Bidding hydropower generation: Integrating short- and long-term scheduling

Stein-Erik Fleten and Daniel Haugstvedt and Jens Arne Steinsbø and Michael Belsnes and Franziska Fleischmann

Norwegian University of Science and Technology, SINTEF Energy, University of Duisburg-Essen

2011

Online at http://mpra.ub.uni-muenchen.de/44450/
MPRA Paper No. 44450, posted 18. February 2013 14:02 UTC
Bidding hydropower generation: Integrating short- and long-term scheduling

Abstract - Bidding of flexible reservoir hydropower in day-ahead (spot) auctions needs to be done under uncertainty of electricity prices and inflow to reservoirs. The presence of reservoirs also means that the short-term problem of determining bids for the next 12–36 hours is a part of a long-term problem in which the question is whether to release water now or store it for the future. This multi-scale challenge is usually addressed by using several models for hydropower planning, at least one long-term model and one short-term model. We present a multistage stochastic mixed integer programming model that has a fine time resolution on near term, and a coarser resolution going forward. It handles price as a stochastic parameter and assumes deterministic inflow as it is intended for use in the winter season.

Keywords - Hydropower scheduling, bidding strategies, stochastic programming.

1 Introduction

For hydropower producers with reservoirs, an important issue in the daily planning is that of choosing whether to generate power tomorrow or at some future time. This challenge is manifested in restructured electricity markets in the form of the bidding problem, in which the producers need to decide their willingness to produce for each of the next 12–36 hours, or half-hours in some markets. For price-taking producers, the optimal bid is marginal cost, however, for reservoir hydropower, this marginal cost is characterized by the interaction of production relationships such as startup costs, a possibly non-linear and non-convex production function, flow- and time dependent operating constraints (this also holds for thermal generation), a possibly complex topology of reservoirs, rivers and power stations, and the value of production tomorrow versus more distant production. For this reason the bidding problem is tightly integrated with short-term scheduling. In this paper we limit the bidding problem to the reservoir winter season of a price-taking producer, and inflow is assumed low enough to be assumed deterministic. The sensitivity analysis in Section 3.3 justifies this assumption. This leaves price as the only stochastic variable in the model.

In the Nordic power system the total reservoir capacity is 217 TWh, and the average-year hydropower generation is 213 TWh, i.e. around 52 % of total energy generation. We use the Leiråda power plant for the case study, because the topology is relatively simple. It consists of one reservoir, Tunsbergsdalsvatnet, and one power station, which has a single generator and turbine. Simplicity of the power plant allows us to focus on the tradeoffs involved in the timing of water releases and disregard the physical complexity often encountered in practice. The framework we use, stochastic multistage mixed integer programming, is able to deal with more complex systems, so the model should be extensible.

Our work can be regarded as an extension of [4]. The most important novelties include the modeling horizon, and the treatment of water values. As in [4] the model presented in this paper uses binary variables to describe the state of the generator in terms of whether it is running or not. This allows for modeling startup- and shutdown costs, minimum production level and non-convex efficiency curves. Our modeling horizon spans from the day ahead to the end of the drawdown season, up to six months ahead. Making the water values, that is the opportunity costs of the water in €/MWh, endogenous is our main scientific contribution and makes the model less dependent of input in the form of water values from long-term models, which is an important practical advantage. In practical planning ([7]), there might even be intermediate-term models that link water values in one-reservoir equivalents in a long-term model to multiple-reservoir water values that serve as input to short-term planning models such as ([8, 19]). Since the long-term model is not updated every day information gets lost. In practice, the information from the long-term model is often simplified in order to be interpretable, e.g. in the form of a single number for the marginal water value, as opposed to a function of reservoir level and other state variables.

Multiple reservoirs in long-term hydro planning can also be handled directly as shown in the seminal paper by [16], however, their approach assumes randomness in the right-hand side only, and cannot directly be used when there is exogenous price uncertainty. A paper that is similar in spirit to ours is [17], who use dynamic programming to study the multi-scale aspect of the reservoir hydropower bidding problem under uncertainty of spot prices and inflow. The approach in [18] is also relevant, but may be more suitable for small reservoirs. For a two-hour-ahead market with piecewise constant bids, [3] introduce a stochastic programming model supporting the bidding process. The day-ahead aspect of our model, presented in Section 2, is based on [4]. Recently, [11] combines a stochastic programming approach a la [4] for the day-ahead problem, with a Markov decision process approach for approximating the value of water in storage for future generation [13].
Optimization models supporting the bidding process for price-taking electricity producers are also given in [1, 2, 12, 14]. Reviews are given by [10, 20].

A mathematical model is presented in Section 2 before the result from the case study is presented. The paper concludes in Section 4.

## 2 The model

### Definitions

Each **stage** is a day or one week in the model. The first seven stages are days followed by twenty-one weeks, thus the model consists of twenty-eight stages.

**Price points** are sixty-four given prices for each hour and the producer/the model has to bid a generation volume for each of the sixty-four prices at the first stage without knowing the clearing price.

A **price segment** is a set of hours with similar price level, irrespective of the chronological order of the hours.

Our aim is to support the bidding process of hydropower generation while integrating the long-term aspects, taking into account that information is likely to be lost in the process of transferring information about water values from long-term models into the short-term bidding problem. We implement this by extending the planning horizon from long-term models into the short-term bidding problem. Because the level of detail needed to achieve realistic bid curves in the first stage of the formulation is unnecessary in later stages, there is a detailed formulation applied for the first 12–36 hours, and a more rough and flexible multistage formulation designed for variable stage length used for the rest of the horizon. We denote these the day-ahead part and long-term parts of the model.

The details of the bidding process in the Nordic electricity market are explained in [4] and [5]. The long-term part of the model, as opposed to the day-ahead part, does not operate with hours as an index, but rather price segments. I.e., time chronology is not respected within each stage, and the reservoir balance is only enforced at the end of each stage. Furthermore, bidding and market clearing process at Elspot is abstracted from; we assume that the prices within the stage are known, that is we have pre-fixed price points, and the producer needs only to decide upon the generation level. Startup- and shutdown cost aspects are not as important on the long-term time scale, and are neglected since each stage is rather long compared to the effect of single startups or shutdowns. Finally, turbine efficiencies are characterized in less detail as compared to the day-ahead part.

Table 1 lists the definition of the sets and their corresponding indexes, and Table 2 lists the decision variables determined in the model.

### Table 1: The sets and indexes in the optimization model

<table>
<thead>
<tr>
<th>Set</th>
<th>Index</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H} )</td>
<td>( n )</td>
<td>Sets of day-ahead hours.</td>
</tr>
<tr>
<td>( \mathcal{I} )</td>
<td>( i )</td>
<td>Pre-fixed price points.</td>
</tr>
<tr>
<td>( \mathcal{S} )</td>
<td>( s )</td>
<td>Price segments in each stage.</td>
</tr>
<tr>
<td>( \mathcal{N} )</td>
<td>( n )</td>
<td>Nodes of scenario tree.</td>
</tr>
<tr>
<td>( \mathcal{T} )</td>
<td>( t )</td>
<td>Stages.</td>
</tr>
<tr>
<td>( \mathcal{N}_t )</td>
<td>( n )</td>
<td>Nodes for stage ( t ).</td>
</tr>
</tbody>
</table>

### Table 2: Decision variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u^n_h )</td>
<td>1 if the generator is running, 0 else.</td>
</tr>
<tr>
<td>( v^n_h )</td>
<td>1 if there is a startup or a shutdown in hour ( h ), 0 else.</td>
</tr>
<tr>
<td>( w^{(b.p.)}_n )</td>
<td>The amount generated at best point production level in price segment ( s ) in node ( n ).</td>
</tr>
<tr>
<td>( w^{(max)}_n )</td>
<td>The amount generated at maximum production level in price segment ( s ) in node ( n ).</td>
</tr>
<tr>
<td>( x^n_{i,h} )</td>
<td>Hourly bid at price point ( i ) in hour ( h ).</td>
</tr>
<tr>
<td>( r^n_h )</td>
<td>Reservoir level in hour ( h ) of node ( n ).</td>
</tr>
<tr>
<td>( q^n_h )</td>
<td>Flow rate of water for hour ( h ) of node ( n ).</td>
</tr>
<tr>
<td>( r^n_h )</td>
<td>Spill over the dam during hour ( h ) of node ( n ).</td>
</tr>
</tbody>
</table>

The following constraints are enforced:

\[
\begin{align*}
\dot{Q} x_{i=\mathcal{I},1} & \leq L^{start}, \quad \text{with } \dot{Q} = \frac{W^{b.p.}}{W^{max}} + \frac{W^{max}}{W^{b.p.}} \\
\dot{Q} x_{i=\mathcal{I},h} & \leq \dot{Q} x_{i=\mathcal{I},h}, \quad n \in \mathcal{N}_t | t = 1, h \in \mathcal{H}, i \in \{2, \ldots, \mathcal{I}\} | P_{i-1} \leq \rho^n_h < P_i \\
\dot{Q} x_{i=\mathcal{I},h} & \leq \dot{Q} x_{i=\mathcal{I},h}, \quad n \in \mathcal{N}_t | t = 1, h \in \mathcal{H}, j \in \{1, 2, 3\} \\
W^{min} u^n_h & \leq W^{max} u^n_h, \quad n \in \mathcal{N}_t | t = 1, h \in \mathcal{H} \quad (7)
\end{align*}
\]

1. Using forward prices does not mean that Leirdøla will get average prices over the delivery periods of the forwards; our price scenarios have hourly resolution and the power plant can and will produce more at higher prices, and less at lower prices.

2. If the model is used for cascaded reservoirs, this way of modelling time will overestimate revenues.
\[ v \geq g(u, u') \quad \text{in} \quad t = 1 \quad (8) \]
\[ l = l' - q - r + F \quad (9) \]
\[ L_{\text{min}} \leq l \leq L_{\text{max}} \quad (10) \]

\[
q^n = \sum_{s \in S} K^t w^{(b.p.)}_s + \sum_{s \in S} K^t w^{(max)}_s, \quad n \in N_t | t \geq T_{t,t},
\]

\[
w^{(b.p.)}_s \leq W^{b.p.}, \quad w^{(max)}_s \leq W^{max}, \quad n \in N_t | t \geq T_{t,t}, s \in S \quad (11)\]

\[
\sum_{s \in S} K^{t} w^{(b.p.)}_s n + \sum_{s \in S} K^{t} w^{(max)}_s n, \quad n \in N_t | t \geq T_{t,t}, s \in S
\]

For hourly bids, (1) links bid volume to committed production during the next day. Price point \( n \) has price level \( P_t \) and \( \rho^n_h \) is the market clearing price in hour \( h \) at node \( n \). \( T_{t.t} = t \) for the first stage at which the long term relationships apply. In the case study we set \( T_{t.t} = 2 \) which means that the day-ahead part of the model uses a two-stage structure. For a more detailed description of these constraints, see [4]. Constraint (2) ensures an increasing bid curve. The first and the last price point is usually set to zero and a very large number, respectively. Thus it is necessary to assure that no volume is bid at the first price point and to restrict the volume bid at the last price point to the maximum available capacity, which is done in (3). Furthermore the constraints (3) to (5) together with the constraints (2) ensure that none of the \( x_{i,h} \) -variables exceeds the available capacity. \( Q \) is a necessary factor in order to distinguish between water used for production and the output generation.

The relationship between water flow rate and generated power is modeled in (6) and (11) for the day-ahead and the long-term part, respectively. In the day-ahead part, we use three supporting cuts of the efficiency curve of the generator, described by the functions \( f_j \), while two sets of variables, one for best point generation and one for maximum generation level, are used in the long-term formulation. The efficiency coefficient is \( W \), and \( K^t \) is the number of hours in each price segment at stage \( t \). Generation bounds (7) and (12) restrict the production in the day-ahead and the long-term part, respectively. Start-up and shutdown variables are linked to unit states \( u \) through (8). The function \( g(u, u') \) is one whenever there is a difference between \( u \) and \( u' \), where \( u' \) represents the status of the generator in the preceding hour or the last hour in the preceding stage. Remember, these constraints hold only in the short-term part of the model (\( t = 1 \)). Reservoir balance is ensured via (9) throughout the horizon. There are special cases of transitions, both between stages and the two parts of the model, however, in general, \( t \) is the reservoir level of a given node in a given hour in the short-term part and the reservoir level in node \( n \) in the simplified long-term model, respectively. \( t' \) is the reservoir level of the preceding hour/node or the last hour/node of the preceding stage. The flow rate of water, spillage and inflow is denoted \( q, r \) and \( F \), respectively. Reservoir bounds are enforced in (10). In addition, there are nonnegativity constraints on all decision variables and \( u \) is binary.

The objective function of the problem is

\[
\max (z_{da} + z_{lt}) \quad (13)
\]
\[
z_{da} = \sum_{n \in N_t | t \geq T_{t.t} \pi^n} \sum_{h \in H} (d^n_h w^n_h - C w^n_h) \quad (14)
\]
\[
z_{lt} = \beta \sum_{n \in N_t | t \geq T_{t.t} \pi^n} \sum_{s \in S} \pi^n \rho^n_t p^n_s (w^{(b.p.)}_s + w^{(max)}_s) \quad (15)
\]

where (14) and (15) are the expected revenues less startup- and shutdown costs from the day-ahead formulation and the discounted (and adjusted by \( \beta^3 \)) revenues from the long-term formulation, respectively. The unconditional probability of being in node \( n \) at the corresponding stage \( t \) is \( \pi^n \). \( C \) is the cost related to startup or shutdown of the generator. The discount interest rate is denoted \( \nu \), and \( \tau^t \) is the time until stage \( t \). In this paper there is no end-of-horizon water value \( \lambda \) because the end of the model horizon is spring and thus there is a lot of inflow at this time. That is why left water in the reservoir at the end of winter would have no value since there is enough inflow.

### 3 Case analysis

This section investigates how the model performs on Leirdøla power station. The model is tested on a base scenario set consisting of 1000 scenarios with prices from 21.11.2006 to 29.04.2007, shown in Figure 1.

Figure 1: A scenario tree with 1000 (fan) scenarios, used as input before scenario reduction. Note the price reduction expected during Christmas and Easter.

First the bid curves and the generation for 21.11.2006 are presented. In subsection 3.2 we will start with a general test concerning the size of the model. The main focus of the case analysis is to justify the assumption of a deterministic inflow and to illustrate the stability of the model. This will be done in the subsections 3.3 and 3.4.

The main focus of this paper is to generate bid curves as this is the only output that depends on the expectation on future prices rather than the actual realizations. The quality of the model is therefore defined as the quality of the bid curves generated. The stability is in the same manner defined as whether the model generates the same bid

\(^3\beta \) is used to compensate the overestimated revenues in the long-term part of the model, thus it is set to \( \beta = 0.9 \).
curves for the same underlying forward curve and assumptions and finally the sensitivity of the model is defined as to which degree changes in the assumptions alter the bid curves.

3.1 Presenting bid curves and generation

In order to present the bid curves and the generation simply, we use two charts in this section. The first one (Figure 2) presents the optimal bidding during the day. The different hours are distinguished by their color. Best point production, that is most efficient water usage for generation, is achieved at approximately 85 MW. Maximum production on best point is at about 93 MW and 115 MW is the maximum generation capacity. The second chart (Figure 3) illustrates the expected clearing price and the expected generation for each hour of the day.

![Figure 2: Bidding curves for 21.11.2006](image1)

We aggregated hours with a similar bidding structure to stabilize the model and, as a secondary effect, to reduce the model size. Due to this it is also easier to show (Figure 2) nearly all bidding curves of the day. At first sight it seems to be surprising that there is no bidding during the hours 1 till 7, 23 and 24. The reason for that is the low expected price during these hours and the low reservoir level during the whole observation period. At the beginning the reservoir level is one third of its maximum level and, since it is winter, there is just a small amount of inflow. There is no production in the optimal solution in these hours and so there is no reason for bidding in these hours.

![Figure 3: Expected price and expected generation for 21.11.2006](image2)

As one can see in Figure 3 there is one production peak in the morning and two peaks in the early evening. This is an expected result, since people need more electricity in the morning and when they come home from work, thus the expected prices are higher, making it profitable to produce during these hours.

3.2 General test

In this test, the model is run with a set of 1000 price scenarios. The aim is to check the size of the model and to test the influence on the model and especially the objective value when block bids are allowed. Block bids are special bids that concern a connected set of from 2 to 24 hours. They are convenient to avoid risk of repeated/extra starts of generators. For further information on block bids and their modelling see [4]. The model will be run both with and without block bids. When block bids are allowed, first a very small set of five blocks are included, four consisting of six consecutive hours and one block representing the entire 24 hour period. In a second test 20 additional blocks are included in the model, which is already a quite high number, thus it will give reliable results.

<table>
<thead>
<tr>
<th>No block bids</th>
<th>25 block bids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>69 128</td>
</tr>
<tr>
<td>Decision variables</td>
<td>1 080 018</td>
</tr>
<tr>
<td>Binary variables</td>
<td>2 496</td>
</tr>
<tr>
<td>Nonzero elements</td>
<td>2 183 121</td>
</tr>
<tr>
<td>Simplex iterations</td>
<td>40 277</td>
</tr>
<tr>
<td>Objective value</td>
<td>5.02431 · 10⁶</td>
</tr>
<tr>
<td>Optimality gap total</td>
<td>€ 90</td>
</tr>
<tr>
<td>Optimality gap %</td>
<td>≈ 0.00001791</td>
</tr>
<tr>
<td>Solution time</td>
<td>30.9 sec</td>
</tr>
</tbody>
</table>

As one can see from Table 3, the performance in terms of the objective function is not very dependent on whether or not block bids are allowed. The difference is even less when only five block bids are included, that is why this case is not demonstrated in the table.

The difference between the objective values in the case where block bids are allowed and where they are not is the value of allowing block bids. We assert that the bound from the linear relaxation of the problem and the objective value is higher when block bids are allowed. This should be expected as the model with allowing block bids is a relaxation of the one disallowing ones. In case of including 25 block bids the upper bound is € 90 higher and the objective value is € 170 higher than in the model without block bids. As shown in Table 3, the optimality gap is small enough for both models to get an acceptable solution. Also worth mentioning is the fact that the computation time is low even with handling 25 block bids.

The conclusion of this section is therefore that the model seems to be able to handle block bids and this in an acceptable time, but the value of allowing such bids is small. The analysis will thus be proceeded without allowing block bids to be able to compare the following results more easily.

3.3 Sensitivity towards inflow

The use of deterministic inflow in the Leirdola case is motivated and justified by the fact that the model is made...

---

Footnote: All hours in an aggregated hour set have the same bidding in each price point. Still one has to keep in mind that the price points of the hours within an hour set differ slightly.
for the winter season, in which the inflow is very low. We test how sensitive the result is to big changes in the inflow. If even large changes do not alter the solution, we argue that a stochastic representation of the inflow is not necessary. In this section, the model will be run for a 25% upward and downward shift in the inflow. Because the inflow during the modeling horizon only constitutes about 31% of the total amount of water (reservoir level at the start of the horizon and total inflow), the change in total amount of water from a 25% change in inflow is only ±7.8%. One should therefore not expect the impact from a change in the inflow to be very large. The Figures 4, 5 and the Figures 6, 7 show the results from the sensitivity analysis on the inflow.

As one can see from the figures 2, 4 and 6, the differences in the bid curves are as expected not very large. We expect that bid curves are more sensitive to inflow towards the end of the drawdown season. Modelling inflow as stochastic is particularly valuable when reservoirs are nearly full or nearly empty [6]. However, in our case, it seems to be justified to use deterministic inflow. In 19 hours of the day our model, Figure 2 and the test model with 25% more inflow have exactly the same bidding curves and the other 5 hours hardly differ as well. If one compares our model with the lower inflow test model, the results become a bit worse, but are still satisfactory. In 11 hours the two models have exactly the same bidding curves and the other hours differ slightly.

The expected generation in case of high inflow is also nearly the same as in our case, see Figures 3 and 5. A comparison between Figures 3 and 7 shows a bigger difference between the generation in case of 75% inflow and in our case. This indicates that even small changes in the bid curves alter the expected generation significantly.

The difference in the objective value is -6.68% and +6.78% in case of low and high inflow, respectively. These are small numbers if one keeps the inflow change of 25% in mind.

Summing up the sensitivity analysis justifies to a large extent the use of deterministic inflow in the winter season. Although the expected generation differs in some hours in case of low inflow, the main results are satisfactory because the bidding curves are the most important indicators.

We also did a sensitivity analysis towards the start-up cost with a 50% upward and downward shift but the results were nearly the same as with the initial start-up costs, thus this is not further mentionable. The reason for this result is the in general low start-up cost for hydro power stations. For further information on the impact of start-up costs see [15].
3.4 Test of stability

Stability tests for scenario trees used in stochastic programming are discussed in [9]. It is distinguished between in sample and out of sample stability and we will test the in sample stability in respect to the bidding curves. The in sample stability is a measure of the stability of the solutions when different scenario trees with the same underlying distribution are used in the model. The same underlying distribution should give fairly the same results. In addition, we will check the stability of the objective value. The aim of the in sample stability test is to figure out which is the smallest number of scenarios that still gives stable results, that is, how many scenarios can be removed from the initial 1000 ones. Figures 8 to 11 show the bid curves for four smaller sets of scenarios. As one can see from the four figures, the stability in the cases of 950, 900 and 800 scenarios are satisfying. A comparison between Figure 2 and Figure 8 shows that 12 hours of the day have exactly the same bidding curves and the other 12 ones are quite similar, which is acceptable. In case of 900 scenarios, Figure 9, there are still 9 hours in which the bidding is exactly the same as in the case of 1000 scenarios and the other 15 hours are also still satisfactory. With 800 scenarios there are 13 hours in which the bidding matches the one with 1000 scenarios and the other 11 hours are similar as well. In case of 750 or less scenarios the results get worse and the bidding curves differ more from the bidding curves in Figure 2. There are still 9 hours in which the bidding curves coincide with the ones in the initial case, but the other hours differ too much. Thus, it is possible to reduce the number of scenarios by 20% of the initial scenario quantity and still get stable bidding curves.

In a second test we compare the initial objective value (1000 scenarios) with the objective value of 18 smaller sets consisting of 100 to 950 scenarios. Figure 12 shows the increasing stability of the objective value in cases of 100 to 1000 scenarios. At the two peaks of the graph, with 200 and 500 scenarios, the change in the objective values amounts only $-2.61\%$ and $+1.73\%$ to the initial objective value (1000 scenarios), respectively.

In conclusion we assert that the stability of the results is satisfying in both tests. Especially in case of the objective value the model seems stable.

4 Conclusion

An advantage of the suggested model is that it is much less dependent on other models with longer time horizon. For the timespan beyond six months into the future, i.e. beyond the horizon of our model, information from long-term models or possibly from the forward market can be used. The result is a model that is satisfactorily detailed for short-term bidding and scheduling and yet solvable for
a rather long time horizon. On the other hand, the model is limited to use in the winter season, and when looking at rivers with cascaded reservoirs, one should be careful about possible overestimation arising when using non-chronological time.

The overall result from the case analysis is that the assumption of deterministic inflow in winter is justified and that the model exhibits a good stability in case of the bidding curves and the objective value. Additionally the model can handle block bids in a short computation time but the benefit of allowing those is slight.

Acknowledgment

The authors would like to thank Lars Holmefjord at Statkraft Energi AS for valuable support. We recognize the Norwegian research centre CenSES, Centre for Sustainable Energy Studies, and acknowledge financial support from the Research Council of Norway through project 190999.

REFERENCES


