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SEIGNIORAGE, TAXATION AND MYOPIA IN EMU

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Abstract
We examine fiscal policy coordination in a two-country world with two types of asymmetries. The first type of asymmetry is a difference in the efficiency of taxation systems, the second is a varying degree of myopia as represented by discount rates. We examine the conditions under which cooperation is beneficial for both countries and we examine how asymmetries affect the loss inflicted by discretion. Our results show that if asymmetries are too great, EMU has poor welfare properties.

JEL Classification: E61, F16 E61

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1. Introduction

Up to the mid 1980s, the benefits that stem from coordination of economic policy on *aggregate* welfare have not been questioned. The view prevailed that since the outcome of any non-cooperative equilibrium could be adopted as the cooperative outcome, the set of sustainable outcomes was enhanced by cooperation and therefore welfare was improved. In a seminal paper, Rogoff (1985) showed that in fact cooperation could lead to lower welfare if policy-makers acted on discretion. In his model there is a temptation to use surprise inflation motivated by a Phillips curve. The penalty associated with inflation is lower in a flexible exchange-rate regime without monetary cooperation. This is perceived by the private sector, that expects higher inflation under cooperation. The distortion that originates in discretion is exacerbated by the cooperative regime.

The paradigm that Rogoff opened has persisted in recent studies of the benefits of cooperation. Levine (1992) studies the effect of fiscal cooperation and arrives at the same conclusion. Van der Ploeg (1991) studies a static monetary model and again emphasizes the effect of reputation on the benefits of cooperation. However, both papers only give approximations of the welfare in discretion. Levine (1992) linearises his model around the steady state and performs numerical simulations. Thus the analysis is only valid in the neighbourhood of the steady state. Van der Ploeg (1991) presents an analytical solution, but with a constant debt/spending ratio.

This paper extends this strand of the literature. We study the influence of parameter asymmetries in a dynamic extension of the model by van der Ploeg (1991). We do not limit our analysis to the steady-state, thereby sacrifice analytical solutions. However our numerical solutions trace the full welfare implications starting from any given initial condition.

We consider two asymmetries within the model. The first one is a difference in the efficiency of taxation systems. This is to account for a popular critique of monetary union raised, among others, by Dornbusch (1988). Basically the argument is that in the southern countries of the European Community the cost of tax collection is higher. These countries rely on seigniorage revenues to a greater extent than the northern countries. In a model in which national governments set the tax rate, the northern countries will find that inflation is excessive.

The second asymmetry is on government's discount rates. Here we extend recent work by Pearlman and Levine (1992), who study the incidence of myopia in a linearised version of the seigniorage model by Obstfeld (1991). Within the context of a single country they find that the equilibrium is only defined if the government is not too myopic. In our two-country context with a shared budget constraint we find exactly the same condition, moreover we find an additional condition for the individual governments' loss to be finite in the reputation equilibrium. In discretion, loss goes to infinity as we approach the Pearlman and Levine limit.

However challenging the study of different discount rates might be for the theorist, it is not easy to justify on empirical grounds because the discount rate is not observable in an objective manner. Short-termism is a matter of mentality and many observers think that it is stronger in the South then in the North of the Community. The extent of environmental damage is one indicator that one might suggest, monetary policy may be another. We consider the discount rate as something inherent in the political system and in the institutions of the country. A coalition government, for example, may find it harder to make long-term decisions if there are different parties with divergent interests. In other countries where government action is bound by constitutional arrangements or is more decentralised, there is a case to argue that the discount rate is lower.

The remainder of the paper is organised as follows. Section 2 presents the model. Section 3 and 4 solve the model in reputation and discretion, respectively. The analyses of the welfare implications are conducted using numerical simulations and are presented in section 5. Section 6 briefly concludes.

2. Presentation of the model

We study a dynamic model where time is discrete and stretches from the current date t to infinity. Each date is indexed by z. The model comprises two countries that differ by the values taken by two macroeconomic parameters. The object of the model is to study the evolution of welfare in one country under the influence of the parameters.

Let us first look at the way we model any one individual country. At date z the treasury has a primary deficit that is the excess of government spending g_z over revenues τ_z from lump-sum taxation. Spending includes interest payment on outstanding nominal debt $r_z d_{z-1}$. The deficit must

be financed by issuing new debt $d_z - d_{z-1}$ or by seigniorage revenues s_z. In each period, the following budget identity holds:

$$d_z + s_z + \tau_z = g_z + (1 + r_z) d_{z-1} \tag{1}$$

Here all quantity variables are expressed as fractions of full-employment national income. The ex-post real interest rate r_z is growth adjusted. The government is solvent in the current period if the stream of future income can finance both the repayment of current debt and future government spending:

$$d_{t-1} \leq \sum_{z=t}^{\infty} \frac{s_z + \tau_z - g_z}{\psi_z} \quad \text{where} \quad \psi_z \overset{\text{def}}{=} \sup \left[\prod_{a=t}^{z} (1 + r_a), 1 \right] \tag{2}$$

Government spending is exogenous but varies in time. Notational convenience suggests to define the permanent level of government spending at time t as the *constant* level that is equivalent to the time-varying spending path:

$$\bar{g}_t = \sum_{z=t}^{\infty} \psi_z g_z \Big/ \sum_{z=t}^{\infty} \psi_z. \tag{3}$$

We use analogous definitions for $\bar{\tau}_t$ and $\bar{s}_t$. If the interest rate is constant, we can write (2) in terms of perpetual bond stocks whose dividends are the permanent amounts:

$$r_t d_{t-1} \leq \bar{\tau}_t + \bar{s}_t - \bar{g}_t. \tag{4}$$

Here r_t is the *constant* interest rate that will prevail from time t onwards. The quantity theory of money holds. Writing M_t for the nominal value of the money stock, we can approximate its growth rate θ_t as follows:

$$\theta_t = \frac{M_t - M_{t-1}}{M_{t-1}} \approx \pi_t + n, \tag{5}$$

where π_t stands for the inflation rate. To ensure compatibility with exogenous government spending we assume that the growth rate n is exogenous as well. For reasons of analytical simplicity, and without loosing any generality, we also assume that it is constant. We finally assume that scarcity of resources prevails, i.e. that government spending can not be financed by growth alone. This is written as:

$$\bar{g}_z > nm \qquad \forall z > t \tag{6}$$

This assumption assures that taxes and seigniorage are positive in equilibrium. Strict positivity of both results from the choice of a quadratic loss function. The loss of the government reflects penalties of inflation and of tax collection. The parameter β expresses the efficiency of the tax system. The lower β is, the more the government will rely on seigniorage in order to finance expenditure.

$$W_t \stackrel{\text{def}}{=} \frac{1}{2}\sum_{z=t}^{\infty}(1+\delta)^{t-z}\{\tau_z^2 + \beta\pi_z^2\}, \quad \text{where } \beta > 0. \tag{7}$$

A time-inconsistency issue can arise if the government's debt is imperfectly indexed on inflation. Without loss of generality, we assume that it is not indexed at all. Hence for any given expected inflation π_t^{e}, the government is interested in surprising the public with inflation in order to decrease the real burden of its debt. Formally, we introduce the distinction between the *ex-post* real interest rate r_t and a constant *ex-ante* real interest rate ρ:

$$r_t = \rho + \pi_t^{\mathrm{e}} - \pi_t. \tag{8}$$

The time-inconsistency problem motivates the study of rule (R) and discretion (D) equilibria. In the rule equilibrium there is a mechanism enforcing that the government will never surprise the public with inflation. Alternatively we can think of the rule equilibrium as one in which the government has a reputation for not using surprise inflation. A government which lacks this reputation will not be able to commit itself to low inflation and, in equilibrium, will suffer a higher loss.

We now introduce the two country world. Each country is characterised by equations (1) to (7). The variables for the first country will be noted with a circumflex accent, whilest the variables of the second will be noted with a háček accent. In what follows we will be constantly using the variables in the "hat" country because the algebra for the other country is identical and the results are symmetrical in this respect. To shorten notation further, we will note the sum of the relevant variables in both countries with a tilde.

The problem of cooperation between economies originates in the fact that the countries share a common central bank. Thus we inevitably have a link between seigniorage extraction and the growth of the money stock. Indeed the budget constraint of the central bank requires:

$$\tilde{s}_t \stackrel{\text{def}}{=} \hat{s}_t + \check{s}_t = \theta_t \tilde{m}. \tag{9}$$

Furthermore, the definition of a monetary union implies that a common interest rate and a uniform inflation rate will prevail in equilibrium. The fact that there is a monetary union is of course common knowledge, hence the uniformity of inflation rates is expected by the public. Formally:

$$\hat{r}_t = \check{r}_t = r_t, \quad \hat{\pi}_t = \check{\pi}_t = \pi_t \quad \text{and} \quad \hat{\pi}_t^{\mathrm{e}} = \check{\pi}_t^{\mathrm{e}} = \pi_t^{\mathrm{e}} \quad \forall t. \tag{10}$$

We draw the well-established distinction between non-cooperative (N) and cooperative (C) equilibria. In the non-cooperative equilibrium, each government minimises its loss with respect to s_z and τ_z, given the action of the other government. A cooperative equilibrium is the one realised by a central planner, who minimises the combined loss of both governments. In the following, we will study the outcome of cooperation and non-cooperation in the cases of rules and discretion.

3. A solution with a rule

In this section, we study equilibria that rely on the existence of a rule that prevents the government from using surprise inflation. Alternatively we can assume that when setting the value of its instruments, the government takes into account the fact that the public forms rational expectations, rather than the value that these expectations take in a particular date. As a consequence, the interest rate ρ will prevail in all dates:

$$r_z^{\mathrm{R}} = \rho \qquad \forall z \tag{11}$$

In a non-cooperative equilibrium, governments minimise their own loss without taking into account the action of the fellow government. Each treasury computes its optimal taxation level and grabs seigniorage from the common central bank up to a level that assures solvency. Formally we minimise (7) under the constraint (2), where we take account of (11). Using the Lagrangian we derive the first-order conditions. Scarcity of resources guarantees that the government budget constraint will bite in equilibrium, however the budget constraint is only summable if the governments are no too myopic, i.e. if:

$$1 + \max(\hat{\delta}, \check{\delta}) < (1+\rho)^2 \tag{12}$$

This condition deserves a few words of explanation. The government's budget constraint is concerned with income and spending in the future as well as at present. Imposing this constraint means to oblige the government to produce a plan for an income stream that covers current and future spending and the repayment of current debt. This is a plan and does not need to be realised, because the optimisation is repeated in the next period. However, if a government is too myopic, it cannot produce such a plan, simply because there is no plan that would both satisfy the first-order conditions *and* the budget constraint. Such a government would postpone the collection of tax and seigniorage to the indefinite future. In the single country case this condition has been discussed more at length in a recent paper by Pearlman and Levine (1992), therefore we will refer to it as the Levine-Pearlman condition. The central bank's budget constraint can only be defined if the optimisation problems of both governments have solutions, therefore we need to impose the above condition that takes care of both discount rates.

If the condition holds, then the budget constraint yields the multiplier, and the first-order conditions derive the values of the instruments in any period as a function of the debt stock at the beginning of the period and the parameters only. For taxation, the relevant expression is:

$$\hat{\tau}_t^{\mathrm{NR}} = \frac{\hat{\beta}(\rho\tilde{d}_{t-1} + \bar{\tilde{g}}_t - n\tilde{m})(\rho^2 + 2\rho - \check{\delta})(\rho^2 + 2\rho - \hat{\delta})}{\left[(\tilde{\beta} + \tilde{m}^2)(\rho^2 + 2\rho) - \tilde{m}^2\tilde{\delta}/2 - \check{\delta}\hat{\beta} - \hat{\delta}\check{\beta}\right]\rho(1+\rho)} \tag{13}$$

Inflation is computed from the level of seigniorage that governments extract:

$$\pi_t^{\mathrm{NR}} = \frac{\tilde{m}(\rho\tilde{d}_{t-1} + \bar{\tilde{g}}_t - n\tilde{m})(\rho^2 + 2\rho - \check{\delta})(\rho^2 + 2\rho - \hat{\delta})}{\left[(\tilde{\beta} + \tilde{m}^2)(\rho^2 + 2\rho) - \tilde{m}^2\tilde{\delta}/2 - \check{\delta}\hat{\beta} - \hat{\delta}\check{\beta}\right]\rho(1+\rho)} \tag{14}$$

We notice that both taxation and inflation are linear functions with a constant of the debt stock. Using the governments' budget identities for each

period, we can write the next date's debt as a function of current debt. If permanent government spending is constant from a date 0 onwards, further tedious manipulation solves a recursive equation that defines the government's debt in time t as a function of an exogenous initial debt level d_0. Debt can then be expressed as:

$$\tilde{d}_t^{\mathrm{NR}} = \frac{n\tilde{m} - \bar{\tilde{g}}_0}{\rho} + \frac{\rho\tilde{d}_0 + \bar{\tilde{g}}_0 - n\tilde{m}}{\rho}\left(\frac{1 + a^{\mathrm{NR}}}{1+\rho}\right)^t \tag{15}$$

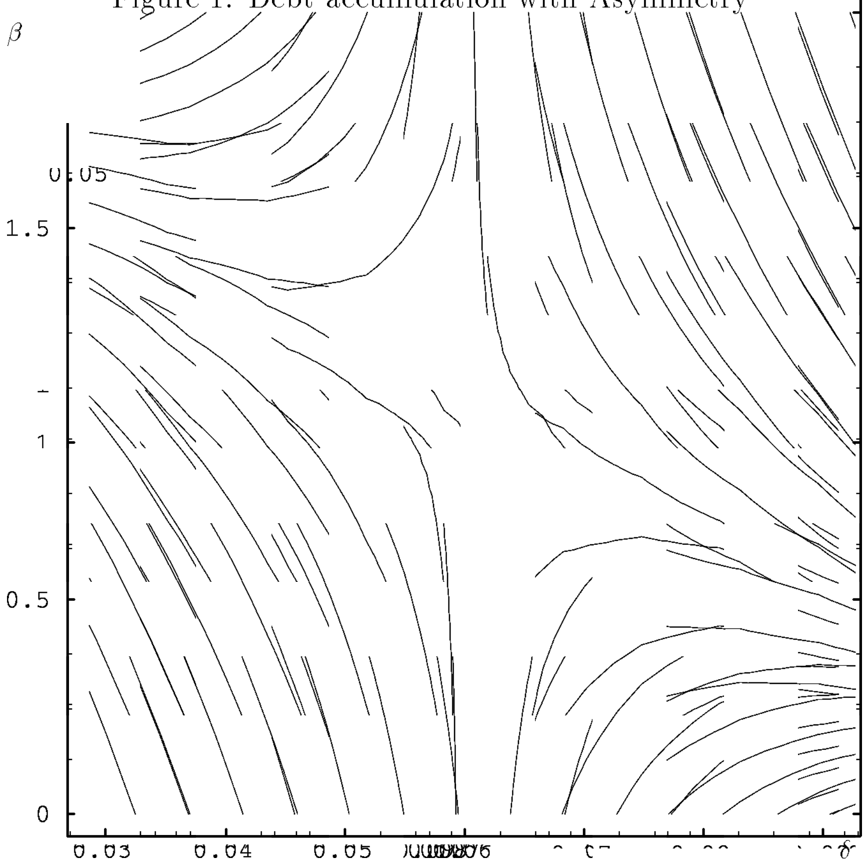
Debt will be equal to the steady-state value plus the distance between the initial state and the steady state multiplied by a factor that changes in geometric ratios. We call a^{NR} the accumulator. Its relation to the interest rate is crucial to determine the dynamics of the model. If the accumulator is smaller then the interest rate, then the steady state will be reached. This condition writes:

$$a^{\mathrm{NR}} = \frac{(\hat{\beta}\hat{\delta} + \check{\beta}\check{\delta} + \tilde{m}^2\tilde{\delta}/2)(\rho^2 + 2\rho) - \hat{\delta}\check{\delta}(\tilde{m}^2 + \tilde{\beta})}{(\tilde{m}^2 + \tilde{\beta})(\rho^2 + 2\rho) - (\hat{\beta}\check{\delta} + \check{\beta}\hat{\delta} + \tilde{m}^2\tilde{\delta}/2)} < \rho \tag{16}$$

The size of the accumulator and the achievement of the steady state will depend on individual countries' myopia, and on the way their myopia is weighted. If the efficiency of a countries tax system is high, then the myopia of that country counts heavily in the weighting. This fact will act as a stabiliser if the country that has the weaker efficiency will also be the more myopic. It has an important implication for Economic and Monetary Union in the sense that it is not necessarily a problem if tax-inefficiency and myopia tend to co-exist in a number of countries. The fact that both come together may help the stability of the system. This idea can be made more precise when looking at a numerical simulation of the influence of asymmetries on the growth rate of debt.

Figure 1 is a surface map of the simulated growth rate of debt, when $\tilde{\beta} = 1$ $\rho = .1$, and $\tilde{\delta} = (1+\rho)^2 - 1$. Note that the last equality means that there is some overall myopia, for the average discount rate is in the centre of the permissible range. We picture a map of the growth rate of debt as a function of δ on the x-axis and β on the y-axis, that both go through the entire interval of permissible values. Given that we hold the total values constant, a modification of the variables corresponds to a change in asymmetry. There are several points to watch. As noticed above, for a given discount rate, the growth rate will increase with tax

Figure 1: Debt accumulation with Asymmetry



efficiency if the country is the more myopic one, and it will decrease if the country under consideration is relatively patient. As expected there is an overall increase in the growth rate of debt if there is a high degree of asymmetry on myopia. The low increases in debt are found not far away from the centre of the x-axis, near the point of equal discount rates. However, it is interesting to note that the lowest growth of debt is not achieved for symmetry in β, but for a value of β that is low if the country is slightly more myopic, and high if the country has the lower discount rate. The term slightly is imprecise but important. If the efficient country is very patient, then the growth rate of debt will be large. If one accepts the conventional wisdom—not modelled here—that myopia and low efficiency

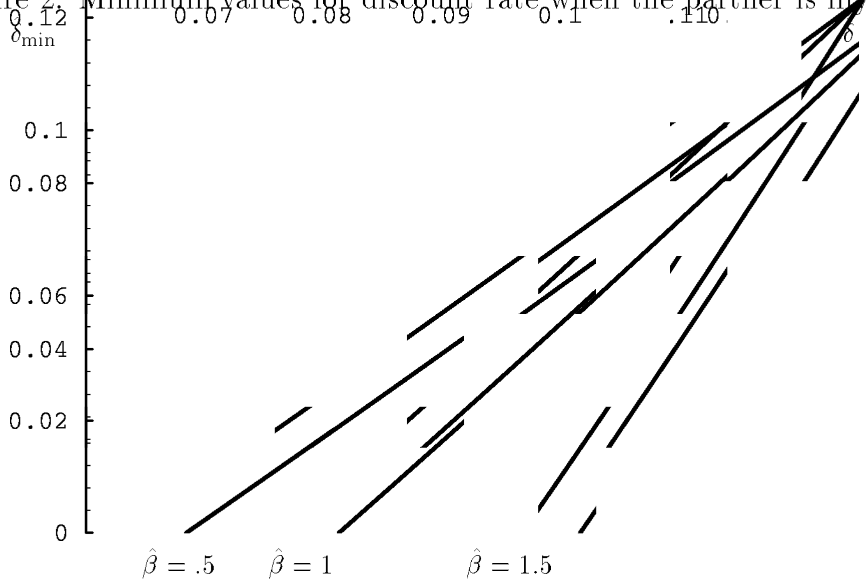
tend to coincide, the above finding is good news for a non-cooperative monetary union. As long as the asymmetries in the discount rates remain moderate, a non-cooperative monetary union can achieve a lower growth rate of debt than the cooperative one.

To obtain the value of the loss that is incurred by the first government, we substitute the solution for debt in the levels of instruments, and replace the instruments in the government's loss function. We discover that the loss for the first country is finite if:

$$1+\hat{\delta} > \left(\frac{1+a^{\mathrm{NR}}}{1+\rho}\right)^2 \tag{17}$$

The finiteness condition for the second country's loss is symmetric. The inverse of the discount factor must be greater then the square of the growth factor of debt. The functional form of this condition corresponds exactly to the functional form of the Levine-Pearlman condition, with the interest rate replaced by the growth rate of debt. This has an important economic interpretation. As in the initial maximisation problem, the government cannot find a solution to its problem when there is a too rapid increase in debt, which will lead to instantaneous welfare losses that rise quicker than what could be compensated for by the discount rate. The rate of expansion of debt is given in equilibrium for each period, because a solution to the problem exists despite the fact that the loss of one player is infinity. Clearly such a monetary union would be impossible to sustain. The origin of non-sustainability is the different degree of myopia across countries. If countries diverge in the rate they discount the future, the result will be that the patient country suffers an infinite loss for its discount term is not high enough to compensate for the ever increasing penalty stemming from the myopic policy of the impatient country. The Levine-Pearlman condition stated that myopia can not be too strong, but we now find a complement in the sense that for a given discount rate of one country, the discount rate of the other country must be sufficiently high to allow for the effect of the myopic action of the first. This can be represented graphically for some numeric values.

The curves show the minimum value $\hat{\delta}_{\min}$ that $\hat{\delta}$ needs to take as a function of $\check{\delta}$ for the monetary union to be sustainable. The computations were done for all curves for $\rho = .06$ and $\tilde{\beta} = 1$, and we let $\hat{\beta}$ take three values $\hat{\beta} \in \{.5, 1, 1.5\}$. We see that for any $\hat{\beta}$, the curves converge in the point of maximum myopia allowed for the existence of a solution. The ure 2: Minimum values for discount rate when the partner is myopic



more myopic one government is, the less divergence between governments is allowed. Furthermore, the greater the patient country's efficiency the smaller is the value of $\hat{\delta}_{\min}$.

If we discard the case where the discounted sum of one-period losses is infinite, a symbolic solution can be found for its value:

$$\hat{W}_1^{\mathrm{NR}} = \frac{-\hat{\beta}(\rho^2+2\rho-\check{\delta})^2(\rho^2+2\rho-\hat{\delta})^2\rho^{-2}}{\left[(\tilde{\beta}+\tilde{m}^2)(\rho^2+2\rho)-(\check{\delta}\hat{\beta}+\check{\beta}\hat{\delta}+\tilde{m}^2\tilde{\delta}/2)\right]^2}$$

$$\times \frac{(\bar{\tilde{g}}_0 - n\tilde{m} + \rho\tilde{d}_0)^2(\hat{\beta}+\tilde{m}^2)}{(1+\hat{\delta})(1+\rho)^2-(1+a^{\mathrm{NR}})^2} \tag{18}$$

The easiest way to formulate the *cooperative* equilibrium is to assume that the central planner discounts the sum of the one-period penalties of both countries at the average rate $\tilde{\delta}/2$. Again, if the central planner is not too myopic:

$$1+\tilde{\delta}/2 < (1+\rho)^2 \tag{19}$$

then a unique solution to the control problem can be found. Taxation is equal in both countries:

$$\hat{\tau}_t^{\mathrm{CR}} = \check{\tau}_t^{\mathrm{CR}} = \frac{(\rho\tilde{d}_{t-1}+\bar{\tilde{g}}_t-n\tilde{m})(\rho^2+2\rho-\tilde{\delta}/2)}{(2+\tilde{m}^2/\tilde{\beta})\rho(1+\rho)} \tag{20}$$

and inflation can be written as:

$$\pi_t^{\mathrm{CR}} = \frac{(\rho\tilde{d}_{t-1}+\bar{\tilde{g}}_t-n\tilde{m})(\rho^2+2\rho-\tilde{\delta}/2)}{\tilde{m}^{-1}(2\tilde{\beta}+\tilde{m}^2)\rho(1+\rho)} \tag{21}$$

These equations—applied to all periods—together with the initial conditions completely characterise the solution. The evolution of debt can now be computed.

$$\tilde{d}_t^{\mathrm{CR}} = \frac{n\tilde{m}-\bar{\tilde{g}}_0}{\rho} + \frac{\rho\tilde{d}_0+\bar{\tilde{g}}_0-n\tilde{m}}{\rho}\left(\frac{1+\tilde{\delta}/2}{1+\rho}\right)^t \tag{22}$$

The functional form is as in the non-cooperative case, but the accumulator simply equates average myopia. Alternatively, we can say that the asymmetry being internalised in a central planning problem, debt grows at the value of the accumulator in the absence of asymmetry. The reader can quickly verify that in a symmetric world $a^{\mathrm{NR}} = \tilde{\delta}/2$. Debt will increase as long as average myopia is greater then the interest rate.

From the point of view of the welfare function applied by the central planner, the cooperation outcome is always better than the non-cooperative outcome. However from the point of view the patient member, cooperation can still result in infinite loss, as long as the following condition is not satisfied:

$$1+\hat{\delta} > \left(\frac{1+\tilde{\delta}/2}{1+\rho}\right)^2 \tag{23}$$

Again the country needs to discount more rapidly then the square of the growth rate of debt. This result that the loss can be infinite stems from our assumption that the central planner discounts at the average rate of the two policy-makers rather then minimising the sum of welfare losses. The latter formulation of the problem is straightforward, however there are no closed forms for the resulting sums, which make it cumbersome to deal with. It is however clear that the infinite loss would not occur for such a central planner. If the loss for the first country is finite, it can be computed as:

$$\hat{W}_1^{\mathrm{CR}} = -\frac{(\tilde{\beta}^2 + \hat{\beta}\tilde{m}^2)(\rho^2 + 2\rho - \tilde{\delta}/2)^2}{(2\tilde{\beta} + \tilde{m}^2)^2\rho^2(1+\rho)^2(1+\hat{\delta})} \times \frac{(\tilde{g}_0 + \rho\tilde{d}_0 - n\tilde{m})^2(1+\tilde{\delta}/2)^2}{(1+\rho)^2(1+\hat{\delta}) - (1+\tilde{\delta}/2)^2} \tag{24}$$

Note that this formulation shows that if $\tilde{\delta}/2 = \rho$, then welfare is a square of the structural parameters over the discount rate, a result that we are familiar with from more simple models. If there is no asymmetry in the inflation penalty, it can be shown that the cooperative equilibrium delivers higher payoff for both governments. The question of the benefit of cooperation in the presence of asymmetries will be addressed in section 5.

4. The solutions in discretion

The introduction of discretion is motivated by the presence of nominal government bonds. If the government creates surprise inflation the real value of its commitments decreases. In equilibrium the public will take account of the intentions of the government and correctly anticipate the price level. For each government, exemplified here by the first, we write the budget constraint in t as:

$$(1 + \rho + \pi_t^{\mathrm{eND}} - \pi^{\mathrm{ND}})\hat{d}_{t-1} \leq \sum_{z=t}^{\infty} \frac{\hat{s}_z^{\mathrm{ND}} + \hat{\tau}_z^{\mathrm{ND}} - \hat{g}_z}{(1+\rho)^{z-t}} \tag{25}$$

This formulation implicitly assumes that when the government is making its decision at period t, it takes into account the fact that all expectations for $z > t$ are rational. Only the current expectations are treated as parametric. If the government were to treat the whole vector of expected prices $(p_z^{\mathrm{e}})_{z=t}^{\infty}$ as parametric, no solution that incorporates the budget constraint would be found for a myopic government. With parametric expectations, the government can manipulate the rate at which any future debt is discounted by driving a wedge between the expected and actual price. Once the discount factors reach 1, the government's budget constraint does not

exist any longer, therefore it could run debt at a sufficiently high level to assure felicity in each successive period.

The earlier static contribution of van der Ploeg (1991) implicitly assumes parametric expectations. Given the fact that the government discounts at the rate of interest in his model, the solution for parametric expectations can be found, however there is an additional existence condition that does not take the form of (17), and that is not mentioned in the paper. More generally, it is true that in equilibrium of a static model, expectations are static, but to introduce a surprise that last from now till Kingdom comes is inconsistent with sequential decision making in a dynamic rational expectations model. Therefore we use (25) to constrain the minimisation of (7). If (12) holds, we compute the non-cooperative taxation as:

$$\hat{\tau}_t^{\mathrm{ND}} = \hat{\beta}\left[\tilde{d}_{t-1} + \frac{\bar{\tilde{g}}_t}{\rho} - \frac{n\tilde{m}}{\rho}\right]\left[\frac{(\tilde{m}^2 + 2\hat{\beta})}{2(\rho^2 + 2\rho - \hat{\delta})} + \frac{\tilde{m} + \hat{d}_{t-1}}{(1+\rho)\tilde{m}^{-1}} + \frac{(\tilde{m}^2 + 2\check{\beta})(\tilde{m} + \hat{d}_{t-1})/2}{(\rho^2 + 2\rho - \check{\delta})(\tilde{m} + \check{d}_{t-1})} + \frac{\tilde{\beta}\tilde{m} + \hat{\beta}\check{d}_{t-1} + \check{\beta}\hat{d}_{t-1}}{(\tilde{m} + \check{d}_{t-1})(1+\rho)}\right]^{-1} \tag{26}$$

Inflation is written as follows:

$$\pi_t^{\mathrm{ND}} = \left[\frac{(\tilde{m}^2 + 2\hat{\beta})(\tilde{m} + \hat{d}_{t-1})^{-1}}{2(\rho^2 + 2\rho - \hat{\delta})} + \frac{\tilde{m}}{1+\rho} + \frac{(\tilde{m}^2 + 2\check{\beta})(\tilde{m} + \check{d}_{t-1})^{-1}}{2(\rho^2 + 2\rho - \check{\delta})} + \frac{(\tilde{\beta}\tilde{m} + \hat{\beta}\check{d}_{t-1} + \check{\beta}\hat{d}_{t-1})/(1+\rho)}{(\tilde{m} + \hat{d}_{t-1})(\tilde{m} + \check{d}_{t-1})}\right]^{-1}\left[\tilde{d}_{t-1} + \frac{\bar{\tilde{g}}_t - n\tilde{m}}{\rho}\right] \tag{27}$$

For there is a non-linearity, we cannot find a corresponding formula for the evolution of debt. However, if permanent government spending remains constant over time, and we will assume this to hold, then we can compute steady state debt. We find two steady states:

$$\bar{\tilde{d}}_1^{\mathrm{ND}} = -\frac{(\rho^3 + 2\rho^2 + \hat{\delta}\check{\delta})(\tilde{\beta} + \tilde{m}) + (\hat{\delta}\rho^2 + 2\hat{\delta}\rho + \rho\check{\delta})\hat{\beta}}{\tilde{m}(\rho^4 + 4\rho^3 + 4\rho^2 - 2\rho\tilde{\delta} - \rho^2\tilde{\delta} + \hat{\delta}\check{\delta})/2} - \frac{(\check{\delta}\rho^2 + 2\check{\delta}\rho + \rho\hat{\delta})\check{\beta} + (\rho^2 + 3\rho)\tilde{m}\tilde{\delta}/2}{\tilde{m}(\rho^4 + 4\rho^3 + 4\rho^2 - 2\rho\tilde{\delta} - \rho^2\tilde{\delta} + \hat{\delta}\check{\delta})/2} \tag{28}$$

This is a steady state that does *not* depend on spending, neither is it a function of initial debt. It only depends on $\tilde{m}$, ρ and the utility parameters. In this situation debt acts purely as a buffer that smooths out receipts to adjust them over time to a form that suits the government.

$$\bar{\tilde{d}}_2^{\mathrm{ND}} = \frac{n\tilde{m} - \bar{\tilde{g}}}{\rho} \tag{29}$$

This is the steady state of the reputation equilibrium. It coincides with the one suggested by Obstfeld (1991). In that case, whatever myopia and for any tax efficiency, the government will accumulate assets that will allow it to cover spending. To assess the stability of both steady states we compute two coefficients that are the first derivation of debt in the point where debt reaches the steady state value. If the absolute value of this coefficient is smaller then unity, then the steady state will be both locally stable and stable in the sense of Lyapunov. A short piece of computer algebra shows that in fact the coefficients associated with both states multiply to unity. This means that either one steady state is stable or the other is, with the exception of the situation where, by coincidence of parameters, both coefficients would be unity. The first steady state, $\bar{\tilde{d}}_1^{\mathrm{ND}}$ is stable if:

$$\left| \frac{-(1+5\rho)\tilde{\delta}\tilde{m}^2 - (\bar{\tilde{g}} - n\tilde{m})(\rho^3 + 4\rho^2 + 4\rho - \tilde{\delta}\rho - 2\tilde{\delta} + \hat{\delta}\check{\delta}/\rho)\tilde{m}}{\left[2\rho(\rho^2+3\rho+2)(\tilde{\beta}+\tilde{m}^2) - 2(1+\rho)(\check{\delta}\hat{\beta} + \hat{\delta}\check{\beta} + \tilde{\delta}\tilde{m}^2/2)\right](1+\rho)} \right.$$
$$- \frac{\left[(3\check{\delta}\rho + 2\rho\hat{\delta} + \check{\delta})\hat{\beta} + (3\hat{\delta}\rho + 2\rho\check{\delta} + \hat{\delta})\check{\beta}\right](1+\rho)^{-1}}{\rho(\rho^2+3\rho+2)(\tilde{\beta}+\tilde{m}^2) - (1+\rho)(\check{\delta}\hat{\beta} + \hat{\delta}\check{\beta} + \tilde{\delta}\tilde{m}^2/2)}$$
$$\left. + \frac{(\rho^4 + 5\rho^3 + 7\rho^2 - \tilde{\delta}\rho^2 + 2\rho + \hat{\delta}\check{\delta})(\tilde{\beta}+\tilde{m}^2)(1+\rho)^{-1}}{\rho(\rho^2+3\rho+2)(\tilde{\beta}+\tilde{m}^2) - (1+\rho)(\check{\delta}\hat{\beta} + \hat{\delta}\check{\beta} + \tilde{\delta}\tilde{m}^2/2)} \right| < 1 \tag{30}$$

This term is complicated because of the underlying asymmetries. If these are absent, the condition would read $(\bar{\tilde{g}} - n\tilde{m})\tilde{m} < 4(1+\rho)(\tilde{\beta} + \tilde{m}^2)$. In essence, the *more* government spending is important when compared to the resources that are provided by growth, the governments will try to accumulate assets rather then keeping a buffer debt. However the higher the interest rate the more the government will tend to keep a buffer debt, because the interest rate represents the cost of accumulating the asset stock.

We now look at the cooperative solution in discretion. Here we make the analogous assumption to (25). As in the reputation case, the amount of taxation is equal across countries:

$$\hat{\tau}_t^{\mathrm{CD}} = \check{\tau}_t^{\mathrm{CD}} = \frac{\tilde{\beta}(1+\rho)(\rho\tilde{d}_{t-1} + \bar{\tilde{g}}_t - n\tilde{m})(\rho^2 + 2\rho - \tilde{\delta}/2)}{\rho(1+\rho)^2(2\tilde{\beta} + \tilde{m}^2) + \tilde{m}\rho\tilde{d}_{t-1}(\rho^2 + 2\rho - \tilde{\delta}/2)} \tag{31}$$

and inflation is:

$$\pi_t^{\mathrm{CD}} = \frac{(\tilde{m} + \tilde{d}_{t-1})(1+\rho)(\rho\tilde{d}_{t-1} + \bar{\tilde{g}}_t - n\tilde{m})(\rho^2 + 2\rho - \tilde{\delta}/2)}{\rho(1+\rho)^2(2\tilde{\beta} + \tilde{m}^2) + (\tilde{m}\rho\tilde{d}_{t-1})(\rho^2 + 2\rho - \tilde{\delta}/2)} \tag{32}$$

Again it is not possible to find a closed form for period 1 utility because of non-linearities in the optimal seigniorage and taxation. The first steady state is:

$$\bar{\tilde{d}}_1^{\mathrm{CD}} = \frac{(\tilde{\delta}/2 - \rho)(2\tilde{\beta} + \tilde{m})(1+\rho)}{\tilde{m}(\rho^2 + 2\rho - \tilde{\delta}/2)} \tag{33}$$

Here we can clearly identify debt as dependent on the arbitration of time. If the discount rate of the planner equals the interest rate, there is neither debt not asset accumulation in this steady state. In this case, the discretion and rule case behave in the same way. There is also the familiar asset-accumulation steady state.

$$\bar{\tilde{d}}_2^{\mathrm{CD}} = \frac{n\tilde{m} - \bar{\tilde{g}}}{\rho} \tag{34}$$

Here is the discriminating condition between both steady states. The first steady state will prevail if:

$$1 > \left| \frac{1+\rho}{1+\tilde{\delta}/2} - \frac{\tilde{m}(\bar{\tilde{g}} - n\tilde{m})(\rho^2 + 2\rho - \tilde{\delta}/2)}{\rho(1+\rho)(1+\tilde{\delta}/2)(2\tilde{\beta} + \tilde{m}^2)} \right| \tag{35}$$

In essence this means that the spending that needs to be financed should not be too high if a buffer debt is to be kept. This becomes again clearer when we consider the case without asymmetries, in which the condition is $\tilde{m}(\bar{\tilde{g}} - n\tilde{m}) < 4(1+\rho)(\tilde{\beta} + \tilde{m}^2/2)$. We therefore conclude that, ceteris paribus, a central planner is more likely to accumulate assets.

To sum up we note that there are two steady states in discretion in this model, but only one will be stable, depending on the values of the parameters. The asset accumulation property—that has so little empirical support—will only hold for a limited set of parameter values, where the gap between government spending and growth dividend is particularly high.

5. Numerical Simulations

In this section we present graphically the results of simulations of the model. Surface plots allow to capture the influence of parameters in two countries simultaneously. We focus on the two asymmetries in the model. The first one is the efficiency of the tax system, or inflation-aversion, represented by the pair $(\hat{\beta}, \check{\beta})$. The other is the pair of discount factors $(\hat{\delta}, \check{\delta})$. All graphs depict the parameter for one country—the "home" country—on the x-axis, and the same parameter for the other country on the y-axis.

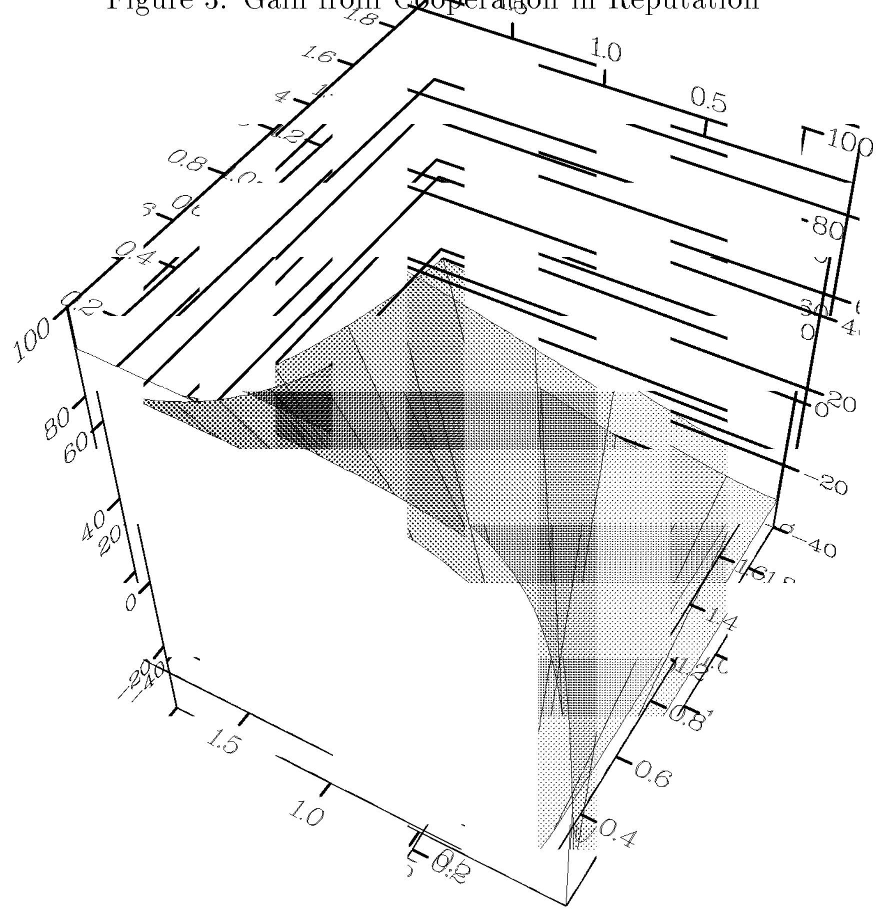
We study the influence of asymmetries on two variables. The first is the *gain* associated with cooperation for the "home" country. Here we plot the difference of the loss in non-cooperation minus the loss of cooperation. This variable can be expected to be positive most of the time. Note that even in reputation it is not positive everywhere because we consider the loss of a particular government, rather then the global loss that both suffer. This variable is inspected separately in the cases of reputation and discretion.

The second variable on which we focus is the natural complement of the first. Here we answer the question what is the *loss* occurred in discretion when compared with reputation. In our formulation this variable should be expected to be negative, i.e. that discretion induces a welfare loss. However, the situation depends on whether we consider the loss in non-cooperation or the individual loss in cooperation, hence we inspect both cases separately. There are 8 cases in total.

Because we concentrate on *relative* gains/losses, calibration can be kept very simple. For all runs, we chose $\tilde{g}_t = 25\%$, $n = 2\%$, $\rho = .06\%$ and $\tilde{d}_0 = 60\%$, but other values show similar functional forms as far as the individual welfare is concerned. In the computations for efficiency aversion we have varied both β between 0 and 2, excluding the borders, while keeping the discount rate at the same level as the interest rate. When considering the discount rate, we varied the rate over the whole set for which an equilibrium is defined, i.e. from 0 to 12.36%, where again the borders are excluded. In these cases we kept $\hat{\beta} = \check{\beta} = 100\%$. Concentrating on one asymmetry at a time greatly simplifies the interpretation of the results. For every graph, we computed 100×100 points. All numerical values are taken as percentages.

For the reputation values, the computation is straightforward. Plugging the numerical values in (18) and (24) gives the answer for the case

Figure 3: Gain from Cooperation in Reputation



where the loss is finite. The discretion values were computed using a loop over the periods in the future whose instantaneous loss was discounted and added to welfare. We kept on adding periods until the change in intertemporal welfare brought by adding the last period represented a percentage change of less then 10^{-7}. If after 1000 iterations the criterion was not fulfilled, no finite equilibrium was supposed to exist. This case did not occur. †

† All computations were carried out with a single Maple V script. It is available on request from the author.

Figure 4: Gain from Cooperation in Discretion

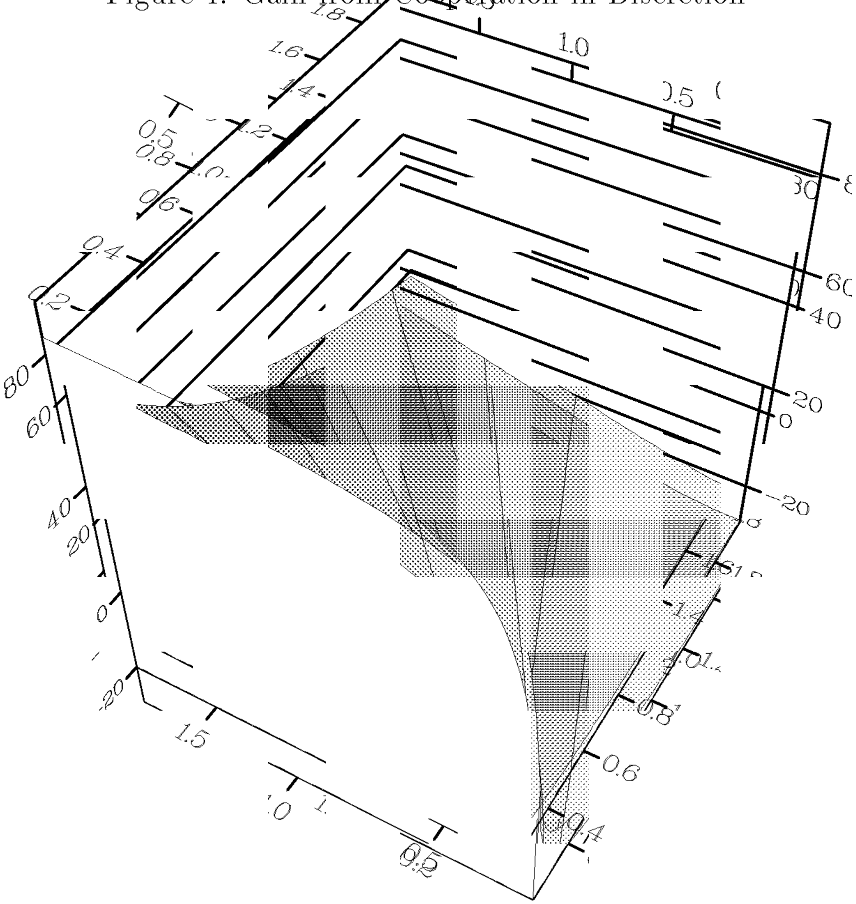
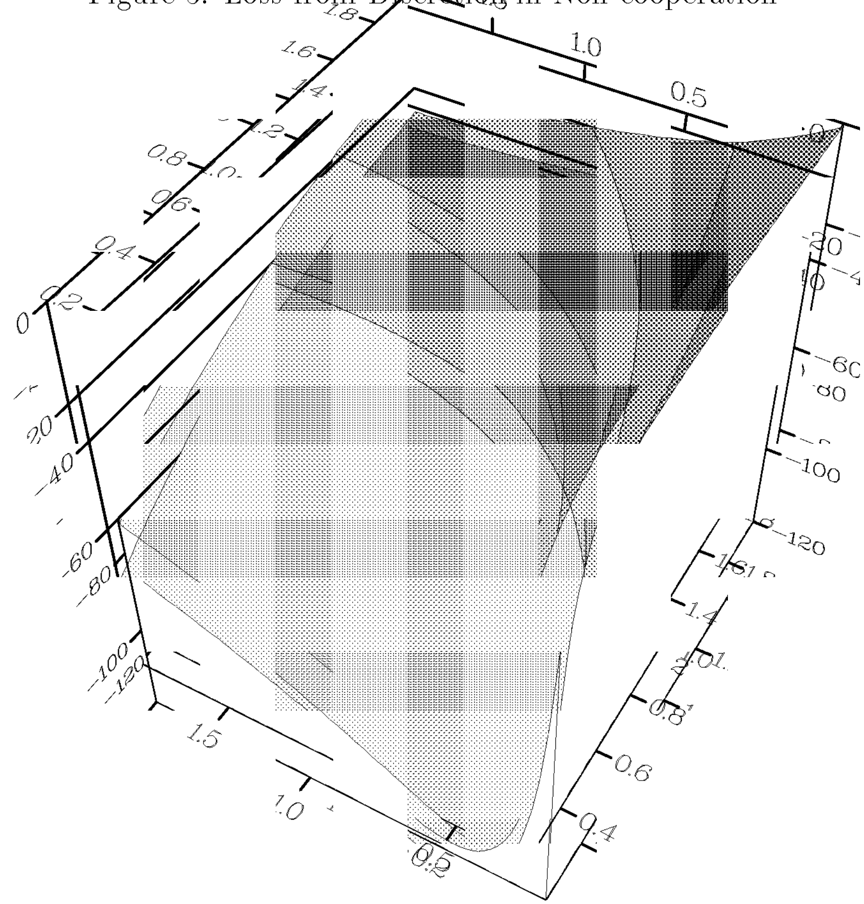


Figure 3 shows the effect of differential taxation penalty on the benefits of cooperation in reputation. We see that from the point of view of the home country, whose β is depicted on the x-axis, the more β is high the more cooperation is desirable. Moreover, the benefits of cooperation are enhanced if the second country is inefficient. If countries share a common β, cooperation is always beneficial. Although this is not easy to detect it on the graph, inspection of the raw data suggests the isoline for 0 runs to the right of the loci where $\hat{\beta} = \check{\beta}$. Therefore cooperation is beneficial under reputation when both countries have the same efficiency. However if the national country is less efficient then its partner, it will prefer non-

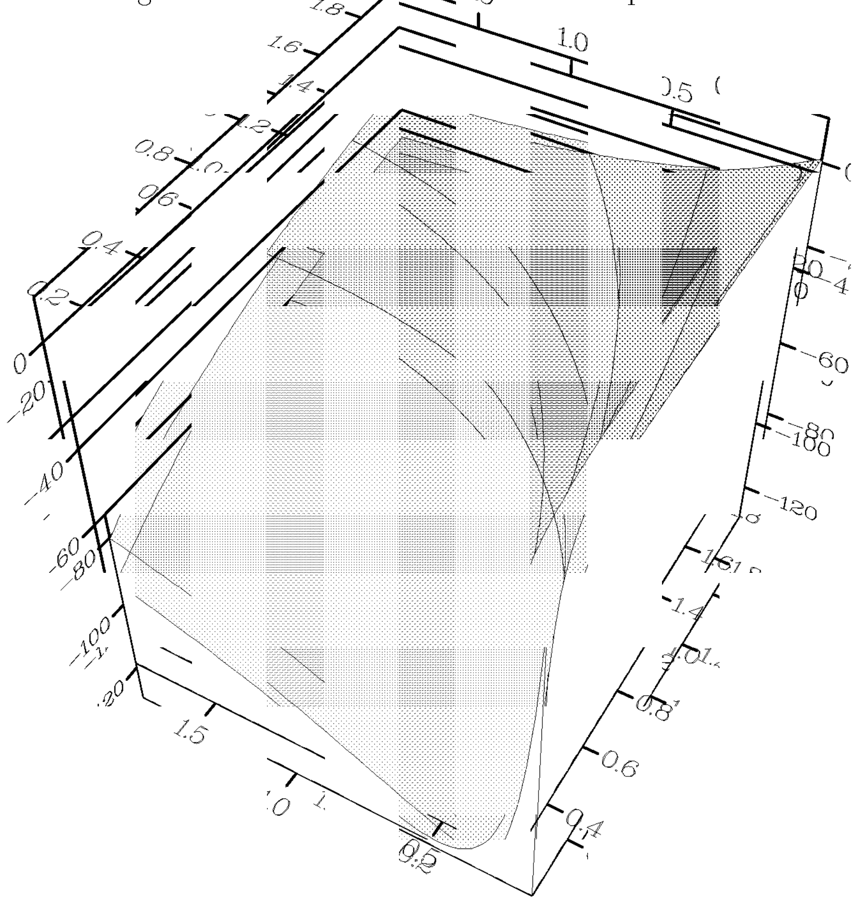
Figure 5: Loss from Discretion in Non-cooperation



cooperation. From a collective viewpoint, the amount the country gains by refusing to cooperate is more then compensated by what the partner country gains, because the curve is much steeper to the left of the line of equal efficiency then it is to the right. If a system of transfers can be implemented, then non-cooperations is Pareto inefficient.

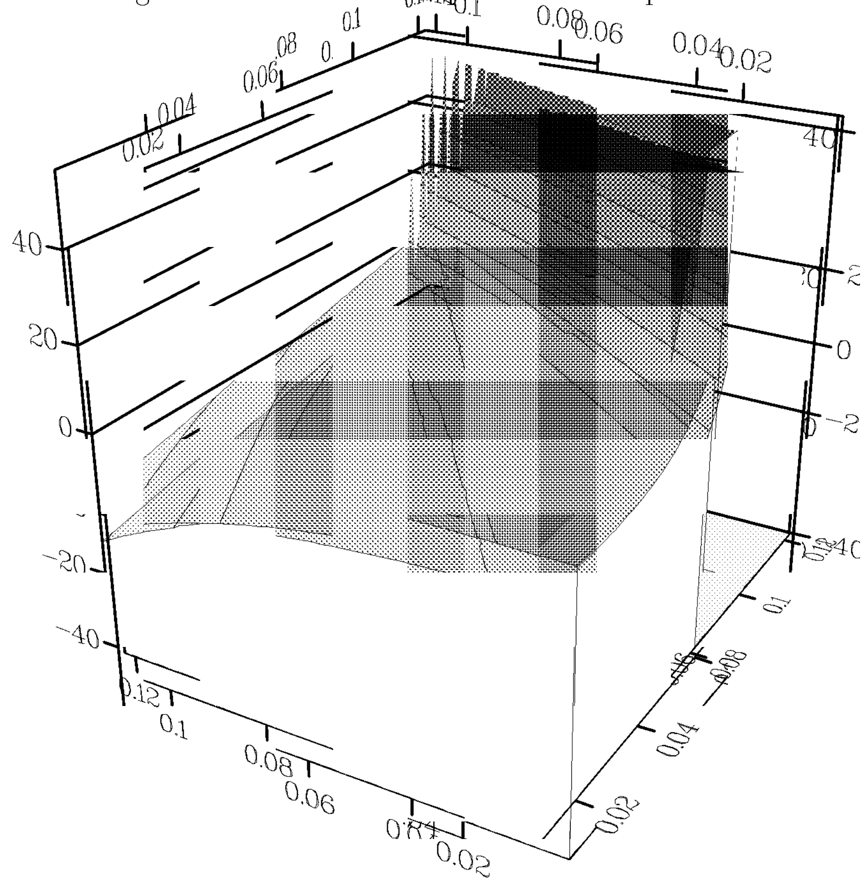
The next figure depicts the same computations as Figure 3, but for discretion. Casual inspection suggests that the functional form of the gain of welfare through cooperation as a function of $\hat{\beta}$ and $\check{\beta}$ is the same. However there are two major changes. Firat the curve is less steep as a whole. If the efficiency is asymmetric in a discretion regime, the country that has

Figure 6: Loss from Discretion in Cooperation



a higher taxation penalty will stimulate the money supply more strongly but because there is a real gain from the monetary expansion in terms of collected seigniorage. This increase in real resources moderates the inconvenience of differential taxation efficiency, because there is a positive spillover from the non-coordinated expansion by one-country on the other. Therefore the curve is less steep then the curve for non-cooperation. The second change concerns the isoquant for zero. In this case it passes on the left of the loci of efficiency symmetry, which suggests that non-cooperation dominates for small departures from equal efficiency. Although the graph here displays the welfare fof one country, it is nevertheless clear that

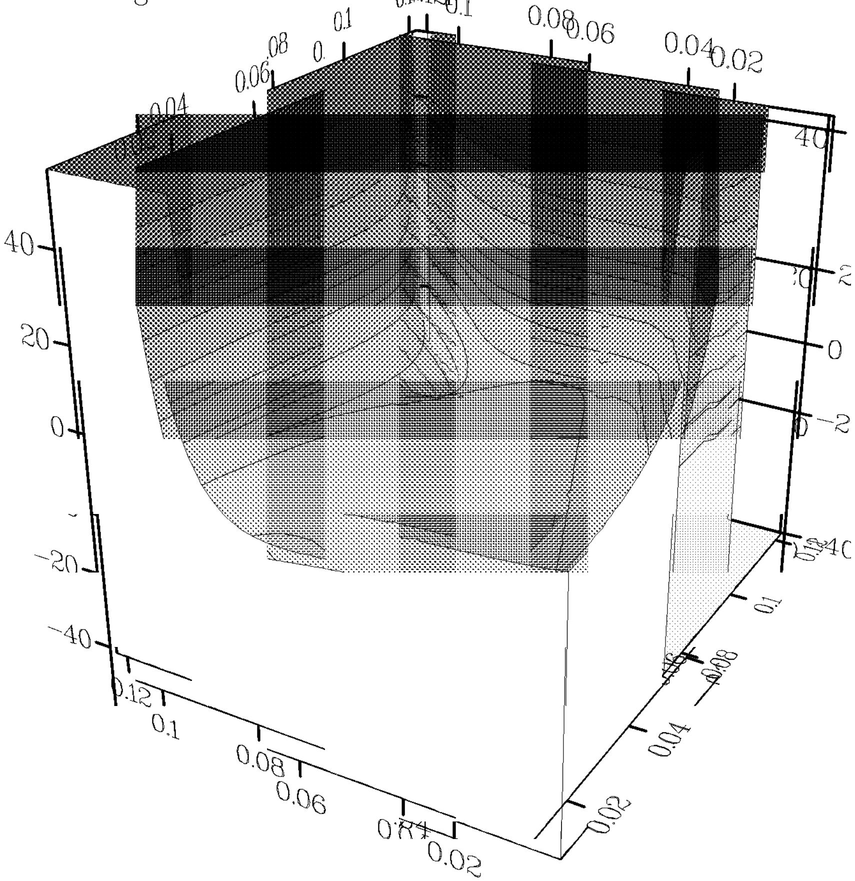
Figure 7: Gain from Cooperation in Reputation



that the “Rogoff paradox” of Pareto dominated cooperation appears in the vicinity of equal efficiency. A cooperative decison-maker facing two different taxation systems would opt for a more moderate expansionist policy, than a cooperative policy-maker that faces two similar counties, because with rising asymmetry, ceteris paribus, the penalty suffered by the efficient member rises more quickly than the penalty suffered by the inefficient. The cooperative policy-maker with asymmetric would therefore adopt a less expansionist policy, that would be expected by the public, and cooperation ends up to yield higher welfare.

We now study if reputation can be sustained, by comparing the loss

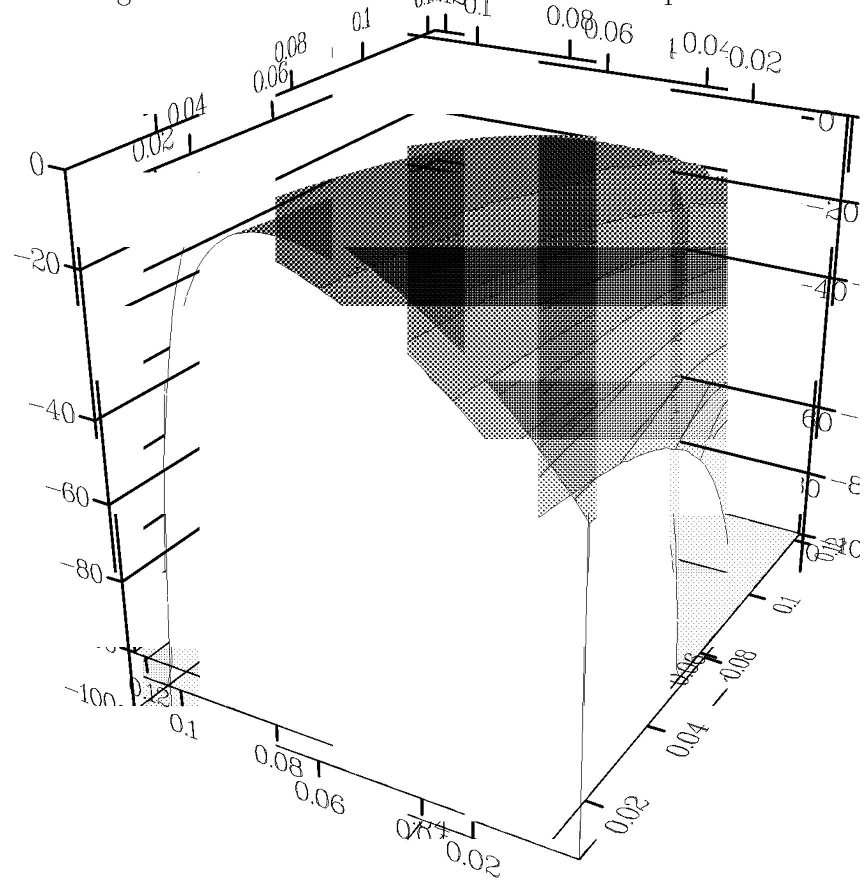
Figure 8: Gain from Cooperation in Discretion



suffered under rules minus the loss under discretion. This number is negative when rules are preferred. Figure 5 shows that this is the case for non-cooperation. For any level of the foreign country, the penalty increases with efficiency. As intuition suggests, for any fixed efficiency of the home country however, the loss of discretion decreases when the other country is more efficient.

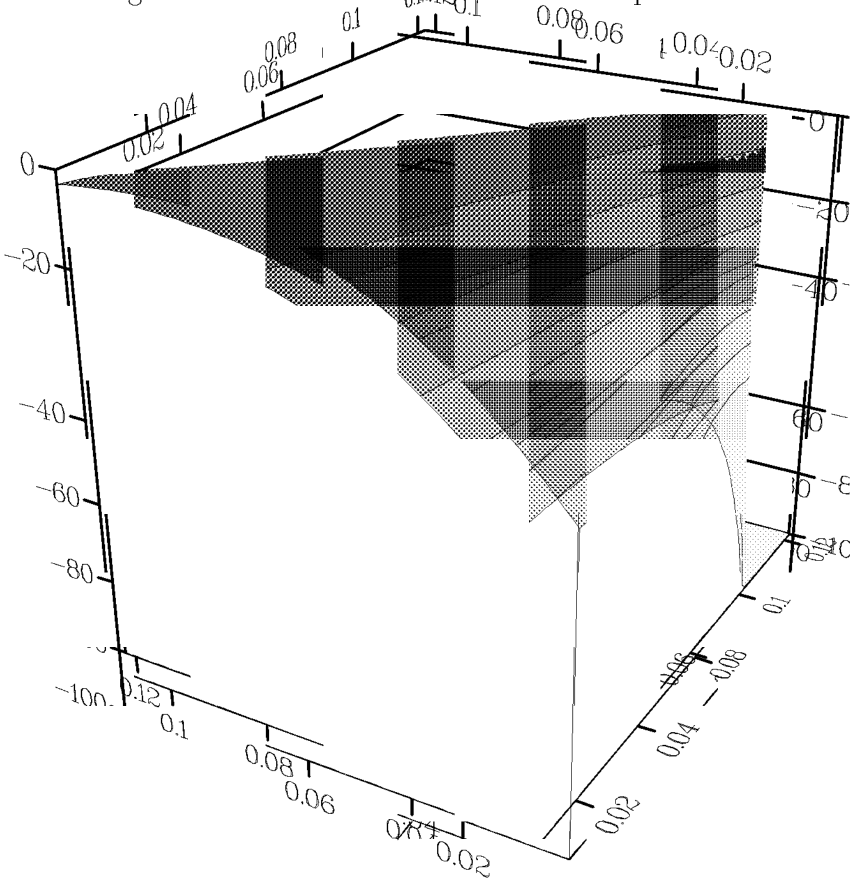
Figure 6 looks at the same question but for the case of cooperation. Cooperation does not seem to matter for the broad picture, but we do recognise that there is a small range to the right of the graph where the home country prefers discretion to rules. That is the case if the home

Figure 9: Loss from Discretion in Non-cooperation



country inefficient, whilest the other country is very efficient. As we have noticed earlier, in that case the central planner will be “biased” towards the efficient country, for its penalty rises more quickly than the loss of the inefficient country declines as the policies become more expansionist. For this reason cooperation is not Parteo dominant for strong inefficiencies. We now turn to the effects of patience and myopia. Here we are facing the problem that the loss of an individual country may not be finite, even if an equilibrium exists. When passing to numerical simulations the loss tends to infinity when a particular border is approached. To obtain any meaningful plot at all, values have to be truncated and equalised to some

Figure 10: Loss from Discretion in Cooperation



extremum value. For consistency in scaling between graphs, our choice was to use a 100% span for all variables.

Figure 7 reports the gains from cooperation in reputation for all allowable pairs of discount rates when $\rho = 6\%$. Beyond the isoline for 50, there is a space of roughly triangular shape for which values are defined, but not displayed on the plot. This region can be recognised as the part when values are constant. It borders the region for which no equilibrium exists. Cooperation is overall beneficial to both countries within a neighbourhood of the line where countries share a common discount rate.

The situation in discretion is more complex. As we can see from figure

8, in the vicinity of the region where the myopia is the same, cooperation is Pareto dominated. Here the Rogoff paradigm applies. This is consistent with earlier findings for discretion. If asymmetires are strong then the myopic country will find it advantageous to replace their discount rate with the common average discount rate of the central planner. The patient country will need a higher discount rate to compensate of the increasing penalties inflicted by the myopic policy of its partner country. The myopic country will find that the lower discount rate of the central planner yields a higher welfare.

We now turn our attention to the loss in discretion as a function of the discount rate. In Figure 9 and Figure 10, we see the plot in the case of non-cooperation and cooperation, respectively. Overall we can be assured that reputation is stable in the sense that a government that can choose between reputation and discretion would always choose reputation, if that reputation equilibrium exists. We note however that the myopic country will become almost indifferent between reputation and discretion in the cooperative case.

6. Conclusions

Having two asymmetric countries in a monetary union. If we call the strong country the one that is patient and efficient, then the strong country will prefer cooperation and reputation solution. Although the weak country will prefer non-cooperation, there is only a very small part of the parameter set where the myopic country would prefer the discretion solution.

Overall the findings are rather pessimistic. Asymmetries have a large impact on the outcome in this type of games. It is difficult to conceive how asymetric countries can stay together in a monetary union, even be it a cooperative one. The de facto freezing of the monetary intergration process in Europe following the break-down of the exchange rate mechanism in July 1993.

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