Towards a new theory of economic policy: Continuity and innovation

Nicola Acocella and Giovanni Di Bartolomeo

University of Teramo

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Nicola Acocella

Università di Roma La Sapienza

Giovanni Di Bartolomeo

Università di Teramo

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Abstract

This paper outlines the evolution of the theory of economic policy from the classical contributions of Frisch, Hansen, Tinbergen and Theil to situations of strategic interaction. Andrew Hughes Hallett has taken an active and relevant part in this evolution, having contributed to both the development and recent rediscovery of the classical theory, with possible relevant applications for model building.

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1. Introduction

The purpose of this paper is to honor Andrew Hughes Hallett. We think that a convenient way to do this, is to outline the evolution of the theory of economic policy, in which he has taken such a decisive part, from the classical contributions of Frisch, Hansen, Tinbergen and Theil to the present day when a sort of "new" theory of economic policy seems to have emerged from the ashes of the old one.

The new theory of economic policy is first consistent with the criticisms raised against the old one, in particular the Lucas critique, and in line with the evolution of economic analysis in terms of rational expectations and micro-foundations. In addition, it plays a decisive role also in defining the conditions for the existence of equilibria in policy games, i.e. in a context where many agents interact. Unlike the old theory, the new one is not centred around the problem of a single policymaker, but is concerned with the more general issue of the interactions of different (public, private or both public and private) 'policymakers'. It gives the necessary and sufficient conditions for the existence of an equilibrium for these interactions (i.e. the conditions according to which the optimal choices of each policymaker are mutually compatible) as well as the particular conditions necessary for obtaining some specific properties associated with such interactions, e.g. short-run fiscal or monetary policy neutrality or non-neutrality.

The paper is organized as follows. The next section describes the evolution of the theory of economic policy from its initial formulation to its recent rediscovery in a strategic context after its dark age because of the Lucas critique. Section 3 briefly outlines the main contents of the new theory of economic policy and clarifies its fields of application and potentialities, which are rather different from those of the traditional theory. The new theory is in fact focused not only on problems of policy effectiveness, but also on the existence of the equilibrium of the economic system, which was considered to be outside the theory of economic policy in non-strategic contexts. Section 3 also presents intuitions for some extensions. Section 4 concludes and hints at further generalizations and applications. The appendix describes the main argument of the new theory in formal terms, in both a static and dynamic context.

2. The Tinbergen-Theil approach and the Lucas critique

2.1. The classical approach to the theory of economic policy

As Andrew reminded us (Hughes Hallett, 1989), the theory of economic policy has its roots in

Tinbergen's econometric models of the Dutch and the US economy (Tinbergen, 1936, 1939) and was developed by Tinbergen himself when serving as the first director of the Dutch Central Planning Bureau, 1945 on.

In the early 1950s he addressed in formal terms the issue of the *controllability* of a fixed set of independent targets by a policymaker facing a parametric context (i.e. facing an economy represented by a system of linear equations) and endowed with given instruments, he was able to state some well-known general conditions for *policy* existence (see Tinbergen, 1952, 1956), in terms of number of instruments and targets. A similar approach was developed by Bent Hansen in the same years (see Hansen, 1958).[1]

Tinbergen's theory deserves the merit of having raised the problem of conditions for the existence of a first-best policy, i.e. a vector of instruments ensuring the solution to the policy problem when addressed in its simplest way of fixed targets.

Among the many issues left unsolved by Tinbergen's theory, Theil (1956) cited four main difficulties: uncertainty as to data; model uncertainty; uncertainty as to the variables controlled by other decision-makers; choice of target values. As underlined by Hughes Hallett (1989), in later works Theil (1954, 1956, 1964) gave a solution for most of these difficulties and for others as well. In particular, by prescribing that the policymaker should maximize a preference function subject to constraints describing the functioning of the economy, he accomplished a lot of tasks: he avoided the sub-optimality of an *a priori* choice of target values; he also avoided the difficulties facing the policymaker when endowed with a lower number of instruments than the number of targets; finally, he gave a certain and positive answer to the issue of the *existence* of a solution for the *policy* problem also in non-Tinbergen systems.[2]

In doing so Theil arrived at a solution of the policy problem formally very similar to that predicated by Ragnar Frisch (Frisch, 1949, 1950, 1957, 1961), who had first conceived policy problems in terms of maximizing a social preference function, derived by interviewing policymakers.

Theil also overcame the rigid distinction between targets and instruments, allowing the latter to be relevant *per se* and directly introducing them into the objective function,[3] and developed the theory of economic policy in a dynamic setting.[4] Further improvements and advancements

[1] As Petit (1990: 5) reminds us, the Swedish edition of Hansen's book is dated 1955.
[2] These are systems where the number of independent instruments is lower than that of independent targets.
[3] This, however, may be necessary partly because of a misspecification of the objective function (Petit, 1990: 148).
[4] Hansen (1958) also developed a dynamic approach to the policy problem.

of the classical theory as to the *existence*, *uniqueness* and *design* of economic policies are due to a number of authors and accounted for in Leontief (1964, 1976), Heal (1973), Johansen (1977, 1978), Preston and Pagan (1982), Hughes Hallett and Rees (1983) and Petit (1990).

Development of the modern methods of control theory[5] complemented this strand of literature to give an apparently very powerful set of instruments for designing and implementing policy issues.[6]

Tinbergen, Theil, and the other founding fathers of economic policy were only partly concerned with analyzing the *effectiveness of specific policy instruments*, which has been raised by the subsequent economic literature with reference to specific instruments, monetary policy, fiscal policy or others.[7]

Indeed, by starting from the structural form and comparing it with the reduced one, in the framework of the classical theory of economic policymaking it is not difficult – in some cases at least – to find the counterpart of the concepts of policy ineffectiveness. We can in fact have an instrument that is ineffective for some specific values of some parameters or is effective only apparently, since it has an impact on some variable no different from that of another instrument. As Andrew puts it, "(i)n Tinbergen's theory, it is important to distinguish the simple necessary condition that there must be at least as many instruments as targets … from the more complicated necessary and sufficient condition that those instruments must also be linearly independent … The reason is obvious: the instruments may be sufficient in number but unable to generate separate effects" (Hughes Hallett, 1989: 195). If we concentrate on reduced forms, instead, and assume independent instruments (i.e. a matrix of full rank in the reduced form, policy effectiveness becomes merely a model assumption and the only problems left are of an econometric kind.

2.2. The critique to the classical theory of economic policy

The classical theory of economic policy has been the object of fierce criticism from a number of points of view. The introduction of rational expectations led to an assertion of the ineffectiveness of monetary policy, which is more forceful than that famously stated by Milton Friedman in his 1968 *American Economic Association Presidential Address* (Sargent and Wallace, 1975). In a similar way, with rational expectations fiscal policy was considered

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[5] See Bellman (1957, 1961), Kalman (1960), Pontryagin *et al*. (1962), Athans and Falb (1966), Pyndick (1973), Aoki (1976).

[6] These methods prove to be particularly useful in a dynamic setting for finding a unique control path or when the system is not point-controllable, as they enable us to get a second best solution. On differences and equivalences between the Theil-type controllability method and optimal control theory see Hughes Hallett (1989) and Petit (1990).

[7] Hansen (1958) is an exception, as he deals extensively with fiscal policy.

to be ineffective on income (Barro, 1974). A proposition of policy neutrality or "invariance" was then stated.

Apart from the critiques advanced with reference to the effectiveness of specific instruments, the more general and forceful argument was raised by Lucas (1976) according to which a Tinbergen-type decision model is inconsistent with the assumption of rational expectations. The importance of this contribution lies in the fact that it denies the validity of the solution given by Tinbergen, Theil and others to the existence of an (optimal) policy vector (or a sequence of vectors) which can achieve policy targets (or get close to it), assuming the private sector behavior to be invariant to the vector itself.

2.3. Policy games and the neutrality proposition

In the 1980s, pioneered by Barro and Gordon (1983), a new approach to the analysis of economic policy was developed, that of policy games, allowing us to overcome the Lucas critique. The main argument of policy invariance of the private sector's behavior, raised by Lucas, can in fact be tackled when the issue facing the policymaker is built in a context (that of games) where the private sector's behavior is explicitly modeled from its preferences, thus taking account of the different (expected) economic policies.

With Barro and Gordon (1983) the emphasis of the policy debate was still far from the search for conditions of existence of an instrument vector that could guarantee satisfaction of some fixed targets (Tinbergen's fixed-target approach) or an optimal policy that maximizes a given preference function (Theil's flexible-target approach). The discussion concentrated instead on issues of the *effectiveness* (or neutrality) of specific instruments, continuing in the new setting the debate started in the previous two decades.

Barro and Gordon (1983) analyzed the effectiveness of monetary policy in terms of a (Stackelberg) game between the central bank and the private sector, where the latter is the leader and trades off real wage and employment when setting the nominal wage rate. They delivered again the well-known assertion of monetary neutrality as a result of the private sector expectations of the monetary policy. The private sector forms rational expectations and fully crowds-out monetary effects on real output. In addition, Kydland and Prescott's (1977) time inconsistency implies a socially inefficient inflation bias in their model.[8]

The debate on monetary policy neutrality has been developed in different ways. In an influential article, Rogoff (1985) shows how uncertainty can break the neutrality mechanism, in terms of second moments by creating a trade off between the variances of inflation and

[8] See also Stokey (1990) and Sargent (2002: Chapter 3).

output (or employment). Another line of discussion has been about the preferences of the private sector, and this is strictly related to the issue of micro-foundations. Gylfason and Lindbeck (1994) suggest that monetary policy non-neutrality arises whenever the private sector (labor unions) shares the objective of price stability with the central bank.[9] However, this rule seems to lose ground when non-competitive markets are introduced into the picture: Soskice and Iversen (2000), Coricelli *et al*. (2006), Cukierman and Lippi (2001), and other studies[10] show in fact that non-neutrality of monetary policy can derive from the interaction between imperfectly competitive goods and labor markets even when unions do not explicitly share a common objective with the monetary authorities. Acocella and Di Bartolomeo (2004) suggest that all these cases can be generalized and non-neutrality only emerges under specific conditions in terms of number of instruments and targets, when unions share some targets with the monetary authorities either directly or indirectly.

The traditional Barro-Gordon mechanism and the non-neutrality proposition have been further extended to a dynamic context based on general equilibrium frameworks. In this vein Ireland (1999) implemented the Barro-Gordon mechanism of monetary neutrality and inflation bias in a general equilibrium environment, which considered a cash-in-advance constraint.[11] Instead, the non-neutrality proposition was introduced by considering a micro-founded version of Rogoff (1985) with price stickiness.[12] Moreover, the frontier of the New Keynesian approach to monetary policy has moved in the direction of the policy game literature, in line with the pioneering paper by Gylfason and Lindbeck (1986), by explicitly introducing an analysis of the labor union behavior (see e.g. Erceg *et al.*, 2000; Blanchard and Galì, 2005, 2006; Gnocchi, 2006). In all these contexts, the aforementioned observations about non-neutrality in terms of number of instruments and targets still hold.

3. Towards a "new" theory

3.1 Two fundamental propositions

Until a couple of years ago the economic policy debate seemed to focus on conditions for the effectiveness of specific policies in a context of strategic interactions, but the conditions suggested were apparently very specific, regarding the particular model used. An

[9] Acocella and Ciccarone (1997) generalize the above result by taking into consideration also public debt. Jerger (2002) demonstrates also in a different setting that the traditional paradigm of classical dichotomy does not hold if wage setters are inflation averse.

[10] See Cukierman (2004) for a survey.

[11] The Barro-Gordon model has also been extended to the case of multiple equilibria in order to explain different empirical stylized facts about periods of persistently high or low inflation (Albanesi *et al*., 2003b).

[12] See e.g. Clarida *et al*. (1999).

advancement in the theory of economic policy needed further steps to be taken in the direction of the generalization.

This advancement was achieved on realizing that:

a) those conditions for policy effectiveness that were stated since the 1980s hold only under specific circumstances, which have to do with the requirements for controllability of an economic system asserted by the classical theory of economic policy;
b) the general conditions for policy (in)effectiveness in a strategic multi-player context can be expressed in terms of the requirements for controllability laid down in the classical theory of economic policy;
c) in the new setting of policy games, conditions under b) are also relevant for the existence of an equilibrium of the economic system.

The advancement in the theory so far realized can be summarized in two simple, but fundamental, propositions, which have been developed with the contribution of Andrew.[13]

Before stating these propositions, we first recall Tinbergen's classical *golden rule*, according to which a policymaker can reach its (fixed) targets if the number of its independent instruments equals the number of its independent targets. Second, we redefine *policy ineffectiveness*[14] by saying that a policy is ineffective if the equilibrium values of the targets are never affected by changes in the parameters of its preference function.[15]

The two fundamental propositions that characterize the new theory of economic policy can now be stated as:

Proposition 1 (ineffectiveness): *If one (and only one) player satisfies the golden rule, all the other players' policies are ineffective.*

Proposition 2 (existence): *Existence of the game equilibrium requires that two or more players do not satisfy the golden rule (unless they share the same target values).*

These propositions appear to be particularly relevant from a methodological point of view. In fact, in order to build a viable model, a check of *mutual consistency* between the optimal decisions of the agents must be performed. In other terms, we need the interaction between the players to guarantee a solution, i.e. equilibrium existence. In any game with all kinds of

[13] See Acocella, Di Bartolomeo (2005, 2006a, b), Acocella, Di Bartolomeo and Hughes Hallett (2006a, b).

[14] The classical definition of policy ineffectiveness implies that autonomous changes in the policymaker's instruments have no influence on the targets. This definition cannot be maintained in the realm of policy games as policy instruments are here endogenous variables, whose values really depend on the preferences of the decision-makers.

[15] See Gylfason and Lindbeck (1994).

players (all public, all private or both public and private) having overlapping targets, the existence of an *equilibrium of the whole system* must exclude controllability of the same targets by more than one player. Proposition 2 must then be satisfied.

In addition, if we want to ensure a particularly important feature of the policy game we are going to build, in the form of some action taken by a player (in particular that player to which – in a narrow definition of economic policy – we want to attribute the quality of a policymaker) to be effective, we need that player to satisfy the golden rule of economy policy, as stated in Proposition 1.

As an example, consider the case in which we want to model monetary policy. In order to overtake the Lucas critique, we must take account of the interaction with the private sector, whose action must be modeled. To ensure consistency of the public and private sector choices, if they have overlapping targets, we need to check that they are not able to control the system at the same time (proposition 2). In addition, if we want to comply with empirical evidence, which – not without some disagreement – predicts that monetary policy is neutral in the long-run, while being non-neutral in the short run, we must ensure that the private sector is able to control output (the set of real variables) in the long, but not in the short, run (proposition 1).

3.2 Extensions

The two fundamental propositions of policy ineffectiveness and existence of equilibrium are so far mainly limited to the common case of quadratic preferences and linear constraints and refer to Nash policy games as well as any hierarchical equilibrium.

They can easily be extended to other richer information structures or model frameworks without affecting the basic underlying intuition. For instance, if we consider linear-quadratic preferences[16] under linear constraints, the results are only slightly different. By redefining the golden rule in terms of quadratic target variables (i.e. a policymaker satisfies the golden rule if the number of its independent instruments equals the number of its independent quadratic targets), both propositions still hold for simultaneous (Nash) games. Proposition 1 also applies to the hierarchical case under linear quadratic preferences, but proposition 2 does not hold. Conditions for a Nash Feedback Equilibrium in LQ-difference games have also been found.[17]

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[16] By linear quadratic preferences we mean that some (target) variables are second order entries in the player's preference function and others enter it only linearly, i.e. are first order entries.

[17] See Acocella, Di Bartolomeo and Hughes Hallett (2006b). The extension to the dynamic case is also proved in the Appendix to the present paper.

What is more, the above propositions have been further extended to the case of sparseness,[18] which is very important for most economic models. Sparseness means that in a structural form each endogenous variable is related to just one or two other endogenous variables and then to one or two lagged endogenous variables or control (predetermined) variables. In the case of sparse economies, the two propositions can be rewritten as follows. In the case of sparse matrices, if the targets of one (and only one) player which are directly subject to dynamic adjustments also satisfy the golden rule among themselves, then the policies of all the other players will be ineffective with respect to their dynamic targets. Conversely, no Nash feedback equilibrium exists in the case of sparse matrices, if two or more players satisfy the golden rule for their dynamic targets – unless they happen to share the target values for those variables. But the Nash equilibrium may still exist if the golden rule is satisfied and the target values for the non-dynamic targets differ across players; and the policies of the other players will still be effective for those targets even if one (or some) player satisfies the golden rule.

4. Conclusions

The importance of the new theory of economic policy lies first in the fact that it settles an old issue (policy controllability) in the only setting (policy games) in which it can be consistently placed to overcome the Lucas critique. In addition, it does so by going back somehow to the propositions of the classical theory, which is of interest from the point of view of the history of economic thought. Third and more importantly, its two fundamental propositions appear to be essential for model building, as they state the conditions for the consistency of the optimal strategies of all the players (and thus the existence of the equilibrium of the game) as well as the effectiveness of policy instruments.

The importance of this theory is further enlightened if we think of the possibility for it to ‘naturally’ accommodate issues of micro-foundations of macroeconomic relations, since the strategies of the different players are the outcomes of a maximizing process. In this sense it is particularly suitable for the recent development of macroeconomic literature, where the linear-quadratic approach is predominant, even if not exclusive, because of the log-linearization procedure. This is the case of the cited standard New Keynesian literature and cash-in-advance versions of Barro and Gordon (1983) as well as of the recent developments that, in the spirit of Gylfason and Lindbeck (1986), consider the interaction between monetary

[18] See Acocella, Di Bartolomeo and Hughes Hallett (2006b).

and fiscal authorities and labor unions, as in, e.g., Blanchard and Galì (2005, 2006). Further grounds of applicability lie in the evolution of the literature about dynamic interactions among many monetary and fiscal authorities both in the new open macro-economy and in monetary union contexts;[19] and the promising field of application of robust control to macroeconomics, which can be modeled as a policy game between a policymaker and a fictitious evil agent.[20]

The propositions outlined in this paper, or similar ones to be developed, can be applied to all these models, in principle. In some cases, they raise both further difficulties but at the same time, promise further fields of developments in analytical frameworks. For instance, the non-linearity existing in many of the aforementioned models would apparently make it difficult to apply our propositions, which have been introduced only for the linear-quadratic case, and can thus be generalized one-for-one only to log-linear-quadratic versions of non linear models. Robust control policy games strongly call for the introduction of uncertainty in a more complex manner than additive disturbs, which can be easily managed in a linear-quadratic context.

Andrew is certainly in a vantage position to challenge these problems and thus to play an important role for future developments. He is, in fact, an exceptional and active witness of the evolution in the theory of economic policy from its first steps, having contributed both to bringing the classical theory to its full power and to its recent rediscovery and reformulation in a strategic context.

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[19] See Pappa (2004) or Aarle *et al*. (2006) for a survey.
[20] See, among others, Söderström (2002) and Leitemo and Söderström (2004).

Appendix[21]

A.1 The static case

From now onwards, we use the following notation. All vectors are real column vector defined by their dimension; all matrices are real matrices defined by their two dimensions. Considering two vectors, a and b, (a, b) is a column vector; considering matrices A and B with the same number of rows, [A:B] is a matrix formed by merging the two matrices.

We refer to an economy where n policymakers strategically interact by minimizing a quadratic loss function. Losses can be formally written as:[22]

(1) $$J_i(u_1, u_2, ..., u_n) = \tfrac{1}{2}(x_i - \bar{x}_i)' Q_i (x_i - \bar{x}_i) \qquad \forall i \in N$$

where N is the set of the n policymakers; $x_i \in \mathbb{R}^{M(i)}$ is a vector of target variables; $u_i \in \mathbb{R}^{m(i)}$ is the (control variable) vector that player i can manipulate; $\bar{x}_i \in \mathbb{R}^{M(i)}$ is a vector of target values; Q_i is an appropriate diagonal matrix. Note that Q_i is a full rank square matrix by assumption. We refer to $\bar{x}_i$ and Q_i as the parameters of player i's criterion. All the control vectors are sub-vectors of u and all the target variable vectors are sub-vectors of x. Each player i controls a sub-vector of u, i.e. $u_i \in \mathbb{R}^{m(i)}$. Of course, $\sum_{i \in N} m(i) = m$, since a control cannot be set by more than one player by definition. For the sake of simplicity, we also assume that each player cannot control more instruments than its targets, i.e. $m(i) \leq M(i)$. By contrast, players can share some target variables, i.e. $\sum_{i \in N} M(i) \geq M$.

The reduced form of the underlying economy is described by a linear system of M equations:

(2) $$x = Bu + F$$

where entry (i, j) of matrix $B \in \mathbb{R}^{M \times m}$ measures the instrument j elasticity of target variable i; vector $F \in \mathbb{R}^M$ is a vector of given constants that are outside the players' control.[23] To keep things simple, we assume that the basis of B is the identity matrix, which means that system (2) cannot be reduced to many independent sub-systems.[24]

[21] This appendix surveys research in Acocella and Di Bartolomeo (2005, 2006a, b) and Acocella, Di Bartolomeo and Hughes Hallett (2006a,b).

[22] We keep targets and instruments formally separate. However, in order to take account of the costs of some instruments, we could simply introduce additional targets into equation (1) as well as equality constraints between them and the instruments into equation (2) below.

[23] It can also contain white noise shocks. In this case our results hold in expected terms.

[24] This assumption can be easily relaxed and results generalized, see Acocella and Di Bartolomeo (2005).

From equation (2), we can extract x_i and $\tilde{x}_i$, obtaining the relevant sub-system for player *i*:

$$(3) \qquad x_i = B_{ii}u_i + \sum_{j \in N/i} B_{ij}u_j + F_i$$

where $B_{ij} \in \mathbb{R}^{M(i)\times m(i)}$, and $F_i \in \mathbb{R}^{M(i)}$ are appropriate matrices and vectors. In the single-player case (*n* = 1), the decision-maker is always able to obtain its first best (i.e. the targets), if and only if the golden rule is satisfied. It is worth noticing that satisfaction of the golden rule implies that Q_{ii} is a square matrix.

The Nash equilibrium can be decoupled in a set of *n* traditional Tinbergen-Theil problems (i.e. minimizing equation (1) subject to (3) for each of the *n* players), which is the set of the reaction correspondences. With reference to this decoupled representation of the policy game, we can then prove the propositions stated in section 3.

Proof of proposition 1. The optimization problem of each player implies the following *n* focs:

$$(4) \qquad \frac{\partial J_i}{\partial u_i} = B'_{ii}Q_iB_{ii}u_i + B'_{ii}Q_i\sum_{j\in N/i} B_{ij}u_j - B'_{ii}\left(Q_i\bar{y}_i - Q_iF_i\right) = 0 \qquad \forall i \in N .$$

Now, let us consider the case of player 1 without loss of generality and assume that $m(1) = M(1)$ and that a solution $\left(u_1^*, u_j^*\right)$ exists, where u_j^* represents the controls of all the players other than player one. If $\left(u_1^*, u_j^*\right)$ is the solution, given u_j^*, u_1^* must satisfy the first order condition (4) for player 1, but no finite value of u_1 could, since $\det\left(B'_{11}Q_1B_{11}\right) = 0$ (thus, cannot be inverted) if $m(1) = M(1)$. ■

Proof of proposition 2. Let us focus on the first two players without any loss of generality. Assume that they share all their target variables and satisfy the golden rule for their sub-systems and assume that a solution $\left(u_1^*, u_2^*, u_j^*\right)$ exists. Then, given u_j^*, $\left(u_1^*, u_2^*\right)$ must satisfy the system of first-order conditions of the first two players, i.e.:

$$(5) \qquad \begin{bmatrix} B'_{11}Q_1B'_{11} & B'_{11}Q_1B'_{12} \\ B'_{22}Q_2B'_{21} & B'_{22}Q_2B'_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} B'_{11}Q_1 \sum_{j\in N/\{1,2\}} B_{1j}u_j^* - B'_{11}\left(Q_1\bar{y}_1 - Q_1F_1\right) \\ B'_{22}Q_2 \sum_{j\in N/\{1,2\}} B_{2j}u_j^* - B'_{22}\left(Q_2\bar{y}_2 - Q_2F_2\right) \end{bmatrix}$$

However, the first matrix of (5) cannot be inverted since, according to our assumptions, $B_{11} = B_{21}$ and $B_{22} = B_{12}$. Hence, no $\left(u_1^*, u_2^*\right)$ can satisfy (5) and $\left(u_1^*, u_2^*, u_j^*\right)$ cannot be the solution. Finally, consider the case where the first two players do not share all their targets. It can be simply solved by considering the problem of each player as two independent problems: a) minimize quadratic deviations from shared targets plus all first-order targets with respect to an arbitrary set of instruments equal in number to the quadratic variables; b) minimize quadratic deviations from non-shared targets plus all first-order targets with respect to the other instruments (which equal the targets because of the golden rule).[25] As above, now the impossibility of a solution now emerges for the first-order condition of the problems a). Summarizing, as claimed, if at least two players satisfy the golden rule and share at least one target variable, the Nash equilibrium does not exist. ■

In an unusual way, we have derived the necessary and sufficient condition (proposition 2 and proposition 1, respectively) for the existence of a Nash equilibrium in terms of a counting rule for the number of instruments and targets. It is then useful to compare our results with a well-known theorem of the existence of the Nash equilibrium. In an LQ-context, a sufficient condition for the Nash equilibrium existence[26] is that the strategy space of each player is convex and compact. If players' controls are unbounded, the Nash equilibrium may not exist. The introduction of quadratic instrument costs would make them bounded, thus assuring the existence of equilibrium. In our terms, this would imply that the dimensions of matrices Q_i become $M(i)+m(i)$. Thus, the number of instruments would always be less than the number of targets, the golden rule will be satisfied by no player and equilibrium would exist. Propositions 1 and 2 are a generalization of the aforementioned theorem of existence, since that of instrument costs can be derived as a particular case.

A.2 The dynamic case

We extend our argument to the dynamic case by considering a dynamic economy and the Nash feedback equilibrium (NFE). We consider the problem where *n* players try to minimize their individual quadratic, now inter-temporal, criterion.

(6) $$J_i\left(u_1,u_2,...,u_n\right)=\sum_{t=0}^{+\infty}\left(x_i(t)-\overline{x}_i\right)' Q_i\left(x_i(t)-\overline{x}_i\right) \qquad \forall i \in N$$

where $x \in \mathbb{R}^M$, is the vector of the states of the system; $u_i \in \mathbb{R}^{m(i)}$ is the (control variable) vector that player *i* can manipulate; and $\overline{x}_i \in \mathbb{R}^{M(i)}$ is a vector of target values.

We extend the previous section economy (2) to a dynamic context, now each player controls a different set of instruments, which affects the economy dynamics, described by the following difference equation system:

(7) $$x(t+1)=Ax(t)+\sum_{i\in N} B_i u_i(t)$$

[25] Notice that the result is independent of the assignment of instruments because of the golden rule satisfaction.
[26] See e.g. Dasgupta and Maskin (1986).

where $A \in \mathbb{R}^{M \times M}$ and $B_i \in \mathbb{R}^{M \times m(i)}$ are full-rank matrices describing the system parameters which (for simplicity) are constant.

For player i, the relevant sub-system of (7) is:

(8) $$x_i(t+1) = A_i x_i(t) + \sum_{j \in N} B_{ij} u_j(t)$$

where $A_i \in \mathbb{R}^{M(i) \times M(i)}$ and $B_{ij} \in \mathbb{R}^{M(i) \times m(i)}$ are appropriate sub-matrices of A and B_i. We assume that all matrices are of full rank, and that $M(i) \geq m(i)$. The economic interpretation of these assumptions is straightforward.

We consider the NFE defined as follows. A vector $u^*(t) = \left(u_1^*(t), u_2^*(t), ..., u_i^*(t) ..., u_n^*(t)\right)$ is a NFE if $J_i\left(u^*(t)\right) \geq J_i\left(u_1^*(t), u_2^*(t), ..., u_i(t) ..., u_n^*(t)\right)$, for any $u_i(t)$ and for any player *i*, where $u_i(t)$ is a feedback strategy, which means that a contingent rule (dependent on the system's state vector) is provided for each player, and that the rules themselves can be obtained from the backward recursions of dynamic programming (Holly and Hughes Hallett, 1989: 176-179).

As in the section above, by decoupling the problem of finding the Nash equilibrium in a set of *n* traditional Tinbergen-Theil problems, we can then prove the propositions stated in section 3.

Proof of proposition 1. We start by *assuming* that the policymakers' value functions are quadratic,[27] $V_i(x) = \left(x_i(t) - \bar{x}_i\right)' P_i \left(x_i(t) - \bar{x}_i\right)$, where P_i are negative definite symmetric matrices so that there are no redundant targets (and for the sake of simplicity, time indexes are omitted). By using the transition law to eliminate the next period state, the *n* Bellman equations become:

(9) $$\left(x_i - \bar{x}_i\right)' P_i \left(x_i - \bar{x}_i\right) = \max_{u_i} \left\{ \left(x_i - \bar{x}_i\right)' Q_i \left(x_i - \bar{x}_i\right) + \left(A_i x + \sum_{j \in N} B_{ij} u_j\right)' P_i \left(A_i x + \sum_{j \in N} B_{ij} u_j\right) \right\}$$

A NFE must satisfy the first-order conditions:

(10) $$\left(B_{ii}' P_i B_{ii}\right) u_i = -B_{ii}' P_i \left(A_i \left(x_i - \bar{x}_i\right) + \sum_{j \in N/i} B_{ij} u_j \right)$$

which yields the following policy rule:

[27] Indeed, we know that the value function must be convex for a solution to exist (see e.g. Başar and Olsder, 1995; Sargent, 1987: 42-48; Dockner *et al.*, 2000). See also Engwerda (2000a, b) for a more advanced exposition.

(11) $u_i = -\left(B_{ii}'P_iB_{ii}\right)^{-1} B_i'P_iA_ix_i - \left(B_{ii}'P_iB_{ii}\right)^{-1} B_i'P_i \sum_{j\in N/i} B_{ij}u_j$

Now, to demonstrate Proposition 1, we focus (without loss of generality) on player 1. If player 1 satisfies the golden rule, then $m(1) = M\left(1\right)$ and $B_{11} \in \mathbb{R}^{M(1)\times M(1)}$ is square and nonsingular. Equation (11) then becomes:

(12) $u_1 = -B_{11}^{-1}A_1\left(x_1 - \overline{x}_1\right) - B_{11}^{-1}\sum_{j=2}^{n} B_{1j}u_j$

since P_1 is also nonsingular. This implies:

(13) $x_1(t+1) = \overline{x}_1$ for all $t \in \left[0,+\infty\right]$

Thus, if a NFE exists, the value of the target vector x_1 is time invariant and only depends on the preferences of player 1, since in that case condition (12) will hold for all periods $t \in \left[0,+\infty\right]$. This completes the proof of Proposition 1. ■

<u>*Proof of proposition 2.*</u> To prove the proposition we only need to show that if another player (e.g. player 2) also satisfies the golden rule, the equilibrium does not exist. Assume a solution exists and that this solution implies the following optimal policy vector $u^* = \left(u_1^*, u_2^*, ..., u_n^*\right)$ at time *t*. Then, given $u_3^*\left(t\right),...,u_n^*\left(t\right)$, $u_1^*\left(t\right)$ and $u_2^*\left(t\right)$ must satisfy the following system (that is obtained from (10)):

(14) $$\begin{bmatrix} B_{11}'P_1B_{11} & B_{22}'P_2B_{12} \\ B_{11}'P_1B_{21} & B_{22}'P_2B_{22} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} B_{11}'P_1 & \varnothing \\ \varnothing & B_{22}'P_2 \end{bmatrix}\begin{bmatrix} A_1\left(x_1 - \overline{x}_1\right) + \sum_{j\in N/1} B_{1j}u_j^* \\ A_2\left(x_2 - \overline{x}_2\right) + \sum_{j\in N/2} B_{2j}u_j^* \end{bmatrix}$$

Notice that the first-partitioned matrix of (14) is always square; and that if both players satisfy their golden rule, then all the matrices therein are also square. Now *assume that both players share the same target variables*, i.e. $x_1 = x_2$. In this case, we have $A_1 = A_2$ and $B_{ij} = B_{ij}$ for $i \in \left\{1,2\right\}$ and $j \in N$. The first-partitioned matrix of (14) therefore has a zero determinant ($B_{11} = B_{21}$ and $B_{12} = B_{22}$) and cannot be inverted. Hence, a couple $\left(u_1^*, u_2^*\right)$ satisfying (14) does not exist and, therefore, u^* cannot be the solution, as claimed by the proposition.

Conversely, consider now target space instead of instrument space. If the first two players both satisfy the golden rule, it is easy to show that by substituting the first order condition for u_2 from (5) into (7) for u_1, the first order conditions for both players cannot both be satisfied unless they both share the same target values, i.e. unless the following holds:

(15) $A(\overline{x}_1 - \overline{x}_2) = 0$ or $\overline{x}_1 = \overline{x}_2$.[28]

Next, consider the case where *the first two players do not share all their targets*. When the system can be controlled, this case can be solved by decomposing the problem of each player into two mutually interdependent problems: (A) minimize the quadratic deviations of the shared targets from their shared target values using an equal number of (arbitrary selected) instruments from u_i, assuming that non-shared target values can be reached; (B) minimize the quadratic deviations of the non-shared targets from their target values with respect to the remaining instruments, assuming that the shared targets are satisfied (and equal to their target values because of the golden rule). Given (10), the impossibility of a solution now emerges from the first-order conditions for the first of the two problems (A).[29] Hence, as claimed, if at least two players control their sub-systems and share at least one target variable, the NFE cannot exist. ■

[28] Here $\overline{x}_1 \neq \overline{x}_2$ is not possible because A is of full rank. We consider the case where the rank of A is strictly smaller than M in the next section.

[29] Notice that, because the targets are controllable, this result is independent of the assignment of the instruments.

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