CVA, FVA (and DVA?) with stochastic spreads. A feasible replication approach under realistic assumptions.

García Muñoz, Luis Manuel

BBVA

7 February 2013

CVA, FVA (and DVA?) with stochastic spreads. A feasible replication approach under realistic assumptions.

Luis Manuel García Muñoz
BBVA. CVA Quant Team

February 7, 2013

Abstract

In this paper we explore the components that should be incorporated in the price of an uncolateralized [1] derivative. We assume that one counterparty will act as the derivatives hedger while the other will act as the investor. Therefore, the derivative's price will reflect the replication costs from the hedger's perspective, which will not be equal to the replication price from the investor's perspective [2]. We will also assume that the hedger only has the incentive to hedge the changes in value that the derivative experiences while the hedger remains not defaulted. We assume that both the investor's and the hedger's credit curves are stochastic, so that the hedger is not only concerned with the default event of the investor (but not of his own), but also with spread changes of both counterparties.

We conclude that CVA and FVA (funding value adjustment, which include both funding cost and benefit) are the only components to be incorporated in the price of financial derivatives. Of course, since we will follow pure hedging arguments, every pricing term can be hedged under reasonable assumptions. The hedging of both components will not only leave the hedger immune to both spread changes and the default event of the investor, but also to spread changes of the hedger. The latter will imply that the debt structure of the hedger will remain unchanged when the new derivative transaction is traded.

[1] Although the results can be easily generalized to partially collateralized transactions

[2] Notice that the same happens with any manufactured product. That is, the price of a car reflects the manufacturing costs of the car manufacturer and has nothing to do with the manufacturing cost of the car buyer if he was to build his own car.

Contents

1 Introduction

Over the last years, the finance community has come up with what could be considered on of the most controversial concepts of all time: that is DVA (debit value adjustment). While conceived by some as economically meaningful, since it contemplates counterparty credit risk in a symmetrical way, others argue that it is meaningless due to the fact that it cannot be hedged.

Apart from that, derivatives hedgers have also become concerned about funding costs (FCA) or benefits (FBA) incurred in the dynamic replication process. This has produced a big debate lately about which of this four components (CVA, DVA, FCA, FBA) should be incorporated into the pricing of financial derivatives.

In order to give an answer to these questions, we will make the following assumptions:

- The price of a derivative should reflect all of the hedging costs faced by the hedger in the replication process.
- A price component that can not be hedged, that represents a profit and that drifts to zero as the portfolio ages, should never be included. If included, current share and bond holders are favored at the expense of future ones.
- Since nowadays a very high percentage (if not all) of uncollateralized transactions imply a counterparty acting as an investor (risk taker) and a hedger (risk hedger), the derivative's price should just reflect the hedging costs borne by the hedger.
- The hedger will only be willing to hedge the fluctuations in the derivative's price that he will experience while being alive, that is, while not having defaulted.
- There is neither CVA none FVA to be made to fully collateralized derivatives(with continuous collateral margining in cash, symmetrical collateral mechanisms and no thresholds, minimum transfer amounts, ...).

Markets assumptions:

- There is a liquid CDS (credit default swap) curve for the investor.
- There is a liquid curve of bonds issued by the hedger.
- The derivative's underlying asset can be repoed on an overnight basis.
- Continuous hedging is possible, unlimited liquidity, no bid-offer spreads, no trading costs.
- Recovery rates are either deterministic or there are recovery locks available so that recovery risk is not a concern.

Model assumptions:

- Both the hedger and the investor are defaultable. Simultaneous default is possible.

- The underlying asset follows a difusion process under the real world measure.
- The derivative's underlying asset is unaffected by the default event of any of the counterparties.
- Both the credit spreads of the investor and of the hedger are stochastic following correlated diffusion processes under the real world measure.

In order to incorporate funding costs into the pricing formula of financial derivatives we will assume that an uncollateralized derivative is hedged with a collateralized derivative (a REPO on the underlying) although in section 5 we will consider the generic situation in which the hedging instrument is a generic derivative collateralized in cash. In order to simplify the algebra we will assume that interest rates are not stochastic, although the results achieved are also valid under stochastic interest rates.

2 The hedgeable risks

We will assume that under the real world measure $\mathbb{P}$, the evolution of the relevant market variables (price of the derivative's underlying asset and credit spreads of the investor and the hedger) are governed by the following stochastic differential equations:

$$
\begin{aligned}
&dS_t = \mu_t^S S_t dt + \sigma_t^S S_t dW_t^{S,\mathbb{P}} \\
&dh_t^I = \mu_t^I dt + \sigma_t^I dW_t^{I,\mathbb{P}} \\
&dh_t^H = \mu_t^H dt + \sigma_t^H dW_t^{H,\mathbb{P}}
\end{aligned}
\tag{1}
$$

Where S_t represents the price of the derivative's underlying asset at time t, h_t^I the short term CDS spread of the investor, h_t^H the short term CDS spread of the derivative's hedger, μ_t^S, μ_t^I, μ_t^H the real world drifts of the 3 processes and σ_t^S, σ_t^I, σ_t^H their volatilities. $W_t^{S,\mathbb{P}}$, $W_t^{I,\mathbb{P}}$, $W_t^{H,\mathbb{P}}$ are brownian motions under the real world measure $\mathbb{P}$.

We will assume that the 3 processes are correlated with time dependent correlations:

$$
\rho_t^{S,I} dt = dW_t^{S,\mathbb{P}} dW_t^{I,\mathbb{P}}, \quad \rho_t^{H,I} dt = dW_t^{H,\mathbb{P}} dW_t^{I,\mathbb{P}}, \quad \rho_t^{S,H} dt = dW_t^{S,\mathbb{P}} dW_t^{H,\mathbb{P}}
$$

Notice that although we could have assumed a n-dimensional Heath Jarrow Morton model for credit spreads, we have assumed that the evolution of the credit curves is governed by one factor models in order to simplify the algebra.

The other two sources on uncertainty are the default indicator processes $N_t^{I,\mathbb{P}} = 1_{\{\tau_I \leq t\}}$, $N_t^{H,\mathbb{Q}} = 1_{\{\tau_H \leq t\}}$ with real world default intensities $\lambda_t^{I,\mathbb{P}}$, $\lambda_t^{H,\mathbb{P}}$. Parameters associated with the investor will carry a superscript I whereas those of the hedger a superscript H. τ_I and τ_H will represent the default times of the investor and the hedger.

The cash flows that the derivative's hedger will face in the replication process will depend on each and every one of the sources of uncertainty $(S_t, h_t^I, h_t^H, N_t^{I,\mathbb{P}}, N_t^{H,\mathbb{P}})$. Therefore $V_t = V(t, S_t, h_t^I, h_t^H, N_t^{I,\mathbb{P}}, N_t^{H,\mathbb{P}})$ (V_t represents the derivative's value from the investor's perspective). Assuming that both the investor and the hedger have not defaulted by time t, the change in value from t to $t+dt$ experienced by V_t will be given by (applying Itô's Lemma for jump diffusion processes)

$$dV_t = \underbrace{\frac{\partial V_t}{\partial S_t} dS_t}_{\text{Delta risk}} + \underbrace{\frac{\partial V_t}{\partial h_t^I} dh_t^I}_{\text{Spread risk to I}} + \underbrace{\frac{\partial V_t}{\partial h_t^H} dh_t^H}_{\text{Spread risk to H}} + \underbrace{\Delta V_t^I dN_t^{I,\mathbb{P}}}_{\text{Default risk to I}} + \underbrace{\Delta V_t^H dN_t^{H,\mathbb{P}}}_{\text{Default risk to H}} + \underbrace{\Delta V_t^{H,I} dN_t^{I,\mathbb{P}} dN_t^{H,\mathbb{P}}}_{\text{Simultaneous default risk}} + \underbrace{O(dt)}_{\text{Theta}} \tag{2}$$

ΔV_t^I represents the jump in the value of the derivative if default of the investor happened at time t, ΔV_t^H the jump if the hedger defaulted and $\Delta V_t^{H,I}$ the jump under a simultaneous default.

Of all the risk terms in (2) the hedger will only be exposed to $\frac{\partial V_t}{\partial S_t} dS_t$, $\frac{\partial V_t}{\partial h_t^I} dh_t^I$, $\frac{\partial V_t}{\partial h_t^H} dh_t^H$ and $\Delta V_t^I dN_t^{I,\mathbb{P}}$. Keep in mind that the others are all conditional on the hedger having defaulted. Since the hedger will not be there to experience the change in value of the derivative, there will be no incentive at all to hedge them.

Nevertheless we will analyze whether each one of the components of (2) can actually be hedged:

- $\frac{\partial V_t}{\partial S_t} dS_t$: This component can be hedged by trading in the underlying asset. We assume that there exists a REPO market on S_t, therefore the hedger will be able to go long or short the underlying asset without having a net cash flow.
- $\frac{\partial V_t}{\partial h_t^I} dh_t^I$ and $\Delta V_t^I dN_t^{I,\mathbb{P}}$: In order to be hedged to both spread and default risks of the investor, the hedger will have to trade in two credit default swaps written on the investor with different maturities. Notice that this is because we have assumed a one factor model for the evolution of the credit spread curve. Had we assumed an n factor model, then the hedger would have to trade in CDSs with $n+1$ different maturities. If we assume that the investor is not perceived by the market as correlated with the hedger, the hedger will be able to either buy or sell protection on the investor. Notice that this hedging component will imply a zero net cash flow, since CDSs are collateralized market instruments [3].
- $\Delta V_t^{H,I} dN_t^{I,\mathbb{P}} dN_t^{H,\mathbb{P}}$: In order to hedge this component, the hedger will have to trade a basket derivative written on both the hedger and the investor. Of

[3] When a market participant enters into a collateralized transaction with a positive value (respectively negative) pays (receives) the value of the deal to (from) the counterparty, but receives (posts) the value as collateral. This produces a net cash flow of zero.

course, the market will never be willing to let the hedger sell protection on a basket for which the investor is one of its components. Therefore this term can not be hedged in general.

- $\frac{\partial V_t}{\partial h_t^H} dh_t^H$ and $\Delta V_t^H dN_t^{H,\mathbb{P}}$ altogether: The hedger will have to trade on two different credit instruments written on himself (or $n+1$ under a n factor model for the evolution of its credit curve). In general he will have to go long or short its own credit risk. Since the market will never be willing to buy protection written on the hedger from the hedger, the hedging of this two components will have to be done by trading on the hedger's own debt. Notice that the hedging will imply a net purchase of debt, so that it could never be done unless V_t was positive (the hedger has received funds from the investor) and enough to purchase the net debt, which will not happen in general. If it was not enough, then the hedging would not be possible. Notice that issuing debt to purchase the hedger own debt is not an option, since the issuance of debt will generate DVA with the funding provider, leaving the overall DVA unaffected.
- $\frac{\partial V_t}{\partial h_t^H} dh_t^H$: Notice that no matter whether the hedger makes the unrealistic assumption of being default free, in the process of replicating the derivative it will be exposed to its own funding spread. This implies that the pricing equation would depend on its current funding curve and, unless the hedger unrealistically believes it to be non stochastic, the hedger should have an incentive to hedge this source of risk. As we will see in section 4, this source of risk can always be hedged by trading on two bonds with different maturities while forcing the net purchase to be zero. Notice also that when a hedger enters into a non collateralized derivative, the hedger will modify its liability structure. Hedging this component will leave the liability structure of the hedger unafected, which should be convenient since it would not make sense that closing a derivative changes the liability structure of the firm.

It is important to stress that the same sources of risk that the hedger will not be able to hedge are the same sources of risk whose cash flows will never be paid or received by the issuer (since it will already be defaulted). Therefore it is convenient to get rid of these sources of risk if we define price as the value of the replicating portfolio.

3 Pricing and hedging counterparty credit risk

In this section we will assume that the hedger is risk free while considering the counterparty as risky. Nevertheless, we will assume that the hedger has a funding spread $\bar{h}_t^H$ over the OIS rate c_t. Notice that $\bar{h}_t^H$ represents the short term funding spread over the OIS rate whereas h_t^H represents the short term credit default swap spread. In general $h_t^H \neq \bar{h}_t^H$. In this case the hedging equation will be given by:

$$V_t = \alpha_t S_t + \beta_t + \gamma_t CDS(t,T) + \epsilon_t \underbrace{CDS(t,t+dt)}_{=0}$$

Remember that V_t represents the derivative's value from the investor's perspective, α_t the number of stock units to purchase and β_t cash in collateralized transactions plus credits or debits. Notice that regarding counterparty credit risk, the hedger is exposed to two different sources of uncertainty (default risk and credit spread risk). Therefore the hedger will have to trade on two CDSs written on the investor with different maturities.

CDS(t,t+dt) is the value of an overnight credit default swap (with unit notional) under which the protection buyer pays a premium at time $t+dt$ equal to $h_t^I dt$. If the default time of the investor $t < \tau^I \leq t+dt$, then the protection buyer receives $(1-R_I)$ (R_I represents the investor's recovery rate) at time $t+dt$. We will assume that $h_t^I dt$ is such that $CDS(t,t+dt) = 0$. CDS(t,T) is a credit default swap maturing on a later date $T > t$. In general $CDS(t,T) \neq 0$. γ_t and ϵ_t represent the notional to trade on each CDS. Both $CDS(t,t+dt)$ and $CDS(t,T)$ will represent the NPV from the protection seller and from the hedger's perspective.

Assume that $V_0 < 0$ (deal inception). Then the hedger will have to pay $-V_0$ to the investor. In order to do so, he will have to borrow an amount equal to $-V_0$ unsecured. At a later time t, if V_t remains negative, the hedger will have a net asset with value $-V_t$ with the client and a liability of equal value that will have to be borrowed unsecured. So that when $V_t < 0$, the hedger still pays a interest rate of $\left(c_t + \bar{h}_t^H\right)$. $\bar{h}_t^H$ represents the hedger's short term funding spread over the OIS rate c_t.

When $V_t > 0$, the hedger should have this proceeds available in cash as a byproduct of an effective replication process (so that its net assets are equal to the liabilities compromised with the investor). We have to make an assumption regarding what the hedger does with this cash. Investing in a risky instrument is not an option, since this would generate an unhedged risk. Another possibility would be to invest it in a risk free asset. We will assume that there is not such thing as a risky asset, so that the most similar to investing funds on a risk free asset is to leave it as collateral in a fully collateralized derivative transaction receiving an interest rate of c_t on it. Another possibility would be to reduce the short term unsecured funding needs of the hedger, so that he stops paying an interest rate of $\left(c_t + \bar{h}_t^H\right)$ (what can be seen as receiving a positive interest on V_t). Another possibility would be to lend it collateralized with an asset through a REPO receiving the REPO rate. In order to reflect the most general situation, whenever $V_t < 0$, the hedger will pay $f_t^C = \left(c_t + \bar{h}_t^H\right)$ and whenever $V_t > 0$, the hedger will receive an interest of f_t^B (C represents cost while B represents benefit).

β_t is comprised of the following:

- V_t: will either represent cash borrowed or lent at interest rates of f_t^C and f_t^B respectively.
- $\alpha_t S_t$: will either represent cash borrowed or lent through a REPO on S_t at a rate r_t.
- $\gamma_t CDS(t,T)$ will either have been posted as collateral by the CDS counterparty to the hedger (if positive) or the opposite (if negative). The collateral will accrue at a rate of c_t.

Therefore

$$d\beta_t = f_t^B V_t^+ dt + f_t^C V_t^- dt - r_t \alpha_t S_t dt - c_t \gamma_t CDS(t,T) dt$$

Remember that we have assumed that $CDS(t, t+dt) = 0$. That is the reason why there's no component in $d\beta_t$ related to it.

Expressing the hedging equation in differential form yields

$$\begin{aligned} dV_t &= \alpha_t dS_t + \alpha_t q_t S_t dt + f_t^B V_t^+ dt + f_t^C V_t^- dt - r_t \alpha_t S_t dt - c_t \gamma_t CDS(t,T) dt \\ &\quad + \gamma_t dCDS(t,T) + \epsilon_t dCDS(t, t+dt) \end{aligned} \tag{3}$$

The term $\alpha_t q_t S_t dt$ comes from the stream of dividends paid by the underlying asset.

If $CDS(t, t+dt)$ and $CDS(t,T)$ represent the NPVs from the protection seller's perspective

$$dCDS(t, t+dt) = h_t^I dt - (1 - R_I) dN_t^{I,\mathbb{P}}$$

$$dCDS(t,T) = \frac{\partial CDS(t,T)}{\partial t} dt + \frac{\partial CDS(t,T)}{\partial h_t^I} dh_t^I + \frac{1}{2}\left(\sigma_t^I\right)^2 \frac{\partial^2 CDS(t,T)}{\partial {h_t^I}^2} dt + \Delta CDS(t,T) dN_t^{I,\mathbb{P}}$$

Where $\Delta CDS(t,T)$ represents the chance in value experienced by $CDS(t,T)$ upon default of the investor

And since V_t is a function of $t,\ S_t,\ h^I_t,\ N^{I,\mathbb{Q}}_t$, applying Itô's Lemma for jump diffusion processes

$$dV_t = \frac{\partial V_t}{\partial t}dt + \frac{\partial V_t}{\partial h^I_t}dh^I_t + \frac{1}{2}\left(\sigma^I_t\right)^2 \frac{\partial^2 V_t}{\partial {h^I_t}^2}dt + \frac{\partial V_t}{\partial S_t}dS_t + \frac{1}{2}\left(S_t\sigma^S_t\right)^2 \frac{\partial^2 V_t}{\partial S^2_t}dt + S_t\sigma^S_t\sigma^I_t\rho^{S,I}_t \frac{\partial^2 V_t}{\partial s_t \partial h^I_t}dt + \Delta V^I_t dN^{I,\mathbb{P}}_t$$

So that the hedging equation is

$$\begin{aligned}
&\tfrac{\partial V_t}{\partial t}dt + \tfrac{\partial V_t}{\partial h^I_t}dh^I_t + \tfrac{1}{2}\left(\sigma^I_t\right)^2 \tfrac{\partial^2 V_t}{\partial {h^I_t}^2}dt + \tfrac{\partial V_t}{\partial S_t}dS_t + \tfrac{1}{2}\left(S_t\sigma^S_t\right)^2 \tfrac{\partial^2 V_t}{\partial S^2_t}dt + S_t\sigma^S_t\sigma^I_t\rho^{S,I}_t \tfrac{\partial^2 V_t}{\partial s_t \partial h^I_t}dt + \Delta V^I_t dN^{I,\mathbb{P}}_t = \\
&= \alpha_t dS_t + \alpha_t q_t S_t dt + f^B_t V^+_t dt + f^C_t V^-_t dt - r_t\alpha_t S_t dt - c_t\gamma_t CDS(t,T)dt \\
&+\epsilon_t\left(h^I_t dt - (1-R_I)dN^{I,\mathbb{P}}_t\right) \\
&+\gamma_t\left(\tfrac{\partial CDS(t,T)}{\partial t}dt + \tfrac{\partial CDS(t,T)}{\partial h^I_t}dh^I_t + \tfrac{1}{2}\left(\sigma^I_t\right)^2 \tfrac{\partial^2 CDS(t,T)}{\partial {h^I_t}^2}dt + \Delta CDS(t,T)dN^{I,\mathbb{P}}_t\right)
\end{aligned} \tag{4}$$

In order to be hedged, $\alpha_t,\ \gamma_t$ and ϵ_t will have to be determined so that the sources of risk $dS_t,\ dh^I_t$ and $dN^{I,\mathbb{Q}}_t$ are canceled. Therefore

$$\begin{aligned}
\alpha_t &= \tfrac{\partial V_t}{\partial S_t} \\
\gamma_t &= \frac{\frac{\partial V_t}{\partial h^I_t}}{\frac{\partial CDS(t,T)}{\partial h^I_t}} \\
\epsilon_t &= \gamma_t \tfrac{\Delta CDS(t,T)}{1-R_T} - \tfrac{\Delta V^I_t}{1-R_I}
\end{aligned} \tag{5}$$

Plugging (15) into (7)

$$\begin{aligned}
&\tfrac{\partial V_t}{\partial t} + \left(r_t - q_t\right)S_t\tfrac{\partial V_t}{\partial S_t} + \tfrac{1}{2}\left(\sigma^I_t\right)^2 \tfrac{\partial^2 V_t}{\partial {h^I_t}^2} + \tfrac{1}{2}\left(S_t\sigma^S_t\right)^2 \tfrac{\partial^2 V_t}{\partial S^2_t} + S_t\sigma^S_t\sigma^I_t\rho^{S,I}_t \tfrac{\partial^2 V_t}{\partial s_t \partial h^I_t} + h^I_t\tfrac{\Delta V^I_t}{1-R_I} = \\
&= +f^B_t V^+_t + f^C_t V^-_t \\
&+\frac{\frac{\partial V_t}{\partial h^I_t}}{\frac{\partial CDS(t,T)}{\partial h^I_t}}\left(\tfrac{\partial CDS(t,T)}{\partial t} + \tfrac{1}{2}\left(\sigma^I_t\right)^2 \tfrac{\partial^2 CDS(t,T)}{\partial {h^I_t}^2} + h^I_t\tfrac{\Delta CDS(t,T)}{1-R_T} - c_t CDS(t,T)\right)
\end{aligned} \tag{6}$$

Since $CDS(t,T)$ is a credit derivative written on the investor it must follow the PDE (partial differential equation) followed by any credit derivative on the same underlying credit reference (see appendix A), therefore

$$\frac{\partial CDS(t,T)}{\partial t} + \frac{1}{2}\left(\sigma^I_t\right)^2 \frac{\partial^2 CDS(t,T)}{\partial {h^I_t}^2} + h^I_t\frac{\Delta CDS(t,T)}{1-R_T} - c_t CDS(t,T) = -\left(\mu^I_t - M^I_t\sigma^I_t\right)\frac{\partial CDS(t,T)}{\partial h^I_t}$$

Where M^I_t is the investor's credit market price of risk. Therefore

$$\frac{\partial V_t}{\partial t} + (r_t - q_t) S_t \frac{\partial V_t}{\partial S_t} + \left(\mu_t^I - M_t^I \sigma_t^I\right) \frac{\partial V_t}{\partial h_t^I} + \frac{1}{2}\left(\sigma_t^I\right)^2 \frac{\partial^2 V_t}{\partial {h_t^I}^2} + \frac{1}{2}\left(S_t \sigma_t^S\right)^2 \frac{\partial^2 V_t}{\partial S_t^2} + S_t \sigma_t^S \sigma_t^I \rho_t^{S,I} \frac{\partial^2 V_t}{\partial s_t \partial h_t^I} + h_t^I \frac{\Delta V_t^I}{1-R_I} =$$

$$= +f_t^B V_t^+ + f_t^C V_t^- \tag{7}$$

As is proved in **??**, the solution to the last PDE is equal to calculating the following expected value

$$\begin{aligned} V_t \quad = \quad & \underbrace{E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right)\Big|\mathcal{F}_t\right]}_{\text{Fully collateralized price}} \\ & \underbrace{-E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I > s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)\left(k_s^I V_s^+ + \bar{h}_s^I V_s^-\right) ds\Big|\mathcal{F}_t\right]}_{\text{Funding value adjustment}} \\ & + \underbrace{E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I > s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)(R_I - 1)\left(V_s^C\right)^- dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_t\right]}_{\text{CVA}} \end{aligned} \tag{8}$$

Where V_t^C is the value of the completely cash collateralized transaction (from the investor's perspective). We have assumed that upon default of the investor V_t jumps to $R_I V_t^C$ if $V_t^C < 0$ and to V_t^C if $V_t^C \geq 0$.

4 Pricing and hedging the hedger's spread risk

In this section we will assume that the hedger is trading with a default free investor, although the hedger itself is a defaultable counterparty.

As we saw in the previous section, whenever $V_t > 0$, the hedger will have cash available that will be invested generating an interest rate of f_t^B and when $V_t < 0$ the hedger will have to borrow funds paying an interest rate of $f_t^C = \bar{h}_t^H + c_t$. Notice that even if $V_t > 0$, in general, there will be a possibility for the sign of V_t to change in the future, so that the hedger will be potentially exposed to its funding spread. This will make the price to depend on the spread curve of the hedger, therefore the hedger will have an incentive to hedge this source of risk. Notice that this borrowing/investing will tipically be done on an overnight basis.

Remember that the hedger's default event will have an impact in both the value of the derivative and the value of the liabilities incurred by the hedger in the replication process. Nevertheless, the hedger will not be exposed to these price changes, since it will have already defaulted. Apart from that, as we saw in section 2, this risk is not hedgeable. Hence, in this section we will assume that regarding its own credit risk, the hedger will only hedge the risk that it is really exposed to, that is spread risk. We will also see that contrary to what happens with default risk (jump to default risk), spread risk can be hedged.

In order to hedge the spread risk, apart from the investing/borrowing already described, the hedger will have to modify its debt structure (by either issuing short term debt and buying back long term debt or issuing long term debt and buying back short term debt) so that the sensitivity of its debt structure to spread changes remains unchanged before and after closing an uncollateralized deal. It is very important that this modification does not imply a net issuance or buy back of debt, so that in dollar terms, the amount of debt to be issued cancels exactly the amount of debt to buy back. Notice that a net issuance or buy back of debt apart from what the term related to V_t (described in the 2nd paragraph of this section) would represent a net cash inflow or outflow that would break the self financing condition of the replication strategy.

The hedging equation will be:

$$V_t = \alpha_t S_t + \beta_t + \gamma_t \underbrace{\left(B(t,T) - \frac{B(t,T)}{B(t,t+dt)} B(t,t+dt) \right)}_{=0}$$

Where, again, V_t is the derivative's value from the investor's perspective, α_t the number of shares purchased (or sold if negative) to delta hedge the position (done through a REPO transaction), β_t represents credits or debits.

$B(t,t+dt)$ represents short term debt issued by the hedger (that matures at time t, dt) and that pays an interest rate of $c_t + \bar{h}_t^H$. $B(t,T)$ represents long term debt. The term $\frac{B(t,T)}{B(t,t+dt)}$ represents the number of shares issued/bought back in $B(t,t+dt)$ per share bought back/issued in $B(t,T)$, so that there is no net issuance/bay back of debt, apart from what is included in β_t. γ_t represents the number of shares bought back (or issued if $\gamma_t < 0$) in $B(t,T)$.

β_t will be comprised by the following terms:

- If $V_t > 0$, the proceeds will be invested in an asset generating an interest rate of f_t^B, if $V_t < 0$, the hedger will have to issue debt on an overnight basis paying an interest of $f_t^C = c_t + h_t^H$.
- $-\alpha_t S_t$ borrowed (or lent if $\alpha_t < 0$) through a REPO on the underlying, generating an interest of r_t.

So that

$$d\beta_t = f_t^B V_t^+ dt + f_t^C V_t^- dt - r_t \alpha_t S_t dt$$

And conditional on the hedger having not defaulted at time $t + dt$ (remember that the hedger will not be really concerned about what happens once he is defaulted)

$$dB(t, t+dt) = \left(c_t + \bar{h}_t^H\right) B(t, t+dt)dt$$

$$dB(t,T) = \frac{\partial B(t,T)}{\partial t}dt + \frac{\partial B(t,T)}{\partial h_t^H}dh_t^H + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 B(t,T)}{\partial {h_t^H}^2}dt$$

$$dV_t = \frac{\partial V_t}{\partial t}dt + \frac{\partial V_t}{\partial h_t^H}dh_t^H + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 V_t}{\partial {h_t^H}^2}dt + \frac{\partial V_t}{\partial S_t}dS_t + \frac{1}{2}\left(S_t\sigma_t^S\right)^2 \frac{\partial^2 V_t}{\partial S_t^2}dt + S_t\sigma_t^S\sigma_t^H\rho_t^{H,S}\frac{\partial^2 V_t}{\partial s_t \partial h_t^H}dt$$

Notice that the 3 processes carry a jump component upon default of the hedger. We have omitted them since the hedger will want to be hedged only on those paths in which he finds himself alive.

Remember that h_t^H represents the short term CDS spread whereas $\bar{h}_t^H$ the short term financing spread over the OIS rate c_t. In section A we see that there is a relationship between h_t^H, $\bar{h}_t^H$ and the short term REPO rate associated with short term debt issued by the hedger $r_t^{H,t+dt}$ [4]:

$$f_t^C = c_t + \bar{h}_t^H = r_t^H + h_t^{H,t+dt}$$

So that the hedging equation on every path under which the hedger is alive at time $t + dt$ is

$$\frac{\partial V_t}{\partial t}dt + \frac{\partial V_t}{\partial h_t^H}dh_t^H + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 V_t}{\partial {h_t^H}^2}dt + \frac{\partial V_t}{\partial S_t}dS_t + \frac{1}{2}\left(S_t\sigma_t^S\right)^2 \frac{\partial^2 V_t}{\partial S_t^2}dt + S_t\sigma_t^S\sigma_t^H\rho_t^{H,S}\frac{\partial^2 V_t}{\partial s_t \partial h_t^H}dt =$$

$$= f_t^B V_t^+ dt + f_t^C V_t^- dt + q_t\alpha_t S_t dt - r_t \alpha_t S_t dt$$

$$+\gamma_t \left(\frac{\partial B(t,T)}{\partial t}dt + \frac{\partial B(t,T)}{\partial h_t^H}dh_t^H + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 B(t,T)}{\partial {h_t^H}^2}dt - \frac{B(t,T)}{B(t,t+dt)}\left(c_t + \bar{h}_t^H\right) B(t,t+dt)dt\right) \quad (9)$$

[4] $r_t^{H,t+dt}$ represents the REPO rate of a REPO transaction with maturity $t + dt$ on a short term bond issued by the hedger with maturity $t + dt$

In order to be hedged

$$\alpha_t = \frac{\partial V_t}{\partial S_t}, \quad \gamma_t = \frac{\frac{\partial V_t}{\partial h_t^H}}{\frac{\partial B(t,T)}{\partial h_t^H}}$$

So that

$$\begin{aligned} &\tfrac{\partial V_t}{\partial t} + \tfrac{1}{2}\left(\sigma_t^H\right)^2 \tfrac{\partial^2 V_t}{\partial {h_t^H}^2} + \tfrac{1}{2}\left(S_t \sigma_t^S\right)^2 \tfrac{\partial^2 V_t}{\partial S_t^2} + S_t \sigma_t^S \sigma_t^H \rho_t^{H,S} \tfrac{\partial^2 V_t}{\partial s_t \partial h_t^H} = \\ &= f_t^B V_t^+ + f_t^C V_t^- + (q_t - r_t) S_t \tfrac{\partial V_t}{\partial S_t} \\ &+ \frac{\frac{\partial V_t}{\partial h_t^H}}{\frac{\partial B(t,T)}{\partial h_t^H}} \left(\tfrac{\partial B(t,T)}{\partial t} + \tfrac{1}{2}\left(\sigma_t^H\right)^2 \tfrac{\partial^2 B(t,T)}{\partial {h_t^H}^2} - \left(r_t^{H,t+dt} + h_t^H\right) B(t,T)\right) \end{aligned} \tag{10}$$

Since $B(t,T)$ is a bond issued by the hedger with a short term financing cost of $r_t^{H,T}$ [5], as seen in appendix A, it must follow the following PDE:

$$\frac{\partial B(t,T)}{\partial t} + \left(\mu_t^H - \sigma_t^H M_t^H\right) \frac{\partial B(t,T)}{\partial h_t^H} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 B(t,T)}{\partial (h_t^H)^2} + \frac{h_t^H}{1-R_H}\Delta B(t,T) - r_t^{H,T} B(t,T) = 0$$

Where M_t^H is the market price of credit risk of bonds issued by the hedger and credit derivatives also written on the hedger. $\Delta B(t,T)$ represents the jump experienced by $B(t,T)$ on default of the hedger. R_H represents the recovery rate for short term debt $B(t,t+dt)$.

Therefore

$$\frac{\partial B(t,T)}{\partial t} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 B(t,T)}{\partial (h_t^H)^2} = -\left(\mu_t^H - \sigma_t^H M_t^H\right) \frac{\partial B(t,T)}{\partial h_t^H} - \frac{h_t^H}{1-R_H}\Delta B(t,T) + r_t^{H,T} B(t,T)$$

Which implies

[5] $r_t^{H,T}$ represents the rate of a REPO that matures at $t+dt$ and that has $B(t,T)$ as underlying.

$$\frac{\partial V_t}{\partial t} + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 V_t}{\partial {h_t^H}^2} + \frac{1}{2}\left(S_t\sigma_t^S\right)^2 \frac{\partial^2 V_t}{\partial S_t^2} + S_t\sigma_t^S\sigma_t^H\rho_t^{H,S}\frac{\partial^2 V_t}{\partial s_t \partial h_t^H} =$$

$$= f_t^B V_t^+ + f_t^C V_t^- + \left(q_t - r_t\right) S_t \frac{\partial V_t}{\partial S_t}$$

$$+ \frac{\frac{\partial V_t}{\partial h_t^H}}{\frac{\partial B(t,T)}{\partial h_t^H}} \left(-\left(\mu_t^H - \sigma_t^H M_t^H\right)\frac{\partial B(t,T)}{\partial h_t^H} - \frac{h_t^H}{1-R_H}\Delta B(t,T) + r_t^{H,T}B(t,T) - \left(r_t^{H,t+dt} + h_t^H\right)B(t,T)\right) \tag{11}$$

If we assume that the short term REPO rate does not depend on the underlying bond $r_t^{H,T} = r_t^{H,t+dt}$ and if we assumed that on default $B(t,T)$ jumped to $R_H B(t,T)$ so that $\Delta B(t,T) = (R_H - 1)B(t,T)$

$$\frac{\partial V_t}{\partial t} + \left(\mu_t^H - \sigma_t^H M_t^H\right)\frac{\partial V_t}{\partial h_t^H} + \frac{1}{2}\left(\sigma_t^H\right)^2\frac{\partial^2 V_t}{\partial {h_t^H}^2} + \left(r_t - q_t\right)S_t\frac{\partial V_t}{\partial S_t} + \frac{1}{2}\left(S_t\sigma_t^S\right)^2\frac{\partial^2 V_t}{\partial S_t^2} + S_t\sigma_t^S\sigma_t^H\rho_t^{H,S}\frac{\partial^2 V_t}{\partial s_t\partial h_t^H} =$$

$$= f_t^B V_t^+ + f_t^C V_t^- \tag{12}$$

With boundary condition $V_T = g(S_T)$.

The solution of the last PDE is equal to the following expected value

$$\begin{aligned} V_t \quad = \quad & \underbrace{E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^T c_s ds\right)\Big|\mathcal{F}_t\right]}_{\text{Fully collateralized price}} \\ & \underbrace{-E_{\mathbb{Q}}\left[\int_{s=t}^T \exp\left(-\int_{h=t}^s c_h dh\right)\left(k_s^I V_s^+ + \bar{h}_s^I V_s^-\right)ds\Big|\mathcal{F}_t\right]}_{\text{Funding value adjustment}} \end{aligned} \tag{13}$$

In a measure $\mathbb{Q}$ under which the drifts of S_t and h_t^H are $(r_t - q_t)S_t$ and $\mu_t^H - M_t^H$ respectively. In order to prove it, define the process $X_t = \exp\left(-\int_{s=0}^t c_s ds\right)V_t$ ($V_t = V(t, h_t^H, S_t)$), apply Itô's Lemma to V_t under $\mathbb{Q}$, integrate between t and T and take the expected value conditional on $\mathcal{F}_t$.

Notice that the adjustment to be made to the fully collateralized price is the same as the one obtained in section 3 and that was identified as the funding value adjustment. Here the only difference is that we do not have the investor's survival indicator function, since the investor is assumed to be default free.

5 All costs being priced and hedged simultaneusly

In this section we consider the most general situation in which both the issuer and the investor can default. Therefore, the hedger will hedge the risk factors that he is exposed to on every path under which he finds himself not defaulted (that are in fact the only that are hedgeable). These risk factors are:

- Market risk due to changes in S_t.
- Investor's spread risk due to changes in h_t^I.
- Investor default event.
- Hedger's spread risk due to changes in h_t^H.

The hedging equation will be

$$V_t = \alpha_t H_t + \beta_t + \gamma_t CDS(t,T) + \epsilon_t \underbrace{CDS(t,t+dt)}_{=0} + \omega_t \underbrace{\left(B(t,T) - \frac{B(t,T)}{B(t,t+dt)} B(t,t+dt) \right)}_{=0}$$

Where again V_t represents the NPV from the investor's perspective, H_t represents the NPV (from the hedger's perspective) of a fully collateralized derivative written on S_t, β_t represents debits/credits and cash in collateral accounts, $CDS(t,t+dt)$ and $CDS(t,T)$ short term and long term credit default swaps written on the investor and $B(t,t+dt)$ and $B(t,T)$ short term and long term bonds issued by the hedger.

The change in β_t will be given by:

$$d\beta_t = V_t^+ f_t^B dt + V_t^- f_t^C dt - c_t \alpha_t H_t dt - c_t \gamma_t CDS(t,T) dt$$

Where all the terms have already been defined

In every path in which the hedger has not defaulted before $t+dt$ and conditional on both the investor and the hedger being alive at time t the change in V_t will be given by

$$dV_t = \mathcal{L}_{SIH} V_t dt + \frac{\partial V_t}{\partial S_t} S_t \sigma_t^S dW_t^S + \frac{\partial V_t}{\partial h_t^I} \sigma_t^I dW_t^I + \frac{\partial V_t}{\partial h_t^H} \sigma_t^H dW_t^H + \Delta V_t^I dN_t^{I,\mathbb{P}}$$

Where

$$\mathcal{L}_{SIH}V_t = \frac{\partial V_t}{\partial t} + \mu_t^S S_t \frac{\partial V_t}{\partial S_t} + \mu_t^H \frac{\partial V_t}{\partial h_t^H} + \mu_t^I \frac{\partial V_t}{\partial h_t^I} + \frac{1}{2}\frac{\partial^2 V_t}{\partial S_t^2} S_t^2 (\sigma_t^S)^2 + \frac{1}{2}\frac{\partial^2 V_t}{\partial {h_t^H}^2}(\sigma_t^H)^2 + \frac{1}{2}\frac{\partial^2 V_t}{\partial {h_t^I}^2}(\sigma_t^H)^2$$
$$+ \frac{\partial^2 V_t}{\partial S_t h_t^H} S_t \sigma_t^S \sigma_t^H \rho_t^{S,H} + \frac{\partial^2 V_t}{\partial S_t h_t^I} S_t \sigma_t^S \sigma_t^I \rho_t^{S,I} + \frac{\partial^2 V_t}{\partial h_t^I h_t^H} \sigma_t^I \sigma_t^H \rho_t^{I,H}$$

The differential change in H_t

$$dH_t = \mathcal{L}_S H_t dt + \frac{\partial H_t}{\partial S_t} S_t \sigma_t^S dW_t^S$$

where

$$\mathcal{L}_S H_t = \frac{\partial H_t}{\partial t} + \mu_t^S S_t \frac{\partial H_t}{\partial S_t} + \frac{1}{2} S_t^2 \left(\sigma_t^S\right)^2 \frac{\partial^2 H_t}{\partial S_t^2}$$

The differential change in $CDS(t, t+dt)$ and in $B(t, t+dt)$

$$dCDS(t, t+dt) = h_t^I dt - (1 - R_I) dN_t^{I,\mathbb{P}}$$
$$dB(t, t+dt) = \left(c_t + \bar{h}_t^H\right) B(t, t+dt) dt$$

Notice that the jump to default component of $B(t, t+dt)$ has been omitted since it will not be experienced by the hedger.

The differential change of CDS(t,T)

$$dCDS(t, T) = \mathcal{L}_I CDS(t, T) dt + \frac{\partial CDS(t, T)}{\partial h_t^I} \sigma_t^I dW_t^I + \Delta CDS(t, T) dN_t^{I,\mathbb{P}}$$

with

$$\mathcal{L}_I CDS(t, T) = \frac{\partial CDS(t, T)}{\partial t} + \mu_t^I \frac{\partial CDS(t, T)}{\partial h_t^I} + \frac{1}{2}\left(\sigma_t^I\right)^2 \frac{\partial^2 CDS(t, T)}{\partial {h_t^I}^2}$$

And finally

$$dB(t,T) = \mathcal{L}_H B(t,T)dt + \frac{\partial B(t,T)}{\partial h_t^H}\sigma_t^H dW_t^H$$

where

$$\mathcal{L}_H B(t,T) = \frac{\partial B(t,T)}{\partial t} + \mu_t^H \frac{\partial B(t,T)}{\partial h_t^H} + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 B(t,T)}{\partial {h_t^H}^2}$$

Again, we have omitted the jump component in $B(t,T)$ since it will not be experienced by the hedger

So that the hedging equation in differential form will be given by

$$\begin{aligned}
&\mathcal{L}_{SIH} V_t dt + \tfrac{\partial V_t}{\partial S_t} S_t \sigma_t^S dW_t^S + \tfrac{\partial V_t}{\partial h_t^I}\sigma_t^I dW_t^I + \tfrac{\partial V_t}{\partial h_t^H}\sigma_t^H dW_t^H + \Delta V_t^I dN_t^{I,\mathbb{P}} = \\
&= V_t^+ f_t^B dt + V_t^- f_t^C dt - c_t \alpha_t H_t dt - c_t \gamma_t CDS(t,T)dt \\
&+\alpha_t \left(\mathcal{L}_S H_t dt + \tfrac{\partial H_t}{\partial S_t} S_t \sigma_t^S dW_t^S\right) \\
&+\gamma_t \left(\mathcal{L}_I CDS(t,T)dt + \tfrac{\partial CDS(t,T)}{\partial h_t^I}\sigma_t^I dW_t^I + \Delta CDS(t,T) dN_t^{I,\mathbb{P}}\right) \\
&+\epsilon_t \left(h_t^I dt - (1-R_I) dN_t^{I,\mathbb{P}}\right) \\
&+\omega_t \left(\mathcal{L}_H B(t,T)dt + \tfrac{\partial B(t,T)}{\partial h_t^H}\sigma_t^H dW_t^H - \left(c_t + \bar{h}_t^H\right) B(t,T)dt\right)
\end{aligned} \tag{14}$$

In order to be hedged

$$\begin{aligned}
\alpha_t &= \frac{\frac{\partial V_t}{\partial S_t}}{\frac{\partial H_t}{\partial S_t}} \\
\gamma_t &= \frac{\frac{\partial V_t}{\partial h_t^I}}{\frac{\partial CDS(t,T)}{\partial h_t^I}} \\
\epsilon_t &= \gamma_t \frac{\Delta CDS(t,T)}{1-R_T} - \frac{\Delta V_t^I}{1-R_I} \\
\omega_t &= \frac{\frac{\partial V_t}{\partial h_t^H}}{\frac{\partial B(t,T)}{\partial h_t^H}}
\end{aligned} \tag{15}$$

So that every risk factor disappears from the hedging equation

$$\widetilde{\mathcal{L}}_{SIH}V_t = V_t^+ f_t^B + V_t^- f_t^C$$

$$+\alpha_t \left(\widetilde{\mathcal{L}}_S H_t - c_t H_t\right)$$

$$+\gamma_t \left(\widetilde{\mathcal{L}}_I CDS(t,T) - c_t CDS(t,T)\right)$$

$$+\epsilon_t h_t^I$$

$$+\omega_t \left(\widetilde{\mathcal{L}}_H B(t,T) - \left(c_t + \bar{h}_t^H\right) B(t,T)\right)$$

Where

$$\begin{aligned} \widetilde{\mathcal{L}}_{SIH}V_t &= \tfrac{\partial V_t}{\partial t} + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial S_t^2} S_t^2 (\sigma_t^S)^2 + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial {h_t^H}^2} (\sigma_t^H)^2 + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial {h_t^I}^2} (\sigma_t^H)^2 \\ &+ \tfrac{\partial^2 V_t}{\partial S_t h_t^H} S_t \sigma_t^S \sigma_t^H \rho_t^{S,H} + \tfrac{\partial^2 V_t}{\partial S_t h_t^I} S_t \sigma_t^S \sigma_t^I \rho_t^{S,I} + \tfrac{\partial^2 V_t}{\partial h_t^I h_t^H} \sigma_t^I \sigma_t^H \rho_t^{I,H} \end{aligned}$$

$$\widetilde{\mathcal{L}}_S H_t = \frac{\partial H_t}{\partial t} + \frac{1}{2} S_t^2 \left(\sigma_t^S\right)^2 \frac{\partial^2 H_t}{\partial S_t^2}$$

$$\widetilde{\mathcal{L}}_I CDS(t,T) = \frac{\partial CDS(t,T)}{\partial t} + \frac{1}{2}\left(\sigma_t^I\right)^2 \frac{\partial^2 CDS(t,T)}{\partial {h_t^I}^2}$$

$$\widetilde{\mathcal{L}}_H B(t,T) = \frac{\partial B(t,T)}{\partial t} + \frac{1}{2}\left(\sigma_t^H\right)^2 \frac{\partial^2 B(t,T)}{\partial {h_t^H}^2}$$

Substituting ϵ_t by its value and grouping terms

$$\widetilde{\mathcal{L}}_{SIH}V_t + \tfrac{h_t^I}{1-R_I}\Delta V_t^I = V_t^+ f_t^B + V_t^- f_t^C$$

$$+\alpha_t \left(\widetilde{\mathcal{L}}_S H_t - c_t H_t\right)$$

$$+\gamma_t \left(\widetilde{\mathcal{L}}_I CDS(t,T) + \tfrac{h_t^I}{1-R_I}\Delta CDS(t,T) - c_t CDS(t,T)\right)$$

$$+\omega_t \left(\widetilde{\mathcal{L}}_H B(t,T) - \left(c_t + \bar{h}_t^H\right) B(t,t+dt)\right)$$

H_t is a collateralized derivative written on S_t, therefore it must meet the following PDE

$$\widetilde{\mathcal{L}}_S H_t + (r_t - q_t)\, S_t \frac{\partial H_t}{\partial S_t} - c_t H_t = 0$$

$CDS(t,T)$ is a collateralized credit derivative written on I, therefore it must follow

$$\widetilde{\mathcal{L}}_I CDS(t,T) + \left(\mu_t^I - M_t^I \sigma_t^I\right) \frac{\partial CDS(t,T)}{\partial h_t^I} + \frac{h_t^I}{1-R_I} \Delta CDS(t,T) - c_t CDS(t,T) = 0$$

And $B(t,T)$ must follow

$$\widetilde{\mathcal{L}}_H B(t,T) + \left(\mu_t^H - M_t^H \sigma_t^H\right) \frac{\partial B(t,T)}{\partial h_t^H} + \frac{h_t^H}{1-R_H} \Delta B(t,T) - r_t^{H,T} B(t,T) = 0$$

So that the hedging equation is given by

$$\begin{aligned}
&\widetilde{\mathcal{L}}_{SIH} V_t + \tfrac{h_t^I}{1-R_I} \Delta V_t^I = V_t^+ f_t^B + V_t^- f_t^C \\
&+ \tfrac{\frac{\partial V_t}{\partial S_t}}{\frac{\partial H_t}{\partial S_t}} \left(- (r_t - q_t)\, S_t \tfrac{\partial H_t}{\partial S_t} \right) \\
&+ \tfrac{\frac{\partial V_t}{\partial h_t^I}}{\frac{\partial CDS(t,T)}{\partial h_t^I}} \left(- \left(\mu_t^I - M_t^I \sigma_t^I\right) \tfrac{\partial CDS(t,T)}{\partial h_t^I} \right) \\
&+ \tfrac{\frac{\partial V_t}{\partial h_t^H}}{\frac{\partial B(t,T)}{\partial h_t^H}} \left(- \left(\mu_t^H - M_t^H \sigma_t^H\right) \tfrac{\partial B(t,T)}{\partial h_t^H} - \tfrac{h_t^H}{1-R_H} \Delta B(t,T) + r_t^{H,T} B(t,T) - \left(r_t^{H,t+dt} + h_t^H \right) B(t,T) \right)
\end{aligned}$$

Where we have taken into account that $c_t + \bar{h}_t^H = r_t^{H,t+dt} + h_t^H$

If as we did in the previous section we assumed that the short term REPO rate does not depend on the underlying bond $r_t^{H,T} = r_t^{H,t+dt}$ and if we assumed that on default $B(t,T)$ jumped to $R_H B(t,T)$ so that $\Delta B(t,T) = (R_H - 1) B(t,T)$ then

$$\widehat{\mathcal{L}}_{SIH}V_t + \frac{h_t^I}{1-R_I}\Delta V_t^I = V_t^+k_t + V_t^-\bar{h}_t^H + c_tV_t$$

Where

$$\begin{aligned}
\widehat{\mathcal{L}}_{SIH}V_t &= \frac{\partial V_t}{\partial t} + (r_t - q_t)S_t\frac{\partial V_t}{\partial S_t} + (\mu_t^H - M_t^H\sigma_t^H)\frac{\partial V_t}{\partial h_t^H} + (\mu_t^I - M_t^I\sigma_t^I)\frac{\partial V_t}{\partial h_t^I} \\
&+\frac{1}{2}\frac{\partial^2 V_t}{\partial S_t^2}S_t^2(\sigma_t^S)^2 + \frac{1}{2}\frac{\partial^2 V_t}{\partial {h_t^H}^2}(\sigma_t^H)^2 + \frac{1}{2}\frac{\partial^2 V_t}{\partial {h_t^I}^2}(\sigma_t^H)^2 \qquad (16) \\
&+\frac{\partial^2 V_t}{\partial S_t h_t^H}S_t\sigma_t^S\sigma_t^H\rho_t^{S,H} + \frac{\partial^2 V_t}{\partial S_t h_t^I}S_t\sigma_t^S\sigma_t^I\rho_t^{S,I} + \frac{\partial^2 V_t}{\partial h_t^I h_t^H}\sigma_t^I\sigma_t^H\rho_t^{I,H}
\end{aligned}$$

The solution to (22) with terminal condition given by $V_T = g(S_T)$ is equal to calculating the following expected value

$$\begin{aligned}
V_t &= \underbrace{E_{\mathbb{Q}}\left[V_T\exp\left(-\int_{s=t}^{T}c_s ds\right)\middle|\mathcal{F}_t\right]}_{\text{Fully collateralized price}} \\
&\underbrace{-E_{\mathbb{Q}}\left[\int_{s=t}^{T}1_{\{\tau^I>s\}}\exp\left(-\int_{h=t}^{s}c_h dh\right)\left(k_s^IV_s^+ + \bar{h}_s^IV_s^-\right)ds\middle|\mathcal{F}_t\right]}_{\text{Funding value adjustment}} \qquad (17) \\
&+\underbrace{E_{\mathbb{Q}}\left[\int_{s=t}^{T}1_{\{\tau^I>s\}}\exp\left(-\int_{h=t}^{s}c_h dh\right)(R_I-1)\left(V_s^C\right)^- dN_s^{I,\mathbb{Q}}\middle|\mathcal{F}_t\right]}_{\text{CVA}}
\end{aligned}$$

In a measure $\mathbb{Q}$ in which the drifts of S_t, h_t^H and h_t^I are given by $(r_t - q_t)S_t$, $\mu_t^H - M_t^H\sigma_t^H$ and $\mu_t^I - M_t^I\sigma_t^I$ respectively. Under this measure, the default intensity of the default event of the investor is $\frac{h_t^I}{1-R_I}$. Notice that we have obtained the same result as in section 3, where we assumed the hedger to be default free. The only difference is that h_t^H is stochastic in this context.

6 Conclusion

We have seen that assuming that the derivative's price incorporates the hedging costs borne by the hedger (and not those of the investor if he was to hedge the derivative) and that the hedger has only the incentive to hedge the risks that he will be exposed to while he remains not defaulted, the only adjustments to be made to the risk free price (that is, the price of a fully colateralized transaction) are an unilateral CVA (that does not depend on the hedger's spread curve) and a funding adjustment (FVA). We have also seen that both components can be hedged under reasonable assumptions and that the hedging of those components leaves the hedger's debt structure unchanged after a new uncollateralized transaction is traded. We have done so in the realistic assumption of stochastic spreads.

A Modeling credit in a PDE framework

In this section our aim is to derive the PDE followed by both bonds issued by and collateralized credit derivatives written on a generic credit reference. We will assume a one factor model assumed for credit spreads and non stochastic interest rates.

Let's assume that we wanted to hedge a credit derivative written on a particular credit reference. h_t represents the credit reference short term credit default swap spread. We assume that under the real world measure $\mathbb{P}$ h_t follows

$$dh_t = \mu_t^{\mathbb{P}} dt + \sigma_t dW_t^{\mathbb{P}}$$

$\mu_t^{\mathbb{P}} t$ represents the drift and σ_t the volatility. $W_t^{\mathbb{P}}$ is a $\mathbb{P}$ brownian process.

E_t will represent the value of the derivative from the investor perspective. It will both depend on the spread h_t and of the default indicator function $N_t^{\mathbb{P}} = 1_{\{\tau \leq t\}}$, where τ is the default time of the credit reference. Therefore

$$dE_t = \frac{\partial E_t}{\partial t} dt + \frac{\partial E_t}{\partial h_t} dh_t + \frac{1}{2}\sigma_t^2 \frac{\partial^2 E_t}{\partial h_t^2} dt + \Delta E_t dN_t^{\mathbb{P}}$$

Where ΔE_t represents the change in E_t on default.

The two sources of randomness will have to be hedged with two different credit derivatives. One of them will be a short term credit default swap whose value from the protection seller will be represented by $CDS(t, t + dt)$. h_t will be such that $CDS(t, t + dt) = 0$. Its differential change will be given by:

$$dCDS(t, t + dt) = h_t dt - (1 - R) dN_t^{\mathbb{P}}$$

R will represent the recovery rate.

Appart from trading on $CDS(t, t+dt)$, that will only have sensitivity to the default events, the hedger should also trade on another collateralized credit derivative H_t (NPV as seen by the hedger) such that

$$dH_t = \frac{\partial H_t}{\partial t}dt + \frac{\partial H_t}{\partial h_t}dh_t + \frac{1}{2}\sigma_t^2\frac{\partial^2 H_t}{\partial h_t^2}dt + \Delta H_t dN_t^{\mathbb{P}}$$

Where ΔH_t represents the change in H_t on default.

The hedging equation will be

$$E_t = \alpha_t H_t + \gamma_t CDS(t, t+dt) + \beta_t$$

Where β_t represents cash held in collateral accounts. We assume both E_t and H_t to be collateralized in cash, so that:

$$d\beta_t = c_t E_t dt - c_t \alpha_t H_t dt$$

So that the hedging equation in differential form is

$$\begin{aligned}
&\tfrac{\partial E_t}{\partial t}dt + \tfrac{\partial E_t}{\partial h_t}dh_t + \tfrac{1}{2}\sigma_t^2\tfrac{\partial^2 E_t}{\partial h_t^2}dt + \Delta E_t dN_t^{\mathbb{P}} = \\
&\alpha_t\left(\tfrac{\partial H_t}{\partial t}dt + \tfrac{\partial H_t}{\partial h_t}dh_t + \tfrac{1}{2}\sigma_t^2\tfrac{\partial^2 H_t}{\partial h_t^2}dt + \Delta H_t dN_t^{\mathbb{P}}\right) \\
&+\gamma_t\left(h_t dt - (1-R)dN_t^{\mathbb{P}}\right) \\
&+c_t E_t dt - c_t \alpha_t H_t dt
\end{aligned} \tag{18}$$

In order to be hedged, the random terms dh_t and $dN_t^{\mathbb{P}}$ should be canceled. In order to do so

$$\alpha_t = \frac{\frac{\partial E_t}{\partial h_t}}{\frac{\partial H_t}{\partial h_t}} \quad \gamma_t = \alpha_t\frac{\Delta H_t}{1-R} - \frac{\Delta E_t}{1-R}$$

So that

$$\frac{\frac{\partial E_t}{\partial t}+\frac{1}{2}\sigma_t^2\frac{\partial^2 E_t}{\partial h_t^2}+\frac{h_t}{1-R}\Delta E_t-c_tE_t}{\frac{\partial E_t}{\partial h_t}}=\frac{\frac{\partial H_t}{\partial t}+\frac{1}{2}\sigma_t^2\frac{\partial^2 H_t}{\partial h_t^2}+\frac{h_t}{1-R}\Delta H_t-c_tH_t}{\frac{\partial H_t}{\partial h_t}} \tag{19}$$

Adding $\mu_t^{\mathbb{P}}$ and dividing by σ_t both sides of the last equation we obtain what could be interpreted as the expected excess return of the derivative over the collateral rate divided by the the derivatives volatility factor, therefore

$$\frac{\frac{\partial E_t}{\partial t}+\mu_t^{\mathbb{P}}\frac{\partial E_t}{\partial h_t}+\frac{1}{2}\sigma_t^2\frac{\partial^2 E_t}{\partial h_t^2}+\frac{h_t}{1-R}\Delta E_t-c_tE_t}{\sigma_t\frac{\partial E_t}{\partial h_t}}=\frac{\frac{\partial H_t}{\partial t}+\mu_t^{\mathbb{P}}\frac{\partial H_t}{\partial h_t}+\frac{1}{2}\sigma_t^2\frac{\partial^2 H_t}{\partial h_t^2}+\frac{h_t}{1-R}\Delta H_t-c_tH_t}{\sigma_t\frac{\partial H_t}{\partial h_t}}=M(t,h_t) \tag{20}$$

Since the ratio must be valid for any credit derivative (H_t and E_t are two generic payoffs), then it must be just a function of t and h_t. $M_t = M(t,h_t)$ will be called the market price of credit risk. Therefore, the PDE followed by any credit derivative must be

$$\frac{\partial E_t}{\partial t}+\left(\mu_t^{\mathbb{P}}-\sigma_tM_t\right)\frac{\partial E_t}{\partial h_t}+\frac{1}{2}\sigma_t^2\frac{\partial^2 E_t}{\partial h_t^2}+\frac{h_t}{1-R}\Delta E_t-c_tE_t=0$$

When dealing with bonds, things are a little bit different. First we have to establish a relationship between the short term financing rate f_t and the short term CDS rate h_t. In order to do so, we compare two different strategies:

- Selling protection at time t with maturity $t+dt$.
- Buying a bond at t maturing at time $t+dt$ through a REPO transaction maturing also at time $t+dt$.

Both strategies imply a net cash flow at time t equal to 0. At time $t+dt$, the net cash flows are (assuming $\tau > t$):

CDS: $h_tdt-(1-R)1_{\{\tau\le t+dt\}}$

REPO: $(1+f_tdt)1_{\{\tau>t+dt\}}+R1_{\{\tau\le t+dt\}}-(1+r_tdt)=$

$$=(1+f_tdt)-(1+r_tdt)-(1-R+f_tdt)1_{\{\tau\le t+dt\}}=(f_t-r_t)dt-(1-R)1_{\{\tau\le t+dt\}}$$

Where r_t is a short term REPO rate on a short term bond maturing at time $t+dt$. Therefore:

$$h_t = f_t - r_t$$

In order to obtain the PDE followed by defaultable bonds and derivatives that are replicated with bonds we should keep in mind that collateralized credit derivatives are financed at the collateral rate used to remunerate collateral accounts in cash no matter the volatility of the underlying derivative, whereas bonds are purchased at REPO rates that might differ between different bonds. Therefore the PDE will be

$$\frac{\partial B_t}{\partial t} + \left(\mu_t^{\mathbb{P}} - \sigma_t M_t\right)\frac{\partial B_t}{\partial h_t} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 B_t}{\partial h_t^2} + \frac{h_t}{1-R}\Delta B_t - r_t^B B_t = 0$$

Where r_t^B represents the short term REPO rate for bond B_t. Notice that h_t is again the short term CDS spread and not the financing spread over EONIA.

B Link between the hedging PDE and the expected value of the derivative

Let's assume a function $V_t = V(t, S_t, h_t^I, h_t^H)$ that follows the PDE:

$$\widehat{\mathcal{L}}_{SIH} V_t + h_t^I \tfrac{\Delta V_t^I}{1-R_I} = +f_t^B V_t^+ + f_t^C V_t^- \tag{21}$$

With terminal condition $V(T, S_T) = g(S_T)$

where

$$\begin{aligned}\widehat{\mathcal{L}}_{SIH} V_t \;=\; & \tfrac{\partial V_t}{\partial t} + (r_t - q_t) S_t \tfrac{\partial V_t}{\partial S_t} + (\mu_t^H - M_t^H \sigma_t^H)\tfrac{\partial V_t}{\partial h_t^H} + (\mu_t^I - M_t^I \sigma_t^I)\tfrac{\partial V_t}{\partial h_t^I} \\ & + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial S_t^2} S_t^2 (\sigma_t^S)^2 + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial {h_t^H}^2}(\sigma_t^H)^2 + \tfrac{1}{2}\tfrac{\partial^2 V_t}{\partial {h_t^I}^2}(\sigma_t^H)^2 \\ & + \tfrac{\partial^2 V_t}{\partial S_t h_t^H} S_t \sigma_t^S \sigma_t^H \rho_t^{S,H} + \tfrac{\partial^2 V_t}{\partial S_t h_t^I} S_t \sigma_t^S \sigma_t^I \rho_t^{S,I} + \tfrac{\partial^2 V_t}{\partial h_t^I h_t^H} \sigma_t^I \sigma_t^H \rho_t^{I,H}\end{aligned} \tag{22}$$

In this section we want to prove that the solution to (21) is equal to the following expected value

$$V_t = \underbrace{E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right)\Big|\mathcal{F}_t\right]}_{\text{Fully collateralized price}}$$

$$\underbrace{-E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)\left(k_s^I V_s^+ + \bar{h}_s^I V_s^-\right) ds\Big|\mathcal{F}_t\right]}_{\text{Funding value adjustment}} \tag{23}$$

$$+\underbrace{E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)(R_I-1)\left(V_s^C\right)^- dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_t\right]}_{\text{CVA}}$$

In a measure $\mathbb{Q}$ in which the drifts of S_t, h_t^H and h_t^I are given by $(r_t-q_t)S_t$, $\mu_t^H - M_t^H\sigma_t^H$ and $\mu_t^I - M_t^I\sigma_t^I$ respectively. Under this measure, the default intensity of the default event of the investor is $\lambda_t^{I,\mathbb{Q}} = \frac{h_t^I}{1-R_I}$.

In order to see the equivalence between the solution of the PDE and an expected value, let's define the process:

$$X_t = V_t \exp\left(-\int_{s=0}^{t} c_s ds\right) 1_{\{\tau^I>t\}}$$

Let's apply Itô's Lemma for jump diffusion processes to X_t in $\mathbb{Q}$

$$dX_t = 1_{\{\tau^I>t\}} \exp\left(-\int_{s=0}^{t} c_s ds\right)\left(-c_tV_tdt + \widetilde{\mathcal{L}}_{SIH}V_tdt + \frac{\partial V_t}{\partial S_t}S_t\sigma_t^S dW_t^S + \frac{\partial V_t}{\partial h_t^I}\sigma_t^I dW_t^I + \frac{\partial V_t}{\partial h_t^H}\sigma_t^H dW_t^H - V_t dN_t^{I,\mathbb{Q}}\right)$$

Taking (21) into account

$$\widetilde{\mathcal{L}}_{SIH}V_t = \left(f_t^B - c_t\right)V_t^+ + \left(f_t^C - c_t\right)V_t^- + c_tV_t - \lambda_t^{I,\mathbb{Q}}\Delta V_t^I = k_t^I V_t^+ + \bar{h}_t^I V_t^- + c_tV_t - \lambda_t^{I,\mathbb{Q}}\Delta V_t^I$$

Where k_t^I is the funding benefit spread.

So that

$$
\begin{aligned}
dX_t &= 1_{\{\tau^I>t\}} \exp\left(-\int_{s=0}^t c_s ds\right)\left(k_t^I V_t^+ dt + h_t^I V_t^- dt - \lambda_t^{I,\mathbb{Q}} \Delta V_t^I dt \right.\\
&\quad \left. + \frac{\partial V_t}{\partial S_t} S_t \sigma_t^S dW_t^S + \frac{\partial V_t}{\partial h_t^I} \sigma_t^I dW_t^I + \frac{\partial V_t}{\partial h_H^I} \sigma_t^H dW_t^H - V_t dN_t^{I,\mathbb{Q}}\right) = \\
&= 1_{\{\tau^I>t\}} \exp\left(-\int_{s=0}^t c_s ds\right)\left(k_t^I V_t^+ dt + h_t^I V_t^- dt - \lambda_t^{I,\mathbb{Q}} \Delta V_t^I dt \right.\\
&\quad \left. + \Delta V_t^I dN_t^{I,\mathbb{Q}} + \frac{\partial V_t}{\partial S_t} S_t \sigma_t^S dW_t^S + \frac{\partial V_t}{\partial h_t^I} \sigma_t^I dW_t^S + \frac{\partial V_t}{\partial h_t^I} \sigma_t^H dW_t^H - (V_t + \Delta V_t^I) dN_t^{I,\mathbb{Q}}\right)
\end{aligned} \tag{24}
$$

Integrating between t and T and assuming $\tau^I > t$

$$
\begin{aligned}
V_T \exp\left(-\int_{s=t}^T c_s ds\right) 1_{\{\tau^I>T\}} - V_t &= \int_{s=t}^T 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^s c_h ds\right)\left(k_s^I V_s^+ ds + h_s^I V_s^- ds \right.\\
&\quad -\lambda_s^{I,\mathbb{Q}} \Delta V_s^I ds + \Delta V_s^I dN_s^{I,\mathbb{Q}} + \frac{\partial V_s}{\partial S_s} S_s \sigma_s^S dW_s^S + \frac{\partial V_s}{\partial h_s^I} \sigma_s^I dW_s^S + \frac{\partial V_t}{\partial h_s^H} \sigma_t^H dW_t^H \\
&\quad \left. -(V_s + \Delta V_s^I) dN_s^{I,\mathbb{Q}}\right)
\end{aligned} \tag{25}
$$

And taking the expectation conditional on $\mathcal{F}_t$

$$
\begin{aligned}
E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^T c_s ds\right) 1_{\{\tau^I>T\}} \middle| \mathcal{F}_t\right] - V_t &= E_{\mathbb{Q}}\left[\int_{s=t}^T 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^s c_h dh\right)\left(k_s^I V_s^+ ds + h_s^I V_s^- ds \right.\right.\\
&\quad -\lambda_s^{I,\mathbb{Q}} \Delta V_s^I ds + \Delta V_s^I dN_s^{I,\mathbb{Q}} + \frac{\partial V_s}{\partial S_s} S_s \sigma_s^S dW_s^S + \frac{\partial V_s}{\partial h_s^I} \sigma_s^I dW_s^I + \frac{\partial V_t}{\partial h_s^H} \sigma_t^H dW_t^H \\
&\quad \left.\left. -(V_s + \Delta V_s^I) dN_s^{I,\mathbb{Q}}\right) \middle| \mathcal{F}_t\right]
\end{aligned} \tag{26}
$$

In the right hand side of (27), the expected values of the terms in $\frac{\partial V_s}{\partial S_s} S_s \sigma_s^S dW_s^S$, $\frac{\partial V_s}{\partial h_s^I} \sigma_s^I dW_s^I$ and $\frac{\partial V_t}{\partial h_s^H} \sigma_t^H dW_t^H$ are zero since they represent the expected values of Itô integrals. The expected value of the term $-\lambda_s^{I,\mathbb{Q}} \Delta V_s^I ds + \Delta V_s^I dN_s^{I,\mathbb{Q}}$ also vanishes, since we have the integral of a Cox process less its compensator.

Therefore

$$
\begin{aligned}
V_t &= E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^T c_s ds\right) 1_{\{\tau^I>T\}} \middle| \mathcal{F}_t\right] \\
&\quad -E_{\mathbb{Q}}\left[\int_{s=t}^T 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^s c_h dh\right)\left(k_s^I V_s^+ ds + h_s^I V_s^-\right) ds \middle| \mathcal{F}_t\right] \\
&\quad +E_{\mathbb{Q}}\left[\int_{s=t}^T 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^s c_h dh\right)(V_s + \Delta V_s^I) dN_s^{I,\mathbb{Q}} \middle| \mathcal{F}_t\right]
\end{aligned} \tag{27}
$$

But

$$E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right) 1_{\{\tau^I>T\}}\Big|\mathcal{F}_t\right] = E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right)\Big|\mathcal{F}_t\right] - E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right) 1_{\{\tau^I\leq T\}}\Big|\mathcal{F}_t\right]$$

and

$$E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{s=t}^{T} c_s ds\right) 1_{\{\tau^I\leq T\}}\Big|\mathcal{F}_t\right] = E_{\mathbb{Q}}\left[\int_{s=t}^{T} V_T \exp\left(-\int_{u=t}^{T} c_u du\right) 1_{\{\tau^I>s\}} dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_t\right] =$$

$$E_{\mathbb{Q}}\left[\int_{s=t}^{T} E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{u=t}^{T} c_u du\right) 1_{\{\tau^I>s\}} dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_s\right]\Big|\mathcal{F}_t\right] =$$

$$= E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I>s\}} dN_s^{I,\mathbb{Q}} E_{\mathbb{Q}}\left[V_T \exp\left(-\int_{u=t}^{T} c_u du\right)\Big|\mathcal{F}_s\right]\Big|\mathcal{F}_t\right] =$$

$$= E_{\mathbb{Q}}\left[\int_{s=t}^{T} 1_{\{\tau^I>s\}} dN_s^{I,\mathbb{Q}} V_s^C \exp\left(-\int_{u=t}^{s} c_u du\right)\Big|\mathcal{F}_t\right]$$

Where V_t^C represents the time t value of a completely collateralized derivative with payoff function $g(S_T)$. We have applied the fact that a collateralized derivative divided by the current account that accrues at the collateral rate is a martingale under $\mathbb{Q}$.

Going back to (27), we have

$$\begin{aligned} V_t &= E_{\mathbb{Q}}\left[V_T \exp\left(-\textstyle\int_{s=t}^{T} c_s ds\right)\Big|\mathcal{F}_t\right] \\ &\quad - E_{\mathbb{Q}}\left[\textstyle\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)\left(k_s^I V_s^+ ds + h_s^I V_s^-\right) ds\Big|\mathcal{F}_t\right] \qquad (28) \\ &\quad + E_{\mathbb{Q}}\left[\textstyle\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)(V_s + \Delta V_s^I - V_t^C) dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_t\right] \end{aligned}$$

And if we assumed that after default $V_s + \Delta V_s^I = R_I V_s^C$ then

$$\begin{aligned} V_t &= E_{\mathbb{Q}}\left[V_T \exp\left(-\textstyle\int_{s=t}^{T} c_s ds\right)\Big|\mathcal{F}_t\right] \\ &\quad - E_{\mathbb{Q}}\left[\textstyle\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)\left(k_s^I V_s^+ ds + h_s^I V_s^-\right) ds\Big|\mathcal{F}_t\right] \qquad (29) \\ &\quad + E_{\mathbb{Q}}\left[\textstyle\int_{s=t}^{T} 1_{\{\tau^I>s\}} \exp\left(-\int_{h=t}^{s} c_h dh\right)(R_I - 1) V_t^C dN_s^{I,\mathbb{Q}}\Big|\mathcal{F}_t\right] \end{aligned}$$

References

[1] D. Brigo, A. Pallavicini and D. Perini. Funding, Collateral and Hedging: Uncovering the Mechanics and the Subtleties of Funding Valuation Adjustments. http://papers.ssrn.com/sol3/papers.cfm?abstract id=2161528, October, 2012.

[2] C. Burgard and M. Kjaer. Partial differential equation representations of derivatives with counterparty risk and funding costs. The Journal of Credit Risk, Vol. 7, No. 3, 1-19, 2011.

[3] C. Burgard, M. Kjaer. In the balance, Risk, Vol 11, 72-75, 2011.

[4] J. Gregory. Being Two-faced over Counterpartyrisk. Risk, February, 2009.

[5] J. Gregory. Counterparty credit risk and credit value adjustment. Wiley, 2nd edition, 2012.

[6] J. Hull, A. White. The FVA debate, Risk, Aug 2012.

[7] J. Hull, A. White. The FVA debate continued, Working paper, Sep 2012.

[8] J. Hull, A. White. CVA, DVA, FVA and the Black-Scholes-Merton Arguments, Working paper, Sep 2012.

[9] M. Kjaer. A generalized credit value adjustment. The Journal of Credit Risk, Vol. 7, No. 1, 1-28, 2011.

[10] C. Burgard, M. Kjaer. Generalised CVA with funding and collateral via semi-replication, Working paper. Dec 2012. http://papers.ssrn.com/sol3/papers.cfm?abstract id=2027195

[11] M. Morini and A. Pramploni. Risky funding with counterparty and liquidity charges. Risk, March, 70-75, 2011.

[12] V. Piterbarg. Funding beyond discounting: Collateral agreements and derivatives pricing. Risk, February, 97-102, 2010.

[13] V. Piterbarg. Cooking with collateral. Risk, August, 2012.

[14] Antonio Castagna. On the Dynamic Replication of the DVA: Do Banks Hedge their Debit Value Adjustment or their Destroying Value Adjustment?. July, 2012. http://www.iasonltd.com/FileUpload/files/DVA

[15] A. Castagna. Funding, liquidity, credit and counterparty risk: Links and implications. Iason research paper. http://iasonltd.com/resources.php, 2011.