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Abstract

This paper modifies the standard Hodrick Prescot Filter in order to reduce the problem of misleading predictive outcome when used with updated information. The modification allow for a more accurate estimation of output gap, as well as the introduction of confidence intervals that permit a better understanding of the uncertainty related to the estimation of the filter. Also improve the efficiency using a correction for autocorrelation in the errors of estimation.

Key Words: Economic cycles, Low-pass filter.

JEL Classification: E32, C22, C52

Resumen

El presente documento modifica el filtro estándar de Hodrick Prescott, con el fin de reducir el problema de los datos extremos en el filtro al ser usado dinámicamente. La modificación permite una estimación más precisa de la brecha de producto, y al a vez la introducción de intervalos de confianza permite mejorar el entendimiento de la incertidumbre presente en la estimación de estos filtros.

Palabras clave: Ciclos Económicos, Filtros.

Clasificación JEL: E32, C22, C52

Managing the Uncertainty in the Hodrick Prescott Filter

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1 Introduction

The filter proposed by Hodrick and Prescott, the so-called HP filter, has been very useful in economic times series analysis. The main idea is to decompose a time series into its high and low frequency components.

The HP filter is the most popular filter for extracting the trend and cycle components from an observed time series. Many researchers consider the smoothing parameter $\lambda = 1600$ as something like an universal constant. It is well known that the HP filter is an optimal filter under some restrictive assumptions, especially that the 'cycle' is white noise.

There are in the literature some theoretical articles and many applications for such filter; being potential GDP estimation the most widely discussed application. Actually, this application involves the largest amount of empirical works using HP.

The filter was first applied in economics by Robert J. Hodrick and Edward C. Prescott. Though Whittaker (1923) was the first to propose the method.

Many methods are available for accomplishing a decomposition of the series into the trend and the cycle. But much of the business cycle literature has applied the Hodrick Prescott Filter, because of this the Hodrick Prescott Filter method is the focus in this paper

Many empirical studies have applied the Hodrick-Prescott Filter in cross-country comparisons of business cycle fluctuations.

The main contribution of this paper is the development of an alternative to the Hodrick Prescott filter, this alternative has the advantage of symmetric non-negative weights for the estimation of the trend. This modified HP filter allows to reduce the uncertainty related with end of periods estimation. Also a correction for autocorrelation in the errors is able to reduce significantly the errors of estimated output gap for end period observations. Another advantage is related to the fact that this modified version is almost identical to the output gap generated by the standard HP filter in the centre of the sample, making comparisons possible to previous work using the HP filter.

The paper is organized as follows. First, Section 2 briefly reviews the theoretical literature related to the Hodrick Prescott Filter, and the literature from the Central Bank of Costa Rica that has used the Hodrick Prescott Filter. In Section 3 a description of the construction of the Hodrick Prescott filter is done, and a discussion about the uncertainty of the output gap is also presented. Section 4 describes the data and the sample used for the estimation. In Section 5 the empirical framework is presented, with the discussion on the proposed modification of the HP filter. The main results and a forecast accuracy analysis is presented on section 6. Section 7 concludes.

2 Literature Review

2.1 Review of theoretical papers about the HPF

Marcet and Ravn (2003) develop two procedures that allow the comparison of the variability of trends between different countries, starting from the value proposed by Hodrick and Prescott for λ in the case of the U.S. Both methods minimize the sum of squares of the trend deviations from the original series, but differ in the applied restrictions. These approaches try to endogenously obtain a value for λ that is consistent with the imposed restrictions, according to the characteristics of each country.

2.2 Use of Hodrick Prescott Filter in Costa Rica

Esquivel and Rojas (2007) where production data from 1991 to 2006 is used to estimate the most appropriate values of λ for Costa Rica, following a methodology proposed by Marcet and Ravn (2003).

Segura and Vasquez (2011) amplifying the information used in the previous work by Esquivel and Rojas (2007), analysing an alternative methodology proposed by Marcet and Ravn (2003) and comparing its results with the formerly used methodology.

3 Theoretical Framework

3.1 Decomposition of Time Series

Typical time series with monthly and quarterly frequency can be decompose into a trend component, a cyclical component, seasonality and an irregular component. As specified in equation(1).

$$y_t = T_t + C_t + S_t + I_t \tag{1}$$

The trend T_t represent the long run movement of the series y_t. While the cyclical component C_t captures the sequence of a non-periodic fluctuations, referred in the literature as economic cycles, also known as transitory deviations. Some series also present a seasonal component which repeat itself every year. Finally the I_t describes random, irregular influences, also called 'noise'.

This components are not observed, therefore any decomposition must be built on a conceptual artefact.

3.2 Hodrick Prescott Filter

The Hodrick Prescott Filter extracts the trend ,T_t, by minimizing the following loss function:

$$min_{\tau_t} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \tag{2}$$

Where λ is the smoothing parameter that controls the smoothness of the adjusted trend series. [1] The first part of the minimization refers to the goodness of fit, while the second part is the penalty for roughness.

The HPF does is to maximize the fit of the trend to the actual series, while minimizing the changes in the trend's slope with a penalty. Where λ increase the weight of the changes in the trend.

Note that equation(2) could also be written as:

$$min_{\tau_t} \sum_{t=1}^{T} \varsigma^2 + \lambda \sum_{t=3}^{T} (\nabla^2 \tau_t)^2 \tag{3}$$

Where $\nabla = (1 - L)$ is the standard differencing operator and L is the standard lag operator. [2]

3.2.1 The Lambda Parameter

Hodrick and Prescott (1997, p.4) state that: *'If the cyclical components and the second differences of the growth components were identically and independently distributed, normal variables with means zero and variances σ_1^2 and σ_2^2 (which they are not), the conditional expectation of the τ_t , given the observations, would be the solution to program (2) when $\sqrt{\lambda} = \frac{\sigma_1}{\sigma_2}$', ...'our prior view is that a 5 percent cyclical component is moderately large, as is a one-eight of 1 percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda} = \frac{5}{\frac{1}{8}}$ or $\lambda = 1,600.$ '*

[1] As $\lambda \to 0$ the trend mimic the actual series y_t, while as $\lambda \to \infty$ the trend becomes a linear trend.

[2] i.e., $\nabla^2 \tau_t = (\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})$

3.2.2 The Generalized-Ridge Regression

Consider $y = y_1, y_2, \ldots$ and $y = I\tau + \varsigma$ then a generalized-ridge [3] regression rule would estimate the trend as:

$$\hat{\tau} = [\mathbf{I'I} + \lambda\mathbf{A}]^{-1}\mathbf{I'y} = [\mathbf{I} + \lambda\mathbf{A}]^{-1}\mathbf{y} \tag{4}$$

Where **A** is a symmetric, positive-definite matrix.

The solution of equation(2) could be solved using the Generalized-Ridge regression, as shown by Danthine and Girardin (1989). This solution can be expressed as follows:

$$\hat{\tau}_t = [\mathbf{I} + \lambda\mathbf{K'K}]^{-1}\mathbf{y} \tag{5}$$

where $\mathbf{y} = [y_1, ..., y_T]$, $\tau = [\tau_1, ..., \tau_T]$, **I** is a TxT identity matrix, and $\mathbf{K} = [k_{i,j}]$ is a $(T-2)xT$ matrix. Also note that if $\mathbf{K'K} = \mathbf{A}$ it becomes apparent that equation(5) is a particular case of equation(4).

$$\mathbf{K} = \left\{ \begin{array}{ccccccccc} 1 & -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{array} \right\} \tag{6}$$

Following the example presented by Ley (2006), but with a $T = 7$ the matrix

$$\mathbf{I} + \lambda\mathbf{K'K} = \left\{ \begin{array}{ccccccc} 1+\lambda & -2\lambda & \lambda & 0 & 0 & 0 & 0 \\ -2\lambda & 1+5\lambda & -4\lambda & \lambda & 0 & 0 & 0 \\ \lambda & -4\lambda & 1+6\lambda & -4\lambda & \lambda & 0 & 0 \\ 0 & \lambda & -4\lambda & 1+6\lambda & -4\lambda & \lambda & 0 \\ 0 & 0 & \lambda & -4\lambda & 1+6\lambda & -4\lambda & \lambda \\ 0 & 0 & 0 & \lambda & -4\lambda & 1+5\lambda & -2\lambda \\ 0 & 0 & 0 & 0 & \lambda & -2\lambda & 1+\lambda \end{array} \right\} \tag{7}$$

[3] Ridge Regression is a variant of ordinary Multiple Linear Regression whose goal is to circumvent the problem of predictors collinearity. It gives-up the Least Squares (LS) as a method for estimating the parameters of the model, and focusses instead of the X'X matrix. This matrix will be artificially modified so as to make its determinant appreciably different from 0. By doing so, it makes the new model parameters somewhat biased (whereas the parameters as calculated by the LS method are unbiased estimators of the true parameters). But the variances of these new parameters are smaller than that of the LS parameters and in fact, so much smaller than their Mean Square Errors (MSE) may also be smaller than that of the parameters of the LS model. This is an illustration of the fact that a biased estimator may outperform an unbiased estimator provided its variance is small enough.

When computing $\mathbf{I} + \lambda\mathbf{K'K}$ for $\lambda = 9$ [4]

If we take $\lambda = 9$ then

$$[\mathbf{I} + \lambda\mathbf{K'K}]^{-1} = \left\{ \begin{array}{ccccccc} 0.57203 & 0.35114 & 0.17781 & 0.06056 & -0.01181 & -0.05728 & -0.09247 \\ 0.35114 & 0.30389 & 0.21762 & 0.13067 & 0.05717 & -0.00323 & -0.05728 \\ 0.17781 & 0.21762 & 0.23768 & 0.19404 & 0.12747 & 0.05717 & -0.01181 \\ 0.06056 & 0.13067 & 0.19404 & 0.22943 & 0.19404 & 0.13067 & 0.06056 \\ -0.01181 & 0.05717 & 0.12747 & 0.19404 & 0.23768 & 0.21762 & 0.17781 \\ -0.05728 & -0.00323 & 0.05717 & 0.13067 & 0.21762 & 0.30389 & 0.35114 \\ -0.09247 & -0.05728 & -0.01181 & 0.06056 & 0.17781 & 0.35114 & 0.57203 \end{array} \right\} \tag{8}$$

From equation(8) it is possible to observe some important characteristics of the matrix of weights ($[\mathbf{I} + \lambda\mathbf{K'K}]^{-1}$):

(1) Weights add up to one.

(2) The weights do not depend on the data itself. But they depend on the length of the data that is used.

(3) Negative weights do occur for some periods in the extremes of the data. And also if λ is smaller than T. When the series is increasing this negative weights will bias downwards the trend obtained.

(4) The filter is asymmetric except for the fourth observation, because it has the equal number of observations before and after.

(5) The endpoints have very large weight.

(6) Observations next to the endpoints have larger weights that the themselves.

For $\lambda = 1,600$ and $T = 101$ there are 48 negative weights that sum up to almost 7 percent. If the filtered series follows a deterministic trend or a unit root with a drift the filter will be bias upwards. (downwards) if the trend or drift increases (decreases).

A simple simulation for the matrix $[\mathbf{I} + \lambda\mathbf{K'K}]^{-1}$ with $T = 101$ and $\lambda = 1600$ provides more details about the behaviour of the weights obtained by the Hodrick Prescott Filter. Figure(1) shows graphically the weights for the 101 observations.

A seventh characteristic of the HP estimator is that weights change depending on the length of the sample. (See figure(16) in the Annexes)

If we take a detail look to the negative part of the weights. As shown in figure(2) it is possible to observe that for every estimated trend there are negative weights, even for the symmetric distribution in the middle of the sample.

[4] λ should be larger than the number of observations, otherwise the weights in the symmetric part will be negative for the observations at the extreme.

Figure 1: Weights of the HP for 100 observations

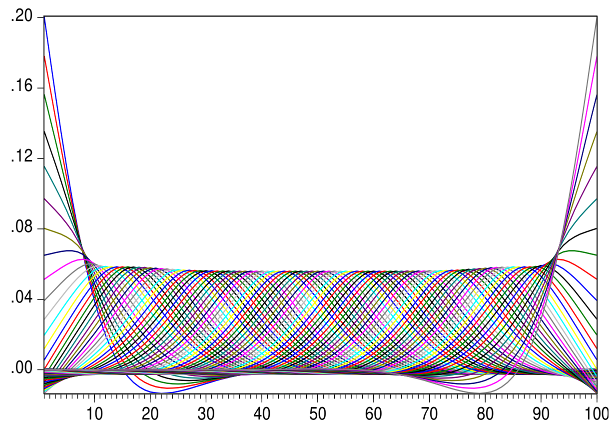
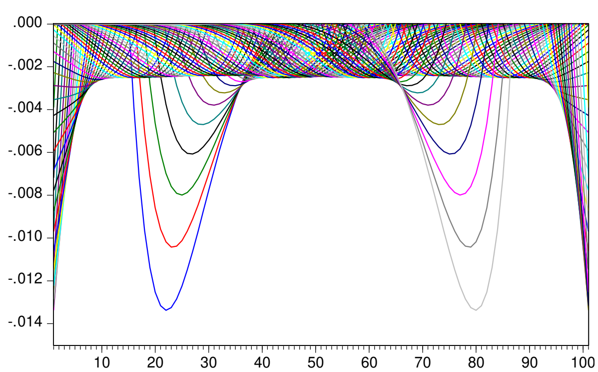


Figure 2: Negative Weights of HP for 100 observations



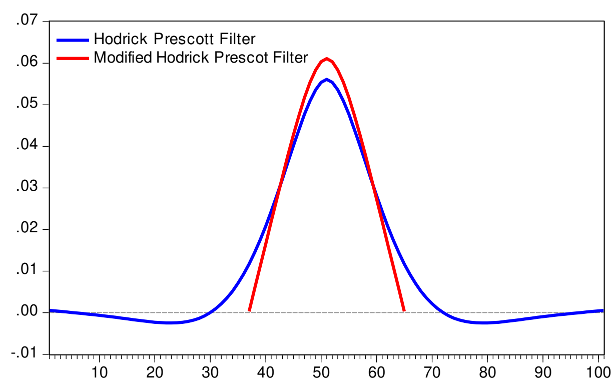
Even if we extract the symmetric part, for $\lambda = 1,600$ and $T = 101$, which is the 51 observation has 48 negative weights that sum up to almost 7 percent. This negative weights would generate a bias in the estimation if the series has a trend. And will put more weight to the central observations, because it sum up to one.

3.3 Proposition

The proposed weights are obtained by diminishing the number of T until all estimated weights are positive. In the particular case of $\lambda = 1,600$, $T = 29$.

This approximation allow to have weights that adds to unity. That do not depend on the length of the data used. By construction there are no negative weights, also the filter become symmetric. These features will generate an unbiased estimator for series with positive trends. This estimator will be also more efficient since it reduces the number of input need to obtain the trend in the data. Regarding the endpoint sample problem, it is possible to backcasts for the early data. And to forecast for the future data.

Figure 3: Proposed HP Filter versus standard HP filter



3.4 HP Filter as an unbiased estimator

The unobserved components (UC) representation is fairly general, as many popular decompositions, including the HP filter, can be formulated within its framework. As noted by Harvey and Jaeger (1993) and King and Rebelo (1993), the HP filter can be interpreted as the optimal estimator in the UC model.

3.5 Uncertainty of the Output Gap

Output gap is general estimated as the logarithm of the observed GDP minus the logarithm of the potential GDP.

$$gap_t = y_t - y_t^{pot} \tag{9}$$

$$E_t[gap_t] = E_t[y_t - y_t^{pot}] \tag{10}$$

$$g\hat{a}p_t - gap_t = \hat{y}_t - y_t - (\hat{y_t^{pot}}) - y_t^{pot} \tag{11}$$

$$g\hat{a}p_t - gap_t = \epsilon_t^{rev} + \epsilon_t^{mea} - \varepsilon_t^{f} - \varepsilon_t^{w} \tag{12}$$

Potential GDP could be estimated using different methods.

When realizing an estimation of output gap using the standard Hodrick Prescott Filter there are four possible sources of uncertainty:

(1) Revision of observed GDP (ϵ_t^{rev})

(2) Measurement errors on the GDP (ϵ_t^{mea}).

(3) Forecast uncertainty (ε_t^{f}).

(4) Variability of the weights (ε_t^{w}).

In this paper I will concentrate on point (4). While for point (1) and (2) it is possible to assume that $\epsilon_t^{rev} \sim N(0, \sigma^{\epsilon_t^{rev}})$ and $\epsilon_t^{mae} \sim N(0, \sigma^{\epsilon_t^{mae}})$

Regarding the point (3), in order not to bias the estimation I will use the same forecast for the forecast accuracy measurements.

4 Data

Quarterly data is used given its relevance to the decision making process of monetary policy by the authorities. Nevertheless this frame work could be applied to higher and lower frequencies. Annual data is usually more difficult to obtain an updated observation, making the estimation of the HP filter more spread out (it could only be estimated once a year). While the monthly data is subject to more revisions, making it less reliable for policy-making decisions.

In order to make the analysis more robust I use data from both Costa Rica and the United States. Also because the optimal λ for both countries are different.

For the United States the data is obtained from FRED on a quarterly frequency. Real Gross Domestic Product. Source: U.S. Department of Commerce: Bureau of Economic Analysis. In billions of Chained 2005 Dollars. Seasonally Adjusted Annual Rate. The sample is from 1947.q1 to 2012.q2.

For Costa Rica the data is obtained from the Costa Rica Central Bank also in quarterly data. Millions of colones of 1991. The sample is from 1980.q1 to 2012.q2.

For the case of the GDP of the U.S. unit root tests suggest that is a I(1) process, while for the GDP of Costa Rica the tests suggest that is a I(2) process.

5 Empirical Framework

Nowadays, it is very common to use the method proposed by Hodrick and Prescott (1980) to split a time series in a trend and a cyclical component. Its use concentrates primarily on the fluctuation analysis of the economic cycles, which were defined by Lucas (1977) as deviations of the real product from a trend.

In this section I estimate the output gap for the United States and Costa Rica (maybe even a pool of countries, at least from Central America) This output gap is measured as the percent difference between the actual GDP and an estimated potential GDP using the constructs of the previous section.

Forecast accuracy of contemporaneous estimation of output gap will use standard Mean Square Error and Mean Absolute Error.

Three ways of estimating the potential output gap: (i) Standard HPF without forecast (ii) Standard HPF with forecast and (iii) Modified HPF with forecast. (iv) Modified HPF with forecast and correction for the autocorrelation in the errors.

In theory the modified HPF should outperform the other two methods (i) and (ii). While the modified HPF with forecast and correction for the autocorrelation in the errors should outperform all of them.

5.1 Forecast Equation for GDP

In order to make a forecast for GDP a simple ARMA equation is estimated using OLS.

$$y_t = \beta_0 + \beta_1 tr_t + \beta_2 AR(1) + \beta_1 MA(1) + \xi_t \tag{13}$$

The result of the estimation are presented figure(4).

5.2 Modified HP Filter

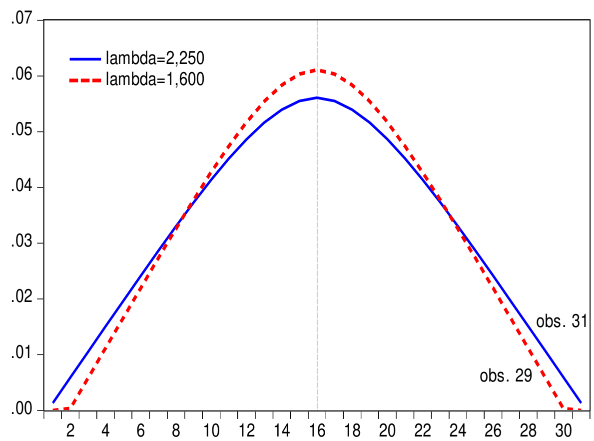
For the case of the U.S. I use the standard $\lambda = 1,600$, which implies a total of 29 observation to construct the symmetric, non negative weights. In the case of Costa Rica, following Segura and Vasquez (2011),

Figure 4: Estimated Equation for GDP Growth

	United States		Costa Rica	
	Coefficient	t-Statistic	Coefficient	t-Statistic
β_0	-4457.99	-1.04	-61535.40	-0.47
β_1	67.33	5.10	4997.15	4.84
β_2	0.99	165.89	0.97	52.03
β_2	0.31	5.21	-0.40	-4.64
Adjusted R-squared	0.999787		0.995014	
F-statistic	404909.2		8449.544	
Akaike info criterion	10.81103		21.15004	
Schwarz criterion	10.86581		21.23917	
Durbin-Watson stat	1.807075		1.728565	
	Sample: 1947Q2 2012Q1		Sample: 1980Q2 2012Q1	

the $\lambda = 2,250$ that length the observations to 31, to obtain a symmetric non negative weights. Figure(5) shows the weights used in the estimation of the modified HP filter. (See also figure(17) in the Annexes).

Figure 5: Symmetric Non-Negative Weights for the U.S. and Costa Rica



5.3 Rolling Window

In order to analyse the results of using the modified HP filter an estimation of the actual GDP gap is done. Then another observation is included and the modified HP filter is recalculated. This recursive estimation of GDP gap allow to estimate the error with respect to the 'final' GDP gap. with respect to t_0 that refers to the period of the last observed GDP, and $t_1, t_2, t_3, t_4, t_5, t_6, t_7$ which are the errors due to new data for the previous estimated gaps at $t = -1, -2, -3, -4, -5, -6 and -7$ respectively. This is equivalent to measure the correction that is made to the output gap due to new observation until two years backwards.

$$\hat{gap}_t^{t_0} = y_t - y_t^{MHP,t=0} \tag{14}$$

$$gap_t^f = y_t - y_t^{MHP,t=T} \tag{15}$$

$$er_t^{t_0} = \hat{gap}_t^{t_0} - gap_t^f \tag{16}$$

Where $er_t^{t_0}$ is the error of the estimated output gap $\hat{gap}_t^{t_0}$ with respect to the gap_t^f at time t when t is the last observed data.

While:

$$er_t^{t_i} = \hat{gap}_t^{t_i} - gap_t^f \tag{17}$$

is the error of the estimated output gap $\hat{gap}_t^{t_0}$ with respect to the gap_t^f at time t when t is the $-i$ observed data.

6 Results

In this section a summary of the results for the US and Costa Rica are presented and commented.

To measure the accuracy I use the Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) [5]

6.0.1 United States

In this part I describe the main results for the U.S. which has a more stable GDP and also a longer period.

In figure(6) three estimated potential GDP are presented. The potential GDP obtained by using the standard HP filter without forecast, the standard HP filter with forecast, and the modified HP filter, this last one includes confidence intervals of 95 percent regarding forecast uncertainty.

Figures (7) (8) show that the modified HP filter outperform the standard HP filter by a small margin.

The errors are not normally distributed. And they also have autocorrelation problems.

[5] The RMSE is defined as: $RMSE = \sqrt{\frac{\sum_{t=1}^{T}(\hat{x}_t - x_t)^2}{T}}$ and the MAE is equal to: $MAE = \frac{\sum_{t=1}^{T}|\hat{x}_t - x_t|}{T}$

Figure 6: Estimation of Potential GDP U.S.

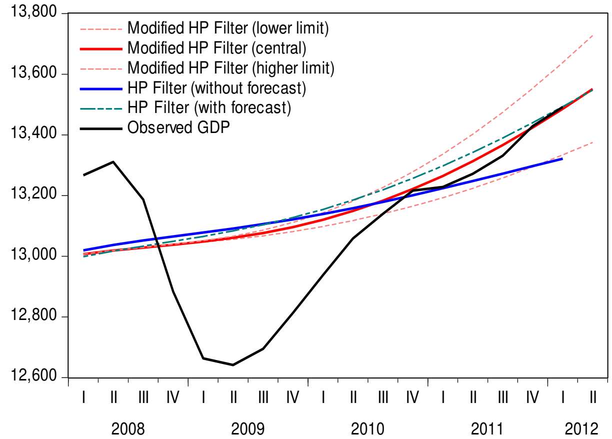


Figure 7: Mean Absolute Error of GAP for the U.S.

	With Respect to the HP Filter		With Respect to the Modified HP Filter	
t	HP Filter	Modified HP Filter	HP Filter	Modified HP Filter
0	**0.889%**	0.898%	0.826%	**0.814%**
-1	**0.773%**	0.782%	0.699%	**0.685%**
-2	**0.661%**	0.671%	0.579%	**0.565%**
-3	**0.560%**	0.571%	0.470%	**0.457%**
-4	**0.470%**	0.483%	0.379%	**0.361%**
-5	**0.390%**	0.406%	0.305%	**0.277%**
-6	**0.321%**	0.341%	0.248%	**0.205%**
-7	**0.261%**	0.292%	0.206%	**0.144%**

Figure 8: Root Mean Square Error of GAP for the U.S.

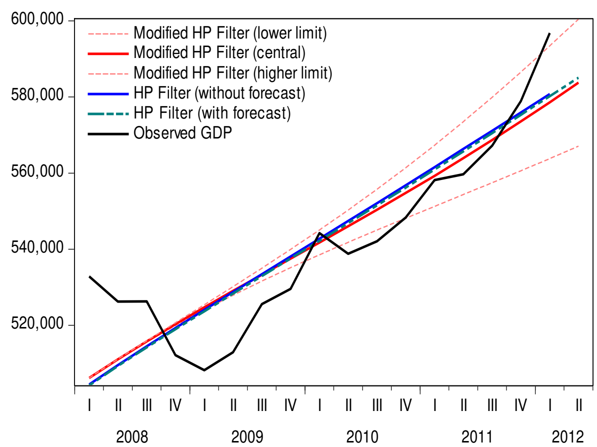
	With Respect to the HP Filter		With Respect to the Modified HP Filter	
t	HP Filter	Modified HP Filter	HP Filter	Modified HP Filter
0	**1.100%**	1.119%	1.018%	**1.014%**
-1	**0.956%**	0.975%	0.865%	**0.857%**
-2	**0.820%**	0.840%	0.721%	**0.710%**
-3	**0.695%**	0.718%	0.591%	**0.576%**
-4	**0.583%**	0.609%	0.478%	**0.457%**
-5	**0.483%**	0.514%	0.382%	**0.353%**
-6	**0.396%**	0.434%	0.306%	**0.263%**
-7	**0.321%**	0.368%	0.250%	**0.187%**

6.0.2 Costa Rica

The case for Costa Rica is discussed here.

In figure(9) three estimated potential GDP are presented. The potential GDP obtained by using the standard HP filter without forecast, the standard HP filter with forecast, and the modified HP filter, this last one includes confidence intervals of 95 percent regarding forecast uncertainty.

Figure 9: Estimation of Potential GDP Costa Rica



Figures (7) (8) show that the modified HP filter outperform the standard HP filter. Even if we compare it to the output gap estimated using the standard HP filter.

Figure 10: Mean Absolute Error of GAP for Costa Rica

	With Respect to the HP Filter		With Respect to the Modified HP Filter	
t	HP Filter	Modified HP Filter	HP Filter	Modified HP Filter
0	1.913%	**1.863%**	2.072%	**1.936%**
-1	1.815%	**1.797%**	1.950%	**1.840%**
-2	**1.712%**	1.717%	1.825%	**1.741%**
-3	**1.598%**	1.624%	1.688%	**1.627%**
-4	**1.487%**	1.530%	1.548%	**1.506%**
-5	**1.378%**	1.434%	1.407%	**1.382%**
-6	**1.268%**	1.337%	1.266%	**1.251%**
-7	**1.154%**	1.234%	1.128%	**1.117%**

The errors are not normally distributed. And they also have autocorrelation problems.

Figure 11: Root Mean Square Error of GAP for Costa Rica

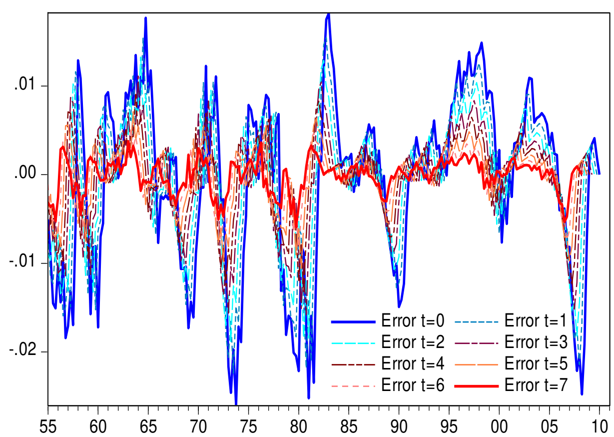
	With Respect to the HP Filter		With Respect to the Modified HP Filter	
t	HP Filter	Modified HP Filter	HP Filter	Modified HP Filter
0	2.352%	**2.307%**	2.498%	**2.373%**
-1	2.235%	**2.219%**	2.357%	**2.262%**
-2	**2.111%**	2.119%	2.209%	**2.139%**
-3	**1.979%**	2.006%	2.053%	**2.000%**
-4	**1.843%**	1.886%	1.891%	**1.850%**
-5	**1.705%**	1.763%	1.725%	**1.696%**
-6	**1.564%**	1.637%	1.560%	**1.538%**
-7	**1.423%**	1.505%	1.395%	**1.374%**

6.0.3 Correction for Autocorrelation

Errors in the estimation of the GDP gap have autocorrelation, this means that it is possible to improve the accuracy of the GDP gap estimation by using the information available.

In theory errors should behave as white noise (normally distributed).

Figure 12: Errors of Estimation



Equation for fitting the errors and improve the accuracy. for the U.S. is:

$$er_t^{t_0} = \gamma_0 + \gamma_1(y_t^{MHP,t=1-j} - y_t^{MHP,t=-j}) + \gamma_3 \triangle y_{t-1} + \xi_t^{t_0} \tag{18}$$

For Costa Rica the equation is:

$$er_t^{t_0} = \gamma_0 + \gamma_1(y_t^{MHP,t=1-j} - y_t^{MHP,t=-j}) + \gamma_3 \triangle y_{t-1} + \gamma_4 \triangle y_t + \xi_t^{t_0} \tag{19}$$

The estimated gap with the correction will be for the U.S. (equation(20)) and Costa Rica (equation(21)) respectively:

$$gap_t^{t_j} = y_t - y_t^{MHP,t=j} + \gamma_0 + \gamma_1(y_t^{MHP,t=1-j} - y_t^{MHP,t=-j}) + \gamma_3 \triangle y_{t-1} \tag{20}$$

$$gap_t^{t_j} = y_t - y_t^{MHP,t=j} + \gamma_0 + \gamma_1(y_t^{MHP,t=1-j} - y_t^{MHP,t=-j}) + \gamma_3 \triangle y_{t-1} + \gamma_4 \triangle y_t \tag{21}$$

The results of the estimation of equation(17) for the U.S. and Costa Rica are presented in figure(13)

Figure 13: Estimation Equation for Errors

	United States		Costa Rica	
	Coefficient	t-Statistic	Coefficient	t-Statistic
γ_0	0.001	0.880	-0.031	-9.873
γ_1	1.095	8.677	0.634	4.506
γ_2	-0.075	-3.085	0.223	3.307
γ_3			0.261	3.944
Adjusted R-squared	0.271546		0.501631	
F-statistic	42.56385		66.1923	
Akaike info criterion	-6.463844		-5.406027	
Schwarz criterion	-6.418152		-5.299854	
Durbin-Watson stat	0.296633		0.45852	
	Sample: 1954Q1 2011Q1		Sample: 1985Q2 2011Q1	

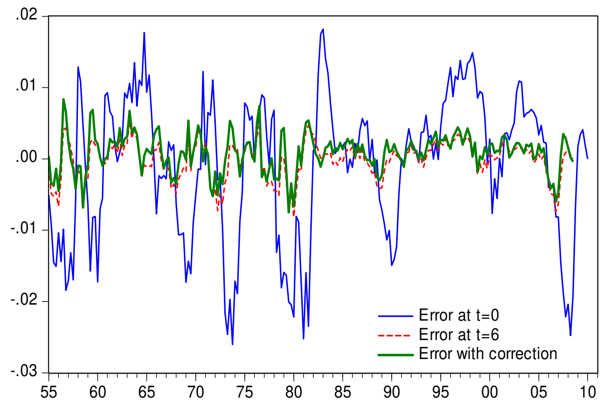
Figure(14)shows an important improvement in the accuracy of forecast when taking into account the correction due to autocorrelation in the errors.

Figure 14: RMSE and MAE for GAP with Correction

	Costa Rica		United States	
Correction	With Respect to the Modified HP Filter			
t	MAE	RMSE	MAE	RMSE
0	1.295%	1.556%	0.761%	0.943%
-1	1.295%	1.556%	0.761%	0.943%
-2	1.188%	1.423%	0.662%	0.812%
-3	1.188%	1.435%	0.562%	0.696%

It is important to note that even-thought the correction does reduce the level of autocorrelation, it do not eliminate the problem. Furthermore errors are still not normally distributed. So there is space for improvement (although marginal).

Figure 15: Errors for the U.S.



7 Conclusion

The modified HPF will provide an important tool for policy-makers in order to accurately estimate the output gap. The output gap is one of the main explanatory variables in the Phillips Curve. The more exact the estimation of the output gap is, the more informed an effective the monetary policy will be.

The autocorrelation in the errors affect the accuracy of the calculated output gap. This autocorrelation can be model. It is an important result that the errors in the estimation of output gap are not normally distributed. This indicate that there are systematic errors in the estimation that have to be address.

Using a correction that takes into account the autocorrelation in the errors dramatically improves the accuracy. It almost reduce in half the RMSE and the MAE. This is specially true for the case of the U.S. Nevertheless for the case of Costa Rica the improvement over the other estimation is still significant.

The use of the modified HP filter corrects for an downwards bias of the trend due to negative weights in series with a positive drift or trend. Even if the bias is marginal, this result indicate that the standard HP filter is not an unbiased estimator of the trend for a non stationary process.

The use of symmetric non-negative weights allow for the potential GDP to become stable, it does not varies with the length of the sample or the addition of more observation. And allow the practitioner to actually know how many periods should forecast.

It will be a negligence from the technical personal and policy makers not to take into consideration the correction presented on this paper of the output gap. Specially if the information of the output gap or potential GDP is part of the statistics that the Central Bank provide. This statistics should be as robust and unbiased as possible.

The framework discussed in this paper is general, and could be implemented to data on monthly or annual frequency.

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8 Annexes

Figure 16: Weights of HP according to sample length

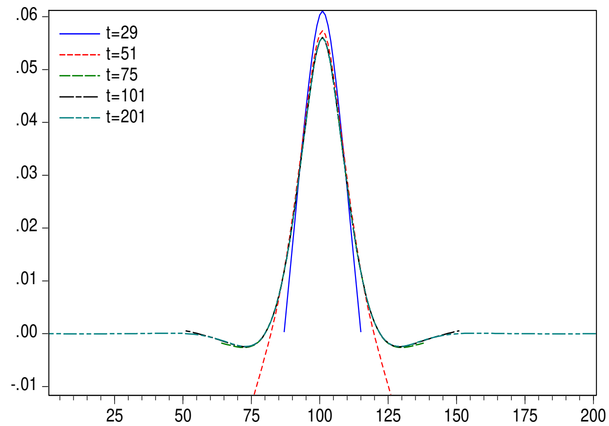


Figure 17: Proposed Weights of HP

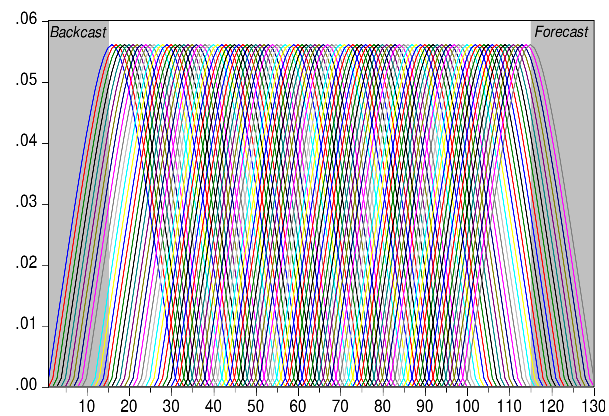
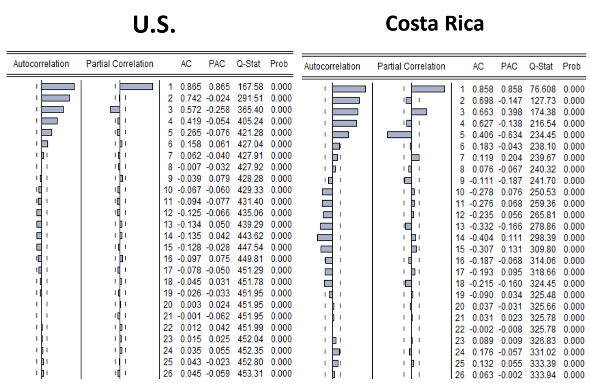


Figure 18: Autocorrelation and Partial Correlation



U.S.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.865	0.865	167.58	0.000
		2	0.742	-0.024	291.51	0.000
		3	0.572	-0.258	365.40	0.000
		4	0.419	-0.054	405.24	0.000
		5	0.265	-0.076	421.28	0.000
		6	0.158	0.061	427.04	0.000
		7	0.062	-0.040	427.91	0.000
		8	-0.007	-0.032	427.92	0.000
		9	-0.039	0.079	428.28	0.000
		10	-0.067	-0.060	429.33	0.000
		11	-0.094	-0.077	431.40	0.000
		12	-0.125	-0.066	435.06	0.000
		13	-0.134	0.050	439.29	0.000
		14	-0.135	0.042	443.62	0.000
		15	-0.128	-0.028	447.54	0.000
		16	-0.097	0.075	449.81	0.000
		17	-0.078	-0.050	451.29	0.000
		18	-0.045	0.031	451.78	0.000
		19	-0.026	-0.033	451.95	0.000
		20	0.003	0.024	451.95	0.000
		21	-0.001	-0.062	451.95	0.000
		22	0.012	0.042	451.99	0.000
		23	0.015	0.025	452.04	0.000
		24	0.035	0.055	452.35	0.000
		25	0.043	-0.023	452.80	0.000
		26	0.045	-0.059	453.31	0.000

Costa Rica

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.858	0.858	76.608	0.000
		2	0.698	-0.147	127.73	0.000
		3	0.663	0.398	174.38	0.000
		4	0.627	-0.138	216.54	0.000
		5	0.406	-0.634	234.45	0.000
		6	0.183	-0.043	238.10	0.000
		7	0.119	0.204	239.67	0.000
		8	0.076	-0.067	240.32	0.000
		9	-0.111	-0.187	241.70	0.000
		10	-0.278	0.076	250.53	0.000
		11	-0.276	0.068	259.36	0.000
		12	-0.235	0.056	265.81	0.000
		13	-0.332	-0.166	278.86	0.000
		14	-0.404	0.111	298.39	0.000
		15	-0.307	0.131	309.80	0.000
		16	-0.187	-0.068	314.06	0.000
		17	-0.193	0.095	318.66	0.000
		18	-0.215	-0.160	324.45	0.000
		19	-0.090	0.034	325.48	0.000
		20	0.037	-0.031	325.66	0.000
		21	0.031	0.023	325.78	0.000
		22	-0.002	-0.008	325.78	0.000
		23	0.089	0.009	326.83	0.000
		24	0.176	-0.057	331.02	0.000
		25	0.132	0.055	333.39	0.000
		26	0.063	-0.002	333.94	0.000