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Predicting bankruptcy using neural networks and other classification methods: the influence of variable selection techniques on model accuracy

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Abstract

We evaluate the prediction accuracy of models designed using different classification methods depending on the technique used to select variables, and we study the relationship between the structure of the models and their ability to correctly predict financial failure. We show that a neural network based model using a set of variables selected with a criterion that it is adapted to the network leads to better results than a set chosen with criteria used in the financial literature. We also show that the way in which a set of variables may represent the financial profiles of healthy companies plays a role in Type I error reduction.

Keywords: Financial failure, Variable selection, Neural network

1. Introduction

Since the early 1990's, neural networks, and multi-layer perceptron neural networks in particular, have been widely used to design bankruptcy prediction models [1]. These neural networks make it possible to get around the statistical constraints of discriminant analysis, the main technique used to design such models since Altman [2]. In addition, their ability to represent non-linear relationships makes them well-suited to modeling the frequently non-linear relationship between the likelihood of bankruptcy and commonly used variables (i.e. financial ratios) [3].

Improving accuracy is certainly one of the most frequently addressed issues in bankruptcy prediction. The aim is to assess the conditions under which a model performs well. Some authors focused on identifying the methods of creating accurate models [2, 4, 5]. Others have studied the role of variables [6, 7] to assess whether some predictors are better than others (financial variables, ratios, non-financial variables). Still others have analyzed the types of failure a model is able to forecast. Indeed, traditional models are dichotomous and can classify only two groups. For this reason, a few authors have attempted to develop multi-state models, such as models to predict a final bankruptcy resolution (liquidation, reorganization, takeover) [8] or events that may affect the financial situation of a firm (non-payment of a debt, reduction of dividend payments)

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[9]. Other authors have also analyzed the influence of sample size, group size, or the cost of misclassification [10]. Only a very few have attempted to assess the contribution of variable selection techniques to the performance of a model [11, 12, 13]. To date, to our knowledge, nobody has done a comparative analysis of variable selection methods. Indeed, when the goal of the research is to find an effective means of improving prediction accuracy, whatever the modelling methods used to design models, one of the following strategies is used to choose explanatory variables: variables are selected because they are considered "good" predictors in the financial literature, for their performance on univariate statistical tests (t test, F test, correlation test), or as a result of automatic search procedures using evaluation criteria tailored to discriminant analysis (Wilks's lambda) or logistic regression (likelihood ratio). Only very few studies used other criteria. Table 1 illustrates the variable selection methods or criteria used by the main bankruptcy or financial failure studies that used a neural network. These criteria are rarely optimal when they are used in conjunction with a multi-layer perceptron neural network. When the aim of the research is to determine the conditions of replaceability of an existing model using new data or of a set a variables optimized for a given method with another [14, 15], the choice is perhaps understandable. But when the aim is to study how to improve prediction accuracy, the methods used to select predictors are far from suitable for a non-linear method [16].

The aim of our research is therefore to analyze the influence of variable selection techniques on model accuracy, and particularly the fit of the evaluation criteria commonly used in the financial literature and a neural network, and to study the relationship that may exist between the structure of a model and its ability to correctly classify failing companies. The remainder of this paper is organized as follows. In Section 2, we present a literature review that explains our research questions. In Section 3, we describe the samples and methods used in our experiments. Finally, in Section 4, we present and discuss our results and, in Section 5, we summarize our main findings.

2. Literature review

Table 1 presents the studies devoted to assessing the efficiency of classification or regression methods (neural networks, in particular), to designing bankruptcy or financial failure prediction systems, or to comparing the accuracy of various techniques, including neural networks, studies that attempt to identify the method best suited to designing prediction models. Table 1 also indicates methods or criteria used to select variables to be included in the models.

The main difficulty in choosing a set of predictors is the large number of variables that may be used. Indeed, the indicators most often chosen are based on financial ratios and then computed using balance sheets and income statements. There is almost no limit to designing financial ratios which indicate a given firm's performance. We have analyzed nearly 200 scientific papers dealing with corporate failure prediction published in the last 50 years and found that more than 500 different ratios were used in the final models. It is for precisely this reason that variables are usually chosen from among those used in previous studies, that is, from among variables that have proven to be reliable failure predictors in the past. Indeed, often chosen are variables that were identified by the very first authors to assess the usefulness of financial ratios in predicting corporate failure and by those who contributed to an understanding of the role played by multivariate statistical methods in the field of bankruptcy prediction, between the 1930's and the 1970's [2, 17, 18, 19, 20, 21, 22].

The second method of selecting variables is to choose them on the basis of univariate statistical tests. These tests are usually tests for differences between means or correlation tests because the authors assume that a "good" model is made up of variables that demonstrate a good individual discrimination ability and low redundancy.

The final method of selecting predictors is to use an automatic search procedure to mine a large set of previously defined variables for the best subset(s), depending on a likewise predefined evaluation criterion. This procedure is usually a stepwise search alternating between backward and forward steps to explore the variable space; a Wilks's lambda or a likelihood criterion is often used to evaluate the solutions.

Nevertheless, these three methods all have major drawbacks. The first method assumes that a variable that has proven to be a reliable indicator in a specific context will always be so in any other. All the experiments done to test the replicability of an existing model, to examine its performance with data not used during its design, show that the variables lose their initial properties when applied to other samples or used with methods other than those originally intended [23].

The second method assumes that a statistical test used to assess the individual discrimination ability of a variable is necessary for the design of a "good" model. However, a "good" model depends both on the interaction of the variables in a set, interaction that cannot be assessed through univariate tests, and on the way the variables fit the modelling method [24, 25], a fit that cannot be evaluated with such statistical tests.

Finally, the third method of selecting variables assumes that criteria optimized for discriminant analysis or logistic regression are suitable for other methods, and especially for neural networks. There is absolutely no reason to think that this assumption is valid. Moreover, Leray and Gallinari [16] have stated that since many parametric variable selection methods rely on the hypothesis that input-output variable dependence is linear or that input variable redundancy is well measured by the linear correlation of these variables, such methods are clearly ill-suited to non-linear methods, and hence to neural networks, since they are non-linear.

To overcome these limitations, some authors have explored other techniques such as genetic algorithms [11, 26, 27, 12, 28, 13, 29] or methods that fit a neural network [30, 31, 32, 33]. But these examples are very few and no comparative study has analyzed the influence of a variable selection technique on the predictive performance of a model. For this reason, this study intends to tackle this issue, in particular the influence of the evaluation criteria most often used with neural networks. To our knowledge, only one study [26] has compared a pair of sets of variables optimized for a discriminant analysis (stepwise method and F test), a logistic regression (stepwise method, Rao's score test to add variables and a likelihood ratio test to discard variables) and a neural network (genetic algorithm), but this only to analyze the differences between the models in terms of accuracy over different prediction timeframes (one, two or three years).

We have also analyzed the relationship between the structure of the models we designed and their ability to classify failing companies. Indeed, when we look at the results of the research mentioned in Table 1 and presented in Table 2, we see that only nine of the 36 studies that note prediction rates of both healthy and failed firms correctly predict failing firms at a rate higher than that at which they predict healthy firms. In other words, three-fourths of the models turn out to be more accurate when they predict that a company will remain healthy in the near future than when they predict that it will fail. This is a constant in the financial literature, regardless of the modelling technique. Of course, for investors or creditors, who may use the model as a decision tool, the cost of having a failing company classified as healthy (Type I error) is far greater than the cost of healthy company classified as failing (Type II error). A Type I error involves the loss of an investment or debt that will not be reimbursed as a result of bankruptcy, while a Type II error involves the loss of a potential bargain. Thus, models should above all avoid Type I errors. For this reason we have also analyzed what, in our models, might lead to an over classification of healthy companies.

3. Samples and methods

3.1. Samples and variables

Our experiments used data selected in 2006, and our results were computed in 2006 and 2007. We first selected French companies in the retail sector because in France, this sector traditionally accounts for the largest percentage of failed firms. Within this set of companies, we selected firms with an asset structure as homogenous as possible to control for the size effect and to allow comparisons of ratios. As there is no a priori rule for drawing a homogenous sample, we ran an Anova and a Mann-Whitney test on several breakdowns to find the most homogenous group, but also a group large enough to allow a relatively large sample size. These tests were computed on both failing and non-failing companies. Finally, the breakdown of companies into one group with assets of more than €750,000 and another with assets of less than €750,000 was the breakdown in which the differences between the two groups, measured with all ratios, were the largest while allowing for a relatively large sample size. This is why we finally chose companies with assets of less than €750,000.

We then selected accounting data and computed only financial ratios. Data were collected within a single year, 2002, and we included just one variable (shareholder funds) from the previous year 2001. When we selected healthy companies, we chose only companies in very good shape, that is, companies that were still in business in 2005. Moreover, we selected companies in operation for at least four years, because during the very first years of their lives, young, healthy companies often have a financial structure similar to that of failing companies. We also took into account the history of companies so as to select healthy firms that did not fail in the previous four years. Indeed, for several years after being reorganized, firms that went bankrupt and were then reorganized may reflect this bankruptcy in their financial statements and hence may look like failing companies.

Bankrupt companies were selected only if they were liquidated or reorganized in 2003, and at least 16 months after the publication of the annual report from 2002 so as to avoid any intentional distortion of financial statements.

We then try to design a well-balanced sample of young and old firms because young companies usually have a much higher probability of bankruptcy than older ones. Finally, we selected bankrupt companies for which accounting data were available in 2002, and shareholder funds available in 2001, and for which bankruptcy was declared (liquidation or reorganization) by court decision in 2003.

This first sample (validation sample), made up of 250 healthy and 250 bankrupt companies, was used to select variables and estimate the neural network parameters. Unsound firms were selected from among 1,548 failed firms in the retail sector and stored in the French database, Diane (in 2003, 10,136 firms in the retail sector went bankrupt in France, according to Insee). We then selected a second sample (test sample), made up of companies from the same sector and with the same amounts of assets, but data were from 2003, with one variable from 2002 (shareholder funds). Bankrupt companies were selected from among companies that were liquidated or reorganized by court decision in 2004, and at least 16 months after the publication of the annual reports from 2003. Healthy companies were randomly selected from among those that were still active in 2005. This second sample was made up of 260 healthy and 260 bankrupt firms in the Diane database. Companies in this second sample were not already included in the first sample.

This sample was used to estimate model accuracy. None of these companies used consolidated data.

Finally we selected a set of 41 initial variables (Table 3) that can be broken up into seven categories that best describe company financial profiles: liquidity-solvency, financial structure, profitability, efficiency, turnover, withdrawal and contribution.

Table 4 shows the means and standard deviation of the distribution of each variable used to describe the discrepancy of the deviations that exist within and between the two groups of companies (figures computed using standardized data with 0 mean and unit variance). This table also indicates the results of a Shapiro-Wilks normality test and the results of two tests for differences between the means of each variable within each group. The normality test indicates that none of the variables are normally distributed at the conventional significance level of 5%. As a consequence, the non-parametric test (Mann-Whitney U test) is more reliable than the parametric one (Student t test). This test highlights that all variables except Total Sales/Total Assets, Current Assets/Total Sales and Labor Expenses/Total Sales, present significant differences between the two groups.

3.2. Modelling techniques

In the 200 papers dealing with financial failure prediction that we analyzed and that have been published since the late 1960's, more than 50 classification or regression techniques are used. Three of these techniques predominate: discriminant analysis, logistic regression and multi-layer perceptron neural network. For this reason we selected these three methods.

All neural network parameters were defined a priori (topology, learning rate, momentum term, weight decay). To define these parameters, we used the following procedure. First, we randomly selected 50 sets of variables. Second, for each set of variables, we analyzed the results achieved with different values for each parameter: the learning rate varied from 0.1 to 0.5, with a step equal to 0.1, the momentum term from 0.5 to 0.9 (step = 0.1), the weight decay from $10 - 5$ to $10 - 2$ (step = $10 - 1$), and the number of hidden nodes from two to ten (step = 1). We used only one hidden layer. Third, for each combination of parameters and each set of variables, we used a ten-cross validation technique to compute the error, and those that led to the lowest out-of-sample error calculated over the 50 sets were then selected for our experiments. We used data from 2002. Each of these 50 sets included on average 20 variables, with a minimum of 9 and a maximum of 27. It took roughly five days to compute all network parameters with 30 PCs running Windows, and an additional day to calculate and check the final results.

3.3. Variable selection methods

The variable selection methods we chose were those most commonly used with discriminant analysis, logistic regression and MLP.

First, we chose a technique that relies on a forward search procedure to explore a (sub) space of possible variable combinations, a Fisher F test to interrupt the search, and a Wilks's lambda to compare variable subsets and determine the "best" one. This technique is especially optimized for discriminant analysis.

We then selected two other techniques optimized for logistic regression: a forward stepwise search and a backward stepwise search, with a likelihood statistic as an evaluation criterion of the solutions and a $Chi2$ as a stopping criterion.

Finally we selected three of the most commonly used [16] methods especially designed for neural networks, two of them to evaluate the variables without using the inductive algorithm (filter methods) and one using the algorithm as an evaluation function (wrapper method). The first is a zero-order technique, Eq. (1), which uses the evaluation criteria designed by Yacoub and Bennani [34] and the second, Eq. (2), is a first-order method that uses the first derivatives of network parameters with respect to variables as an evaluation criterion. The last one relies on the evaluation of an out-of-sample error calculated with the neural network. We do not choose a second-order method, based on second derivatives of network parameters, in order to look into an equivalent number of points of comparison. With all these criteria, we use only a backward search procedure, rather than a forward or a sequential search, and the network is retrained after each variable removal.

The zero and first-order criterion were calculated as follows. With a network composed of n inputs, one hidden layer with h neurons and one output, where w_{ji} is the weight between input i and neuron j in the hidden layer, and w_j the weight between neuron j and the output, the relevance or the saliency s of a variable i may be defined as:

$$S_i = \sum_{j=1}^{h} \left(\frac{|w_{ji}|}{\sum_{k=1}^{n} |W_{jk}|} \cdot \frac{|w_j|}{\sum_{j'=1}^{h} |w_{j'}|} \right) \tag{1}$$

$$S_i = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{\partial y_i}{\partial x_{ji}} \right| \tag{2}$$

Where x_i is a variable, y is the output of the network calculated with only one neuron and N the sample size.

To select variables, $1,000$ random bootstrap samples were drawn with replacement from the dataset of year 2002 (500 companies). Each bootstrap sample included 500 companies. We used the following three-step procedure to select variables:

- During the first step, each selection method was used to select variables with these $1,000$ bootstrap samples.

- Then, to identify important variables, those that were included in more than 70% of the selection results were selected. But this procedure might lead to remove highly correlated variables. Indeed, if two variables are correlated, the selection results may contain one or the other of these two variables while none of them will be included in 70% of the results. To avoid discarding potentially relevant but highly correlated variables, we used a second step. During the second step, variable pairs in which one or both variables were included in more than 90% of the bootstrap selections were considered pairs containing a relevant variable. Then, for each identified pair, the variable that occurs in most of the selection results was chosen.

- Finally, during the third step, variables that were selected during the first and second step were used to choose the final subsets. To choose these final subsets, the process used during the first step was repeated once.

3.4. Model development

We used the following three-step procedure to develop the models:

- During the first step, the sample of year 2002 was randomly divided into two sub-samples: a learning sample A of 450 companies and a validation sample V of 50 companies.

- During the second step, twenty-five bootstrap samples were drawn with replacement from A, each bootstrap sample included 450 companies and, for each selected set of variables, used to estimate as many models as bootstrap samples.

- Finally, during the third step, the resulting models were used to classify the observations of sample V thanks to a majority voting scheme. The cut-off point for classifying a firm as healthy or bankrupt was set as that which maximizes the overall rate of correct classifications. These three steps were repeated 100 times and the out-of-sample error was first estimated, along with a validation sample, and then re-estimated using the 25×100 models, along with a test sample of 520 companies.

- We used such a procedure to reduce the variance of prediction error caused by data instability. Indeed, financial ratios are always far from being normally distributed and contain many outliers. As a consequence, a small change in the data may produce a substantial change in the results. It is for this reason that we have chosen this bootstrap scheme [35, 36].

Moreover, the figures used to implement the bootstrap scheme (proportion of companies belonging to the learning sample and test sample, number of replicates of the procedure) were inspired by those used by Breiman [35] for a similar procedure. The computational time required to achieve the development of all models was quite large as it took several weeks, using 30 PCs running Windows.

4. Results and discussion

4.1. Selected variables

Tables 5, 6 and 7 show the six sets of variables that appeared in the selection results. Those that were chosen in more than 70% of the results were included in the final models. Each variable name is followed by its frequency of selection.

4.2. Selected variables and individual discrimination power

Table 8 ranks the variables by frequency of appearance in the six sets of variables, and Table 9 shows the same ranking, but only for variables that are identified with the criteria optimized for a neural network. This ranking is compared in Table 10, where the variables are ranked by their discrimination ability, as assessed by an F test. In this table, we have added their rank as it appears in the previous table. The first half of Table 10 (line 1 to line 21) shows the variables for which the F test reveals the highest discrimination power. This part of the table also contains 13 of the 14 variables selected with the neural network. This result indicates that there is a relationship between a parametric measure of discrimination and all the others we used in this study and which are non-parametric. However, this relationship is fairly rough because the two rankings are quite different. For instance, as Table 10 shows, the six variables that are most frequently selected with a neural network (EBITDA/Total Assets, Change in Equity Position, Shareholder Funds/Total Assets, (Cash + Marketable Securities)/Total Assets, EBIT/Total Assets, and Cash/Current Liabilities) are ranked 4th, 20th, 12th, 3rd and 13th respectively. By contrast, variables with high discrimination ability, such as EBITDA/Total Sales, Cash/Total Assets, Current Liabilities/Total Assets, or Cash/Total Debt, are not selected with any selection techniques.

As a consequence, it appears that using a t or an F test for a selection or pre-selection of the inputs of a neural network is unreliable as these tests may lead to the choice of unnecessary variables as well as to the removal of variables of great interest. This might well have been the case here, with the Change in Equity Position, for which the F test is quite low even though this variable is in fact relevant according to the neural network. Indeed, selection with a Wilks's lambda removes this variable. But when the value of an F test falls below a certain level, the only other variable selected is Accounts Receivable/Total Sales (which is selected only once).

4.3. Model accuracy

We then analyzed the relationship between modelling techniques and variable selection methods. The aim was to ascertain whether any pairs perform better than others and to study the behavior of a neural network while using sets of variables that were optimized for other methods.

We first measured the accuracy of "modelling method/selection technique" combinations, but only for those for which the evaluation criterion suited the classification technique. We compared the results of the following six pairs of methods: discriminant analysis-Wilks's lambda, logistic regression-likelihood criterion (with two search procedures), and neural network-zero-order, first-order, and error criteria. As Tables 11 and 12 show, the neural network outperforms discriminant analysis and to a lesser extent logistic regression. Indeed, the best result−93.85%−is achieved with a neural network on the test samples, followed by that for logistic regression with 90.77% and discriminant analysis with 85.19%. We then analyzed the results obtained when a modelling technique is used with a selection procedure for which the fit is not deemed acceptable. The results were computed on the validation samples. Table 13 shows the results obtained with the set of variables selected with a Wilks's lambda and those selected with a likelihood criterion. Table 14 shows the results calculated with the three sets of variables optimized for a neural network

Table 13 shows that variable selection based on a variance criterion (i.e., Wilks's lambda) leads to poor results; the adequate classification rate of 87.20% achieved with discriminant analysis is slightly lower with the other two methods. The criterion used here relies on assumptions that dovetail with those on which discriminant analysis is founded. It is not surprising that variables that cannot satisfactorily classify a high percentage of firms with discriminant analysis are unable to provide good results with other methods; the models built with logistic regression and the neural network lead to very similar results. Therefore, this criterion is clearly ill-suited to non-linear techniques.

The sets of variables that are selected with a likelihood criterion lead to less accurate results with discriminant analysis than with logistic regression −86.06%− as opposed to 92.01% with a backward search, and 85.20% as opposed to 89.20% with a forward search. However, with a neural network, the results of these two sets are fairly good−91.21% and 89.61% respectively−similar to the results obtained with logistic regression. As it turns out, the network leads to better results in one case out of two. With the likelihood criterion, logistic and neural models lead to broadly similar results, but this is no longer the case with neural network-based criteria. The error criterion achieved an accuracy of 94.03%, compared with 90.00% for logistic regression, and only 84.39% for discriminant analysis. The discrepancy between the results of the three methods is nearly the same with a zero-order criterion, with respective figures for correct classification of 93.59%, 88.01% and 83.60%. However, with a first-order criterion there is a decrease, with figures of 92.82%, 89.19% and 84.45%.

Thus, we can see that, the neural network leads to far better results than other methods, especially with an error criterion, which is not really surprising since this criterion is both the evaluation criterion of the variable relevance and the measure of this relevance. This is a very characteristic feature of wrappers because the inductive algorithm is used directly during variable selection. This result is consistent with what we might expect. The zero-order criterion's outperformance of a first-order criterion can be put down primarily to chance as there is no evidence that the former is better than the latter.

Neural models, when developed with appropriate variables, are thus much more reliable than logistic or discriminant models. Nevertheless, logistic models seem to better fit the data than discriminant models, whatever the variables used. In addition, with an error criterion, a logistic model produces 90.00% accuracy, whereas the neural model achieves 94.03%, leaving the logistic model−at 84.38%−far behind.

The accuracy of a model is in part the result of the intrinsic characteristics of the modelling technique and in part that of the fit of this technique and the variable selection procedure involved in its design. In bankruptcy prediction, all the experiments that have been done with large samples show that both financial ratios and a probability of bankruptcy behave in a non-linear manner. It is precisely for this reason that, as long as this non-linearity cannot be taken into account, it is hard to develop accurate models. Although using a selection criterion that fits logistic regression to design a neural model may be relevant, the choice of a criterion that fits discriminant analysis for the same purpose should not be recommended. It is necessary, at the very least, to consider other solutions.

4.4. Relationship between the structure of the models and their ability to correctly classify failing companies

As stated in the literature review, financial failure prediction models lead to better results when they predict that a company will remain healthy in the future as opposed to when they predict that it will fail. We can observe the same results with discriminant analysis (Table 11) and, to a lesser extent, with logistic regression. It appears that sound companies have a much wider variety of financial profiles than failing companies, with some of them having profiles similar to those of failing companies. Failing firms may continue to do business, but it is much more unusual for healthy firms to go suddenly bankrupt. This suggests there are sub-groups of sound companies whose financial profiles are so similar to those of failing companies that they move the boundary between the two groups in such a way that the models tend to identify bankrupt firms as healthy, and as a consequence lead to weaker results when classifying bankrupt firms.

To study this issue, we have used a Kohonen map to analyze the profiles of each group of firms and data from 2002. To determine the form of the map (i.e., the number of rows and columns), we first used Sammon's mapping method to examine the topology of the data. This map provides a general overview of the shape of the data and makes it possible to determine whether we may use a rectangular or square map. We have chosen a square map as there was no evidence that a rectangular one was better; it is made up of 100 neurons, 10 per line and 10 per column, one-fifth of the year 2002 sample.

To ensure that the order relationship of the input space will remain in the output space, and that the input space will be rather well approximated with a two-dimensional map, we made sure that, using Sammon's mapping method, no significant folding or stretching was visible on the map at the end of the learning phase. We first analyzed the distribution of the neurons by group of companies, whether they represent sound or unsound firms. Each map was designed using one of the six sets of variables we selected. Figures 1 to 6 show the number of neurons for each class in each each map (one map per set of variables). The dark grey part of the figures represents healthy companies, the light grey, bankrupt companies, and the white corresponds to neither. To assign a neuron to a class, we used a majority voting scheme, depending on the class of the nearest observations to a neuron.

The characteristics of the map are shown in Table 15. For each map (i.e., each set of variables), Table 15 presents the number of neurons in each group, the number of variables, the quantization error, that is, the information loss because of the projection of the input space onto a plan, the standard deviation of this error and the p-value of a Student t test for differences between the quantization error of the two groups. The error corresponds to the mean of the Euclidean distances computed between the neurons belonging to the same class and the firms that were the closest to these neurons. In Table 15, we can see that each group is represented by a somewhat different number of neurons. On average, healthy companies are coded using 52 neurons, compared with 45 neurons for bankrupt companies. The tiniest difference, which appears twice, can be observed with the set of variables selected using a Wilks's lambda and the set selected using a first-order criterion, where healthy firms are coded with 50 neurons and bankrupt firms with 49. The largest difference is due to the set of variables chosen on the basis of the error criterion, where healthy companies are coded with 56 neurons and bankrupt companies with 40. This finding reinforces the idea we mentioned in our look at the results of many bankruptcy prediction models published in the financial literature. Indeed, in this literature healthy firms are more often correctly classified than failing firms, as if the financial profiles of the former were much more complex and multiform than the profiles of the latter. As a consequence, the profiles of some healthy companies seem to be similar to the profiles of failing companies. Using a Kohonen map and financial ratios to develop a typology of companies, Pérez [37] noted that healthy firms would present a much wider spectrum of profiles than failing firms, without further analysis.

We can also notice in Table 15 that the quantization error associated with bankrupt companies is larger than the one associated with healthy companies, and the difference is statistically significant at the threshold of 5% in three cases out of six, and nearly significant in a fourth one if we consider the first-order criterion. This error is partly due to the difference between the number of neurons in each class when this difference exists. Once a group is quantified by a number of prototypes lower than another one, we may consider that the quality of its quantization is less than the quality of the other. This is probably the case in our experiments. However another factor may have an influence when the number of neurons in each group is similar. We used factor analysis in order to design sub-groups in each class (healthy vs bankrupt) and we found that the difference we can observe in the quantization error between healthy and bankrupt companies is also partly due to a few sub-groups of bankrupt companies (which are much heterogenous than the others). However, on average, healthy companies present a wider variety of financial profiles than bankrupt firms.

Now, if we analyze the structure of the maps in the light of model accuracy, we can notice that the accuracy of a neural network model is all the more important as the number of neurons encoding sound companies is large, as shown in Table 16. The same is true for the two models designed using a likelihood criterion. However, the hierarchy is not preserved when the types of criteria are no longer taken into account. If we analyze the performance of the models while taking into account the correct classification rates for each group of firms, we can observe in

Table 17 that there is a relationship between the ability to classify failing firms correctly and the structure of the maps. Indeed, the models that achieve much higher correct classification rates for healthy firms than for bankrupt firms are designed on profiles where healthy companies can express all their diversity. As a result, the corresponding maps are based on a slightly different number of neurons encoding each class. The first three lines in Table 17−two of three models are far better at identifying failed firms than at identifying healthy ones−summarize the situation well. At the bottom of Table 17, we find models that achieve similar or less good results when classifying healthy companies, for which the underlying profiles are encoded using roughly the same number of neurons.

However, these results should be considered with caution since they rely on a single sample, and they only express a trend. Indeed, the relationship we have just analyzed is not true in all cases. Our results should therefore be confirmed by further studies, and if they are confirmed, they would make it possible to show that some financial profiles (i.e., sets of ratios) may intrinsically better discriminate between failing firms and sound firms than do others. For the moment, one of the main techniques used to improve the classification rate of failing firms is to look for an optimal cut-off point that will be able to maximize this rate while keeping the broader classification rate in a suitable range. Therefore, it will be valuable if some variables result in improved classification of failing firms; the advantage of this improvement is that it would make possible a simplified calculation to take into account the asymmetry of the costs of mistaken identification of failing and healthy companies.

5. Conclusion

We have shown that there is a relationship between the discrimination ability of a variable, as measured with a t test or an F test, and its susceptibility to selection by an automatic procedure that relies on other measures, but we have also found a discrepancy in this relationship which indicates that such statistical tests should not be used alone if the purpose of the selection is to create a neural model. As a consequence, we may use them-but with extreme caution-to build non-linear models, and if we intend to do so, we would do well to use them in conjunction with other techniques. However, even if criteria that fit neural networks lead to accurate classification results, in practice, it is hard to choose the right criterion. Indeed, the performance of a criterion depends on data structure. For instance, Leray and Gallinari [16] have demonstrated that the accuracy of a model designed with a specific criterion strongly depends on the level of correlation between variables, the degree of linear separability of data and the level of noise. These results clearly indicate the drawbacks of an approach based only on a single selection criterion, even is this criterion is well-suited to the modelling techniques used.

We have also shown that a neural-network-based model for predicting bankruptcy performs significantly better when designed with appropriate variable selection techniques than when designed with methods commonly used in the financial literature. Unlike the latter, the former are slow and hard to use, which may account for their under-utilization. However, a few studies have looked into other techniques, mainly genetic algorithms. So the reasons for the failure of neural network-based variable selection methods to be adopted more widely must be found elsewhere, perhaps in the absence of cross-disciplinary approaches to this particular field. Neural network algorithms are in exactly the same situation: while many types are commonly used in many scientific disciplines, only one is systematically used in corporate finance. Also variable selection techniques face the same issue: they come from a field of knowledge that has little to do with corporate finance. Of course, these results will need to be confirmed by additional studies in a variety of other settings, such as other samples, types of firms, sectors, etc..Nevertheless, they all point to the need to use relevant variable selection techniques to develop neural models. As it turns out, the most recent research papers continue to rely on traditional methods: variables are still selected for their performance on univariate statistical tests [38] or as a result of their popularity in the field of financial analysis [39]. Finally, we have shown that healthy companies have a much wider variety of financial profiles than failing firms, as already stated by Pérez [37]. Our models suggest that the ability of a prediction rule to classify failing firms correctly is related to its ability to account precisely for the entire spectrum of healthy firms.

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6. Figures

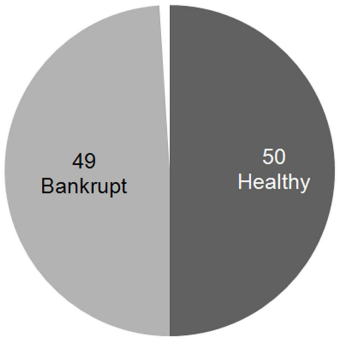

Figure 1: Number of neurons in the Kohonen map per class. Set of variables selected using a Wilks's lambda criterion and a stepwise search

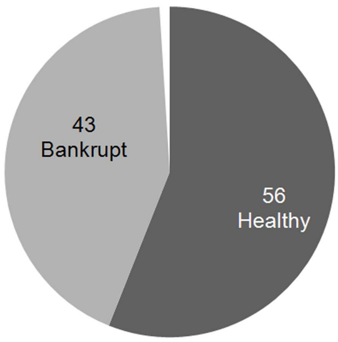

Figure 2: Number of neurons in the Kohonen map per class. Set of variables selected using a likelihood criterion and a backward stepwise search

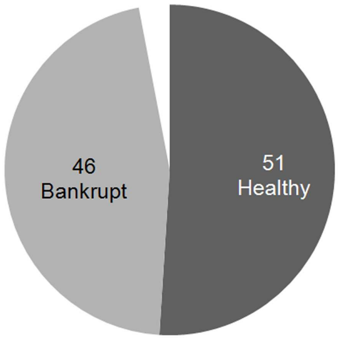

Figure 3: Number of neurons in the Kohonen map per class. Set of variables selected using a likelihood criterion and a forward stepwise search

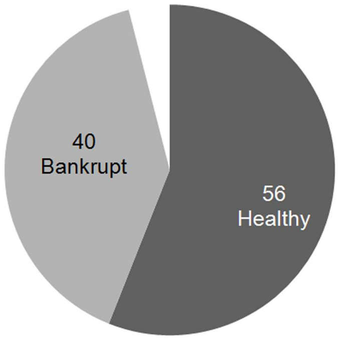

Figure 4: Number of neurons in the Kohonen map per class. Set of variables selected using an error criterion and a backward search

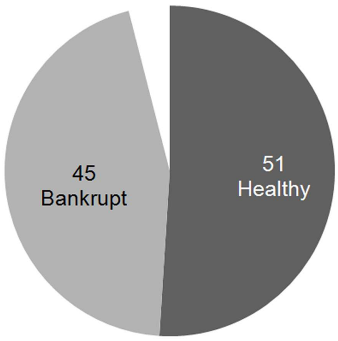

Figure 5: Number of neurons in the Kohonen map per class. Set of variables selected using a zero order criterion and a backward search

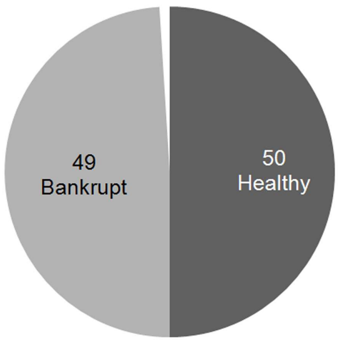

Figure 6: Number of neurons in the Kohonen map per class. Set of variables selected using a first order criterion and a backward search

7. Tables

Table 1: Variable selection methods used in major studies which aimed to design bankruptcy prediction models with neural network models

Authors	Final variable selection method or criterion	Neural network used
Agarwal [40]	Variables used in studies by Altman [2] and Wilson and Sharda [41] and one other study	MLP-BP
Alfaro et al. [39]	Variables used in previous studies	MLP-BP
Altman et al. [9]	Method and criterion not indicated	MLP-BP
Anandarajan et al. [42]	Variables used in previous studies	MLP-BP - MLP-GA
Back et al. [11]	Genetic algorithm applied to a set of variables used in previous studies	MLP-BP - SOM - BM
Back et al. [26]	Genetic algorithm applied to a set of variables used in studies by Altman [2], Altman [17], Blum [19], Beaver [18], Deakin [20], Merwin [21], Ramser and Foster [22] and five other studies	MLP-?
Back et al. [27]	Genetic algorithm applied to a set of variables used in studies by Altman [2], Altman [17], Blum [19], Beaver [18], Deakin [20], Merwin [21], Ramser and Foster [22] and five other studies	MLP-BP
Bell et al. [43]	Univariate analysis applied to variables used in previous studies	MLP-NCB
Berg [44]	Signification coefficient tests applied to variables in one previous study	MLP-?
Boritz and Kennedy [45]	Variables used in studies by Altman [2] and Ohlson [5]	MLP-BP - MLP-FLBP - MLP-PBP - MLP-PCBP
Bose and Pal [30]	Method (not mentioned) that fits a neural network applied to variables used in a previous study and new variables	MLP-BP
Boyacioglu et al. [38]	t test and factor analysis applied to a set of variables commonly used in financial analysis	MLP-BP
Brabazon and Keenan [12]	Genetic algorithm applied to a set of variables used in studies by Altman [2, 17], Back et al. [26], Ohlson [5], Serrano-Cinca [46], five other studiesand	MLP-GA
Brockett et al. [47]	Canonical analysis, correlation test and stepwise search with criteria optimized for discriminant analysis and logistic regression applied to variables used in previous studies	MLP-BP
Charalambous et al.[31]	Method that fits a neural network applied to variables used in previous studies	MLP-CG
Charitou et al. [48]	Backward and forward search with a criterion optimized for logistic regression, and coefficient signification tests applied applied to variables used in previous studies	MLP- CG
Dorsey et al. [49]	Variables used in previous studies	MLP-GA

Etheridge and Sriram [50]	Not mentioned	SOM - PNN
Fan and Palaniswami [51]	Distance measure between the centroid of the groups applied to variables used in studies by Altman [2], Ohlson [56], and one other study, and variables commonly used in financial literature	MLP-BP - LVQ
Fanning and Cogger [52]	Variables used in one previous study	MLP-BP - GANN
Goss and Ramchandani [53]	Variables used in one previous study	MLP-BP
Huang et al. [54]	Variables used by insurance company regulation authority	MLP-GA
Jo et al. [55]	t test applied to variables used in studies by Beaver [18], Altman [2], Blum [40], Deakin [20], Odom and Sharda [4], Ohlson [5], and fourteen other studies	MLP-BP
Kim and McLeod [28]	Judgment of an expert applied to variables computed with a factor analysis and other variables commonly used in the financial literature	MLP-GA
Kiviluoto [56]	Not mentioned	SOM - SOM-RBF
Kotsiantis et al. [57]	Relief method applied to a set of variables commonly used in the literature	MLP-RBF
Kumar et al. [8]	Stepwise search with a criterion optimized for discriminant analysis applied to variables used by Altman [2] and in two other studies	MLP-BP
Lacher et al. [41]	Variables used in study by Altman [2]	CasCor
Laitinen and Kankaanpaa [58]	Variables used in previous studies	MLP-BP
Lee et al. [59]	Stepwise search with a criterion optimized for discriminant analysis applied to variables used in previous studies	MLP-BP - SOM-MLP
Lee et al. [60]	Variables used in study by Altman [2]	MLP-LM - SOM
Leshno and Spector [61]	Judgment and correlation test applied to variables used in previous studies and variables used in study by Altman [2]	MLP-BP - MLP-?
Li and Gupta [62]	Variables used in one previous study	MLP-GA
McKee and Greenstein [63]	Variables used in one previous study	?
Min et al. [64]	Stepwise search with criteria optimized for discriminant analysis and logistic regression, and t test applied to variables used in previous studies	?
Min and Lee [65]	Stepwise search with criteria optimized for discriminant analysis logistic regression, and t test and	MLP-BP
Min and Lee [66]	Factor analysis, t tests and stepwise search with criteria optimized for discriminant analysis and logistic regression	MLP-BP
Odom and Sharda [4]	Variables used in study by Altman [2]	MLP-BP
Pendharkar [67]	Variables used in studies by Altman [2] and Altman et al. [17]	MLP-BP - MLP-GA
Piramuthu et al. [68]	Classification tree and symbolic rules	MLP-BP
Pompe and Feelders [69]	Judgment of an expert applied to financial variables commonly used by financial experts to predict corporate failure	MLP-BP
Salchenberger et al. [70]	Multiple regression applied to variables commonly used in financial analysis	MLP-BP
Sen et al. [32]	Tests applied to the weight of the neural network	MLP-BP

Serrano-Cinca [46]	Variables used in one previous study	MLP-BP
Sexton et al. [13]	Genetic algorithm applied to variables commonly used in financial analysis	MLP-GA
Shin et Lee [71]	Stepwise search with a criterion optimized for discriminant analysis and t test	MLP-BP
Tam and Kiang [72]	Variables that belong to the following financial categories: asset, income, liquidity	MLP-BP
Tan and Dihardjo [73]	Variables used in two previous studies	MLP-BP
Tung et al. [74]	Correlation test applied to variables used in previous studies	MLP-BP - GSOFNN
Tyree and Long [33]	Method that fits a neural network (sensitivity measure) and stepwise search with a criterion optimized for discriminant analysis	MLP-BP - PNN
Vieira et al. [75]	Variables used in previous studies	HLVQ - G-Prop
Wallrafen et al. [74]	Genetic algorithm applied to variables that belong to the following financial categories: liquidity, profitability, asset, solidity, debt and reimbursement ability	MLP-GA
West et al. [76]	Variables used in study by Altman [2]	MLP-BP
Wilson and Sharda [41]	Variables used in study by Altman [2]	MLP-BP
Wu et al. [77]	Variables used in previous studies	MLP-BP
Yang and Harrison [78]	Stepwise search with a criterion optimized for discriminant analysis applied to variables commonly used in financial analysis	MLP-BP - PNN - RHPNN
Yang et al. [79]	Variables used in one previous study	MLP-BP - PNN
Yim and Mitchell [80]	Variables used in previous studies	MLP-BP
Yim and Mitchell [81]	Variables that belong to the following financial categories: liquidity, profitability, structure, asset	MLP-BP

This table indicates, in addition to the variable selection methods or criteria that were used, the name of the neural network and the name of the algorithm used during the learning process. For example, MLP-BP corresponds to a multi-layer perceptron neural network trained using a back-propagation method. This learning algorithm is used in more than 75% of the research works mentioned here, whereas the MLP is used in almost all experiments. The acronyms are those used by the authors. A question mark (?) means that no indication was given.

Neural networks used:

BM: Boltzman Machine
CasCor: Cascade Correlation
G-Prop: MLP used in conjunction with a genetic algorithm during the learning process
GANN: Generalized Adaptive Neural Network
GSOFNN: Generic Self-Organizing Fuzzy Neural Network
HLVQ: Hidden Layer Vector Quantization (MLP used in conjunction with a Kohonen Map
LVQ: Learning Vector Quantization
MLP: Multi-Layer Perceptron
PNN: Probabilistic Neural Network (Parzen Window)
RHPNN: Robust Heteroscedastic Probabilistic Neural Network
SOM: Self-Organizing Map (Kohonen)

Algorithms used during the learning process:

GA: Genetic Algorithm
BP: Back-Propagation
FLBP: Functional Link Back-Propagation
GC: Conjugate Gradient

GRG2: Generalized Reduced Gradient
LM: Levenberg-Marquardt
NCB: Normalized Cumulative Back-Propagation
OET: Optimal Estimation Theory
PBP: Pruned Back-Propagation
PCBP: Predictive Cumulative Back-Propagation
RBF: Radial Basis Function

Table 2: Prediction results obtained with a neural network

Authors	Best result obtained with a neural network and data taken from the most recent period before bankruptcy		
	Non-bankrupt firms correctly classified	Bankrupt firms correctly classified	Total of correct classification
Agarwal [40]	99.00%	93.70%	96.38%
Alfaro et al. [39]	92.37%	82.20%	87.29%
Altman et al. [9]	89.40%	86.20%	87.80%
Anandarajan et al. [42]	93.75%	95.45%	95.19%
Back et al. [11]	100.00%	98.00%	99.00%
Back et al. [26]	100.00%	94.25%	97.30%
Back et al. [27]	100.00%	94.40%	97.30%
Bell et al. [43]	97.70%	61.80%	94.00%
Berg [44]	?	? 69.50%	
Boritz and Kennedy [45]	84.03%	74.27%	?
Bose and Pal [30]	75.62%	68.21%	71.88%
Boyacioglu et al. [38]	?	?	95.50%
Brabazon and Keenan [12]	82.67%	78.67%	80.67%
Brockett et al. [47]	94.50%	73.30%	89.30%
Charalambous et al. [31]	84.21%	84.21%	84.21%
Charitou et al. [48]	76.19%	90.48%	83.33%
Dorsey et al. [49]	?	?	?
Etheridge and Sriram [50]	92.71%	100.00%	92.85%
Fan and Palaniswami [51]	?	?	66.11%
Fanning and Cogger [52]	76.00%	94.00%	85.00%
Goss and Ramchandani [53]	75.00%	75.00%	75.00%
Huang et al. [54]	?	?	?
Jo et al. [55]	?	?	83.79%
Kim and McLeod [28]	?	?	78.70%
Kiviluoto [56]	88.30%	69.50%	86.50%
Kotsiantis et al. [57]	?	?	71.17%
Kumar et al. [8]	?	?	69.73%
Lacher et al. [82]	97.20%	91.50%	94.70%
Laitinen and Kankaanpaa [58]	89.50%	84.20%	86.80%
Lee et al. [59]	?	?	80.48%
Lee et al. [60]	76.19%	95.24%	85.22%

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Table 2 – Continued

Authors	Best result obtained with a neural network and data taken from the most recent period before bankruptcy		
	Non-bankrupt firms correctly classified	Bankrupt firms correctly classified	Total of correct classification
Leshno and Spector [61]	91.00%	90.90%	90.05%
Li and Gupta [62]	?	?	78.15%
McKee and Greenstein [63]	86.00%	75.00%	86.00%
Min et al. [64]	?	?	75.96%
Min and Lee [65]	85.71%	79.36%	82.54%
Min and Lee [66]	?	?	71.72%
Odom and Sharda [4]	?	?	81.81%
Pendharkar [67]	?	?	?
Piramuthu et al. [68]	92.70%	85.40%	89.10%
Pompe and Feelders [69]	?	?	72.60%
Salchenberger et al. [70]	90.00%	96.00%	92.50%
Sen et al. [32]	?	?	?
Serrano-Cinca [46]	?	?	93.94%
Sexton et al. [13]	97.40%	95.65%	?
Shin et Lee [71]	?	?	74.10%
Tam and Kiang [72]	85.00%	89.00%	87.00%
Tan and Dihardjo [73]	92.35%	64.70%	92.23%
Tung et al. [74]	99.31%	54.54%	73.12%
Tyree and Long [33]	100.00%	94.55%	97.95%
Vieira et al. [75]	?	?	90.00%
Wallrafen et al. [29]	?	?	?
West et al. [76]	?	?	87.27%
Wilson and Sharda [41]	98.00%	97.50%	97.75%
Wu et al. [77]	99.40%	83.30%	
Yang and Harrison [83]	95.50%	92.37%	95.27%
Yang et al. [79]	90.00%	63.00%	84.00%
Yim and Mitchell [80]	94.00%	80.00%	90.96%
Yim and Mitchell [81]	100.00%	89.00%	96.55%

?: Results not mentioned The results presented in this table correspond to the best results achieved with a neural network when many results were presented. In every situation, the results were computed using a classification criterion that maximized the overall classification rate.

Table 3: Initial set of variables

Index	**Liquidity-Solvency**
1	Current Assets/Current Liabilities
2	Current Assets/Total Assets
3	(Current Assets-Inventory)/Tot. Assets
4	Quick Ratio
5	Current Liabilities/Total Assets
6	Financial Debt/Cash Flow
7	(Cash + Mark. Sec.)/Total Sales
8	(Cash + Mark. Sec.)/Total Assets
9	EBITDA/Total Sales
10	Cash/Current Liabilities
11	Cash/Total Assets
12	Cash/Total Debt
Index	**Financial Structure**
13	Net Op. Work. Capital/Total Assets
14	Shareholder Funds/Total Assets
15	Long Term Debt/Shareholder Funds
16	Long Term Debt/Total Assets
17	Total Debt/Shareholder Funds
18	Total Debt/Total Assets
Index	**Profitability**
19	EBITDA/Permanent Assets
20	EBITDA/Total Assets
21	Profit before Tax/Shareholder Funds
22	EBIT/Total Assets
23	Net Income/Shareholder Funds
24	Net Income/Total Assets
Index	**Efficiency**
25	Total Sales/Shareholder Funds
26	Total Sales/Total Assets
27	Operating Cash Flow/Total Assets
28	Operating Cash Flow/Total Sales
29	Gross Trading Profit/Total Sales
30	EBIT/Total Sales
31	Value Added/Total Sales
Index	**Rotation**
32	Current Assets/Total Sales
33	Net Op. Work. Capital/Total Sales
34	Accounts Receivable/Total Sales
35	Accounts Payable/Total Sales
36	Inventory/Total Sales
37	Cash/Total Sales
Index	**Withdrawal**
38	Change in Other Debts
39	Change in Equity Position
Index	**Contribution**
40	Financial Expenses/Total Sales

Continued on Next Page…

Table 3 – Continued

41	Labor Expenses/Total Sales

Table 4: Characteristics of the variables belonging to the learning and validation samples (Means - Normality test and tests for differences between the two groups)

	Mean		Standard deviation		S-W			
Index	B	NB	B	NB	B	NB	t	U
Liquidity-Solvency								
1	-0.46	0.46	0.69	1.05	0.000	0.000	0.000	0.000
2	0.12	-0.12	1.00	0.99	0.000	0.000	0.008	0.000
3	-0.46	0.46	0.54	1.14	0.000	0.000	0.000	0.000
4	-0.08	0.08	1.01	0.98	0.000	0.000	0.063	0.039
5	0.51	-0.51	1.17	0.33	0.000	0.024	0.000	0.000
6	0.02	-0.02	1.41	0.12	0.000	0.000	0.669	0.000
7	-0.48	0.48	0.48	1.15	0.000	0.000	0.000	0.000
8	-0.51	0.51	0.52	1.10	0.000	0.000	0.000	0.000
9	-0.55	0.55	0.99	0.64	0.000	0.000	0.000	0.000
10	-0.50	0.50	0.33	1.18	0.000	0.000	0.000	0.000
11	-0.53	0.53	0.75	0.94	0.000	0.095	0.000	0.000
12	-0.48	0.48	0.32	1.20	0.000	0.000	0.000	0.000
Financial Structure								
13	-0.23	0.23	1.29	0.48	0.000	0.473	0.000	0.000
14	-0.54	0.54	1.15	0.29	0.000	0.000	0.000	0.000
15	-0.02	0.02	1.41	0.13	0.000	0.000	0.681	0.000
16	0.19	-0.19	1.28	0.54	0.000	0.000	0.000	0.000
17	-0.54	0.54	0.42	1.12	0.000	0.000	0.000	0.000
18	0.54	-0.54	1.16	0.30	0.000	0.000	0.000	0.000
Profitability								
19	0.01	-0.01	1.41	0.04	0.000	0.000	0.743	0.000
20	-0.55	0.55	1.02	0.60	0.000	0.000	0.000	0.000
21	-0.13	0.13	1.37	0.29	0.000	0.000	0.003	0.000
22	-0.55	0.55	1.08	0.47	0.000	0.000	0.000	0.000
23	-0.02	0.02	1.41	0.05	0.000	0.000	0.651	0.026
24	-0.54	0.54	1.11	0.43	0.000	0.000	0.000	0.000
Efficiency								
25	-0.07	0.07	1.41	0.09	0.000	0.000	0.142	0.000
26	-0.01	0.01	1.00	1.00	0.000	0.000	0.878	0.994
27	-0.23	0.23	1.17	0.72	0.000	0.000	0.000	0.000
28	-0.19	0.19	1.20	0.70	0.000	0.000	0.000	0.000
29	-0.14	0.14	1.04	0.94	0.000	0.005	0.001	0.006
30	-0.55	0.55	1.04	0.56	0.000	0.000	0.000	0.000
31	-0.35	0.35	0.94	0.94	0.000	0.001	0.000	0.000
Rotation								

Continued on Next Page…

Table 4 – Continued

	Mean		Standard deviation		S-W			
Index	B	NB	B	NB	B	NB	t	U
32	0.10	-0.10	1.13	0.84	0.000	0.000	0.028	0.121
33	-0.20	0.20	1.25	0.60	0.000	0.000	0.000	0.000
34	0.16	-0.16	1.11	0.85	0.000	0.000	0.000	0.001
35	0.38	-0.38	1.20	0.52	0.000	0.000	0.000	0.000
36	0.18	-0.18	1.16	0.77	0.000	0.000	0.000	0.001
37	-0.51	0.51	0.75	0.96	0.000	0.000	0.000	0.000
Withdrawal								
38	0.09	-0.09	1.17	0.78	0.000	0.000	0.046	0.000
39	0.04	-0.04	1.07	0.93	0.000	0.000	0.431	0.995
Contribution								
40	0.07	-0.07	1.33	0.49	0.000	0.000	0.139	0.026
41	0.29	-0.29	0.97	0.95	0.000	0.000	0.000	0.000

S-W: p-value of a Shapiro-Wilks normality test
t: p-value of a Student t test for differences between the means of the two groups
U: p-value of a Mann-Whitney test for the equality of the sum of ranks of each group
B: Bankrupt
NB: Non Bankrupt
Index: the same as in Previous Table

Table 5: Selected variables using a Wilks Lambda criterion

Variables included into the model	Freq.1
Cash/Total Assets	93.4%
Total Debt/Shareholder Funds	91.1%
Cash/Total Debt	88.7%
(Cash + Mark. Sec.)/Total Assets	87.5%
EBIT/Total Assets	81.2%
EBITDA/Total Assets	76.8%
Shareholder Funds/Total Assets	72.2%

Table 10: Rank of the variables according to an F test

	Variables	F	p-val	*Rank*[1]
1	EBIT/Total Sales	220.15	0.000	7
2	EBITDA/Total Sales	219.49	0.000	
3	EBIT/Total Assets	218.96	0.000	3
4	EBITDA/Total Assets	213.91	0.000	1

Continued on Next Page…

Table 10 – Continued

	Variables	F	p-val	*Rank*[1]
5	Net Income/Total Assets	210.01	0.000	7
6	Shareholder Funds/Total Assets	207.59	0.000	3
7	Total Debt/Total Assets	202.20	0.000	7
8	Total Debt/Shareholder Funds	201.14	0.000	7
9	Cash/Total Assets	195.01	0.000	
10	Cash/Total Sales	179.60	0.000	7
11	Current Liabilities/Total Assets	179.32	0.000	
12	(Cash + Mark. Sec.)/Total Assets	171.62	0.000	3
13	Cash/Current Liabilities	168.19	0.000	3
14	Cash/Total Debt	150.50	0.000	
15	(Cash + Mark. Sec.)/Total Sales	145.63	0.000	
16	Current Assets/Current Liabilities	133.77	0.000	7
17	Quick Ratio	131.30	0.000	
18	Accounts Payable/Total Sales	85.95	0.000	
19	Value Added/Total Sales	68.37	0.000	
20	Change in Equity Position	44.29	0.000	1
21	Operating Cash Flow/Total Sales	28.57	0.000	7
22	Net Operating Working Capital/Total Assets	27.21	0.000	
23	Net Operating Working Capital/Total Sales	21.10	0.000	
24	Operating Cash Flow/Total Assets	19.40	0.000	
25	Long Term Debt/Total Assets	19.32	0.000	
26	Inventory/Total Sales	16.00	0.000	
27	Accounts Receivable/Total Sales	13.38	0.000	7
28	Gross Trading Profit/Total Sales	10.53	0.001	
29	Profit before Tax/Shareholder Funds	8.97	0.003	
30	Current Assets/Total Assets	7.13	0.008	
31	Current Assets/Total Sales	4.83	0.028	
32	Financial Expenses/Total Sales	4.04	0.045	
33	(Current Assets-Inventory)/Total Assets	3.47	0.063	
34	Change in Other Debts	2.20	0.139	
35	Total Sales/Shareholder Funds	2.16	0.142	
36	Labour Expenses/Total Sales	0.62	0.431	
37	Net Income/Shareholder Funds	0.20	0.651	
38	Financial Debt/Cash Flow	0.18	0.669	
39	Long Term Debt/Shareholder Funds	0.17	0.681	
40	EBITDA/Permanent Assets	0.11	0.743	
41	Total Sales/Total Assets	0.02	0.878	

[1] Rank of the variables in Table 5

Table 6: Selected variables using a Likelihood criterion

(a)

Search: Backward Stepwise	
Variables included into the model	Freq.
Shareholder Funds/Total Assets	94.0%
Profit before Tax/Shareholder Funds	89.3%
Change in Equity Position	87.6%
(Cash + Mark. Sec.)/Total Assets	86.1%
(Cash + Mark. Sec.)/Total Sales	81.5%
EBITDA/Total Assets	73.9%
Cash/Total Sales	70.2%

(b)

Search: Forward Stepwise	
Variables included into the model	Freq.
Change in Equity Position	83.6%
Shareholder Funds/Total Assets	81.2%
Cash/Total Debt	77.3%
EBITDA/Total Assets	72.1%
EBIT/Total Sales	70.8%

Table 7: Selected variables using neural network criteria

(a)

Error	
Variables included into the model	Freq.
Shareholder Funds/Tot. Assets	91.8%
EBIT/Total Assets	86.2%
Cash/Current Liabilities	83.1%
Change in Equity Position	81.8%
EBITDA/Total Assets	76.6%
EBIT/Total Sales	76.1%
(Cash + Mark. Sec.)/Tot. Assets	74.9%
Accounts Receivable/Tot. Sales	70.6%

(b)

0 Order	
Variables included into the model	Freq.
Net Income/Total Assets	86.7%
(Cash + Mark. Sec.)/Tot. Assets	84.3%
Shareholder Funds/Tot. Assets	83.8%
EBITDA/Total Assets	80.9%
Cash/Current Liabilities	78.2%
Total Debt/Total Assets	74.5%
Change in Equity Position	73.9%
Cash/Total Sales	71.5%

(c)

1^{st} Order	
Variables included into the model	Freq.
Total Debt/Shareholder Funds	91.2%
Current Assets/Current Liabilities	84.7%
Change in Equity Position	81.9%
EBIT/Total Assets	77.1%
EBITDA/Total Assets	76.8%
Operating Cash Flow/Total Sales	70.5%

Table 8: Rank of the variables

Variables	Number of selections	Rank of appearance in the six models					
EBITDA/Total Assets	6	4	4	5	5	6	6
Shareholder Funds /Total Assets	5	1	1	2	3	7	
Change in Equity Position	5	1	3	3	4	7	
(Cash + Mark. Sec.)/Total Assets	4	2	4	4	7		
EBIT/Total Assets	3	2	4	5			
Total Debt/Shareholder Funds	2	1	2				
Cash/Total Debt	2	3	3				
Cash/Current Liabilities	2	3	5				
EBIT/Total Sales	2	5	6				
Cash/Total Sales	2	7	8				
Net Income/Total Assets	1	1					
Cash/Total Assets	1	1					
Current Assets/Current Liabilities	1	2					
Profit before Tax/Shareholder Funds	1	2					
(Cash + Mark. Sec.)/Total Sales	1	5					
Operating Cash Flow/Total Sales	1	6					
Total Liabilities/Total Assets	1	6					
Accounts Receivable/Total Sales	1	8					

Table 9: Rank of the variables selected with a neural network

Rank	Variables	Number of selections
1	EBITDA/Total Assets	3
1	Change in Equity Position	3
3	Shareholder Funds/Total Assets	2
3	(Cash + Mark. Sec.)/Total Assets	2
3	EBIT/Total Assets	2
3	Cash/Current Liabilities	2
7	Current Assets/Current Liabilities	1
7	Accounts Receivable/Total Sales	1
7	Operating Cash Flow/Total Sales	1
7	EBIT/Total Sales	1
7	Net Income/Total Assets	1
7	Cash/Total Sales	1
7	Total Debt/Shareholder Funds	1
7	Total Debt/Total Assets	1

Table 11: Model accuracy for "modelling technique/variable selection method" pairs calculated on validation samples

	DA Wilks-S	LR Lik.-S	LR Lik.-FS	NN Error-B	NN 0 order-B	NN 1^{st} order-B
NB	91.20%	93.60%	89.56%	92.78%	91.96%	92.82%
B	83.20%	90.42%	88.84%	95.28%	95.22%	92.82%
Total	87.20%	92.01%	89.20%	94.03%	93.59%	92.82%

DA: Discriminant analysis, LR: Logistic regression, NN: Neural network
Lik.: Likelihood, B: Backward, F: Forward, S: Stepwise
NB: Non-bankrupt, B: Bankrupt

Table 12: Model accuracy for "modelling technique/variable selection method" pairs calculated on test samples

	DA Wilks-S	LR Lik.-S	LR Lik.-FS	NN Error-B	NN 0 order-B	NN 1^{st} order-B
NB	89.62%	91.15%	88.85%	93.08%	92.69%	91.15%
B	80.77%	90.38%	88.46%	94.62%	91.92%	88.85%
Total	85.19%	90.77%	88.65%	93.85%	92.31%	90.00%

DA: Discriminant analysis, LR: Logistic regression, NN: Neural network
Lik.: Likelihood, B: Backward, F: Forward, S: Stepwise
NB: Non-bankrupt, B: Bankrupt

Table 13: Model accuracy according to modelling techniques and two variable selection criteria (Wilks's lambda-Likelihood) calculated on validation samples

	Wilks's lambda Stepwise			Likelihood Backward stepwise			Likelihood Forward stepwise		
	DA	LR	NN	DA	LR	NN	DA	LR	NN
NB	91.20%	88.06%	90.02%	87.28%	93.60%	89.68%	87.98%	89.56%	88.08%
B	83.20%	79.18%	77.20%	84.84%	90.42%	92.74%	82.42%	88.84%	91.14%
Total	87.20%	83.62%	83.61%	86.06%	92.01%	91.21%	85.20%	89.20%	89.61%

DA: Discriminant analysis, LR: Logistic regression, NN: Neural network
NB: Non-bankrupt, B: Bankrupt

Table 14: Model accuracy according to modelling techniques and three variable selection criteria (Error, 0 and 1st-order) calculated on validation samples

	Error Backward			0 Order Backward			1^{st} Order Backward		
	DA	LR	NN	DA	LR	NN	DA	LR	NN
NB	83.38%	90.44%	92.78%	83.20%	86.38%	91.96%	87.06%	88.16%	92.82%
B	85.38%	89.56%	95.28%	84.00%	89.64%	95.22%	81.84%	90.22%	92.82%
Total	84.28%	90.00%	94.03%	83.60%	88.01%	93.59%	84.45%	89.19%	92.82%

DA: Discriminant analysis, LR: Logistic regression, NN: Neural network
NB: Non-bankrupt, B: Bankrupt

Table 15: Characteristics of the groups according to their representation on the maps

Criteria	Number of neurons per group		Number of variables	Quantization error		Standard deviation of the error		t
	NB	B		NB	B	NB	B	
Wilks's lambda	50	49	7	0.37	0.43	0.20	0.26	0.002
Likelihood - Backward	56	43	7	0.47	0.52	0.17	0.23	0.004
Likelihood - Forward	51	46	5	0.40	0.43	0.18	0.22	0.164
Error	56	40	8	0.77	0.79	0.26	0.30	0.554
0 order	51	45	8	0.51	0.56	0.21	0.27	0.019
1^{st} order	50	49	6	0.48	0.52	0.20	0.23	0.056

t: p-value of a Student t test for differences between the quantization error of the two groups

Table 16: Structure of the maps and model accuracy

Criteria	Number of neurons per group		Difference	Correct classification rate (validation sample)
	NB	B		
Error	56	40	16	94.03%
0 order	51	45	6	93.59%
1st order	50	49	1	92.82%
Likelihood - Backward	56	43	13	92.01%
Likelihood - Forward	51	46	5	89.20%
Wilks's Lambda	50	49	1	87.20%

Table 17: Structure of the maps and model accuracy according to groups

Criteria	Number of neurons per group		Difference	Correct classification rate (validation sample)	
	NB	B		NB	B
Error	56	40	16	92.78%	95.29%
Likelihood - Backward	56	43	13	93.60%	90.42%
0 order	51	45	6	91.96%	95.21%
Likelihood - Forward	51	46	5	89.56%	88.85%
1st order	50	49	1	92.81%	92.83%
Wilks's lambda	50	49	1	91.20%	83.19%