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Free Entry and Social Inefficiency under Co-opetition*

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Abstract

We investigate the social desirability of free entry in the co-opetition model in which firms compete in a homogeneous product market while sharing common property resources that affect market size or consumers' willingness to pay for products. We show that free entry leads to socially excessive or insufficient entry into the market in the case of non-commitment co-opetition, depending on the magnitude of "business stealing" and "common property" effects of entry. On the other hand, in the case of pre-commitment co-opetition, free entry leads to excess entry and a decline in the common property resources. Interestingly, in the latter case, the excess entry result of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) holds even when there are no entry (set-up) costs for entrants. These results have important policy implications for entry regulations.

Keywords Excess entry; Free entry; Co-opetition; Entry regulations; Common property resource

JEL Code L13; D43; L51

1 Introduction

In many industries, firms compete for market share while cooperating in the management of "common property resources" that affect the market size or consumers' willingness to pay for products. This simultaneous competition and cooperation is called "co-opetition" (Brandenburger and Nalebuff 1996). For example, retail stores in shopping malls, tourist sites, and food courts share common property resources such as parking lots, historic ruins and a natural environment, and dining areas, respectively. The quality of the common property resources affects the market size and/or consumers' willingness to pay for the products or services, and high quality of the resources generates non-excludable benefits for firms in the industry. Therefore, each firm's investment in the common property resources—such as eliminating congestion by expanding parking lots in shopping malls, preserving historic ruins and natural environment of tourist sites, and maintaining a clean, hygienic environment in food courts—create public goods from which all the firms benefit. Another example of such co-opetitive behavior is generic advertising for various commodities, such as tea, oranges, milk, butter, cheese, beef, fish, and eggs. Creating a better product image generates non-excludable benefits for all producers who provide the same products. In

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this case, firms share the product image as a common resource and, sometimes, voluntarily contribute to improve the image.[1]

Is free entry into such co-opetitive industries desirable from a social welfare point of view? To answer this question, we need to identify how firm entry into such a market can affect social efficiency. One way is the well-known "business stealing" effect of entry that creates production inefficiency in an industry (Mankiw and Whinston 1986). This generally results in socially excessive entry. Another is the effect of entry on the amount of common property resources, which we call the "common resource" effect of entry. On the one hand, an increase in the number of firms may increase the total investment in the common property resources. In this case, free entry may result in socially insufficient entry because firms do not consider the positive external effect of their investment in the common property resources on other firms. On the other hand, an increase in the number of firms may exacerbate the under-provision of common property resources among firms and, thus, may lead to a tragedy-of-commons situation. For instance, an increase in tourism firms can deteriorate the quality of tourist attractions (such as wild life and historic ruins) and can eventually destroy tourism itself. As Puppim de Oliveira (2003) indicates, places like Acapulco in Mexico, the French Rivera and Mallorca and Torremolinos in Spain have faced tourism-related environmental problems. In this case, the common property effect contributes to socially excessive entry. These conjectures lead us to the question of whether government should regulate or encourage firm entry into the co-opetitive industry.

In this paper, we formulate a simple model of co-opetition with endogenous entry to present a welfare analysis of free entry equilibrium. In particular, we consider whether the number of firms that can enter a co-opetitive market is excessive or insufficient from the viewpoint of social welfare. We distinguish two types of co-opetitive investment in common property resources: investment with and without commitment. In the case of non-commitment investment, firms are modeled to choose their output and investment at the same stage. We call the game "simultaneous co-opetition game." In the case of pre-commitment investment, firms decide their investment before they choose outputs. We call the game "sequential co-opetition game." In either game, firms' entry decisions are made at the first stage. The difference between non-commitment and pre-commitment investments reflects the difference in reversibility and persistency in investment. When investment has long-term impacts and is difficult to reverse (e.g., renovating historic building and expanding parking lots), the investment has strategic commitment value, which can be described by sequential co-opetition game. On the other hand, when investment has short-term impacts only and is easy to reverse (e.g., providing generic advertising on the daily newspaper and cleaning up the shopping mall or food courts), the investment has no strategic commitment value, which can be described as a simultaneous co-opetition game.

We show that whether free entry into a co-opetitive industry is socially excessive or insufficient depends on the following two effects: business stealing and common property effects of entry. The former, as is well known from previous studies such as Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), originates from the fact that an entrant firm does not take into account its negative impact (externality) on the profits of other firms. Therefore, when firms face a fixed entry (set-up) cost, the business stealing effect leads to excess entry of firms in the market. The latter effect is novel and depends on the effect of entry on the total amount (or quality) of common property resources. An increase in the number of entrants will increase the incentive to free ride on investments in common property resources (public

[1] Other possible examples of co-opetition include development of open source software and rent-seeking or lobbying to get permission to sell product to certain groups (e.g., the permission to sell tobacco to under-age people, specific medicines to wide mass of people, and financial products to inexperienced consumers).

goods) made by other firms. However, the quality of common property resources may be increased by a rise in the number of entrants. If that is the case, entry generates positive external effects on other firms. Because private firms do not take the positive externality into account when deciding whether to enter a market, this common property effect leads to insufficient entry. On the other hand, if market entry results in a decline in the common property resources, entry causes negative external effects on other firms. In this case, a negative common property effect leads to excess entry.

We find that, in the simultaneous co-opetition game, an increase in the number of firms increases the total amount of investment in common property resources while reducing individual investment. Therefore, the business stealing and common property effects work in opposite directions. In other words, whether free entry is socially excessive or insufficient depends on the relative magnitude of the two effects. In particular, by providing two concrete examples that assume linear and constant elasticity demand, we show that free entry is more likely to result in socially insufficient entry when initial market size, investment cost, and production cost are smaller and/or the demand is more elastic.

However, the business stealing and common property effects work in the same directions in the sequential co-opetition game in which the investment has a commitment value. The important thing here is that the total amount investment in common property resources is decreased by an increase in the number of entrants, that is, the common property effect is negative. This is in contrast to the result of the simultaneous co-opetition game. This is because, when the investment has a commitment value, an increase in firm's investments induces rival firms to respond more aggressively by increasing their outputs in the subsequent stage. Therefore, this pre-commitment effect of investment reduces the incentive for investment in common property resources. Because an increase in the number of rival firms strengthens the pre-commitment effect, the sequential co-opetition game gives rise to a negative common property effect of entry. In other words, a marginal entry actually decreases the total amount of public goods. As a result, excess entry holds in sequential co-opetition. Interestingly, we show that excess entry results hold for the sequential co-opetition game even when there are no entry (set-up) costs.

Our results enrich the established excess entry theorem in theoretical industrial organization literature, developed by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).[2] Their studies show that in a Cournot model with homogenous products, free entry is socially excessive when firms have fixed entry costs.[3] First, our results from the simultaneous co-opetition game suggest that free entry may lead to socially insufficient entry when firms share common property resources that affect the market size or the consumers' willingness to pay for the product. Second, excess entry holds in the sequential co-opetition game even when firms have no entry costs.

The excess entry theorem has been extended in various directions. For example, Konishi et al. (1990) extend the traditional Cournot model with free entry to a general equilibrium model and explore Pareto-improving tax-subsidy policies. Incorporating strategic cost-reducing R&D activities into Cournot model with free entry, Okuno-Fujiwara and Suzumura (1993) show that the existence of R&D investment strengthens the tendency of excess entry in a free-entry equilibrium.[4] There is a critical difference between R&D investment in their study and investment in common property resources in our study: in the former, investment generates private benefits for the investing firm, while in the latter, investment

[2]See also von Weizsacker (1980) and Perry (1984).

[3]Berry and Waldfogel (1999) empirically examine the problem of excess entry into U.S. commercial radio broadcasting and estimate the welfare loss from excess entry.

[4]Haruna and Goel (2011) also consider the problem of excess entry in the presence of cost-reducing R&D with spillovers and show that whether free entry is socially excessive or insufficient depends on the degree of research spillovers.

generates public benefits for all firms.

Some previous studies find that free entry can result in socially insufficient entry (e.g., Spence 1976, Dixit and Stiglitz 1977, Kühn and Vives 1999, and Ghosh and Saha 2007). Ghosh and Morita (2007) consider a vertical relationship between industries in a homogeneous Cournot model and show that free entry in the upstream sector can lead to socially insufficient entry. The driving force behind their insufficient entry result is that entry in the upstream sector has positive external effect on the downstream sector's profit. On the other hand, the driving force behind our insufficient-entry result is that entry may have positive external effect on other firms' profits through changes in the quality of common property resources. Incorporating a constant elasticity demand into a standard Salop (1979) spatial framework, Gu and Wenzel (2009) show that the excess entry theorem does not hold when the price elasticity of demand is large. Although their study differs from ours in several respects, their conclusions are similar: insufficient entry occurs when the price elasticity of demand is large. Therefore, the degree of price elasticity of demand may serve as a guideline for entry regulation policy.[5][6]

This paper is organized as follows. Section 2 presents the basic structure of the model. Section 3 considers a simultaneous co-opetition game, investigates the properties of a free-entry equilibrium, and compares it with the second-best solution. In addition, we present two examples that specify the functional form of demand (linear and constant elasticity demands) and cost to provide a more concrete results. Section 4 investigates the same for a sequential co-opetition game. Section 5 concludes the paper.

2 Basic Framework

Consider n firms producing a homogenous good. Firms compete in their outputs in a market while they can invest in common property resources which has a public good nature among all the competing firms. Profits of firm i ($i = 1, \cdots, n$) is given by

$$\pi_i = P(Q, Z) \cdot q_i - C(q_i) - D(z_i) - K, \tag{1}$$

where $P(Q, Z)$ is the market price (or inverse demand) of the product, $q_i \geq 0$ is firm i's output, $Q \equiv \sum_i^n q_i$ is the industry output, $z_i \geq 0$ is the amount of firm i's investment (or individual contribution to common property resources), $Z \equiv \sum_i^n z_i$ is the total amount of investment (or quality of common property resources), $C(q_i)$ is the cost function for production, $D(z_i)$ is the cost function for investment, and $K \geq 0$ is the fixed entry (set-up) costs. The inverse demand $P(Q, Z)$ has the property of $P_Q < 0$, $P_Z > 0$, $P_{ZZ} \leq 0$. The second and third properties mean that increasing the total amount of investment enlarges the willingness to pay of consumers, but by a non-increasing rate. The production cost function $C(\cdot)$ has $C' > 0$ and $C'' \geq 0$, and the investment cost function $D(\cdot)$ has $D' > 0$ and $D'' \geq 0$.

We consider the following two types of co-opetition behavior of firms: simultaneous and sequential co-opetition. In simultaneous co-opetition, firms' investments have no commitment value and are modeled to be decided simultaneously with outputs. Therefore, the simultaneous co-opetition is modeled as

[5] Matsumura and Okamura (2006) show that the equilibrium number of firms can be either excessive or insufficient in a spatial price discrimination model. Mukherjee and Mukherjee (2008) theoretically show that free entry can result in socially insufficient entry in the presence of technology licensing.

[6] For the welfare evaluation of entry regulation, Kim (1997) considers the strategic behavior of firms and the government and shows that entry regulation to prevent excess entry induces the incumbent to behave strategically against the government. As a result, the final outcome is socially suboptimal compared to the case without government intervention.

a two-stage game: in the first stage, firms make entry decisions and the number of firms in the industry is endogenously decided; in the second stage, each firm non-cooperatively decides on its investment and outputs. On the other hand, in sequential co-opetition, firms' investments are committed and are modeled to be decided before choosing outputs. Therefore, the sequential co-opetition is modeled as a three-stage game: in the first stage, firms make entry decisions; in the second stage, each firm non-cooperatively decides on its investment; in the last stage, each firm engages in Cournot competition. Within the above framework, we derive the number of firms in a free-entry equilibrium and then compare it with the socially optimal number of firms.

To obtain clear and intuitive results, we employ specific functional forms. We consider the following two types of demands. One is linear demand expressed by

$$P(Q, Z) = (a + Z) - b\,Q, \tag{2}$$

where a and b are positive constants. Obviously, $P_Q = -b < 0$, $P_Z = 1 > 0$, and $P_{ZZ} = 0$, which satisfy our assumptions stated before. The other is constant elasticity demand expressed by

$$P(Q, Z) = \left(\frac{a + Z}{Q}\right)^{1/\epsilon}, \tag{3}$$

where a is positive constant and ϵ is a price elasticity of demand. We further assume $\epsilon \geq 1$ to satisfy $P_{ZZ} \leq 0$. Furthermore, we employ the constant marginal cost of production $C(q_i) = c\,q_i$, where $c > 0$ and the quadratic investment cost function $D(z_i) = (d/2)(z_i)^2$, where $d > 0$.

3 Free entry under simultaneous co-opetition

In this section, we consider the simultaneous co-opetition where firms' investments have no commitment power. This situation can be modeled by a procedure where each firm decides upon its outputs and investment simultaneously.

3.1 Production and investment decisions

The game is solved by backward induction. The first-order conditions for profit maximization are

$$P_Q \cdot q_i + P - C' = 0, \tag{4}$$

$$P_Z \cdot q_i - D' = 0. \tag{5}$$

We assume that a symmetric Nash equilibrium exists.[7] We denote the symmetric Nash equilibrium amount of output and investment per firm as $\hat{q}(n)$ and $\hat{z}(n)$, and the total amount of output and investment as $\hat{Q}(n) = n\hat{q}(n)$ and $\hat{Z}(n) = n\hat{z}(n)$.

[7] For the existence and uniqueness of symmetric Nash equilibrium, we should assume $(1 + n)P_Q + QP_{QQ} < 0$ (see Vives 1999) and the Hessian matrix to be negative devinite.

In a symmetric equilibrium, from (4) and (5), we have the following comparative static results:

$$\begin{aligned}
\frac{d\hat{q}}{dn} &= -\frac{1}{\Delta}\Big\{\big(P_{ZZ}\hat{Q}-D''\big)\Big[\hat{q}\,(P_{QQ}\hat{q}+P_Q)+\hat{z}(P_{QZ}\hat{q}+P_Z)\Big] \\
&\quad -\big(P_{QZ}\hat{Q}+nP_Z\big)\big(P_{QZ}\hat{q}^2+P_{ZZ}\,\hat{q}\,\hat{z}\big)\Big\}, \\
\frac{d\hat{z}}{dn} &= -\frac{1}{\Delta}\Big\{(-P_{QZ}\hat{Q}-P_Z)\Big[\hat{q}\,(P_{QQ}\hat{q}+P_Q)+\hat{z}(P_{QZ}\hat{q}+P_Z)\Big] \\
&\quad -\Big[P_{QQ}\hat{Q}+P_Q\,(n+1)-C''\Big]\big(P_{QZ}\hat{q}^2+P_{ZZ}\,\hat{q}\,\hat{z}\big)\Big\}, \\
\frac{d\hat{Q}}{dn} &= \frac{1}{\Delta}\Big\{\hat{q}(D''-P_{ZZ}\hat{Q})(C''-P_Q)+(P_{QZ}\hat{Q}+nP_Z)(\hat{z}D''-D')\Big\} \\
\frac{d\hat{Z}}{dn} &= \frac{1}{\Delta}\Big\{\hat{q}(P_{QZ}\hat{Q}+P_Z)(C''-P_Q)+\big[P_{QQ}\hat{Q}+(n+1)P_Q-C''\big](\hat{z}D''-D')\Big\},
\end{aligned}$$

where the determinant is

$$\Delta \equiv \Big[P_{QQ}\hat{Q}+(n+1)P_Q-C''\Big](P_{ZZ}\hat{Q}-D'')-(P_{QZ}\hat{Q}+nP_Z)(P_{QZ}\hat{Q}+P_Z)>0.$$

From the above comparative statics, we find that $d\hat{Z}/dn>0$ holds for (a) $P_{QZ}\hat{Q}+P_Z>0$ and (b) $\hat{z}D''-D'\le 0$. The condition (a) holds naturally because it only requires the marginal profit of production to be increasing function of $\bar{z}$. The condition (b) requires the investment cost function not to be too convex. In general, these signs are ambiguous, but $d\hat{q}/dn<0$, $d\hat{z}/dn<0$, $d\hat{Q}/dn>0$, and $d\hat{Z}/dn>0$ are quite likely to hold in practice. In the following, we confirm that they hold for linear and constant elasticity demand cases.

■ Linear demand case

In the linear demand case, we obtain the symmetric equilibrium of the second stage as

$$\hat{q}=\frac{(a-c)d}{\hat{\Delta}},\quad \hat{z}=\frac{a-c}{\hat{\Delta}}, \tag{6}$$

where the determinant Δ in this linear demand case is $\hat{\Delta}=bd(1+n)-n>0$ by assumption, which also means $bd>1$ and $d\hat{\Delta}/dn>0$. Then, we have

$$\begin{aligned}
\frac{d\hat{q}}{dn}&=-\frac{(a-c)(bd-1)d}{\hat{\Delta}^2}<0,\quad \frac{d\hat{Q}}{dn}=\frac{(a-c)bd^2}{\hat{\Delta}^2}>0,\\
\frac{d\hat{z}}{dn}&=-\frac{(a-d)(bd-1)}{\hat{\Delta}^2}<0,\quad \frac{d\hat{Z}}{dn}=\frac{(a-c)bd}{\hat{\Delta}^2}>0,
\end{aligned}$$

which indicates that individual outputs and investments decrease, whereas the total output and investments increase as the number of firms increases. Furthermore, we have

$$\lim_{n\to\infty}\hat{q}=\lim_{n\to\infty}\hat{z}=0,\ \lim_{n\to\infty}\hat{Q}=\frac{(a-c)d}{bd-1}>0,\ \lim_{n\to\infty}\hat{Z}=\frac{a-c}{bd-1}>0,$$

which implies that individual outputs and investments converge to zero as the number of firms approaches infinity, while the total output and investment converges to positive and finite values.

■ **Constant elasticity demand case**

In the constant elasticity demand case, we obtain the symmetric equilibrium of the second stage as

$$\hat{q} = \frac{\Lambda^{\epsilon}(a\,d\,\epsilon + \Lambda^{\epsilon-1})}{n\,d\,\epsilon},\ \hat{z} = \frac{\Lambda^{\epsilon-1}}{n\,d\,\epsilon}, \tag{7}$$

where

$$\Lambda \equiv \frac{n\epsilon - 1}{c\,n\,\epsilon} > 0,$$

and $d\Lambda/dn > 0$. We can also confirm that the determinant Δ is positive because

$$\Delta = \frac{c\,n\,d^2\,\Lambda^{1-2\epsilon}}{1 + a\,d\,\epsilon\Lambda^{1-\epsilon}} > 0.$$

Then, we have the following comparative statics:

$$\begin{aligned}
\frac{d\hat{q}}{dn} &= -\frac{\Lambda^{2\epsilon-1}\{(n-2) + ad\,[(n-1)\,\epsilon - 1]\,\Lambda^{1-\epsilon}\}}{n^2\,d(n\epsilon - 1)} < 0,\\
\frac{d\hat{z}}{dn} &= -\frac{(n-1)\Lambda^{\epsilon-1}}{n^2 d(n\epsilon - 1)} < 0,\\
\frac{d\hat{Q}}{dn} &= \frac{\Lambda^{2\epsilon-1}\left[(2\epsilon - 1) + ad\epsilon^2\Lambda^{1-\epsilon}\right]}{nd\epsilon(n\epsilon - 1)} > 0,\\
\frac{d\hat{Z}}{dn} &= \frac{(\epsilon - 1)\Lambda^{\epsilon-1}}{nd\epsilon(n\epsilon - 1)} > 0.
\end{aligned}$$

As the number of firms increases, individual outputs and investments decrease, whereas the total output and investment increases. Also, we have

$$\lim_{n\to\infty} \hat{q} = \lim_{n\to\infty} \hat{z} = 0,\ \lim_{n\to\infty} \hat{Q} = \frac{c^{-2\epsilon}(c + ac^{\epsilon}d\epsilon)}{d\epsilon} > 0,\ \lim_{n\to\infty} \hat{Z} = \frac{c^{1-\epsilon}}{d\epsilon} > 0,$$

which implies that individual outputs and investments converge to zero as the number of firms approaches infinity, while the total output and investment converges to positive and finite values.

3.2 Entry decisions and the second best

In the first stage, firms enter the market until their profits fall to zero. Therefore, the free entry number of firms is defined as $\hat{n}_f$ such that

$$\hat{\pi}\left(\hat{n}_f\right) = P\left(\hat{Q}(\hat{n}_f), \hat{Z}(\hat{n}_f)\right)\hat{q}(\hat{n}_f) - C\left(\hat{q}(\hat{n}_f)\right) - D\left(\hat{z}(\hat{n}_f)\right) - K = 0. \tag{8}$$

We then consider the second-best problem for a social planner who can control the number of firms entering the market. Let $\widehat{W}(n)$ denote the total surplus as

$$\widehat{W}(n) \equiv \int_0^{\hat{Q}} P(s, \hat{Z})ds - nC(\hat{q}) - nD(\hat{z}) - nK.$$

Using (4) and (5), we have

$$\begin{aligned}
\widehat{W}'(n) &= P\left(\hat{q} + n\frac{d\hat{q}}{dn}\right) + P_Z\hat{Q}\left(\hat{z} + n\frac{d\hat{z}}{dn}\right) - C - nC'\frac{d\hat{q}}{dn} - D - nD'\frac{d\hat{z}}{dn} - K\\
&= \hat{\pi} - P_Q\hat{Q}\frac{d\hat{q}}{dn} + P_Z\hat{Q}\left[\frac{d\hat{z}}{dn}(n-1) + \hat{z}\right].
\end{aligned}$$

The social planner chooses the second-best number of firms $n = \hat{n}_{sb}$ that maximizes $\widehat{W}(n)$, which implies

$$\widehat{W}'(\hat{n}_{sb}) = 0 \quad \text{if} \;\; \hat{n}_{sb} > 1.$$

We assume that the second-order condition should be satisfied, that is, $\widehat{W}''(n) < 0$. Because $\hat{\pi} = 0$ when $n = \hat{n}_f$, we have

$$\widehat{W}'(\hat{n}_f) = \underbrace{-P_Q \hat{Q}\frac{d\hat{q}}{dn}}_{\text{business stealing}} + \underbrace{P_Z \hat{Q}\frac{d\hat{Z}_{-1}}{dn}}_{\text{common property}}, \tag{9}$$

where $\hat{Z}_{-1} \equiv (n-1)\hat{z}$. Thus, we find that $\hat{n}_f > \hat{n}_{sb}$ holds when (9) is negative. In this case, free entry leads to excess entry. On the other hand, $\hat{n}_f < \hat{n}_{sb}$ holds when (9) is positive. In this case, free entry leads to insufficient entry.

The first term on the right-hand side of (9) is the well-known "business stealing" effect of entry (Mankiw and Whinston 1986). Firms enter the market without taking into account the negative impact of their entry on the profitability of their rivals. As shown below, the term is usually negative. The second term represents the "common property" effect of entry. Firms do not take into account the positive impact of their investment (or contribution to common property resources) on the profitability of their rivals. As is shown above ($d\hat{Z}/dn > 0$ holds for both linear and constant elasticity demand cases), the term is usually positive in this simultaneous co-opetition case. Therefore, we have the following proposition.

Proposition 1

In a simultaneous co-opetition game, free entry results in socially insufficient entry when $\widehat{W}'(\hat{n}_f) > 0$ and socially excessive entry when $\widehat{W}'(\hat{n}_f) < 0$. In particular, insufficient entry results hold when the common property effect dominates the business stealing effect of entry.

In the following, we clearly demonstrate under what conditions the excess or insufficient entry theorem applies in linear and constant elasticity demand cases.

■ Linear demand case

From (6), the free-entry equilibrium number of firms, $\hat{n}_f$, satisfies

$$\hat{\pi}(\hat{n}_f) = \frac{(a-c)^2(2bd-1)d}{2\left[bd(1+\hat{n}_f) - \hat{n}_f\right]^2} - K = 0.$$

Because $\hat{\pi}(n)$ is strictly decreasing in n and $\lim_{n\to\infty} \hat{\pi} = -K$, we confirm that

$$\lim_{K\to 0} \hat{n}_f = \infty.$$

Therefore, the number of firms under free entry goes to infinity when there are no entry costs.

The socially optimal (second-best) number of firms, $\hat{n}_{sb}$, satisfies,

$$\widehat{W}'(\hat{n}_{sb}) = \hat{\pi}(\hat{n}_{sb}) - \frac{(a-c)^2 d\hat{n}_{sb}}{\left[bd(1+\hat{n}_{sb}) - \hat{n}_{sb}\right]^3}\left(1 - 3bd + b^2d^2\right) = 0.$$

We have

$$\widehat{W}'(n)\big|_{K=0} = \frac{(a-c)^2 d[n(3bd-1) + bd(2bd-1)]}{2\left[bd(1+\hat{n}_{sb}) - \hat{n}_{sb}\right]^3} > 0,$$

which implies that

$$\lim_{K\to 0} \hat{n}_{sb} = \infty.$$

Therefore, the second-best number of firms goes to infinity when there are no entry costs. Then, we have

$$\widehat{W}'(\hat{n}_f) = -\frac{(a-c)^2 d\hat{n}_f}{[bd(1+\hat{n}_f)-\hat{n}_f]^3}\left(1-3bd+b^2d^2\right) \lesseqgtr 0 \quad \Leftrightarrow \quad bd \gtreqless \frac{3+\sqrt{5}}{2}.$$

Thus, we have the following corollary.

Corollary 1 *Under linear demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the simultaneous co-opetition game yields excessive (insufficient) entry for greater (smaller) investment costs and/or steeper (flatter) inverse demand.*

The greater (smaller) b and/or d, the more likely free entry leads to excess (insufficient) entry. This result is quite intuitive. When b is large (or the demand is less elastic), the equilibrium price is greatly decreased by firm entry, which leads to a greater business stealing effect of entry. Also, when d is large (or the investment is more costly), the total investment is less sensitive to firm entry, which leads to a smaller common property effect of entry.

We provide a numerical example: in the case of $a = 10,\ b = 1,\ c = 1$, and $K = 2$, we find that $\hat{n}_f \approx 9 < \hat{n}_{sb} \approx 12$ for $d = 2$, which corresponds to the insufficient entry theorem. On the other hand, we find that $\hat{n}_f \approx 6 > \hat{n}_{sb} \approx 4$ for $d = 6$, which corresponds to the excess entry theorem.

■ Constant elasticity demand case

From (12), the free-entry equilibrium number of firms, $\hat{n}_f$, satisfies

$$\hat{\pi}(\hat{n}_f) = \frac{c\Lambda^{2\epsilon-1}\left(1+2ad\epsilon\Lambda^{1-\epsilon}\right)}{2nd\epsilon(n\epsilon-1)} - K = 0.$$

Because $\hat{\pi}(n)$ is strictly decreasing in n and $\lim_{n\to\infty}\hat{\pi} = -K$, we confirm

$$\lim_{K\to 0} \hat{n}_f = \infty.$$

Thus, the free-entry number of firms goes to infinity when there are no entry costs as in the linear demand case.

The socially optimal number of firms, $\hat{n}_{sb}$, satisfies,

$$\widehat{W}'(\hat{n}_{sb}) = \hat{\pi}(\hat{n}_{sb}) - \frac{c\Lambda^{\epsilon}\left\{ad\left(\hat{n}_{sb}\epsilon-1\right)\left[\epsilon\left(\hat{n}_{sb}-1\right)-1\right] - c\hat{n}_{sb}\left[\epsilon\left(\hat{n}_f+1\right)-\hat{n}_{sb}\right]\Lambda^{\epsilon}\right\}}{(\hat{n}_{sb}\epsilon-1)^3 d\hat{n}_{sb}} = 0.$$

We also find that

$$\lim_{K\to 0} \hat{n}_{sb} = \infty,$$

which implies that the second-best number of firms goes to infinity as K approaches zero.

Then we have

$$\begin{aligned}\widehat{W}'(\hat{n}_f) \quad &= \quad -\frac{c\Lambda^{\epsilon}\left\{ad\left(\hat{n}_f\epsilon-1\right)\left[\epsilon\left(\hat{n}_f-1\right)-1\right] - c\hat{n}_f\left[\epsilon\left(\hat{n}_f+1\right)-\hat{n}_f\right]\Lambda^{\epsilon}\right\}}{(\hat{n}_f\epsilon-1)^3 d\hat{n}_f} \lesseqgtr 0 \\ &\Leftrightarrow\ d \gtreqless \frac{c\,\hat{n}_f[\epsilon(\hat{n}_f+1)-\hat{n}_f]\,\Lambda^{\epsilon}}{a(\hat{n}_f\epsilon-1)[\epsilon(\hat{n}_f-1)-1].}\end{aligned}$$

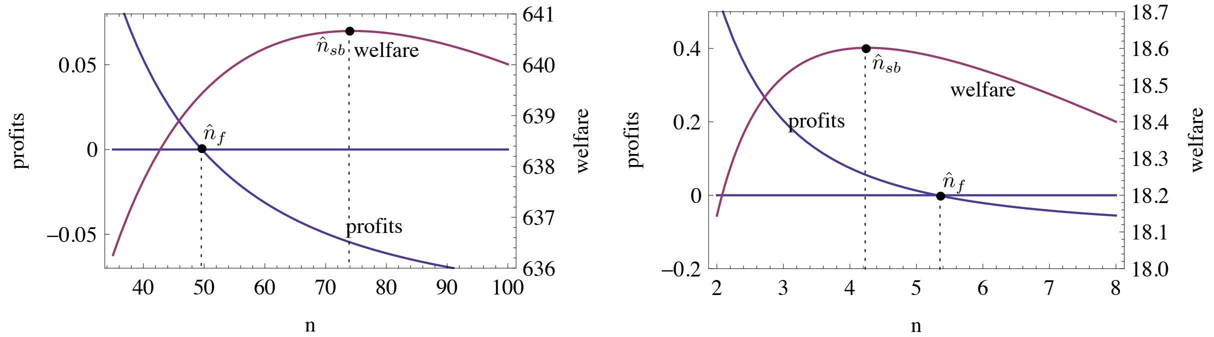

Figure 1: Equilibrium number of firms and the second-best: the case for $\epsilon = 3$ (left) and $\epsilon = 1.2$ (right).

Therefore, we find that the excess (insufficient) entry theorem applies for larger (smaller) values of a, d and c. We cannot analytically derive the impact of ϵ on the sign of $\widehat{W}'(\hat{n}_f)$, but the numerical examples demonstrate the tendency that the excess entry theorem is more likely to hold for the smaller value of ϵ, that is, when the demand is less elastic.

Corollary 2 *Under constant elasticity demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the simultaneous co-opetition game yields excessive (insufficient) entry for larger (smaller) market size, greater (smaller) investment and production costs, and smaller (greater) price elasticity of demand.*

Varian (1995) demonstrates that excessive entry results hold for constant elasticity demand cases in his simple Cournot model with endogenous entry. Our result extends his result by allowing firms co-opetitive investment and shows that free entry leads to either excessive or insufficient entry depending on the value of the price elasticity and investment costs.

We provide some numerical examples. In the case of $a = 2$, $c = 0.1$, $\epsilon = 2$, and $K = 0.1$, we find that $\hat{n}_f \approx 14$ and $\hat{n}_{sb} \approx 19$ for $d = 1$, and $\hat{n}_f \approx 10$ and $\hat{n}_{sb} \approx 8$ for $d = 8$. Therefore, free entry leads to excess (insufficient) entry when d is large (small). In the case of $a = 2$, $c = 0.1$, $d = 3$, and $K = 0.1$, we find that $\hat{n}_f \approx 49 < \hat{n}_{sb} \approx 74$ for $\epsilon = 3$, and $\hat{n}_f \approx 5 > \hat{n}_{sb} \approx 4$ for $\epsilon = 1.2$. Figure 1 depicts this situation. In each panel of Figure 1, profits and welfare at the second-stage equilibrium are depicted. The left (right) panel depicts the case in which the price elasticity of demand is high (low). We see from the figure that the excess (insufficient) entry occurs in the right (left) panel.

4 Free entry under sequential co-opetition

In this section, we consider the sequential co-opetition. Here we think of a three-stage game. In the first stage, firms enter the market. In the second stage, firms decide upon their investment, and then in the third stage, firms choose output (in a Cournot fashion). In contrast to the previous simultaneous co-opetition game, investment in this game has a strategic nature in the sense that each firm strategically chooses its investment taking into account its effect on market competition in the subsequent stage.

4.1 Production decisions

As before, the game is solved by backward induction. In the third stage, firms choose their outputs. The first-order conditions are given by (4). Then, a symmetric Nash equilibrium output per firm is given by $\tilde{q}(n, Z)$ with $\partial\tilde{q}/\partial n < 0$ and $\partial\tilde{q}/\partial z_i = \partial\tilde{q}/\partial z_j > 0$, for $i \neq j$. In addition, the total output in a symmetric equilibrium is $\tilde{Q}(n, Z)$ with $\partial\tilde{Q}/\partial n > 0$ and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$, for $i \neq j$.[8]

4.2 Investment decisions

In the second stage, each firm chooses the amount of investment by solving the following maximization problem given other firms' investment $Z_{-i} \equiv \sum_{j\neq i} z_j$:

$$\max_{z_i} \pi_i(z_i, Z_{-i}) = P\left(\tilde{Q}, z_i + Z_{-i}\right)\tilde{q} - C(\tilde{q}) - D(z_i) - K.$$

Using (4), the first-order conditions are given as follows:[9]

$$\frac{\partial\pi_i}{\partial z_i} = \left[P_Q\frac{\partial\tilde{q}}{\partial z_i}(n-1) + P_Z\right]\tilde{q} - D' = 0. \tag{10}$$

The comparison of (5) with (10) clarifies the difference between investment choices in simultaneous and sequential co-opetition games. In the sequential co-opetition game, firms choose investment with anticipation that their own investment will make the rival aggressive (increases rival's output) as represented by the first term in the parentheses of (10). This "pre-commitment" effect of investment reduces the incentive for investment. Therefore, ceteris paribus, firms' incentives to invest are smaller in the sequential co-opetition game, as compared to the simultaneous co-opetition game.

Solving (10) for all $i = 1, \cdots, n$, we derive the equilibrium amount of investment in a symmetric subgame perfect Nash equilibrium at the second stage as denoted by $\bar{z}(n)$ and $\bar{Z}(n) \equiv n\bar{z}$. In addition, we denote the equilibrium output in a symmetric subgame perfect Nash equilibrium at the second stage as $\bar{q}(n) \equiv \tilde{q}(n, \bar{Z})$ and $\bar{Q}(n) \equiv \tilde{Q}(n, \bar{Z})$.

The effects of entry on $\bar{q}$, $\bar{z}$, $\bar{Q}$, and $\bar{Z}$ are quite complex. Therefore, in the following, we provide the comparative static results for linear and constant elasticity demand cases.

■ Linear demand case

Specifying the inverse demand as (2), we obtain the third-stage equilibrium:

$$\tilde{q}(n, Z) = \frac{a - c + Z}{b(n+1)}, \quad \tilde{Q}(n, Z) = \frac{n(a - c + Z)}{b(n+1)}.$$

[8] In detail, we have the comparative static results:

$$\frac{\partial\tilde{q}}{\partial n} = -\frac{P_{QQ}\tilde{q}^2 + P_Q\tilde{q}}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} < 0, \quad \frac{\partial\tilde{q}}{\partial z_i} = \frac{\partial\tilde{q}}{\partial z_j} = -\frac{P_Z}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0,$$
$$\frac{\partial\tilde{Q}}{\partial n} = \frac{(P_Q - C'')\tilde{q}}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0, \quad \frac{\partial\tilde{Q}}{\partial z_i} = \frac{\partial\tilde{Q}}{\partial z_j} = -\frac{nP_Z}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0.$$

[9] We assume that the second-order conditions are satisfied, i.e.,

$$\frac{\partial^2\pi_i}{\partial z_i^2} = \left[P_Q\frac{\partial\tilde{q}}{\partial z_i}(n-1) + P_Z\right]\frac{\partial\tilde{q}}{\partial z_i} + \left[P_{QQ}\frac{\partial\tilde{Q}}{\partial z_i}\frac{\partial\tilde{q}}{\partial z_i}(n-1) + P_Q\frac{\partial^2\tilde{q}}{\partial z_i^2}(n-1) + P_{ZZ}\right]\tilde{q} - D'' < 0.$$

We can easily confirm that $\partial\tilde{q}/\partial n < 0$, $\partial\tilde{q}/\partial z_i > 0$, $\partial\tilde{Q}/\partial n > 0$, and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$.

Solving the second-stage problem for firm i, we have the following reaction function:

$$z_i = R_i(Z_{-i}) \equiv \frac{2(a-c)}{(n+1)^2bd-2} + \frac{2}{(n+1)^2bd-2}Z_{-i},$$

which indicates that the investment choices are strategic complement. The second-stage equilibrium is characterized as:

$$\bar{q}(n) = \frac{(a-c)(n+1)d}{\Theta}, \quad \bar{z}(n) = \frac{2(a-c)}{\Theta}, \tag{11}$$

where $\Theta \equiv (n+1)^2bd - 2n > 0$ from the stability of Nash equilibrium in the second stage. Therefore, we find that

$$\begin{aligned} \frac{d\bar{q}}{dn} &= -\frac{(a-c)d[(n+1)^2bd-2]}{\Theta^2} < 0, \\ \frac{d\bar{z}}{dn} &= -\frac{4(a-c)[(n+1)bd-1]}{\Theta^2} < 0, \\ \frac{d\bar{Q}}{dn} &= \frac{(a-c)^2d[(n+1)^2bd-2n^2]}{\Theta^2}, \\ \frac{d\bar{Z}}{dn} &= -\frac{2bd(a-c)(n^2-1)}{\Theta^2} < 0. \end{aligned}$$

We should notice that $d\bar{Z}/dn < 0$ holds for any n, which indicates that an increase in the number of firms actually decreases total investment as well as individual investment. Furthermore, we have

$$\lim_{n\to\infty} \bar{q} = \lim_{n\to\infty} \bar{z} = \lim_{n\to\infty} \bar{Z} = 0, \ \lim_{n\to\infty} \bar{Q} = \frac{a-c}{b} > 0,$$

that is, as the number of firms increases, the total outputs converge to the perfect competition outcome without investment activities because the amount of common property resources converge to zero. This is quite contrasting to the case of simultaneous co-opetition.

■ Constant elasticity demand case

Specifying the inverse demand as (3), we obtain the third-stage equilibrium:

$$\tilde{q}(n,Z) = \frac{(a+Z)\Lambda^{\epsilon}}{n}, \quad \tilde{Q}(n,Z) = (a+Z)\Lambda^{\epsilon}.$$

We obtain $\partial\tilde{q}/\partial n < 0$, $\partial\tilde{q}/\partial z_i > 0$, $\partial\tilde{Q}/\partial n > 0$, and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$.

Solving for the second-stage problem, we have

$$\bar{z} = \frac{c\Lambda^{\epsilon}}{(n\epsilon-1)nd}, \quad \bar{q} = \frac{a\Lambda^{\epsilon}}{n} + \frac{c\Lambda^{2\epsilon}}{(n\epsilon-1)dn}. \tag{12}$$

The comparative static yields the following:

$$\begin{aligned} \frac{d\bar{q}}{dn} &= -\frac{\Lambda^{2\epsilon-1}\left[\{2(n-1)\epsilon-1\} + adn\epsilon\left\{\epsilon(n-1)-1\right\}\Lambda^{1-\epsilon}\right]}{dn^2\epsilon(n\epsilon-1)} < 0, \\ \frac{d\bar{z}}{dn} &= -\frac{[\epsilon(2n-1)-1]\Lambda^{\epsilon-1}}{dn^2\epsilon(n\epsilon-1)} < 0, \\ \frac{d\bar{Q}}{dn} &= \frac{\epsilon\Lambda^{2\epsilon-1}\left[adn\epsilon\Lambda^{1-\epsilon}-(n-2)c\right]}{dn^2\epsilon(n\epsilon-1)}, \\ \frac{d\bar{Z}}{dn} &= -\frac{(n-1)\Lambda^{\epsilon-1}}{dn^2(n\epsilon-1)} < 0. \end{aligned}$$

In addition, we have

$$\lim_{n\to\infty}\bar{q}=\lim_{n\to\infty}\bar{z}=\lim_{n\to\infty}\bar{Z}=0,\ \lim_{n\to\infty}\bar{Q}=ac^{-\epsilon}>0,$$

Therefore, we find that, as the number of firms increases, the total output converges to the perfect competition outcome without investment activities and the total amount of investment converges to zero, as in the linear demand case.

4.3 Entry decisions and the second best

In the first stage, firms enter the market until their profits fall to zero. Therefore, the free entry number of firms is defined as $\bar{n}_f$ such that

$$\bar{\pi}\left(\bar{n}_f\right)=P\left(\bar{Q}\left(\bar{n}_f\right),\bar{Z}\left(\bar{n}_f\right)\right)\cdot\bar{q}\left(\bar{n}_f\right)-C\left(\bar{q}\left(\bar{n}_f\right)\right)-D\left(\bar{z}\left(\bar{n}_f\right)\right)-K=0$$

We then consider the second-best problem for a social planner who can control the number of firms entering the market. Let $\overline{W}(n)$ denote the total surplus as

$$\overline{W}(n)\equiv\int_0^{\bar{Q}}P(s,\bar{Z})ds-nC(\bar{q})-nD(\bar{z})-nK.$$

Then we have

$$\begin{aligned}\overline{W}'(n) &= P\left(\bar{q}+n\frac{d\bar{q}}{dn}\right)+P_Z\bar{Q}\left(\bar{z}+n\frac{d\bar{z}}{dn}\right)-C-nC'\frac{d\bar{q}}{dn}-D-nD'\frac{d\bar{z}}{dn}-K\\ &= \bar{\pi}-P_Q\bar{Q}\left[\frac{d\bar{q}}{dn}+\frac{\partial\tilde{q}}{\partial z}\frac{d\bar{z}}{dn}(n-1)\right]+P_Z\bar{Q}\left[\frac{d\bar{z}}{dn}(n-1)+\bar{z}\right].\end{aligned}$$

The social planner chooses $n=\bar{n}_{sb}$ that maximizes $\overline{W}(n)$, which implies

$$\overline{W}'(n)|_{n=\bar{n}_{sb}}=0\quad\text{if}\quad\bar{n}_{sb}>1.$$

We assume that the second-order condition should be satisfied ($\overline{W}''<0$). Thus we have

$$\overline{W}'(\bar{n}_f)=\underbrace{-P_Q\bar{Q}\left[\frac{d\bar{q}}{dn}+\frac{\partial\tilde{Q}_{-1}}{\partial z}\frac{d\bar{z}}{dn}\right]}_{\text{business stealing}}+\underbrace{P_Z\bar{Q}\frac{d\bar{Z}_{-1}}{dn}}_{\text{common property}}.\tag{13}$$

The first term is the business stealing effect of entry. The sign of this term is negative. Private firms consider neither the negative direct impact of their entry on rivals' outputs (that is represented by $d\bar{q}/dn<0$) nor the negative indirect impact through the change in rivals' investments (this is represented by $(d\tilde{Q}_{-1}/dz)(d\bar{z}/dn)<0$). The second term is the common property effect of entry. Different than the case of non-commitment investment, the sign of the effect is negative when $d\bar{Z}/dn<0$ holds in the second stage. Private firms do not take into account the negative impact of their entry on the profitability of their rivals through the decrease in total amount of investment or public good. Then we have the following proposition.

Proposition 2
In a sequential co-opetition game, the free entry more likely to result in socially excessive entry and the depletion of common property resources.

The proposition contrasts with the result in the simultaneous co-opetition case shown by Proposition 1. In the sequential co-opetition case, each firm chooses its investment with anticipation that its own investment increases not only its own output but also rivals' outputs in the subsequent stage due to the pre-commitment effect of investment as in (10). Therefore, each firm's investment is strategically chosen to be smaller than that in the simultaneous case. In addition, as the number of firms increases, the strategic effect is strengthened. As a result, the total amount of investment is a decreasing function of the number of firms in the market. In general, the total provision of voluntarily provided public goods is an increasing function of the number of players, while the individual contribution to public goods is a decreasing function of it. However, in our case, the total amount of public goods (investment) is also decreasing function of the number of firms. This is because when n increases, there are two channels to reduce firms' incentive to invest: firms tend to free ride on the contributions of others and individual outputs become small, which reduces the marginal profits of investment.

In the following, we confirm the proposition for the linear and constant elasticity demand cases. We obtain a strong result that excess entry results hold for the sequential co-opetition game even if the entry costs are zero.

■ **Linear demand case**

From (11), the free-entry equilibrium number of firms, $\bar{n}_f$, satisfies

$$\bar{\pi}(\bar{n}_f) = \frac{(a-c)\left[(\bar{n}_f+1)^2 bd - 2\right] d}{\Theta^2} - K = 0.$$

From the fact that $\bar{\pi}(n)$ is strictly decreasing in n and $lim_{n\to\infty}\bar{\pi} = -K$, we confirm

$$\lim_{K\to 0} \bar{n}_f = \infty,$$

which indicates that the free-entry number of firms goes to infinity when there are no entry costs. After some tedious manipulation, we find that the second-best number of firms, $\bar{n}_{sb}$, satisfies,

$$\overline{W}'(\bar{n}_{sb}) = \bar{\pi}(\bar{n}_{sb}) - \frac{(a-c)^2\left[2bd\left(\bar{n}_{sb}^3 + \bar{n}_{sb}^2 - 6\bar{n}_{sb} - 6\right) + (\bar{n}_{sb}+1)^3 b^2 d^2 + 8\right] nd}{\Theta^3} = 0.$$

In addition, we have

$$\overline{W}'(n)\big|_{K=0} = \frac{(a-c)^2 d}{\Theta^3}\left[b^2d^2(n+1)^3 - 2bd(n^4+2n^3-3n^2-3n+1) - 4n\right] = 0$$

when

$$bd = \frac{n^3+n^2-4n+1+\sqrt{n^6+2n^5-7n^4-6n^3+22n^2-4n+1}}{(n+1)^2}, \tag{14}$$

and $W''(n) < 0$. Therefore, we find that

$$\lim_{K\to 0} \bar{n}_{sb} = n^*,$$

such that n^* satisfies the condition of (14). In other words, the second-best number of firms is positive and finite even when there are no entry costs.

Then, we find that

$$\overline{W}'(\bar{n}_f) = -\frac{(a-c)^2\left[2bd\left(\bar{n}_{sb}^3 + \bar{n}_{sb}^2 - 6\bar{n}_{sb} - 6\right) + (\bar{n}_{sb}+1)^3 b^2 d^2 + 8\right] nd}{\Theta^3} < 0,$$

which indicates that free entry necessarily results in excessive entry.

Corollary 3 *Under linear demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the sequential co-opetition game yields excessive entry. Furthermore, this result holds even when there are no entry costs.*

We provide some numerical examples. In the case of $a = 10$, $b = 1$, $c = 1$, $d = 2$, and $K = 2$, then $\bar{n}_f \approx 6 > \bar{n}_{sb} \approx 2$. In the case of $a = 10$, $b = 1$, $c = 1$, $d = 8$, and $K = 0$, then $\bar{n}_f = \infty > \bar{n}_{sb} \approx 5$, which clearly shows that excess entry property holds even when $K = 0$.

■ Constant elasticity demand case

From (12), the free-entry equilibrium number of firms, $\bar{n}_f$, satisfies

$$\bar{\pi}(\bar{n}_f) = \frac{c\Lambda^{2\epsilon}\left[c(2\bar{n}_f - 1) + 2ad\bar{n}_f(\bar{n}_f\epsilon - 1)\Lambda^{-\epsilon}\right]}{2d\bar{n}_f^2(\bar{n}_f\epsilon - 1)^2} - K = 0.$$

From the fact that $\bar{\pi}(n)$ is strictly decreasing in n and $lim_{n\to\infty}\bar{\pi} = -K$, we confirm

$$\lim_{K\to 0} \bar{n}_f = \infty,$$

which indicates that the free-entry number of firms goes to infinity when there are no entry costs. After some tedious manipulation, we find that the second-best number of firms, $\bar{n}_{sb}$, satisfies,

$$\overline{W}'(\bar{n}_{sb}) = \bar{\pi}(\bar{n}_{sb}) - \frac{c\Lambda^{2\epsilon}}{(n\epsilon - 1)^3 d\bar{n}_{sb}^2}\left[\begin{array}{l} ad\bar{n}_{sb}\Lambda^{-\epsilon}\left(\bar{n}_{sb}\epsilon - 1\right)\{\epsilon(\bar{n}_{sb} - 1) - 1\} \\ \quad + c\{\bar{n}_{sb}\epsilon(\bar{n}_{sb}^2 - 4) + \bar{n}_{sb}(\bar{n}_{sb}\epsilon - 1) + 1 + \epsilon\} \end{array}\right] = 0.$$

Then, we find that

$$\overline{W}'(\bar{n}_f) = -\frac{c\Lambda^{2\epsilon}}{(n\epsilon - 1)^3 d\bar{n}_f^2}\left[\begin{array}{l} ad\bar{n}_f\Lambda^{-\epsilon}\left(\bar{n}_f\epsilon - 1\right)\{\epsilon(\bar{n}_f - 1) - 1\} \\ \quad + c\{\bar{n}_f\epsilon(\bar{n}_f^2 - 4) + \bar{n}_f(\bar{n}_f\epsilon - 1) + 1 + \epsilon\} \end{array}\right] < 0,$$

which indicates that free entry necessarily results in excessive entry.

Corollary 4 *Under constant elasticity demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the sequential co-opetition game yields excessive entry. Furthermore, this result holds even when there are no entry costs.*

In this sequential co-opetition game, as Corollaries 3 and 4 indicate, the well-known excess entry theorem, developed by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), applies even when there are no entry (set-up) costs for entrants. In their model, free entry results in the socially optimal number of firms when there are no entry costs. Our results reflect that as the number of firms increases, the total amount of socially beneficial investment is reduced, which is independent of the existence of fixed set-up costs.[10]

In this constant elasticity demand case, the second-best number of firms when $K = 0$ cannot be analytically derived, so we provide some numerical examples to show that excess entry property holds even when $K = 0$. In the case of $a = 20$, $c = 0.1$, $d = 4$, and $\epsilon = 2$, $\bar{n}_f \approx 31 > \bar{n}_{sb} \approx 7$ holds for $K = 0.1$ and $\bar{n}_f \approx \infty > \bar{n}_{sb} \approx 9$ holds for $K = 0$.

[10]Ghosh and Saha (2007) also show the possibility of excess entry without fixed entry costs but in the presence of marginal cost difference.

5 Concluding Remarks

In many industries, firms share a common property resource that affects the market size or consumers' willingness to pay for the products and contributes to the common property resources. This paper investigates whether free entry leads to socially excessive or insufficient entry in a co-opetitive model where firms compete and cooperate with each other at the same time. We explore two approaches to modeling firms' co-opetitive behavior: simultaneous and sequential co-opetition. We find that, in the simultaneous co-opetition case, in which firms simultaneously decide upon their investments and outputs, free entry leads to insufficient or excessive entry depending on the relative magnitude of the business stealing and common resource effects of entry. In particular, free entry is more likely to result in socially insufficient entry when production and investment costs are smaller and price elasticity is greater. On the other hand, in the sequential co-opetition case, in which firms can use investment as a commitment, free entry leads to excess entry due to the negative common property effect. Interestingly, this excessive-entry result holds even when there are no entry (set-up) costs. These findings contribute to the literature on excess-entry property in oligopoly markets.

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