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Koray, Semih and Saglam, Ismail

Bilkent University

1997

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Semih Koray and Ismail Saglam

*Department of Economics, Bilkent University, Bilkent, 06533 Ankara, Turkey.*

**Abstract**

We show that in oligopolistic markets the social choice correspondence which selects all socially efficient outcomes is Nash implementable if the number of firms is at least two. Thus, monopoly regulation whenever consumers are favored by the designer or the society is the only framework, among all oligopolistic regulatory models, where Bayesian approach is indispensable.

*Key Words:* Bayesian Implementation, Nash Implementation, Asymmetric Information, Oligopoly, Regulation.

*JEL Descriptors:* L50, D70, D82.

## 1 Introduction

Implementation is designing mechanisms (game forms) through which noncooperative actions of agents in a society yield to a set of equilibrium allocations (or allocation rules) which coincides with the set of social choice functions.

In situations where the number of agents is greater than one, the equilibrium concepts according to which a game can be resolved are not unique, giving rise to the problem of selecting the most desirable (simple) solution concept, hence the implementation technique it will induce, from the viewpoint of the designer or the society, depending upon the distribution of information in the society as well as the mode of behaviour that the agents

*The second author gratefully acknowledges the financial supports of Scientific and Technical Research Council of Turkey (TUBITAK) and Center for Economic Design (CED). The usual disclaimer applies.

in the society are endowed with. Among many alternatives, implementation in dominant equilibrium which requires no information on the part of any agent or the designer about the characteristics of agents in the society when determining his or her strategy is, of course, the most desirable one. Besides, whenever a mechanism is designed to implement in dominant equilibrium, 'rational' agents ex-post behave in the way the designer (or the mechanism) ex-ante predicts them to do. Moreover, whenever the social choice rule can be implemented in dominant equilibrium, the designer can restrict himself by the Revelation Principle w.l.o.g. to direct-revelation mechanisms which ask all agents to report their characteristics and which give them no incentives to lie, making the strategy (message) space, and thus the mechanism, as simple as possible. (See Gibbard and Satterthwaite (1977), Dasgupta, Hammond and Maskin (1979), Harris and Townsend (1981) and Myerson (1979), among others.)

If a particular social choice rule is desired to satisfy Pareto efficiency and non-dictatorship, which are minimal requirements that can be imposed on a social choice rule regarding the distribution of power among the agents, then implementation in dominant equilibrium is, unfortunately, impossible, as shown by Gibbard (1973) and Satterthwaite (1975), when the domain of agents' preferences over the feasible outcomes is unrestricted (or sufficiently rich). This negative result in implementation under universal domain makes it inevitable either to work in sufficiently restricted domains or to use implementation in other solution concepts, such as Nash equilibrium or Bayesian (Nash) equilibrium, for which the impossibility result does not prevail under some conditions.

Nash equilibrium concept requires a strategy to be in equilibrium if no agent has any incentive to change his or her strategy unilaterally whenever all other agents stick to their equilibrium strategies. Under this solution concept, an agent needs to know not only his own characteristics (and his preference relation associated with his characteristics) but the characteristics (and hence preferences) of everyone else in order to determine his equilibrium strategy. Maskin (1977, 1985) showed that a social choice rule is Nash implementable only if it is monotonic. Furthermore, if for a given set of preference profiles of agents in the society, the social choice rule satisfies monotonicity and weak no-veto power conditions, then it is Nash implementable whenever the number of agents is at least three. (See Maskin (1977, 1985), Williams (1984), Saijo (1988), Moore and Repullo (1990), Danilov (1993)). Whenever, the number of agents is two, no Pareto optimal social choice rule is Nash implementable under the universal domain of preference profiles unless it is dictatorial (Maskin (1977) and Hurwicz and Schmeidler (1978)). The characterization of the necessary and sufficient conditions, so called condition $\beta$, for two-person Nash implementation is due to Dutta and Sen, who showed that on restricted domains, such as economic environments or "cardinal utility, lottery" framework, one can avoid the impossibility result. (See also Roberts (1979) and Laffont and Maskin (1982)).

Bayesian equilibrium concept (due to Harsanyi (1967-68)) is basically an extension of Nash equilibrium to a case in which each agent only knows his preferences and has incomplete information about the preferences of everyone else. Each agent has a belief about the preferences of other agents, which is assumed to be originating from the same and commonly known prior. A strategy profile is then said to be in Bayesian equilibrium if every agent's strategy under his or her beliefs about the other agents preferences (and hence strategies) conditional upon his preferences, is in Nash equilibrium. It was shown by Ledyard (1978) that in classical economic environments with a finite number of agents, there exists no mechanism which implements any given social choice rule in Bayesian equilibrium and which satisfies no-trade option (individual-rationality) and Pareto efficiency for all prior beliefs about the state of the society. Nevertheless, it is also shown by Jackson (1991) that if in an economic environment, a social choice function satisfies the necessary conditions, namely, closure, incentive-compatibility and Bayesian monotonicity then it is implementable in Bayesian equilibrium. (See also Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1987, 1989)).

In the literature where Bayesian approach has been used, the social objective function is defined as the expected value (under the common prior belief about the preferences of all agents) of what the society would agree upon as to maximize under the complete information situation. It is obvious that whenever the social choice rule associated with an actual social objective function chosen under the complete information case can be implemented in some equilibrium concept, then the common prior belief (of the designer or the society) about the preference profile of agents is inconsequential to the outcomes induced by the selected mechanism whether the equilibrium concept is Bayesian or not. However, in situations where there exists no equilibrium concept in which the social choice rule that would be selected under complete information can be implemented, the Bayesian approach which avoids from the impossibility result by redefining the social objective function and hence its associated social choice rule seems to be the only appropriate and meaningful solution.

The Bayesian approach in oligopoly regulation has been introduced by Baron and Myerson (1982) who examined the problem of regulating a natural monopoly with unknown costs. They showed that when the social objective function assigns equal weights to both consumers' and producer's welfare, the socially efficient outcome that would be chosen under complete information can be implemented by a mechanism in dominant equilibrium. Whenever, the society assigns more weight to consumers' welfare than producer's welfare, however, the socially efficient outcome that would be chosen under full information case is no longer implementable by any mechanism, even though the domain of admissible preferences of the firm is very thin; so in such a case redefining the social objective in the Bayesian sense becomes inevitable.

An important point to note is that in the monopoly regulation problem as well as in any generalized principal-agent framework where there is a single agent with private information, every equilibrium concept according to which the implementation will take place reduces to the singleton, namely, single agent's utility maximization. So, the Bayesian approach in a principal-agent framework is inevitable as the only normative approach when the social objective function favors principal's welfare. However, when the number of players in any game under incomplete information is greater than one, we can talk about some conjectural equilibrium concepts as well. The lack of a general characterization result for dominant (equilibrium) implementation, therefore, does not restrict us completely. Indeed, to make a justifiable generalization of the inevitability of Bayesian implementation in the principle-agent framework to a multi-agent (oligopoly) model, one needs to show that there exists no equilibrium concept in which the social choice rule that would be chosen under complete information in a multi-agent model can be implemented.

Of course, one may argue at this point that the existence of non-Bayesian mechanisms in a multi-agent framework does not necessarily imply that one can avoid Bayesian mechanisms under all circumstances, as each equilibrium concept, except for dominant equilibrium, may be meaningful and valid for only a particular informational structure of the society. That is, Bayesian implementation may be still the only appropriate approach if the agents have incomplete information about each other's relevant characteristics.

But, it should be also noticed that even in environments where agents completely know each other and thus may be induced to play their Nash strategies through a non-Bayesian mechanism yielding to an ex-post efficient social outcome, a designer who has incomplete information about the state of the society may enforce a Bayesian mechanism, by redefining the social objective as maximizing the expected value of the social welfare function, the society has agreed upon, in the presence of his prior beliefs about the state of the society. The Bayesian nature of the mechanism, however, may lead to ex-post inefficient outcomes as well as render the mechanism open to the *manipulation* of a dishonest regulator whenever his beliefs are not common knowledge.[1]

In the light of the criticisms raised in the literature against Bayesian implementation, it is then a natural attitude to find out the cases in which Bayesian implementation is indispensable. This paper attempts to considers the justifiability of Bayesian regulatory mechanisms in a special kind of economic environment, namely, an oligopoly.

Section 2 presents implementation in general environments. Section 3, discusses oligopoly regulation as a special kind of implementation problem. Section 4 contains results on Nash implementation in our oligopolistic framework, and finally, Section 5 concludes.

# 2 Implementation in General Environments

Consider an environment in which there are a finite number, $n$, agents. Let $A$ denote the set of alternatives (outcomes) for the agents. A utility function, $u$, on $A$ is a real-valued function from $A$ to $\Re$ (reals). Let $U^i$ is the set of all utility functions for the agent $i = 1, ..., n$. An $n$-person social choice correspondence (SCC) on $U = (U^1, ..., U^n)$ is a correspondence $F : U \twoheadrightarrow A$, which lists the desirable outcomes depending upon the utility profile of the society.

A mechanism (or a game form) $\Gamma = (M, h)$ is a pair consisting of a strategy (message) space $M = M^1 \times ... \times M^n$ and an outcome function

[1] The manipulability of Bayesian mechanisms was pointed out by many authors, including Crew and Kleindorfer (1986), Vogelsang (1988), Koray and Sertel (1990), Laffont (1994) and Koray and Saglam (1995a,b).

$h : M \to A$. Given the profile $u \in U$ of the society, the mechanism $\Gamma$ leads to a normal form game $\Gamma[u]$.

Finally, $\sigma$ is a solution concept according to which the game $\Gamma[u]$ is resolved.

A mechanism $\Gamma$ fully implements a social choice rule $F$ if the following diagram commutes, i.e., $h(\sigma(\Gamma[u])) = F(u)$ for all $u \in U$.

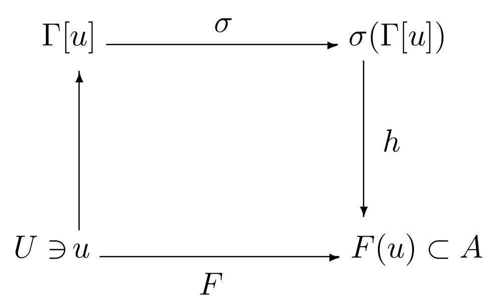


Diagram 1

# 3 Oligopoly Regulation

We will now introduce a regulatory mechanism design problem for an oligopolistic market as a special kind of the construction in Section 2. Consider a single-product industry consisting of a (nonempty) finite set $N = \{1, ..., n\}$ of firms.

The set of alternatives $A$ is given by

$$A = \{(q,t) \in [0,\bar{q}]^n \times [t^l, t^u]^n \mid \sum_{i \in N} q^i \leq \bar{q}, \text{ and } \sum_{i \in N} t^i \leq t^u\},$$

where $\bar{q} > 0$ is the maximum amount of industry output, and $t^l$ and $t^u$ are respectively the lower and upper bounds of the set of feasible transfers, which bounds will be characterized later. The $i$th component $q^i$ of $q \in [0, \bar{q})^n$ represents the output level of firm $i$ and the $i$th component $t^i$ of $t \in [t^l, t^u]^n$, the transfer made to firm $i$ from the consumers in the presence of the output vector $q$ for each $i \in N$.

We assume that each firm $i \in N$ has a cost function $C^i(\theta^i, q^i)$ where $\theta^i$ is the private cost parameter of the firm $i$ which is known to lie in the interval $\Theta^i = [\theta_0^i, \theta_1^i]$ with $\theta_0^i < \theta_1^i$ for all $i \in N$. The form of the cost function $C^i(.,.)$ for all $i \in N$ is assumed to be common knowledge, so all possible states of the society are given by $\Theta = \Theta^1 \times ... \times \Theta^n$. We further assume that[2] $C^i(\theta^i, 0) = 0$ and $C^i_{q^i} > 0, C^i_{\theta^i} > 0, C^i_{q^i,\theta^i} > 0$, and $C^i_{q^i,q^i} > 0$ for all $i \in N$ at all $q^i \in [0, \bar{q})$ and $\theta^i \in \Theta^i$.

Let $P(Q)$ be the inverse demand function of our oligopoly, where $P$ is continuous, decreasing and concave in the total output $Q = \sum_{i\in N} q^i \in [0, \bar{q})$ produced by $N$. Moreover we assume that $P(\bar{q}) = 0$.

The net gain of each firm $i \in N$ is given by

$$NPG^i(\theta^i; q, t^i) = P(Q)q^i - C^i(\theta^i, q^i) + t^i, \tag{1}$$

leading to the total net producers' gain

$$NPG(\theta; q, t) = \sum_{i\in N} NPG^i(\theta^i; q, t^i), \tag{2}$$

while the net consumers' gain is

$$NCG(\theta; q, t) = \int_0^Q (P(x) - P(Q))dx - \sum_{i\in N} t^i. \tag{3}$$

Now we define the set of feasible transfer payments as $[t^l, t^u] = [\bar{t} - z - \epsilon, \underline{t} + \epsilon]$ where $\epsilon > 0$ and

$$\begin{aligned}
\bar{t} &= -\max_i \max_{\theta^i} \max_q \; P(Q)q^i - C^i(\theta^i, q^i) \\
\underline{t} &= -\min_i \min_{\theta^i} \min_q \; P(Q)q^i - C^i(\theta^i, q^i) \\
z &= \max_{\theta^i} \max_q \; u^2(\theta^2; (q^2, t^u))
\end{aligned}$$

As usual, we define the social welfare function as of the form

$$\begin{aligned}
SW(\theta; q, t) &= NCG(\theta; q, t) + \alpha\, NPG(\theta; q, t) \\
&= \int_0^Q P(x)dx - \sum_{i=1}^n C^i(\theta^i, q^i) - (1-\alpha)NPG(\theta; q, t) \qquad (4)
\end{aligned}$$

[2] We also assume that demand and cost functions are such that it is socially optimal to allow all firms to produce under complete information.

with $\alpha \in [0,1]$. For any $\alpha \in [0,1]$, $F_\alpha : \Theta \twoheadrightarrow A$ is the social choice correspondence where for each $\theta \in \Theta$, $F_\alpha(\theta)$ is the set of alternatives in $A$ which maximize the social welfare function $SW(\theta; ., .)$ subject to the participation (individual rationality) constraints for consumers and for each producer, respectively,

$$NCG(\theta; q, t) \geq 0, \tag{5}$$

$$NPG^i(\theta; q, t^i) \geq 0, \quad \text{for all } i \in N. \tag{6}$$

Formally,

$$F_\alpha(\theta) = \{arg \max_{(q,t)\in A} SW(\theta; q, t) | (5) \text{ and } (6) \text{ hold}\}. \tag{7}$$

Using (4) we obtain

$$\begin{aligned} F_\alpha(\theta) \; = \; & \{(q,t) \in A | P(Q) = C^i_{q^i}(\theta^i, q^i), \text{ and} \\ & (1-\alpha) NPG^i(\theta; q, t) = 0, \; \forall i\} \end{aligned} \tag{8}$$

Both Loeb and Magat (1979) and Baron and Myerson (1982) (B-M) showed that when $n = 1$ in our setting (i.e., when the market is monopoly) the social choice correspondence $F_\alpha$ can be implemented if $\alpha = 1$. Later Koray and Saglam (1995a, 1995b) showed that $\alpha = 1$ is both a necessary and a sufficient condition for the social choice rule of a similar yet more general form as $F_\alpha$ to be implementable in a non-Bayesian equilibrium under a generalized principle-agent framework which includes B-M model as a special case. To avoid the negative result arising when $\alpha \in [0,1)$, B-M introduced a Bayesian regulator who implements a *second-best* social choice rule associated with an *expected* social welfare function.

Later, Saglam (1997) extended the Bayesian approach of B-M to an oligopoly consisting $n \geq 2$ firms. Koray and Saglam (1995a, 1995b) showed that in the Bayesian regulation model of B-M as well as in any Bayesian principal-agent framework, a non-benevolent regulator can manipulate the mechanism the society expects him to design and hence obtain dishonest gains by misrepresenting his beliefs whenever they are not verifiable, since there exists admissible beliefs the designer may choose which are almost[3] best or worst in either ex-ante or ex-post sense from the viewpoint of consumers (principal) and (or) the producer (agent). Therefore, if the possibility of such an opportunistic behaviour of regulators are not taken into consideration in designing Bayesian mechanisms, the realized outcome of the selected mechanism may be totally different from what the traditional theory predicts unless the regulator has a belief which is commonly agreed and verified by the society, which is a condition too strong to be held under many circumstances.

Note that in the situation where $n = 1$ all the equilibrium concepts reduce to a single agent's utility maximization. So, the Bayesian implementation, inspite of all the existing criticisms, is inevitable and seems to be the only normative approach in a natural monopoly regulation when $\alpha \neq 1$, owing to the negative result in non-Bayesian implementation. However, when number of players in any environment with incomplete information is greater than one, to make a generalization of the negative result for the monopoly case to the (nondegenerate) oligopoly case, one needs to show that there exists no equilibrium concept via which the social choice correspondence $F_\alpha$ in the oligopolistic market can be implemented.

As a natural but not necessarily complete step to arrive at a conclusion as to whether Bayesian implementation is indispensable in oligopolistic markets, in what follows, we will consider implementation through Nash equilibrium, which is the most intensely used solution concept in the literature.

# 4 Results

We have already discussed that in a monopolistic market (i.e., $n = 1$) the SCC $F_\alpha$ is not implementable unless $\alpha = 1$.

We will now check whether $F_\alpha$ is Nash implementable in a 'literally' oligopolistic market (i.e., $n > 1$). We have to consider two cases, namely $n = 2$ and $n \geq 3$, separately as the necessary and sufficient conditions for Nash implementation are different in those two cases.

[3] Although the best belief from the viewpoint of the principal or the worst belief from the viewpoints of both the principal and agent, hence the society, are not admissible in an incomplete information as they are in the form of a Dirac function, there exists some admissible beliefs which lead to, for almost all possible states of the society, almost same (arbitrarily close) welfare implications on the part of both the principal and the agent.

## 4.1 Implementation with Three or More Firms

**Definition 1:** (Maskin (1977, 1985)) An SCC $f$ satisfies weak no-veto power if, for all $(u^1, ..., u^n) \in U^1 \times ... \times U^n$ and $a \in A$, $a \in F(u^1, ..., u^n)$ whenever there exists $i$ such that for all $j \neq i$ and all $b \in A$, $u^j(a) \geq u^j(b)$.

**Definition 2:** ( Muller and Satterthwaite (1977) and Moulin and Peleg (1982)) An SCC $f$ is monotonic if, for all $(u^1, ..., u^n), (\bar{u}^1, ..., \bar{u}^n) \in U^1 \times ... \times U^n$ and $a \in A$, $a \in f(\bar{u}^1, ..., \bar{u}^n)$ whenever (i) $a \in f(u^1, ..., u^n)$ and, (ii) for all $b \in A$ and $i$, $u^i(a) \geq u^i(b)$ implies $\bar{u}^i(a) \geq \bar{u}^i(b)$.

**Theorem:** (Maskin (1977, 1985)) *Suppose that $f$ is an n-person SCC. If $n \geq 3$ and $f$ satisfies weak no-veto power and monotonicity, then it is fully implementable.*

It is known that in economic environments the Pareto correspondence, which selects all Pareto optima corresponding to a given utility profile, is Nash implementable. Note that weak no-veto power is vacuously satisfied in economic environments as agents cannot be simultaneously satiated at the same outcome. So to show that Pareto correspondence is Nash implementable, it is left to check they are monotonic. Let $f$ be the Pareto correspondence in an economic environment. Take $u \in U$ and $a \in f(u)$. Then $a$ is Pareto efficient at $u$. Take $\bar{u} \in U$ such that for all $i$, and $b \in A$, $u^i(a) \geq u^i(b)$ implies $\bar{u}^i(a) \geq \bar{u}^i(b)$ but $a \notin f(\bar{u})$. Since $f$ is Pareto correspondence, there exists some $c \in A$ such that $\bar{u}^i(c) \geq \bar{u}^i(a)$ for all $i$ and $\bar{u}^j(c) > \bar{u}^j(a)$ for some $j$. Then it must be true that $u^i(c) \geq u^i(a)$ for all $i$ and $u^i(c) > u^i(a)$ for some $j$, contradicting that $a$ is Pareto efficient at $u$.

Now using Maskin's theorem we will show that the social choice correspondence $F_\alpha$ associated with the oligopoly model in Section 3 is Nash implementable for all $\alpha \in [0, 1]$, whenever $n \geq 3$. Let $u^i(\theta; (q, t))$ represent $NPG^i(\theta; q, t^i)$ for all $i \in N$. Note that consumers' utility function $NCG$ is common knowledge; thus under any mechanism which implements the social choice rule $F_\alpha$ consumers do not occur as players, though the social choice rule depends on their utility function.

**Proposition 1:** *Assume $\alpha \in [0, 1]$ and $n \geq 3$. Then $F_\alpha(.)$ is implementable in Nash equilibrium.*

**Proof:** We will first show that $F_\alpha$ satisfies weak no-veto power. Let $\theta \in \Theta$. Define $a_{(i)} = (q_{(i)}, t_{(i)}) \in A$ for all $i \in N$ as follows:

$$P(Q_{(i)}) = -P'(Q_{(i)})q^i_{(i)} + C^i_{q^i}(\theta^i, q^i_{(i)}), \quad t^i_{(i)} = t^u, \quad q^j_{(i)} = t^j_{(i)} = 0, \quad \forall j \neq i.$$

It is obvious that for all $i \in N$ and $b \in A$, $u^i(\theta; a_i) \geq u^i(\theta; b)$ and for all $i \neq j$ we have $a_{(i)} \neq a_{(j)}$. Thus, no two firms will agree that any given alternative is top ranked, since each (most) prefers being a monopolist and to get maximal amount of transfer. As there are at least three firms in our oligopolistic market by assumption, weak no-veto power is (vacuously) satisfied.

To check monotonicity condition, we first note that $F_\alpha$ coincides with Pareto correspondence whenever $\alpha = 1$, (as seen from equation (9)). So $F_1$ is monotonic.

Whenever $\alpha \in [0, 1)$, $F_\alpha$ is single-valued and hence it does not coincide with Pareto correspondence, implying that monotonicity condition should be checked. Let $\theta \in \Theta$ and $x = (q, t) = F_\alpha(\theta)$. Now take any $\theta_1 \in \Theta$ such that $\theta_1 \neq \theta$. Then there exists $i \in N$ such that $\theta^i_1 \neq \theta^i$. Fix the firm $i$. Let $a = (q_a, t_a),\ b = (q_b, t_b) \in A$ be such that[4]

$$\begin{array}{ll} q^i_a = q^i/2, & u^i(\theta; a) = 0, \quad \text{and} \\ q^i_b = (q^i + \bar{q})/2, & u^i(\theta; b) = 0. \end{array}$$

Now define $c = (q_c, t_c)$ as follows:

$$c = \begin{cases} a & \text{if } \theta^i_1 > \theta^i \\ b & \text{if } \theta^i_1 < \theta^i \end{cases}$$

We have $u^i(\theta; x) \geq u^i(\theta; c)$ satisfied. Note that

$$\begin{aligned} u^i(\theta_1; c) &= P(Q_c)q^i_c - C^i(\theta^i_1, q^i_c) + t^i_c \\ &= C^i(\theta^i, q^i_c) - C^i(\theta^i_1, q^i_c) \end{aligned}$$

as $u^i(\theta; c) = 0$ by construction. Similarly, we have

$$u^i(\theta_1; x) = C^i(\theta^i, q^i) - C^i(\theta^i_1, q^i)$$

[4]Note that $t_a, t_b \in [t^l, t^u]$ guarantees for the existence of such a construction.

as $u^i(\theta; x) = 0$ by the supposition that $x = F_\alpha(\theta)$. But then $u^i(\theta_1; x) < u^i(\theta_1; c)$ due to the assumption $C^i_{\theta^i, q^i} > 0$ and the construction that the sign of $(q^i_c - q^i)(\theta^i_1 - \theta^i)$ is negative. Thus, hypothesis (ii) of monotonicity is not satisfied, and therefore the SCC $F_\alpha(.)$ is monotonic.

Since the SCC $F_\alpha$ satisfies both monotonicity and weak no-veto power for all $\alpha \in [0, 1]$, by Maskin's theorem it is implementable in Nash equilibrium for all $\alpha \in [0, 1]$ and $n \geq 3$. □

## 4.2 Implementation in a Duopoly

We will use two-person implementation results by Dutta and Sen (1991) to show that social choice rule $F_\alpha(.)$ can be implemented in a duopolistic market.

**Definition 3:** For any $i \in I$, $u \in U$ and $a \in A$, let $L^i(u, a)$ and $SL^i(u, a)$ denote the sets $\{c \in A | u^i(a) \geq u^i(c)\}$ and $\{c \in A | u^i(a) > u^i(c)\}$, respectively.

**Definition 4:** For any $i \in I$, $u \in U$ and $D \subseteq A$, let $M^i(u, D)$ denote the maximal elements in $D$ for agent $i$ according to utility function $u^i$, that is $M^i(u, D) = \{a \in D | u^i(a) \geq u^i(d),\ \forall d \in D\}$.

**Definition 5:** (Dutta and Sen (1991)) An SCC $f$ satisfies Condition $\beta$ if there exists a set $A^*$ which contains the range of $f$, and for each $i \in I$, $u \in U$ and $a \in f(u)$ there exists a set $D^i(u, a) \subseteq A^*$, with $a \in D^i(u, a) \subseteq L^i(u, a)$ such that for all $u_1 \in U$, we have

(i) (a) for all $b \in f(u_1)$, $D^1(u, a) \cap D^2(u_1, b) \neq \emptyset$. (b) Moreover, there exists $x \in D^1(u, a) \cap D^2(u_1, b)$ such that if for some $u_2 \in U$, $x \in M^1(u_2, D^1(u, a)) \cap M^2(u_2, D^2(u_1, b))$, then $x \in f(u_2)$.

(ii) if $a \neq f(u_1)$, there exists $j \in I$ and $b \in D^j(u, a)$ such that $b \notin L^j(u_1, a)$.

(iii) $[M^i(u_1, D^i(u, a)) \backslash \{a\}] \cap M^j(u_1, A^*) \subseteq f(u_1)$ $\forall i \in I$ and $j \neq i$.

(iv) $M^1(u_1, A^*) \cap M^2(u_1, A^*) \subseteq f(u_1)$.

**Theorem:** (Dutta and Sen (1991)) *The SCC $f$ is implementable if and only if it satisfies condition $\beta$.*

**Proposition 2:** *Assume $\alpha \in [0, 1]$ and $n = 2$. Then $F_\alpha(.)$ is implementable in Nash equilibrium.*

**Proof:** Let $A^* = A$ and $\theta \in \Theta$. Define $a \equiv (q_a, t_a) \in F_\alpha(\theta)$. Let $D^1(\theta, x) = \{x\} \cup SL^1(\theta, x)$ and $D_2(\theta, x) = \{x\} \cup SL^2(\theta, x)$ for all $x \in A$. Take any $\theta_1 \in \Theta$, and let $b \equiv (q_b, t_b) \in F_\alpha(\theta_1)$.

**Part 1:** Let $c \equiv (q_c, t_c) \in A$ be such that $q_c = q_a$ and $t_c < min\{t_a, t_b + u^2(\theta_1; b) - u^2(\theta_1, (q_a, t_b))\}$. We have $c \in D^1(\theta, a)$ since $u^1(\theta, c) = u^1(\theta; (q_a, t_c))$ $= u^1(\theta; (q_a, t_a)) + (t_c - t_a) < u^1(\theta; a)$. Moreover, we have $c \in D^2(\theta_1, b)$ since $u^2(\theta_1; c) = u^2(\theta_1; (q_a, t_c)) = u^2(\theta_1; (q_a, t_b)) + (t_c - t_b) < u^2(\theta_1; b)$. So $\beta$-(i)-(a) is satisfied. To check $\beta$-(i)-(b), note that

$$M^1(\theta_2, D^1(\theta, a)) = \begin{cases} a & \text{if } \theta_2^1 = \theta^1 \\ \emptyset & \text{otherwise} \end{cases}$$

and

$$M^2(\theta_2, D^2(\theta_1, b)) = \begin{cases} b & \text{if } \theta_2^2 = \theta_1^2 \\ \emptyset & \text{otherwise} \end{cases}$$

It is clear that $a = b$ if and only if $\theta = \theta_1$ since $C_{\theta^i, q^i} > 0$. Thus,

$$M^1(\theta_2, D^1(\theta, a)) \cap M^2(\theta_2, D^2(\theta_1, b)) = \begin{cases} a & \text{if } \theta^2 = \theta_1 = \theta \\ \emptyset & \text{otherwise} \end{cases}$$

If $\theta_2 = \theta_1 = \theta$, then $M^1(\theta_2, D^1(\theta, a)) \cap M^2(\theta_2, D^2(\theta_1, b)) = a \in F_\alpha(\theta^2)$. Therefore, $\beta$-(i)-(b) is also satisfied.

**Part 2:** Take $\theta_1 \in \Theta$ such that $a \notin F_\alpha(\theta_1)$. Since $C_{\theta_i, q^i} > 0$ for all $i$ by assumption, we must have $\theta_1 \neq \theta$, and so there exists some $j$ such that $\theta_1^j \neq \theta^j$. Take such a firm $j$. Note that

$$u^j(\theta_1; (q, t)) = u^j(\theta; (q, t)) + C^j(\theta^j, q^j) - C^j(\theta_1^j, q^j).$$

Let $q_d$ be such that

$$q_d^j = \begin{cases} q_a^j/2 & \text{if } \theta_1^j > \theta^j, \\ (q_a^j + \bar{q})/2 & \text{if } \theta_1^j < \theta^j. \end{cases}$$

Denote $K = C^j(\theta^j, q_d^j) - C^j(\theta_1^j, q_d^j) - [C^j(\theta^j, q_a^j) - C^j(\theta_1^j, q_a^j)]$. We know that $K$ is positive since $sign(\theta_1^j - \theta^j)(q_d - q_a) < 0$, and $C^j_{\theta^j, q^j} > 0$. There exists some $t_d \in R^n$ such that $-K < u^j(\theta;(q_d.t_d)) - u^j(\theta;a) < 0$. Take such a transfer $t_d$ and define $d \equiv (q_d, t_d)$. We note that $d \in D^j(\theta, a)$. Now we have

$$\begin{aligned} u^j(\theta_1;d) - u^j(\theta_1;a) &= u^j(\theta;d) - u^j(\theta;a) \\ &\quad + C^j(\theta^j, q^j_d) - C^j(\theta^j_1, q^j_d) - [C^j(\theta^j, q^j_a) - C^j(\theta^j_1, q^j_a)] \\ &\geq 0, \end{aligned}$$

implying that $d \notin L^j(\theta_1, a)$. Therefore, $\beta$-(ii) is satisfied.

**Part 3:** Take any $i$. Note that for $j \neq i$, we have $M^j(\theta_1, A^*) = (q,t)$ such that $q^i = t^i = 0$, $P(q^j) = -P'(q^j)q^j + C_{q^j}(\theta_1, q^j)$ and $t^j = t^u$. We also have

$$M^i(\theta_1, D^i(\theta, a)) = \begin{cases} a & \text{if } \theta^i_1 = \theta^i \\ \emptyset & \text{otherwise.} \end{cases}$$

Thus $[M^i(\theta_1, D^i(\theta,a)) \backslash a] \cap M^j(\theta_1, A^*) = \emptyset$, for all $i$ and $j \neq i$. Therefore, $\beta$-(iii) is (vacuously) satisfied.

**Part 4:** We have $M^i(\theta_1, A^*) = (q_{(i)}, t_{(i)})$ such that $q^j_{(i)} = t^j_{(i)} = 0$ for $j \neq i$, $P(q^i_{(i)}) = -P'(q^i_{(i)})q^i_{(i)} + C_{q^i_{(i)}}(\theta_1, q^i_{(i)})$ and $t^i_{(i)} = t^u$. Thus $M^1(\theta^1, A^*) \cap M^2(\theta^1, A^*) = \emptyset$. Therefore, $\beta$-(iv) is (vacuously) satisfied.

Since condition $\beta$ is satisfied, $F_\alpha$ is Nash implementable for all $\alpha \in [0,1]$, whenever $n = 2$. $\square$.

# 5 Conclusions

Bayesian implementation, or rather changing the social objectives in Bayesian sense, may be the only possible solution, even in situations where all agents in the society but the designer have the complete information about the state of the society, if the ex-post socially efficient outcome can not be implemented in any equilibrium concept. In this study, we showed that in oligopolistic markets with at least two firms this is certainly not the case, and hence Bayesian implementation in oligopoly regulation is not inevitable under all circumstances regarding the informational asymmetries among the firms about the state of the market. Indeed, the monopoly market where consumers are favored by the designer or the society, is the only oligopolistic framework where Bayesian implementation is *indispensable*.

As a future research one may consider the problem of implementing social optima in oligopolies when demand is unknown. One may also extend the results of this paper to a general principal-multi agent framework.

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