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Abatement Costs of Alternative Tax Systems to Regulate Agricultural Nitrogen Loss

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Abstract

Nitrogen emissions from agriculture are considered an important environmental problem in Denmark motivating consideration of different tax schemes as regulatory instruments. In this paper, input/output behaviour of Danish pig farmers is estimated with farm level panel data using the dual profit function approach, and emission functions for nitrogen loss are derived. With the estimated model we are able to compare cost-effectiveness of a comprehensive Pigouvian tax on nitrogen loss with simpler tax schemes that focus on nitrogen use. We find that both a fertilizer tax and a feed tax generate substantially higher abatement costs than Pigouvian incentives. A tax on nitrogen in all inputs will, on the other hand, only generate a marginal increase in abatement costs.

These results are of interest because a tax on all nitrogen inputs is easier to implement than a comprehensive nitrogen loss tax. Our result implies that even a limited administrative cost advantage may make the input tax preferable to implementing Pigouvian incentives through an nitrogen loss tax.

1. Introduction

Nitrogen emissions from agriculture are considered an important environmental problem in Denmark and substantial reductions in nitrogen loss have been called for. Reduction goals have so far not been met and incentive schemes are being considered as an alternative to the existing command and control regulation in order to increase regulatory cost-effectiveness.

A tax on nitrogen fertilizer has been on the political agenda for several years and recently a tax on nitrogen loss has been suggested in different variants by e.g. Huang and LeBlanc (1994), Fontein *et al.* (1994) and Hansen (1999). If nitrogen loss is accepted as an indicator of the environmental effect then taxing nitrogen loss corresponds to a Pigouvian externality tax and will therefore minimise abatement costs. However, the administrative costs of taxing nitrogen loss may be substantially higher than the administrative costs of taxing nitrogen fertilizer inputs. It is therefore important to evaluate the magnitude of the abatement cost advantage of the nitrogen loss tax relative to simpler tax schemes. If the abatement cost advantage is small, simpler tax systems may be preferable when administrative costs are taken into account.

In this paper, input demand and output supply behaviour of Danish pig farmers is estimated with farm level panel data using the dual profit function approach. Our panel allows estimation with a number of farm-specific effects and we exploit estimation of the unrestricted profit function to derive long-run effects. The data also allow calculation of farm level nitrogen loss so that it becomes possible to quantify (an indicator of) the environmental effect of different tax policies as well as their abatement costs, i.e. allowing us to compare cost-effectiveness.

Although the analysis only covers part of the Danish agricultural sector, pig production is a major contributor to Danish agriculture's nitrogen loss as well as being an economically important agricultural sub-sector (see footnotes 1 and 2 for references). Since nitrogen loss from pig production is also considered an important environmental problem in a number of other European countries and some US states, results regarding regulatory efficiency within the Danish pig sector may be of more general interest.

Our analysis may also be of methodological interest since we utilize the emission function approach instead of the standard externality model used in other comparable studies (to our knowledge the only other micro econometric study of nitrogen emission from pig production is Fontein *et al.* (1994)). The emission function approach solves a consistency problem that arises when the standard approach is applied to nitrogen loss from Pig production. The consistency problem pointed out here is, however, not specific to this emission problem and the emission function approach may therefore be preferred for analysis of a wider class of emissions.

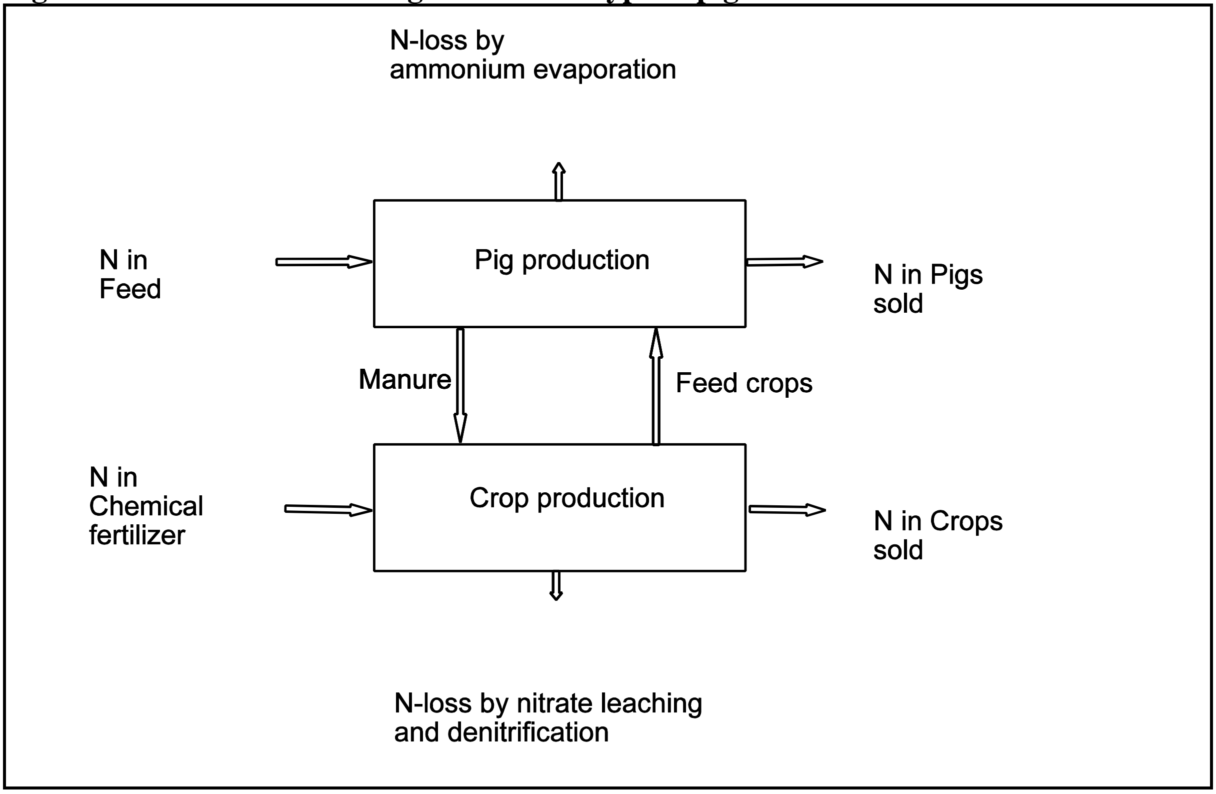
In the next section Danish pig farming is described. In section three, we demonstrate the consistency problem of the standard externality model and develop an alternative approach. After a data description in section four, model estimation and control is presented in section five. Section six summarises the results, and conclusions are drawn in section seven.

2. Danish Pig Production, Nitrogen Emissions and Regulation

Denmark is, along with the Netherlands and the US, one of the world's major pork meat exporters and the total number of pigs delivered to slaughterhouses was close to 22 million in 1999 - a number that has been steadily rising over the last decade. Pig farms account for about

40% of total agricultural income on full time farms[1]. In addition, a comparable proportion of total agricultural nitrogen loss in Denmark is attributed to pig production making it a key sub-sector in this regard as well[2]. Nitrogen flows, through a typical Danish pig farm are illustrated in figure 1.

Figure. 1. Illustration of nitrogen flow on a typical pig farm[3]



Note: N=Nitrogen

Nitrogen is imported to the farm in feed and fertilizer and exported as part of pig and crop output. Finally, nitrogen is lost through the production process. A substantial part of nitrogen emissions from pig farms originates from the production of pig manure. Part of the nitrogen content of manure is lost in stalls and during storage through ammonium evaporation. When the remaining manure bound nitrogen is utilized in plant production, the utilisation level is substantially lower than the utilisation level for chemical fertilizer. Surplus nitrogen in manure (i.e. the part of the nitrogen content that is not ultimately incorporated into crops) is washed out as nitrate or denitrified along with surplus nitrogen from chemical fertilizer. Thus, when disregarding changes in nitrogen stocks, (i.e. nitrogen bound in organic form in humus,

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[1] See Danmarks Statistik (1999a and 1999b).

[2] See e.g. Skop and Schou (1996) and Andersen et al. (forthcoming).

[3] Nitrogen fixating leguminous crops are not an important part of the Danish pig farm crop mix and are ignored in the figure.

weeds, cover crops etc.), nitrogen loss is equal to nitrogen imported in fertilizer and feed less nitrogen exported in products.

Nitrogen emissions are widespread in Denmark, (where almost two thirds of the total land area is cultivated), and have given rise to local groundwater contamination and regional eutrofication[4] of coastal sea areas, (effects primarily originating from nitrate leaching and ammonium evaporation), in addition to global warming affects attributed to denitrified di-nitrogen-oxide. The intensity of emissions and the regional/global nature of some of the environmental effects have motivated the Danish Parliament to set national goals for reduction of aggregate nitrogen emissions that supplement local statutes addressing local contamination problems.

Although nitrogen loss is not a good indicator of the total environmental effect of emissions, (known to vary from locality to locality and across emitted compound types), it may be a usable indicator of the baseline local and regional/global effects targeted in national environmental action plans and we will use this indicator in the following[5].

Since the end of the 1980s a national goal for reducing aggregate nitrogen emissions has been part of the Danish environmental action plan and all Danish farmers have been subject to national regulations that have restricted when and how manure can be applied to fields, required minimum manure storage capacity and winter plant cover etc. As mentioned, the reductions called for in the action plans have not been reached and incentive regulation is being considered as a way of meeting the planned emission reductions at lower costs.

A tax on nitrogen in chemical fertilizer is one possibility being considered. This tax is easy to implement and control (only requiring implementation and control at the level of wholesale fertilizer suppliers). However, when considering figure 1 it is clear that such a tax does not target nitrogen loss directly, and that perfect correlation between fertilizer use and nitrogen loss cannot be expected to hold (e.g. a fertilizer tax gives farmers an incentive to reduce fertilizer consumption by increasing manure nitrogen application/feed imports which ceteris paribus increases nitrogen loss).

Recently, a tax on nitrogen loss has been proposed. Clearly such a tax corresponds to a Pigouvian externality tax if, as we have assumed, nitrogen loss is an acceptable measure of the targeted environmental effect of nitrogen emissions[6]. By utilising the mass balance condition, (nitrogen loss = nitrogen imported in inputs - nitrogen exported through outputs), the tax can be implemented by requiring farmers to keep an account of nitrogen in farm

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[4] Eutrofication is a process where algie growth spurred by excess nutrient loading causes oxygen depletion and death/migration of higher level life forms.

[5] For a detailed presentation of nitrogen loss from Danish agriculture, its environmental effects and use of the nitrogen loss indicator as a proxy for environmental effects see e.g. Hansen (1999).

[6] Here, and in the following, we use the term Pigouvian to indicate that the structure of this tax system allows/is consistent with implementation of welfare maximizing Pigouvian incentives. By itself, the tax system only ensures that the abatement cost of reaching any given reduction goal is minimized. Attainment of Pareto optimum also requires that the emission reduction goal in the national action plan be set at the welfare maximising level.

imports and exports, thus allowing nitrogen loss to be calculated residually[7]. This would, however, necessitate costly inspection and control of nitrogen accounts at the farm level. It may be possible to implement such a tax through a nitrogen deposit refund system, whereby farm level inspection can be avoided[8]. But a deposit-refund system would still require a complex system of taxes and refunds at the level of wholesale suppliers, dairies, slaughterhouses etc. involving substantially higher administrative costs than, for example, a simple fertilizer tax.

In conclusion it is not clear that implementation of Pigouvian incentives will be the most efficient tax alternative when administrative costs are taken into account. An efficiency ranking requires quantification of administrative costs as well as abatement costs. In the following, we quantify abatement costs of the Pigouvian tax on nitrogen loss and three simpler alternatives (a feed tax, a fertilizer tax, and a tax on nitrogen in all inputs) for the key agricultural sub-sector of pig production.

3. Modelling Pig Farm Production and Nitrogen Loss

We are aware of one published microeconometric study of nitrogen loss from pig production (Fontein et al., 1994)[9]. In this paper the authors follow the standard externality specification (see e.g. Baumol and Oates, 1988) and model nitrogen loss as a production input. Since nitrogen loss is not observed directly it calculated from observed input and output flows using the mass balance condition. However, this approach implies a consistency problem since the

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[7] Recently, the Netherlands has implemented a farm level tax on nitrogen loss based on a farm level nutrient accounting system (MINAS).

[8] See e.g. Hansen (1999) for details.

[9] Other related studies include Poit-Lepetit and Vermersch (1998) where the non-parametric distance function approach is used on micro data from French pig farms and several econometric studies of aggregated time series data e.g. Hallam and Zanoli (1993) and Kuiper and Meulenberg (1997).

assumed production function does not satisfy the mass balance condition[10]. This is clearly so in the short run where inputs and outputs may vary while nitrogen loss is constant. When the usual flexible functional forms are used (e.g. quadratic, trans-log, generalized Liontief) the mass balance condition will not be satisfied in the long run either.[11].

The emission function approach

To overcome these problems, this paper uses the emission function approach (see e.g. Holtermann, 1976 for an early presentation and e.g. Weaver *et al.* (1996) for a more recent application). In this modelling approach, the production function is specified independently of nitrogen loss i.e. $f(x,z)$ where x is a vector of variable inputs (negatively signed) and outputs (positively signed) and z is a vector of fixed inputs. This is supplemented with an emission function $n = g(x,z)$ where n is nitrogen loss. Emissions are not a production input nor do they enter into the firms' maximisation problem (in the pre-regulation state). Instead emissions are derived ex post the production decision as a function of firm decision variables. Specification of the emission function is given by the mass balance condition so that we have $n = g(x,z) = -\alpha x$ where α is a vector indicating unit nitrogen content of outputs and inputs.

This approach ensures mass balance consistency and avoids the somewhat awkward assumption that nitrogen loss is constant in the short run. Again, letting θ be a vector of parameters, denote $\pi(p_x,z,\theta)$ as the dual profit function associated with $f(x,z)$ so that the system

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[10] In Fontein et al.(1994) a multi-input multi-output production function $f(x,n,z)$ is assumed where x is a vector of variable inputs (negatively signed) and outputs (positively signed), n is nitrogen loss and z is a vector of fixed inputs. Fontein et al. assume that nitrogen loss is fixed in the short run and specify the dual profit function $\pi(p_x,n,z,\theta)$ where p_x is the vector of variable input/output prices and θ is a vector of parameters. Parameters are estimated using the system of derived short run demand and supply equations:

$$\pi \square \pi(p_x,n,z,\theta)$$
$$x \square \frac{d\pi}{dp_x}(p_x,n,z,\theta)$$

Nitrogen loss cannot be observed, but must be calculated utilising the mass balance condition $n = -\alpha x$ where α is a vector indicating unit nitrogen content of outputs and inputs. The shadow value of increasing nitrogen loss $\hat{p}_n$ is derived as $\hat{p}_n \square \frac{d\pi}{dn}(p_x,n,z,\hat{\theta})$ where $\hat{\theta}$ is the vector of estimated parameters. The model implies that a tax (t_n) on nitrogen loss has no effect in the short run but has a long run effect that is calculated assuming that farmers adjust n (holding z constant) so that $\hat{p}_n \square\square t_n \square \frac{d\pi}{dn}(p_x,n,z,\hat{\theta})$ is satisfied (with corresponding long-run adjustments in x).

[11] This issue is not peculiar to the modelling of pig production. There is a potential consistency problem whenever the standard externality model (combined with the usual functional forms) is applied to an emissions that is known to satisfy the mass balance condition (or in fact any known functional relationship with production inputs/outputs).

of derived short run demand and supply functions to be estimated becomes:

$$\begin{aligned}\pi &= \pi(p_x, z, \theta)\\ x &= \frac{d\pi}{dp_x}(p_x, z, \theta)\end{aligned} \tag{1}$$

while nitrogen loss is given by the emission function:

$$n = -\alpha x. \tag{2}$$

Any tax system based on inputs and outputs can be characterised by the vector of changes in the prices farmers face. Denoting this vector as t_x , the input demand and output supply induced in the short run are found by inserting the new price vector $p_x + t_x$ into the estimated system (1) keeping fixed inputs constant[12].

The effects of taxes on inputs and outputs in the x vector can be calculated directly from (1), but since nitrogen loss is not modelled as an input, the effects of a tax (t_n) on nitrogen loss are simulated by the following tax system:

$$t_x = \alpha t_n \tag{3}$$

i.e. all input and output prices are raised in proportion to their nitrogen content (i.e. inputs are taxed and outputs subsidised in proportion to their nitrogen content). By the mass balance condition/emission function this is equivalent to a tax of t_n on nitrogen loss[13].

Short and long run cost effectiveness

We derive short run effects as well as long run reactions where fixed inputs are allowed to adjust. Noting that π is a gross margin from which the cost of fixed inputs must be deducted to find the farmers' profit (denoted Π) we have:

$$\Pi = \pi - p_z z \tag{4}$$

where p_z is a vector of fixed input prices.

The short run abatement costs induced by a tax system t_x are, by definition, equal to the induced profit loss less net tax payment. Letting superscript 0 indicate the base period and superscript 1 the period just after implementation of any given tax vector t_x , the abatement

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[12] This notation implies that an input tax has a positively signed tax rate (raising the input price) while an output tax has a negatively signed tax rate (lowering the output price). For subsidies the reverse sign rule applies.

[13] This is easily verified: total tax payed by the farmer is $-t_x x$ which by (3) is $-\alpha t_n x = t_n(-\alpha x)$ and by (2) is equal to $t_n n$ (see Holtermann, 1976 for a general presentation of Pigouvian incentives through indirect taxation).

costs (denoted ΔC^{sr}) generated in the short run become:

$$\Delta C^{sr} = \Pi^1 - \Pi^0 + t_x x = \pi(p_x^0 + t_x, z^0, \hat{\theta}) - \pi(p_x^0, z^0, \hat{\theta}) + t_x x \tag{5}$$

and by (2) and (3), the corresponding short run change in nitrogen emissions is

$$\Delta n^{sr} = n^1 - n^0 = (\alpha x^1) - (\alpha x^0) = -\alpha\left[\frac{d\pi}{dp_x}(p_x^0 + t_x, z^0, \hat{\theta}) - \frac{d\pi}{dp_x}(p_x^0, z^0, \hat{\theta})\right] \tag{6}$$

Even when the vector of fixed input prices p_z is not observed, long-run effects of a tax system can be calculated if the profit function is fully estimated. The shadow values of fixed inputs $\hat{p}_z$, for the chosen base period, can be derived from the estimated profit function as

$$\hat{p}_z^0 = \frac{d\pi}{dz}(p_x^0, z^0, \hat{\theta}) \tag{7}$$

Assuming that farmers are in long-run equilibrium in the base year, the derived shadow price will be equal to the marginal cost of acquiring an additional unit of the given fixed input so that $p_z = \hat{p}_z^0$. This allows full specification of the farmers' profit function i.e.:

$$\Pi(p_x, z, \hat{\theta}) = \pi(p_x, z, \hat{\theta}) - \hat{p}_z^0 z \tag{8}$$

giving the following first order conditions for optimal fixed input allocation under the tax system t_x:

$$\hat{p}_z^0 = \frac{d\pi}{dz}(p_x^0 + t_x, z, \hat{\theta}) \tag{9}$$

Let z^2 denote the solution to (9) and superscript 2 indicate a period after long-run adjustment of the fixed inputs.

The long run abatement costs then become:

$$\Delta C^{lr} = \Pi^2 - \Pi^0 + t_x x = \pi(p_x^0 + t_x, z^2, \hat{\theta}) - \pi(p_x^0, z^0, \hat{\theta}) - \hat{p}_z^0(z^2 - z^0) + t_x x \tag{10}$$

where the change in fixed costs is added to the long run effect on the gross margin to find the long-run change in profits. The corresponding long-run change in nitrogen loss is:

$$\Delta n^{lr} = n^2 - n^0 = (\alpha x^2) - (\alpha x^0) = -\alpha\left[\frac{d\pi}{dp_x}(p_x^0 + t_x, z^2, \hat{\theta}) - \frac{d\pi}{dp_x}(p_x^0, z^0, \hat{\theta})\right] \tag{11}$$

As already noted, only when farmers are in long run equilibrium in the estimation period will the estimated marginal profit effect of fixed inputs equal the fixed input cost. If the true price is lower/higher than $\hat{p}_z^0$ use of this proxy will lead to over/underestimation of long run marginal abatement costs. However, relative cost evaluations (that are the focus of the current paper) will be less sensitive to deviations from the assumption, since cost evaluations of different regulatory schemes are always biased in the same direction.

Specification of the production function

Since the focus here is nitrogen loss, it is crucial that the estimated model describes supply and demand behaviour for inputs and outputs that are important for nitrogen flow, while at the same time keeping the model reasonably simple. The production function assumed here for each of the two groups of farms has two outputs: Pigs (x_1) i.e piglets/fattening pigs and crops (x_2), two inputs: feed (x_3) and nitrogen fertilizer (x_4) and two fixed intermediate inputs: cultivated land (z_5) and pig stall capacity (z_6) i.e. sow sties/fattening pigsties. The production of intermediate fixed input is specified as follows:

$$z_5 = z^5(\text{capital, labour, materials, land})$$
$$z_6 = z^6(\text{capital, labour, materials})$$

so that the specified primary inputs can be dropped from the model if intermediate inputs are observed in the data.

We assume that cultivated land is produced using land, tractor and harvesting equipment, fuel and labour as inputs (i.e. the cost of cultivating a hectare of land is assumed independent of the amount of fertilizer applied and the crop yield harvested). The remaining non-nutrient cropping input (drying costs, sowing seed and pesticide costs) is assumed to be proportional to crop production (i.e. a limitation production relationship). Proportionality to crop output is also assumed for phosphate and kali fertilizer since they under Danish farming conditions usually are applied so to insure an ample stock of nutrients for growing crops (i.e. the nutrient restricting crop yield is normally nitrogen). The cost of heating stalls, along with capital and labour are treated as an input to producing pig stall capacity (and therefore assumed independent of the actual number of pigs produced). Whereas veterinary costs and the cost of piglets for fattening pigs is assumed proportional to output. Thus, only feed and nitrogen fertilizer inputs enter into a complex production relationship with output.

This specification seems a reasonable simplification while retaining flexible estimation of behaviour with respect to key nitrogen flows. In addition, the specification allows us to utilise more reliable data on intermediate inputs (cultivated land area and number of pigsties) instead of capital accounts and self-reported labour inputs. We do, however, estimate alternative specifications in order to test the assumed production structure.

4. The Data

Estimations are based on a panel data set provided by the Danish Agricultural Advisory Centre (Landbrugets Rådgivningscenter). The panel contains annual data, and is unbalanced covering twelve growing seasons (1980 to 1991) with, on average, 1350 farms represented each year with each farm participating for an average of 3.9 years. Data are sampled from detailed gross margin accounts through a voluntary programme where only a small proportion of the more business oriented farmers participate. On the one hand, voluntary participation is an advantage in that participating farmers a priori are motivated and have an incentive to provide data of high quality. On the other hand, the sample of farmers in the data set is not representative of the population of Danish farmers.

For this analysis two groups of specialized pig farms were selected. The criteria for selection was that the farmer had no cattle, that at least 20% of gross revenue originated from pig production and that pig production is specialized (i.e. either piglets or fattening pigs are produced, but not both). Specialised pig farms comprise about half of all pig farms and about 10% of all farms in the data set.

For each farm the data include detailed annual accounts of variable costs, along with corresponding accounts of quantitative flows of most nitrogen relevant inputs and outputs (e.g. fertilizer, feed, crop yield, meat production, etc.). This allows an analysis of production and calculation of input and output prices at the farm level for most nitrogen relevant inputs and outputs. Coefficients indicating average nitrogen content of different inputs and outputs have been added enabling us to calculate annual farm level mass balances for nitrogen and residual nitrogen loss.

Prices for pig output and nitrogen input were calculated directly for each farmer as income (net of proportional input costs) and costs respectively, divided by volume. Using a common base observation (containing all feed and crop types) Fisher price indexes for crops and feed were constructed based on each individual farmer's price. These were again calculated as income (net of proportional input costs) and costs respectively divided by volume. Thus prices for all inputs and outputs vary across farms as well as over time. Mean values of the price index are reported in Table 1.

Table 1 Means of farm specific price index for specialized pig farms

	Piglet producers				**Fattening pig producers**			
Year	*Pigs*	*Crops*	*Feed*	*N-fertilizer*	*Pigs*	*Crops*	*Feed*	*N-fertilizer*
1980	0.78	0.83	0.56	0.65	0.91	0.86	0.54	0.74
1981	0.79	0.90	0.63	0.82	No observations			
1982	0.97	1.02	0.72	0.95	1.20	1.02	0.79	0.93
1983	1.07	1.11	0.77	1.17	1.20	1.13	0.88	1.17
1984	1.03	1.19	0.83	1.23	1.29	1.21	0.96	1.23
1985	1.25	1.10	0.84	1.39	1.29	1.12	0.90	1.38
1986	1.00	1.05	0.73	1.25	1.11	1.08	0.82	1.21
1987	0.97	0.97	0.71	0.93	1.08	0.98	0.75	0.94
1988	0.98	0.97	0.68	0.96	1.12	0.97	0.76	0.92
1989	1.20	1.02	0.69	0.98	1.26	1.03	0.79	0.99
1990	1.06	0.91	0.62	1.01	1.06	0.93	0.70	1.00
1991	1.15	0.86	0.62	1.13	1.20	0.90	0.65	1.13

All indexes exhibit substantial variation over the data period though no trend is apparent.

Mean values for the two data sets of key production variables and environmental indicators are shown in Table 2. As noted, applied nitrogen fertilizer volume is registered in farm accounts while nitrogen loss is calculated using registered volumes and standard (average) nitrogen coefficients.

Table 2 Means of production and emission variables for specialized pig farms

	Piglet producers	Fattening pig producers
Profit share - Pigs*	1.47	1.81
Profit share - Crops*	0.37	0.51
Profit share - Feed*	-0.82	-1.30
Profit share - N-fertilizer*	-0.02	-0.02
Pig stall capacity	126.7 sow stalls	717.6 pig stalls
Cultivated area	38.4 hectares	53.9 hectares
Applied nitrogen fertilizer	118.6 kg/hectare	114.1 kg/hectare
Nitrogen loss	263.8 kg/hectare	255.1 kg/hectare
Number of farms	220	220
Number of observations	610	676

* When calculating profit shares income from outputs (positively signed) and costs from inputs (negatively signed) are measured as proportions of profit. Thus the sum of positively signed shares is greater than 1 so that when negatively signed profit shares originating from costs are added the resulting profit is by definition 1.

While pig production is the main revenue generator, crop production is an important income source for both groups (i.e. Danish pig farms are less specialised than e.g Dutch pig farms). The sizable cultivated area (by European standards) on Danish pig farms also explains why a substantial amount of chemical nitrogen fertilizer is applied to fields in addition to manure and why N-loss per area is relatively low by European standards (see Brouwer et al. (1995) for a useful cross country comparison).

5. Estimation

Separate models were estimated for each of the two groups of farmers. The profit function was assumed to have the trans-log[14] functional form with the following specification:

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[14] The trans-log profit function resembles the trans-log cost function in that functional form and parameters are identical. However instead of dependant variables being positively signed cost shares (that sum to 1) the profit function has profit shares as the dependants that represent both costs of inputs and income from outputs. Corresponding to this the underlying cost minimisation assumption has been replaced by a profit maximisation assumption. The profit shares also sum to 1 but while income from outputs (measured as a proportion of profit) are positively signed costs of inputs (measured as a proportion of profit) are negatively signed. The sum of positively signed profit shares from income is typically greater than 1 so that when negatively signed profit shares from costs are added the resulting profit is (by definition) equal to 1. Se e.g Chambers (1988) for the basics and e.g. Fontein et al. (1994) for an application.

$$\begin{aligned}\ln(\pi_{n,t}) = a_n + \sum_{i=1}^{4}\ln(p_{i,n,t})b_{i,n} + \sum_{i=5}^{6}\ln(z_{i,n,t})b_{i,n} +\\ ½\sum_{i=1}^{4}\sum_{j=1}^{4}\ln(p_{i,n,t})c_{i,j}\ln(p_{j,n,t}) + ½\sum_{i=5}^{6}\sum_{j=5}^{6}\ln(z_{i,n,t})c_{i,j}\ln(z_{j,n,t}) +\\ ½\sum_{i=1}^{4}\sum_{j=5}^{6}\ln(p_{i,n,t})c_{i,j}\ln(z_{j,n,t}) + ½\sum_{i=5}^{6}\sum_{j=1}^{4}\ln(z_{i,n,t})c_{i,j}\ln(p_{j,n,t})\end{aligned} \qquad (12)$$

yielding derived profit share equations for i=1,...,4 of the following form:

$$s_{i,n,t} = b_{i,n} + \sum_{j=1}^{4}c_{i,j}\ln(p_{j,n,t}) + \sum_{j=5}^{6}c_{i,j}\ln(z_{j,n,t}) \qquad (13)$$

where i and j subscripts indicate input/output types, n is a farm index and t indicates the time period and $s_{i,n,t}$ is the profit share of input/output i (defined as $s_{i,n,t} = \frac{p_{i,n,t}x_{i,n,t}}{\pi_{n,t}}$). This is a quite flexible model with fixed effects in each budget share equation as well as in the profit equation. Thus, the model allows for substantial heterogeneity with five firm specific parameters for each farm.

The complete system was estimated in two steps. First the system of derived profit share equations (without the profit function) was estimated using an iterated SUR procedure (corresponding to maximum likelihood estimation). Restricting the parameters to ensure symmetry and homogeneity in prices and fixed inputs (implying $\sum_{i=1}^{4} b_{i,n} = 1$ and $\sum_{i=5}^{6} b_{i,n} = 1$ for all n, $\sum_{j=1}^{4} c_{i,j} = 0$, $\sum_{j=5}^{6} c_{i,j} = 0$ for all j and $c_{i,j} = c_{j,i}$ for all i,j) we eliminate the feed equation (S_3) to avoid singularity (the maximum likelihood procedure ensures that estimates are invariant as to which equation is eliminated). Technically $c_{i,j}$ for i = 1,..,4 and j = 1,..,6 in (13) are estimated using within farm transformed variables, eliminating time invariant constants $b_{i,n}$, which are then estimated as $\hat{b}_{i,n} = \bar{s}_{i,n,t} - \sum_{j=1}^{4}\hat{c}_{i,j}\overline{ln}(p_{j,n,t}) - \sum_{j=5}^{6}\hat{c}_{i,j}\overline{ln}(z_{j,n,t})$ for all n and i = 1,..,4, where ¯ indicates the mean of the variable over the time periods that farm n participates in the panel. Results of this estimation are reported in Tables 3 and 4 (unique parameters for the feed equation were calculated residually as indicated).

Table 3 Common parameters estimated in the first step

Parameter	Piglet producers: Estimate	Piglet producers: Approx Std Err	Piglet producers: Approx Prob	Fattening pig producers: Estimate	Fattening pig producers: Approx Std Err	Fattening pig producers: Approx Prob
c_{11}	-0.3742	0.0593	0.0001	-0.3411	0.1198	0.0046
c_{12}	-0.2334	0.0239	0.0001	-0.4212	0.0322	0.0001
c_{13}	0.5854	0.0521	0.0001	0.7354	0.1099	0.0001
c_{14}	0.0223	0.0029	0.0001	0.0269	0.0046	0.0001
c_{15}	-0.4119	0.0380	0.0001	-0.2186	0.0678	0.0013
c_{16}	0.4119	0.0380	0.0001	0.2186	0.0678	0.0013
c_{22}	0.3070	0.0294	0.0001	0.3951	0.0356	0.0001
c_{23}	-0.0663	0.0242	0.0064	0.0236	0.0267	0.3768
c_{24}	-0.0072	0.0034	0.0329	0.0024	0.0053	0.6522
c_{25}	0.1661	0.0139	0.0001	0.0216	0.0145	0.1366
c_{26}	-0.1661	0.0139	0.0001	-0.0216	0.0145	0.1366
c_{33}*	-0.5170			-0.7433		
c_{34}	-0.0019	0.0029	0.5050	-0.0157	0.0040	0.0001
c_{35}*	0.2501			0.1963		
c_{36}*	-0.2501			-0.1963		
c_{44}	-0.0130	0.0021	0.0001	-0.0136	0.0026	0.0001
c_{45}	-0.0042	0.0016	0.0093	0.0006	0.0021	0.7763
c_{46}	0.0042	0.0016	0.0093	-0.0006	0.0021	0.7763

Eq	Piglet producers: DF	Piglet producers: R^2_{within}	Piglet producers: R^2	Fattening pig producers: DF	Fattening pig producers: R^2_{within}	Fattening pig producers: R^2
S_1	387	0.1769	0.7980	453	0.0305	0.8241
S_2	387	0.2891	0.8492	453	0.2052	0.8499
S_4	387	0.1133	0.7439	453	0.0821	0.6670

Piglet producers: Correlation of Residuals

	S_1	S_2	S_4
S_1	1.000	-0.362	-0.242
S_2	-0.362	1.000	-0.284
S_4	-0.242	-0.284	1.000

Fattening pig producers: Correlation of Residuals

	S_1	S_2	S_4
S_1	1.000	-0.346	-0.164
S_2	-0.346	1.000	-0.331
S_4	-0.164	-0.331	1.000

Note: * indicates residual calculation using model restrictions.

In Table 3 we present common parameters as well as key regression statistics and the residuals' correlation matrix for the estimated system, and Table 4 contains the mean, median and standard deviation of the distribution of residually calculated farm specific fixed effects.

Table 4 Farm specific fixed effects estimated in the first step

	Piglet producers			Fattening pig producers		
Parameter	*Mean of Estimates*	*Median of Estimates*	*Std. Dev. of Estimates*	*Mean of Estimates*	*Median of Estimates*	*Std. Dev. of Estimates*
$b_{1,n}$	1.1656	1.1355	0.2331	1.4843	1.3736	0.6272
$b_{2,n}$	0.5600	0.5622	0.0865	0.6283	0.6293	0.1466
$b_{3,n}$*	-0.6989	-0.6718	0.2361	-1.0828	-0.9786	0.5381
$b_{4,n}$	-0.0268	-0.0257	0.0111	-0.0299	-0.0272	0.0140

Note: * indicates residual calculation using model restrictions.

Fixed effects are highly significant in all derived equations. The common parameters are also highly significant in most cases. Estimated share equations are consistent with monotonicity and convexity. Inspection of error correlation matrices did not reveal serious serial correlation in any of the equations. Statistical tests showed error distributions significantly different from the normal distribution for all equations though residual plots indicated that deviations are not substantial. Residual plots for all equations in both data sets showed signs of heteroscedastic error terms most clearly in equation S_4. Though non-normality and heteroscedasticity may invalidate inference tests, parameter estimates are still unbiased.

In the second step, the remaining parameters of the profit function were estimated treating fixed effects and parameters estimated in the derived system as known, i.e. :

$$\left[\ln(\pi_{n,t}) - \sum_{i=1}^{4}\ln(p_{i,n,t})\hat{b}_{i,n} - ½\sum_{i=1}^{4}\sum_{j=1}^{4}\ln(p_{i,n,t})\hat{c}_{i,j}\ln(p_{j,n,t}) - \sum_{i=1}^{4}\sum_{j=5}^{6}\ln(p_{i,n,t})\hat{c}_{i,j}\ln(z_{j,n,t})\right] = a_n + \sum_{i=5}^{6}\ln(z_{i,n,t})b_{i,n} + ½\sum_{i=5}^{6}\sum_{j=5}^{6}\ln(z_{i,n,t})c_{i,j}\ln(z_{j,n,t}) \qquad (14)$$

If the parameters estimated in the first step are unbiased, this will also be the case for parameters estimated in the second step, however, the procedure may generate heteroscedasticity and invalidate the usual inference statistics[15]. Results of the second estimation step are reported in Table 5.

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[15] Estimating the complete system, including the profit function, with fixed effects requires explicit representation of all fixed effects for each farm (as dummy variables) and twas not feasible. The alternative of specifying the five farm specific parameters (a_n , $b_{i,n}$ for i = 1,..4) as random effects was not attempted. The assumption of independence of individual effects and explanatory variables necessary for the random effects specification to be valid is not likely to hold.

Table 5 Parameters estimated in the second step

	Piglet producers			**Fattening pig producers**		
Parameter	*Estimate*	*Approx Std Err**	*Approx Prob**	*Estimate*	*Approx Std Err**	*Approx Prob**
b_5	0.5641	0.0398	0.0001	0.6489	0.0450	0.0001
b_6	0.4358	0.0398	0.0001	0.3510	0.0450	0.0001
c_{55}	-0.1853	0.1431	0.1957	-0.1285	0.0565	0.0235
c_{56}	0.1853	0.1431	0.1957	0.1285	0.0565	0.0235
c_{66}	-0.1853	0.1431	0.1957	-0.1285	0.0565	0.0235
	Mean of Estimates	*Median of Estimates*	*Std. Dev. of Estimates*	*Mean of Estimates*	*Median of Estimates*	*Std. Dev. of Estimates*
a_n	8.9408	8.9430	0.4098	8.2365	8.2847	0.4941
	DF	R^2_{within}	R^2	DF	R^2_{within}	R^2
	388	0.3698	0.9135	454	0.1075	0.8790

Note: * inference statistics conditional on known first step parameters, i.e. a lower bound for true standard errors.

Error correlation matrices and residual plots for the second step estimation did not indicate serious serial correlation or heteroscedasticity whereas significant (but not serious) non-normality was detected.

All estimated own price elasticities have the expected signs (consistent with profit maximisation) in mean and median[16] and depending on the own price elasticity this also holds for 61%-99% of all single observations in the fattening pig data set and for 77%-100% of all single observations in the piglet producer data set (see appendix). Cross-price elasticities of outputs with corresponding variable inputs (i.e. pigs with feed and crops with fertilizer) all have the expected signs in mean and median and depending on the price elasticity for 88% - 100% of individual observations in the two data sets. In both data sets, shadow rents for fixed inputs are positive in mean and median as are all single observations of rents of cultivated land while pig stall rents are positive for 98% of the piglet producer observations and 87% of the fattening pig producer observations. As one would expect the mean/median shadow rent of land is almost the same in the two data sets and as expected the shadow rent for a sow stall is substantially larger than for a fattening pigsty. Shadow rents also have plausible orders of magnitude when compared with market prices of land and building costs of sties.

One peculiarity seems notable. Both of the estimated models have a relatively large positive cross price elasticities between the two outputs. This may be explained by e.g. the fertilizer effect of pig manure. We have, nevertheless, estimated alternative models with parameter restrictions (on $c_{1,2}$) ensuring moderate cross price elasticities. These and a number of other alternative specifications where the homogeneity assumption for fixed inputs is dropped and where trend and labour input variables are included as fixed inputs, are presented

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[16] With one exception: mean own price elasticity of fattening pig fertilizer is positive due to a small number of relatively large positive elasticity observation. The median of this own price is negative as are 78% of the single observations.

in the appendix. Price elasticities are not affected dramatically by these specification changes whereas shadow rents of fixed inputs are - in some cases becoming wrongly signed or having implausible/inconsistent orders of magnitude.

In conclusion, the estimated model performs well statistically and own and cross-price elasticities have the correct signs, plausible magnitudes and are robust to plausible specification changes. Shadow rents of fixed inputs on the other hand, are highly dependent on the assumed specification but attain more plausible values with the model presented than with alternative specifications.

6. Results

In Table 6 short and long run own price elasticities for each farm type are reported.

In the short run fixed inputs are held constant so that elasticities for goods i = 1,..,4 are given by the usual trans-log formula:

$$s_i + c_{ii}/s_i - 1 \tag{15}$$

When calculating long-run elasticities, a choice must be made about which fixed inputs are allowed to adjust and whether the costs of fixed inputs are affected by this adjustment.

When considering marginal investments in pig stall capacity, it seems reasonable to assume elastic supply functions for producers of the relevant buildings and equipment, so that the price of pig stall capacity can be assumed not to change.

This is not the case for land. Aggregate land supply available for farming is highly inelastic. This means that the ceteris paribus assumption will not hold for land prices when analysing a change in the price of a variable input/output that affects all farmers. Land prices will adjust so as to neutralise the initial change in aggregate land demand. Thus holding land input constant when calculating long-run elasticities may, off hand, seem a reasonable choice. However, changes in input/output prices will probably affect pig-producing and non-pig-producing farmers differently so that some reallocation of aggregate land input between the two farm groups is probable.

Modelling land market price reactions is beyond the scope of this paper and we therefore assume constant land input and full pig stall adjustment when calculating long run elasticities. Under these assumptions the long run elasticity for goods i = 1,..,4 becomes (see appendix for the derivation):

$$\left[s_i + c_{ii}/s_i - 1\right] - \left[s_6 + c_{i6}/s_i\right]\left[\frac{s_i + c_{i6}/s_6}{s_6 + c_{66}/s_6 - 1}\right] \tag{16}$$

where s_6 is defined corresponding to the profit share equations (13):

$$s_6 = b_6 + \sum_{j=1}^{4} c_{j6}\ln(p_j) + \sum_{j=5}^{6} c_{j6}\ln(z_j) \tag{17}$$

The reported long run elasticities should be interpreted in the light of these assumptions. It should also be stressed that while our short run elasticities are robust to model specification changes, our long run elasticities depend critically on parameters that are not robust to plausible specification changes.

Table 6 Own price input demand and output supply elasticities

	Piglet producers		Fattening pig producers	
	Short run	*Long run*	*Short run*	*Long run*
Pig output	0.27	1.42	0.62	1.02
Crop output	0.19	0.22	0.27	0.28
Feed input	-1.28	-1.98	-1.75	-2.12
N-fertilizer input	-0.38	-0.38	-0.45	-0.45

Note: calculated for the 1987 median observation which generates elasticities close to the corresponding medians over all observations.

In their study of Dutch pig farms, Fontein et al. (1994) report feed own price elasticities of -3.2 for fattening pigs and -2.8 for piglets when N-loss is allowed to adjust while other fixed inputs are held constant (elasticities for output are not reported). It is not clear whether these should be compared with our short run or long run elasticities. In addition to this paper we are aware of two studies of pig production generating comparable results. Hallam and Zanoli (1993) estimate an error correction model for UK piglet production using aggregate time series data finding a short-run own price elasticity of 0.16 for piglet output. Kuiper and Meulenberg (1997) use the Johansen co-integration method on aggregate time series for Dutch fattening pigs finding a long run own price elasticity of 3.37. While short run elasticities derived from aggregate data probably can be compared with those derived from micro studies, long run elasticities from aggregate studies include the effects of farm entry-exit and land adjustment that are held constant in our study. Given this, methodological differences and the substantial structural differences between pig farms of the different countries our results seem consistent with previous studies.

The results on which we focus here are the abatement costs of reducing nitrogen loss under different tax schemes that can be derived from the estimated model. In table 7 abatement costs with a 10% reduction in nitrogen loss are reported for four tax schemes:

- A tax on nitrogen loss
- A tax on nitrogen content in input (feed as well as chemical fertilizer)
- A tax on nitrogen content in feed, and
- A tax on nitrogen content in chemical fertilizer.

Abatement costs and N-loss reductions in the short-run adjustment column are calculated according to (5) and (6) respectively. Long-run abatement costs and reductions are calculated under the same assumption as the reported long-run elasticities (fixed land input and full pig stall adjustment) according to (10) and (11) respectively, where the long run adjustment in pig stall capacity (z_6) is found numerically by iterating equation (9). The difference between the two columns is the extra cost of insisting on quick achievement of the reduction goal.

Table 7 Average abatement costs (DKK per kilo N-loss reduction) when N-loss is reduced by 10%

	Piglet Producers		Fattening pig producers	
Incentive scheme:	*10% reduction after short-run adjustment*	*10% reduction after long-run adjustment*	*10% reduction after short-run adjustment*	*10% reduction after long-run adjustment*
Tax on N-loss	0.87	0.66	0.77	0.72
Tax on all input	0.87	0.66	0.78	0.74
Tax on feed	1.5	0.98	1.44	1.27
Tax on N-fertilizer	1.62	1.51	1.5	1.35

Note: calculated for the 1987 median observation.

By design the N-loss tax generates incentives that minimize abatement costs. As expected both a fertilizer tax and a feed tax distort incentives and generate substantial increases in abatement costs (with the fertilizer tax generally doubling abatement costs while the cost increase of a feed tax is somewhat smaller). The input tax, on the other hand, only implies a very small increase in abatement costs compared to the N-loss tax.

Efficiency of the N-loss tax is consistent with the theoretical result of Holtermann (1976) which is built in to our empirical model. The real news here is that the input tax is almost as efficient as the N-loss tax while both the fertilizer and feed taxes are substantially more costly. Part of the reason for this is the larger elasticities for inputs then for outputs seen in table 6. However, the main explanation is that nitrogen content per kroner value is substantially larger for inputs then for outputs. Thus a uniform rate tax/subsidy implies substantially smaller *relative* (per cent) effects on output prices then on input prices. Since the input tax (relative to the efficient N-loss tax) only distorts prices on the output side the resulting efficiency loss is small. On the other hand both the fertilizer and the feed tax also imply distortion of input prices resulting in a substantially larger efficiency loss.

Calculations have also been carried out for price/fixed input combinations spanning the variations of observations in the two data sets and for alternative model specifications. Though the absolute magnitude of abatement costs varies substantially in all cases, we find the same pattern of abatement cost ratios, i.e. substantially higher abatement costs for the feed and fertilizer tax and only marginal increases for the input tax. Specifically, irrespective of farm type and time horizon, we found that in no case did the abatement cost increase, implied by the input tax, exceed 3% of the Pigouvian cost level.

7. Conclusions

Using panel data we have estimated flexible form profit functions and nitrogen loss for Danish piglet and fattening pig producers with several farm specific effects. The resulting system emits plausible short run price elasticities that are robust to model specification changes and consistent with other studies. Using a shadow price appraisal of fixed input costs we are able to derive long run elasticities. These also seem plausible and are consistent with other studies, but depend critically on parameters that are not robust to plausible model specification changes.

The main result of this study is the quantification of abatement costs for reducing nitrogen loss under different tax systems. We compare four tax systems: a tax on nitrogen loss, a tax on nitrogen in all input, a tax on nitrogen in feed and a tax on nitrogen in chemical fertilizer. By design the N-loss tax generates Pigouvian incentives that minimize abatement costs. As expected both a fertilizer tax and a feed tax distort incentives and generate substantial increases in abatement costs. The input tax, on the other hand, only implies a marginal increase in abatement costs.

This is interesting because an input tax is easier to implement than an N-loss tax. Our result implies that even a limited administrative cost advantage would make the input tax preferable to implementing Pigouvian incentives through an N-loss tax.

It should be stressed, however, that the estimated model is based on aggregated crop output and feed input and so does not capture the effects of incentives to substitute within these aggregates. Taking account of these effects will tend to increase the abatement cost advantage of Pigouvian incentives. In addition, the model only covers pig farms. Since the abatement cost pattern of dairy and crop farms may differ our results do not necessarily apply to the Danish agricultural sector as a whole. Further since our data are the result of a voluntary program the sample of pig farms used here may not be fully representative of the entire population of Danish pig farmers.

Appendix

Table A: Price elasticities of model

Price elasticities for variable inputs and outputs and shadow rents for fixed inputs are calculated for expected input/output levels for each data observation. The mean and median of the distribution of elasticities/rents and the proportion of positive elasticities/rents are reported for all input/output price combinations.

Input/ output	***Piglet producers*** *Prices*				***Fattening pig producers*** *Prices*			
	p_1	p_2	p_3	p_4	p_1	p_2	p_3	p_4
x_1								
Mean	0.20	0.21	-0.41	-0.005	0.60	0.255	-0.85	-0.008
median	0.19	0.20	-0.39	-0.004	0.50	0.259	-0.76	-0.005
%>0	73%	95%	11%	38%	79%	97%	7%	33%
x_2								
Mean	0.68	0.42	-1.05	-0.046	0.82	0.441	-1.25	-0.019
median	0.73	0.22	-0.99	-0.043	0.80	0.282	-1.14	-0.016
%>0	95%	99%	0%	0%	97%	100%	0%	7%
x_3								
Mean	0.68	0.46	-1.13	-0.019	1.15	0.495	-1.64	-0.010
median	0.71	0.44	-1.15	-0.017	1.08	0.511	-1.56	-0.008
%>0	88%	99%	2%	2%	92%	100%	0%	25%
x_4								
Mean	-0.22	0.93	-0.68	-0.033	-0.72	0.289	0.18	0.256
median	0.24	0.79	-0.67	-0.349	0.41	0.404	-0.45	-0.382
%>0	62%	99%	1%	27%	67%	93%	25%	22%
		shadow rents				shadow rents		
		Dkk/hectare	Dkk/stall			Dkk/hectare	Dkk/stall	
Mean		13447.99	1668.70			11786.40	152.65	
median		11181.28	1246.02			10356.85	77.45	
%>0		100%	98%			100%	87%	

Table B: Own price elasticities and shadow rents of model and alternative specifications

Own price elasticities for variable inputs and outputs and shadow rents for fixed inputs are calculated for expected input/output levels for each data observation. The mean and median of the distribution of elasticities/rents and the proportion of positive elasticities/rents are reported for all own prices and shadow rents.

Input/ output	***Piglet producers***					***Fattening pig producers***				
Price elast:	Base	Alt1	Alt2	Alt3	Alt4	Base	Alt1	Alt2	Alt3	Alt4
x_1										
Mean	0.20	0.26	0.28	0.21	0.28	0.60	0.65	0.72	0.59	0.78
median	0.19	0.23	0.26	0.19	0.26	0.50	0.54	0.62	0.50	0.66
%>0	73%	78%	79%	74%	80%	79%	84%	88%	88%	93%
x_2										
Mean	0.42	0.45	0.30	0.38	0.68	0.44	0.46	0.42	0.44	0.80
median	0.22	0.24	0.13	0.19	0.41	0.28	0.29	0.26	0.28	0.54
%>0	99%	99%	99%	99%	100%	100%	97%	100%	100%	100%
x_3										
Mean	-1.13	-1.18	-1.26	-1.14	-1.12	-1.64	-1.68	-1.78	-1.63	-1.69
median	-1.15	-1.20	-1.28	-1.16	-1.13	-1.56	-1.60	-1.72	-1.56	-1.61
%>0	2%	2%	1%	3%	3%	0%	1%	0%	1%	0%
x_4										
Mean	-0.03	-0.02	0.17	0.10	0.00	0.26	-0.44	0.44	-0.11	-0.74
median	-0.35	-0.35	-0.24	-0.29	-0.35	-0.38	-0.35	-0.21	-0.34	-0.39
%>0	27%	26%	34%	32%	28%	22%	24%	37%	24%	22%
shadow rents:										
Dkk/hectare										
Mean	13447	-157564	2505	10971	13561	11786	-153443	9646	8652	11781
median	11181	-135914	2695	9346	11253	10356	-140796	8317	7927	10354
%>0	100%	0%	95%	100%	100%	100%	0%	100%	99%	100%
Dkk/stall										
Mean	1668	-15067	4071	2152	1646	152	-4490	359	396	153
median	1246	-12478	3859	1802	1218	77	-3809	226	272	78
%>0	98%	1%	100%	100%	98%	87%	0%	93%	93%	87%
Dkk/workyear										
Mean	.	.	.	107579	.	.	.	.	989269	.
median	.	.	.	98806	.	.	.	.	905529	.
%>0	.	.	.	96%	.	.	.	.	100%	.

Note: Alt1 is the base model without homogeneity restrictions on fixed inputs.
Alt2 is the base model with a time trend added as a fixed input.
Alt3 is the base model with a variable indicating labour use added as a fixed input.
Alt4 is the base model with $c_{1,2}$ restricted to reduce output cross price elasticities

Derivation of long run own price elasticity when one fixed input adjusts

Generally the long-run elasticity of input *i* can be decomposed as follows:

$$\frac{dx_i}{dp_i}\frac{p_i}{x_i} = \left[\frac{\delta x_i}{\delta p_i} + \frac{\delta x_i}{\delta z_6}\frac{dz_6}{dp_i}\right]\frac{p_i}{x_i} = \left[\frac{\delta x_i}{\delta p_i}\frac{p_i}{x_i}\right] + \left[\frac{\delta x_i}{\delta z_6}\frac{z_6}{x_i}\right]\left[\frac{dz_6}{dp_i}\frac{p_i}{z_6}\right] \tag{i}$$

The first parenthesis is the short run price elasticity while the second is input *i* 's elasticity with respect to the fixed input. For the trans-log specification these are $(s_i + c_{ii}/s_i - 1)$ and $(s_6 + c_{i6}/s_i)$ respectively. In the third parenthesis (dz_6/dp_i) is given by the assumption that the fixed input is adjusted so as to hold its shadow rent constant i.e. (dz_6/dp_i) is given by equation (9) that for the trans-log specification (when variables that are held constant are dropped from the functional expressions) is:

$$\hat{p}_6^0 = \frac{\pi(p_i,z_6)\, s_6(p_i,z_6)}{z_6} \tag{ii}$$

Defining

$$F(p_i,z_6) = \frac{\pi(p_i,z_6)\, s_6(p_i,z_6)}{z_6} - \hat{p}_6^0 \tag{ii}$$

we find the derivative by implicit differentiation i.e.:

$$\frac{dz_6}{dp_i} = -\frac{\dfrac{dF}{dp_i}}{\dfrac{dF}{dz_6}} = -\frac{\dfrac{\pi}{z_6}\dfrac{c_{6,i}}{p_i} + \dfrac{s_6}{z_6}\dfrac{d\pi}{dp_i}}{\dfrac{\pi}{z_6}\dfrac{c_{6,6}}{z_6} + \dfrac{s_6}{z_6}\dfrac{d\pi}{dz_6} - \dfrac{\pi\, s_6}{(z_6)^2}} \tag{iii}$$

which after multiplying by p_i/z_6 and inserting $\frac{d\pi}{dp_i} = \frac{\pi}{p_i}s_i$ and $\frac{d\pi}{dz_6} = \frac{\pi}{z_6}s_6$ reduces so that:

$$\frac{dz_6}{dp_i}\frac{p_i}{z_6} = -\left[\frac{s_i + c_{i6}/s_6}{s_6 + c_{66}/s_6 - 1}\right] \tag{iv}$$

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