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Patents as Collateral*

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Abstract

This paper studies how the assignment of patents as collateral determines the savings of firms and magnifies the effect of innovative rents on investment in research and development (R&D). We analyse the behaviour of innovative firms that face random and lumpy investment opportunities in R&D. High growth rates of innovations, possibly higher than the real rate of interest, may be achieved despite financial constraints. There is an optimal level of publicly funded policy by the patent and trademark office that minimizes the legal uncertainty surrounding patents as collateral and maximizes the growth rate of innovations.

Keywords: Collateral, Patents, Research and Development, Credit rationing, Growth, Innovation.

JEL Codes: D92, G24, G32, O16, O41, O34.

“It is the uncertainty created by this legal and regulatory structure [in the United States] which leads to the very market imperfections

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and inefficiencies currently minimizing the ability to leverage the value of intellectual property assets and consequently stunting the economic growth of inventors and entrepreneurs,” Murphy (2002) report to the United States Patent and Trademark Office (USPTO).

1. Introduction

The practice of using a valuable patent portfolio as collateral for a debt assignment is slowly becoming more and more important in the United States and elsewhere.¹ It follows a start-up stage financed by venture capital where the firm obtained at least one valuable patent. After an initial public offering, many innovators still lack the capital necessary to develop new research and must turn to outside sources for funding. For these innovative firms, there is now considerable empirical evidence that variables related to financing constraints such as debt/assets ratio and/or cash flow availability are correlated with R&D investment (see Hall’s (2002) survey). Due to the limited availability of physical capital as collateral, innovators face an external finance constraint. With asymmetric information on the state of the R&D project, additionally, problems of adverse selection and moral hazard occur. Blundell, Griffith and Van Reenen (1999) explain: *“A more traditional interpretation of the innovation-market power correlation is that failures in financial markets force firms to rely on their own supra-normal profits to finance the search for innovation. The availability of internal sources of funding (‘deep pockets’) are useful for all forms of investment, but may be particularly important for R&D”*. In the knowledge economy, wealth creation is increasingly based on innovation that, in turn, can give rise to important intellectual property rights. For many companies, these intellectual property rights represent their most valuable assets. Patent-backed loans increase the availability of external funds and the return on equity for the shareholders.

In the United States, the potential effect of patent-backed loans on the growth of innovation is estimated to be important for the following reasons. First, the stock of potential untapped intangible collateral is by now huge. Corrado, Hulten

¹Since the end of the 1990s, several IPR intermediaries services (Ocean Tomo and Patent Ratings, M-CAM, PLX, etc.) provide valuations of patents as collateral information. The Development Bank of Japan since 1995 and the Landesbank Rheinland-Pfalz in Germany use patents as collateral for loans to venture firms (Kamiyama, S., J. Sheehan and C. Martinez (2006)).

and Sichel (2006)² estimate investment in intangible assets to be \$1.2 trillion per year for the period 2000-2003 (a level of investment that roughly equals the gross investment in corporate tangible assets), including \$230 billion in innovative property of scientific R&D, besides innovative property of non-scientific R&D and computer software. Depending on its depreciation rate, the stock of intangible assets may be five to ten times this level of investment. Second, post initial public offering (IPO) shareholders of innovative firms have a strong monetary incentive to use patent-backed debt instead of new share issues: no dilution of capital and a leverage multiplier effect on their return on equity. Third, the patent backed loan industry is fostered by intellectual property rights (IPR) lawyers, IPR valuation, technology and financial intermediaries and IPR insurance firms, who lobby for the required legal and regulatory changes to be supported by the USPTO. Fourth, the share of measured innovations (patents, R&D spending) by older firms owning at least one valuable patent (that could be used as collateral) is much larger than the one of start-ups financed by venture capital in the United States.³ The pool of innovative firms with new projects, likely to use patent-backed loans, is broad.

Kiyotaki and Moore's (1997) and Kiyotaki's (1998) seminal papers deal with the magnifying effects of collateral availability constraints in order to explain business cycles movements. Their framework paved the way to new studies of monetary policy and housing prices (Iacoviello (2005)) or asset prices, the credit channel, liquidity in closed or open economies (e.g. Faia and Monacelli (2007), Gertler, Gilchrist and Natalucci (2007), Kato (2006), Kunieda and Shibata (2005), Moretto and Tamborini (2007), Bougheas, Mizen and Yalcin (2006), Cordoba and Ripoll (2004), Amable, Chatelain and Ralf (2004), Chatelain (2001)). Kiyotaki and Moore's (1997) framework also tackles the issue of lumpy and irreversible investment, leading to recent extensions by Caggese (2007) and Sveen and Weinke (2007). Lumpiness is also an observed characteristic of R&D investment in lab equipment (Geroski, Van Reenen and Walters (1997), Aghion et al. (2007)). But collateral issues remained confined to business cycles theory.

This paper introduces patents as collateral in the context of expanding variety growth models dealing with intellectual property rights (Rivera Batiz and Romer

²In their table 2.

³Kamiyama, S., J. Sheehan and C. Martinez (2006) mention that high value patents are among the most important factors (along with good management) that venture capitalists consider in their investment decisions in shares. In the pre-investment phase, the availability of a patent might also be seen as a liquidation benefit for the venture shareholders, given that the key patents may still be sold or redistributed if the company does not succeed. See also Keuschnigg (2004) on venture capital driven growth.

(1991), Amable et Chatelain (1995), Kwan and Lai (2003), Boucekkinne and de la Croix (2003), Donoghue and Zweimüller (2004), Barro and Sala-I-Martin (2004), Gancia and Zilibotti (2005), Strulik (2007), Furukawa (2007)). These papers already discussed various arguments for and against intellectual property rights granting monopoly rights to patent holders. The novel point here is to consider the prospective consequences of a large development of the use of patents as collateral on economic growth, as this practice is likely to spread in the next decades.

The paper has three goals: Firstly, it provides the condition for a significant leverage effect of the collateral assignment of patents on the growth of innovation. It shows in particular that the dependence of innovations on past innovations increases with innovative rents *relatively more* than in standard expanding variety growth models based on R&D (Romer and Rivera Batiz (1990), Barro and Sala-I-Martin (2004)). Secondly, it models the financial constraints on individual firms savings, the aggregate debt-equity ratio, and economic growth. Finally, we show that the rate of return of innovation is higher than the credit interest rate in a growing economy and that the growth of patents is a decreasing function of the interest rate. This is not the case in the standard R&D endogenous growth models. The model differs from the Kiyotaki and Moore's (1997) credit cycle model in various ways: the size of the aggregate capital stock is no longer fixed, but may grow over time, and expected monopoly rents on existing patents are used as collateral, so that they increase the value of collateral, the available amount of loans and economic growth. The model is the first one dealing with the assignment of patents as collateral in the economic literature.

The model recommends an institutional policy, which has been much less advocated by economists than by lawyers (Murphy (2002)), improving the laws dealing with security interest in patents in order to greatly reduce the uncertainty surrounding the use of patents as collateral. For example, the United States Patent and Trademark Office registers transactions transferring patents to different owners than the inventor, so-called assignments, in the Patent Assignments Database (Serrano (2008)), but it does not hold a registry of collateral assignment of patents conditional to borrowers' default (Murphy (2002)), paving the way for increased uncertainty of transfers of IPR. Transfers of property rights over the income of patents should become enforceable, not only against the debtor, but also against competing creditors at low cost. Lenders have to be protected against the borrower's ability to transfer, abandon or license the patent collateral and against the borrower's lack of continued patent maintenance, prosecution and exploitation. These legal improvements are a way to rise the aggregate debt ceiling and

the growth of innovations. Moreover, the model specifies how such a leverage may lead to high speed growth of the knowledge economy. A credit constraint based on the value of already existing patents rules out Ponzi finance, so that a high speed growth rate of innovations may exceed the interest rate on patent backed loans in equilibrium.

The paper is organized as follows. The microeconomic behaviour of agents is described in section 2. Section 3 provides the conditions for steady state aggregate growth. Section 4 concludes the paper with a discussion of the results and related research.

2. The model

We consider a lab-equipment model of expanding variety (Rivera-Batiz and Romer (1991), Barro and Sala-I-Martin (2004)), which is directly related to R&D investment equations estimated in applied work (Blundell et al. (1999)). As in other “increasing product variety” models (Romer (1990), Grossman and Helpmann (1991)), the economy has three sectors of production: a final goods sector, whose price is taken as numeraire, an intermediate goods sector, whose output is used in the production of the final good and an R&D sector in which blueprints allowing the creation of new intermediate goods are discovered.

2.1. Households

A constant population of wage-earning households is distributed on $[0, L]$. A household maximizes utility over an infinite time horizon:

$U_t = \sum_{\tau=0}^{+\infty} \beta^\tau u(c_{t+\tau})$ with $u(c_t) = (c_t^{1-\sigma} - 1)/(1 - \sigma)$ for $\sigma > 0$ and $\sigma \neq 1$ or with $u(c_t) = \ln(c_t)$ for $\sigma = 1$. $c_t \geq 0$ is consumption in t , $\rho \geq 0$ is the rate of time preference, $\beta = 1/(1 + \rho)$ is the discount rate and σ is the relative fluctuation aversion. Households supply inelastically one unit of labour used in the final goods industry and are paid a real wage rate w_t . They have no disutility of labour. They lend to entrepreneurs and earn a rate of return r_t on their wealth b_t^h . The law of motion of their wealth is $b_t^h = (1 + r_{t-1})b_{t-1}^h + w_t - c_t$. The initial wealth b_0^h is given and identical for all households. Taking into account the optimum consumption plan of the households, consumption growth g_c is given by

$$1 + g_{c,t+1} = c_{t+1}/c_t = C_{t+1}/C_t = (\beta R_t)^{\frac{1}{\sigma}}, \quad (1)$$

where $C_t = c_t L$ denotes aggregate household consumption and $R_t = 1 + r_t$. The growth rate of household consumption increases with the return on savings and decreases with the rate of time preference and with the relative fluctuation aversion σ .

2.2. Production of the final good

A large number of producers of the final good, indexed by j , operate in perfect competition. Producer j produces the quantity Y_{jt} according to a constant return to scale production function:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^{N_t} X_{jt}(i)^\alpha di \text{ with } 0 < \alpha < 1. \quad (2)$$

A producer uses labour and intermediates as inputs that are fully used up within the period. $X_{jt}(i)$ is the amount of intermediate good i used by producer j on a set $\{X_{jt}(i), i \in [0, N_t]\}$. N_t represents the most recently invented intermediate good, so that the interval $[0, N_t]$ is the variety of intermediate goods available in the economy. The representative producer of final goods maximizes profit while buying intermediate goods. This leads to the following demand function for intermediate inputs:

$$X_{jt}(i) = \left(\frac{\alpha}{p_{it}} \right)^{1/(1-\alpha)} L_{jt} \text{ for } i \in [0, N_t]. \quad (3)$$

2.3. Production of intermediate goods

The firm producing an intermediate non-durable good, indexed by i , acts as a monopolist selling to final good producers at a price which adds a mark-up to marginal costs. A rent π_{it} has to be paid to the innovator for using his blueprint at each date t . Production of intermediate goods takes place at constant marginal cost, normalized to 1. Taking into account the demand for intermediate inputs (see equation (3)), the producer of an intermediate good maximizes

$$\max_{p_{it}} \pi_{it} = (p_{it} - 1) \sum_j X_{jt}(i). \quad (4)$$

The solution for the monopoly price is

$$p_{it} = p = \frac{1}{\alpha} > 1. \quad (5)$$

Hence, the price p_{it} is constant over time and the same for all intermediate goods i . The aggregate quantity of each intermediary good produced and the monopoly profit are also constant over time, whereas the aggregate level of final output Y_t is proportional to the number of intermediate goods N_t . Output net of intermediate goods is then:

$$Y_t - XN_t = \left(\frac{1}{\alpha} + 1 \right) \pi N_t. \quad (6)$$

2.4. R&D sector: technology and finance

Every period, a continuum of risk neutral entrepreneurs distributed over the interval $[0, 1]$ is engaged in the R&D activity. On date 0, each entrepreneur k receives an initial dividend $d_{0,k}$ that he spends on consumption and has an initial endowment of $n_{0,k}$ of *valuable* blueprints.⁴ The aggregate number of patents on date 0 is denoted N_0 and aggregate dividends D_0 .

Utility is given as the expected present value V_0 of dividends $d_t \geq 0$. Future dividends are discounted using the market interest rate r_t . E_0 is the expectation operator at date 0:

$$\text{Choose } n_t, b_t, i_t, d_t \text{ that } \max V_0 = E_0 \left[d_0 + \sum_{\tau=1}^{\tau=+\infty} \frac{d_\tau}{\prod_{T=1}^{T=\tau} (1 + r_T)} \right]. \quad (7)$$

The entrepreneur maximizes his utility. He chooses the state variables which are the stock of patents n_t and the stock of debt b_t , (with given initial endowments $b_{0,k}$ and $n_{0,k}$) by changing the control variables: new patents i_t and dividends d_t . His utility is subject to constraints. First, there are two equality constraints: the law of motion of patents derived from the innovation process, the law of motion of debt (or net worth) derived from the flow of funds constraint. Second, there are two inequality constraints: the saving ceiling (or minimal consumption, or positive dividend) and the debt ceiling.

Innovation on date t depends on two factors: The entrepreneur has to have new ideas and he has to provide his firm specific labour. The first factor accounts for the fact that the entrepreneur may find a number of positive net present value ideas leading to new valuable patents ($\tilde{1}_{i_t > 0} = 1$, where i_t is the number of new blueprints obtained in a period) only with probability θ ($0 < \theta \leq 1$). With probability $1 - \theta$, he will have no ideas ($\tilde{1}_{i_t > 0} = 0$). This factor is motivated by the

⁴The following model of aggregate growth of innovations holds for *all* types of initial distributions of patents $n_{0,k}$ among entrepreneurs.

observation that R&D lab-equipment investment is lumpy (Geroski, Van Reenen, Walters (1997)). The value of the random variable $\tilde{1}_{i_t > 0}$ is known by the entrepreneur at the beginning of the period t , but is not observable for the creditor. The second factor accounts for the fact that the entrepreneur may supply inelastically one unit of R&D specific labour ($h_t = 1$) in her own firm, without disutility of labour, or withdraw her labour ($h_t = 0$) leading to zero investment in R&D. This assumption captures the informational asymmetry between lenders and innovators describing a problem of moral hazard. It is too costly for lenders to enforce loan repayment with an ex post assessment discriminating zero investment related to “not working” versus zero investment related to “having no ideas” ($\tilde{1}_{i_t > 0} = 0$). The fact that the labour input of the entrepreneur is R&D specific and cannot be carried out by a hired worker enables us to avoid labour sectoral allocation problems as in Romer (1990). The stock of blueprints of the entrepreneur evolves then according to the following equation:

$$n_t = \left(\tilde{1}_{i_t > 0} \cdot h_t \right) \cdot i_t + (1 - \delta) n_{t-1} \quad (8)$$

where $\delta \geq 0$ is a hazard rate related to the opposition and litigation due to an identical, close or overlapping prior innovation, so that the related intermediate goods disappear.⁵

The flow of funds constraint states that dividends are equal to the profit at date t earned from previously discovered blueprints plus new debt net of interest repayment minus cost of investment in R&D:

$$d_t = \left(\frac{\pi}{q} - \delta \right) q n_{t-1} - r_{t-1} b_{t-1} + b_t - b_{t-1} - \left(\tilde{1}_{i_t > 0} \cdot h_t \right) \cdot q \cdot (n_t - n_{t-1}). \quad (9)$$

where q is the unit cost per patent granted: the cost function of R&D investment is linear when the entrepreneur does invest:

$$\left(\tilde{1}_{i_t > 0} \cdot h_t \right) \cdot q \cdot (n_t - (1 - \delta) n_{t-1}).$$

⁵A patent examiner must find evidence of prior art which can be elusive. Hence, litigation for close prior innovation is frequent when a new patent turns out to be profitable. One may also assume that some varieties would be eventually produced by competitive firms (Kwan and Lai (2003), Barro and Sala-I-Martin (2004)), which complicates this model in adding a fourth state variable for the stock of these varieties. The results of the model are unchanged when we assume $\delta = 0$.

Minimal consumption constraint: Entrepreneurs consume a minimum amount $d_m \geq 0$ which is written as a proportion $(1 - \beta')$ of firms net worth (equity). $\beta' \leq 1$ is then the entrepreneurs savings rate. If $\beta' = 1$, the entrepreneur may not consume anything and we get the usual assumption that dividends are greater than or equal to zero. Dividends have to be larger than the minimum consumption required by the entrepreneur:

$$d_t \geq d_m = (1 - \beta') (\Pi q n_{t-1} - R_{t-1} b_{t-1}) \geq 0, \quad (10)$$

where $\Pi = 1 + \frac{\pi}{q} - \delta$ (assuming a positive net return of innovation: $0 \leq \delta \leq \frac{\pi}{q} < 1$), $R_{t-1} = 1 + r_{t-1}$.

Financing: patents as collateral. A large number of risk neutral and perfectly competitive financial intermediaries pool households savings in order to diversify the risks related to the use of patents as collateral. An entrepreneur has always the ability to threaten its creditors to withdraw his labour input ($h_t = 0$), repudiate his debt contract, consume at the end of the period the stock of debt b_t and its rate of return $r_t b_t$ and find other creditors for next periods (Kiyotaki and Moore (1997)). This incentive to cheat exists as soon as the end of period value of the *stock* of debt is larger than the expected value of the *flow* of new patents denoted $v_{t+1}(i_t)$: $(1 + r_t) b_t > v_{t+1}(i_t)$. In case of default, a lender receives a random proportion, μ , $0 \leq \mu \leq 1$ of the value of the collateral. The value of the debt, therefore, should not exceed the discounted market value of the sum of existing patents for each entrepreneur. When patents are used as collateral, the entrepreneur has no longer an incentive to withdraw labour, because he will lose his stream of future incomes, so that: $h_t = 1$.

The loan to patent portfolio value $\mu/(1 + r_t)$ depends on the Patent Office public spending. This key assumption takes into account that lenders are not perfectly protected against the borrower's ability to transfer, abandon or license the patent collateral to a third party or to competing creditors at no legal costs (Schavey, 2003; Murphy, 2002). Lenders pool patent backed loans so that they take into account the expected proportion $0 \leq \mu \leq 1$ of the next period value of the current patent portfolio assigned as collateral $V_{t+1}(n_t)$ when deciding the amount to lend. When the existing patent portfolio is fairly valued, default never

occurs because lenders protect themselves by limiting the amount of the loan:⁶

$$(1 + r_t) b_t \leq \mu V_{t+1}(n_t) \text{ with} \quad (11)$$

$$V_{t+1}(n_t) = n_t \pi \left[1 + \sum_{\tau=1}^{+\infty} \frac{(1 - \delta)^\tau}{\prod_{T=1}^{T=\tau} (1 + r_T)} \right].$$

The patent portfolio n_t provides its first return πn_t on date $t + 1$. When the collateral constraint is not binding, we assume that there is free entry into the business of being an inventor so that any entrepreneur can pay the R&D cost qn_t to secure the net present value $\frac{V_{t+1}}{1+r_t}$. Free entry leads to a zero profit condition:

$$qn_t = \frac{V_{t+1}(n_t)}{1 + r_t}. \quad (12)$$

The equilibrium interest rate with free entry is $r^F = \frac{\pi}{q} - \delta$. When borrowing constraints hinder entry, then one has:

$$qn_t = \frac{V_{t+1}(n_t)}{1 + r^F} < \frac{V_{t+1}(n_t)}{1 + r_t}. \quad (13)$$

The limitations on borrowing at the interest r_t prevent that an infinite amount of resources would be channelled into R&D at time t . The credit constraint may be written as a “leverage” or debt/patent ratio x_t bounded by an endogenous ceiling x^c (the last equality holds for a steady state interest rate: $r_t = r$):

$$x_t = \frac{b_t}{qn_t} \leq x^c = \frac{\mu}{1 + r_t} \frac{V_{t+1}(n_t)}{qn_t} = \mu \frac{\pi}{q(r + \delta)}. \quad (14)$$

Innovators' behaviour: To sum up, entrepreneurs maximize their utility, with control variables i_t and d_t , and state variables n_t and b_t , n_0 and b_0 given:

$$V_0 = E_0 \left[d_0 + \sum_{t=1}^{t=+\infty} \frac{d_t}{\prod_{T=1}^{T=t} (1 + r_T)} \right]$$

⁶This assumption corresponds to current practice. Financial and intellectual property rights intermediaries providing patent backed loans diversify the collateral risk in pooling loans and/or obtaining infringement enforcement insurance or defence cost reimbursement insurance from insurance companies, such as Swiss Re and Intellectual Property Insurance Services Corporation.

subject to equality constraints

$$\begin{aligned} n_t &= \tilde{1}_{i_t > 0} \cdot i_t + (1 - \delta) n_{t-1}, \\ d_t &= \left(\frac{\pi}{q} - \delta \right) q n_{t-1} - r_{t-1} b_{t-1} + b_t - b_{t-1} - \tilde{1}_{i_t > 0} q (n_t - n_{t-1}), \end{aligned}$$

and to inequality constraints

$$\begin{aligned} 0 &\leq q n_t x^c - b_t, \\ 0 &\leq d_t - (1 - \beta') (\Pi q n_{t-1} - R_{t-1} b_{t-1}). \end{aligned}$$

Substituting consumption from the flow of funds equation (9) into the saving ceiling constraint (10) and using the debt ceiling constraint (14), one finds an upper limit on the growth of innovations determined by the ceiling of internal savings and by the debt ceiling:

$$(1 - x^c) q n_t + \left(\tilde{1}_{i_t > 0} - 1 \right) q (n_t - n_{t-1}) \leq \beta' (\Pi q n_{t-1} - R_{t-1} b_{t-1}). \quad (15)$$

When an innovator has an opportunity to invest ($\tilde{1}_{i_t > 0} = 1$), and when $x^c \geq 1$ (that is when $\mu \frac{\pi}{q} - \delta > r_t$), the above constraint does not set a ceiling on the stock of patents n_t , but a negative floor which is not constraining patents.

Condition 1: $\mu \frac{\pi}{q} - \delta < r_t < \frac{\pi}{q} - \delta$.

When the interest rate satisfies condition 1, patents are limited by:

$$q n_t \leq \frac{\beta' (\Pi q n_{t-1} - R_{t-1} b_{t-1})}{1 - x^c}. \quad (16)$$

The growth rate of patents is also limited due to the collateral constraint on the number of patents (11) and the flow of funds equation (9).

The solution of the optimization program of the entrepreneur is summarized in the following proposition:

Proposition 1. Optimal R&D Investment, Saving and Borrowing at the Entrepreneur Level.

In each period, innovating firms can be in one of three regimes:

(i) *Perfect capital market regime: $r_t^F = \frac{\pi}{q} - \delta$. The free entry interest rate equals the marginal gain of R&D investment.*

(ii) *Currently binding credit constraint regime: With probability θ the innovator has an opportunity to invest at date t and faces binding debt and patents ceilings:*

$$b_t = x^c q n_t = \frac{\mu V_{t+1}(n_t)}{1 + r_t} \quad (17)$$

$$q n_t = \frac{\beta' (\Pi q n_{t-1} - R_{t-1} b_{t-1})}{1 - x^c}. \quad (18)$$

(iii) *Anticipated credit constraint regime: With probability $1 - \theta$, the innovator has no opportunity to invest and saves as much as possible in order to decrease debt. Then:*

$$b_t = q n_{t-1} - \beta' (\Pi q n_{t-1} - R_{t-1} b_{t-1}) < b_{t-1}. \quad (19)$$

In regime (i), the debt ceiling never binds, debt policy does not affect R&D investment.

Financially constrained regimes (ii) and (iii) are obtained under the condition 1 that the credit interest rate is below the marginal return of R&D investment $r_t < \frac{\pi}{q} - \delta$. The entrepreneur consumes his minimal level of consumption ($d_t = d_m$). In regime (iii), the size of the patent portfolio declines due to depreciation:

$$n_t = (1 - \delta) n_{t-1}. \quad (20)$$

If an entrepreneur has a long history of no opportunity to invest in R&D, he may eventually become a net creditor. When an entrepreneur which has built “deep pockets” over a history of no profitable ideas faces an opportunity to invest, he invests *as much as allowed by the financial constraint* due to his linear cost function and because the marginal return on R&D exceeds the credit interest rate. A proof of proposition 1 can be found in appendix 1.

Additional remarks: The assumption of discounting firms’ dividends by the interest rate is common to most of neo-classical and endogenous economic growth models. This assumption leads to a straightforward comparison of the financially constrained growth regime with the perfect capital market growth regime of the lab-equipment model presented in Barro and Sala-I-Martin (2004). In business cycles models with collateral constraints, the usual assumption is to assume that financially constrained entrepreneurs discount the logarithm of their dividends by a subjective discount factor denoted ξ distinct from households discount factor (e.g. Iacoviello (2005)). It can be shown by using a variant of this model that changing this assumption does not alter the results. (Appendix 2)

3. High Growth of Innovations with Collateral Constraints

In this section we will analyse the steady state solution of the above model. In order to do this, we calculate first the steady state aggregate debt/patent ratio (debt and patents grow at the same rate in a steady state). Secondly, we find the equilibrium interest rate such that households' consumption grows also at the steady state growth rate. In what follows, we assume additionally that R&D investment is not lumpy ($\theta = 1$).⁷ Then the innovator can only be in the perfect capital market regime or in the current binding credit regime. In the latter, debt is a linear function of the stock of patents (equation 17) at the microeconomic level. Hence, the dynamics of aggregate debt is identical to the dynamics of aggregate patents. The patents equations are linear in patent and debt, so that aggregation across entrepreneurs does not require to keep track of the distribution of the individual entrepreneurs' patents and debt. Aggregate patents and debt are denoted by capital letters N_t and B_t . Since the population of entrepreneurs is unity, the aggregate number of patents is limited by the sum of patent ceiling inequalities (16):

$$qN_t \leq \frac{\beta' (\Pi qN_{t-1} - R_{t-1}B_{t-1})}{1 - x^c}. \quad (21)$$

The above inequality is an equality when condition 1 is fulfilled (financially constrained regime).

Equilibrium on the final goods market occurs if total consumption, i.e. aggregate consumption of households plus aggregate consumption of entrepreneurs, equals output in the final goods sector net of intermediate goods minus the amount of resources devoted to R&D activity:

$$C_t + D_t = Y_t - N_t X - q(N_{t+1} - N_t).$$

Aggregate consumption of the entrepreneur is equal to aggregate dividends and can be expressed as a proportion of the number of patents.⁸ Since output net of

⁷The case of lumpy R&D investment ($\theta < 1$) is treated elsewhere by the authors.

⁸Expressing the debt level as a fraction of the debt ceiling $B_t = \mu_0 x^* qN_t$, with $0 < \mu_0 \leq 1$ and $\mu_0 = 1$ in the financially constraint regime we get:

$$\begin{aligned} D_t &= (1 - \beta'_0) (\Pi qN_{t-1} - R_{t-1}B_{t-1}) \\ D_t &= (1 - \beta'_0) (\Pi - R_{t-1}\mu_0 x^*) \frac{1}{1 + g} qN_t. \end{aligned}$$

intermediate goods is also proportional to the number of patents (see equation (6)) and $q(N_{t+1} - N_t) = qgN_t$, where g is the growth rate of patents, aggregate household consumption is proportional to the number of patents. In a steady state growth path all these aggregates therefore grow at the same constant rate.

Households' aggregate consumption growth rate follows from $C_{t+1} = (\beta(1 + r_t))^{\frac{1}{\sigma}} C_t$. The endogenous interest rate r_t ensures the equality of demand and supply of funds. When the interest rate is equal to the marginal return of innovation $r_t = \frac{\pi}{q} - \delta$, the free entry steady state growth factor of patents is equal to $N_{t+1}/N_t = (\beta\Pi)^{1/\sigma}$. The growth rate is positive as long as the interest rate is higher than the households' rate of time preference. This growth level can be reached only if it is below the maximal rate of growth of patents allowed by the financial constraint when $r_t = \frac{\pi}{q} - \delta$ is equal to $\beta'\Pi$. Else, the financially constrained growth rate prevails.

Condition 2 for financially constrained growth:

$$\beta' < \beta^{\frac{1}{\sigma}} \Pi^{\frac{1}{\sigma}-1} \tag{22}$$

*Entrepreneurs' saving rate of equity β' is sufficiently low (or equivalently the entrepreneurs' rate of time preference is sufficiently high).*⁹

Proposition 2: Steady State Growth Regimes

When condition 2 is not fulfilled, the free entry equilibrium prevails. The interest rate is equal to the return of R&D investment and there exists a unique steady state growth factor $(\beta\Pi)^{1/\sigma}$. When condition 2 is fulfilled, there exists a unique steady state interest rate r^ that is lower than the marginal return on R&D investment and there exists a unique financially constrained steady state growth factor $G^* = G_C(r^*) = G_N(r^*)$.*

Proof Under condition 2, an equilibrium interest rate r^* determines a constrained steady state growth rate g^* . The growth rate of consumption equals the

⁹If the households' $\frac{1}{2}$ relative fluctuation aversion, σ is equal to one, condition 2 boils down to entrepreneurs' saving rate of net worth lower than households' saving rate of net worth: $\beta' < \beta$. Although several recent papers dealing with collateral constraints assumed $\beta' < \beta$, in national accounts, firms' savings rate is often higher than households' savings rate (a key assumption of Kaldorian models of growth and distribution (Bertola, Foellmi and Zweimüller (2006))). Here, the entrepreneurs' $\frac{1}{2}$ savings rate of net worth can be higher than households savings rate when households relative fluctuation aversion exceeds unity.

maximal growth rate of patents:

$$H(r) = \beta' \left(\Pi + (\Pi - R) \left(\frac{1}{1 - x^c(r)} - 1 \right) \right) - (\beta(1+r))^{1/\sigma} = 0.$$

As H is a continuous and strictly decreasing function of the interest rate over the interval

$$\left] \max \left(\rho, \mu \frac{\pi}{q} - \delta \right), \frac{\pi}{q} - \delta \right[, \text{ as } \lim_{r \rightarrow \mu \frac{\pi}{q} - \delta} H(r) = +\infty > 0$$

and as $H\left(\frac{\pi}{q} - \delta\right) < 0$ (condition 2), there exists an unique equilibrium interest rate, corresponding to an unique strictly positive patent growth rate, according to the intermediate value theorem.

Remark: When dividends are always zero ($\beta' = 1$), condition 2 $\left(\Pi < (\beta\Pi)^{\frac{1}{\sigma}}\right)$ is the equivalent of a no-Ponzi-game hypothesis, ruling out chain-letter debt finance. Chain-letter debt finance can be seen as taking ever larger amounts of debt in order to pay off the debt of the previous period plus interest. Then, the steady state growth rate with perfect capital market is below the free entry rate of interest $r^F = \frac{\pi}{q} - \delta$. This particular no-Ponzi-game condition sets a minimal level for the relative fluctuation aversion (strictly below unity) in order to slow down the free entry steady state growth (Barro and Sala-I-Martin (2004)), so that chain-letter debt finance is not feasible:

$$\sigma \geq 1 + \frac{\ln(\beta)}{\ln(\Pi)}. \quad (23)$$

However, this condition on a minimal level of the relative fluctuation aversion is no longer necessary when there is a binding collateral constraint, which also rules out chain-letter debt finance, hence proposition 3:

Proposition 3: Financially constrained growth rate higher than the real interest rate is feasible.

A steady state financially constrained growth rate higher than the real interest rate is feasible because collateral backed debt rules out Ponzi finance. The collateral constraints extend the set of feasible high growth rates and interest rates currently accepted in the endogenous growth literature. Contrary to a commonly

held view, financial constraints are compatible with high level of the growth of innovation, because collateral leverages growth. However, the growth rate cannot exceed $(1 + \frac{1}{\alpha}) \frac{\pi}{q}$, else consumption is zero.

As debt is always fully backed by a correct evaluation of the value of available collateral at all future periods, lenders avoid repayment problems related to Ponzi finance (see also Araujo et al. (2002)). It is not necessary to add an alternative no-Ponzi finance condition such that, in an equilibrium, the growth rate has to be lower than the interest rate, nor to assume overlapping and growing generations. In the real world, Ponzi finance arises when lenders lend *more* than the expected value of collateral and/or when they *systematically over-estimate* the long-term value of collateral when there is an asset price bubble. There remains a ceiling (higher than the interest rate) to the growth rate, because R&D investment should not totally crowd out households consumption, which has to remain positive.¹⁰

Figure 1 below presents a graphical representation of the steady state with the growth rate on the vertical axis and the interest rate on the horizontal axis.

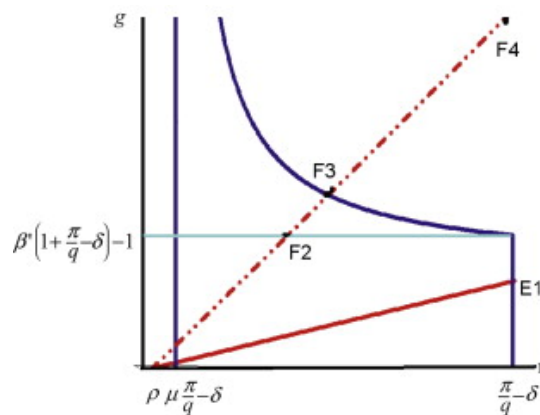


Fig. 1. Equilibrium growth rates as a function of real interest rates.

We distinguish two cases of relative risk aversion of households: the rising line starting from the value of time preference $\rho = 1\%$ with the higher slope rising line corresponds to $\sigma = 0.5$, the one with the lower slope corresponds to $\sigma = 2$. They show all possible growth rates for a given interest rate. The parameters for innovative firms are set as follows: $\frac{\pi}{q} - \delta = 5\%$, $\theta = 1$, $\beta' = 0.98$. A vertical line

¹⁰If there are some firms that are not able to invest, $\theta < 1$, we get transitional dynamics for the aggregate debt patent ratio. After an exogenous shock, convergence is regular without any cyclical or chaotic patterns. Because of financial constraints, however, the transitional dynamics of the interest rate and of the marginal productivity is no longer necessarily the same.

on the left side of the figure represent the asymptote of the financially constrained patent growth rate: $r = \mu \frac{\pi}{q} - \delta = 1.25\%$ for $\mu = 0.25$. On the right of this asymptote, the patent growth curve for $\mu = 0.25$ is first represented by a curve decreasing with interest rate as long as:

$$\mu \frac{\pi}{q} - \delta = 1.25\% < r < \frac{\pi}{q} - \delta = 5\%$$

with a growth rate higher than $\beta' \Pi - 1 = 2.9\%$. For growth rate below $\beta' \Pi - 1 = 2.9\%$, the patent growth rate curve is represented by a vertical line: $r = \frac{\pi}{q} - \delta = 5\%$, because of the free entry condition in capital markets. We can consider four steady state growth regimes (Table 1).

	σ	μ	$1.25\% < r \leq 5\%$	g
E1	2	$0 \leq \mu \leq 1$	$5\% = \frac{\pi}{q} - \delta$	$2\% < \beta' \Pi - 1$
F2	0.5	$\mu \rightarrow 0$	2.5%	$2.9\% = \beta' \Pi - 1$
F3	0.5	25%	2.9%	3.8%
F4	0.5	$\mu \rightarrow 1$	$5\% = \frac{\pi}{q} - \delta$	8, 1%

Table 1. Steady state growth rates and interest rates.

In equilibrium E1 there is a high relative fluctuation aversion ($\sigma = 2$) and condition 2 is not fulfilled: the free entry steady state growth prevails. For equilibria with a low relative fluctuation aversion of households that is households do not satisfy the bounded utility condition (23), the growth rate increases with μ , from the “no debt” regime with $\mu = 0$ (F2), to the $\mu = 25\%$ (F3). The theoretical limit case of perfect collateral ($\mu = 100\%$) is not possible in practice with patents as collateral, where the free entry growth rate level can be reached even when the growth rate exceeds the real interest rate (F4).

Proposition 4: Policy effects of Murphy’s (2002) legal reforms on patents growth. *Public expenditures may shift the economy from equilibrium F2 to equilibrium F3, with a sharp increase of the growth of innovations.*

Murphy (2002) proposes to raise public expenditures in the USPTO and in the IPR public legal system in order to reduce the uncertainty surrounding the use of patents as collateral for lenders and the costs of litigation on transfers of IPR, for example, by funding a public registry of patents used as collateral. The proportion of the value of patents transferred to lenders $\mu (G/N) = \mu_0 (G/N)^a$ (with $\mu_0 > 0$ and with elasticity $a > 0$) increases with G/N , which measures public funding per patent devoted to grease the wheels of the transfers of property rights in IPR when patents are used as collateral (with $\mu' > 0$). G/N is financed by taxing

with a proportion τ ($0 \leq \tau < 1$) the rent π of each patent currently registered at the USPTO. An alternative public funding leading to similar results consists of a corporate income tax with a deduction of interest payments. This taxation also decreases the collateral market value of each patent by a factor $(1 - \tau)$. The tax policy maximizing the growth of patents amounts to maximize the after tax rate of return on equity of innovative firms (written in a way to simplify the derivative with respect to the tax rate):

$$\max_{\tau} \left[r + \frac{(1 - \tau) \frac{\pi}{q} - (r + \delta)}{1 - \mu_0 (\tau \pi)^a \frac{(1 - \tau) \frac{\pi}{q}}{r + \delta}} \right].$$

The tax rate increases the proportion of the value of patents transferred to lenders, but decreases the value of patents, and decreases the flow of profits. The trade-off between the first two effects leads to the tax rate maximizing the debt/patent ratio x^c (at the denominator of the growth rate) which is

$$\tau^* = \frac{a}{1 + a}.$$

Because the tax rate also decreases the flow of profits and hence the growth rate, the growth maximizing tax rate τ^{**} is below the loan/patent ratio maximizing tax rate: $0 < \tau^{**} < \tau^*$. This solution is an interior solution as long as the after tax profits remain positive, that is when: $\tau \leq 1 - q(r + \delta)/\pi$, else this upper bound prevails as a corner solution. This corner solution disappears when one taxes corporate income after interest payments. After tax profits (net of interest payments) are then always positive. The growth maximizing tax rate τ^{**} is given by the implicit equation:

$$\frac{\pi}{q} (1 - x^c(\tau)) = \left((1 - \tau) \frac{\pi}{q} - (r + \delta) \right) \left(\frac{\partial x^c(\tau)}{\partial \tau} \right).$$

The reform is such that the economy shifts from $\mu \approx 0$ (very few patent backed loans, with a suboptimal level of public expenditures equal to zero: $G/N = \tau = 0$) to a widespread practice of patent backed loans, with an aggregate loan to patent ratio $\mu(\tau^{**})/(1 + r)$, expected to be close to 25% in practice (Edwards (2002)). This effect of this policy is evaluated using the growth differential following a shift from equilibrium F2 to equilibrium F3: $G(\mu(\tau^{**})) - \beta' \Pi$. Under condition 2, satisfied by a low relative fluctuation aversion, such as $\sigma = 0.5$ in Example 2,

the policy effect on the growth rate of innovation may be huge. The growth rate increases from 2.9% to 3.9% and the interest rate increases from 2.5% to 2.9%.

Additional information can be gained when we compute the marginal effect of policy reforms on growth:

$$\frac{\partial G_N^*}{\partial \mu} = (\beta(1+r^*))^{\frac{1}{\sigma}-1} \frac{\frac{\partial g_N}{\partial \mu}}{\frac{\partial g_C}{\partial r} - \frac{\partial g_N}{\partial r}} > 0 \quad (24)$$

with

$$\frac{\partial g_N}{\partial \mu} = \beta' \left(\frac{\pi}{q} - \delta - r^* \right) \frac{\frac{\partial x^*}{\partial x^c} \frac{\pi}{q(r^*+\delta)}}{(1-x^*)^2} > 0.$$

A large effect of a marginal change of the loan to patent ratio μ is obtained for a large gap between the equilibrium credit interest rate and the marginal return on innovation. Graphically, the closer the equilibrium interest rate is to the value of the real interest rate determining the vertical asymptote of the patent growth curve, the larger the marginal effect on growth of reducing the legal uncertainty surrounding patents as collateral.

4. Conclusion

This paper describes an endogenous growth model with lenders limiting credit up to the collateralizable value of existing patents and with a composition between innovative firms facing a probability to find a positive net present value R&D investment opportunity or not each period.

First, at the entrepreneur level, financial constraints and lumpiness lead to a specific entrepreneurs savings behaviour where they build “deep pockets” by anticipating future financial constraints. When a lumpy R&D investment opportunity occurs, the dependence of the persistence of R&D investment on the mark-up rewarding innovations is amplified by the debt/patent collateral constraint.

Secondly, the aggregation of entrepreneurs behaviour determines a steady state endogenous aggregate leverage (or debt/patent ratio) below the leverage ceiling.

Thirdly, this financially constrained steady state occurs only for relatively large growth rates. In this regime, a large effect on growth of reforms protecting lenders using patents as collateral occurs for low values of the equilibrium interest rate with respect to the rate of return on innovation; a factor which depends also on the growth of credit supply and not only on the behaviour of innovative firms.

Extensions suggest that collateral assignment of patents may be detrimental to open source, because it adds incentives to value patents portfolios. Leverage driven growth is a necessary characteristic of high speed growth of innovation.

Appendix A. Proof of proposition 1

The Lagrangian of the entrepreneur program is:

$$(n_t, b_t) \in \text{Arg max } E_0 \sum_{t=1}^{+\infty} \frac{L_t}{\prod_{T=1}^{T=t} (1 + r_T)}$$

$$\text{with } L_t = (1 + \lambda_t^d) d_t + \lambda_t^b (qx^c n_t - b_t) \\ + \lambda_t^d (-(1 - \beta') (\Pi q n_{t-1} - R_{t-1} b_{t-1}))$$

where λ_t^b is the Lagrange multiplier related to the debt ceiling constraint, λ_t^d is the Lagrange multiplier related to the minimal consumption constraint, and with consumption d_t given by the flow of funds constraint:

$$d_t = (\Pi q n_{t-1} - R_{t-1} b_{t-1}) + b_t - q n_t - \left(\tilde{1}_{i_t > 0} - 1 \right) q (n_t - n_{t-1}).$$

The Euler equation on debt b_t is $\frac{\partial L_t}{\partial b_t} = 0$, for any date t :

$$0 = 1 + \lambda_t^d - \lambda_t^b + E_t \left(\frac{-(1 + r_t) (1 + \lambda_{t+1}^d) + \lambda_{t+1}^d (1 - \beta') (1 + r_t)}{1 + r_{t+1}} \right) \Rightarrow$$

$$\lambda_t^d = \lambda_t^b - 1 + (1 + r_t) E_t \left(\frac{1 + \beta' \lambda_{t+1}^d}{1 + r_{t+1}} \right).$$

The first order condition with respect to the stock of patents is $\frac{\partial L_t}{\partial n_t} = 0$, that is:

$$0 = (1 + \lambda_t^d) (-q) + qx^c \lambda_t^b \\ + E_t \left(\frac{1}{1 + r_{t+1}} \right) (1 + \lambda_{t+1}^d) \left[\Pi q + \left(\tilde{1}_{i_{t+1} > 0} - 1 \right) q \right] \\ + E_t \left(\frac{1}{1 + r_{t+1}} \right) \lambda_{t+1}^d (-(1 - \beta') \Pi q).$$

One divides by q and substitutes λ_t^d using the first order condition for debt:

$$\begin{aligned} 0 &= -\lambda_t^b - (1 + r_t) E_t \left(\frac{1 + \beta' \lambda_{t+1}^d}{1 + r_{t+1}} \right) + x^c \lambda_t^b \\ &+ \left(1 + \frac{\pi}{q} - \delta \right) E_t \left(\frac{1 + \beta' \lambda_{t+1}^d}{1 + r_{t+1}} \right) \\ &+ E_t \left(\frac{1 + \lambda_{t+1}^d}{1 + r_{t+1}} \right) \left(\tilde{\mathbf{1}}_{i_{t+1} > 0} - 1 \right). \end{aligned}$$

Hence:

$$\frac{\pi}{q} - \delta - r_t = (1 - x^c) \frac{\lambda_t^b}{E_t \left(\frac{1 + \beta' \lambda_{t+1}^d}{1 + r_{t+1}} \right)} + E_t \left(\frac{(1 + \lambda_{t+1}^d) (1 - \tilde{\mathbf{1}}_{i_{t+1} > 0})}{1 + \beta' E_t \lambda_{t+1}^d} \right).$$

The sufficient conditions as stated in Chow (1997), p.29, are also fulfilled, since the functions are concave and either the Lagrange parameter is equal to zero and the inequality condition not binding or the Lagrange parameter is different from zero and the inequality condition is binding, see Chatelain (2000).

Appendix B. Logarithmic Utility for Entrepreneurs

This appendix considers the case where entrepreneurs maximize a (concave) logarithmic utility, with a discount factor $\xi = 1/(1 + \rho')$ which may differ from households discount factor¹¹:

$$\sum_{\tau=0}^{+\infty} \xi^\tau \ln(d_{t+\tau}) \quad (25)$$

subject to the flow of funds constraint, the collateral constraint, and the positive consumption constraint $d_t \geq 0$. The Euler equation on debt b_t is $\frac{\partial L_t}{\partial b_t} = 0$, for any date t :

$$\begin{aligned} 0 &= \left(\frac{1}{d_t} + \lambda_t^d \right) - \lambda_t^b + \frac{E_t \left(-(1 + r_t) \left(\frac{1}{d_{t+1}} + \lambda_{t+1}^d \right) \right)}{1 + \rho'} \Rightarrow \\ \lambda_t^d &= \lambda_t^b - \frac{1}{d_t} + (1 + r_t) \xi E_t \left(\left(\frac{1}{d_{t+1}} + \lambda_{t+1}^d \right) \right). \end{aligned} \quad (26)$$

¹¹This discount factor plays the same role as the maximal saving rate of equity when entrepreneurs' utility is linear

The first order condition with respect to the stock of patents is $\frac{\partial L_t}{\partial n_t} = 0$, that is:

$$\begin{aligned} 0 &= \left(\frac{1}{d_t} + \lambda_t^d \right) (-q) + qx^c \lambda_t^b \\ &+ q\xi (\Pi) E_t \left(\frac{1}{d_{t+1}} + \lambda_{t+1}^d \right) \\ &+ q\xi E_t \left(\frac{1}{d_{t+1}} + \lambda_{t+1}^d \right) \left(\tilde{1}_{i_{t+1}>0} - 1 \right). \end{aligned}$$

One divides by q and substitutes λ_t^d using the first order condition for debt:

$$\frac{\pi}{q} - \delta - r_t = (1 - x^c) \frac{\lambda_t^b}{E_t \left(\frac{\frac{1}{d_{t+1}} + \xi \lambda_{t+1}^d}{1+r_{t+1}} \right)} + E_t \left(\frac{(1 + \lambda_{t+1}^d) (1 - \tilde{1}_{i_{t+1}>0})}{E_t \left(\frac{1}{d_{t+1}} + \xi \lambda_{t+1}^d \right)} \right).$$

a) Case where $\lambda_t^b = \lambda_{t+1}^d = 0$ and $\tilde{1}_{i_{t+1}>0} = 1$, the Euler equation for patents and on debt leads to:

$$\frac{d_{t+1}}{d_t} = \xi \Pi = \xi (1 + r_t).$$

Hence $r_{t-1} = \frac{\pi}{q} - \delta = r_t$, so that the flow of funds constraint can be written as a function of the entrepreneur's equity:

$$qn_t - b_t = R_{t-1} (qn_{t-1} - b_{t-1}) - d_t. \quad (27)$$

Using the Euler equation for debt, one has:

$$\xi R_{t-1} = R_{t-1} - \frac{d_t}{qn_{t-1} - b_{t-1}}.$$

Hence:

$$d_t = (1 - \xi) \Pi (qn_{t-1} - b_{t-1})$$

with the entrepreneur's net worth on date t defined by $a_t = \Pi (qn_{t-1} - b_{t-1})$. The savings of an entrepreneur are a fraction ξ of her firm net worth a_t . A balanced growth where the growth rate of households consumption is equal to the growth factor of entrepreneurs consumption $\xi \Pi$ is obtained when households utility is also logarithmic with an identical discount factor $\beta = \xi$, or in the particular case where $[\beta \Pi]^{\frac{1}{\sigma}} = \xi \Pi$.

b) Case where $\lambda_t^b > 0$ and $\lambda_{t+1}^d > 0$ and $\tilde{1}_{i_{t+1}>0} = 1$, $r_{t-1} < \frac{\pi}{q} - \delta$: entrepreneurs invest by borrowing up to the credit limit because the rate of return on their R&D investment exceeds the real interest rate. The debt ceiling constraint can be written as:

$$a_{t+1} = \Pi q n_t - (1 + r_t) b_t = \left(1 - \mu \frac{\pi}{q(r + \delta)} \frac{1 + r_t}{\Pi} \right) \Pi q n_t.$$

When the debt ceiling constraint binds, the flow of funds constraint can be written as:

$$q n_t = \frac{\Pi q n_{t-1} - R_{t-1} b_{t-1} - d_t}{1 - x^c}.$$

Hence, the entrepreneur net worth is:

$$a_{t+1} = \frac{\left(1 - x^c \frac{1+r_t}{\Pi} \right)}{1 - x^c} (a_t - d_t).$$

Maximizing the log utility of dividends determined by the entrepreneur net worth constraint implies that the saving of an entrepreneur is a fraction ξ of her firm net worth a_t . When all firms do invest and are financially constrained on date t and date $t - 1$, and the growth factor of patents is equal to the growth rate of households consumption in the steady state, which may lead to an equilibrium interest rate below its free entry level $r_{t-1} < \frac{\pi}{q} - \delta$:

$$\xi \left[\Pi + \left(\frac{\pi}{q} - \delta - r_{t-1} \right) \frac{x^c}{1 - x^c} \right] = [\beta R_{t-1}]^{\frac{1}{\sigma}}$$

under condition 2:

$$\xi \Pi < [\beta \Pi]^{\frac{1}{\sigma}}.$$

When $\sigma = 1$ (households utility is also logarithmic), then condition 2 means that the discount factor of entrepreneurs is lower than the discount factor of households $\xi < \beta$ (entrepreneurs rate of time preference discount the future more heavily than households).

Appendix C. Steady state debt/patent ratio

The law of motion of the debt/patent ratio x_t as a function of its previous value: $x_t = M(x_{t-1})$ is computed using the aggregate patent growth factor:

$$\frac{N_t}{N_{t-1}} = G_\theta = \frac{\theta}{1 - x^c} \beta' (\Pi - R_{t-1} x_{t-1}) + (1 - \theta) (1 - \delta), \quad (28)$$

and the aggregate debt (or equity) law of motion, which can be written as:

$$\frac{N_t}{N_{t-1}} = G_F = \frac{\beta' (\Pi - R_{t-1}x_{t-1})}{1 - x_t} = \beta' \left[\Pi + \left(\Pi - R_{t-1} \frac{x_{t-1}}{x_t} \right) \left(\frac{1}{1 - x_t} - 1 \right) \right].$$

The steady state leads to the implicit relation $N(x_t, x_{t-1}) = G_F - G_\theta = 0$:

$$N(x_t, x_{t-1}) = \frac{\beta' (\Pi - R_{t-1}x_{t-1})}{1 - x_t} - \frac{\theta}{1 - x^c} \beta' (\Pi - R_{t-1}x_{t-1}) - (1 - \theta)(1 - \delta) = 0.$$

which can be written as this explicit equation:

$$\begin{aligned} x_t &= M(x_{t-1}, r_{t-1}) = 1 - \frac{\beta' (\Pi - R_{t-1}x_{t-1})}{\left(\frac{\theta}{1-x^c}\right) \beta' (\Pi - R_{t-1}x_{t-1}) + (1 - \theta)(1 - \delta)} \\ &= 1 - \frac{1 - x^c}{\theta} \left(1 - \frac{1}{1 + \left(\frac{\theta}{1-x^c}\right) \left(\frac{\beta' (\Pi - R_{t-1}x_{t-1})}{(1 - \theta)(1 - \delta)}\right)} \right). \end{aligned} \quad (29)$$

A sufficient condition for $\frac{\partial M}{\partial x_{t-1}} < 1$:

$$\begin{aligned} \frac{\partial M}{\partial x_{t-1}} &= \frac{\beta' R_{t-1} (1 - \theta)(1 - \delta)}{\left[\left(\frac{\theta}{1-x^c}\right) \beta' (\Pi - R_{t-1}x_{t-1}) + (1 - \theta)(1 - \delta)\right]^2} < 1 \\ r_{t-1} &< \frac{1}{\beta' (1 - \theta)(1 - \delta)} - 1 < \frac{G_\theta^2}{\beta' (1 - \theta)(1 - \delta)} - 1 \Rightarrow \\ \frac{\partial M}{\partial x_{t-1}} &< 1. \end{aligned} \quad (30)$$

The steady state debt/patent ratio x is given by the following quadratic equation:

$$\frac{N(x, x)}{\beta'} = \frac{\Pi - R_{t-1}x}{1 - x} - \frac{\theta}{1 - x^c} (\Pi - R_{t-1}x) - \frac{(1 - \theta)(1 - \delta)}{\beta'} = 0.$$

The function $N(x, x)$ is continuous on the interval $[0, x^c]$ and strictly increasing:

$$\frac{\partial N(x, x)}{\partial x} = \beta' \left(\frac{\Pi - R_{t-1}}{(1 - x)^2} + \frac{\theta}{1 - x^c} R_{t-1} \right) > 0.$$

According to the intermediate value theorem, a unique solution exist for a positive steady state debt/patent ratio $0 < x^* \leq x^c < 1$ under the conditions $N(0, 0) < 0$ and $N(x^c, x^c) > 0$. First, $N(x^c, x^c) > 0$ is always fulfilled as long as $\theta < 1$:

$$N(x^c, x^c) = (1 - \theta) \left[\beta' \left(\Pi + (\Pi - R_{t-1}) \left(\frac{x^c}{1 - x^c} \right) \right) - (1 - \delta) \right] > 0.$$

Second, $N(0, 0) < 0$ leads to condition 2, such that θ should not be too low:

$$\begin{aligned} N(0, 0) &= (1 - x^c) [\beta' \Pi - (1 - \theta)(1 - \delta)] - \theta \beta' \Pi < 0 \\ \Rightarrow x^c &= \frac{\mu \frac{\pi}{q}}{r + \delta} > x_{\min}^c = (1 - \theta) \left(\frac{\beta' \Pi - (1 - \delta)}{\beta' \Pi - (1 - \theta)(1 - \delta)} \right). \end{aligned}$$

Condition 2 for the steady state debt/patent ratio to be strictly positive implies that the interest rate should be below the ceiling r_{\max} :

$$r < r_{\max} = \frac{\mu}{x_{\min}^c} \frac{\pi}{q} - \delta.$$

The explicit solution x^* is found by solving the quadratic equation $N(x, x) = 0$.

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