Regional income convergence in India: A Bayesian Spatial Durbin Model approach

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5 March 2013

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Abstract

A debate on the regional disparity is always an interesting topic. This study analysed the regional income disparity in India during 1980 – 2010, which contains pre, early and later reform periods. The study used per capita GSDP data from Central Statistical Organisation. First, the study reviewed various growth models and suggests that spatial durbin model of Fingleton and Lopez-Bazo(2006) is empirically useful. Second, this study estimated parameters of Bayesian Spatial Durbin Model and disscussed the convergence hypothesis in the light of LeSange and Fischer (2008) formulation. The study concludes that the later reform period has witnessed beta convergence due to feedback effect.

Keywords: Convergence, Regional, Spatial Durbin Model and Bayesian.
JEL Classification Code: R12, C11, C21

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Introduction

The debate on the economic disparities among both people and regions has always been a sensitive political issue and evokes intense response from many quarters, for which India is not an exception. The outcome of studies on disparities found to be too sensitive to samples, variables, measures and approaches. The variation in outcome may be attributed to underlying assumptions. The common regional income disparity models found to assume spational independence and heterogeneity. This paper attemptes to estimate the growth regression (Barro and Sala-i-Martin, 1996) model acknowledging the presence of spatial dependence and spatial heterogeneity. This paper estimates and presents the results of spatial durbin model, a variant of spatial autoregression model for statewise per capita income data during 1980-2010.

The popular empirical strategies to analyse economic disparity are testing for β and σ convergence hypotheses. Among the two, regression based β convergence hypothesis testing is widely used compared to dispersion based σ convergence testing. [1] The coefficient of $y_{i,0}$, her β is assessed for its statistical significance and for its sign to infer about convergence. When the estimate for β is negative and statistically significant, in other words the lower initial income region has a higher growth rate as compared to regions with a higher initial income, the β – convergence is accepted. The statistical insignificance or the positive co-efficient and its significance would suggest rejection of β convergence.

A sophisticated version of growth regression involve lograrithm differences and more explanatory variables in addition to the initial income variable. The presence of β convergence in this case is taken as the income of all regions converge to each of its steady state (conditional β-convergence).

$\frac{1}{T}\ \ln\left[\frac{y_{i,T}}{(y_{i,0})}\right] = \alpha + \beta\ln\,[y_{i,0}] + x'_i\gamma + \varepsilon_{i,}$ where i (i=1, …, n), 0, and T are the indices that denote region, initial period, and final period, respectively; y denotes the income; $T^{-1}\times\ln(y_{i,T}/y_{i,0})$ is the growth rate; x_i is a vector of structural control variables of the region that contains $m\times1c_i$; ε_i's are i.i.d. errors; α and β are the parameters; and γ the m×1 and parameter vectors, respectively.

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[1] In a typical β convergence hypothesis testing approach, a neo classical growth equation, described below on cross sectional data is used. $\left(\frac{y_{i,t} - y_{i,0}}{y_{i,0}}\right) = \alpha + \beta\ y_{i,0} + u_i$, where, $y_{i,t}$ – is the income of i^{th} state at time 't'; $y_{i,0}$ – is the income of i^{th} state at the initial year. $[y_{it} - y_{io}/y_{io}]$ is the growth of i^{th} state.

The importance of inclusion of spatial effect viz., spatial dependence and spatial heterogeity within the growth equation framework was stressed in few studies (see Seya et al., 2012). It was pointed out that the spatial dependence issue was handled in an adhoc manner in the general econometric models (Fingleton and Lopez-Bazo, 2006). A systematic effort was made to include the spatial depenendence using economic spillover models (Egger and Pfaffermayr, 2006). It was found that various spatial autoregression models(SAR) offer sufficient scope for the inclusion of spatial dependence or spatial spillover effects into growth equation models.

Different spatial auto regression models (SAR) were considered in the literature. The difference was charecterised by the inclusion of spatial lag terms for the different explanatory variables components in the growth regression namely, initial income variable, structural variable and control variables(Lopez-Bazo et al., 2004; Ertur and Koch, 2007; Basile, 2008). Kakamu(2007) has favoured the inclusion of spatial lag for all the explanatory variable to effectively address the issue of spatial dependence. This type of models in literature is called spatial durbin models (SDM).

The growth equation model in SDM frame work is likely to be afflicted with specific error as the growth determination by a large number of unknown factors that may be difficult to specify (Seya et al., 2012). The specification error is likely to result in heteroscedastic stochastic error term in the growth equation. The estimates in the presence of hertoscedasticity would be inefficient. This is serious in β convergence testing as statistical significance of β is prime concern in deciding on the issue of convergence. The inclusion of spatial lag variables is SDM tend to increase the risk of multicollinearity problem in the growth regression (Kakamu, 2009).

Different approaches to address various issue of estimation in this framework was considered. One strategy suggested to address the concerns in the estimation was panel data approach (Lopez-Rodrigues, 2008; Parent and LeSage 2010). But this approach suffers from data availability as preparing a data set of explained and explanatory variables for all the years was not always possible. The second approach to address the issue of spatial heterogeneity in the spatial durbin framework was using MLE but was found to suffer from loss of degrees of freedom (Seya et al., 2012). Severe loss of degrees of freedom arising from the need to estimate error variance for each spatial unit included for analysis in this approach. The third approach that uses Bayesian statistics found to provide strategy to address the issue of spatial dependence, spatial heteroscedasticity and loss of degrees of freedom at once (Geweke 1993). This strategy is also found to provide robust estimates in the presence of multicollinearity. For the estimation, the third approach observed to be intuitive, The details of the methodology used in this study is discussed below:

The Bayesian approach to estimate spatial durbin model was described by Seya et al.(2012). The SDM model is defined as,

$$Y^* = \rho WY^* + \alpha\iota + \beta Y_o + \theta WY_o + X\gamma + WX\xi + \varepsilon,$$

Where Y^* is an $n\times1$vector whose elements y_i * are given by $T^{-1}\times\ln(y_{i,T}/y_{i,0})$; ι is an $n\times x$ vector with all elements equal to 1; Y_0 is an $n\times1$ vector whose elements are given by $\ln(y_{i,0})$; WY^* is the spatial lag for Y^*, ρ is the spatial dependence parameter. If the estimate for ρ is positive (negative) and statistically significant, positive spatial autocorrelation is (not) implied; X is an $n\times m$ structural and control variables matrix; ε is an $n\times1$ vector of iid. errors; W is a row- standardized spatial weight matrix.

In this framework the issue of spatial dependence is accounted by the spartial lag terms of explained and explanatory variables and the issue of spatial heterogeneity is addressed through employing the bayesian estimates (LeSage, 1997; Pace and Barry, 1998).

Bayesian estimation SDM

The Bayesian approach consists of three entities namely, the prior distribution, likelihood function and the posterior distribution. The prior distribution, is used to capture the prior beliefs of the researcher on the para and to formalize those beliefs a probability distribution. Each of the parameters in the model is assigned a prior. The priors are of two types namely non informative / diffuse / ignorant priors and informative priors. The informations about each of the parameters may be defined interms of appropriate well known prior distributions, viz., normal and inverse gamma distributions.

The probability density function of growth equation error term charecterise the likelihood function. The product of likelihood of each sample point would give likelihood of sample. The parameters of the likelihood functionwould be functions of regression co-efficients. In the present model,

$L(\beta,\sigma,\rho,Y^*,X)=(2\pi)^{-n/2}\sigma^{-n}|I_n-\rho W|\exp\left\{-\frac{1}{2\sigma^2}(\varepsilon'\varepsilon)\right\}$ where $\varepsilon = [I_n-\rho W](Y^*-X\beta)$.

The posterior distributions summarize all informations about different parameters of the model and is the focus of estimation and its statistical inference. The posterior distributions are derived by multiplying the likelihood function with the prior distribution function. The conditional posterior distribution of each parameter is derived using either Gibbs Sampling Algorithm or Metropolis-Hastings Algorithm.

Deriving posterior density for growth model in SDM:

The spatial durbin model could be rewritten as

$[I_n - \rho W]\, Y^* = Z\varphi + \varepsilon$, where, $Z = [\iota\ Y_o\ WY_o\ X\ WX]$ and $\varphi = [\alpha\ \beta\ \theta\ \gamma\ \xi]$

The parameters of interest in this model consists of regression co-efficient, spatial dependence, error variance and variance co-variance of stochastic error term.

The full prior distribution of this model

The joint prior distribution of the parameters used in the model may be given as $\pi(\varphi, \rho, \sigma_\varepsilon^2, V) = \pi(\varphi).\pi(\rho).\pi(\sigma_\varepsilon^2).\pi(V)$ [since, the prior distributions are assumed to be independent]

(i). $\rho \sim$ unif(-1,+1) , uniform prior

(ii). $\varphi \sim$ diffuse prior

(iii). $\sigma_\varepsilon^2 \sim$ standard diffuse prior

(iv). $v_i^{-1} \mid q \sim$ iid $\chi^2(q)$, v_i is the i^{th} element in V, the variance covariance matrix.

(v). $q \sim \Gamma(a_q, b_q)$, Gamma prior

Joint posterior distribution function of the parameters may be got from the product of the respective prior and likelihood functions. Full conditional prior for various parameters in the model may be derived as given below:

(a). The fuel conditional prior for φ

$\pi(\varphi \mid \rho, \sigma_\varepsilon^2, V, q) \propto N(r, S)$, Normal distribution -

where $r = [\sigma_\varepsilon^{-2} Z'V^{-1}\widetilde{Y}]$; $S = [\sigma_\varepsilon^{-2} Z'V^{-1}Z]^{-1}$

(b). The full conditional prior for σ_ε^2

$$\pi(\sigma_\varepsilon^2 \mid \varphi, \rho, V, q) \propto IG\left[\frac{n}{2}, \frac{e'V^{-1}e}{2}\right], \text{ Inverse Gamma distribution}$$

(c). The full conditional posterior for v_i

$\pi\left[\frac{-\sigma_\varepsilon^2 e_i^2}{v_i} \mid \varphi,\rho,\sigma_\varepsilon^2, v_{-i}, q\right] \propto$ iid $\chi^2(q+1)$ Chi square distribution; e_i – is the ith element of e & v_{-i} denotes the vector of all diagonal elements except v_i.

(d). The full conditional posterior for ρ.

$\pi(\rho \mid \varphi, \sigma_\varepsilon^2, V, q) \propto |I - \rho W| \exp\left\{-\frac{1}{2\sigma^2}(e'V^{-1}e)\right\}$ is a kernel of distribution.

(e). The log of the full condition posterior distribution for q

$\pi(q \mid \rho, \varphi, \sigma_\varepsilon^2, V) = \text{constant} + \frac{nq}{2}\ln\left(\frac{q}{2}\right) - n\ln\Gamma\left(\frac{q}{2}\right) - Kq - (a_q - 1)\ln(q)$; $K = \frac{1}{2}\sum_{i=1}^{n}\left\{\ln(v_i)+\frac{1}{v_i}\right\} + b_a$

The samples from the distribution [a]-[c] with Gibbs Sampler, and the distribution [d]-[e] with Metropolis – Hastings Algorithm (M-H Algorithm) are usef for the analysis.

Interpreting the Spatial Durbing Model:

The traditional beta convergence approach draws its inference from the coefficient of initial income variable β. For spatial durbin model this interpretation is not valid (LeSage and Fischer 2008; Fischer 2010). In this model there would be two effects; one described by Y_0 and the other described by Y_0, as Y is affected directly by any change in Y and is also affected by the feedback effect through $Y_{(j,o)}$ and thus, the impact of the initial value varies with location and the neighborhoods described byW. The former effect is the direct effect while the later is the indirect effect. They may be measured using the following:

$M_{direct}=n^{-1}tr(S(w))$, $S(w) = [1-\rho w]^{-1}[\beta I - \theta w]$; $M_{total}=n^{-1}\iota' S(w)\ \iota$; $M_{indirect}=M_{total}-M_{direct}$, where, $S(w)=(I-\rho w)-1(\beta I+\theta w)$.

Data source

This study analysed the regional disparity among 17 major states viz., AndraPradesh, Assam, Bihar, Goa, Gujarat, Hariyana, HimachalPradesh, Karnataka, Kerala, MadhyaPradesh, Maharashtra, Orissa, Punjab, Rajasthan, TamilNadu, UttarPradesh, WestBengal with the statewise data on Gross State Domestic Product (GSDP) at 2004-05 constant prices, obtained from the Ministry of Statistics and Programme Implementation and the percapita income was calculated using the projected statewise population data from the report of the Registrar General of Census, Government of India. The spatial weight matrix was computed based on row standardized binary contiquity matrics.

The model was estimated with the help of Bayesian MCMC sampler for all the three periods. This study used the structural variables viz., proportion of agriculture, proportion of industry and tertiary to industrial sector ratio as independent variables apart from the usual growth equation variables. The marginal likelihood was computed using method developed by Gelfand and Dey (1994).

The present study applied the Bayesian approach to spatial durbin model using the statewise percapita GSDP data at 2004 – 05 prices. The convergence hypothesis is tested for the periods. [a] 1980 – 1991 (pre reform period); [b] 1991 – 2000 (early reform period); [c] 2000 – 2010 (later reform period).

Results and discussion:

The results are given in the table 1 for all the 3 periods and the t value of the same is given. The statistic values suggests that the samples were successfully converged to the posterior distribution.

Table 1: Results of parameter estimation of Spatial Durbin Model for various time periods

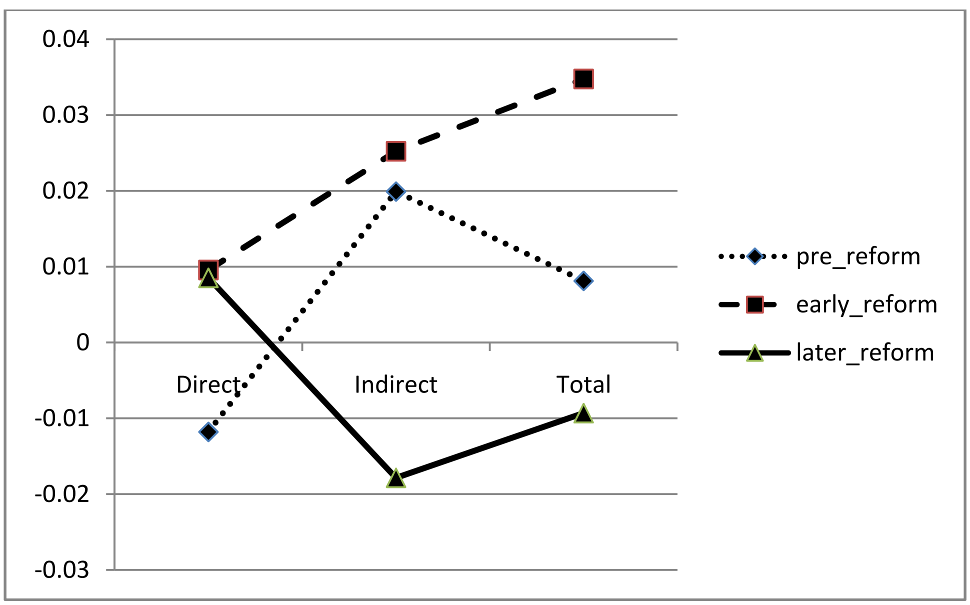
	Pre Reform period	Early Reform period	Later Reform period
Initial Income	-0.0123 (6.2709)	0.0121 (9.1792)	0.0078 (9.7654)
Proportion of Agriculture	-0.0514 (2.1626)	-0.0909 (13.3185)	-0.0908 (6.8347)
Proportion of Industry	0.0016 (0.0319)	-0.0135 (1.0391)	-0.0319 (1.2785)
Tertiary – Industry Ratio	-0.0054 (1.5013)	0.0048 (3.4152)	-0.0026 (1.0669)
Spatially lagged initial income	0.0196 (4.7171)	0.0336 (7.0966)	-0.0184 (7.4311)
Spatially lagged proportion of agriculture	0.035 (0.9213)	-0.0969 (3.2758)	-0.1727 (8.4939)
Spatially lagged proportion of industry	-0.1741 (1.6124)	-0.1239 (2.3915)	-0.3973 (7.0927)
Spatially lagged tertiary industry ratio	-0.0287 (3.6079)	0.0066 (1.3946)	-0.0402 (7.3162)
Constant	0.0636 (0.6921)	-0.3184 (4.6158)	0.4222 (6.3473)
Spatial Dependence measure - Rho (ρ)	0.1030 (14.0658)	-0.3143 (36.4325)	-0.1462 (16.6499)
Error variance (σ_ε^2)	0.0027 (3.0117)	0.0012 (1.8254)	0.0015 (2.9036)
R^2	0.7894	0.4583	0.4279

Note: The t – values of the respective coefficients are given in the paranthesis.

The estimation of ρ was positive for the first period. This could mean that the neighbouring regions have evolved similarly especially over this period. The estimate for the initial income was negative only for the pre reform period. But for the other two periods the coefficients were positive and significant. However, the β convergence hypothesis should not be tested with these estimates. For all the periods the coefficient of agricultural proportion was negative and significant. In the first period coefficient of industrial proportion and of the tertiary-industry ratio the same was positive and negative respectively but not significant. In the early reform period, the coefficient of industrial proportion was found to be negative and insignificant. The tertiary – industry ratio was significantly positive. In the later reform period, the proportion of agriculture was found to be negative and statistically significant but for the other two variables it was not statistically significant.

As mentioned in the methodology, in the spatial durbin model β convergence hypothesis cannot be tested using the values of β in the growth regression. Therefore, in this study we have calculated the direct, indirect and total effects.

Fig. 1. Decomposition of the effects (1980–2010).



The figure suggests that in the pre reform period direct effect was negative but the indirect effect was found to positive and the overall effect was positive. In the early reform period all the effects (direct / indirect / total) were positive. In the later reform period, though the direct effect was found to be positive, the indirect and over all effect was found to negative and hence a confirmation of beta convergence. In the pre reform and early reform periods the total effect suggesting the negation beta convergence. The direct effect or the outcome of the growth equation which is observed and interpreted in most of the studies, indicating an acceptance of beta convergence in pre reform and its rejection in the post reform periods. However, due to the feedback effect / indirect effect, the later reform period alone witnessed convergence while the convergence outcome was reversed in pre reform period. The negative indirect effect suggest the non existence of spill over effects. Thus this paper can conclude that the income disparities have increased since the earlier reforms period but have converged in the later reform period.

Conclusion

This study analysed the regional income disparity at the subnational level in India during the pre early and later reform periods. The data used in this research are per capita GSDP during 1980 to 2010. First, the study reviews various growth models and contents that spatial durbin model of Fingleton and Lopez-Bazo(2006) was empirically useful. Second, this study estimated parameter Bayesian Spatial Durbin Model for the three periods that is pre reform (1980-1991), early reform (1991-2000) and later reform (2000-2010) periods. Finally, the convergence hypothesis is tested in the light of LeSange and Fischer (2008) formulation.

The results suggest that the β convergence does not hold since the pre-reform period. An interesting observation is that all the effects are positive except one single negative direct effect during the pre-reform period, which was overwhelmed by the indirect effect resulting a positive total effect. The later reform period witnessed beta convergence due to feedback effect.

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