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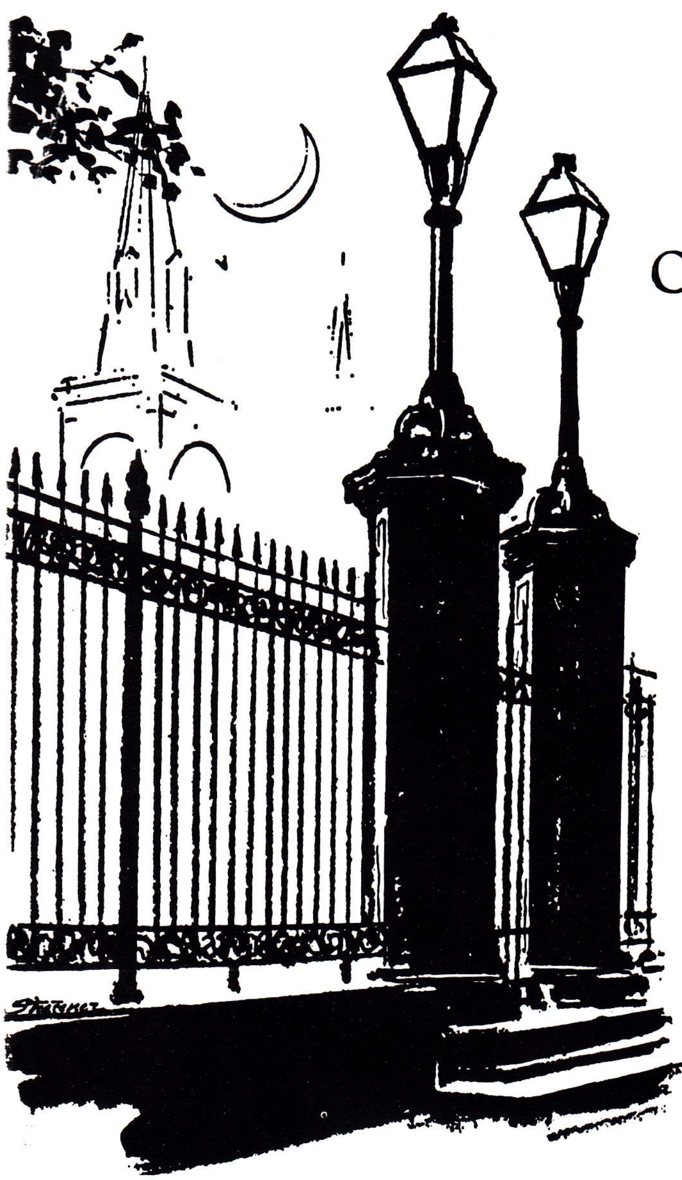

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OPTIMAL PRICE ADJUSTMENT: TESTS OF A PRICE EQUATION IN U.S. MANUFACTURING

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I. SUMMARY

The following description and analysis of a firm in atomistic competition is motivated by the need to specify a dynamic equation of price behavior to be tested on U.S. manufacturing time-series data. It will be shown that uncertainty of price information in a market composed of many competing firms leads to a model which is more or less in the Evans tradition of dynamic monopoly theory.[3]

The key dynamic element is the firm's reaction to customer behavior in an uncertain price situation. Price uncertainty forces newcomers to the market to search for an acceptable price which is less than the marginal utility of the good. Old customers may decide to search after a price increase, if the expected difference in search costs and price is less than the recently experienced price change.

The implications of the theory are examined using a phase diagram analysis. Of particular interest for empirical study are the effects of changes in model parameters on the time path of the optimal price control equation. In line with the conclusions of the theoretical model the estimation results seem to suggest that price adjusts to a moving equilibrium path in a variable manner determined by cyclical factors in the economy.

II. THE DYNAMICS OF MARKET DEMAND

A central fact motivating the dynamics of the model is the existence of uncertainty and the cost of information. In order to determine the most favorable price a prospective buyer must canvass the market. The probability of ending up with the lowest price offer increases with the amount of search. Since search costs increase with effort, the buyer must determine how long he should search before making a decision. The decrease in the expected minimum price is a decreasing function of additional search, given any distribution function of price quotations. If the cost of search is increasing or constant, there is a unique reservation price which a buyer will accept.

Buyers who are accustomed to making their purchases at one particular firm may have a different preference set than new customers searching for good deals. But sufficient increases in price can cause them to lose their loyalty and engage in search which may now have become profitable. If the firm is aware of this it has an incentive to vary price less than if buyers were insensitive to price.

Market search is determined as follows. Given his preferences and a subjective price distribution, the individual engages in a process of random sampling until a price no higher than his reservation price is offered, unless, of course, he has already bought the product somewhere and is satisfied with the price. The reservation price is determined as that price which minimizes the expected cost of the good, including search costs. It also has the property of leaving the person at least as well off as he is not buying at all or paying the old price.

Stigler[8] and McCall[6] modeled market search models for newcomers or for durable goods. Here the analysis is broadened to cover repeated purchases. We associate a marginal utility ψ with a unit of the commodity. The subjective probability density function on prices charged in the industry is $g(n)$. The cost per search is S. It is due to lost wages, leisure time, transportation costs and the like. For the density $g(n)$ the probability of receiving an offer p is ${}_0\int^p g(n)dn$ which we denote by $1/N(p)$. The expected number of searches required to find such an offer is thus $N(p)$. To determine his reservation price $\hat{p}$ the individual minimizes the expected price $E(p) = {}_0\int^p ng(n)dn$ plus search costs after $N(p)$ searches, subject to one of two possible restrictions. If he is a newcomer we require that $\psi \geq p$. If he is already in the market he will not want to search, as long as he is satisfied with the price he is or has been paying. However, assuming his subjective density to be stationary over time, he may decide to search if his price has risen by $u = p_1 - p_o$, where p_o and p_1 are the old and the new price, respectively. Given that p_o was previously determined in an optimal manner, he will search only if $u \geq p - p_o$. We thus have

$$\min_p N(p)[E(p) + S] = K(p,g(n),S) \tag{2.1}$$

$$\text{s.t } 0 \geq h(p) \begin{cases} 0 \geq p - \psi & \text{newcomer} \\ 0 \geq p - p_1 & \text{old customer} \end{cases}$$

The Kuhn-Tucker conditions are

$${}_0\int^{\hat{p}} (\hat{p}-n)g(n)dn - S \geq 0 \qquad \text{as } h(\hat{p}) \leq 0 , \tag{2.2}$$

equality if $h(\hat{p}) < 0$. For the newcomer the criterion is that the marginal benefit of additional search must be at least as great as the extra search cost, given that the reservation price is less than marginal utility. In contrast, the old customer is motivated to search only if the expected difference in cost is less than the price change in his current store. Substituting (2.1) into (2.2) the optimal reservation price turns out to be

$$\hat{p} \geq K \quad h(\hat{p}) \leq 0 \text{ , equality if } h(\hat{p}) < 0. \tag{2.3}$$

We call $\hat{u}$ the reservation price variability if $\hat{p} = K$. For any $u \leq \hat{u}$, there is no further search; the customer remains loyal for one more period. Clearly, it is in the interest of a firm to manage its price policy such that, *ceteris paribus*, u is as small as possible in order to minimize the loss of customers.

Since individuals and their reservation prices differ, the demand facing a particular firm is based on a distribution on p and u of old customers and potential newcomers. Arbitrarily associating one unit of the good with each person the potential demand for a firm's commodity x arises from a tri-variate density function $\varphi(x,p,u)$ which is assumed to be identically and independently distributed over time. Dealing with continuous time we let $u = dp/dt$. The firm sets price p at a rate u. The quantity demanded by customers whose reservation price and variance are at least equal to $\hat{p}$ and $\hat{u}$, respectively, is the conditional density $\varphi(x|\hat{p},\hat{u})$. Expected demand is $D(p,u) = E(x|p,u) = \int_X x\varphi(x|p,u)dx$. For a cost function $C(x)$, with properties,

$$C(0) \geq 0,\ C' > 0,\ C'' \geq 0 \tag{2.A}$$

expected profit is

$$V = E(V^*) = \int_X [xp - C(x)]\varphi(x|p,u)dx , \tag{2.4}$$

where V^* is actual profit.

It is assumed that the density φ is normal and that C is at most a polynomial of second order in x. Thus, the highest term in $E[C(x)]$ is $C''\text{var}(x|p,u)$. Since in this case the variance is independent of p and u [1], the partials C_p and C_u reduce to the certainty equivalents $C'D_1$ and $C'D_2$, respectively.

As is the convention, we assume further that expected marginal revenue $MR(x)$ and demand $D(p,u)$ are downward sloping, convex to the origin, and finite

$$D_1 < 0,\ 0 \leq D_{11} \leq 2D_1^2/D,\ \inf_{p,u} D(p,u) = 0 \tag{2.B}$$

$$D_2 < 0,\ u \gtrless 0,\ D_{22} < 0 \tag{2.C}$$

The sign assumption on D_2 is a generalization. It covers absolute price sensitivity of those who prefer price stability *per se*. We finally assume that price is at least equal to the competitive price $p > C'$, and that the monopoly price, equating expected marginal revenue and cost occurs at a positive output such that $p > C'(0)$.

III. THE FIRM'S OPTIMAL CONTROL POLICY

The optimum feasible price path maximizes the integral of the discounted instantaneous cash-flow function $V(p,u,t)$. By feasibility we mean p satisfies the differential equation

$$dp/dt = \dot{p} = u, \quad p(0) = p_o \geq 0, \quad p \geq 0, \tag{3.1}$$

where u is piece-wise continuous but otherwise unrestricted. The Hamiltonian

$$H(p,u,\lambda,t) = e^{-\delta(t)}(V + \lambda u) \tag{3.2}$$

is piece-wise differentiable in (p,u), and $\lambda(t)$ is a piece-wise differentiable, non-negative function of time such that (p^*,u^*) maximizes

$$\overline{H} = H(p,u,\lambda,t) + \lambda p(\dot{\lambda}/\lambda - r(t)) \tag{3.4}$$

at each moment of time t, where $r(t) = (d/dt)\delta(t)$, and

$$\lim_{t\to\infty} e^{-\delta(t)}\lambda(t) = 0.$$

If V, and therefore H, is concave in p, sufficiency of the conditions below is assured. See Arrow[2], Kamien and Schwartz[4], Mangasarian[5] and Mirrlees[7]. Differentiating $\overline{H}$ with respect to p and u, the first-order conditions are

$$H_1(p^*,u^*,\lambda,t) + \dot{\lambda} = \lambda r(t) \tag{3.B}$$

$$-V_2 = \lambda \tag{3.C}$$

Equations (3.1)-(3.C) and the assumptions listed in the preceding section make up the basic structure of a price-setting firm in atomistic competition. The aim in this section is to establish H-maximizing patterns of behavior in (u,p) space satisfying (3.1) and (3.B). Within that set we are interested in those trajectories that lead to equilibrium. Thus, the stationary paths of (3.1) and (3.B) are singled out for analysis. Optimal stationary paths in turn are those that result by invoking the transversality condition (3.A). At points of intersection the system is in equilibrium. The task is to determine if there is a multiplicity of such points and which ones, if any, exhibit some form of stability. Finally, the investigation will turn on the behavior of the system if it is subjected to exogenous shocks from outside. The conclusions of that last analysis will be of importance in specifying and interpreting the empirical example.

From (3.C) we note that $\lambda(t)$ is a shadow price equal to the cost of changing price. Condition (3.B) is in fact a marginal condition equating the present control cost with a discounted stream of future benefits from the new price level.

$$\lambda(t) = \int_t e^{-\delta(t-s)} V_1 ds > 0, \tag{3.5}$$

where $V_1 = D_1(p + D/D_1 - C')$ by the assumptions of Section II. The first-order conditions and the transversality condition thus lead us to a very familiar economic criterion for choice: make a change if all the benefits, present and future, are at least as large as the costs of instituting the change. The augmented Hamiltonian $\overline{H}$ can be interpreted as the current expected profit plus the gains $\dot{\lambda}p$ from a given price level less interest payments λrp.

Combining (3.B) and (3.C) the first-order conditions produce the Euler equation relating the rate of change of the imputed control value to the long-run pay-off

$$V_1 = D_1(p + D/D_1 - C') = -(r(t) + \dot{V}_2/V_2)V_2 \tag{3.6}$$

This expression reveals the non-optimality of always charging the static monopoly price in this kind of model. The first term is marginal profit (with respect to price) which is ordinarily equal to zero at the optimum. The monopoly price p_m is given when marginal revenue = marginal cost inside the first parenthesis. This can clearly only be true when we have $V_2 = \dot{V}_2 = 0$, or in the exceptional case when the percentage rate of change of the shadow price equals the interest rate.

The behavior of the system is illustrated in (u,p) space using the phase diagram Figure 1. Since the Euler equation must always be satisfied, it must hold along stationary paths. The curve $\dot{\lambda} = 0$ is then the locus of points (u,p) for which

$$V_1 + rV_2 = 0. \tag{3.7}$$

It is the optimal stationary path if it satisfies (3.A), hence leads to p_m. The slope of this curve is

$$du/dp|_{\dot{\lambda}=0} = -(V_{11} + rV_{12})/(V_{12} + rV_{22}) . \tag{3.8}$$

The sign of this derivative is clearly of importance to uniqueness and stability of stationary points. The $\dot{\lambda} = 0$ curve defines a set of tangencies between the market rate and a set of isoprofit lines $V(p,u,\overline{t}) =$ constant, where $\overline{t}$ is a particular moment in time,

$$r = -(V_1/V_2) = du/dp|_{V=\text{const}} > 0 \quad p \gtrless p_m . \tag{3.9}$$

Along the stationary $\dot{\lambda} = 0$ the path slope of an isoprofit curve must equal the market rate of interest. Clearly, if r, V_1 or V_2 change, the points of tangencies change, hence the curve changes position implying different adjustment rates and/or different equilibrium. The other equation of relevance in this phase diagram is the horizontal axis $\dot{p} = u = 0$. Two questions must be answered. (1) Is there an optimal stationary path which is stable? (2) Is such a path unique?

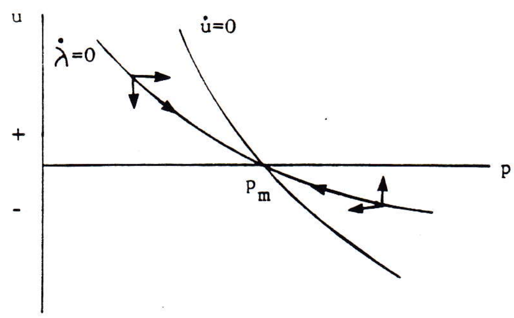


Figure 1.

To answer the first question we approximate the Euler equation to first order near the monopoly price p_m at some particular time $\overline{t}$.

$$\ddot{p} - r\dot{p} - ap + ap_m - V_1(p_m,0,\overline{t}) - rV_2(p_m,0,\overline{t}) = 0 , \tag{3.10}$$

where $a = V_{22}^{-1}(V_{11} + rV_{12})$. Since $V_1 = V_2 = 0$ in equilibrium, the last two terms drop out. If $a > 0$, the roots are real (r is real), have opposite signs and are centered on r. Hence, p_m is a saddlepoint at $\overline{t}$. Clearly, $a > 0$ if $V_{11} + rV_{12} < 0$ holds for all values of (u,p). If H is concave with respect to p ($H_{11} = V_{11} < 0$) the saddlepoint condition is satisfied whenever V_{12} is non-positive or is small, if it is positive. Note that, first of all $V_{22} = (p-C')D_{22} - C''D_2^2 < 0$ by (2.5). In addition $V_{11} < 0$ if $D_{11}(p-D/D_1-C') + D_1(2 + DD_{11}/D_1^2 - C''D_1) < 0$. The second term is negative by assumption. The first term can be positive if p is much higher than the price at which marginal revenue equals marginal cost. If $D_2 > 0$ whenever $p > p_m$ such a situation will not prevail because it would then be in the firm's interest to reduce price as quickly as possible. If $D_2 < 0$ in all cases, we must assume that demand is so elastic that p will never profitably exceed p_m by very much. This would be the normal economic situation. The sign of V_{12} depends on the signs of D_{12} and D_2. Differentiating V_2 with respect to p we have

$$V_{12} = D_2 + (p - C')D_{12} - C''D_1D_2 . \tag{3.11}$$

By previous assumptions this expression is non-positive if D_2 and D_{12} are non-positive. If demand does not become less sensitive to price increases at higher prices, $D_{12} \leq 0$. Since this is a fairly reasonable assumption, the second term is nonpositive. Thus, V_{12} is clearly negative if $D_2 < 0$ as in the case of pure variance aversion. We note therefore that the very assumption which might cause positivity of V_{11} must unequivocally imply a negative V_{12}. This offsetting effect of the two alternative assumptions may be sufficient to imply positivity of the coefficient $\underline{a}$ in all cases, as must be assumed if we are to have a saddlepoint equilibrium.

The answer to the second question raised above is partially implied by the preceding analysis. Multiple stationary points can occur only if the $\dot{\lambda} = 0$ curve changes direction somewhere in (u,p) space, ie., if $du/dp|_{\dot{\lambda}=0}$ changes sign. As shown in Fig. 1 this curve must have a negative slope near a stable equilibrium, if price is to drop whenever $p > p_m$ and to rise whenever $p < p_m$. The saddlepoint assumption means that the numerator of (3.8) is non-positive. This condition must be strengthened to negativity. Similarly, the denominator must be negative. Since V_{22} is negative we merely require that V_{12} be negative or small in absolute value.

By differentiating the Euler equation which is a function of $\dot{u}$,u and p a stationary Euler path $\dot{u} = 0$ is determined. Its slope

$$du/dp|_{\dot{u}=0} = -(V_{11} + rV_{12})/(rV_{22} + 2V_{12}) \tag{3.12}$$

is smaller than that of the $\dot{\lambda} = 0$ curve if $V_{12} < 0$, as shown in Fig. 1. The optimal stationary path thus cuts the stationary $\dot{u} = 0$ curve from below, and the arrows indicate the direction of motion in (u,p) space.

The preceding discussion points to the general behavioral phenomenon that, if price is anywhere but in equilibrium, there exist forces which will return it to equilibrium. Indeed, the amount of adjustment depends on how far away from equilibrium price happens to be. The rate of adjustment of price is thus the distributed lag

$$\dot{p} = F(p_m - p), \quad F(0) = 0, \; F' > 0 , \tag{3.13}$$

where the last two properties follow from the assumptions underlying the phase diagram.

We now turn to the matter of the dependence of adjustment F and equilibrium p_m on time. Markets and technology are undergoing continual change. This fact is reflected in some of the parameters of the model, for instance the market rate r(t), the cost function C and its derivatives, and the demand parameters D_i and D_{ij}. A change in the interest rate r will not affect equilibrium, since $V_1 = 0$ is not a function of r. However, whenever r increases, the iso-profit lines must become steeper. The change in the slope of the equilibrium curve $\dot{\lambda} = 0$ depends on the sign of V_{12}. If $V_{12} > 0$, the adjustment $\dot{p} = u(p)$ to equilibrium is retarded, but if $V_{12} < 0$, F' increases if $|V_{12}|$ is large.

An increase in marginal cost or in the demand elasticity increases equilibrium price. The rate of adjustment is thereby also increased. Finally, the cost of adjustment V_2 plays an important role in how quickly the firm can adapt to changing conditions. If we consider an increase in $|D_{22}|$, hence of $|V_{22}|$, due to a rise in the aversion to price changes, we find that the slope of the $\dot{\lambda} = 0$ curve is raised. Thus, even as equilibrium price remains the same, adjustment to that point is accelerated if price-change sensitivity increases.

IV. APPROXIMATION OF THE OPTIMAL PATH

We take as starting point the linear expansion (3.11) with $ap_m(t)$ serving as a forcing function of the approximate optimal control law. Thus, although the Euler equation in (3.11) applies strictly speaking to a particular time $\bar{t}$, we now assume that the expansion path is a function of time, but, to be truthful, only in a limited manner. Any changes in r and $\underline{a}$ will, for the moment, be conveniently ignored, only to be resurrected later on, where such changes will be taken care of in a somewhat more ad hoc manner.

The roots of (3.10) have the symmetry property

$$\lambda^- + \lambda^+ = r \qquad \lambda^- < 0 < r < \lambda^+ \tag{4.1}$$

As shown by Tinsley[10] a solution satisfying initial and transversality conditions, ie., a solution which is optimal for the approximate problem consists of two parts

$$p(t) = p^T(t) + p^L(t) \quad , \tag{4.2}$$

where the transient component is

$$p^T(t) = e^{\lambda^- t}(p_o - p^L(0)), \tag{4.3}$$

and the long-term trajectory is a convolution of past and future stationary mark-ups $ap_m(t)$:

$$p^L(t) = (\lambda^+ - \lambda^-)^{-1}\Big[\int_0^t e^{\lambda^-(t-s)} ap_m(s)ds + \int_t^\infty e^{\lambda^+(t-s)} ap_m(s)ds\Big] . \tag{4.4}$$

An optimal control law in the present sense is one in which the firm adjusts to equilibrium by taking into consideration the entire planning horizon, discounting future stationary points with a positive root and weighting past mark-up points using the negative root. If time t is large, ie., the firm is on a so-called turnpike, the transitory part in (4.2) becomes unimportant (after a sufficient time initial conditions cease to matter) and the solution simplifies to

$$p(t) \approx p^L(t) = \int_0^t w(t-s)\bar{p}(s)ds \tag{4.5}$$

where w(t) is the weight pattern

$$w(t) = -\lambda^- e^{\lambda^-(t)} \tag{4.6}$$

and $\bar{p}(t)$ is the target of the distributed lag function (4.5),

$$\bar{p}(t) = -(\lambda^-)^{-1} \int_t^\infty e^{\lambda^+(t-s)} ap_m(s)ds . \tag{4.7}$$

A heuristic interpretation of the target $\bar{p}(t)$ can be based on an argument of quasi-myopic behavior. If the firm were completely myopic it would consider only the present value of $ap_m(t)$. In quasi-myopic optimization the firm takes the mark-up values current at each point of time and forms a convoluted target as the forward lead shown in (4.7).

The correspondence between the analysis in this section and the phase-diagram becomes evident if we differentiate (4.5) with respect to time

$$\dot{p}(t) = -\lambda^-[\bar{p}(t) - p(t)] = F(\bar{p}(t) - p(t)). \tag{4.8}$$

Clearly, as before the adjustment properties F(0) = 0 and F' > 0 are preserved. We notice also that the value to which price adjusts in phase space is no longer the simplified stationary monopoly mark-up ap_m but the convolution shown in (4.7). If we now consider the effects of changing interest rates and changing sensitivity to price-variance, we find that adjustment $-\lambda^-$ and the target must be revised. The roots of (3.10), our approximate Euler equation, are determined by the discount rate r(t) and by the coefficient $\underline{a}$ which is a function of model parameters embodied in V. A quick check with (3.10) confirms the result of the phase diagram analysis for the general model, ie.,

$$\partial{-\lambda^-}/\partial r \gtrless 0, \qquad \partial{-\lambda^-}/\partial a > 0 . \tag{4.9}$$

As in Section III, the conclusion is that an increase in the rate of discount tends to speed up adjustment if V_{12} is negative and large in absolute value, while an increase in $\underline{a}$, brought about by increased costs, decreased demand elasticity, or an increased sensitivity to price variation, will accelerate adjustment.

The similarity of equation (4.8) to others that have been proposed as empirical hypotheses for price adjustment is deceptive. As noted, $\bar{p}(t)$ is not a current term but embodies all the information, past and future, which the firm has available concerning its cost and demand structure. Even the adjustment rate which ordinarily is a constant is now determined by conditions affecting costs and opportunities of the firm. Given a changing environment, optimal behavior by the firm will imply continual revision of the adjustment process. The task of the next section will be to find a suitable way of expressing this phenomenon for empirical estimation.

Finally, in real-world applications decisions and measurements are discrete. The discrete approximation of the continous control equation (4.8) is

$$p(t) = \lambda \sum_{0}^{t}(1-\lambda)^{i}\bar{p}(t-i) \quad 0 < \lambda = -\lambda^{-}/(1-\lambda^{-}) < 1. \tag{4.10}$$

These weights are obtained by considering the time derivative of p in (4.8) as the limit

$$\lim_{h\to 0} \frac{p(t) - p(t-h)}{h} = -\lambda^{-}/(1-h\,\lambda^{-})[\bar{p}(t)-p(t-h)] \;.$$

If, however, we let $h = 1$, the result in (4.10) is obtained. The normalization of $h = 1$ will thus be assumed for quarterly data.

V. SPECIFICATION OF A PRICE EQUATION

If the production function is linear homogeneous, the stationary monopoly price is given by

$$p_m = \eta C'(x) = \eta[\omega/\pi]^{k}\pi \tag{5.1}$$

where $\eta = e/1{+}e$ is the static monopoly mark-up, e the demand elasticity pD_1/x, ω the wage rate, π the price of materials, and $k < 1$ the elasticity of output with respect to labor input. $\bar{p}$ is a convolution of forward equilibrium prices $p_m(t)$. We assume the exponential forecasting rule $E[p_m(s)|t] = e^{v(s-t)}p_m(t)$. Let $v \neq \lambda^{+}$, integrate (4.7) and substitute the result and (5.1) into (4.10)

$$p(t) = \sum_{0}^{t} W^{*t-i}(i)[\omega(t-i)/\pi(t-i)]^{k}\pi(t-i) \;, \tag{5.2}$$

where $W^{*t-i}(i) = \eta a(t)(1-\lambda(t))(1-\lambda(i))^{i}/(\lambda^{+}(t)-v)$.

Expanding (5.2) log-linearly about the sample mean $\bar{c} = (\overline{w/\pi})^{k}\bar{\pi}$, subtracting $\ln\pi(t)$ and differencing we obtain a distributed-lag function in logarithmic differences[12]

$$\Delta\ln(p/\pi) = b+k\sum_{0}^{n} W^{t-1}(i)\Delta\ln(\omega/\pi)_{t-i}$$

$$+\sum_{0}^{n} \bar{W}^{t-i}(i)\Delta\ln\pi_{t-i} \tag{5.3}$$

where

$$W^{t-i}(i) = W^{*t-i}(i)/\sum_{0}^{n} W^{*t-i}(i), \text{ and } \bar{W}(i) = W(i),$$

$i=1,2,\ldots,n;$ $= W(i)-1$, $i=0$. b is a constant, if the remainder of the expansion is a linear polynomial of time. The unit-sum restriction assumed to apply to the $W^{*}(i)$ also holds for the $W(i)$. Following the discussion in IV, we single out $r(t)$ and a variable affecting V_{22} as altering the adjustment profile. The ratio of unfilled orders to capacity U represents demand pressures that have no more than transitory effects on price behavior. According to the theory both variables should accelerate adjustment.[12] If Z stands for either r or U, the hypothesis will be tested that Z modifies the weight schedule as follows[9]

$$W^{t}(i) = h(i) + \sum_{0}^{i} m(j,i-j)Z_{t+j}, \quad \sum_{0}^{i} m(j,i-j) = 0,$$

where $W^{t}(i)$ is the i-th period portion of the total effect transmitted due to a change in ω or π in period t. Because of common time trends leading to collinearity problems, we use only 2-period moving averages of Z placed in alternating periods. Assuming a lag of three periods, the weight schedule has the staggered form

$$\begin{aligned} W^{t}(0) &= h(0) + m(0,0)\Delta Z_{t} \\ W^{t}(1) &= h(1) + m(0,1)\Delta Z_{t} \\ W^{t}(2) &= h(2) + m(0,2)\Delta Z_{t} + m(2,0)\Delta Z_{t+2} \\ W^{t}(3) &= h(3) + m(0,3)\Delta Z_{t} + m(2,1)\Delta Z_{t+2} \end{aligned} \tag{5.4}$$

where $m(2,0) = -m(2,1)$ by the zero-sum restriction. The effect in period 3 of a change in $c(t)$ is thus $h(3)$, a constant, plus a constant proportion of the change in Z_t, plus a further portion caused by ΔZ two periods hence. Clearly, as the time path of Z varies, so does the profile of the adjustment pattern.

The weights $h(i)$ and $m(i)$ were estimated using an orthogonal power series approximation $v(i) = b_0 + b_1 i + \ldots + b_d i^{d}$.[11] The best fitting combination of **n** and **d** was taken as the preferred outcome. The results are shown in Tables 1 and 2. In part I of the tables the weight schedules

$$W(i)X_{-i} = [h(i) + m(0,i)\Delta Z_{-i} + m(2,i-2)\Delta Z_{-2}]X_{-i}$$

are denoted by the polynomial

$$P_{w}(X) = P_{h}(X) + P_{0-3}(X) + P_{2-3}(X) + \ldots .$$

The first term indicates the fixed portion of the schedule. P_{i-j} indicates the periods (from i to j) in which $Z(i-j)$ modifies the variable portion of W. Part I of each table gives the t-values of the polynomial coefficients for each of the fixed and variable weight schedules used to construct $W(i)$ in Part II. Bars over m and h mean they pertain to distributed weights on π. (See 5.3). Theoretically, the barred and unbarred weights should be identical, after dividing by k, but were not so restricted. The second part in each of the tables is (5.4) transposed.

The quarterly data are[12]

p: BLS manufacturing wholesale price index

π: An index of materials prices

ω: Compensation per manhour in U.S. manufacturing

r: Corporate bond rate (Moody Aaa)

U: Ratio of unfilled orders to capacity output (real sales less changes in finished goods inventories divided by the Wharton rate of capacity utilization)

The variable influence on the adjustment profile of price to changes in costs exerted by U and r is quite evident. As hypothesized, increases in the ratio of unfilled orders to capacity output accelerate the distributed response. The same is true of the discount rate, but to a somewhat lesser extent. This result does not contradict a possible hypothesis that D_{12} is negative, as suggested in Sections III and IV.

Table 1

I. Interpolation Coefficients Z = U

Polynomial	Degree	t-ratios				
		b^{*}_{o}	b_1	b_2	b_3	
$P_h(\omega/\pi)$	3	11.1	1.01	1.65	1.06	$R^2 = .99$
$P_h(\pi)$	3		5.34	3.85	3.06	SE = .0027
$P_{0-3}(\omega/\pi)$	2		3.23	1.73		DW = 2.11
$P_{0-3}(\pi)$	2		2.59	1.56		Intercept= -.006
$m(2,0)^{a}$			2.02			Span 54II-67IV
$\bar{m}(2,0)$			1.86			

*b = 0 for zero sum restriction.

Table 1 contd.

II. Normalized Weight Schedule (ΔU = $1 Mil.)

i	0	1	2	3	Ch. SUM
$h(i)$	.3140	.2209	.0549	.2336	.8234=k
$\bar{h}(i)^b$	-.5587	.2407	.0909	.2271	.0
$m(0,i)\Delta U$	.5527	-.0215	-.2871	-.2442	.0
$\bar{m}(0,i)\Delta U$	.5081	-.0448	-.2765	-.1868	.0
$m(2,i-2)\Delta U_{+2}$			-.2858	.2858	.0
$\bar{m}(2,i-2)\Delta U_{+2}$			-.2332	.2332	.0
$W(i)^c$	.9341	.2467	-.5062	.3253	1.0

a. $m(2,0)$ is coefficient of $\Delta\ln[\omega/\pi\,\Delta Z_{+2}]_{-2}$.

b. See definition of $\bar{W}$ in (5.3).

c. $W(i) = h(i)/k + \sum_{j=o}^{2} m(j,i)$ to give W in (5.3). The $\bar{W}$ sum applied to the distributed lag on π is very similar.

Table 2

I. Interpolation Coefficients $Z = r$

Polynomial	Degree	t-ratios				
		b_o^*	b_1	b_2	b_3	
$P_h(\omega/\pi)$	2	9.59	.729	1.90		R^2 = .985
$P_h(\pi)$	3		5.49	2.94	8.63	SE = .0029
$P_{0-3}(\omega/\pi)$	2		1.17	.388		DW = 2.13
$P_{0-3}(\pi)$	2		1.24	.356		Intercept= -.0057
$m(2,0)^a$			1.14			Span 54II-67IV
$\bar{m}(2,0)$			1.10			

* See note to Table 1.

II. Normalized Weight Schedule (Δr = .01)

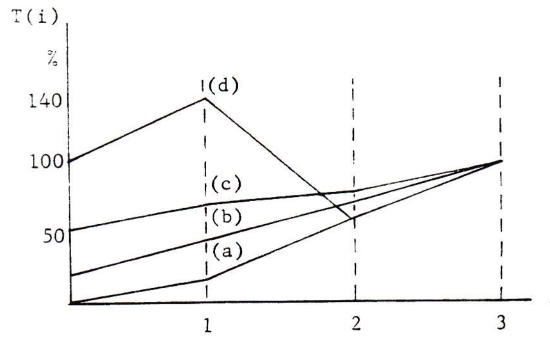
i	0	1	2	3	Ch. SUM
$h(i)$	.3341	.1340	.1022	.2386	.8089=k
$\bar{h}(i)^b$	-.5464	.1281	.1393	.2790	.0
$m(0,i)\Delta r$	.2552	.0078	-.1237	-.1393	.0
$\bar{m}(0,i)\Delta r$	.3062	.0233	-.1414	-.1881	.0
$m(2,i-2)\Delta r_{+2}$			-.2565	.2515	.0
$\bar{m}(2,i-2)\Delta r_{+2}$			-.2331	.2331	.0
$W(i)^c$	.6682	.1735	-.2539	.4072	1.0

a, b, c, see notes to Table 1.

Four cumulative distributions using data from past periods are shown in Fig. 2. The initially large response of curve (d) illustrates the empirical conclusion of this paper in a rather dramatic way.

VI. CONCLUSION

The following points deserve emphasis. (1) If the results are at least qualitatively valid, the variable-weight schedules seem to indicate that, as in the phase-space analysis, the moving equilibrium path and adjustment to it are regulated by the economic features of the model. (2) A fairly strong case can be made for the claim that price adjustment patterns are cyclical to the extent that unfilled orders behave cyclically. This should be of particular significance to policy makers trying to control inflation. (3) This model has excluded the interdependence of other decisions of the firm. The interaction of dynamic adjustments, ie., additional consideration as state and control variables of output, factor inputs, unfilled orders, and inventories has not been represented in this paper.



$$T(i) = 100 \sum_{o}^{i} W(j) / \sum_{o}^{3} W(j)$$

	t	$Mil	t	t+2
(a)	48I	: ΔU =	(-.6,	-.2)
(b)	57I	: ΔU =	(-.3,	-.6)
(c)	66IV:	ΔU =	(.2,	-.2)
(d)	50IV:	ΔU =	(1.2,	1.3)

Figure 2.

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