Does "Okun's Law" state a Pi:1 ratio? Toward a harmonic interpretation of why Okun's Law works

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Does "Okun's Law" State a π:1 Ratio?
Toward a Harmonic Interpretation of Why Okun's Law Works

By Scott A. Albers and Andrew L. Albers*

Abstract: *In Albers & Albers (Spring, 2013) we demonstrated that the historic development of U.S. real GNP, 1869-present, may be structured in recurring 14-year periods. A steady-state rate of growth of 3.4969% is thereby calculated, generating an increase in real GNP proportional to the famous "Golden Mean" (1:φ , or 1:1.6180) every fourteen years on average.*

Building on this foundation we show herein that "Okun's Law," a 3:1 proportion between percent growth in real GNP and percent decrease in the rate of unemployment, is actually a π:1 proportion, created through a form of mathematic / harmonic inverse. The resulting model of economics in the United States is thereby aligned with geometric, harmonic and trigonometric analysis, rather than purely statistical methods.[1],[2]

Okun's Law

"Okun's Law" was first presented in a 1962 paper by Arthur M. Okun entitled "Potential GNP: Its Measurement and Significance." The paper opens with the question: "How much output can the economy produce under conditions of full employment?" Dr. Okun writes:

> The basic technique I am reporting consists of a leap from the unemployment rate to potential output rather than a series of steps involving the several underlying factors (which might impact on potential output). Strictly speaking, the leap requires the assumption that, *whatever the influence of slack economic activity on average hours, labor force participation, and manhour productivity, the magnitudes of all these effects are related to the unemployment rate*. With this assumption, the unemployment rate can be viewed as a proxy variable for all the ways in which output is affected by idle resources. The measurement of potential output then is simplified into an estimate of how much output is depressed by unemployment in excess of four percent.

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[1] This article contains 8,128 words, with an abstract of 117 words. Additional papers toward an understanding of the ideas presented herein is found at "scribd_scott_albers_1" and the Munich Personal Repository.

[2] Acknowledgements. This paper is a reply to Dr. Edward Knotek's rhetorical question "How Useful is Okun's Law?" (*Economic Review* 2007) made possible only because Dr. Knotek has been so generous with his time, his valuable information, his insights as to Okun's Law and his repeated explanations of data and methods which he used in that article.

The leap from an individual consideration of various subsidiary "effects" to the "proxy variable" of a single, all-encompassing "national rate of unemployment" significantly simplifies the approach to the question posed. It is important to note however that the subsidiary "effects" leapt over ("average hours, labor force participation, and manhour productivity") continue to have a vital and independent standing as they are "related to the unemployment rate."

We maintain in this paper that an inverse and reciprocal relationship is implied by this "leap." We maintain that, just as the national unemployment rate will be directly related to the individual "effects" which are disregarded by Okun's approach, so will these subsidiary and far more personal "effects" be related to the unemployment rate in a direct and reciprocal fashion.

One might imagine the inverse relationship implied between the "personal" and the "national" if we let "Government" = 1. In this case the fraction "1/individual" might represent the individual as he/she relates to the nation. Inversely, the fraction "individual/1" would represent the nation as it relates to the individual.

Okun's paper makes the leap that the rate of unemployment is a "proxy variable" for the more specific and personal "effects" which connect the national rate of unemployment with the person who is employed or unemployed. The significance of this inverse relationship is not mentioned by Dr. Okun, nor does he deal in any way with its impact on his analysis.

The fulcrum upon which Okun's Law balances so perfectly lies in the fact that employment affects each and every person in the United States in a tremendously personal way. ("Do you have a job?") This personal connection which each citizen has with their employment is intimately tied to the very concepts – average hours, labor force participation, manhour productivity – which are "leapt over" by Okun's approach. Through the right of the citizen to insist that he / she have some creative, on-going and functional role in the economy a continual relationship between the national economy and the individual worker is maintained.

The risk inherent in Okun's approach is that the inverse relationship between the personal and the national might become obscured, taking on by unacknowledged acquiescence the nature of a "1/1," an impenetrable and un-investigable union. In this paper we will deal extensively with the inverse relationship between the "personal made national," and the "national made personal" as the mathematic foundation of our approach.

Dr. Okun then states his law.

> The answer I have to offer is simple and direct. In the postwar period, on the average, each extra percentage point in the unemployment rate above four percent has been associated with about a three percent decrement in real GNP.
>
> ... My own subjectively weighted average of the relevant coefficients is 3.2, yielding the following estimate of potential:

$$P = A(1 + .032 (U-4))$$

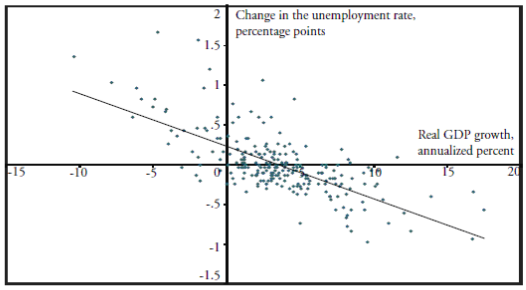
In sum "Okun's Law" notes that for every three percentage points of increase in real GNP the rate of employment increases by one percentage point, and that decreases of both take place at the same rate.[1] This 3 : 1 proportion is generally referred to using a double negative, i.e. an increase of three percent in real GNP will lead to a one percent *decrease* in the rate of *un*employment. Although first stated by Arthur Okun, at the time senior economist of President Kennedy's Council of Economic Advisors, "Okun's Law" has taken on a legend of its own, as "one of the most reliable empirical regularities in macroeconomics."[1] (Tobin, 1983)

Dr. Edward Knotek's article "How Useful Is Okun's Law?" (2007) proposes that Okun's Law is, at best, a helpful rule of thumb. As the title of the article suggests directly, Dr. Knotek describes in detail our present understanding of Okun's Law as both a mathematic equation and as a policy tool.

To make the point of his article Dr. Knotek organizes data sets which follow mainstream econometric methods as applied to well-known and easily available federal data bases covering a 60 year period of American economic history, i.e. the second quarter of 1947 through the third quarter of 2007. Charts One and Two graph the quarterly and annual data sets supporting the regularity of the relationship between changes in the size of real GNP (x-axis) and the corresponding effect this has on the rate of employment (y-axis).

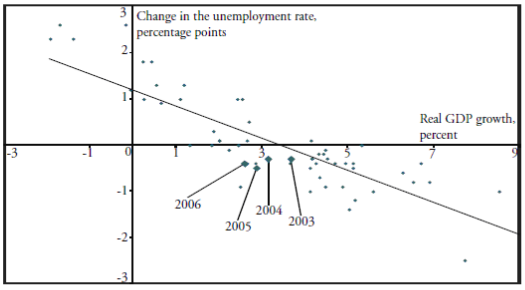
DIAGRAM 2-2.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

Dr. Knotek then takes issue with the straight-forward association presented above between the rate of growth and the rate of unemployment. He makes the point that the historic associations underlying these trend lines merit much closer scrutiny. We will turn to this insight near the end of this paper.

Surprisingly there is at present no theoretic structure to explain the apparently long-standing and vital macroeconomic / mathematic relationship given by Okun's Law. On this point see Owyang and Sekhposyan (2012):

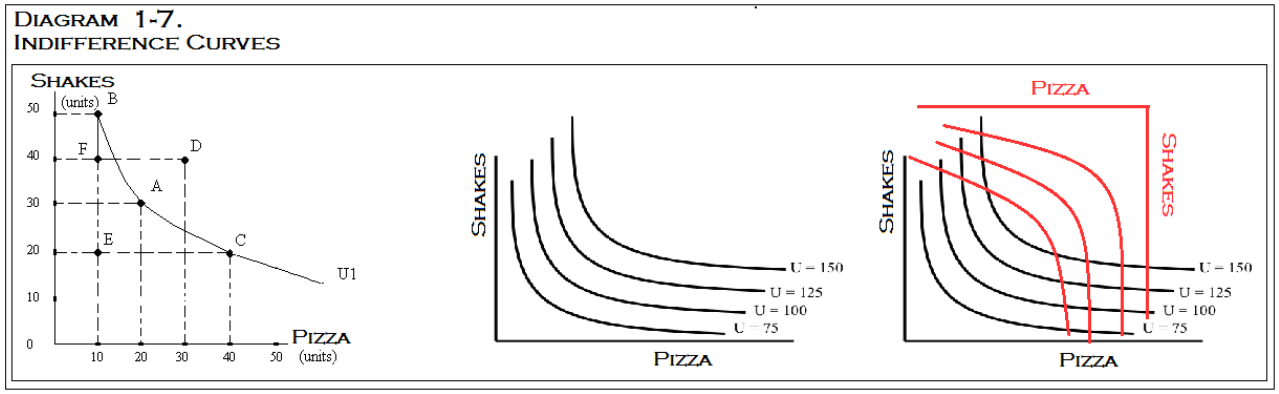
> Many macroeconomic textbooks contain a rule of thumb relating real output growth to changes in the unemployment rate. This relationship, called Okun's law after Okun (1962) typically assigns a 2- to 3- percentage point decrease in real gross domestic product (GDP) growth to a 1-percentage point increase in the unemployment rate. Unlike laws in the physical sciences (e.g. Newton's laws of motion) Okun's law is an (arguably loose) empirical correlation and is, in general, neither theoretically motivated nor strictly adhered to in the data. As many of the reduced-form relationships build strictly on associations and not causation, Okun's law appears to vary depending on the sample period studied.

This paper attempts to provide a theory which explains Okun's Law, both as a matter of micro-economic foundation as well as macro-economic and historic importance. The possibility of deriving a theory which explains such an important relationship must be of great interest, i.e. What causal precedents underlie such an important rule of macro-economics? ... seemingly one of the few macroeconomic observations to ever be denominated a "law" at all.

Why does Okun's Law work? To Buy or Not To Buy.

By way of introduction it is worth noting that in his course on the development of political morality Dr. Shapiro makes clear that the whole of neo-classical economics can be condensed into a study of the indifference curve. (Shapiro, 2012) These curves attempt to demonstrate the microeconomic relationship between consumer preferences as balanced between two different goods.

The curve drawn represents the "indifference" for any consumer as to which combination of good is offered. (left, Diag. 1-7 below) The "indifference curves" generated from this pair of dichotomies represents the willingness to trade one set of goods for different goods.



As increasing levels of affluence at provided, a map of multiple curves becomes possible. (center, Diag. 1-7 above)

The indifference curves of two competing trading partners may be explored by inverting the curve of one of the partners. (right, Diag. 1-7, above)

"Pareto optimality" represents a qualitative evaluation of these relationships. Given an initial allocation of goods among a set of individuals, a change to a different allocation that makes at least one individual better off without making any other individual worse off is called a Pareto improvement. An allocation is defined as "Pareto efficient" or "Pareto optimal" when no further Pareto improvements can be made.

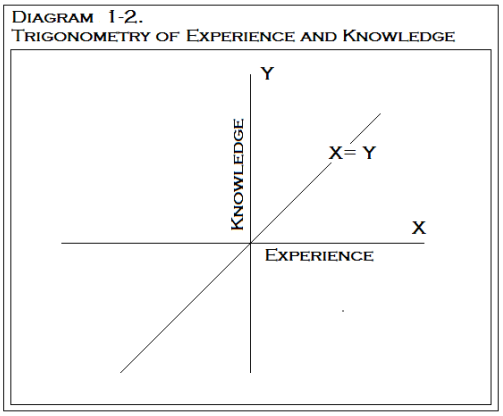
The essential point to notice about each of these curves is that they assume that the "space" lying outside the curve is synonymous with a "refusal to trade" or "saving," and the points within the curve are synonymous with a "willingness to trade" or simply "trade."

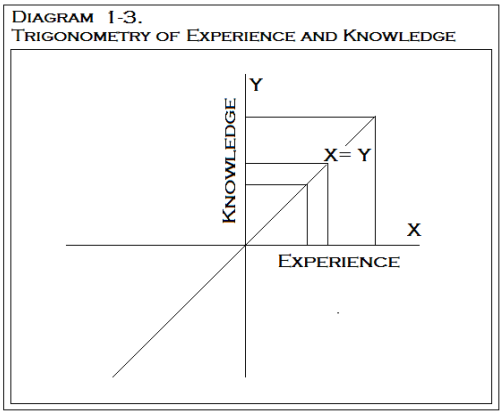
The Theory

For the purposes of these essays we will take as an axiomatic truth that all human life is based upon the presumed equivalence between that which we experience through the senses and that which we know to be real.[3] If "that which we experience" is given the variable "X" and "that which we know to be real" is given the variable "Y", we may state this equivalence as:

$$X = Y.$$

If we place this equation in a Cartesian coordinate system, we have the following 45 degree angle line, beginning at x = 0, y = 0 and extending on toward and infinite number of associations.





This picture represents the outcome of an infinite number of squares, wherein each corner point has a specific meaning. "X" represents our experience of something, "Y" represents our knowledge of the thing experienced, the point "(X, Y)" represents the interaction between our experience of the thing itself and our knowledge of the thing itself, and the origin of the graph "(0, 0)" represents the beginning association we make between experience and knowledge as fundamental assumptions of all inquiry.[4]

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[3] See Boswell, J. (1820). "After we came out of the church, we stood talking for some time together of Bishop Berkeley's ingenious sophistry to prove the nonexistence of matter, and that every thing in the universe is merely ideal. I observed, that though we are satisfied his doctrine is not true, it is impossible to refute it. I never shall forget the alacrity with which (Samuel) Johnson answered, striking his foot with mighty force against a large stone, till he rebounded from it -- "I refute it *thus*."

One might assert that the experience of reading a book and enjoying the imaginary world conveyed is not the same as "experiencing" or "knowing" anything about the world imagined.

Our point here is far more modest and direct. The "experience" referred in this essay is simply that of "reading the book" and the knowledge considered is simply that the person reading knows that he or she is reading a book. The equivalence understood between the *experience* of reading the book, and the *knowledge* that one *is* reading a book, is the equivalence with which we begin this analysis.

[4] See Ornstein, at 63: "In 1268, Roger Bacon, one of the founders of modern science, wrote (in his *Opus Maius....*), 'There are two modes of knowing, through argument and experience. Argument brings conclusion and compels us to concede them, but does not cause certainty nor remove doubts in order that the mind may remain at rest in truth, unless this is provided by experience.' These two modes are complementary (both are "right"), and together form the basis for the complete human consciousness."

Extension to the Jury Trial of a Criminal Case

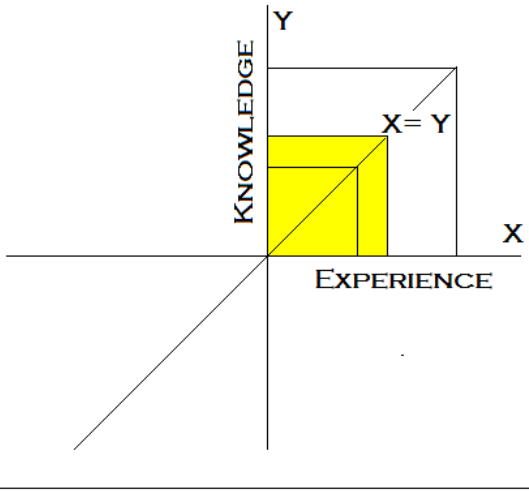
In the United States the jury trial of a case is premised on this same equation "X = Y," "experience" and "knowledge," taken to the next higher social level of the jury. The jury's reception and consideration of the evidence presented indicates that this small group is the expansion of the smaller individual and included minds. In the jury's deliberation the jury demonstrates itself as being the larger, expanded, copied and congruent larger "fractal" of the individual mind.

Specifically, the jury's personal *experience* of the evidence as presented in trial represents the "X" of a trial proceeding.

The jury's evaluation of this evidence as understood through the prism of their own life experiences is the "Y" of the trial proceeding, their collective *knowledge* of the facts presented.

The final verdict given by the jury states its evaluation of the association between the "X" of the trial (the evidence presented) with the "Y" of the trial (the jury's evaluation of this evidence).

DIAGRAM 1-4.
TRIGONOMETRY OF EXPERIENCE AND KNOWLEDGE



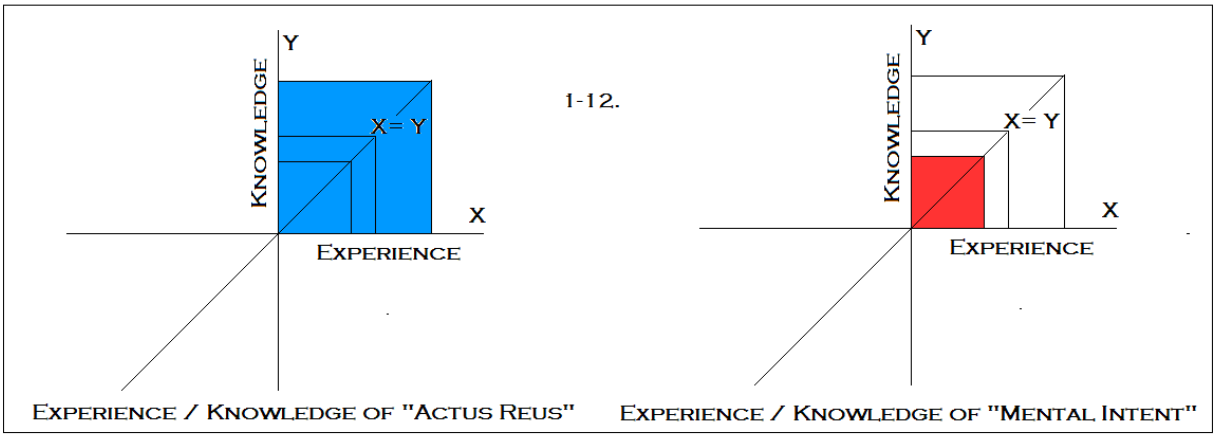
This simple model may be expanded upon.

The criminal law of the United States is based upon a dichotomy between the criminal act alleged to have been committed – the *actus reus* of the offense – and the mental intent –the *mens rea* of the offense – associated with the crime. For example, the act of killing someone is a homicide if done with the intent to kill the individual. If the killing was the result of recklessly driving in a crowded street, the crime is less because the evil of the intent to harm was less. Differences in the consequence to the Defendant can be quite significant, depending upon the nature of the criminal act and mental intent found by the jury.

If we let the "actus reus" of any given offense equal a particular number – for example, 5 – then the jury's experience with the evidence presented as to the criminal act (X = 5) and the jury's understanding of that evidence (Y = 5) may be given as a square, in blue below.

Similarly, if we let the "mens rea" of the same offense equal a different number – for example, 3 – then the jury's experience with the evidence presented as to mental intent (X = 3) and the jury's understanding of that evidence (Y = 3) may be given as a red square, in red below.

DIAGRAM 1-5.
TRIGONOMETRY OF EXPERIENCE AND KNOWLEDGE



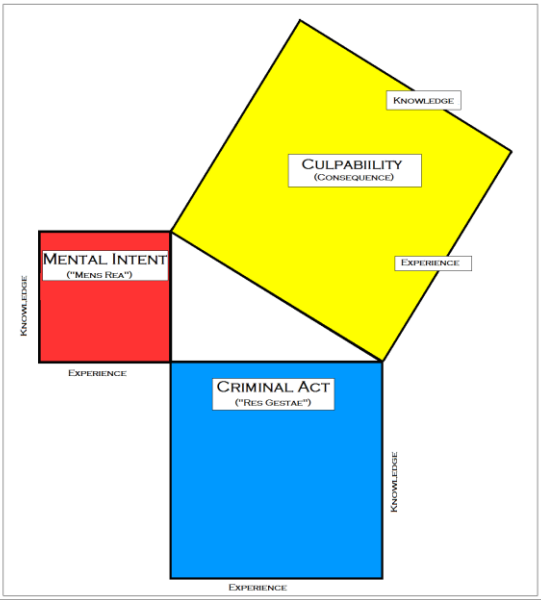
EXPERIENCE / KNOWLEDGE OF "ACTUS REUS" EXPERIENCE / KNOWLEDGE OF "MENTAL INTENT"

The culpability, if any, of the Defendant for a crime is given in accordance with the sum of these two elements of proof. The full experience and knowledge summarized by the case will equal the sum of these two squares. Stating the jury's experience with the evidence of a criminal act as a positive distance "A" and the jury's experience with the evidence of mental intent as a positive distance "B", then the experience / knowledge represented by Culpability (C) associated with the verdict should equal the sum of these two things, or :

$$A^2 + B^2 = C^2$$

Geometrically, this equation may be portrayed with the proportions of the Pythagorean Theorem as follows.

DIAGRAM 1-6.
PYTHAGOREAN THEOREM OF SOCIAL CONDUCT



From the economic point of view, there is no difference between stating that "John purchased x" and "John is guilty of purchasing x." The relationship between the act and the thought which motivates the act, speaking economically, is the same as that of the court considering such an act criminally.

DIAGRAM 1-8.
PYTHAGOREAN THEOREM - SOCIAL AND ECONOMIC ACTIVITY

Micro-economics: The "Chooser – Available Choice" Model

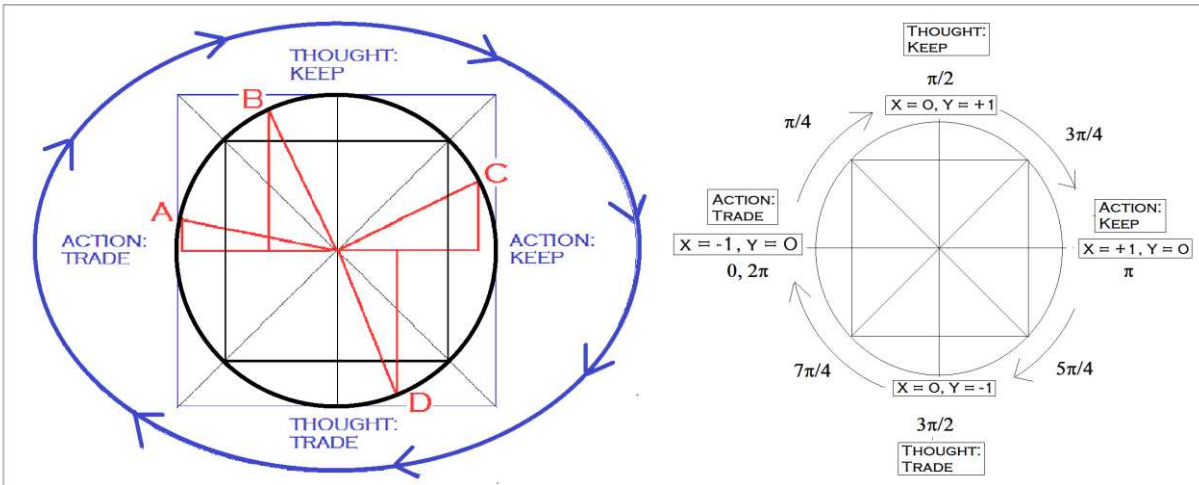
Each of the points within the plane of an indifference curve – both those on the curves and those outside the curve – represents a given decision to trade or to keep various properties. If we contrast the actions of *trading* a good versus *keeping* that same good, a set of dichotomies may be constructed which maybe used to structure our understanding of economic development.

The first dichotomy – action, as comparable to the "actus reus" of criminal law – represents a tension between "Keeping" a particular good vs. "Trading" the good for something else. This is indicated in the circle below by the opposition of "Keep" at 3 o'clock and "Trade" at 9 o'clock. All economic life stems from the core principle that one may *act* freely in choosing either to keep a given property or to trade it for some other piece of property and that these transactions clearly affect the status of the property so owned or traded.

This is contrasted with a secondary dichotomy – thoughts, as comparable to the "mens rea" of criminal law – which represents a tension between one's mental "thoughts in favor of keeping" and "thoughts in favor of trading" a particular property, located at 12 o'clock and 6 o'clock respectively in the circle below. These are the mental pre-dispositions of every owner towards keeping or trading a given piece of property for something else.

Using the Pythagorean Theorem to structure the sum total of possible permutations between the "Action" aspect of a purchase, and the "Thought" aspect of a decision to Purchase, we may structure every possible balancing of these two with the "Purchase" itself.[5]

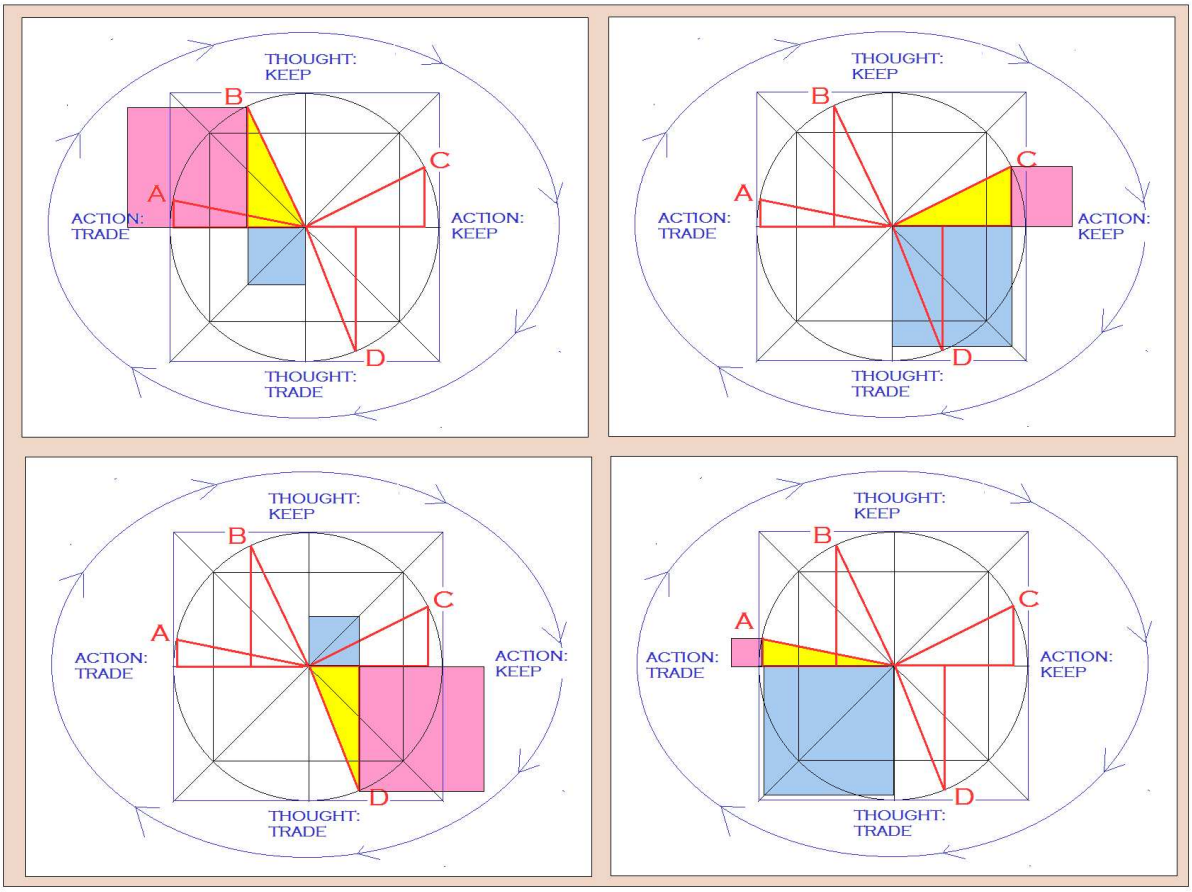
DIAGRAM 1-10.
TRIGONOMETRY OF TRADING VS. KEEPING PROPERTY



[5] The "clock-wise" direction of movement around the unit circle and the "9:00 o'clock" place of beginning the analysis as used in these essays are opposite that taken in most trigonometry textbooks. This approach does not alter the trigonometric identities considered in the slightest and provides an approach to the measurement of time which is consistent with the sense of the hands of a clock.

The Pythagorean relationships inherent in the association of Action and Thought as expressed previously create around the unit circle create an infinite set of mathematic relationships wherein the actual possibility of a Purchase is set as the sum of some combination of Action and Thought.

DIAGRAM 1-11.
TRIGONOMETRIC RATIOS.



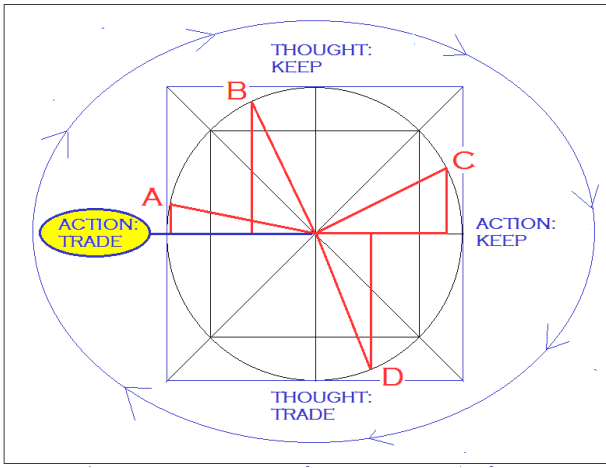
The unity of the underlying ego which selects these various points may be associated with the radius of this circle. If we give this radius the number "1" it represents the "unity" of the ego as a balancing radius between these two dichotomies of Action ("Trading" vs. Keeping") and Thoughts ("Thoughts related to Trading the property," "Thoughts related to Keeping the property"). An internal angle is thus constructed at the origin of the coordinate system.

The Significance of Trading

There is only one point along the Unit Circle where Action is wholly aligned with Trading, i.e. the point at 9:00. All other points along the unit circle are similar to one another in that there is some "Y" component connected to some mental aspect of trading and/ or keeping the object in question. This mental aspect must include some possibility of cancelling the action contemplated. Consequently only at 9:00 o'clock is the possibility of a "Trade" wholly equivalent with Action, and at this point "Thought" is Zero and the Action Trading occurs.

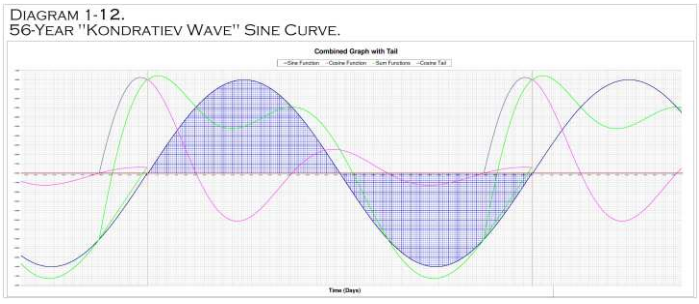
Conversely at 3:00 o'clock the action undertaken is to "Keep" the property in question and the status quo is actively continued.[6]

DIAGRAM 1-13.
TRIGONOMETRY OF TRADE.



6 If we consider the side opposite the internal angle as divided by the hypotenuse of "1" we set up a set of fractions which may be charted against an x-axis wherein the circumference of the circle is superimposed upon the x-axis in divisions associated with 2π. Beginning at 9 o'clock and moving clockwise, we have the following mathematic associations between various points along the unit circle, to wit, the sine curve.

DIAGRAM 1-12.
56-YEAR "KONDRATIEV WAVE" SINE CURVE.



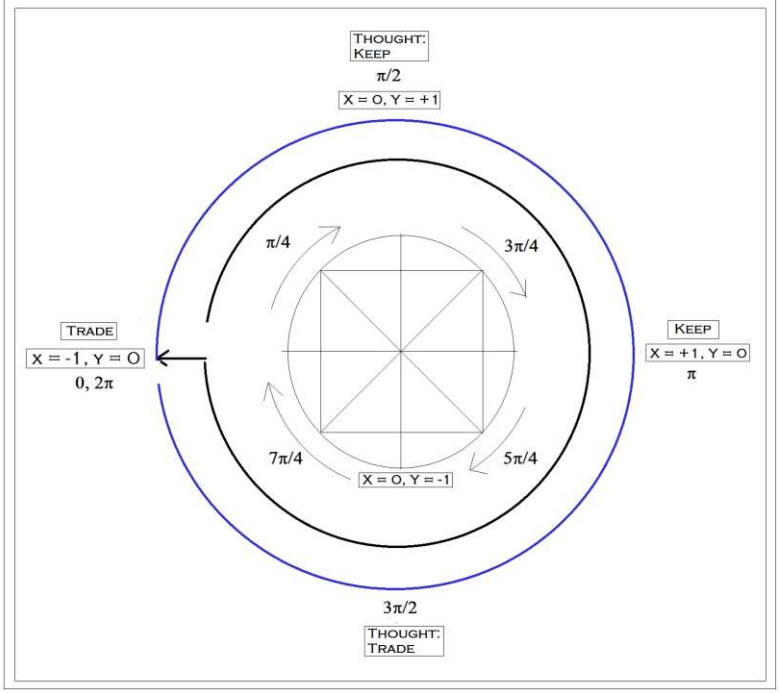
The equation for this wave is:

$$g(y) = \sin(y)$$

The unique aspect of this point at 9:00 o'clock creates an unavoidable change in the overall unit circle. The break which is presented at (x = -1, y = 0) creates a new and unknown element in the unit circle itself. Once the trade is made, the situation is no longer the way it was. Something new has taken place. [7]

In contrast, when the x-axis is directed toward "Keeping" a particular good, the point at which Thought = 0 will be that point most dedicated in favor of the status quo.

DIAGRAM 1-14.
NEW CIRCLE ON TRADE.



The model will be referred to as the "chooser – available choice" model, as a way of presenting the unit circle and its radius of "1" – representing the "chooser" – and the number π – representing the "choices available" – in a simple and direct fashion. Our premise is that a radius originating at the center of the unit circle and moving toward any spot on the circle of possible choices divides the circle at a 1 : π ratio. Half of the circle constitutes "available choices" which will be associated with the point at which the radius and the circle intersect. This relationship will exclude an equivalent set of opposite choices on the opposing side of the circle.

[7] There is an analogy here to quantum mechanics in the "Schrodinger's Cat Thought Experiment." The second half of the third postulate of quantum mechanics states, roughly speaking, that observation changes the physical system. http://vergil.chemistry.gatech.edu/notes/quantrev/node20.html A physical system exists in as many state as possible until it is observed. Once the observation has been made, it changes into another state, one which can be unique or not.

Until one opens the box, the cat is both dead and alive. Opening the box (observing the state of the cat), indicates which state it is, and so changes the state of the physical system. In this essay, trading equates with the observation. By analogy, stating that with trade "something new has happened" one would indicate that the wave function describing the state of the cat has changed.

In other words, one can not simultaneous trade a good and keep the same good, or vice versa. The possible choices which *are* available toward any particular goal are those which are not directly undermining of whatever goal is chosen. The choices which are *not* available are those which are in some negative value, or opposite position, from this chosen goal. This same dynamic applies to any point of psychological consideration along the unit circle.

We conclude that it is possible to construct a simple and mathematically straight-forward model of micro-economic choices which is completely in accord with the available evidence of social behavior as evidenced by universal and legally required social understandings.

By drafting the experience and knowledge of a jury as the larger "fractal" of the individual mind, we have the ability to state a pattern of "mind" itself which is both useful and concrete in its form.

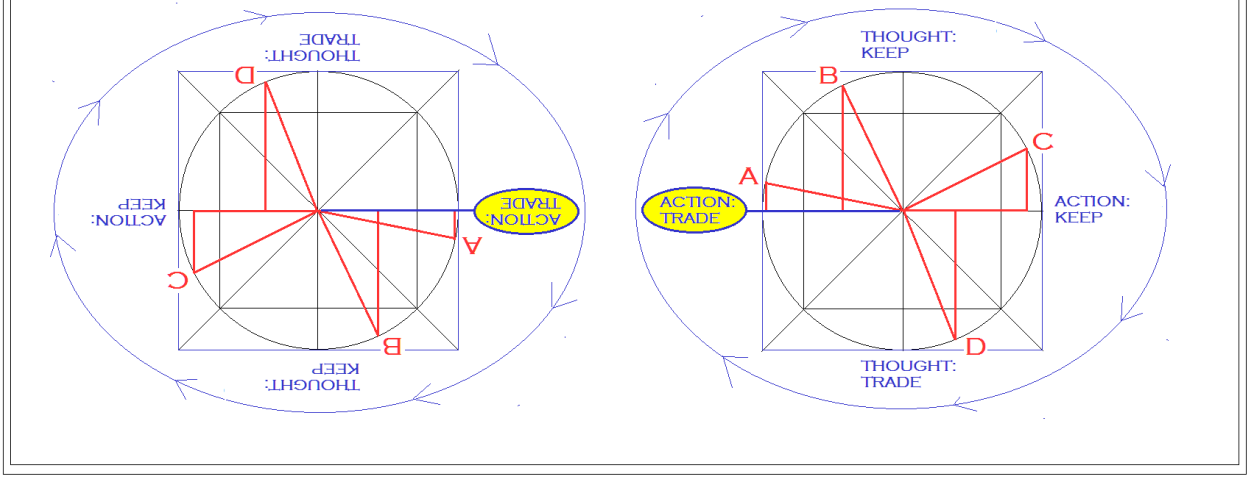
Macro-economics: The "Chooser - Available Choice" model in aggregate

The "chooser – available choice" model is the central point of departure for this model. If we invert this model such that the willingness to "trade" of one person meets the willingness to "trade" of a trading partner, we have a connection between two people indicating a mutual willingness to exchange goods or services with one another. (See discussion of Pareto efficiency *supra*) The willingness and ability of persons to trade goods and their services with one another is the foundation for the entire economy.

Let us begin with a proposed willingness of Farmer Jones to part with two cows in return for three horses. This willingness is met by Farmer Smith who is willing to trade three specific horses which he owns in return for two specific cows belonging to Farmer Jones.

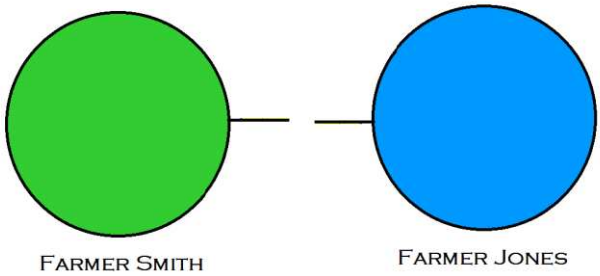
The fact that these two farmers have met with a match which in their minds is favorable to both is indicated by the fact that both have extended the 9:00 axis "Action : Trade" towards one another. As a result of this trade, Farmer Jones' two cows will be handed over to Farmer Smith, and Farmer Smith's three horses will be handed over to Farmer Jones.

DIAGRAM 2-4.
"TRADE" UNDER THE "CHOOSER - AVAILABLE CHOICE" MODEL



The following two circles simplify the basic ideas going into the above trade. Note that the early barter of horses for cows suggested by the circles below depicts trading at its most elementary level. Note that the trade itself must in some fashion state an improvement in the lives of the trading partners. Consequently the act of trading makes more efficient and useful the sum total of property within society because those who own the property are seeking ever more agreeable collections of that property by trading what they have for things which they desire but do not possess.

DIAGRAM 2-5.
A SINGLE TRADE

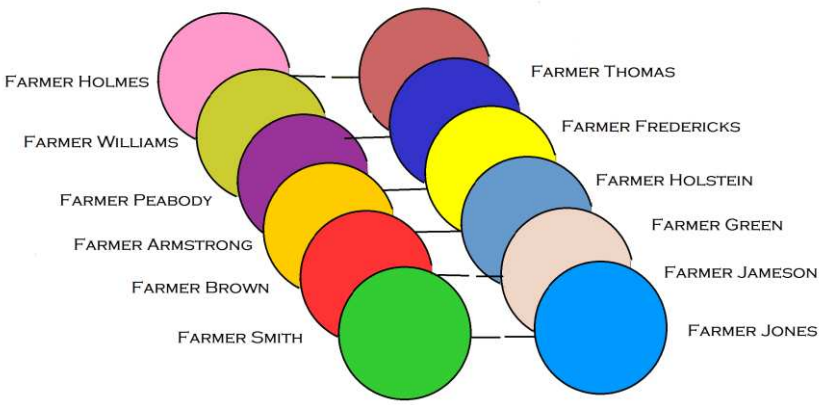


These trades represent a re-arrangement of property amongst those owning property. There is no "expansion" of the economy based upon this trade. However the usefulness of the property exchanged, in combination with the improved efficiency brought about by the trade, suggests that the natural rate of increase in any biologic organism – a farm, a household, a local market – will result from the full set of trades engaged in by all persons.

In short, the same property and the same traders exist after as well as before the trade. However the straight forward exchange of one set of property for another is conveyed by the model above.

There is no limit to the number of such trades which can be done over the course of any particular period of time. We may imagine two pipes running parallel, each suggesting the desire of one of two trading partners to enter into trade. Each trade may be listed in chronologic order and depicted as below.

DIAGRAM 2-6.
THE STREAM OF TRADE



As reliable currency enters into circulation persons engaged in trading have the further ability to makes trades of much greater complexity that a straight-forward barter. By saving the money obtained from prior trades people are able to amass a trading ability to trade which far exceeds the more clumsy and complicated trade of physical objects, herds of cattle or flocks of geese, etc.

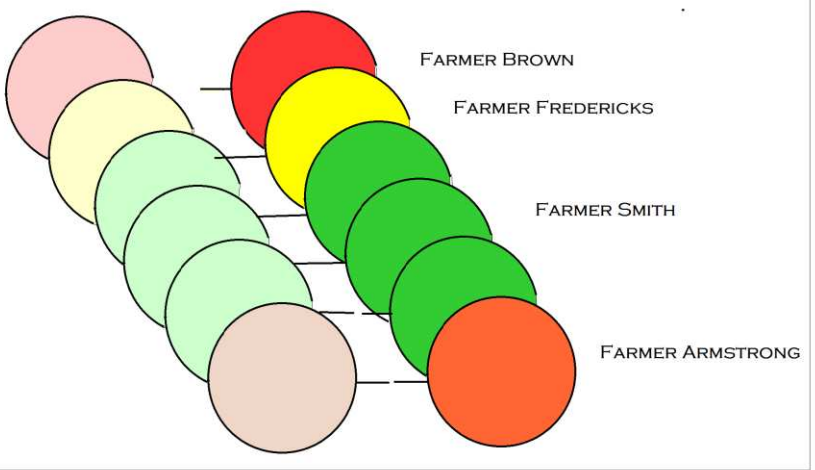
The ability to trade goods and services for currency permits the evaluation of the worth of the trade itself in relative terms visa vi all other trades, however subjective. A trade of $50 might represent an acre of land, a pair of mules, a suit of fine clothes or a suite of furniture. By "mirroring" the value of these various goods (or services), currency permits a much broader extent of trading and trading partners.

The pastel coloration below of the thing traded – money – is available to give a relative value to all the trades of an economy. These "trades" now become "sales," i.e. the surrender of something in return for currency.

The chronology of the trade is given be the difference in color, the red trade being first, the yellow being second, the green third, the orange fourth, etc. The pastel coloration indicates that in this case Farmer Jones did not trade goods for goods but rather money for goods (or services).

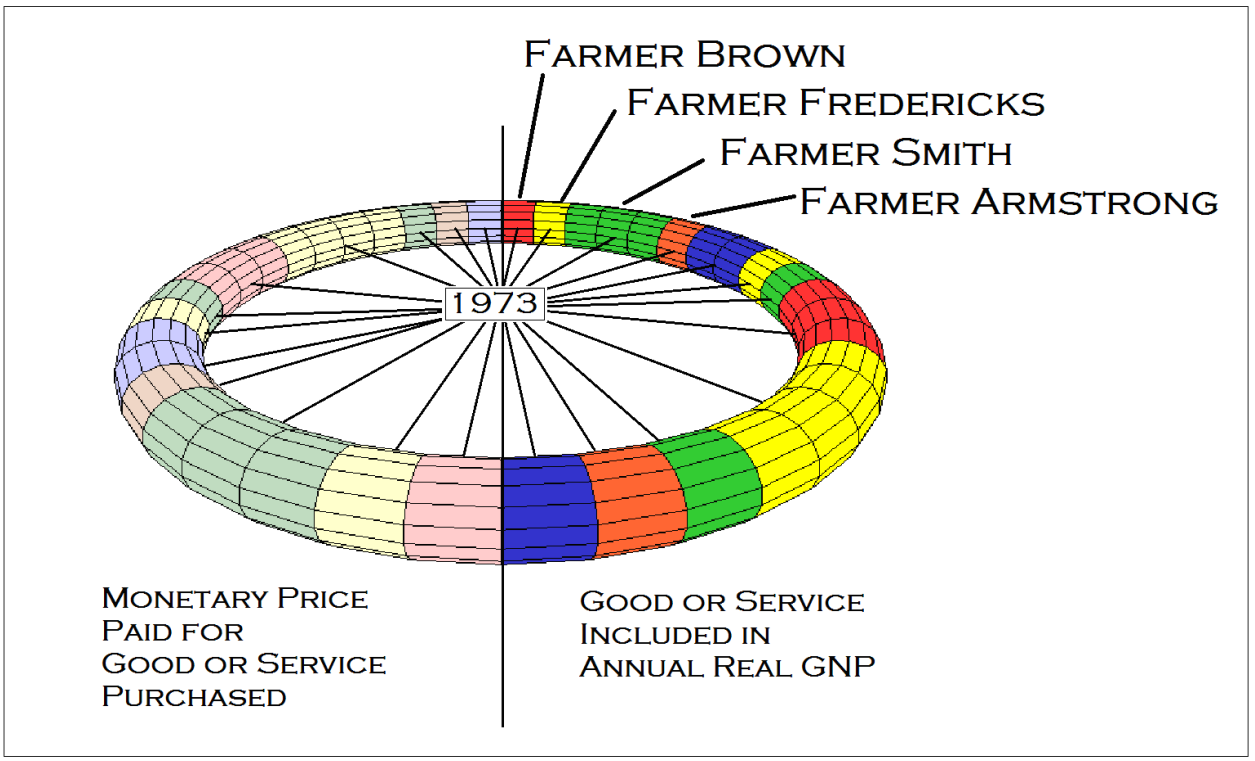
The size of the trade in question, its monetary value, is indicated by the number of circles used. For example Farmer Smith's trade of goods or services for money (three green circles) is three times as valuable in monetary terms as Farmer Brown's trade of goods and serves for money (one red circle), Farmer Frederick's trade of goods or services for money (one yellow circle) and Farmer Armstrong's trade of goods or services for money (one orange circle).

DIAGRAM 2-7.
CHRONOLOGIC SALES OF GOODS AND SERVICES



If we set an arbitrary division of the stream of trade at a single 365-day year, we can place the monetary and the "real" aspects of these sales of goods and services as oppositions from one another. The result is a circle of such sales. The length of half the circle indicates the monetary value of each of the sales of goods or services included in the year. If the size of these transactions is copied into the length of the circuit itself, we have the following. Because the connection of any particular sale of a good or service to the year "1973" is no greater than any other trade, we draw here a circle, i.e. that geometric construct in which all points in a plane lie equidistant from a single point.

DIAGRAM 2-8.
UNITED STATES ANNUAL REAL GNP

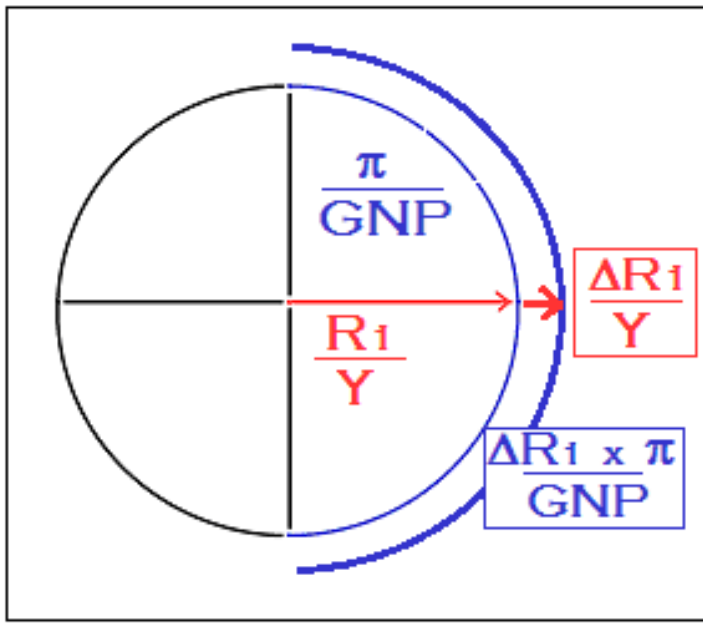


The development of currency and its association with trade given above suggests that the "work" necessary for Farmer Smith or Farmer Jones to possess "trade-able items" has now become the "employment" of Farmer Smith and Farmer Jones as engaged "sales" of these items in a money-based, capitalistic society. In this fashion the use of currency which has turned "trades" into "sales" is in a direct relationship to the rate of employment, i.e. that employment necessary to sustain the full scope of sales given above.

In the above diagram 2-8 we have used the GNP per capita of the United States as a radius "r" of the generating circle and the rate of unemployment as the radius "R" of the generating torus which swings the smaller circle in an arc around the center point "1973." [8]

If this relationship is stated geometrically, it would appear necessary that an increase in the rate of employment from one year to the next (R = the radius of the circle = 1) will correlate geometrically to a necessary increase in the size of GNP (Y = half circumference = π) at the necessary ratio of 1 : π, as follows.

DIAGRAM 2-9.
MODEL OF PRODUCTION VS. EMPLOYMENT



The above diagram is therefore the basis for an understanding of <u>why</u> Okun's Law works. The pi:1 ratio (3.14159:1 ratio) given above between "Percent Change in real GNP" and "Percent Change in the Rate of Unemployment" is a trigonometric outcome of necessary and straightforward social realities within the economic history of the United States.

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[8] The 2010 real GNP for the United States was $2.27 trillion dollars in 1958 dollars with a population in the same year of 308,745,538 residents, for a GNP per capita of $7,355 per resident in 1958 prices. (See Essay Three, Data Set One, for figures as to real GNP. See 2010 Census for population figures.)

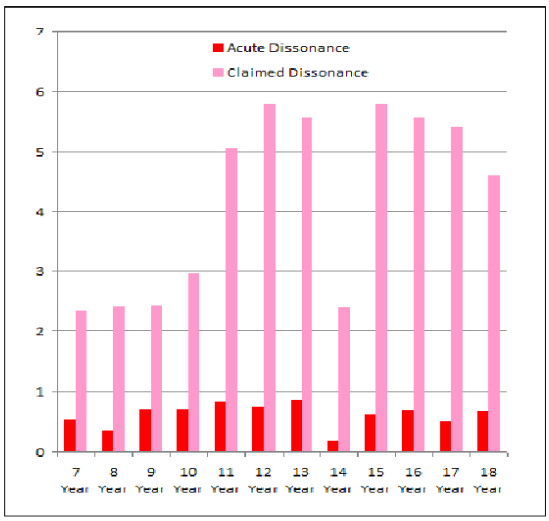
One might picture the relative size of these relationships by noting that if GNP per capita was set as the one inch radius of a pipe and the length of pipe set equal to U.S. real GNP, the pipe would run 406 miles (25,728,794 inches), roughly the distance from Chicago to Kansas City. To bend this pipe into the shape of half a circle would require a radius of 129 miles, roughly the distance from Washington D.C. to Philadelphia.

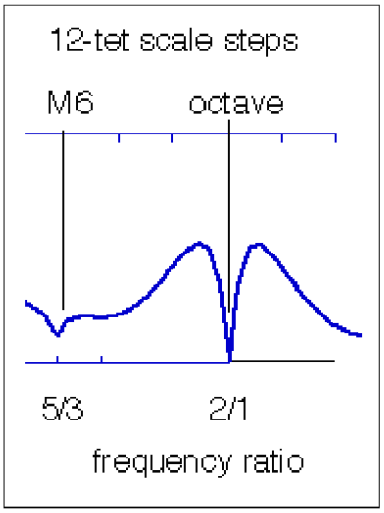
These proportions might be taken on a smaller scale. If a length of string representing 2010 real GNP was set equal to the length of a football field (3600 inches), the equivalent proportional thickness of the string would measure 0.00014 inches in a radial thickness. Spider silk measurements vary from 0.00012 to 0.00032 inches in diameter. The radius would run from the goal line to the 31.8 yard line.

The GNP Spiral

The macro-economic statement of annual GNP takes place in a context of years in sequence. Through the distinct similarity which ratios of U.S. real GNP using various "spreads of years" have with octaves of musical harmony, one may determine "octaves" of mathematic association within the economic data itself, falling at spreads of 14 years. (Albers & Albers, 2013) This is consonant with the onset of reproductive capabilities within the American citizenry; moreover it presents associations of both economics as well as politics.

DIAGRAM 3.11.
"ACUTE DISSONANCE" AND "CLAIMED DISSONANCE" WITH MUSICAL OCTAVE





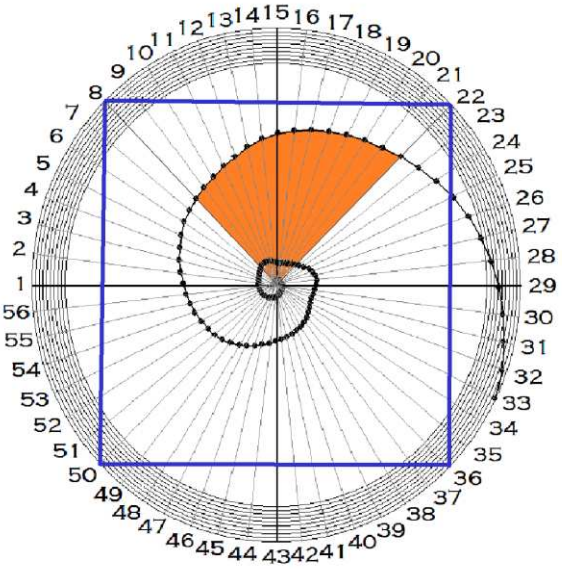
- RED	+ RED		Acute Dissonance	Claimed Dissonance
0	0.539222	7 Year	0.53922225	2.33166
0	0.328323	8 Year	0.32832275	2.39980
0	0.71348	9 Year	0.713480444	2.41405
0	0.702555	10 Year	0.702555	2.95466
-0.82882	0	11 Year	0.828815273	5.05443
-0.07077	0.680828	12 Year	0.751594667	5.77002
-0.07244	0.789312	13 Year	0.861753846	5.55129
-0.01764	0.134157	14 Year	0.151795	2.39229
-0.28313	0.338373	15 Year	0.6214992	5.77038
-0.16258	0.526958	16 Year	0.689539375	5.54327
-0.10785	0.394316	17 Year	0.502165176	5.40061
-0.04362	0.624485	18 Year	0.668103333	4.58002

Using 14 year periods arranged in a square it may be shown that the central quantitative fixture of the economy of the United States is the proportion 1:φ, as demonstrated cursorily in the following diagram. (Albers & Albers, 2011, 2013) The resulting "GNP Spiral" is an exactly 56-year pattern within the political, social and economic history of the United States which correlates generally with the well-known Kondratiev Wave or "Long Wave."

In other words, over the course of 14 years the real GNP of the United States increases on average in a 1 : 1.6180 ratio. This proportion is the famous "Golden Mean" of botanical arrangement, natural selection, pyramid construction, Greek art, Euclidean geometry, Renaissance painting, modular architecture, etc. The biologic, mystical, natural, mathematic, etc. associations, benign and otherwise, brought forward by this unexpected yet quite quantifiable fact are yet to be explored fully.

DIAGRAM 4-7. THE "GNP SPIRAL"

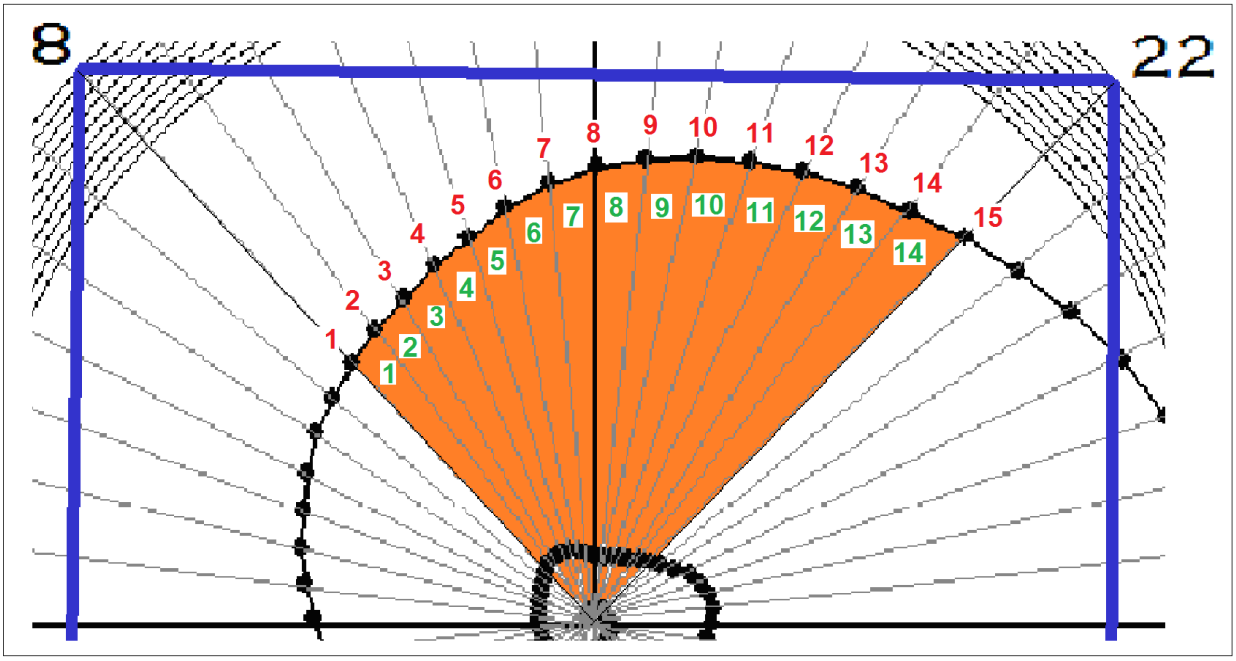
	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015



	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1836	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

Using the above model – "the GNP Spiral" – repetitions of constitutional amendment in the lower left quadrant stand at a 18 liberal : 3 conservative ratio in relation to the upper right quadrant. Moreover the Golden Mean and its association with φ = 1.6180... is stated to within 3.4 parts of 10,000 – and under even more exacting analysis at 5.3 parts of 100,000 – with an explained steady-state rate of growth between 3.496 and 3.499 percent annually.

The above spiral, which mimics the spiral of galaxies and shellfish alike, brings forward numerous questions as to the nature of time in social systems. Here let us note that one of these aspects is that an additional inverse is implied. This inverse suggests that the running of a period of time, like the running of a race, can be looked at from two different but mathematically very complementary points of view.



In so far as the race begins with a starting line and ends with a finish line, the number of lines counted will be one more than the spaces held between the lines. In the above case highlighted in orange we count 15 lines creating 14 spaces. The fourteen spaces themselves contain a specific number days. To begin the count of days we start at the first day, indicating the starting line of the race. It is, however, the second line, not the first, which represents the end of the first year.

Consequently the period of time in orange might be measured as 14/15 (counting the time held within the boundaries). Conversely we may consider the same period as stated at its inverse, 15/14 (counting the number of boundaries holding the time period).

Okun suggests that the unemployment rate is to be taken as a proxy variable for a number of lesser and included features of employment (Okun, 1962: "average hours, labor force participation, and manhour productivity"). If these "included" factors are a form of inverse of the national rate of employment, then these "included" aspects of personal employment must be stated formally in the final calculation and understanding of Okun's Law.

It appears from the data that the inverses 14/15 and 15/14 above represent the personal "race through time" of the American citizenry as they are engaged as members of the national work force. This work force creates U. S real GNP over time through the personal element of the employment rate which was "leapt over" by Okun's approach. This personal aspect of employment is a necessary part of any understanding of *why* Okun's Law works. These inverses of 14/15 and 15/14 are a fundamental part of Okun's Law. In short,

If the GNP Spiral is governed by the "Golden Mean" as associated with the lifespan of American workers, then the 1 : π relationship between employment and GNP must include as well the fractions 14/15 and 15/14 as representing these lifespans.

The Harmonic Inverse

In order to demonstrate the relationship which these different levels of economic activity bear in the consideration of Okun's Law, let us develop more carefully the nature of a mathematic inverse.

Let it first be considered that the positive numbers, $0<x$ may be divided arbitrarily into three groups, which I denominate for the purposes of this essay feminine ($0<F<1$), $1=1$, and masculine ($1<M$). Any $1/x = F$ must and always will have some number $x/1 = M$ by way of a multiplicative inverse, the product of which will be 1.

$$\frac{1}{x} \times \frac{x}{1} = 1$$

Here let the word "Progenic" be introduced, as referring to the product of the above association of feminine and masculine numbers. By "progenic" ("P" as taken from the root word "progeny" signifying "child" or "children") I mean the number which is derived from a member of the feminine numbers and a member of the masculine numbers *as an intended result*, as contrasted with a number which appears in the data through statistical chance.

Second, I propose that a "Harmonic multiplicative inverse" may be created by taking a feminine number and calculating some M as the projenic product, rather than the number "1."

For example, should a "Harmonic multiplicative inverse" be derived for the number ½ about the projenic number π, the algorithm $2/1 \times \pi = 6.28...$, will be the masculine number necessary, as follows:

$$\frac{1}{2} \longrightarrow \pi$$

$$\pi \times \frac{2}{1} = 2\pi$$

$$\frac{1}{2} \times 2\pi = \pi$$

To state clearly: a *proper* multiplicative inverse has as its progenic product the number "1," and a *Harmonic* multiplicative inverse has as its progenic product some number greater than 1, some "P," implying thereby the existence of some masculine father as determined to be always at some multiple greater than x/1.

By way of example, let us consider the simple process whereby a Harmonic multiplicative inverse may be procured for the number 1/46 about the projenic number phi = 1.6180... We would use the following straight-forward calculus:

$$\frac{1}{46} \longrightarrow \varphi$$

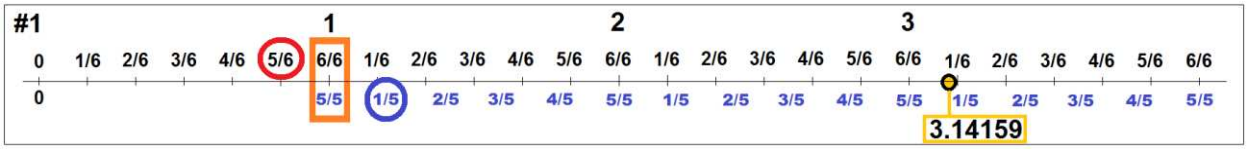
$$\varphi \times \frac{46}{1} = 46\varphi$$

$$\frac{1}{46} \times 46\varphi = \varphi$$

In the same fashion, taking the feminine number "5/6" a proper multiplicative inverse may be created by reversing the numerator and denominator and "6/5" is found to be the proper multiplicative inverse. (see #1, below):

$$\frac{5}{6} \times \frac{6}{5} = 1$$

As this might be placed on a number line, we have:



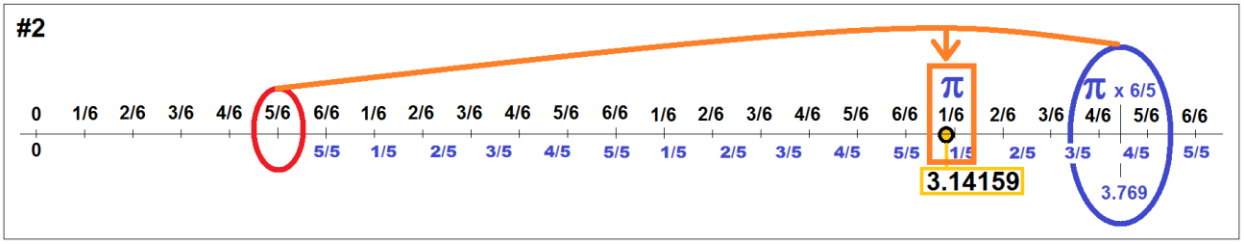
If a Harmonic multiplicative inverse about the progenic number π is intended, then multiplying π x 6/5 yields the following (see #2, below):

$$\frac{5}{6} \longrightarrow \pi$$

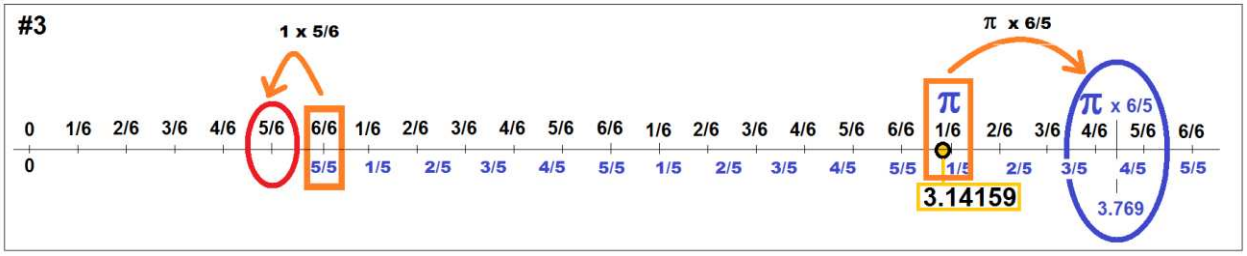
$$\pi \times \frac{6}{5} = \frac{6}{5}\pi$$

$$\frac{5}{6} \times \frac{6}{5}\pi = \pi$$

... or stated in the context of a number line:



Viewed in reverse, the progenic number P = π has been shown to be the progenic product of a Harmonic multiplicative inverse using 5/6 as the feminine number as follows: feminine number = "5/6 x 1," masculine number "6/5 x P." (see #3, below):



The fact that this relationship might be expressed in decimals rather than fractions does not alter the situation in the least. The following example, using decimals, is equivalent for the purposes of this commentary, to wit:

0.8333... x 1.2 = 1

With this discussion of the concept of an inverse, let us consider the data which underlies Dr. Knotek's description of Okun's Law and the tables used.

The following tables indicate the governmental data sets from which are derived these charts and calculations.[9]

[9] For the original data sets used by Professor Knotek to create these graphs, see Appendix 1.

Data Analysis

This first table states the size of GDP as measured quarterly. These numbers form the basis for calculating GDP growth. Annual GDP growth is calculated as 100*((GDP in the fourth quarter of this year)/(GDP in the fourth quarter of last year)-1). Quarterly GDP figures are annualized according to the formula provided by the Bureau of Economic Analysis.[10]

Table 1.1.6. Real Gross Domestic Product, Chained Dollars
[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates
Quarterly data from 1947 To 2007
Bureau of Economic Analysis
Data published September 27, 2007
File created 9/26/2007 9:47:04 AM

		Gross domestic product	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate ((Current Quarter divided by Previous Quarter)^4 minus 1)	Copy of Previous Column (Values Only)	Make Column F a Percent (Multiply by 100)
1947	1	1,570.5				
1947	2	1,568.7	0.999	-0.00458	-0.00458	-0.45767
1947	3	1,568.0	1.000	-0.00178	-0.00178	-0.17837
1947	4	1,590.9	1.015	0.05971	0.05971	5.97106
1948	1	1,616.1	1.016	0.06488	0.06488	6.48818
1948	2	1,644.6	1.018	0.07243	0.07243	7.24282
1948	3	1,654.1	1.006	0.02331	0.02331	2.33069
1948	4	1,658.0	1.002	0.00946	0.00946	0.94645
1949	1	1,633.2	0.985	-0.05850	-0.05850	-5.85020
1949	2	1,628.4	0.997	-0.01170	-0.01170	-1.17043
1949	3	1,646.7	1.011	0.04572	0.04572	4.57156
1949	4	1,629.9	0.990	-0.04019	-0.04019	-4.01886
1950	1	1,696.8	1.041	0.17457	0.17457	17.45697
1950	2	1,747.3	1.030	0.12447	0.12447	12.44685
1950	3	1,815.8	1.039	0.16628	0.16628	16.62782
1950	4	1,848.9	1.018	0.07493	0.07493	7.49336
1951	1	1,871.3	1.012	0.04935	0.04935	4.93491
1951	2	1,903.1	1.017	0.06973	0.06973	6.97265
1951	3	1,941.1	1.020	0.08229	0.08229	8.22939
1951	4	1,944.4	1.002	0.00682	0.00682	0.68176
1952	1	1,964.7	1.010	0.04242	0.04242	4.24195
1952	2	1,966.0	1.001	0.00265	0.00265	0.26493
1952	3	1,978.8	1.007	0.02630	0.02630	2.62982
1952	4	2,043.8	1.033	0.13801	0.13801	13.80097
1953	1	2,082.3	1.019	0.07751	0.07751	7.75058
1953	2	2,098.1	1.008	0.03070	0.03070	3.06982
1953	3	2,085.4	0.994	-0.02399	-0.02399	-2.39934
1953	4	2,052.5	0.984	-0.06163	-0.06163	-6.16277
1954	1	2,042.4	0.995	-0.01954	-0.01954	-1.95385
1954	2	2,044.3	1.001	0.00373	0.00373	0.37263
1954	3	2,066.9	1.011	0.04496	0.04496	4.49592
1954	4	2,107.8	1.020	0.08153	0.08153	8.15329
1955	1	2,168.5	1.029	0.12026	0.12026	12.02633
1955	2	2,204.0	1.016	0.06711	0.06711	6.71087
1955	3	2,233.4	1.013	0.05443	0.05443	5.44347
1955	4	2,245.3	1.005	0.02148	0.02148	2.14837
1956	1	2,234.8	0.995	-0.01857	-0.01857	-1.85749
1956	2	2,252.5	1.008	0.03206	0.03206	3.20591
1956	3	2,249.8	0.999	-0.00479	-0.00479	-0.47861
1956	4	2,286.5	1.016	0.06686	0.06686	6.68643
1957	1	2,300.3	1.006	0.02436	0.02436	2.43611
1957	2	2,294.6	0.998	-0.00987	-0.00987	-0.98750
1957	3	2,317.0	1.010	0.03962	0.03962	3.96237
1957	4	2,292.5	0.989	-0.04163	-0.04163	-4.16299
1958	1	2,230.2	0.973	-0.10435	-0.10435	-10.43510
1958	2	2,243.4	1.006	0.02389	0.02389	2.38860
1958	3	2,295.2	1.023	0.09561	0.09561	9.56082
1958	4	2,348.0	1.023	0.09524	0.09524	9.52424
1959	1	2,392.9	1.019	0.07871	0.07871	7.87128
1959	2	2,455.8	1.026	0.10936	0.10936	10.93633
1959	3	2,453.9	0.999	-0.00309	-0.00309	-0.30911
1959	4	2,462.6	1.004	0.01426	0.01426	1.42571
1960	1	2,517.4	1.022	0.09203	0.09203	9.20271
1960	2	2,504.8	0.995	-0.01987	-0.01987	-1.98708
1960	3	2,508.7	1.002	0.00624	0.00624	0.62426
1960	4	2,476.2	0.987	-0.05082	-0.05082	-5.08214
1961	1	2,491.2	1.006	0.02445	0.02445	2.44517
1961	2	2,538.0	1.019	0.07729	0.07729	7.72887
1961	3	2,579.1	1.016	0.06637	0.06637	6.63659
1961	4	2,631.8	1.020	0.08427	0.08427	8.42734
1962	1	2,679.1	1.018	0.07385	0.07385	7.38513
1962	2	2,708.4	1.011	0.04447	0.04447	4.44689
1962	3	2,733.3	1.009	0.03728	0.03728	3.72847
1962	4	2,740.0	1.002	0.00984	0.00984	0.98411
1963	1	2,775.9	1.013	0.05345	0.05345	5.34478
1963	2	2,810.6	1.013	0.05095	0.05095	5.09472
1963	3	2,863.5	1.019	0.07744	0.07744	7.74387
1963	4	2,885.8	1.008	0.03152	0.03152	3.15165
1964	1	2,950.5	1.022	0.09274	0.09274	9.27418
1964	2	2,984.8	1.012	0.04732	0.04732	4.73178
1964	3	3,025.5	1.014	0.05567	0.05567	5.56688
1964	4	3,033.6	1.003	0.01075	0.01075	1.07521
1965	1	3,108.2	1.025	0.10205	0.10205	10.20532
1965	2	3,150.2	1.014	0.05516	0.05516	5.51560
1965	3	3,214.1	1.020	0.08364	0.08364	8.36400
1965	4	3,291.8	1.024	0.10026	0.10026	10.02623
1966	1	3,372.3	1.024	0.10147	0.10147	10.14659
1966	2	3,384.0	1.003	0.01395	0.01395	1.39502
1966	3	3,406.3	1.007	0.02662	0.02662	2.66210
1966	4	3,433.7	1.008	0.03257	0.03257	3.25660
1967	1	3,464.1	1.009	0.03589	0.03589	3.58868
1967	2	3,464.3	1.000	0.00023	0.00023	0.02310
1967	3	3,491.8	1.008	0.03213	0.03213	3.21325
1967	4	3,518.2	1.008	0.03059	0.03059	3.05870
1968	1	3,590.7	1.021	0.08501	0.08501	8.50116
1968	2	3,651.6	1.017	0.06959	0.06959	6.95875
1968	3	3,676.5	1.007	0.02756	0.02756	2.75560
1968	4	3,692.0	1.004	0.01697	0.01697	1.69708
1969	1	3,750.2	1.016	0.06456	0.06456	6.45620
1969	2	3,760.9	1.003	0.01146	0.01146	1.14617
1969	3	3,784.2	1.006	0.02501	0.02501	2.50125
1969	4	3,766.3	0.995	-0.01879	-0.01879	-1.87870
1970	1	3,760.0	0.998	-0.00667	-0.00667	-0.66741
1970	2	3,767.1	1.002	0.00757	0.00757	0.75746
1970	3	3,800.5	1.009	0.03594	0.03594	3.59394
1970	4	3,759.8	0.989	-0.04215	-0.04215	-4.21533
1971	1	3,864.1	1.028	0.11567	0.11567	11.56667
1971	2	3,885.9	1.006	0.02276	0.02276	2.27584
1971	3	3,916.7	1.008	0.03208	0.03208	3.20833
1971	4	3,927.9	1.003	0.01149	0.01149	1.14874
1972	1	3,997.7	1.018	0.07300	0.07300	7.29985
1972	2	4,092.1	1.024	0.09785	0.09785	9.78529
1972	3	4,131.1	1.010	0.03867	0.03867	3.86707
1972	4	4,198.7	1.016	0.06708	0.06708	6.70789
1973	1	4,305.3	1.025	0.10549	0.10549	10.54887
1973	2	4,355.1	1.012	0.04708	0.04708	4.70776
1973	3	4,331.9	0.995	-0.02114	-0.02114	-2.11387
1973	4	4,373.3	1.010	0.03878	0.03878	3.87795
1974	1	4,335.4	0.991	-0.03422	-0.03422	-3.42169
1974	2	4,347.9	1.003	0.01158	0.01158	1.15829
1974	3	4,305.8	0.990	-0.03817	-0.03817	-3.81724
1974	4	4,288.9	0.996	-0.01561	-0.01561	-1.56076
1975	1	4,237.6	0.988	-0.04699	-0.04699	-4.69929
1975	2	4,268.6	1.007	0.02958	0.02958	2.95845
1975	3	4,340.9	1.017	0.06949	0.06949	6.94914
1975	4	4,397.8	1.013	0.05347	0.05347	5.34715
1976	1	4,496.8	1.023	0.09313	0.09313	9.31315
1976	2	4,530.3	1.007	0.03013	0.03013	3.01336
1976	3	4,552.0	1.005	0.01930	0.01930	1.92980
1976	4	4,584.6	1.007	0.02896	0.02896	2.89560
1977	1	4,640.0	1.012	0.04922	0.04922	4.92189
1977	2	4,731.1	1.020	0.08088	0.08088	8.08778
1977	3	4,815.8	1.018	0.07356	0.07356	7.35574
1977	4	4,815.3	1.000	-0.00042	-0.00042	-0.04152
1978	1	4,830.8	1.003	0.01294	0.01294	1.29379
1978	2	5,021.2	1.039	0.16722	0.16722	16.72230
1978	3	5,070.7	1.010	0.04002	0.04002	4.00198
1978	4	5,137.4	1.013	0.05366	0.05366	5.36633
1979	1	5,147.4	1.002	0.00781	0.00781	0.78088
1979	2	5,152.3	1.001	0.00381	0.00381	0.38132
1979	3	5,189.4	1.007	0.02912	0.02912	2.91153
1979	4	5,204.7	1.003	0.01185	0.01185	1.18455
1980	1	5,221.3	1.003	0.01282	0.01282	1.28189
1980	2	5,115.9	0.980	-0.07833	-0.07833	-7.83339
1980	3	5,107.4	0.998	-0.00663	-0.00663	-0.66294
1980	4	5,202.1	1.019	0.07626	0.07626	7.62553
1981	1	5,307.5	1.020	0.08354	0.08354	8.35407
1981	2	5,266.1	0.992	-0.03084	-0.03084	-3.08380
1981	3	5,329.8	1.012	0.04927	0.04927	4.92700
1981	4	5,263.4	0.988	-0.04891	-0.04891	-4.89095
1982	1	5,177.1	0.984	-0.06399	-0.06399	-6.39895
1982	2	5,204.9	1.005	0.02165	0.02165	2.16528
1982	3	5,185.2	0.996	-0.01505	-0.01505	-1.50538
1982	4	5,189.8	1.001	0.00355	0.00355	0.35533
1983	1	5,253.8	1.012	0.05025	0.05025	5.02475
1983	2	5,372.3	1.023	0.09332	0.09332	9.33190
1983	3	5,478.4	1.020	0.08137	0.08137	8.13690
1983	4	5,590.5	1.020	0.08440	0.08440	8.43954
1984	1	5,699.8	1.020	0.08053	0.08053	8.05276
1984	2	5,797.9	1.017	0.07064	0.07064	7.06423
1984	3	5,854.3	1.010	0.03948	0.03948	3.94821
1984	4	5,902.4	1.008	0.03327	0.03327	3.32720
1985	1	5,956.9	1.009	0.03745	0.03745	3.74488
1985	2	6,007.8	1.009	0.03462	0.03462	3.46194
1985	3	6,101.7	1.016	0.06400	0.06400	6.39998
1985	4	6,148.6	1.008	0.03110	0.03110	3.11018
1986	1	6,207.4	1.010	0.03880	0.03880	3.88048
1986	2	6,232.0	1.004	0.01595	0.01595	1.59465
1986	3	6,291.7	1.010	0.03887	0.03887	3.88725
1986	4	6,323.4	1.005	0.02031	0.02031	2.03064
1987	1	6,365.0	1.007	0.02658	0.02658	2.65758
1987	2	6,435.0	1.011	0.04472	0.04472	4.47216
1987	3	6,493.4	1.009	0.03680	0.03680	3.67986
1987	4	6,606.8	1.017	0.07171	0.07171	7.17069
1988	1	6,639.1	1.005	0.01970	0.01970	1.96995
1988	2	6,723.5	1.013	0.05183	0.05183	5.18282
1988	3	6,759.4	1.005	0.02153	0.02153	2.15296
1988	4	6,848.6	1.013	0.05384	0.05384	5.38398
1989	1	6,918.1	1.010	0.04121	0.04121	4.12143
1989	2	6,963.5	1.007	0.02651	0.02651	2.65095
1989	3	7,013.1	1.007	0.02880	0.02880	2.87973
1989	4	7,030.9	1.003	0.01019	0.01019	1.01911
1990	1	7,112.1	1.012	0.04700	0.04700	4.70025
1990	2	7,130.3	1.003	0.01028	0.01028	1.02754
1990	3	7,130.8	1.000	0.00028	0.00028	0.02805
1990	4	7,076.9	0.992	-0.02989	-0.02989	-2.98940
1991	1	7,040.8	0.995	-0.02025	-0.02025	-2.02488
1991	2	7,086.5	1.006	0.02622	0.02622	2.62168
1991	3	7,120.7	1.005	0.01944	0.01944	1.94445
1991	4	7,154.1	1.005	0.01889	0.01889	1.88946
1992	1	7,228.2	1.010	0.04208	0.04208	4.20789
1992	2	7,297.9	1.010	0.03913	0.03913	3.91326
1992	3	7,369.5	1.010	0.03983	0.03983	3.98255
1992	4	7,450.7	1.011	0.04481	0.04481	4.48073
1994	1	7,715.1	1.010	0.04132	0.04132	4.13197
1994	2	7,815.7	1.013	0.05319	0.05319	5.31865
1994	3	7,859.5	1.006	0.02261	0.02261	2.26056
1994	4	7,951.6	1.012	0.04770	0.04770	4.77036
1995	1	7,973.7	1.003	0.01116	0.01116	1.11637
1995	2	7,988.0	1.002	0.00719	0.00719	0.71929
1995	3	8,053.1	1.008	0.03300	0.03300	3.29996
1995	4	8,112.0	1.007	0.02958	0.02958	2.95783
1996	1	8,169.2	1.007	0.02850	0.02850	2.85049
1996	2	8,303.1	1.016	0.06719	0.06719	6.71930
1996	3	8,372.7	1.008	0.03395	0.03395	3.39536
1996	4	8,470.6	1.012	0.04760	0.04760	4.75978
1997	1	8,536.1	1.008	0.03129	0.03129	3.12911
1997	2	8,665.8	1.015	0.06218	0.06218	6.21765
1997	3	8,773.7	1.012	0.05074	0.05074	5.07429
1997	4	8,838.4	1.007	0.02983	0.02983	2.98251
1998	1	8,936.2	1.011	0.04500	0.04500	4.50015
1998	2	8,995.3	1.007	0.02672	0.02672	2.67178
1998	3	9,098.9	1.012	0.04687	0.04687	4.68705
1998	4	9,237.1	1.015	0.06215	0.06215	6.21528
1999	1	9,315.5	1.008	0.03438	0.03438	3.43847
1999	2	9,392.6	1.008	0.03352	0.03352	3.35194
1999	3	9,502.2	1.012	0.04750	0.04750	4.74984
1999	4	9,671.1	1.018	0.07302	0.07302	7.30176
2000	1	9,695.6	1.003	0.01017	0.01017	1.01719
2000	2	9,847.9	1.016	0.06433	0.06433	6.43287
2000	3	9,836.6	0.999	-0.00458	-0.00458	-0.45819
2000	4	9,887.7	1.005	0.02094	0.02094	2.09420
2001	1	9,875.6	0.999	-0.00489	-0.00489	-0.48860
2001	2	9,905.9	1.003	0.01233	0.01233	1.23293
2001	3	9,871.1	0.996	-0.01398	-0.01398	-1.39784
2001	4	9,910.0	1.004	0.01586	0.01586	1.58566
2002	1	9,977.3	1.007	0.02744	0.02744	2.74425
2002	2	10,031.6	1.005	0.02195	0.02195	2.19478
2002	3	10,090.7	1.006	0.02377	0.02377	2.37746
2002	4	10,095.8	1.001	0.00202	0.00202	0.20232
2003	1	10,126.0	1.003	0.01202	0.01202	1.20192
2003	2	10,212.7	1.009	0.03469	0.03469	3.46908
2003	3	10,398.7	1.018	0.07486	0.07486	7.48649
2003	4	10,467.0	1.007	0.02653	0.02653	2.65325
2004	1	10,543.6	1.007	0.02960	0.02960	2.95959
2004	2	10,634.2	1.009	0.03482	0.03482	3.48171
2004	3	10,728.7	1.009	0.03602	0.03602	3.60223
2004	4	10,796.4	1.006	0.02548	0.02548	2.54806
2005	1	10,878.4	1.008	0.03073	0.03073	3.07284
2005	2	10,954.1	1.007	0.02813	0.02813	2.81269
2005	3	11,074.3	1.011	0.04462	0.04462	4.46200
2005	4	11,107.2	1.003	0.01194	0.01194	1.19364
2006	1	11,238.7	1.012	0.04820	0.04820	4.82043
2006	2	11,306.7	1.006	0.02442	0.02442	2.44226
2006	3	11,336.7	1.003	0.01066	0.01066	1.06555
2006	4	11,395.5	1.005	0.02091	0.02091	2.09087
2007	1	11,412.6	1.002	0.00602	0.00602	0.60159
2007	2	11,520.1	1.009	0.03821	0.03821	3.82134

It is important to mention at the outset that the character of GDP is quite different from the character of employment. Unlike employment, the measure of GDP begins with the fact that, like any object which grows, it has size. Sharing a commonality with the size of a dog, a flower or a tree, the measurement of GDP above is intended to give an estimate of the size of the economy as an objective entity.

This is important to mention because the *growth* of GDP is considered from two standpoints. The first is growth over the course of a year (Annual). The second, relating to quarterly GDP, is figured from a mathematic algorithm. In this algorithm (1) the current quarter is divided by the previous quarter, (2) this is then taken to the fourth power, (3) from this figure we subtract one and (4) make this number a percent by multiplying by 100.

The purpose of these procedures is to find the rate of growth of an object. Using estimates of the growth rate over quarters, which are four times as numerous as annual estimates,

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10 See email of October 10, 2011.

we might expect that these repeated quarterly annual-izations render a much more precise value than is possible for annual data. This is quite unlike the nature of a measurement of quarterly an

The next table below states the employment rate in months. For annual data, the change in the unemployment rate is the current December minus the previous December. For quarterly data, the change in the unemployment rate is the difference between subsequent quarterly averages.[11]

Monthly Unemployment, Bureau of Labor Statistics
Labor Force Statistics from the Current Population Survey

Series Id: LNS14000000
Seasonally Adjusted
Series title: (Seas) Unemployment Rate
Labor force status: Unemployment rate
Type of data: Percent or rate
Age: 16 years and over

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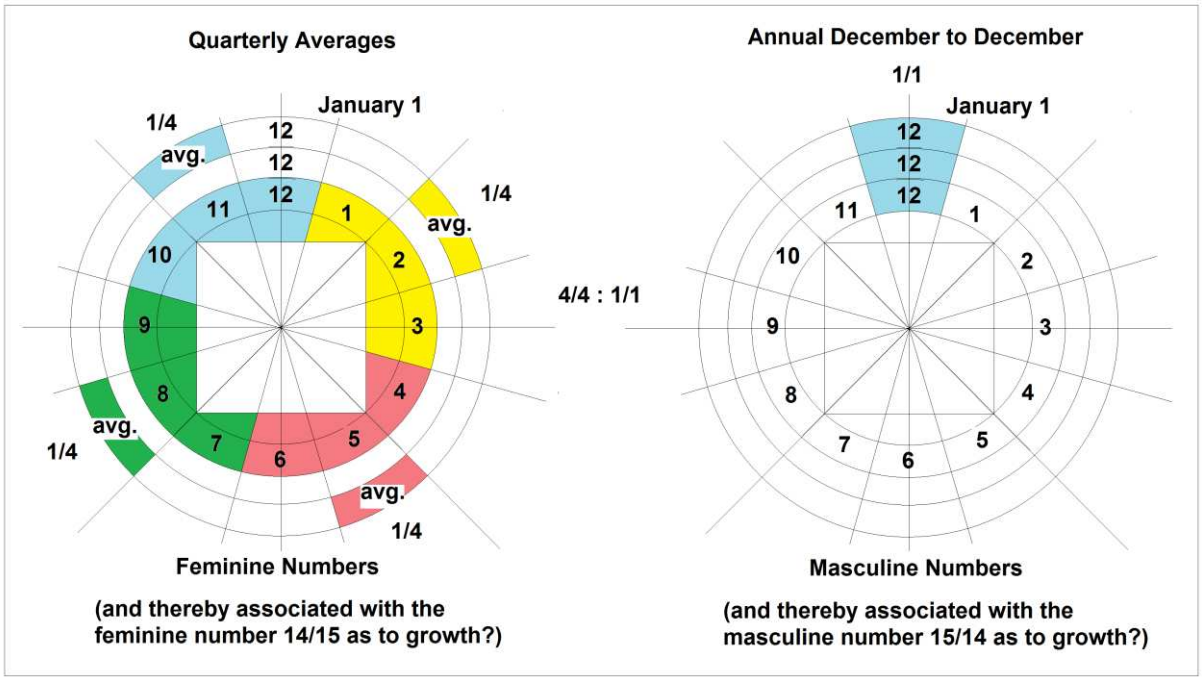
Year	Jan	Feb	Mar	1st Quarter Average (example: "1948.1")	Apr	May	Jun	2nd Quarter Average (example: "1948.2")	Jul	Aug	Sep	3rd Quarter Average (example: "1948.3")	Oct	Nov	Dec	4th Quarter Average (example: "1948.4")	Annual
1948	3.4	3.8	4	3.733333333	3.9	3.5	3.6	3.666666667	3.6	3.9	3.8	3.766666667	3.7	3.8	4	3.833333333	
1949	4.3	4.7	5	4.666666667	5.3	6.1	6.2	5.866666667	6.7	6.8	6.6	6.7	7.9	6.4	6.6	6.966666667	
1950	6.5	6.4	6.3	6.4	5.8	5.5	5.4	5.566666667	5	4.5	4.4	4.633333333	4.2	4.2	4.3	4.233333333	
1951	3.7	3.4	3.4	3.5	3.1	3	3.2	3.1	3.1	3.1	3.3	3.166666667	3.5	3.5	3.1	3.366666667	
1952	3.2	3.1	2.9	3.066666667	2.9	3	3	2.966666667	3.2	3.4	3.1	3.233333333	3	2.8	2.7	2.833333333	
1953	2.9	2.6	2.6	2.7	2.7	2.5	2.5	2.566666667	2.6	2.7	2.9	2.733333333	3.1	3.5	4.5	3.7	
1954	4.9	5.2	5.7	5.266666667	5.9	5.9	5.6	5.8	5.8	6	6.1	5.966666667	5.7	5.3	5	5.333333333	
1955	4.9	4.7	4.6	4.733333333	4.7	4.3	4.2	4.4	4	4.2	4.1	4.1	4.3	4.2	4.2	4.233333333	
1956	4	3.9	4.2	4.033333333	4	4.3	4.3	4.2	4.4	4.1	3.9	4.133333333	3.9	4.3	4.2	4.133333333	
1957	4.2	3.9	3.7	3.933333333	3.9	4.1	4.3	4.1	4.2	4.1	4.4	4.233333333	4.5	5.1	5.2	4.933333333	
1958	5.8	6.4	6.7	6.3	7.4	7.4	7.3	7.366666667	7.5	7.4	7.1	7.333333333	6.7	6.2	6.2	6.366666667	
1959	6	5.9	5.6	5.833333333	5.2	5.1	5	5.1	5.1	5.2	5.5	5.266666667	5.7	5.8	5.3	5.6	
1960	5.2	4.8	5.4	5.133333333	5.2	5.1	5.4	5.233333333	5.5	5.6	5.5	5.533333333	6.1	6.1	6.6	6.266666667	
1961	6.6	6.9	6.9	6.8	7	7.1	6.9	7	7	6.6	6.7	6.766666667	6.5	6.1	6	6.2	
1962	5.8	5.5	5.6	5.633333333	5.6	5.5	5.5	5.533333333	5.4	5.7	5.6	5.566666667	5.4	5.7	5.5	5.533333333	
1963	5.7	5.9	5.7	5.766666667	5.7	5.9	5.6	5.733333333	5.6	5.4	5.5	5.5	5.5	5.7	5.5	5.566666667	
1964	5.6	5.4	5.4	5.466666667	5.3	5.1	5.2	5.2	4.9	5	5.1	5	5.1	4.8	5	4.966666667	
1965	4.9	5.1	4.7	4.9	4.8	4.6	4.6	4.666666667	4.4	4.4	4.3	4.366666667	4.2	4.1	4	4.1	
1966	4	3.8	3.8	3.866666667	3.8	3.9	3.8	3.833333333	3.8	3.8	3.7	3.766666667	3.7	3.6	3.8	3.7	
1967	3.9	3.8	3.8	3.833333333	3.8	3.8	3.9	3.833333333	3.8	3.8	3.8	3.8	4	3.9	3.8	3.9	
1968	3.7	3.8	3.7	3.733333333	3.5	3.5	3.7	3.566666667	3.7	3.5	3.4	3.533333333	3.4	3.4	3.4	3.4	
1969	3.4	3.4	3.4	3.4	3.4	3.4	3.5	3.433333333	3.5	3.5	3.7	3.566666667	3.7	3.5	3.5	3.566666667	
1970	3.9	4.2	4.4	4.166666667	4.6	4.8	4.9	4.766666667	5	5.1	5.4	5.166666667	5.5	5.9	6.1	5.833333333	
1971	5.9	5.9	6	5.933333333	5.9	5.9	5.9	5.9	6	6.1	6	6.033333333	5.8	6	6	5.933333333	
1972	5.8	5.7	5.8	5.766666667	5.7	5.7	5.7	5.7	5.6	5.6	5.5	5.566666667	5.6	5.3	5.2	5.366666667	
1973	4.9	5	4.9	4.933333333	5	4.9	4.9	4.933333333	4.8	4.8	4.8	4.8	4.6	4.8	4.9	4.766666667	
1974	5.1	5.2	5.1	5.133333333	5.1	5.1	5.4	5.2	5.5	5.5	5.9	5.633333333	6	6.6	7.2	6.6	
1975	8.1	8.1	8.6	8.266666667	8.8	9	8.8	8.866666667	8.6	8.4	8.4	8.466666667	8.4	8.3	8.2	8.3	
1976	7.9	7.7	7.6	7.733333333	7.7	7.4	7.6	7.566666667	7.8	7.8	7.6	7.733333333	7.7	7.8	7.8	7.766666667	
1977	7.5	7.6	7.4	7.5	7.2	7	7.2	7.133333333	6.9	7	6.8	6.9	6.8	6.8	6.4	6.666666667	
1978	6.4	6.3	6.3	6.333333333	6.1	6	5.9	6	6.2	5.9	6	6.033333333	5.8	5.9	6	5.9	
1979	5.9	5.9	5.8	5.866666667	5.8	5.6	5.7	5.7	5.7	6	5.9	5.866666667	6	5.9	6	5.966666667	
1980	6.3	6.3	6.3	6.3	6.9	7.5	7.6	7.333333333	7.8	7.7	7.5	7.666666667	7.5	7.5	7.2	7.4	
1981	7.5	7.4	7.4	7.433333333	7.2	7.5	7.5	7.4	7.2	7.4	7.6	7.4	7.9	8.3	8.5	8.233333333	
1982	8.6	8.9	9	8.833333333	9.3	9.4	9.6	9.433333333	9.8	9.8	10.1	9.9	10.4	10.8	10.8	10.66666667	
1983	10.4	10.4	10.3	10.36666667	10.2	10.1	10.1	10.13333333	9.4	9.5	9.2	9.366666667	8.8	8.5	8.3	8.533333333	
1984	8	7.8	7.8	7.866666667	7.7	7.4	7.2	7.433333333	7.5	7.5	7.3	7.433333333	7.4	7.2	7.3	7.3	
1985	7.3	7.2	7.2	7.233333333	7.3	7.2	7.4	7.3	7.4	7.1	7.1	7.2	7.1	7	7	7.033333333	
1986	6.7	7.2	7.2	7.033333333	7.1	7.2	7.2	7.166666667	7	6.9	7	6.966666667	7	6.9	6.6	6.833333333	
1987	6.6	6.6	6.6	6.6	6.3	6.3	6.2	6.266666667	6.1	6	5.9	6	6	5.8	5.7	5.833333333	
1988	5.7	5.7	5.7	5.7	5.4	5.6	5.4	5.466666667	5.4	5.6	5.4	5.466666667	5.4	5.3	5.3	5.333333333	
1989	5.4	5.2	5	5.2	5.2	5.2	5.3	5.233333333	5.2	5.2	5.3	5.233333333	5.3	5.4	5.4	5.366666667	
1990	5.4	5.3	5.2	5.3	5.4	5.4	5.2	5.333333333	5.5	5.7	5.9	5.7	5.9	6.2	6.3	6.133333333	
1991	6.4	6.6	6.8	6.6	6.7	6.9	6.9	6.833333333	6.8	6.9	6.9	6.866666667	7	7	7.3	7.1	
1992	7.3	7.4	7.4	7.366666667	7.4	7.6	7.8	7.6	7.7	7.6	7.6	7.633333333	7.3	7.4	7.4	7.366666667	
1993	7.3	7.1	7	7.133333333	7.1	7.1	7	7.066666667	6.9	6.8	6.7	6.8	6.8	6.6	6.5	6.633333333	
1994	6.6	6.6	6.5	6.566666667	6.4	6.1	6.1	6.2	6.1	6	5.9	6	5.8	5.6	5.5	5.633333333	
1995	5.6	5.4	5.4	5.466666667	5.8	5.6	5.6	5.666666667	5.7	5.7	5.6	5.666666667	5.5	5.6	5.6	5.566666667	
1996	5.6	5.5	5.5	5.533333333	5.6	5.6	5.3	5.5	5.5	5.1	5.2	5.266666667	5.2	5.4	5.4	5.333333333	
1997	5.3	5.2	5.2	5.233333333	5.1	4.9	5	5	4.9	4.8	4.9	4.866666667	4.7	4.6	4.7	4.666666667	
1998	4.6	4.6	4.7	4.633333333	4.3	4.4	4.5	4.4	4.5	4.5	4.6	4.533333333	4.5	4.4	4.4	4.433333333	
1999	4.3	4.4	4.2	4.3	4.3	4.2	4.3	4.266666667	4.3	4.2	4.2	4.233333333	4.1	4.1	4	4.066666667	
2000	4	4.1	4	4.033333333	3.8	4	4	3.933333333	4	4.1	3.9	4	3.9	3.9	3.9	3.9	
2001	4.2	4.2	4.3	4.233333333	4.4	4.3	4.5	4.4	4.6	4.9	5	4.833333333	5.3	5.5	5.7	5.5	
2002	5.7	5.7	5.7	5.7	5.9	5.8	5.8	5.833333333	5.8	5.7	5.7	5.733333333	5.7	5.9	6	5.866666667	
2003	5.8	5.9	5.9	5.866666667	6	6.1	6.3	6.133333333	6.2	6.1	6.1	6.133333333	6	5.8	5.7	5.833333333	
2004	5.7	5.6	5.8	5.7	5.6	5.6	5.6	5.6	5.5	5.4	5.4	5.433333333	5.5	5.4	5.4	5.433333333	
2005	5.3	5.4	5.2	5.3	5.2	5.1	5	5.1	5	4.9	5	4.966666667	5	5	4.9	4.966666667	
2006	4.7	4.8	4.7	4.733333333	4.7	4.6	4.6	4.633333333	4.7	4.7	4.5	4.633333333	4.4	4.5	4.4	4.433333333	
2007	4.6	4.5	4.4	4.5	4.5	4.4	4.6	4.5	4.7	4.6	4.7	4.666666667	4.7	4.7	5	4.8	
2008	5	4.8	5.1	4.966666667	4.9	5.4	5.6	5.3	5.8	6.1	6.2	6.033333333	6.6	6.8	7.3	6.9	
2009	7.8	8.2	8.6	8.2	8.9	9.4	9.5	9.266666667	9.5	9.7	9.8	9.666666667	10.1	9.9	9.9	9.966666667	
2010	9.7	9.7	9.7	9.7	9.8	9.6	9.5	9.633333333	9.5	9.6	9.6	9.566666667	9.7	9.8	9.4	9.633333333	
2011	9	8.9	8.8	8.9	9	9.1	9.2	9.1	9.1	9.1	9.1	9.1					

[11] See email of October 10, 2011.

The above chart is of specific interest as it relates to the calculation of quantities of monthly unemployment in both their "feminine" and "masculine" characteristics, or put another way, their "circumferential" and "radial" characteristics.

To make the distinction plain, let us imagine that the march of months within a year was made congruent to the 12 hours on the face of the clock. The manner in which the data for unemployment is collected and analyzed against itself partakes of the circular nature of a unit circle. In this way the average of each quarter is taken and compared with the average of each other quarter. This is portrayed in the left hand side of the following chart.

Contrast this with a single month, chosen from the twelve, and it alone being contrasted with the same month of the following year, and then the following year, and so on.



On the left we have a circumferential relationship between quarterly data which itself relies upon a circular sense of time, a legitimate apportion-izing of something which itself is taken as a "1." On the right we have a distinctly different and radial view of time, one which does not accept any obvious limitation to its ongoing list of endless Decembers.[12]

Note that the estimation of a "quarterly" rate for unemployment takes as its beginning source of numeric encouragement the idea that it is 1/4th of something else, specifically a sub-part of a 12-month, four-quarter year. If we were to have a full year specified in quarters then numerically we would be interested in a year stated as 4/4 which, according to number theory, would equal a single year.

[1212] It must be noted, however, that the GNP Spiral assumes a further circular aspect of time applying even to annual data. Consequently the 14/15 association of feminine numbers in this regard, and the 15/14 association with masculine numbers remains a connected aspect of this insight. In short, if the annual data itself falls into a larger circumferential relationship, what relationship might this have to the quarterly data which are, at best, a sub-part of the GNP Spiral and its 1:φ ratio over a span of 56 years?

Conversely the statement of an "annual" rate of unemployment seeks not an association between the data and the year itself, but rather to an *on-going set of years in sequence*. Consequently the rate of one December is compared to the rate of the next December and measured. In contrast to the quarterly data – which by definition is part of some other wholeness – we might state annual data as a repeated sense of "1," each point repeating itself in endless time, a 1/1.

Here we enter into the intrigues and quiet thoughts of the numbers themselves. Placing both feminine and masculine numbers together we see above a hinted "radius : 2π" relationship between annual and quarterly approaches using a single data set describing unemployment and a second single data set describing GDP growth. Three questions arise.

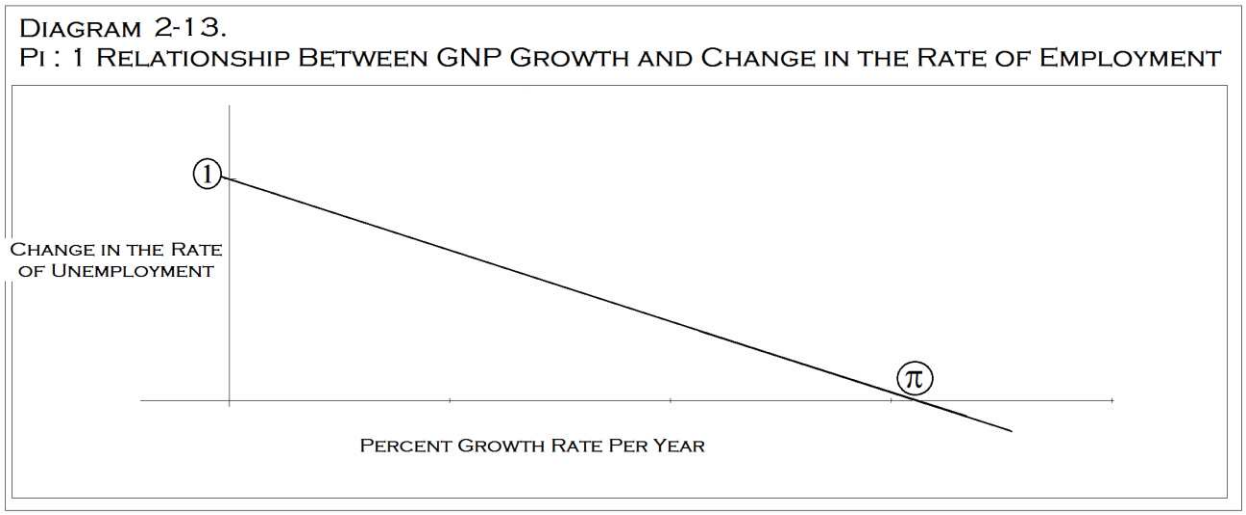
1. Do the feminine ($0<F<1$) numbers maintain a secret relationship with the quarterly employment figures, their circumferential sense of time and the fraction 14/15 as these relate to the GNP Spiral / Kondratiev Wave, perhaps "filling up" the space between moments of time?

2. Do the masculine ($1<M$) numbers share an equally hidden relationship with annual employment figures, their radial sense of time and the fraction 15/14 as these relate to the GNP Spiral / Kondratiev Wave, perhaps setting up "boundaries" separating moments of time?

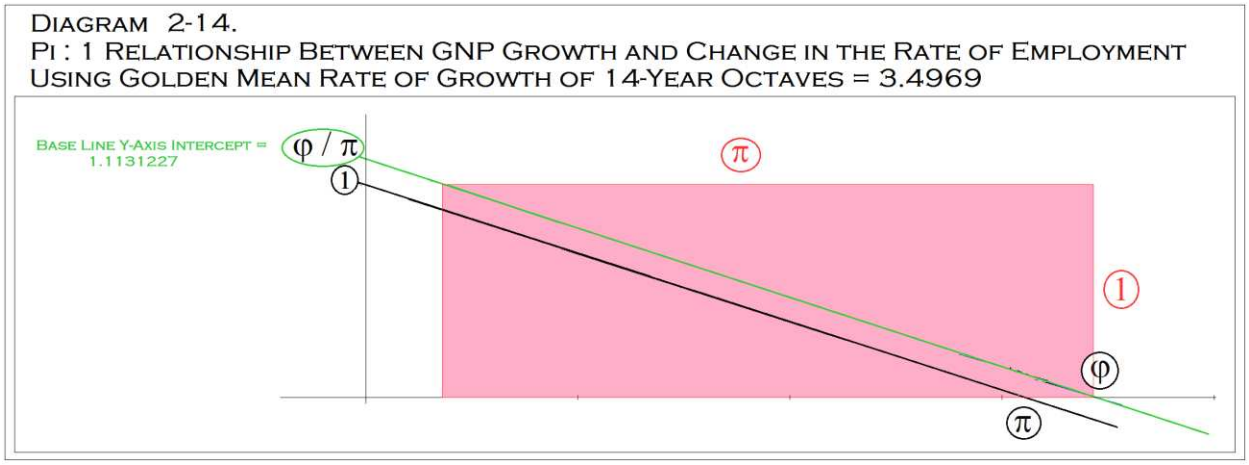
3. Under what circumstances might these secrets be revealed, secrets which although hidden, tentative and circumspect, might actually bear an inverse relationship of some sort to one another?

"The Harmonic Multiplicative Inverse Surprise"

The relationship which we are anticipating is that a 1: π relationship will exist between a percentage change in the rate of unemployment and the percentage growth of GNP. As the rate of growth increases on the x-axis, the rate of unemployment will go down on the y-axis. Setting this relationship as a straight-forward linear relationship, we have the following.



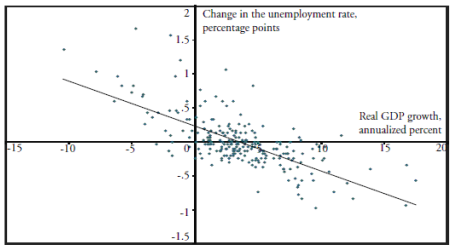
In order to establish a 1 : φ proportion over fourteen years the economy of the United States must possess a steady state rate of growth of approximately 3.4969% per year. As one calculates a 1 : π exchange between rates of unemployment and GDP growth under Okun's Law, one notices that the slope of the 1 : π relationship must remain the same, but that the y-intercept shifts slightly upwards, becoming not "1" but 3.4969 / π = 1.1131227.



Comparing this to the observed data calculated by Dr. Knotek, one notices that Chart One uses quarterly *growth data* which has been annualized. However quarterly *employment data* is *not* annualized.

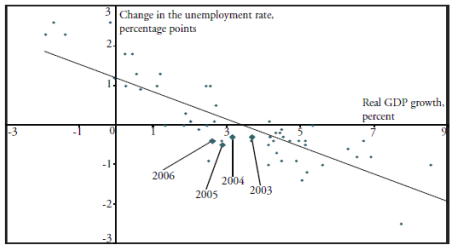
DIAGRAM 2-2.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA

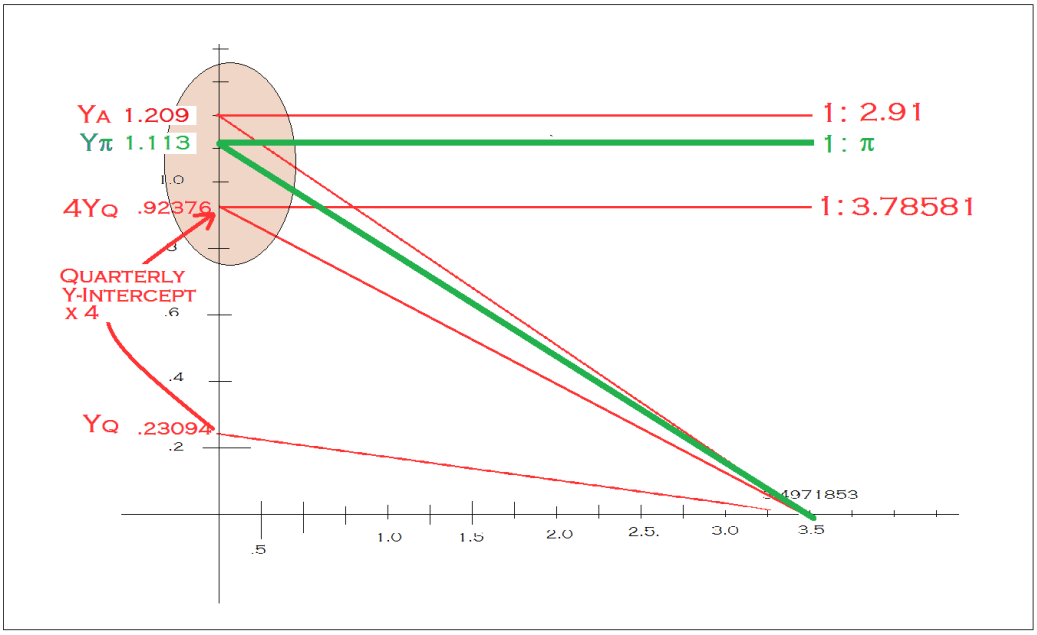


Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

I adjust the trend line for annualized quarterly data by multiplying quarterly employment data by four, thereby "annualizing" quarterly employment data. In this manner annualized quarterly data on growth is matched with "annualized" quarterly data on employment.

DIAGRAM 2-15.
MULTIPLYING QUARTERLY UNEMPLOYMENT RATE X 4

IMPLIED RATIOS BETWEEN
Y-INTERCEPT AND
X = 3.4551266 (ANNUAL X-INTERCEPT)
X = 3.4969781 (GOLDEN MEAN X-INTERCEPT)
X = 3.4972429 (QUARTERLY X-INTERCEPT)

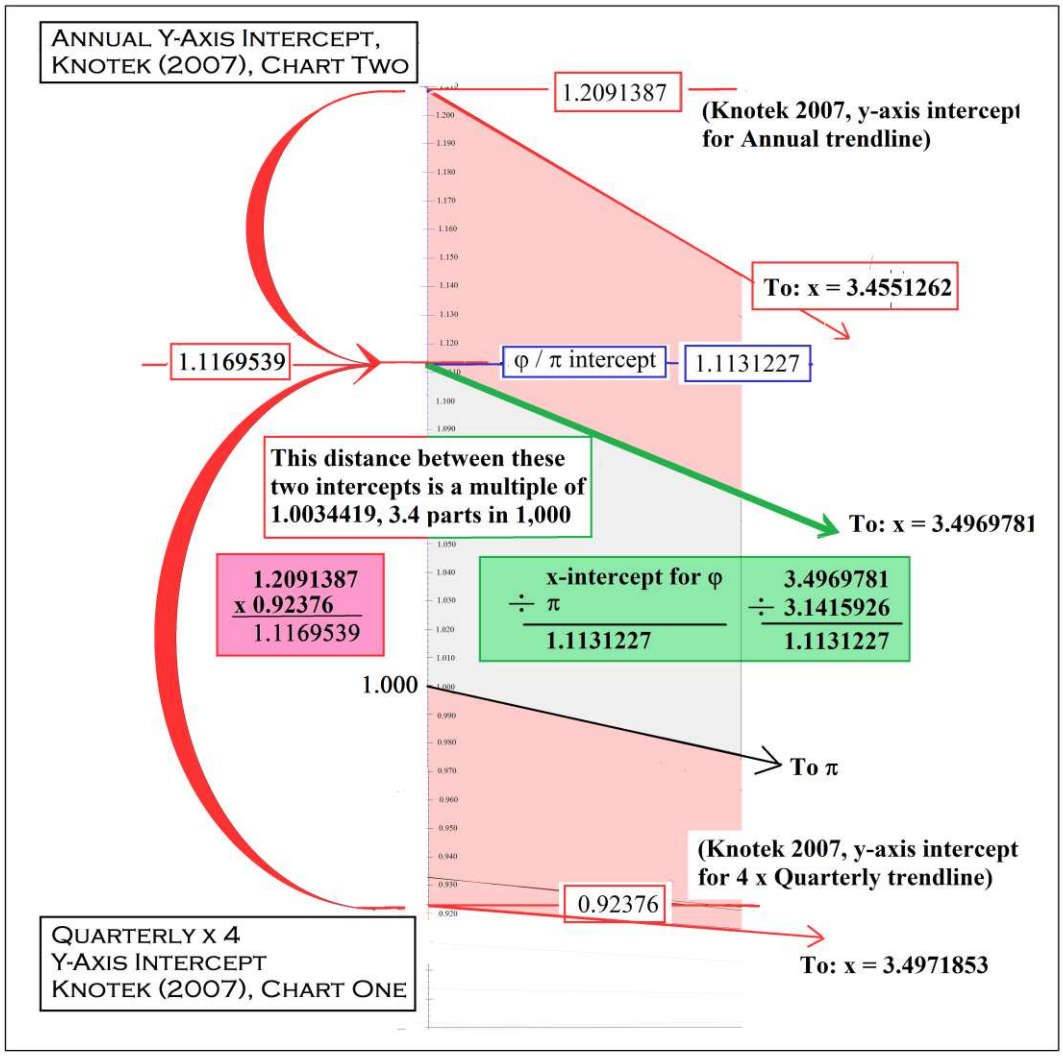


If the steady state rate given for the Golden Mean proportion (3.4969 percent per year) is divided by π, the y-axis intercept is 1.1131227.

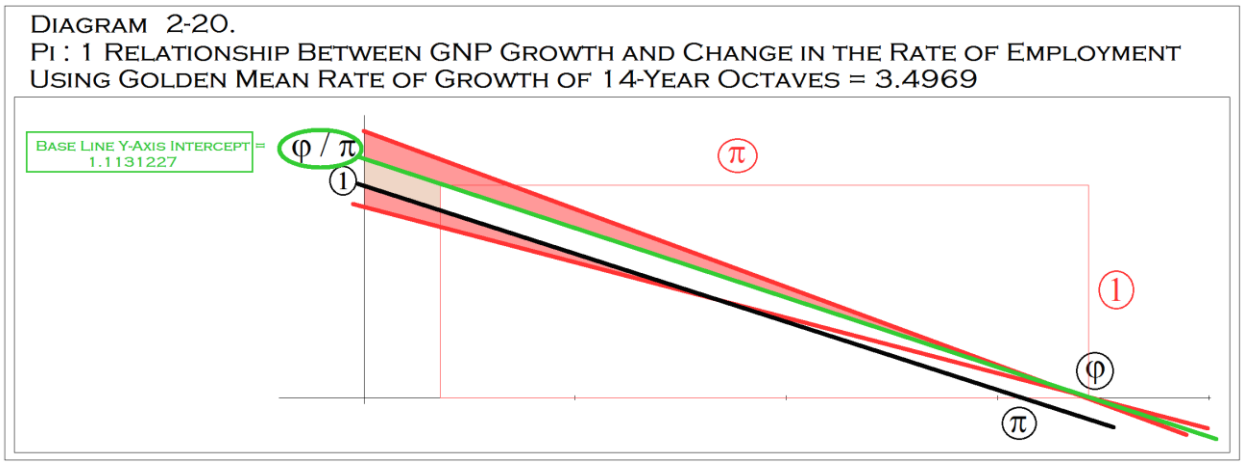
If we accept that the "Annual" y-intercept given in Knotek 2007 as 1.2091387, and that the "4 x Quarterly" y-intercept is 0.92376, we may multiply the two in order to test whether they are inverses around a common point. The multiple of these two intercepts is 1.1169539. The result is remarkable.

In short when the growth rate is zero (the y-axis), the y-axis intercepts for "Annual" and "4 x Quarterly" unemployment rate create a "Harmonic multiplicative inverse" about a progenic y-axis intercept equal to the projected trend line connecting a 1:φ steady-state rate of growth with a π:1 ratio for Okun's Law. This calculation is to an accuracy of 0.34%, 3.4 parts in 1,000, or 99.65%. (See chart below)

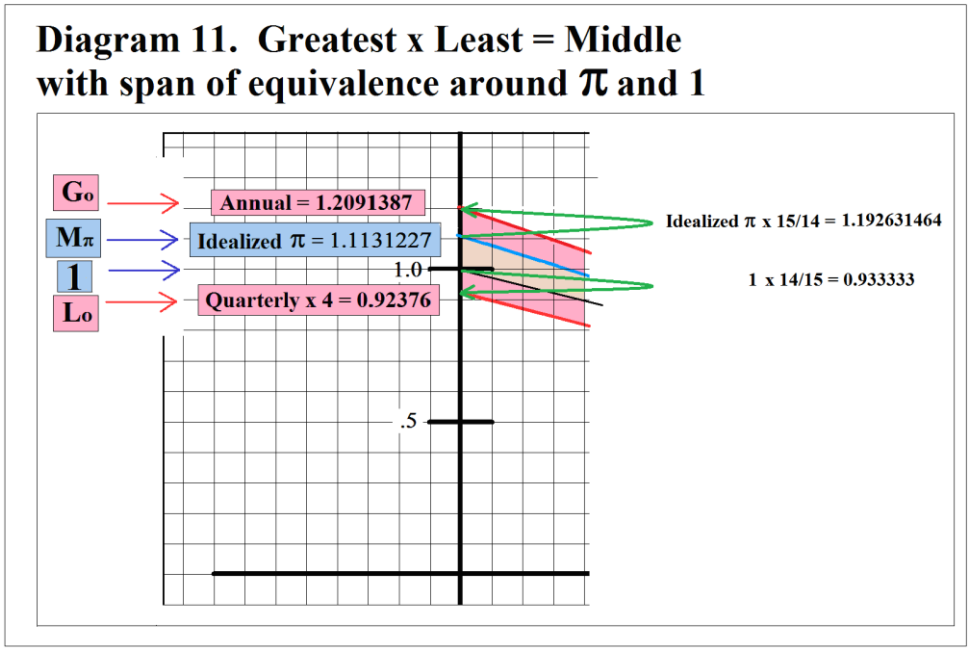
DIAGRAM 2.18. RELATIONSHIP BETWEEN φ AND π INTERCEPT (IN BLUE) AND KNOTEK'S ANNUAL AND "QUARTERLY X 4" Y-AXIS INTERCEPTS



In short, the possibility of two specific sets of numbers – feminine and masculine – as configured in the theory of a Harmonic multiplicative inverse appears to generate a remarkable understanding of the econometric data underlying Okun's Law.[13]

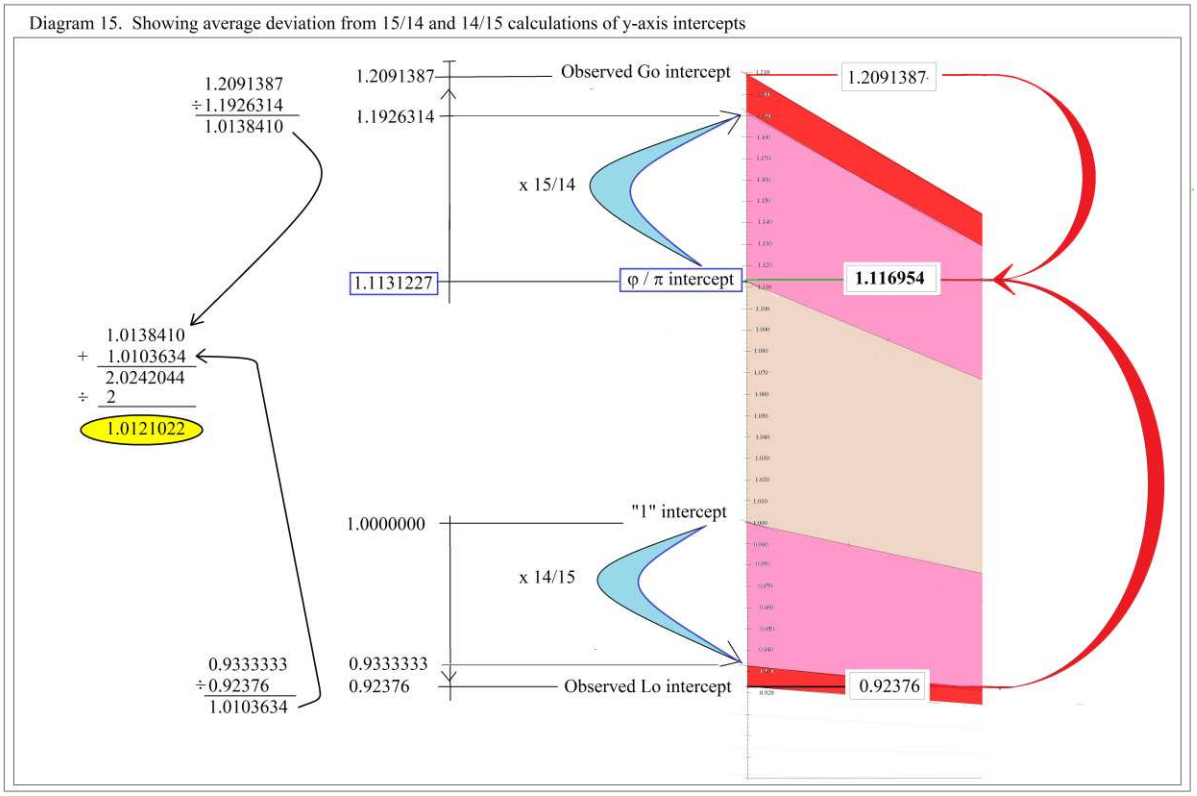


The advantages increase considerably if we connect the "1:π / 1:φ" trend line to an analysis of the Kondratiev Wave. We note, as we must, that the progenic π/φ intercept ("P") may be constructed from a feminine "14/15 x 1" as combined with a masculine "15/14 x P." The resulting projections of Annual and Quarterly intercepts lie at variances from Dr. Knotek's calculations of 1.0% (from the Quarterly unemployment y-axis intercept) and 1.3% (from the Annual unemployment y-axis intercept).

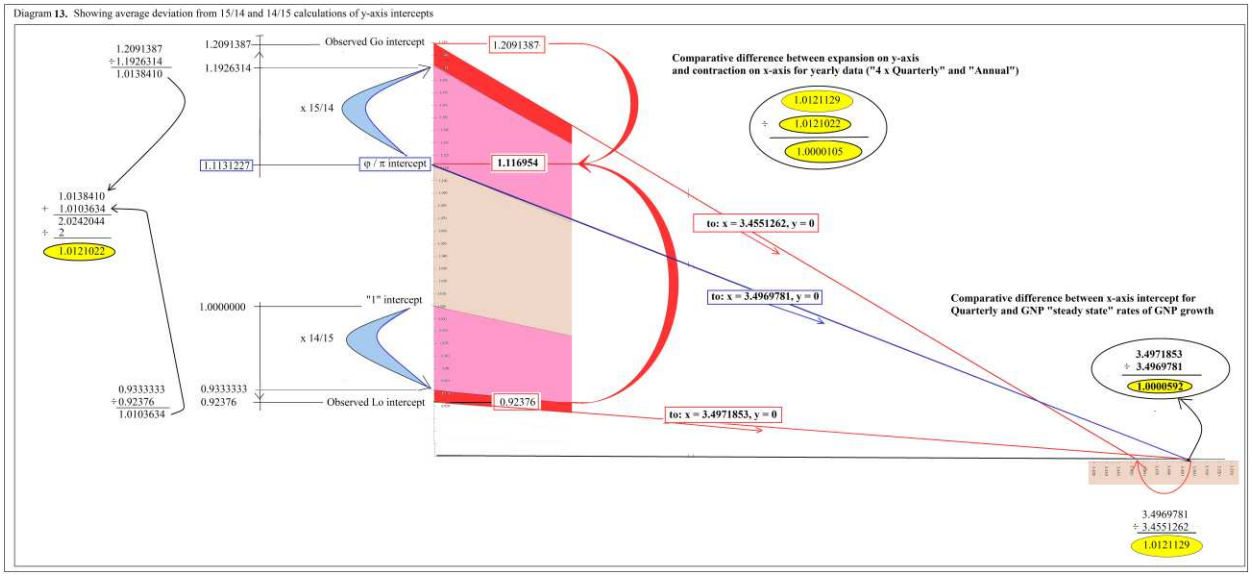


[13] A simple 3:1 ratio, with the same approach used, yields a y-intercept of 1.1656. This is contrasted with a π/φ intercept of 1.1131 / 1.1656 (at 95.49%) and an observed intercept of 1.1169 / 1.1656 (at 95.81%).

This yields an average expansion of 1.2% beyond the masculine and feminine inverses, or more specifically a multiple of 1.0121022, in yellow below.

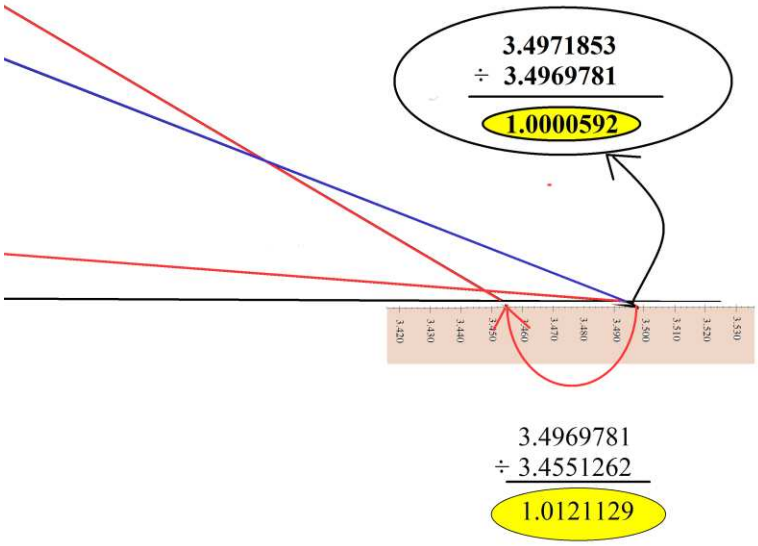


This y-axis deviation balances a similar deviation between growth rates along the x-axis. The steady state rate for Annual Data calculated by Dr. Knotek is 3.4551266. The steady state rate of growth calculated via the GNP Spiral (3.4969781) is greater than this number by a multiple of 1.0121129, virtually identical to the y-axis deviation stated above.



Let us consider more carefully the three rates we have for a steady state rate of growth, each of which constitutes an x-axis intercept. These are Knotek:Annual (3.4551262), Knotek:Annualized Quarterly (3.4971853) and the GNP Spiral (3.4969781).

Comparative difference between x-axis intercept for Quarterly and GNP "steady state" rates of GNP growth



Dr. Knotek's data track slightly more than one complete circuit around the 56-year GNP Spiral, i.e. covering the second quarter of 1947 through the third quarter of 2007, a period of 60 years. This data misses the full range of GNP values available from the Department of Commerce (1869 through 1946), a period of 78 years. No unemployment figures are available for this period. Moreover between 1869 and 1947 very large growth rates are found in GNP ratios. These larger ratios are included as a part of the calculation of the GNP Spiral.

Despite the incongruity of data sets Knotek:Annualized Quarterly (3.4971853) is virtually the same as that given for the GNP Spiral (3.4969781).

When the larger (3.4971853) is divided by the smaller (3.4969781) a multiple of 1.0000592 is found, indicating a proximity between the two numbers of 5.9 parts in 100,000.[14]

Given the absence of GNP data pre-dating 1947, one might expect that the Knotek:Annual must be smaller than that of the growth rate calculated by the GNP Spiral. Indeed the x-axis intercept for Knotek:Annual (3.4551262) retreats from the expected GNP Spiral x-axis intercept (3.4969781), the second being larger by a multiple of 1.0121129.

As noted previously, this compares to an expansion along the y-axis for unemployment averaging between feminine and masculine components of 1.0121022.

[14] This result, as first pointed out by Dr. Knotek in an email of June 24, 2011, was the genesis of the correspondence resulting in this paper.

When the deviation along the x-axis 1.0121129 is divided by the deviation along the y-axis 1.0121022 a multiple of 1.0000105 results. This indicates that a balance between growth and employment along a 1:φ / 1:π trendline is accurate to within 1.05 parts in 100,000. It further suggests that while unemployment states a Harmonic multiplicative inverse, growth is not figured in such a way.

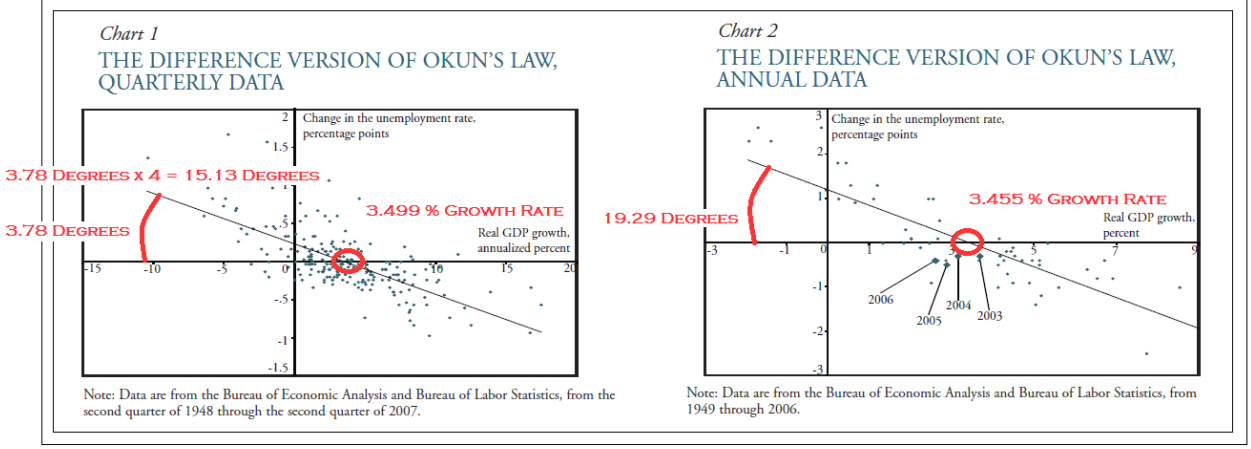
This leads to the following insight as to the operation of the Harmonic multiplicative inverse and its impact upon the analysis of data surrounding Okun's Law.

When change in the rate of unemployment is zero, the rate of growth is seen clearly; there is no inverse at all to found in the growth data.

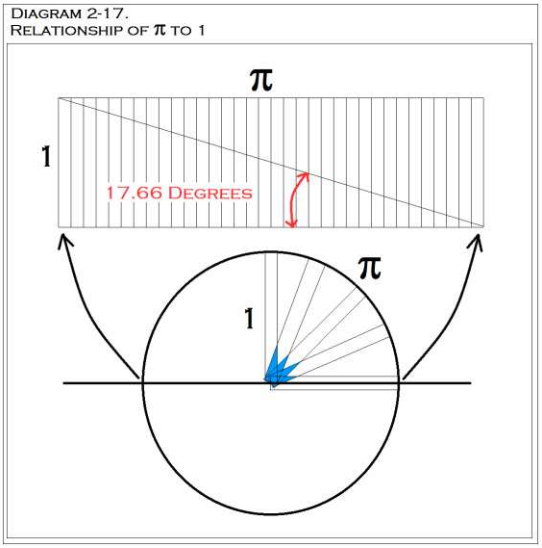
When the growth of GDP is zero, quarterly and annual rates of unemployment at in great flux and we see quite clearly the Harmonic multiplicative inverse in the unemployment data.

A second test of the Harmonic multiplicative inverse may be found in the fact that the π:1 understanding of Okun's Law generates an angle bisecting that of Charts One and Two to within half a degree. These angles are 15.13 degrees for annualized quarterly data and 19.29 degrees for annual data.

DIAGRAM 2-16.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"



The angle created by the rectangle $\pi : 1$ bisects these two within one half of one degree, i.e. 17.66 degrees.



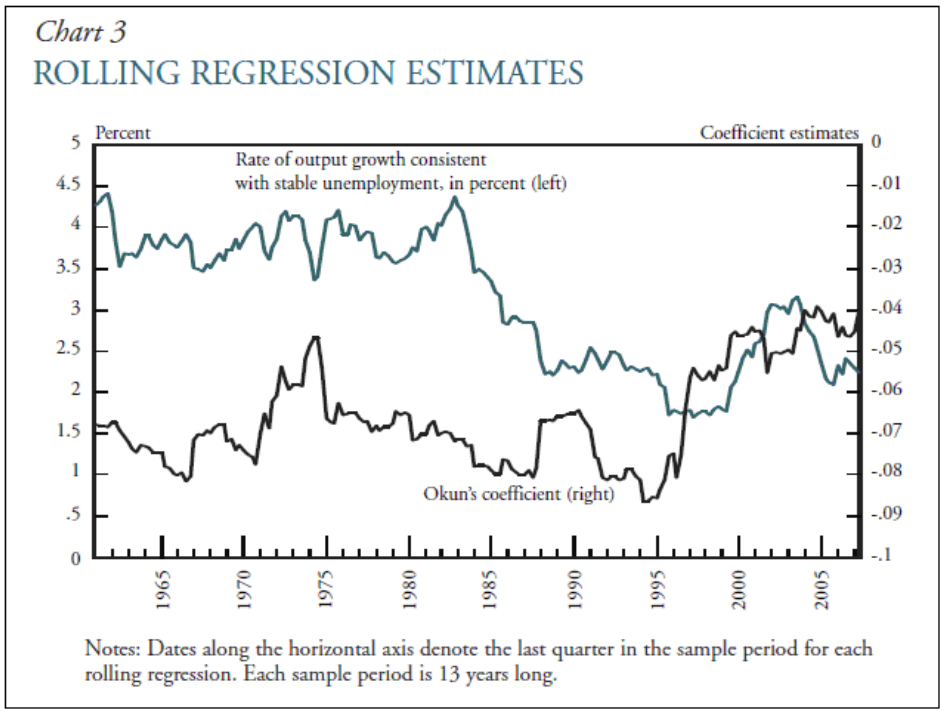
In other words, the slope of the angle bisecting the angles given in Charts One and Two is 17.213 degrees, less than half a degree from the slope of 17.66 degrees of a projected relationship between the constant π and 1 as projected by this approach.

The Kondratiev Wave

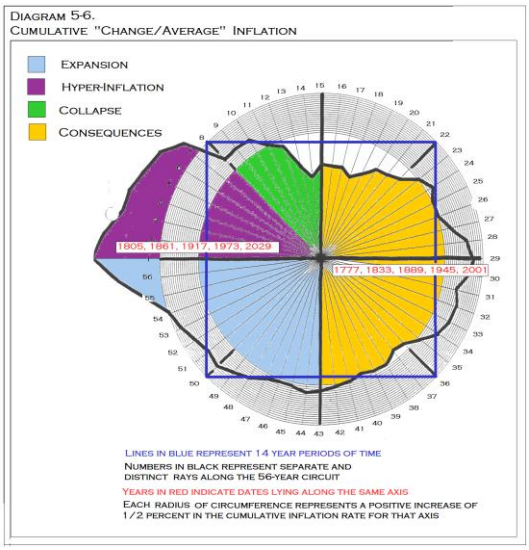
The straightforward discovery of Okun's Law within the data does not, however, fully answer the need for a complete theory. As Dr. Knotek argues, the stability of the trend lines provided in Charts One and Two mask the underlying dynamics of these relationships. He writes:

> One problem with a long time series – such as from 1948 to 2007 – is that history can hide changes in relationships. This is the case for Okun's law. The previous section (Charts 1 and 2) found considerable similarities between Okun's original estimate and an updated regression using a longer time series. This section shows that, when estimated over shorter time horizons, the relationship between changes in the unemployment rate and real output growth has varied considerably.
>
> To capture this variation, this article uses a technique called rolling regressions. A rolling regression estimates a particular relation over many different sample periods. Each regression produces a set of estimated coefficients. If the relationship is stable over time, then the estimated coefficients should be relatively similar from one regression to the next. Variations in the relation will appear as sizeable movements in the estimated coefficients.

DIAGRAM 2-3.
CHART THREE OF "HOW USEFUL IS OKUN'S LAW?"



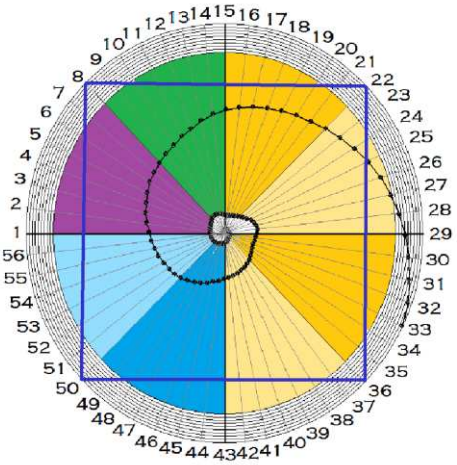
This apparent chaos may be understood by recalling that Okun's Law, being the consequence of the inverse between national and personal aspects of the economy, is itself intimately connected to the GNP Spiral and the passage of time within it. In consequence we are able to divide the time periods shown by Chart Three above into quite specific, distinct *and predictable* periods of definite duration, each of which possesses its own unique political economy.



This discovery, coupled with the necessary use of a steady state rate of growth of between 3.496 and 3.499% per year, permits a reconsideration of Okun's Law as it applies to the economy of the United States.

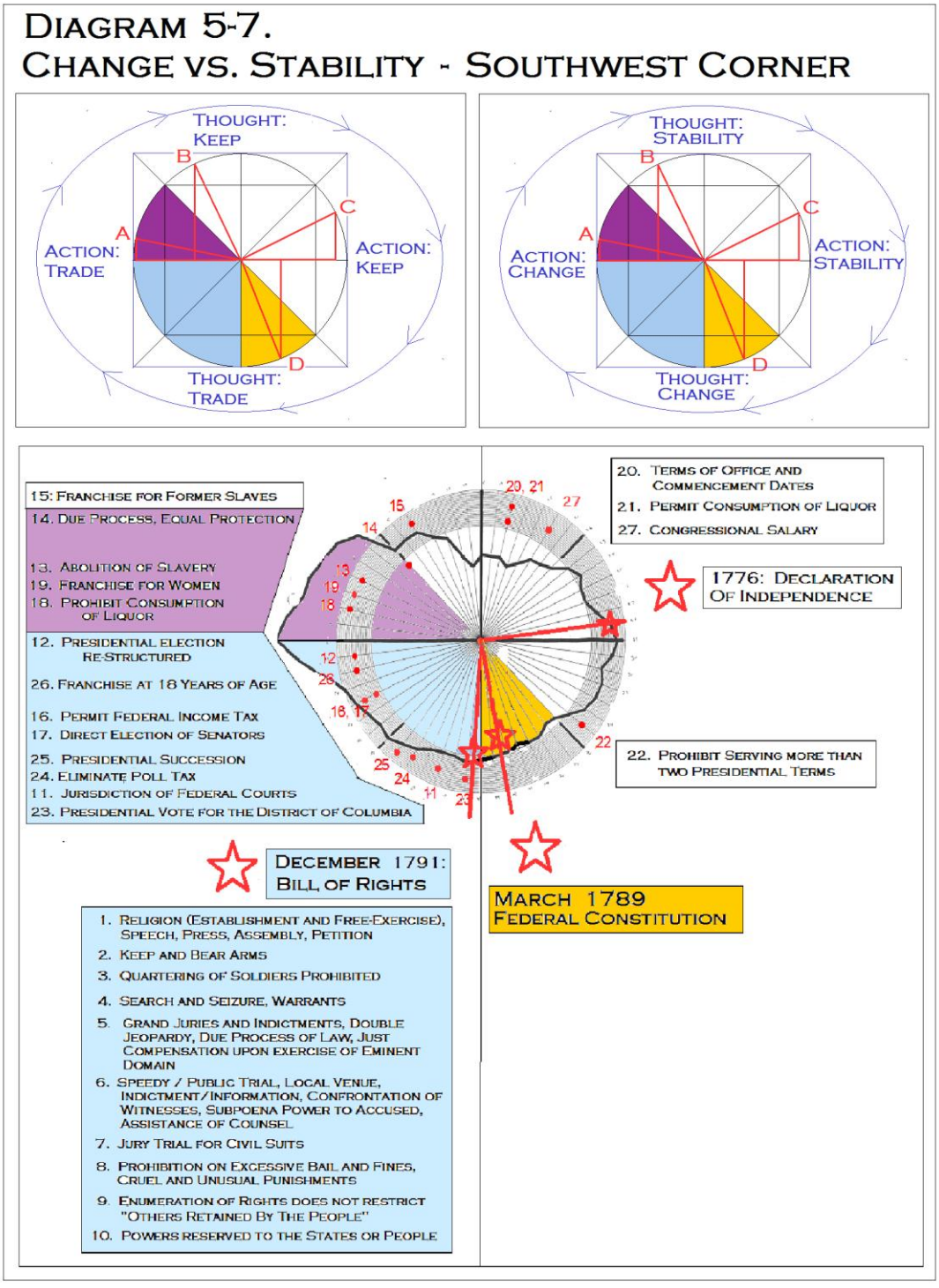
DIAGRAM 4-7. THE "GNP SPIRAL"

	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015

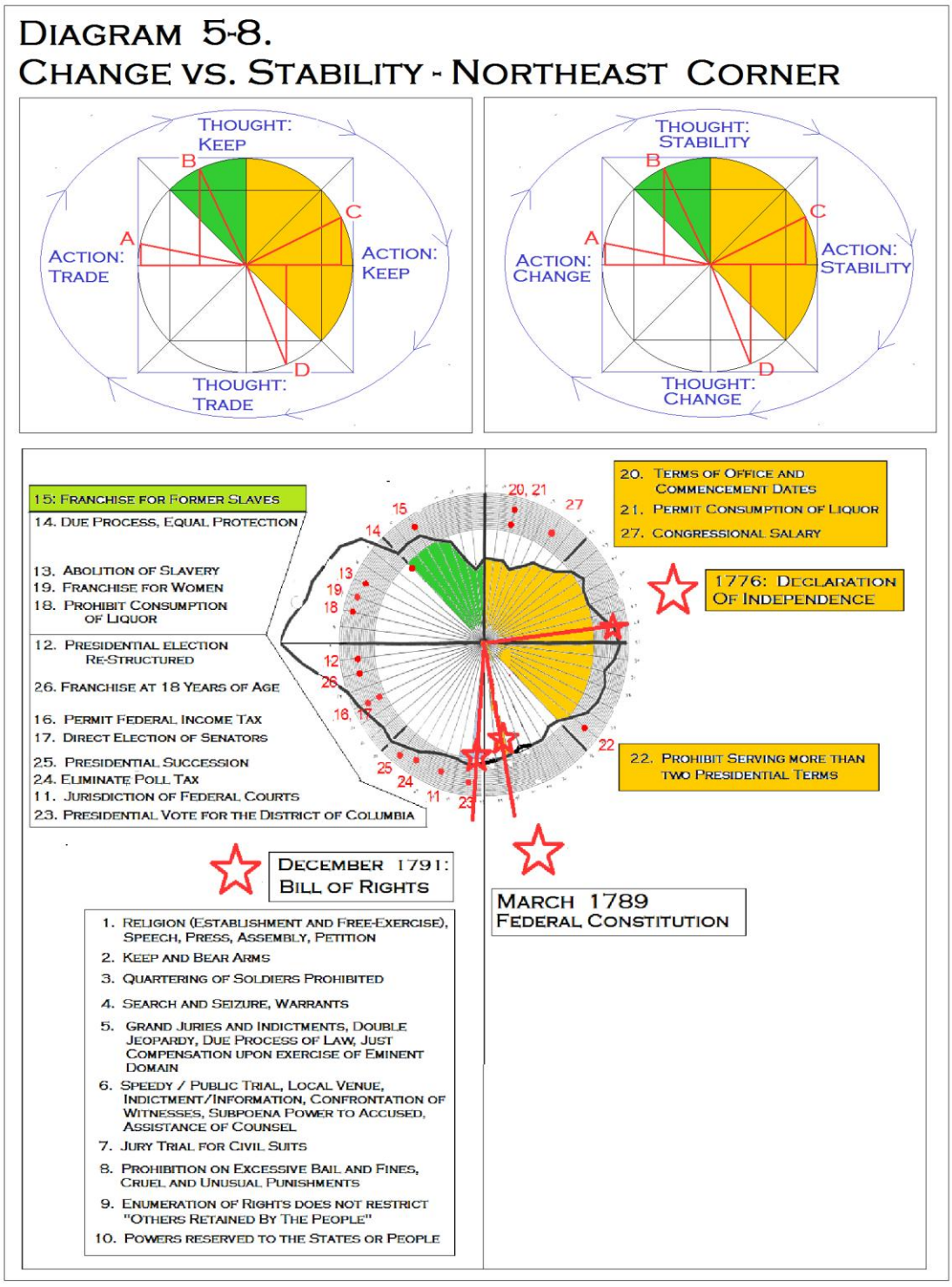


	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1836	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

As mentioned in our previous paper, the portions of the circuit of 56 year circuit are each of dramatically different political emphasis. The period of time which we face most immediately is that of a tremendous sea change in political attitudes. During these periods a frustration with the political status quo leads to significant and enduring constitutional changes. These changes alter the ground rules of economic engagement and permit the capitalistic enterprise to move forward. These are the amendments associated with this period of time.

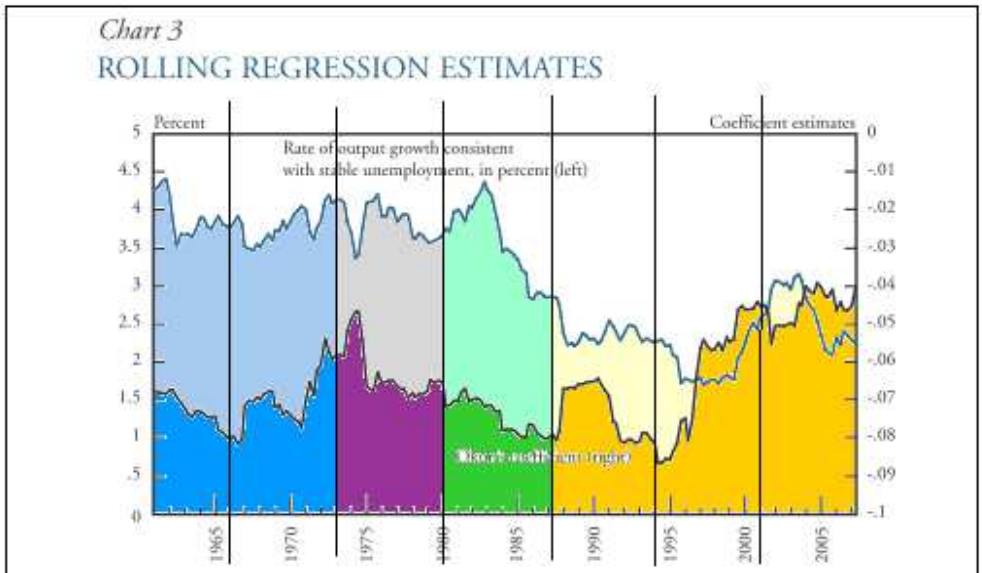


The period of time which we are leaving is one of tremendous conservatism, a period during which the rules previously laid down are made permanent to the satisfaction of a newly empowered political elite.

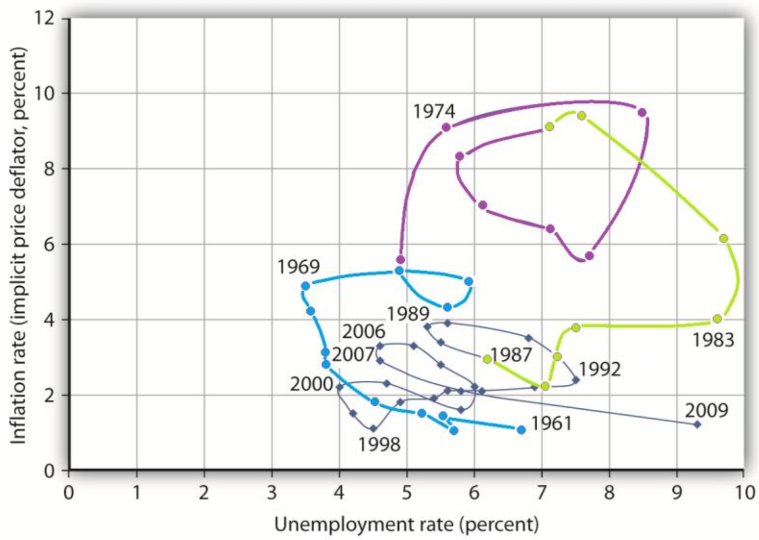


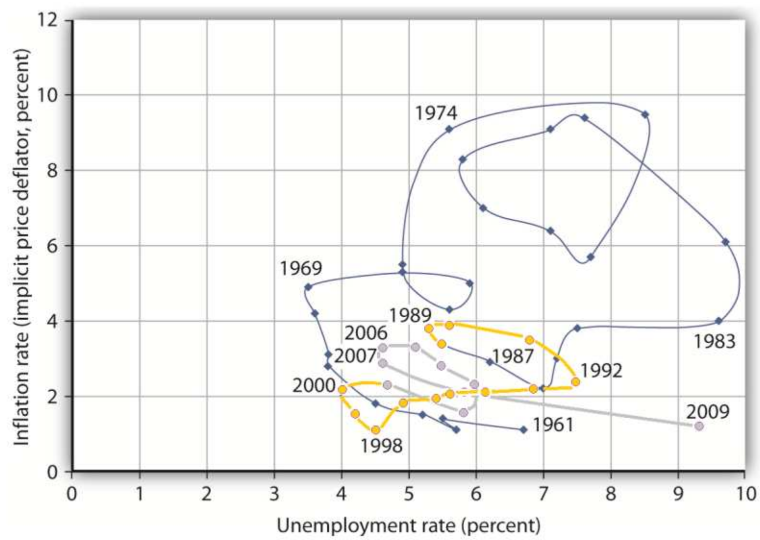
In short, using ideas proposed by Nicholai Kondratiev in 1925 we may develop a theory undergirding the seeming randomness of Knotek's Chart Three, one which brings light to the proper understanding of a short-term calculation of Okun's Law, without diminishing its long-term perspicuity.

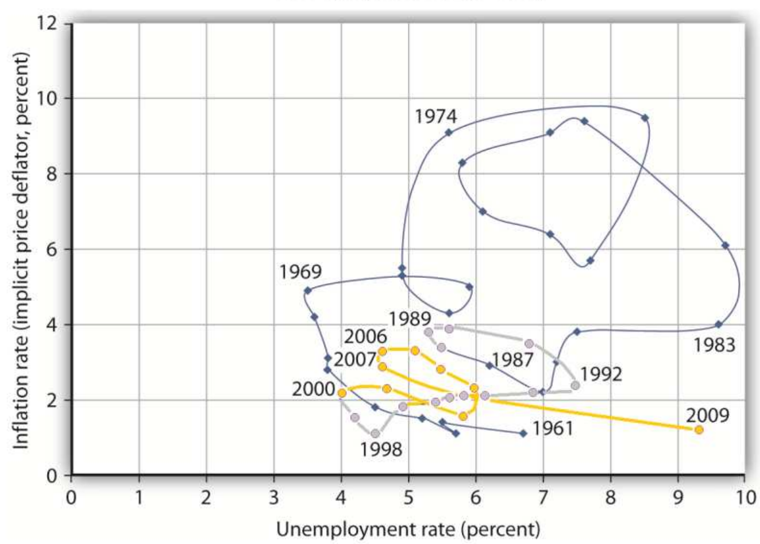
DIAGRAM 5-9.
ROLLING REGRESSIONS WITH HISTORIC CONTEXT



Typical of this period is a steady increase in the inflation rate which is consistent with a citizenry which is ever more involved with the determination and active change in its circumstances. During this period the largest number of constitutional amendments have been ratified, and also during this period a general upswing in radical activity takes place.

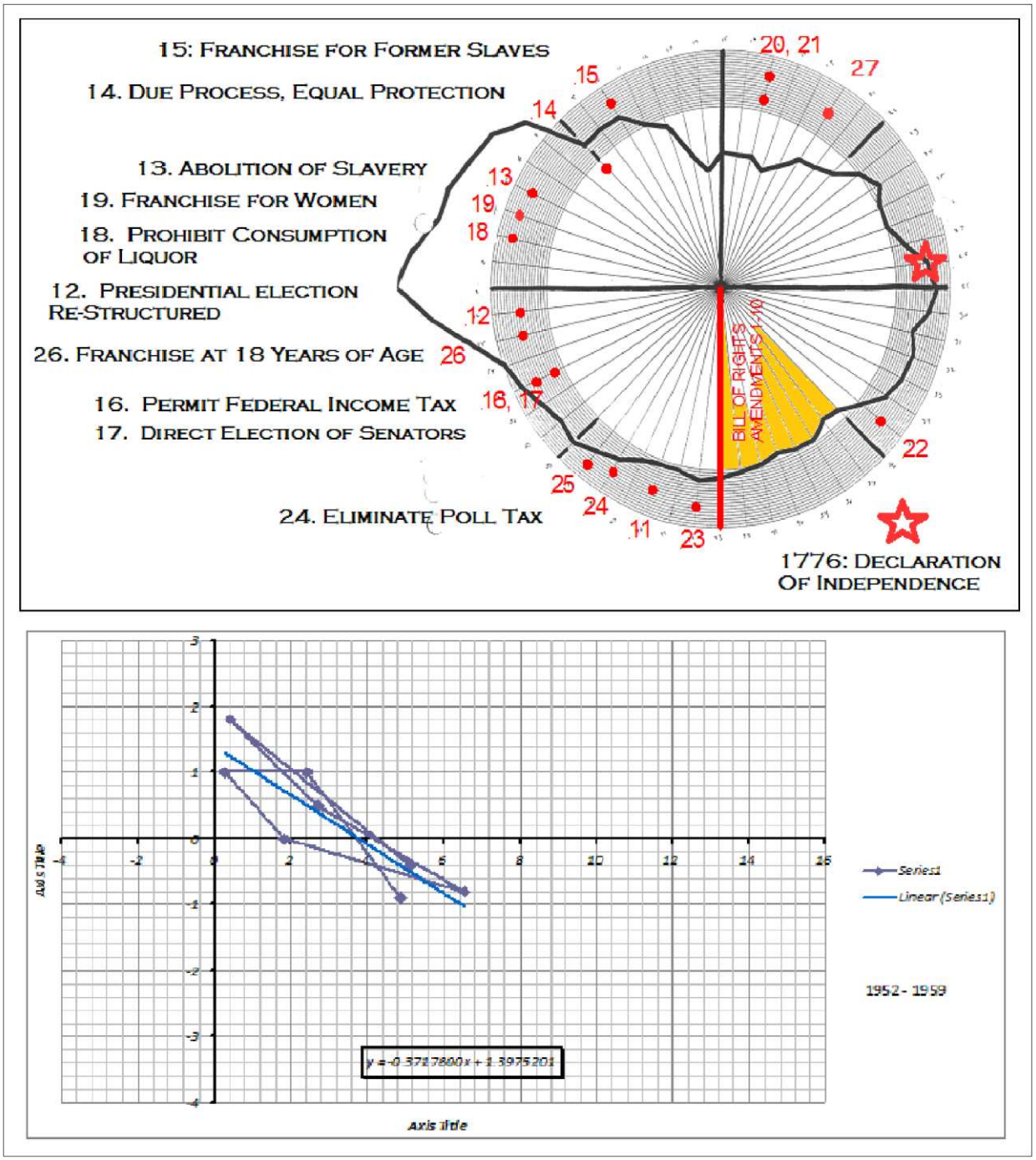

1974
1969
1989
2006
2007
1987
1992
1983
2000
2009
1961
1998
Inflation rate (implicit price deflator, percent)
Unemployment rate (percent)


1974
1969
1989
2006
2007
1987
1992
1983
2000
2009
1961
1998
Inflation rate (implicit price deflator, percent)
Unemployment rate (percent)

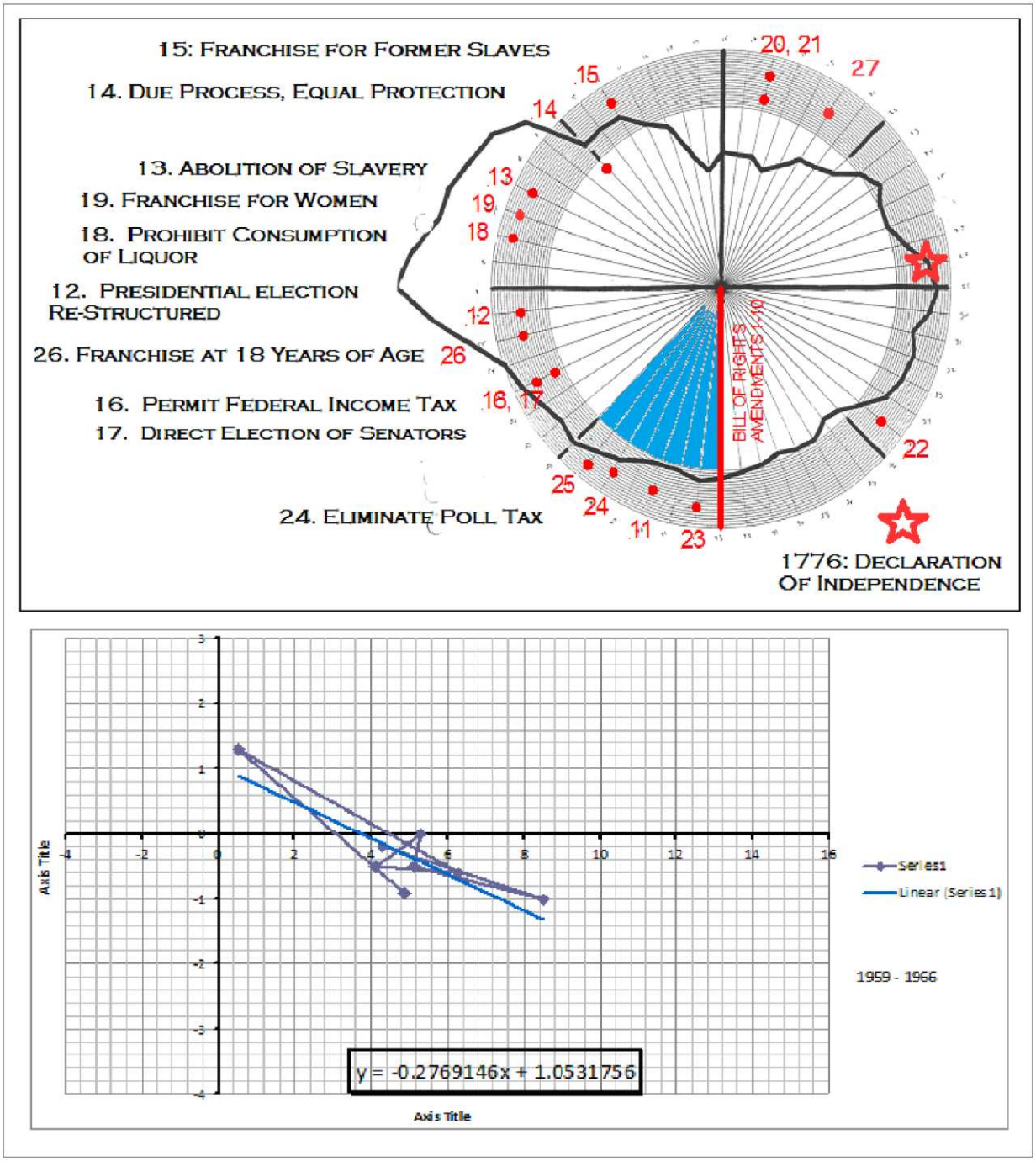

1974
1969
1989
2006
2007
1987
1992
1983
2000
2009
1961
1998
Inflation rate (implicit price deflator, percent)
Unemployment rate (percent)

Using these separate portions of the circuit as guides, let us now re-evaluate Okun's Law as it moves around the 56-year circuit.

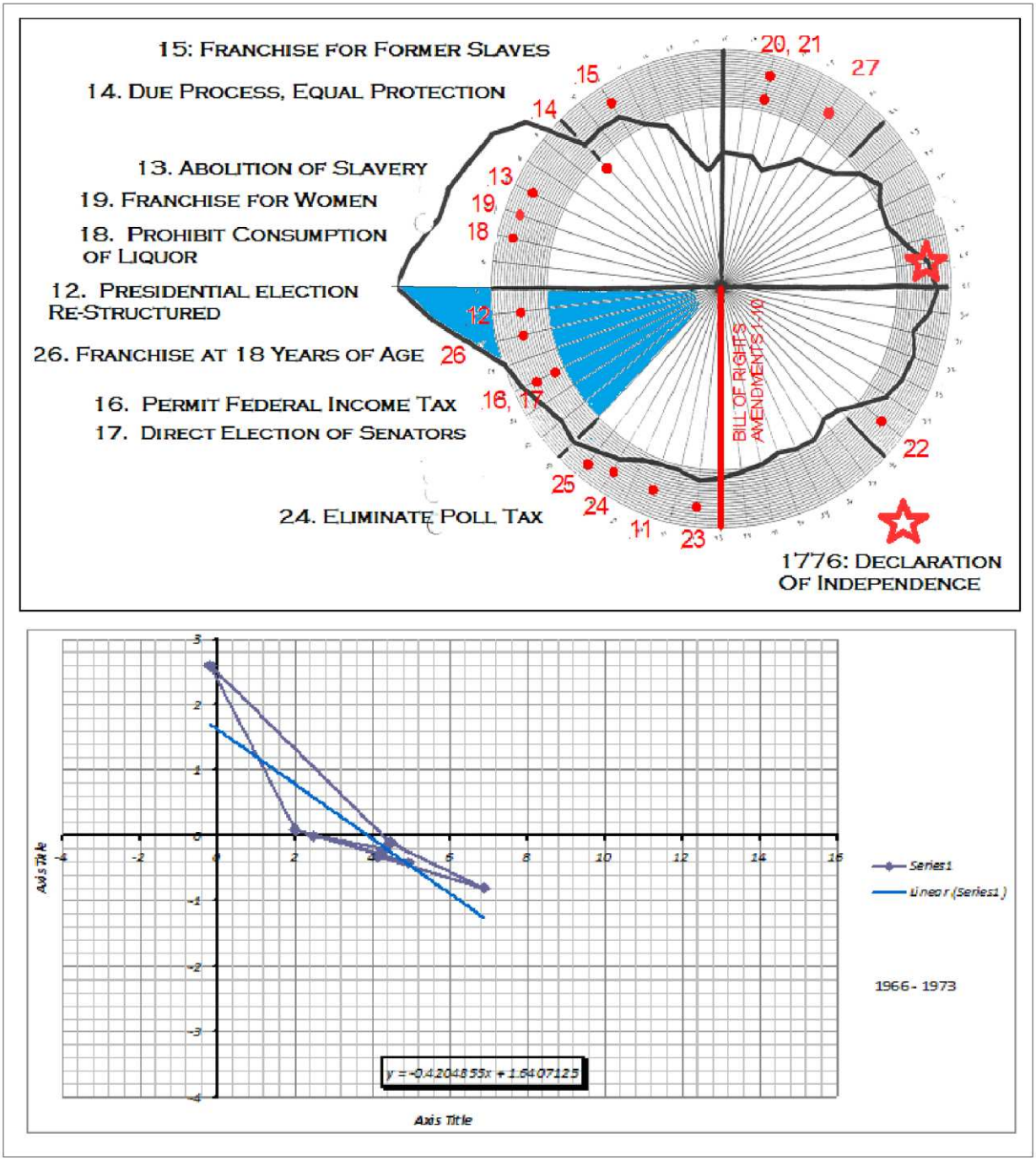
Early Evolving Revolution



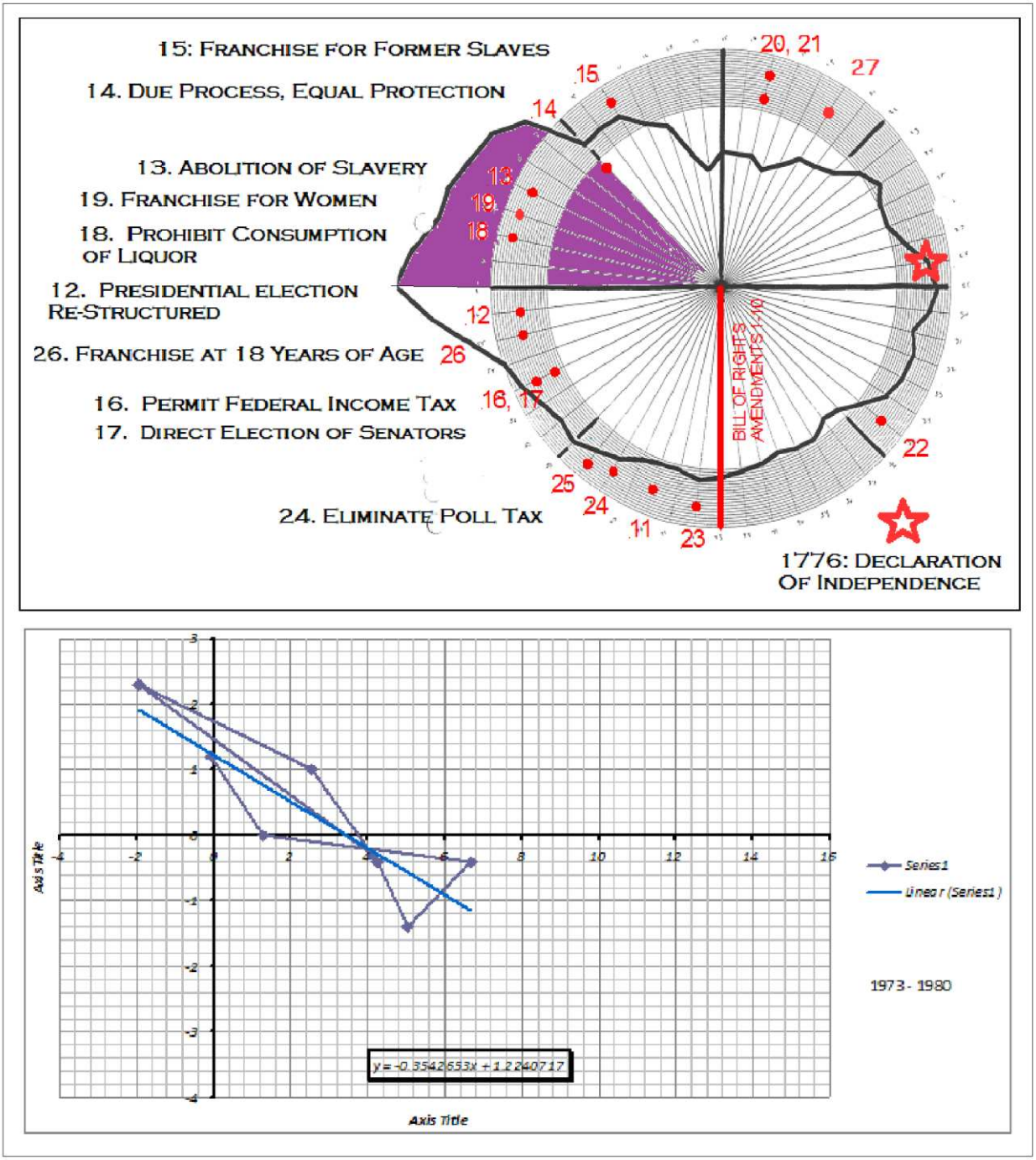
Late Evolving Revolution



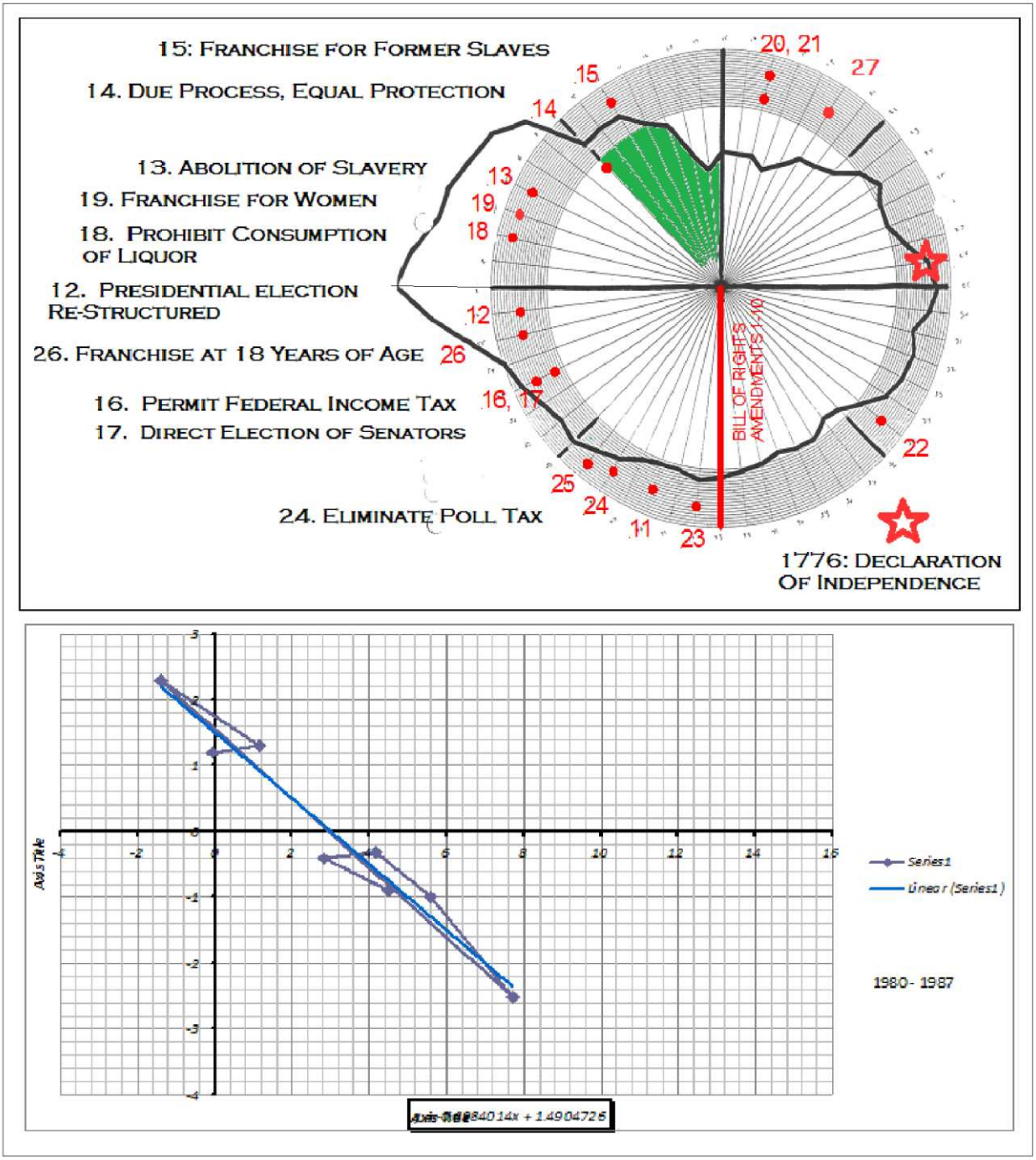
Early Revolution



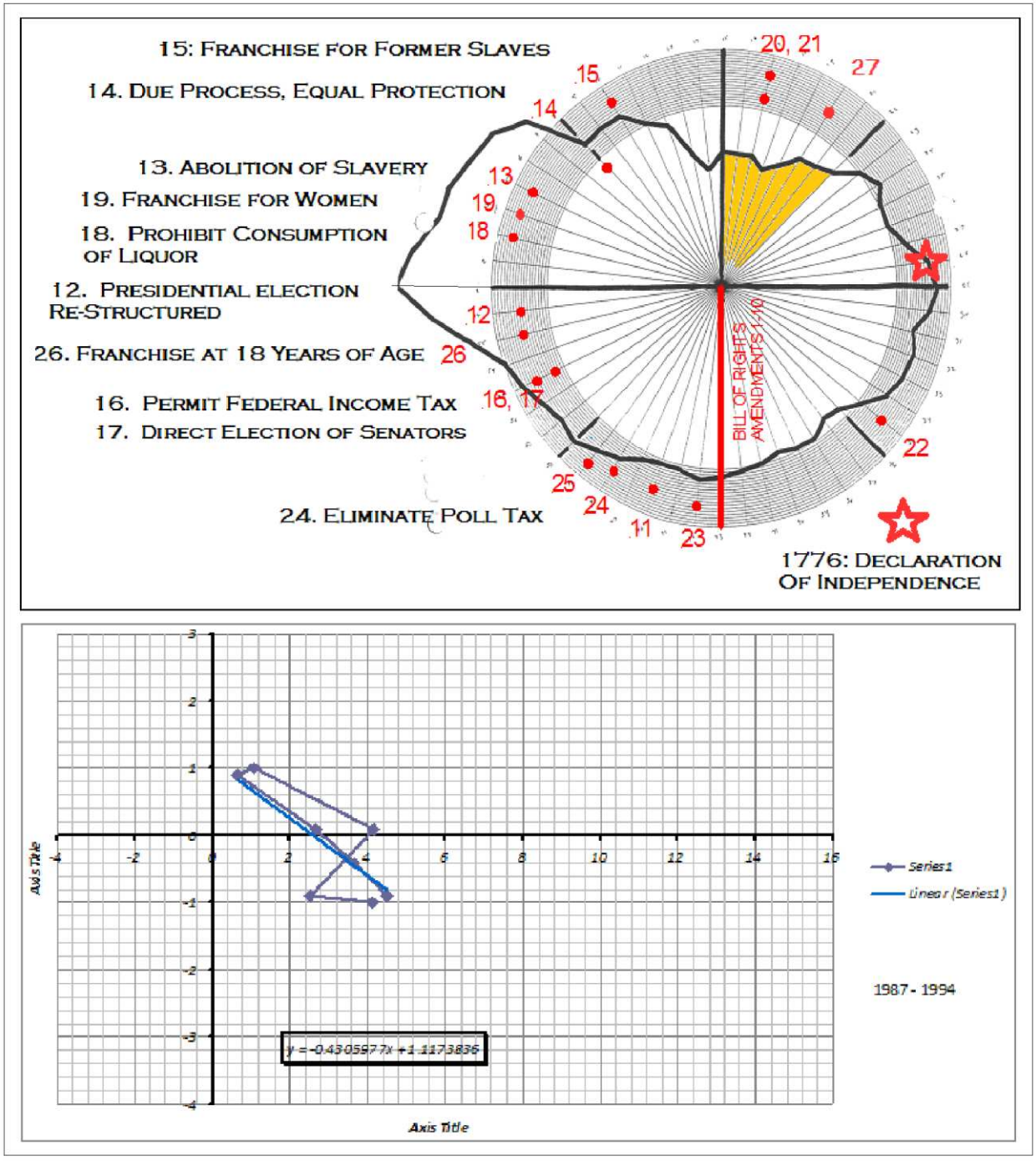
Late Revolution



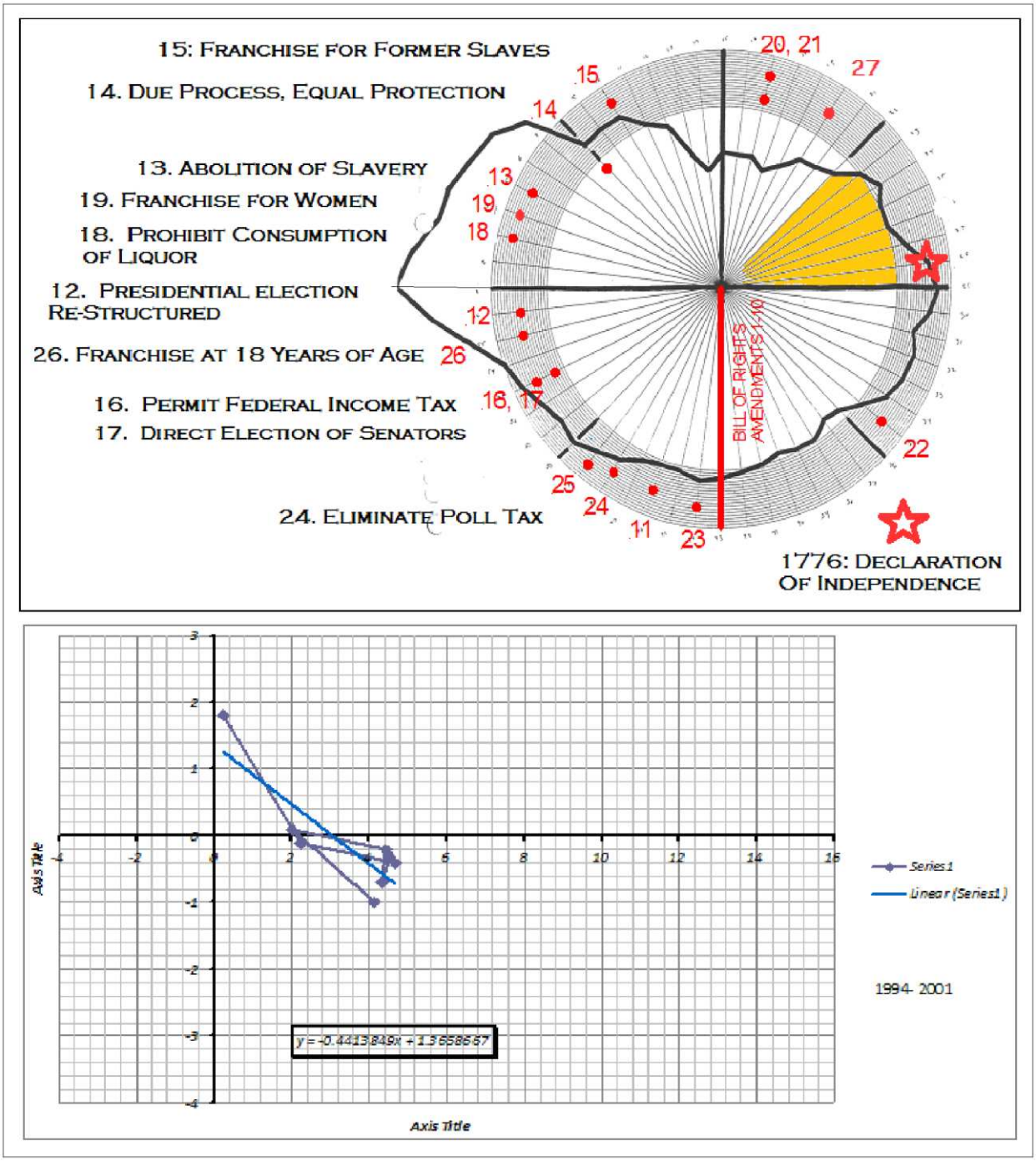
Early Evolving Consolidation



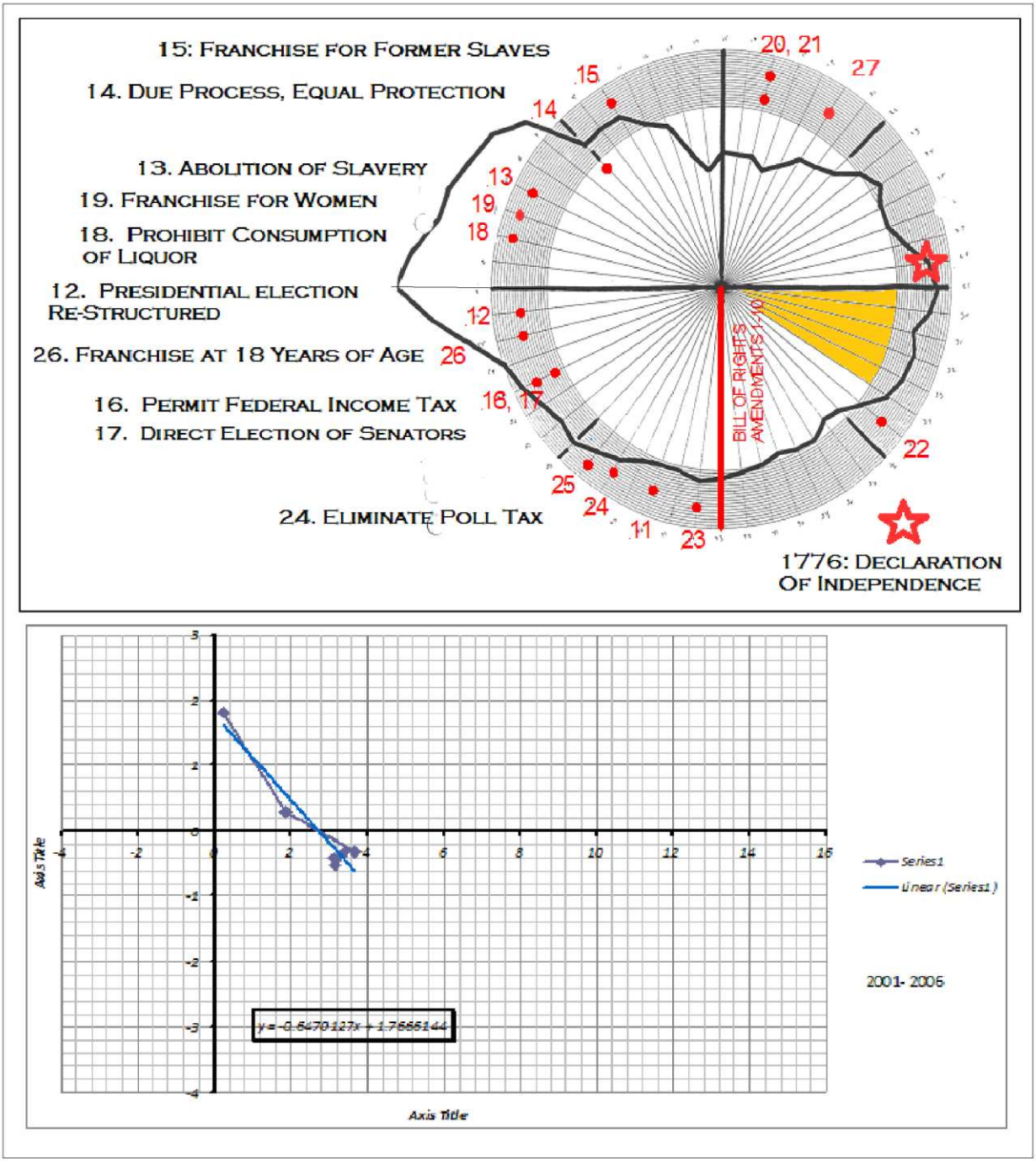
Late Evolving Consolidation



Early Consolidation



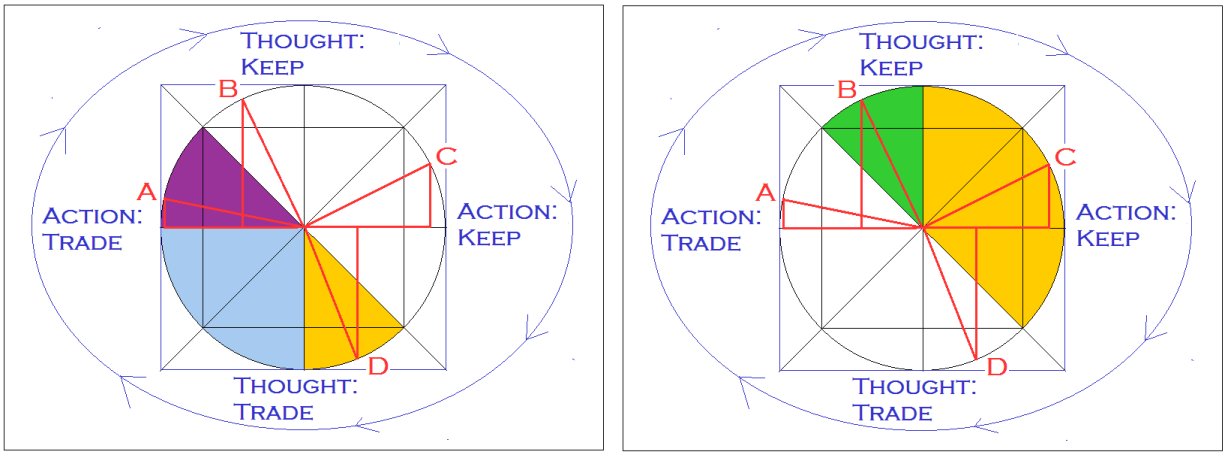
Late Consolidation



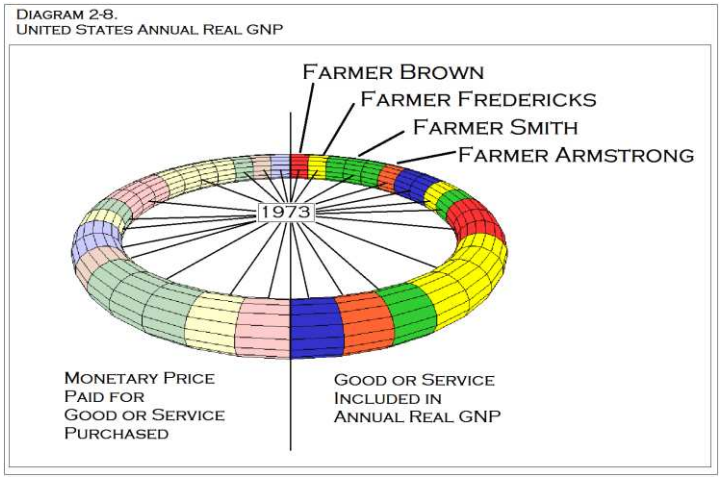
Conclusion

In conclusion, we began our description of Okun's Law with a formal structure combining action and thought in a geometric form,

Micro-economic Unit Circle

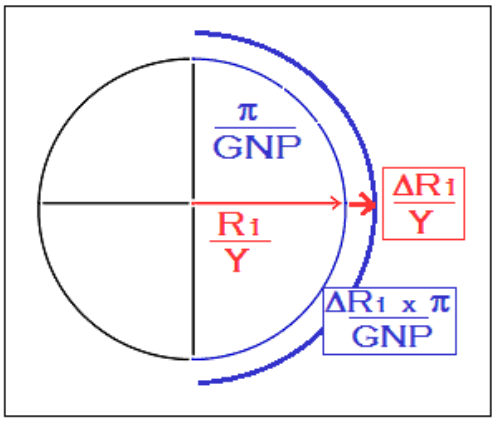


... and then aggregated this form over the period of a single year....



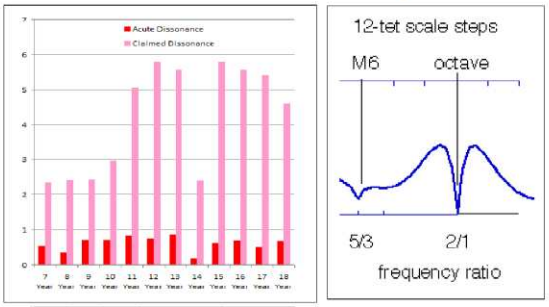
... to establish the 1:pi relationship which is at the heart of the 3:1 ratio pointed out by Okun in his analysis

DIAGRAM 2-9.
MODEL OF PRODUCTION VS. EMPLOYMENT



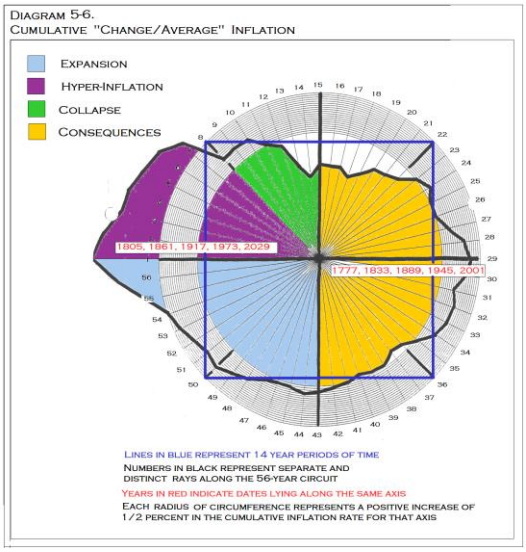
... thereby bringing into play the life-spans of the people responsible for the creation of the real GNP examined

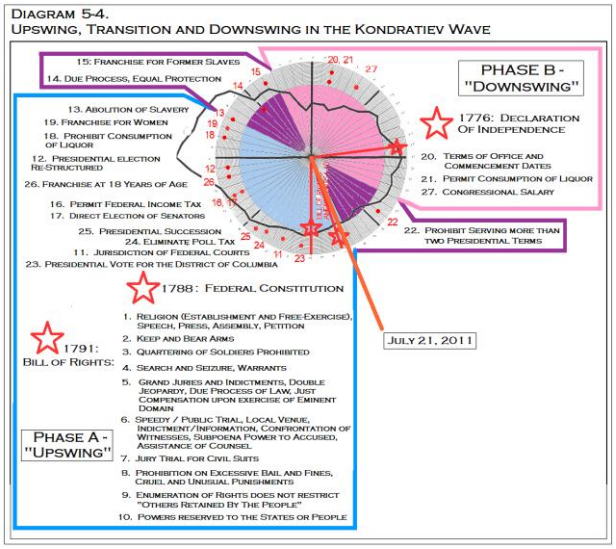
DIAGRAM 3.11.
"ACUTE DISSONANCE" AND "CLAIMED DISSONANCE" WITH MUSICAL OCTAVE



- RED	+ RED		Acute Dissonance	Claimed Dissonance
0	0.539222	7 Year	0.53922225	2.33166
0	0.328323	8 Year	0.32832275	2.39980
0	0.71348	9 Year	0.713480444	2.41405
0	0.702555	10 Year	0.702555	2.95466
-0.82882	0	11 Year	0.828815273	5.05443
-0.07077	0.680828	12 Year	0.751594667	5.77002
-0.07244	0.789312	13 Year	0.861753846	5.55129
-0.01764	0.134157	14 Year	0.151795	2.39229
-0.28313	0.338373	15 Year	0.6214992	5.77038
-0.16258	0.526958	16 Year	0.689539375	5.54327
-0.10785	0.394316	17 Year	0.502165176	5.40061
-0.04362	0.624485	18 Year	0.668103333	4.58002

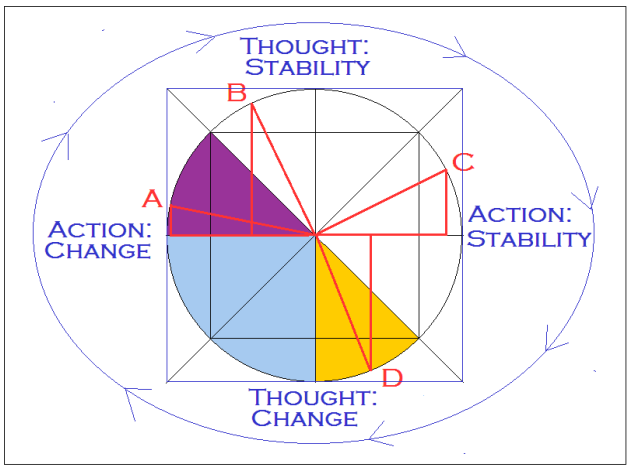
... and the political effect which these lives have on the environment of the United States

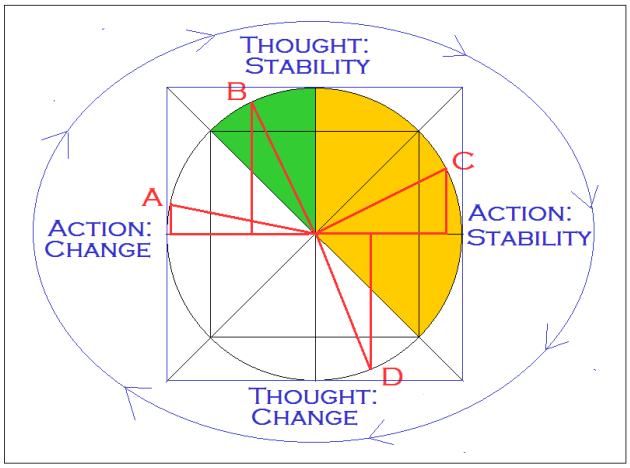




... leading in turn to the political re-statement of the original action vs. thought dichomoty with which we began the analysis. In short, making the larger the fractal of the smaller.

GNP Spiral





It would appear that Okun's Law is in fact a trigonometrically driven proportion. This accounts for its historic stability, heretofore completely un-described in either character or causation. This is demonstrated by a form of number theory engaging the set of feminine ($0<F<1$) and masculine ($1<M$) numbers using a trend line representing a $1:\varphi$ ratio for growth over time and a $1:\pi$ ratio for growth to employment as connected by a Harmonic multiplicative inverse. This view of the relationships is considerably enhanced the central tenets of the GNP Spiral generate masculine (15/14) and feminine (14/15) fractions which can be used to further interpret the interaction of time upon econometric data, however hidden these relationships might appear.

It is at this point wherein the general theory of microeconomic trading matches exactly the sense of macroeconomic of values over time. For the basic theory under which a π:1 relationship exists is that the micro-economic ability to choose builds into a similar macro-economic ability to choose new values over the long term.

And so it is that the two levels appear as fractals of one another, not unlike Russian dolls wherein the smaller builds into the greater and is contained and congruous to it.

DIAGRAM 1-13.
TRIGONOMETRY OF TRADE.

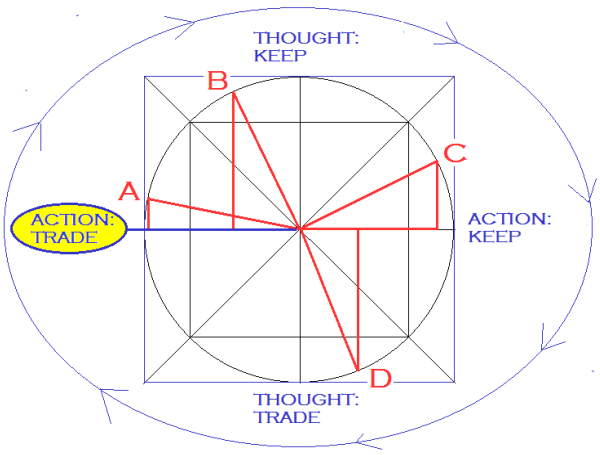
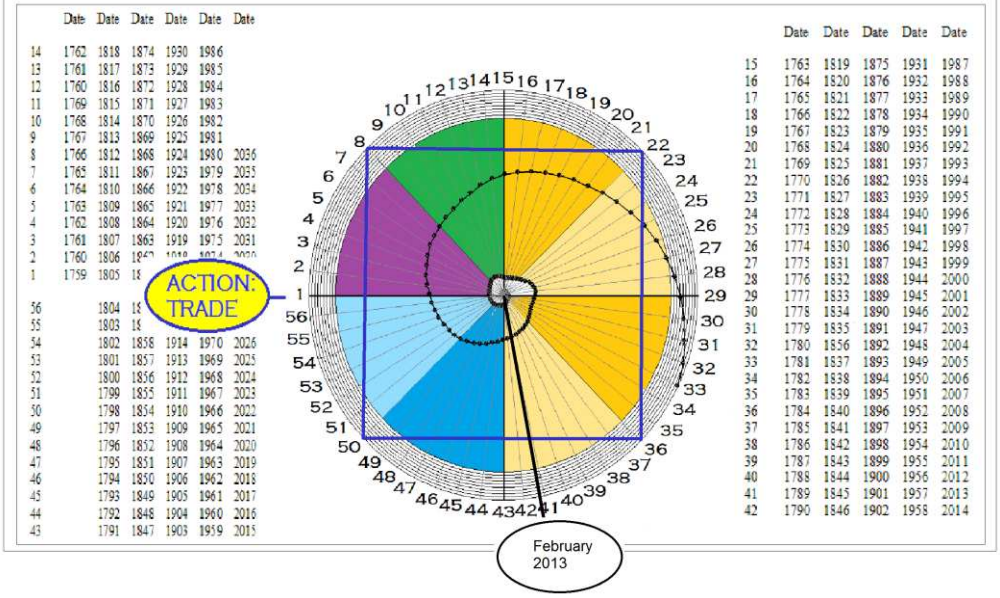
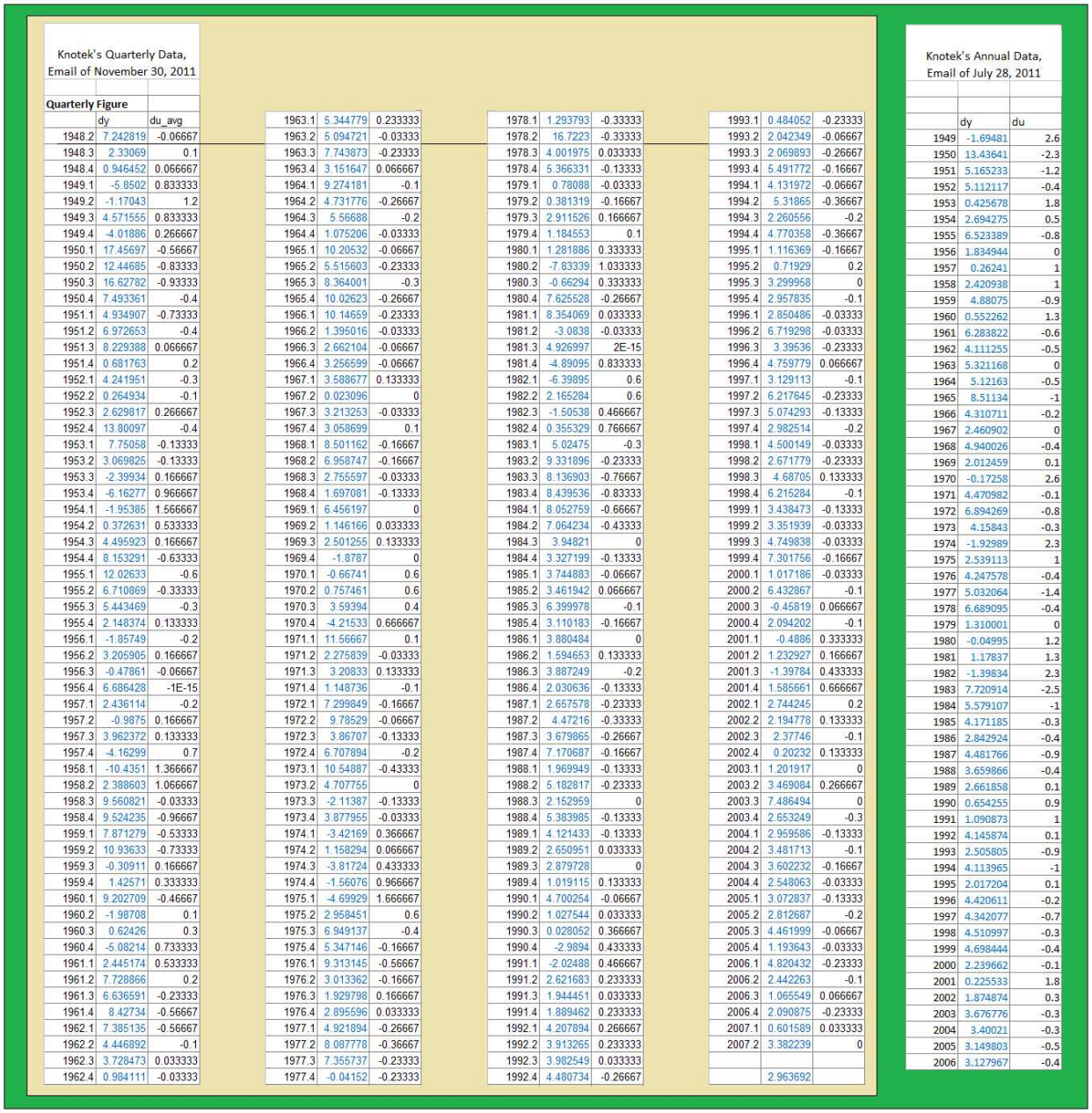


DIAGRAM 4-7. THE "GNP SPIRAL"



Scott A. Albers and Andrew L. Albers
Great Falls and Bozeman, Montana
March 7, 2013

The following two emails provide Dr. Knotek's data sets for the annualized quarterly calculation (Chart One) and annual calculation (Chart Two) of Okun's Law.



Knotek's Quarterly Data, Email of November 30, 2011

Quarterly Figure

	dy	du_avg
1948.2	7.242819	-0.06667
1948.3	2.33069	0.1
1948.4	0.946452	0.066667
1949.1	-5.8502	0.833333
1949.2	-1.17043	1.2
1949.3	4.571555	0.833333
1949.4	-4.01886	0.266667
1950.1	17.45697	-0.56667
1950.2	12.44685	-0.83333
1950.3	16.62782	-0.93333
1950.4	7.493361	-0.4
1951.1	4.934907	-0.73333
1951.2	6.972653	-0.4
1951.3	8.229388	0.066667
1951.4	0.681763	0.2
1952.1	4.241951	-0.3
1952.2	0.264934	-0.1
1952.3	2.629817	0.266667
1952.4	13.80097	-0.4
1953.1	7.75058	-0.13333
1953.2	3.069825	-0.13333
1953.3	-2.39934	0.166667
1953.4	-6.16277	0.966667
1954.1	-1.95385	1.566667
1954.2	0.372631	0.533333
1954.3	4.495923	0.166667
1954.4	8.153291	-0.63333
1955.1	12.02633	-0.6
1955.2	6.710869	-0.33333
1955.3	5.443469	-0.3
1955.4	2.148374	0.133333
1956.1	-1.85749	-0.2
1956.2	3.205905	0.166667
1956.3	-0.47861	-0.06667
1956.4	6.686428	-1E-15
1957.1	2.436114	-0.2
1957.2	-0.9875	0.166667
1957.3	3.962372	0.133333
1957.4	-4.16299	0.7
1958.1	-10.4351	1.366667
1958.2	2.388603	1.066667
1958.3	9.560821	-0.03333
1958.4	9.524235	-0.96667
1959.1	7.871279	-0.53333
1959.2	10.93633	-0.73333
1959.3	-0.30911	0.166667
1959.4	1.42571	0.333333
1960.1	9.202709	-0.46667
1960.2	-1.98708	0.1
1960.3	0.62426	0.3
1960.4	-5.08214	0.733333
1961.1	2.445174	0.533333
1961.2	7.728866	0.2
1961.3	6.636591	-0.23333
1961.4	8.42734	-0.56667
1962.1	7.385135	-0.56667
1962.2	4.446892	-0.1
1962.3	3.728473	0.033333
1962.4	0.984111	-0.03333
1963.1	5.344779	0.233333
1963.2	5.094721	-0.03333
1963.3	7.743873	-0.23333
1963.4	3.151647	0.066667
1964.1	9.274181	-0.1
1964.2	4.731776	-0.26667
1964.3	5.56688	-0.2
1964.4	1.075206	-0.03333
1965.1	10.20532	-0.06667
1965.2	5.515603	-0.23333
1965.3	8.364001	-0.3
1965.4	10.02623	-0.26667
1966.1	10.14659	-0.23333
1966.2	1.395016	-0.03333
1966.3	2.662104	-0.06667
1966.4	3.256599	-0.06667
1967.1	3.588677	0.133333
1967.2	0.023096	0
1967.3	3.213253	-0.03333
1967.4	3.058699	0.1
1968.1	8.501162	-0.16667
1968.2	6.958747	-0.16667
1968.3	2.755597	-0.03333
1968.4	1.697081	-0.13333
1969.1	6.456197	0
1969.2	1.146166	0.033333
1969.3	2.501255	0.133333
1969.4	-1.8787	0
1970.1	-0.66741	0.6
1970.2	0.757461	0.6
1970.3	3.59394	0.4
1970.4	-4.21533	0.666667
1971.1	11.56667	0.1
1971.2	2.275839	-0.03333
1971.3	3.20833	0.133333
1971.4	1.148736	-0.1
1972.1	7.299849	-0.16667
1972.2	9.78529	-0.06667
1972.3	3.86707	-0.13333
1972.4	6.707894	-0.2
1973.1	10.54887	-0.43333
1973.2	4.707755	0
1973.3	-2.11387	-0.13333
1973.4	3.877955	-0.03333
1974.1	-3.42169	0.366667
1974.2	1.158294	0.066667
1974.3	-3.81724	0.433333
1974.4	-1.56076	0.966667
1975.1	-4.69929	1.666667
1975.2	2.958451	0.6
1975.3	6.949137	-0.4
1975.4	5.347146	-0.16667
1976.1	9.313145	-0.56667
1976.2	3.013362	-0.16667
1976.3	1.929798	0.166667
1976.4	2.895596	0.033333
1977.1	4.921894	-0.26667
1977.2	8.087778	-0.36667
1977.3	7.355737	-0.23333
1977.4	-0.04152	-0.23333
1978.1	1.293793	-0.33333
1978.2	16.7223	-0.33333
1978.3	4.001975	0.033333
1978.4	5.366331	-0.13333
1979.1	0.78088	-0.03333
1979.2	0.381319	-0.16667
1979.3	2.911526	0.166667
1979.4	1.184553	0.1
1980.1	1.281886	0.333333
1980.2	-7.83339	1.033333
1980.3	-0.66294	0.333333
1980.4	7.625528	-0.26667
1981.1	8.354069	0.033333
1981.2	-3.0838	-0.03333
1981.3	4.926997	2E-15
1981.4	-4.89095	0.833333
1982.1	-6.39895	0.6
1982.2	2.165284	0.6
1982.3	-1.50538	0.466667
1982.4	0.355329	0.766667
1983.1	5.02475	-0.3
1983.2	9.331896	-0.23333
1983.3	8.136903	-0.76667
1983.4	8.439536	-0.83333
1984.1	8.052759	-0.66667
1984.2	7.064234	-0.43333
1984.3	3.94821	0
1984.4	3.327199	-0.13333
1985.1	3.744883	-0.06667
1985.2	3.461942	0.066667
1985.3	6.399978	-0.1
1985.4	3.110183	-0.16667
1986.1	3.880484	0
1986.2	1.594653	0.133333
1986.3	3.887249	-0.2
1986.4	2.030636	-0.13333
1987.1	2.657578	-0.23333
1987.2	4.47216	-0.33333
1987.3	3.679865	-0.26667
1987.4	7.170687	-0.16667
1988.1	1.969949	-0.13333
1988.2	5.182817	-0.23333
1988.3	2.152959	0
1988.4	5.383985	-0.13333
1989.1	4.121433	-0.13333
1989.2	2.650951	0.033333
1989.3	2.879728	0
1989.4	1.019115	0.133333
1990.1	4.700254	-0.06667
1990.2	1.027544	0.033333
1990.3	0.028052	0.366667
1990.4	-2.9894	0.433333
1991.1	-2.02488	0.466667
1991.2	2.621683	0.233333
1991.3	1.944451	0.033333
1991.4	1.889462	0.233333
1992.1	4.207894	0.266667
1992.2	3.913265	0.233333
1992.3	3.982549	0.033333
1992.4	4.480734	-0.26667
1993.1	0.484052	-0.23333
1993.2	2.042349	-0.06667
1993.3	2.069893	-0.26667
1993.4	5.491772	-0.16667
1994.1	4.131972	-0.06667
1994.2	5.31865	-0.36667
1994.3	2.260556	-0.2
1994.4	4.770358	-0.36667
1995.1	1.116369	-0.16667
1995.2	0.71929	0.2
1995.3	3.299958	0
1995.4	2.957835	-0.1
1996.1	2.850486	-0.03333
1996.2	6.719298	-0.03333
1996.3	3.39536	-0.23333
1996.4	4.759779	0.066667
1997.1	3.129113	-0.1
1997.2	6.217645	-0.23333
1997.3	5.074293	-0.13333
1997.4	2.982514	-0.2
1998.1	4.500149	-0.03333
1998.2	2.671779	-0.23333
1998.3	4.68705	0.133333
1998.4	6.215284	-0.1
1999.1	3.438473	-0.13333
1999.2	3.351939	-0.03333
1999.3	4.749838	-0.03333
1999.4	7.301756	-0.16667
2000.1	1.017186	-0.03333
2000.2	6.432867	-0.1
2000.3	-0.45819	0.066667
2000.4	2.094202	-0.1
2001.1	-0.4886	0.333333
2001.2	1.232927	0.166667
2001.3	-1.39784	0.433333
2001.4	1.585661	0.666667
2002.1	2.744245	0.2
2002.2	2.194778	0.133333
2002.3	2.37746	-0.1
2002.4	0.20232	0.133333
2003.1	1.201917	0
2003.2	3.469084	0.266667
2003.3	7.486494	0
2003.4	2.653249	-0.3
2004.1	2.959586	-0.13333
2004.2	3.481713	-0.1
2004.3	3.602232	-0.16667
2004.4	2.548063	-0.03333
2005.1	3.072837	-0.13333
2005.2	2.812687	-0.2
2005.3	4.461999	-0.06667
2005.4	1.193643	-0.03333
2006.1	4.820432	-0.23333
2006.2	2.442263	-0.1
2006.3	1.065549	0.066667
2006.4	2.090875	-0.23333
2007.1	0.601589	0.033333
2007.2	3.382239	0
	2.963692	

Knotek's Annual Data, Email of July 28, 2011

	dy	du
1949	-1.69481	2.6
1950	13.43641	-2.3
1951	5.165233	-1.2
1952	5.112117	-0.4
1953	0.425678	1.8
1954	2.694275	0.5
1955	6.523389	-0.8
1956	1.834944	0
1957	0.26241	1
1958	2.420938	1
1959	4.88075	-0.9
1960	0.552262	1.3
1961	6.283822	-0.6
1962	4.111255	-0.5
1963	5.321168	0
1964	5.12163	-0.5
1965	8.51134	-1
1966	4.310711	-0.2
1967	2.460902	0
1968	4.940026	-0.4
1969	2.012459	0.1
1970	-0.17258	2.6
1971	4.470982	-0.1
1972	6.894269	-0.8
1973	4.15843	-0.3
1974	-1.92989	2.3
1975	2.539113	1
1976	4.247578	-0.4
1977	5.032064	-1.4
1978	6.689095	-0.4
1979	1.310001	0
1980	-0.04995	1.2
1981	1.17837	1.3
1982	-1.39834	2.3
1983	7.720914	-2.5
1984	5.579107	-1
1985	4.171185	-0.3
1986	2.842924	-0.4
1987	4.481766	-0.9
1988	3.659866	-0.4
1989	2.661858	0.1
1990	0.654255	0.9
1991	1.090873	1
1992	4.145874	0.1
1993	2.505805	-0.9
1994	4.113965	-1
1995	2.017204	0.1
1996	4.420611	-0.2
1997	4.342077	-0.7
1998	4.510997	-0.3
1999	4.698444	-0.4
2000	2.239662	-0.1
2001	0.225533	1.8
2002	1.874874	0.3
2003	3.676776	-0.3
2004	3.40021	-0.3
2005	3.149803	-0.5
2006	3.127967	-0.4

The following states the annual measures of GNP as compared with Dr. Knotek.

Change in Annual Unemployment vs. Change in Annual GDP

Quarterly GDP, Bureau of Economic Analysis

Table 1.1.6. Real Gross Domestic Product, Chained Dollars

[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates

Quarterly data from 1947 To 2007

Bureau of Economic Analysis

Data published September 27, 2007

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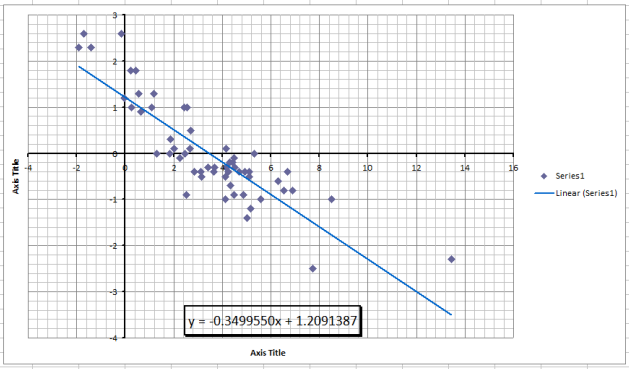
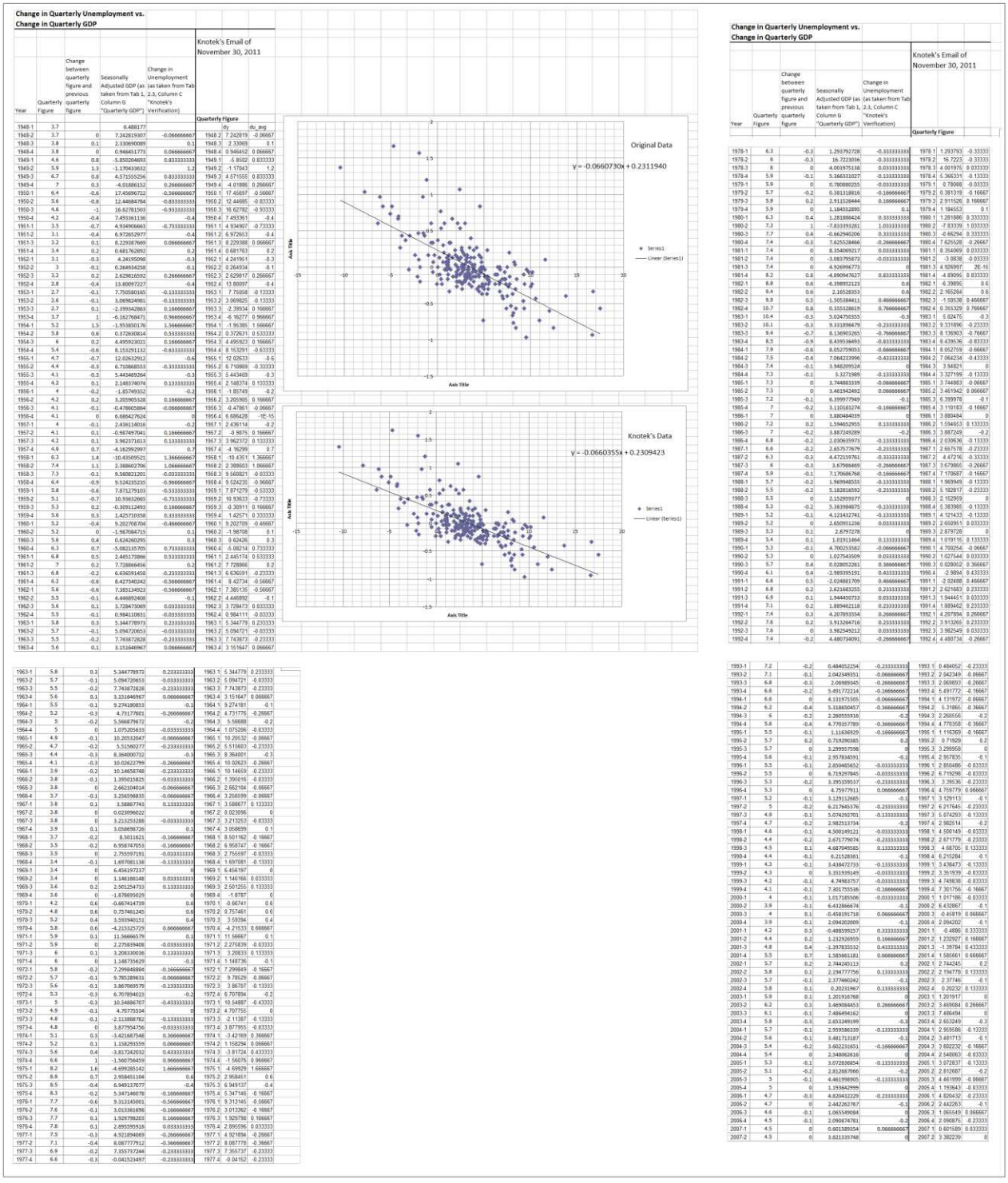
					Albers' GDP Calculations from Table 1.1.6, September 27, 2007			Knotek's Annual Data as provided through correspondence		
		Gross domestic product			Quarter 4 divided by Quarter 4	Minus 1	Multiply 100		dy	du
1947	4	1,590.9								
1948	4	1,658.0	1948	4	1.042177	n=m-1	n=mx100		dy	du
1949	4	1,629.9	1949	4	0.983052	-0.01695	-1.694813	1949	-1.69481	2.6
1950	4	1,848.9	1950	4	1.134364	0.134364	13.436407	1950	13.43641	-2.3
1951	4	1,944.4	1951	4	1.051652	0.051652	5.1652334	1951	5.165233	-1.2
1952	4	2,043.8	1952	4	1.051121	0.051121	5.1121168	1952	5.112117	-0.4
1953	4	2,052.5	1953	4	1.004257	0.004257	0.4256777	1953	0.425678	1.8
1954	4	2,107.8	1954	4	1.026943	0.026943	2.6942753	1954	2.694275	0.5
1955	4	2,245.3	1955	4	1.065234	0.065234	6.5233893	1955	6.523389	-0.8
1956	4	2,286.5	1956	4	1.018349	0.018349	1.8349441	1956	1.834944	0
1957	4	2,292.5	1957	4	1.002624	0.002624	0.2624098	1957	0.26241	1
1958	4	2,348.0	1958	4	1.024209	0.024209	2.4209378	1958	2.420938	1
1959	4	2,462.6	1959	4	1.048807	0.048807	4.8807496	1959	4.88075	-0.9
1960	4	2,476.2	1960	4	1.005523	0.005523	0.5522618	1960	0.552262	1.3
1961	4	2,631.8	1961	4	1.062838	0.062838	6.283822	1961	6.283822	-0.6
1962	4	2,740.0	1962	4	1.041113	0.041113	4.1112547	1962	4.111255	-0.5
1963	4	2,885.8	1963	4	1.053212	0.053212	5.3211679	1963	5.321168	0
1964	4	3,033.6	1964	4	1.051216	0.051216	5.1216301	1964	5.12163	-0.5
1965	4	3,291.8	1965	4	1.085113	0.085113	8.5113397	1965	8.51134	-1
1966	4	3,433.7	1966	4	1.043107	0.043107	4.3107115	1966	4.310711	-0.2
1967	4	3,518.2	1967	4	1.024609	0.024609	2.4609022	1967	2.460902	0
1968	4	3,692.0	1968	4	1.0494	0.0494	4.9400261	1968	4.940026	-0.4
1969	4	3,766.3	1969	4	1.020125	0.020125	2.0124594	1969	2.012459	0.1
1970	4	3,759.8	1970	4	0.998274	-0.00173	-0.1725832	1970	-0.17258	2.6
1971	4	3,927.9	1971	4	1.04471	0.04471	4.4709825	1971	4.470982	-0.1
1972	4	4,198.7	1972	4	1.068943	0.068943	6.8942692	1972	6.894269	-0.8
1973	4	4,373.3	1973	4	1.041584	0.041584	4.15843	1973	4.15843	-0.3
1974	4	4,288.9	1974	4	0.980701	-0.0193	-1.9298928	1974	-1.92989	2.3
1975	4	4,397.8	1975	4	1.025391	0.025391	2.5391126	1975	2.539113	1
1976	4	4,584.6	1976	4	1.042476	0.042476	4.2475783	1976	4.247578	-0.4
1977	4	4,815.3	1977	4	1.050321	0.050321	5.0320639	1977	5.032064	-1.4
1978	4	5,137.4	1978	4	1.066891	0.066891	6.6890952	1978	6.689095	-0.4
1979	4	5,204.7	1979	4	1.0131	0.0131	1.3100012	1979	1.310001	0
1980	4	5,202.1	1980	4	0.9995	-0.0005	-0.0499548	1980	-0.04995	1.2
1981	4	5,263.4	1981	4	1.011784	0.011784	1.1783703	1981	1.17837	1.3
1982	4	5,189.8	1982	4	0.986017	-0.01398	-1.3983357	1982	-1.39834	2.3
1983	4	5,590.5	1983	4	1.077209	0.077209	7.7209141	1983	7.720914	-2.5
1984	4	5,902.4	1984	4	1.055791	0.055791	5.5791074	1984	5.579107	-1
1985	4	6,148.6	1985	4	1.041712	0.041712	4.1711846	1985	4.171185	-0.3
1986	4	6,323.4	1986	4	1.028429	0.028429	2.8429236	1986	2.842924	-0.4
1987	4	6,606.8	1987	4	1.044818	0.044818	4.4817661	1987	4.481766	-0.9
1988	4	6,848.6	1988	4	1.036599	0.036599	3.6598656	1988	3.659866	-0.4
1989	4	7,030.9	1989	4	1.026619	0.026619	2.6618579	1989	2.661858	0.1
1990	4	7,076.9	1990	4	1.006543	0.006543	0.6542548	1990	0.654255	0.9
1991	4	7,154.1	1991	4	1.010909	0.010909	1.0908731	1991	1.090873	1
1992	4	7,450.7	1992	4	1.041459	0.041459	4.1458744	1992	4.145874	0.1
1993	4	7,637.4	1993	4	1.025058	0.025058	2.5058048	1993	2.505805	-0.9
1994	4	7,951.6	1994	4	1.04114	0.04114	4.1139655	1994	4.113965	-1
1995	4	8,112.0	1995	4	1.020172	0.020172	2.0172041	1995	2.017204	0.1
1996	4	8,470.6	1996	4	1.044206	0.044206	4.4206114	1996	4.420611	-0.2
1997	4	8,838.4	1997	4	1.043421	0.043421	4.3420773	1997	4.342077	-0.7
1998	4	9,237.1	1998	4	1.04511	0.04511	4.5109975	1998	4.510997	-0.3
1999	4	9,671.1	1999	4	1.046984	0.046984	4.6984443	1999	4.698444	-0.4
2000	4	9,887.7	2000	4	1.022397	0.022397	2.2396625	2000	2.239662	-0.1
2001	4	9,910.0	2001	4	1.002255	0.002255	0.2255327	2001	0.225533	1.8
2002	4	10,095.8	2002	4	1.018749	0.018749	1.8748739	2002	1.874874	0.3
2003	4	10,467.0	2003	4	1.036768	0.036768	3.6767765	2003	3.676776	-0.3
2004	4	10,796.4	2004	4	1.03147	0.03147	3.1470335	2004	3.40021	-0.3
2005	4	11,107.2	2005	4	1.028787	0.028787	2.8787374	2005	3.149803	-0.5
2006	4	11,395.5	2006	4	1.025956	0.025956	2.5956137	2006	3.127967	-0.4



The following states the quarterly data for annualized real GNP and quarterly employment, as contrasted with that of Dr. Knotek.



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See also the figures for Real GNP, 1947 to present, maintained by the St. Louis Federal Reserve at http://research.stlouisfed.org/fred2/series/GNPC96.

