's expected return function to exclude the interest rate as a rationing device. In fact, a first positive-sloping and then backward-bending loan offer curve is the necessary and sufficient condition for a hump-shaped expected return function for the lender. To see the pint, note that the slope of the offer curve at the turning point, $(\widehat{D}, \widehat{R})$, is

$$-\frac{\partial \pi_l / \partial D}{\partial \pi_l / \partial R} = \infty. \tag{5}$$

It follows $\partial \pi_l / \partial R = 0$, which means that $(\widehat{R}, \widehat{\pi}_l)$, where $\widehat{\pi}_l = \pi_l(\widehat{D}, \widehat{R})$, is the turning point of the expected return function of the lender in the $R - \pi_l$ space, given the loan size $\widehat{D}$. That is, the the turning point of the loan offer curve corresponds to the turning point of the expected return function of the lender (see *Figure* 5 for an illustration). It follows that the expected return function is hump-shaped as long as loan offer curve is backward-bending.

In the following, we will show how the loan offer curve moves upwards as that in *Figure*

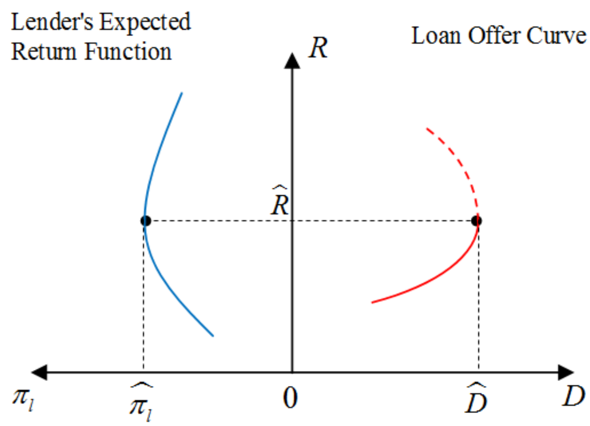

Figure 5: Loan Offer Curve and Lender's Expected Return

4. due to agency problems, including costly state verification, money diversion, risk-shifting and hidden shirking.

Bankruptcy Cost or State Verification Cost

In the literature, bankruptcy cost is sometimes considered as a monitoring cost or state verification cost (e.g. Townsend, 1979; Gale and Hellwig, 1985; Williamson, 1987). Let's now introduce bankruptcy cost to the baseline model. Assume that there is deadweight cost, B, in default at date 1. For example, in default, the lender has to spend an audition cost to know how much left in the firm or liquidates the firm with a cost. Due to this cost, the loan offer curve changes from (2) to

$$\pi_l(D, R) = \int_m^{DR} (x - B)\, dF(x) + DR\,[1 - F(DR)] - \gamma D = 0$$

and the slope of the loan offer curve at any point (D, R) is

$$\left.\frac{dR}{dD}\right|_{\pi_l=0} = \frac{1}{D}\left[\frac{\gamma}{1 - F(DR) - Bf(DR)} - R\right]\Bigg|_{\pi_l=0} \tag{6}$$

Comparing with (2), equation (6) indicates that, in presence of the bankruptcy cost, the offer curve becomes steeper than that when there is no any agency problem. For any given interest rate, the offered loan size is smaller. The loan offer curve has a turning point, $(\widehat{D}, \widehat{R})$, where $1 - F(\widehat{R}\widehat{D}) - Bf(\widehat{R}\widehat{D}) = 0$ or $\frac{1}{B} = \frac{f(\widehat{R}\widehat{D})}{1-F(\widehat{R}\widehat{D})}$. By assuming that the hazard rate of distribution F, $\frac{f(x)}{1-F(x)}$, is an increasing function of x within interval $[m, M]$, the turning point is unique (see e.g. Milde and Riley, 1988), and $\widehat{D}$ is the borrowing capacity. Moreover, the loan offer curve before the turn point, $(\widehat{D}, \widehat{R})$, is positive-sloping, so the equilibrium debt contract (D^*, R^*), if being existing with $D^* > \underline{D}$, is characterized by the *JR-type* credit rationing. Furthermore, $\widehat{D}$ is lower for a higher B. When B is high enough, no contract along the offer curve satisfies the participation constraint of the borrower. That is, the borrower cannot get the loan that she needs at any interest rate. This is the *Tirole-type* rationing.

To sum up, when bankruptcy cost is present, the equilibrium contract may include a smaller loan size and a higher interest rate. For large enough bankruptcy cost, credit rationing by loan size occurs with efficiency loss. This is a simplification of the Williamson (1987) model. The bankruptcy cost can also be interpreted as an enforcement cost or any kind of ex-post deadweight agency cost.

Money Diversion

The second kind of agency cost that we consider is money diversion. Starting from our baseline model, suppose that the borrower is able to divert a proportion, λ, of the project return through hidden perquisite consumption (see e.g. Jensen, 1986), through investing in human capital (e.g. Hart and Moore, 1994), etc. The loan offer curve changes to

$$\pi_l(D, R) = \int_m^{\frac{DR}{1-\lambda}} (1-\lambda)x\,dF(x) + DR\left[1 - F\Big(\frac{DR}{1-\lambda}\Big)\right] - \gamma D = 0$$

and the slope to

$$\left.\frac{dR}{dD}\right|_{\pi_l=0} = \frac{1}{D}\left[\frac{\gamma}{1 - F\big(\frac{DR}{1-\lambda}\big)} - R\right]\Bigg|_{\pi_l=0} \tag{7}$$

The slope in (7) is larger than that in (2) for any give loan size. This is just the situation illustrated in *Figure* 4. Note also that the borrowing capacity is reduced to $(1-\lambda)\overline{D}$ where $\overline{D} = \int_m^M x\,dF(x)/\gamma$. As we argue in the case of bankruptcy cost, money diversion can also induce either the *JR-type* or the *Tirole-type* credit rationing. The latter is the case when λ is large enough.

This model extension simply illustrates the idea of Hart and Moore (1994, 1998) and Stein (1997). In Hart and Moore (1994), the borrower's human capital is not collateralizable due to legal reasons. Such a hold-up problem reduces the borrowing capacity to be the liquidation value of tangible assets or collateralizable assets of the project.

Risk-Shifting

To illustrate credit rationing due to ex-post risk-shifting, we assume that the borrower is able to change the risk of the project after the debt contract is signed. Without loss of generality, let $\overline{E} = 0$. Starting from our baseline model, in addition to the given project, denoted by its payoff distribution F, the borrower can switch to another project, G. Project

G has negative NPV, i.e. $\int_m^M x\,dG(x) < \gamma I$. F second-order-stochastic-dominates G, that is, the latter is "riskier" than the former (Rothschild and Stiglitz, 1970). Due to information asymmetry, the lender is not able to distinguish which project is undertaken. With contract (D, R), suppose the lender and the borrower get $\mu_b(x)$ and $\mu_l(x)$ respectively for a realized payoff, x. $\mu_b(x)+\mu_l(x) = x$, $\mu_b(x) = \max\{0, x-DR\}$ and $\mu_l(x) = \min\{x, DR\}$. The lender's expected payoff by choosing project $i \in \{F, G\}$ is

$$\pi_l^i(D, R) = \int_m^{DR} \mu_l(x)\,di(x). \tag{8}$$

Because $\mu_l(x)$ is concave, from the definition of the second-order stochastic dominance, we know that the expected payoff of the lender from project F is no less than that from project G. The lender always prefers F to G. The loan offer curves for project F and G can be illustrated in *Figure* 4, where the loan offer curve for G is above that for F. Because project G has negative NPV, its borrowing capacity is below I.

For the borrower, which project is preferred is inconclusive.[5] We focus on the case in which the borrower has incentive to shift project risk, i.e. the borrower prefers G to F. In this case, the interest rate charged for the borrower would be according to project G. Since project G has a negative NPV, the borrower cannot get the loan she needs, though project F is the ex-ante planned project. We give a simple example to illustrate the idea.

Example 1: Suppose that project i ($i \in \{F, G\}$) earns X_i with probability p_i and zero otherwise, where $p_F X_F > I > p_G X_G$ and $X_G > X_F$. In this example, F second-order-stochastic-dominates G. If the borrower can commit to undertake project F, the fair interest

[5] For a thorough discussion, see Su (2012).

rate is $R = I/p_F$ and the expected return of the borrower is $p_F X_F - I$. After the contract is signed, the borrower is able to shift the risk of the project to G due to information asymmetry. If she does so, her expected payoff will be $p_G(X_G - I/p_F)$. As long as $p_G(X_G - I/p_F) > p_F X_F - I$, risk-shifting does occur.

Consider a simple numerical example where $I = 1.01$, $p_G = 0.2$, $X_G = 5$, $p_F = 0.5$ and $X_F = 2.4$. $R = I/p_F = 2.02$, $p_F X_F - I = 0.19$, and $p_G(X_G - I/p_F) = 0.596$. The borrower will always choose project G ex post. In this case, the lender refuses lending to the borrower at any interest rate, resulting in the *Tirole-type* credit rationing.

Hidden Shirking

Similar to the above risk-shifting case, we now extend our baseline model to involve hidden shirking. The cash-equivalent effort we studied in the baseline model can be thought of as observed effort, which can be written in the loan contract. Assume that the borrower's ex-post effort has influence on the payoff of the project. More specifically, the distribution F can only be achieved if the borrower works hard. If instead the borrower shirks, the distribution of the payoff of the project is G. Whether the borrower works hard or not is hidden to the lender. For simplicity, let $\overline{E} = 0$. Like that in the risk-shifting case, let the lender and the borrower get $\mu_b(x)$ and $\mu_l(x)$ respectively for a realized payoff, x. When (D, R) is a standard debt contract, $\mu_b(x) = \max\{0, x - DR\}$ and $\mu_l(x) = \min\{x, DR\}$. We consider the simple case in which F first-order stochastic dominates G and $\int_m^M x\, dG(x) < \gamma I$.[6] Moreover,

[6] We assume the first-order stochastic dominance only to illustrate a case in which the cost of hidden shirking does play a crucial role in inducing the borrower to shirk. Obviously, $\mu_b(x)$ and $\mu_l(x)$ are both increasing in x. Recall equation (8) and the definition of the first-order stochastic dominance, we have that both the borrower and the lender prefer project F to G if there is neither cost from working hard nor benefit from shirking.

assume that hard-working incurs a cost ϕ for the borrower and $\pi_b^F(D,R) > \pi_b^G(D,R)$, where

$$\pi_b^F(D,R) = \int_m^M \mu_b(x)\,dF(x) - \phi,$$

and

$$\pi_b^G(D,R) = \int_m^M \mu_b(x)\,dG(x).$$

In this case, no matter how good the project is, the borrower cannot get financed because she will choose to shirk ex post. We also use a simple example to illustrate the existence of this case.

Example 2: Suppose the distribution i ($i =$ "G" or "F") has realization X with probability p_i and zero otherwise, where $p_F X > I > p_G X$. In this example, F first-order stochastic dominates G. If the borrower can commit to work hard, the fair interest rate is $R = I/p_F$ and the expected return of the borrower is $p_F X - I - \phi$. After the contract is signed, if the borrower shirks, her expected payoff will be $p_G(X - I/p_F)$. As long as $p_G(X - I/p_F) > p_F X - I - \phi$, the borrower chooses to shirk ex post and then ex ante the lender rejects financing the project. Credit rationing occurs in the sense that the positive NPV project cannot get financed at any interest rate. A simple numerical case is as follows: $I = 1.02$, $p_G = 0.5$, $p_F = 0.6$, $X = 5$ and $\phi = 0.1$. $R = I/p_F = 1.7$, $p_F X - I - \phi = 0.08$, and $p_G(X - R) = 0.15$.

The above extension of our baseline model, as a variant from Holmstrom and Tirole (1997) and Tirole (2006), shows that moral hazard due to hidden shirking may induce credit rationing. As usual, the idea is shown in *Figure* 4, in which the loan offer curve is on the right side if the borrower works hard.

3.2 Non-price Instruments in Debt

When the lender cannot be compensated through the interest rate alone, the equilibrium loan size or borrowing capacity is reduced. One way to expand borrowing capacity is to use collateral, which is a widely observed non-price debt feature (Berger et al. (2011,?)). In the literature, it is well documented that collateral plays two key roles in mitigating agency costs: the *disciplinary* role to mitigate ex-post moral hazard problem, e.g. Stulz and Johnson (1985), Boot et al. (1991) and Holmstrom and Tirole (1997), etc; the *signaling* role to solve the adverse selection problem through risk-sorting, e.g., Bester (1985, 1987).[7] We focus on the disciplinary role of collateral since only ex-post agency problems are considered in the paper.

Suppose the debt contract is (D, R, C) where C is the amount of outside collateral pledged by the borrower and $C \leq DR - m$.[8] The expected profit of the bank from the contract, (D, R, C) , is

$$\pi_l(D, R, C) = \int_m^{m+C} (m + C)\, dF(x) + \int_{m+C}^{DR} x\, dF(x) + \int_{DR}^{\infty} DR\, dF(x) - \gamma D$$

The loan offer curve is $\pi_l(D, R, C) = 0$ and its slope at (D, R) for a fixed $C = \overline{C}$ is

$$\left.\frac{dR}{dD}\right|_{\pi_l=0} = \frac{1}{D^2}\left.\left[\frac{\int_{m+\overline{C}}^{DR} x\, dF(x)}{1 - F(DR)}\right]\right|_{\pi_l=0} \tag{9}$$

[7]See Coco (2000) for a survey of the theory.

[8]Outside collateral is defined as collateral outside the firm under finance. For example, entrepreneur's personal house can be considered as outside collateral, while inventory or machines in the firm are inside collateral. In this paper, we only consider outside collateral to give a simple illustration. The case with inside collateral is similar.

Comparing (9) with (2), it is clear that pledging collateral reduces the slope of the loan offer curve in the $D - R$ space for any given loan size. Under the setting of our baseline model, the optimal debt contract may lie on the flat part of the loan offer curve for sufficiently large collateral. An obvious case is that, when $C = DR - m$, the loan is risk free. Without default risk, the interest rate is the effective loan price and loan demand equals supply at equilibrium, like those for traditional goods. Furthermore, in any of the cases we discussed in section 3, it is not difficult to show that pledging collateral increases the borrowing capacity, because the use of collateral mitigates agency problems by aligning the wedge of incentives between the borrower and the lender.

4 Concluding Remarks

We construct a model to study the relationship between a borrower and her lender. The borrower requires debt finance to start an endowed project with positive NPV. Information concerning the project under finance is symmetric before the contract is signed. Due to uncertainty of the future payoff of the project and the limited liability of the borrower, debt is risky and the loan offer curve is positive-sloping. We illustrate that the borrower aways prefers a larger loan than the lender is willing to offer at the signed interest rate. That is, credit rationing in the sense of Jaffee and Russell (1976) can occur. The key reason is that the interest rate is not the effective loan price. When increasing the interest rate, the effective loan price is not necessarily increasing and so is the lender's expected payoff. In the absence of agency costs, this kind of rationing is not accompanied by efficiency loss.

We then consider the case with ex-post agency problems. We show that any agency cost may shift the positive-sloping loan offer curve upwards further and reduce the borrowing capacity.

If agency costs are not high enough to exclude any feasible contract, at equilibrium the *JR-type* credit rationing occurs with efficiency loss. If agency cost is sufficiently high, no contract lying on the loan offer curve satisfies the borrower's participation constraint. The borrower cannot get the loan at any interest rate. This is the *Tirole-type* credit rationing, illustrated in many ex-post agency models. In sum, we generalize the two types of credit rationing in a unified framework.

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