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The Intertemporal Cross Price Behavior of Common Stocks: Evidence and Implications

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In a strictly efficient securities market, autocorrelations in the returns of individual securities as well as intertemporal (noncontemporaneous) correlations between pairs of securities' returns should not exist since the securities' prices would adjust fully to new information as soon as it reaches the market. In this case, changes in the prices of individual securities should be independent through time, and autocorrelation will be statistically insignificant since the arrival of new information is a random process. Also, since securities' prices would adjust fully to new information immediately, securities' returns will covary contemporaneously with one another, the intertemporal cross correlations between the returns of securities will be statistically insignificant, and the price movement of any security will neither lead nor lag the price movements of other securities.

The empirical search for the presence of autocorrelation in securities' returns has received considerable attention in the literature starting with Fama's seminal work [5]. Fama investigated the daily price behavior of a sample of 30 common stocks traded on the New York Stock Exchange (*NYSE*) and concluded that they did not exhibit significant autocorrelation [2].

The possible existence of intertemporal cross correlation, however, has not yet been investigated. This paper presents evidence on the presence and causes of daily intertemporal cross correlations among the returns of a value-stratified sample of 50 *NYSE* common stocks and the market (*S&P* 500) and discusses the implications of these findings for empirical work in finance. It is shown that the existence of intertemporal cross correlations is a sufficient condition to explain various phenomena reported in the literature such as positive autocorrelation in market indexes, the sensitivity of estimated systematic risk and other parameters of the capital asset pricing model to changes in the length of the differencing interval over which security returns are measured, and the existence of intertemporal systematic risks in the daily price movements of common stocks.

The meaning of statistically significant intertemporal cross correlation is that the price movements of securities are not contemporaneous; that is, they do not change in unison since some securities may lag behind and others may lead the general market movement. At this point two observations should be made. First, intertemporal cross correlations can be present among securities' returns even if the price movement of each security is **not** autocorrelated. Hence, the absence of autocorelation does not rule out the presence of intertemporal cross correlations. Second, intertemporal cross correlations may exist in an economically efficient market. In this case the intertemporal cross correlations may not be strong enough to enable market participants to formulate abnormally profitable trading strategies. However, they may be strong enough, for example, to affect the estimated value of a security's systematic risk, and to produce positive autocorrelation in market indexes composed of intertemporally cross correlated securities.

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The next section presents various processes that can explain the existence of intertemporal cross correlations. In Section II, a method to measure the intertemporal cross dependence between two time-series is developed. Section III reports evidence of the presence of intertemporal cross correlations between the daily price movements of *NYSE* securities and the movements of the *S & P-500* stock market index. Sections IV and V examine the relationship between intertemporal cross dependence and selected characteristics of the securities. The purpose is to determine whether securities with large market values and a high frequency of trades exhibit a different relationship with the market compared to issues with thinner markets. Section VI examines how the presence of intertemporal cross correlations can explain various phenomena reported in the literature. Finally, Section VII presents concluding remarks.

I. An Explanation for Intertemporal Cross Correlations

A plausible explanation for the existence of noncontemporaneous cross correlations is based on the speed-of-price-adjustment hypothesis. For an explanation of intertemporal cross correlations based on the microstructure of securities market, i.e., various "frictions" in the trading system, see Cohen et al. [3], [4]. Suppose that new information arrives in the market place that raises the price of securities and the value of the market index. Some securities may adjust fully to the new information on the day it reaches the market while others may not adjust fully on the same day. In comparison to the general market, the securities which do not adjust fully on the day news reaches the market will display lagging behavior. The others will display leading behavior. It should be noted that only the lagging behavior is "real." The leading securities do not anticipate the news; they simply exhibit leading behavior because the market is an average of all securities. If some securities lag behind the average, others will lead it.

A question that remains is why different securities have different rates of price adjustment. Three possible explanations are as follows. First, frequently-traded securities may appear to absorb information faster as their prices are recorded more frequently. Second, the speed-of-price-adjustment for a security might be related to the securities' "clientele." Securities attracting a sophisticated clientele may tend to adjust more quickly than securities with a less sophisticated clientele. Last, the market index is composed of the recorded prices at the last transaction for each security. Thus, the index is nonsynchronous with its own components causing lead-lag behavior.

II. Measuring Intertemporal Cross Correlations: The Time-Covariance Function

This section develops a measure of intertemporal cross dependence based on the properties of the Time-Covariance function (*T-C* function). This function also provides an analytical framework which can be used to investigate the implications of intertemporal cross correlations for empirical work in finance.

The *T-C* function establishes the relationship between the time interval over which changes in two random variables are measured and the covariance of these changes. Suppose that the distributions of common stock returns are stationary with finite variance, and that the returns are measured as logarithms of price relatives over differencing intervals (holding periods) of varying length. In this case, returns measured over differencing intervals of T days,[1] $R_{tT} = \ln[(P_t + D_t)/P_{t-T}]$, are expressed as the sum of daily returns, $R_{t1} = r_t = \ln[(P_t + D_t)/P_{t-1}]$, or $R_{tT} = \sum_{k=0}^{T-1} r_{t-k}$. We have shown elsewhere (see Hawawini [10], [11]) that the following relationship holds between the T-day and 1-day covariances:

$$\sigma_{im}(T) = \sigma_{im}\,[T + \sum_{s=1}^{T-1} r_{t-k}(T-s)\, q_{im}^{s}] \tag{1}$$

where:

$$q_{im}^{s} = \frac{(\rho_{im}^{+s} + \rho_{im}^{-s})}{\rho_{im}} = \text{the } q\text{-ratio of order-}s\text{ for security-}i. \tag{2}$$

with: $\sigma_{im}(T)$ = the covariance between the i-th security returns and those of a market index (m), measured over differencing intervals of a length of T days,

σ_{im} = the covariance between securities and market returns, measured over daily differencing intervals,

T = the length of the differencing interval in days,

s = a positive integer such that $1 \le s \le T-1$,

ρ_{im}^{+s} & ρ_{im}^{-s} = the intertemporal cross correlation coefficient in daily returns of order $+s$ and $-s$ for which the returns of the i-th security lead ($+s$) or lag ($-s$) those of a market index, respectively,

ρ_{im} = the contemporaneous cross correlation coefficient in daily returns,

q_{im}^{s} = the q-ratio of order s for the i-th security defined in equation (2) as the sum of the lead and lag intertemporal cross correlation coefficients of order s divided by the contemporaneous cross correlation coefficient.

Note that the T-C function is a generalization of the Time-Variance function which can be derived from equation (1) by substituting $m = i$. For an alternate derivation of the Time-Variance function see Schwartz and Whitcomb [18].
In this case:

$$\sigma_{ii}(T) = \sigma_i^2(T) = \sigma_i^2\,[T + 2\sum_{s=1}^{T-1}(T-s)\rho_i^s] \tag{3}$$

where ρ_i^s is the autocorrelation of order s for the daily returns of the i-th security. The parallel nature of the Time-Variance function (3) and the Time-Covariance function (1) implies that the q-ratio of equation (2) is an appropriate measure of intertemporal cross dependence between two time-series. Note that a security's q-ratio with respect to itself is equal to twice its autocorrelation, $q_{ii} = 2\rho_i$. One should also

[1]In this study the shortest differencing interval is a day. However, the choice of the minimum length of the return interval is arbitrary and will depend on the nature of the investigation.

note that the strength and effects of intertemporal cross dependence as measured with the q-ratio are **per unit** of contemporaneous cross dependence and **not** with intertemporal cross correlation coefficients unadjusted for contemporaneous strength. The validity of this result is tested in Sections IV and V.

III. Evidence from the New York Stock Exchange

Evidence on the presence of intertemporal cross correlations between daily returns of the *S & P-500* and a sample of 50 common stocks listed on the *NYSE* for the period 1 January 1970 to 31 December 1973 is given in Table 1.[2] The list of the fifty firms appears in Table 1 in ascending order of their market value of shares outstanding as of 31 December 1971, the sample period's midpoint. The daily closing prices were adjusted for stock dividends and splits, and cash dividends were added when generating the logarithms of price relatives.

The important results of Table 1 can be summarized as follows. First, the intertemporal cross correlations are generally positive, that is, the lag and lead structure of securities' returns has the same direction as the general market movement. Second, these correlations are all positive and statistically significant for the first order lag. Third, they are generally stronger and more prevalent for lags than for leads. Fourth, they are weaker and less prevalent the higher their order. Last, they are never stronger than their corresponding contemporaneous ($s=0$) cross correlation.

From the information given in Table 1, one can easily compute the value of a security's q-ratios. For example, for the first and the last firms listed in Table 1, the

$$\begin{aligned} &\text{Wayne Gossard:} \quad q\text{-ratio} = \frac{.060 + .106}{.143} = 1.161 \\ &\text{Eastman Kodak:} \quad q\text{-ratio} = \frac{.189 + .094}{.626} = 0.452 \end{aligned} \tag{4}$$

All the securities have significant first order q-ratios, and these are greater than their corresponding q-ratios of higher orders. For the market index (*S & P-500*) the first order q-ratio is:

$$S\,\&\,P\text{-}500\text{: } q\text{-ratio} = \frac{\rho_m^{-1} + \rho_m^{+1}}{1} = 2\rho_m = 2(.285) = .570 \tag{5}$$

where $\rho_m^{-1} = \rho_m^{+1} =$ the first order autocorrelation coefficient, equal to .285, over the period same as that used to estimate securities' intertemporal cross correlations.

[2]The common stocks listed on the NYSE throughout the 4-year interval were ranked according to the market value of shares outstanding as of the last trading day of 1971 and stratified into deciles. A random sample of five securities was obtained from each decile yielding a sample of 50 securities.

TABLE 1. Evidence of Intertemporal Cross-Correlation Between Securities and the Market Daily Return

	Order of Intertemporal Cross-correlations																
	$s=0$	$s=+1$	$s=-1$	$s=+2$	$s=-2$	$s=+3$	$s=-3$	$s=+4$	$s=-4$	$s=+5$	$s=-5$	$s=+10$	$s=-10$	$s=+15$	$s=-15$	$s=+20$	$s=-20$
1. Wayne Gossard	.143*	.060	.106*	.006	.003	.101	.036	.056	−.020	−.031	.053	−.078*	.003	−.037	.008	.026	−.039
2. Washington Steel	.173*	.044	.157*	−.001	.025	−.022	.035	.027	.049	.001	−.037	.015	.014	−.015	−.013	.051	−.044
3. Michigan Seamless Tube	.165*	.041	.177*	.010	.071*	.002	.051	.106*	.025	.034	−.011	−.032	.007	−.044	.010	−.055	.014
4. Keystone Cons. Ind.	.127*	.014	.067*	−.104*	−.054	−.012	−.010	.006	−.002	−.007	.018	.011	.016	−.044	.001	−.012	.048
5. Dictaphone Corp.	.330*	.124*	.098*	−.008	−.005	.004	.038	.084*	.005	.001	.072*	−.057	−.041	−.077*	−.055	−.005	−.037
6. Dymo Industries	.328*	.057	.224*	−.039	.068*	−.060	.060	−.006	.066*	.001	.078*	−.027	−.066*	−.069*	−.028	−.047	.027
7. Publicker Industries	.281*	.018*	.150*	.055	.032	.011	−.032	.030	.044	−.006	.049	−.025	−.002	.025	−.064*	−.010	.028
8. Great Western United	.233*	.096*	.131*	.057	.012	.028	.031	.045	.007	−.001	−.014	−.023	.027	−.019	.004	−.012	.006
9. Allied Products Corp.	.217*	.057	.108*	.013	−.006	−.011	.059	.010	−.006	−.002	.020	−.017	.031	−.026	.008	−.019	−.022
10. Copperweld Steel	.298*	.107*	.160*	.017	.008	−.064*	.000	−.001	.016	−.011	.005	.011	.042	−.036	.037	−.003	.024
11. Family Finance	.248*	.046	.200*	−.059	−.031	−.052	−.004	.054	.035	.016	.009	−.022	−.010	−.021	−.068*	.016	−.022
12. Koehring Co.	.285*	.085*	.144*	.043	.103*	.002	.016	−.034	−.018	−.023	.025	−.023	−.051	.035	.035	.004	.036
13. Bobbie Brooks	.339*	.100*	.108*	.034	−.016	−.023	.023	.028	.003	.034	−.003	.008	−.041	−.006	−.032	−.003	−.002
14. USM Corp.	.224*	.085*	.142*	−.067*	.048	.017	.013	−.024	.035	.014	.030	.008	.024	.022	.031	−.007	−.034
15. Monogram Ind. Inc.	.459*	.166*	.119*	.037	.039	.049	.042	.034	.054	−.025	.078*	−.058	−.001	−.022	−.074*	−.029	.005
16. Allegheny Ludlum	.333*	.046	.196*	−.014	.017	−.013	.012	−.004	.035	−.001	.034	−.018	−.004	.001	−.001	−.036	−.026
17. Faberge Inc.	.337*	.134*	.087*	.032	−.056	.080*	−.005	.061*	.032	−.026	−.028	−.021	−.046	−.053	−.009	.027	.054
18. Alleghany Corp.	.442*	.087*	.190*	.051	.037	.051	.021	−.020	.010	.031	−.010	−.031	−.015	.004	−.005	−.053	−.029
19. Hammerhill Paper	.232*	.057	.180*	.023	.053	−.008	.029	−.022	−.018	−.060	.003	−.058	.005	.008	.010	−.017	−.050
20. Eagle Picher Inds.	.204*	.064*	.194*	−.041	.124*	−.015	.097*	−.032	.038	.029	−.026	.003	−.027	−.040	−.055	.038	.008
21. American Sterilizer	.322*	.073*	.142*	.056	.050	.053	.004	.014	.035	.026	.010	.000	−.012	.013	−.001	.041	−.003
22. Maryland Cup Corp.	.296*	.083*	.264*	.022	.137*	−.043	.081*	−.031	.042	−.028	.047	.039	.038	.046	.026	−.006	.009
23. Benguet Consolidated	.282*	.104*	.173*	−.046	.038	−.004	−.035	−.021	−.010	.047	.035	.056	−.051	−.041	−.029	−.028	−.060
24. Dillingham Corp.	.192*	.047	.119*	−.006	.024	.011	.017	.022	−.018	.034	.058	.033	.031	.026	.038	.016	−.011
25. Vornado Inc.	.391*	.124*	.225*	.008	.039	.007	−.008	.057	.008	.019	−.020	−.046	−.043	−.016	.013	.024	−.099*

Continued

TABLE 1. (Continued)

	Order of Intertemporal Cross-correlations																
	$s=0$	$s=+1$	$s=-1$	$s=+2$	$s=-2$	$s=+3$	$s=-3$	$s=+4$	$s=-4$	$s=+5$	$s=-5$	$s=+10$	$s=-10$	$s=+15$	$s=-15$	$s=+20$	$s=-20$
26. Big Three Ind.	.355*	.090*	.219*	.001	.040	.032	.079*	-.005	.067*	-.003	.046	-.020	.012	-.048	.012	-.011	-.024
27. Thomas & Betts Corp.	.245*	.221*	.063*	.112*	.039	-.055	.086*	.127*	-.040	-.034	.036	-.011	.050	.025	-.002	-.006	-.001
28. Cleveland Cliffs	.301*	.083*	.296*	.020	.191*	.025	.072*	-.081	.047	-.052	.090*	-.068*	-.011	-.068*	.034	.010	-.016
29. Idaho Power Co.	.203*	.113*	.157*	.031	.035	-.004	.006	-.001	.007	-.001	-.016	.066*	-.010	-.034	-.065*	-.019	.009
30. Cabot Corp.	.324*	.037	.175*	.007	.055	-.002	-.015	-.040	.009	-.037	.037	-.067*	.032	.023	.076	.045	-.005
31. General Development	.328*	.132*	.101*	.001	-.022	-.027	.033	-.012	.004	-.055	.097*	-.027	.043	-.013	-.022	-.031	-.015
32 N.Y.S. Gas & Elec.	.205*	.064*	.086*	-.017	.057	.053	-.006	.043	.024	.002	.010	.026	.063*	.019	-.031	.008	.003
33. Addresso-Multigraph	.404*	.168*	.074*	.056	-.014	-.002	.043	.027	.024	-.016	.075*	-.027	.035	.088*	-.005	.026	-.026
34. Texas Oil & Gas	.383*	.073*	.241*	-.017	.071*	-.018	.029	-.015	.001	.035	-.053	.013	-.059	-.010	.045	-.037	.001
35. Trans Union Corp.	.312*	.073*	.154*	.002	.043	-.048	.083*	-.019	.050	-.007	.060	-.025	.024	.008	-.057	-.042	-.008
36. Great Western Finance	.546*	.180*	.176*	.048	.027	.054	.000	-.007	.010	-.056	-.006	-.085	-.085	-.007	-.087*	-.011	-.029
37. Pacific Lighting	.190*	.049	.118*	-.006	.027	.010	.031	.019	.003	.003	-.041	.019	-.016	.005	.020	.006	-.040
38. Great Atl. & Pac. Tea	.215*	.041	.112*	-.049	.031	-.082*	.006	-.007	-.015	-.032	.009	.033	.044	-.025	-.045	-.007	-.019
39. Genuine Parts Co.	.338*	.077*	.231*	-.019	.079*	-.068*	.041	-.025	.023	-.060	-.039	-.075*	-.053	-.061	-.030	-.010	.010
40. Union Electric Co.	.189*	.040	.063*	.013	.011	.015	.021	.077*	.050	.025	-.045	-.007	-.011	-.015	-.027	-.016	-.016
41. Square D Co.	.324*	.089*	.099*	-.037	.057	.016	.014	-.024	.055	-.004	.008	-.003	-.048	-.007	.008	-.053	.012
42. Borden Inc.	.330*	.092*	.142*	.013	.075*	-.006	.070*	.020	.046	.012	.004	-.024	.004	-.042	.023	.030	-.051
43. Colgate-Palmolive	.334*	.047	.164*	.015	.041	-.034	.046	-.090*	-.014	.012	-.014	.031	-.016	-.044	.040	.020	-.001
44. Aluminum Co. of Amer.	.457*	.112*	.148*	-.024	.040	-.003	.012	-.048	.028	-.082*	.032	-.030	-.011	-.073*	-.009	-.042	-.033
45. Searle, G.D.	.390*	.128*	.194*	.034	.064*	.012	.071*	-.032	.000	.038	.000	-.057	-.061	-.062*	-.006	.021	-.014
46. Pacific Gas & Electric	.353*	.164*	.137*	.047	.058	.002	.037	-.015	.014	-.015	.026	-.025	-.042	-.006	-.001	.053	-.032
47. Shell Oil Co.	.394*	.110*	.198*	-.002	.049	-.035	-.018	-.037	.037	-.025	.049	-.073*	.026	.044	.007	-.071	-.033
48. Kresge, S.S. Co.	.502*	.168*	.137*	.083*	-.004	.022	-.020	-.020	.010	.046	.016	-.031	-.037	-.036	-.117*	.008	-.049
49. American Home Prods.	.498*	.112*	.221*	.032	.100*	-.019	-.004	-.054	-.068*	-.028	-.045	-.067*	-.041	-.025	.009	-.032	-.041
50. Eastman Kodak Co.	.626*	.189*	.094*	-.013	.013	-.049	-.029	.004	-.029	.022	-.024	-.075*	-.097*	-.041	-.052	-.061*	-.084*
% of significant correlation	100%	72%	100%	8%	22%	8%	16%	14%	4%	2%	12%	16%	4%	10%	12%	2%	4%

* Astericks indicate significant correlations at the .05 level. The critical value = .064.

IV. The Relationship Between the Intertemporal Cross Correlations and Characteristics of Securities

Securities which adjust fully to new information should display weaker intertemporal cross correlations with the market compared to securities that do not adjust fully to new information. A security's speed-of-price-adjustment could be considered to be a function of the market value of its shares outstanding (*MVSO*) or the market value of its shares traded (*MVST*). Either measure can be justified on the assumption that securities with large *MVSO* and frequent trading adjust to new information faster than securities with smaller *MVSO* and infrequent trading. A high frequency of trading may indicate that prices are adjusted to new information more often.

Under the above assumptions one should expect a security's q-ratio to be inversely related to both its *MVSO* and *MVST*. Securities with relatively large *MVSO* and *MVST* should display relatively low q-ratios since the magnitude of their intertemporal cross correlation per unit of contemporaneous cross correlation should be smaller. One should also expect this inverse relationship to be more significant with *MVST* than with *MVSO* since the former indirectly includes trading frequency while the latter does not. Although an inverse relationship can be predicted on the basis of the speed-of-price-adjustment hypothesis, *a priori* one cannot determine the functional form relating these two variables. Thus, in addition to testing for a linear relationship between q-ratios and *MVSO* or *MVST*, several common transformations were also tested to determine if a better fit could be obtained. The transformations tested were the logarithmic, hyperbolic, exponential, and ordinary power functions.

The empirical results are summarized in Table 2. *MVSO* is measured as of 31 December 1971, the sample period's midpoint. *MVST* is obtained by multiplying a share's price on 31 December 1971 by the number of shares traded during the month of December 1971. Of all the functional relationships tested, q-ratio as the hyperbolic function of *MVSO* or *MVST* produced the best fit; that is, the reciprocal of q-ratio as a linear function of those variables. As expected, there is an inverse relationship and it is more significant with *MVST* and *MVSO*. Other proxies such as number of shares traded and number of shares outstanding were also tested. They generated poor results.

In order to verify the validity of the hypothesis set forth in Section III, according to which one should use q-ratios rather than the individual values of intertemporal cross correlations as a proxy for the speed-of-price-adjustment, regressions were run for four proxies. These were $(\rho_{im}^{-1} / \rho_{im})$, $(\rho_{im}^{+1} / \rho_{im})$, (ρ_{im}^{-1}), and (ρ_{im}^{+1}) as dependent variables and *MVST* or *MVSO* as independent variables. The second, third, and fourth dependent variables produced statistically insignificant fits regardless of the functional form of the equation or the independent variable used. The first variable, however, produced better fits than the q-ratios. This result may be due to the fact that for 100 percent of the securities ρ_{im}^{-1} is significant whereas the corresponding proportion for ρ_{im}^{+1} is only 72 percent (Table 1). Also, the q-ratio is an equally weighted sum of these two variables (equation 2). These results are reported in lines 3 and 4 of Table 2. Thus, as equation (1) implies, the intertemporal cross dependence should not be measured by the cross-serial correlation itself, but by the relative magnitude of this correlation *vis-a-vis* the contemporaneous cross correlation.

TABLE 2. Cross-Sectional Regression Results Based on First Order q-ratios.

(1)
$$q^{1}_{im} = \frac{\rho^{+1}_{im} + \rho^{-1}_{im}}{\rho_{im}} = \frac{1}{1.1708 + 1.4173\,(10^{-6})\,(MVS\,T_i)}$$
Coefficient of determination = .346
F-ratio = 25.4

(2)
$$q^{1}_{im} = \frac{\rho^{+1}_{im} + \rho^{-1}_{im}}{\rho_{im}} = \frac{1}{1.2324 + 6.5791\,(10^{-8})\,(MVSO_i)}$$
Coefficient of determination = .227
F-ratio = 14.1

(3)
$$\frac{\rho^{-1}_{im}}{\rho_{im}} = \frac{1}{1.8324 + 5.3879\,(10^{-6})\,(MVST_i)}$$
Coefficient of determination = .410
F-ratio = 33.4

(4)
$$\frac{\rho^{-1}_{im}}{\rho_{im}} = \frac{1}{2.0446 + 2.7784\,(10^{-7})\,(MVSO_i)}$$
Coefficient of determination = .332
F-ratio = 23.9

Note:
$MVSO$ = Market Value of Shares Outstanding.
$MVST$ = Market Value of Shares Traded.

V. The Relationship Between the Intertemporal Cross Correlations and the Characteristics of Securities: General Case

In the previous section, examination of the relationship between securities' intertemporal cross correlations and characteristics such as $MVST$ and $MVSO$ is based only on the first order q-ratio. However, this approach ignores intertemporal cross correlations of higher orders. This section uses the properties of the T-C function to derive a *single* estimate of the *complete* pattern of intertemporal cross correlations.

In the absence of intertemporal cross correlations in daily returns, a security's q-ratios are zero and the T-C function of equation (1) reduces to:

$$\sigma^{*}_{im}(T) = T\sigma_{im} \tag{6}$$

where the asterisk is associated with zero q-ratios. In this case the T-day covariance is a linear function of the length T of the differencing interval. Any deviation from this *pure* T-C function will indicate a presence of intertemporal cross correlations. There may exist a particular nonzero pattern on intertemporal cross correlations for which $\sigma_{im}(T) = T\sigma_{im}$. However, this is unlikely. Assuming that nonzero intertemporal cross correlations exist only up to the k-th order, with $T > k+1$, then from the

T-C function we get:

$$\lambda_{im}(T) = \frac{\sigma_{im}(T)}{\sigma_{im}} = (1 + \sum_{s=1}^{k} q_{im}^{s})\, T \text{-} (\sum_{s=1}^{k} sq_{im}^{s}) \tag{7}$$

and:

$$\lambda^{*}_{im}(T) = T \qquad \text{if} \quad q_{im}^{s} = 0 \qquad \text{for all } s \geq 1. \tag{8}$$

From Table 1, a k-value of five trading days is sufficiently long to ensure that the q-ratios of orders higher than 5 are statistically insignificant. In order to detect any significant deviations from the pure *T-C* function, the following two-step test can be performed. First, T-day covariances are estimated, for a given security, over varying lengths of the differencing interval and the corresponding $\lambda_{im}(T)$ ratios are computed by dividing the T-day covariances by the daily covariance. For the analysis, fourteen differencing intervals are used. They are of 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, and 20 days. Second, a linear regression is run between $\lambda_{im}(T)$ and the length T of the differencing interval using the model:

$$\lambda_{im}(T) = a_i + b_i \cdot T + e_i(T)\,. \tag{9}$$

In the absence of intertemporal cross correlations in daily returns, the intercepts a_i for all i should be equal to zero and the slopes b_i should be equal to one.[3] Any deviation from the pure *T-C* function will indicate the presence of positive (negative) intertemporal cross correlations if $b_i > 1$ ($b_i < 1$) and/or $a_i < 0$ ($a_i > 0$). If $b_i > 1$, then the sum of the q-ratios is positive. This will be evidence of positive intertemporal cross correlations only if $\rho_{im} > 0$; see equation (2); this being the case for all the fifty stocks in the sample. There may exist a pattern of daily intertemporal cross correlation coefficients of various orders with alternate signs for which $b_i = 1$ or $a_i = 0$. However, if such an intertemporal cross correlation pattern existed, it could not simultaneously result in $b_i = 1$ and $a_i = 0$. This is because if the intercepts $a_i = 0$, that is, $\sum_{s=1}^{k} sq_{im}^{s} = 0$, then $\sum_{s=1}^{k} q^{s}_{im} \neq 0$ and the slopes β_i must be different from one unless $q_{im}^{s} = 0$ for all s. Alternatively, if the slopes $b_i = 1$, then the intercepts a_i must be different from zero unless $q_{im}^{s} = 0$ for all s. Consequently, one can use either the estimated intercept a_i or the estimated slope b_i as a single measure of the *complete* pattern of a security's intertemporal cross correlation with the market movement.

The regression statistics for the fifty securities in the sample are presented in Table 3. Changes in the length T of the differencing interval explain, on average, 95 percent of the variation in individual security's $\lambda_{im}(T)$ ratios. The reported t-statistics indicate that all the estimated slopes $\hat{b}_i$ are significantly greater than one, and that 40 percent of the estimated intercepts are significantly negative at the 5 percent level of

[3]Comparing eqs. (7) and (9) we see:

$$\alpha_i = -(\sum_{s=1}^{k} sq_{im}^{s}) \text{ and } b_i = (1 + \sum_{s=1}^{k} q_{im}^{s}).$$

TABLE 3. Regression Results

Name	Y Intercept	T-Stat. (Inter. = 0)	Slope	T-Stat. (Slope = 1)	R Squared
Eastman Kodak Co.	0.064	0.0533	1.2430	2.820	0.94548
American Home Products	0.085	0.0343	1.5029	2.856	0.85856
Kresge, S.S. Co.	0.515	0.4169	1.6096	6.924	0.96534
Shell Oil Co.	−3.979	−2.3140	2.3940	11.373	0.96950
Pacific Gas & Electric	−5.931	−2.9585	2.8803	13.159	0.97132
Searle, G.D.	−0.839	−0.3181	2.2162	6.474	0.92062
Aluminum Co. Amer.	−1.716	−1.1265	1.7889	7.265	0.95765
Colgate-Palmolive Corp.	−2.448	−1.3056	2.1622	8.695	0.95615
Borden Inc.	−8.096	−3.7035	3.5202	16.176	0.97703
Square D Co.	−2.235	−0.8597	2.3708	7.397	0.93169
Union Electric Co.	2.045	0.3449	2.1531	2.729	0.68394
Genuine Parts Co.	1.430	0.5920	1.7486	4.348	0.89577
Great A & P Tea Co.	−5.524	−2.9152	2.4131	10.463	0.96377
Pacific Lighting	−8.776	−4.0363	3.7048	17.453	0.97943
Great Western Finance	0.978	0.3886	1.4501	2.508	0.84478
Trans Union Corp.	−4.108	−1.5231	2.9129	9.950	0.95032
Texas Oil & Gas Corp.	−0.404	−0.1241	2.0139	4.367	0.86243
Addresso-Multigraph	−5.071	−3.1624	2.4883	13.022	0.97531
N.Y. State Gas & Electric	−5.870	−2.2016	3.1191	11.151	0.95735
General Development	−2.829	−2.8468	2.3143	18.558	0.98889
Cabot Corp.	−0.332	−0.1774	1.7257	5.445	0.93320
Idaho Power Co.	−5.592	−3.8501	3.3952	23.139	0.98897
Cleveland Cliffs Iron	−3.170	−1.7934	3.3852	18.934	0.98365
Thomas & Betts Corp.	−11.131	−4.6841	4.6377	21.479	0.98425
Big Three Inds. Inc.	−0.965	−0.2014	2.7351	5.080	0.84235
Vornado Inc.	−4.315	−2.7291	2.7400	15.440	0.98010
Dillingham Corp.	−10.425	−3.0945	3.9792	12.408	0.95814
Benguet Consolidated Inc.	−4.415	−2.6917	2.6087	13.762	0.97647
Maryland Cup Corp.	−8.571	−2.8563	4.2918	15.391	0.97106
American Sterilizer	−3.781	−1.5296	2.7857	10.137	0.95421
Eagle Picher Inds. Inc.	−6.740	−1.4545	3.7387	8.292	0.91437
Hammerhill Paper Co.	−7.210	−1.8122	3.5536	9.006	0.92903
Alleghany Corporation	−4.577	−5.3536	2.7536	28.781	0.99416
Faberge Inc.	−2.156	−1.3111	2.2549	10.710	0.96861
Allegheny Ludlum Inds.	−2.149	−2.1969	1.9823	14.093	0.98538
Monogram Ins. Inc.	−1.825	−1.0607	2.3036	10.628	0.96709
USM Corp.	−6.763	−2.2452	3.4222	11.282	0.95490
Bobbie Brooks Inc.	−2.433	−1.5258	2.4208	12.501	0.97423
Koehring Co.	−0.071	−0.0314	2.1711	7.284	0.93825
Family Finance Corp.	−2.396	−1.2366	2.0974	7.946	0.95054
Copperweld Steel Co.	−3.926	−1.7705	2.5671	9.916	0.95649
Allied Products Corp.	−3.505	−3.3711	2.5048	20.309	0.98961
Great Western United	−6.048	−3.1937	3.4339	18.032	0.98180
Publicker Inds.	−2.674	−1.4265	2.5606	11.682	0.96837
Dymo Industries	0.141	0.1018	1.8236	8.329	0.96591
Dictaphone Corp.	−1.872	−0.7312	2.3401	7.344	0.93201
Keystone Cons. Ind. Inc.	−3.290	−0.8196	2.7103	5.977	0.88202
Michigan Seamless Tube	−5.565	−2.1805	3.9694	16.323	0.97542
Washington Steel Corp.	−1.316	−0.4768	2.5839	8.053	0.93498
Wayne Gossard Corp.	−3.910	−1.4130	3.2783	11.551	0.95837

TABLE 4. Cross-Sectional Regression Results Based on the T-C Function.

(1)
$$\hat{a}_i = \frac{1}{-.7616 + 1.0956\,(10^{-6})\,(MVSO_i)}$$
Coefficient of determination = .490
F-ratio = 46.1

(2)
$$\hat{a}_i = \frac{1}{-1.2285 + 1.667\,(10^{-5})\,(MVST_i)}$$
Coefficient of determination = .373
F-ratio = 28.6

(3)
$$\hat{b}_i = \frac{1}{.3596 + 6.1852\,(10^{-7})\,(MVST_i)}$$
Coefficient of determination = .444
F-ratio = 38.3

(4)
$$\hat{b}_i = \frac{1}{.3851 + 3.0528\,(10^{-8})\,(MVSO_i)}$$
Coefficient of determination = .329
F-ratio = 23.5

Note:
MVST = Market Value of Shares Traded.
MVSO = Market Value of Shares Outstanding.

significance.[4] These statistical results are evidence of a significant deviation from the pure *T-C* function indicating the presence of significant *positive* intertemporal cross correlations between the daily returns of securities and those of the market. This result is consistent with the direct observation of these correlations in Table 1 and the discussion in Section III.

One can now re-examine the relationship between the intertemporal cross correlations, using $\hat{a}_i$ or $\hat{b}_i$ as a proxy for the pattern of these correlations, and the characteristics of securities, that is, *MVST* and *MVSO*. Such a procedure should yield stronger relationships than those reported in Table 2 because both a_i and b_i capture the complete structure of the intertemporal correlations and not just the first order relationship like the *q*-ratios used in Table 2.

The cross-sectional regression results are presented in Table 4. As expected, the

[4]The critical value for $|t(\hat{a}_i)|$ and $|t(\hat{b}_i - 1)|$, with 14 observations, is 2.160 at the .05 level of significance. Also, it is evident from eqs. (7) and (9) that while b_i contains an unweighted average of the *q*-ratios, a_i contains a weighted average of the same ratios. In a_i, the higher order *q*-ratios have higher weights and this has the effect of magnifying the value of small and probably insignificant *q*-ratios of higher orders. Thus, statistically insignificant negative *q*-ratios of higher orders may offset statistically significant positive *q*-ratios of lower orders yielding statistically insignificant intercepts.

explanatory power of the regression generally increases when either a_i or b_i is used as the dependent variable. The highest value of the coefficient of determination was .346 for the q-ratio (with *MVST*) and .410 for the ratio ρ_{im}^{-1}/ρ_{im} (with *MVST*). It rises to .444 for b_i (with *MVST*) and to .490 for a_i (with *MVSO*). This means that the market value of shares traded (*MVST*) and shares outstanding (*MVSO*) affect the intertemporal cross correlations even more than was conveyed by the tests of Table 2. The signs of all the coefficients are positive indicating that the strength of securities' intertemporal cross dependence is inversely related to *MVST* or *MVSO*.

VI. Some Implications for Empirical Work in Finance

The presence of intertemporal cross correlations in the daily returns of securities is sufficient to explain various phenomena reported in the literature. First, consider the Lawrence Fisher effect. Fisher [6] showed that the returns of stock market indexes exhibit positive autocorrelation even when they are constructed from individual securities which do not exhibit significant autocorrelations. This phenomenon can be attributed to the widespread existence of positive intertemporal cross correlations among the securities that compose the index.

To show that these correlations are the major source of autocorrelation in indexes, consider an index made of N securities each with a weight w_i. The daily return on such an index is equal to $r_{mt} = \sum_{i=1}^{N} w_i r_{it}$. Assuming stationarity, the autocorrelation coefficient of order s in the index return can be written as:[5]

$$\rho_m^s = \frac{1}{\sigma_m^2} \operatorname{Cov}\Big(\sum_{i=1}^{N} w_i r_{i,t}, \sum_{j=1}^{N} w_j r_{j,t-s}\Big)$$

$$\rho_m^s = \frac{1}{\sigma_m^2} \Bigg[\sum_{i=1}^{N} w_i^2 \operatorname{Cov}(r_{i,t}, r_{i,t-s}) + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \operatorname{Cov}(r_{i,t}, r_{j,t-s}) \Bigg] \tag{10}$$

$$\rho_m^s = \frac{1}{\sigma_m^2} \Bigg[\sum_{i=1}^{N} w_i^2 \sigma_i^2 \rho_i^s + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_i w_j \sigma_i \sigma_j (\rho_{ij}^{+s} + \rho_{ij}^{-s}) \Bigg]$$

$$\rho_m^s = \frac{1}{\sigma_m^2} \Bigg[\sum_{i=1}^{N} w_i^2 \sigma_i^2 \rho_i^s + \sum_{j=i+1}^{N} w_i w_j \sigma_{ij} q_{ij}^s \Bigg]$$

where $q_{ij}^s = (\rho_{ij}^{+s} + \rho_{ij}^{-s})/\rho_{ij}$, and ρ_{ij}^{+s} and ρ_{ij}^{-s} are the intertemporal cross correlations of order s for which the $i-th$ security's returns lead and lag those of the $j-th$ security, respectively. It is clear from equation (10) that as the number of securities (N) included in the index increases, the first term becomes negligible in comparison to the second. This is because the number of intertemporal cross correlations rises much faster than the number of autocorrelations as N increases. For an N-security index, there are N autocorrelation coefficients and $(1/2)N(N-1)$ q-ratios of each order.

Note that even if all autocorrelation coefficients for the securities are equal to

[5]See Hawawini [11].

zero, autocorrelation in the market index will not vanish as long as q-ratios are nonzero. Since the daily first order q-ratios of *NYSE* securities were found to be, in general, significantly positive, it follows that the daily returns on an *NYSE* index should display positive autocorrelation of first order. Also, since the daily first order q-ratios were found to increase as the market value of securities outstanding decreases, one should expect broader indexes to display **stronger** autocorrelation than narrowly defined indexes which are generally based on fewer securities with larger market values. As seen next, the empirical results support this contention.

For the same period as for the earlier tests, using daily data, autocorrelation coefficients were computed for three different market indexes: the Dow Jones Industrial (narrow), the *S & P-500* (broad), and the *NYSE* Composite (broadest). Based on these data, the first order autocorrelation for the *DJIA* is +.248, for the *S & P-500* it is +.285, and for the *NYSE* Composite it is +.338. All the three values are statistically significant at 5 percent.

Second, consider a phenomenon which is generating considerable interest in the literature: The effect of changing differencing interval on the estimated value of financial parameters, especially the beta coefficients in the market model. Smith [21], Lee and Morimune [15], Chen [1], and Hawawini [11] examine this issue and present strong evidence of the effect of intervaling on the beta coefficient. A sufficient explanation for the observed intervaling effect is the presence of intertemporal cross dependence.[6] The beta coefficient estimated over a T-day return interval is simply the ratio of the T-day covariance [equation (1)] to the T-day variance [equation (3)] of the market index:

$$\beta_i(T) = \frac{\sigma_{im}(T)}{\sigma_m^2(T)} = \frac{\sigma_{im}}{\sigma_m^2}\left[\frac{T + \sum_{s=1}^{T-1}(T-s)q_{im}^s}{T + 2\sum_{s=1}^{T-1}(T-s)\rho_m^s}\right] = \beta_i(T)\,\phi_i(T) \tag{11}$$

where:

$$\phi_i(T) = \frac{T + \sum_{s=1}^{T-1}(T-s)q_{im}^s}{T + 2\sum_{s=1}^{T-1}(T-s)\rho_m^s}$$

It is clear from equation (11) that as long as intertemporal cross dependence exists in the data, q_{im}^S and q_{mm}^S ($= 2p_m^S$) will be different from zero and $\beta_i(T)$ will be different from $\beta_i(1)$. Furthermore, securities with q-ratios larger than the market ($q_{im}^S > 2\rho_m^S$) will have estimated systematic risks that rise as the length of differencing interval is increased, and securities with q-ratios smaller than the market ($q_{im}^S < 2\rho_m^S$) will have estimated systematic risks that fall as the length of differencing interval is increased. Also, since q-ratios are inversely related to market values, securities with larger market value and therefore $q_{im}^S < 2\rho_m^S$ will have falling betas as T is increased. Empirical evidence supporting these conclusions is reported in Hawawini [11].

Third, Hawawini and Vora [12] and Levhari and Levy [16] present an analysis of

[6] For an alternative explanation see Green and Fielitz [8].

the intervaling effect on the estimated Sharpe [2] - Lintner [17] security market line (SML). Also see Greene and Fielitz [8]. Hawawini and Vora show that the presence of intertemporal cross dependence in the data causes the estimated *SML* to rotate uniformly as the length of the differencing interval is increased. The rotation may be clockwise or counterclockwise depending on the relative strength of the intertemporal cross correlation coefficients of one-day returns. The fact that the estimated *SML*, and hence the market price of risk, is sensitive to changes in the length of the return interval should not come as a surprise given that the beta coefficients are also subject to an intervaling effect.

Last, the existence of intertemporal cross dependence among securities' returns will produce significant noncontemporaneous (lead or lag or both) systematic risks for individual securities as well as for well-diversified portfolios, if daily returns are used, as reported by Hawawini and Vora [13], [14].[7] As securities with larger market values, and hence lower *q*-ratios, tend to be listed on the *NYSE*, on average the *NYSE* securities appear to lead the *AMEX* securities. The list of phenomena discussed in this section is not meant to be exhaustive and the intertemporal cross dependence among the daily returns of securities may be responsible for other phenomena not surveyed here.[8]

VII. Conclusion

In this paper empirical evidence of intertemporal cross dependence among *NYSE* securities' returns and *S & P's* returns was presented. An appropriate measure of these correlations was developed and shown to be inversely related to a security's market value of shares outstanding or shares traded. Finally, these correlations were shown to provide a sufficient explanation for various phenomena reported in the literature among which are the observed positive autocorrelation in market indexes, the intervaling effect on beta and the *SML*, and the existence of intertemporal (leading and lagging) systematic risks for securities and portfolios.

REFERENCES

1. S.N. Chen, "Time Aggregation and Autocorrelation on Systematic Risk Estimates—Additive vs. Multiplicative Assumptions," *Journal of Financial and Quantitative Analysis,* 151-173.
2. K. Cohen, S. Maier, R. Schwartz, and D. Whitcomb, "On the Existence of Serial Correlation in an Efficient Securities Market," *TIMS Studies in Management Science,* (1979).
3. K. Cohen, G. Hawawini, S. Maier, R. Schwartz, and D. Whitcomb, "Implications of Microstructure Theory for Empirical Research in Stock Price Behavior," *Journal of Finance,* (May 1980), 249-257.
4. K. Cohen, G. Hawawini, S. Maier, R. Schwartz, and D. Whitcomb, "Frictions in the Trading Process and the Estimation of Systematic Risk," Working Paper, (1980).
5. E. Fama, "The Behavior of Stock-Market Prices," *Journal of Business* (January 1965), 34-105.
6. L. Fisher, "Some New Stock Market Indexes," *Journal of Business* (January 1966), 394-419.
7. J. Gilster, "Autocorrelation, Investment Horizon, and Efficient Frontier Composition," *The Financial Review* (Winter 1979), 23-39.

[7]Note that $\beta_i^{\pm s} = \rho_i^{\pm s} (\sigma_i/\sigma_m)$.

[8]See Gilster [7] for the effect of correlation on the shape of the efficient frontier, Schwartz and Whitcomb [19] for the effect on the market model residuals and coefficient of determination, and Hawawini [9] for the effect on the moments of the distributions of securities' returns and the evaluation of the investment performance of institutional investors.

8. M.T. Greene and B.D. Fielitz, "The Effect of Long Term Dependence on Risk-Return Models of Common Stocks", *Operations Research,* (Sept.-Oct. 1979), 944-951.
9. G. Hawawini, "On the Time-Behavior of Financial Parameters: An Investigation of the Intervaling Effect," Unpublished Doctoral Dissertation, (October 1977).
10. G. Hawawini, "The Time-Covariance Function: Properties and Application," *Economics Letters* (No. 3, 1979), 235-238.
11. G. Hawawini, "Intertemporal Cross Dependence in Securities Daily Returns and the Short Run Intervaling Effect on Systematic Risk," *Journal of Financial and Quantitative Analysis* (March 1980), 139-149.
12. G. Hawawini and A. Vora, "Sensitivity of the Security Market Line to the Return Interval," Presented at the Annual Meeting of the Financial Management Association, Minneapolis (October 1978).
13. G. Hawawini and A. Vora, "Evidence of Intertemporal Systematic Risks in the Daily Price Movements of *NYSE* and *AMEX* Common Stocks," *Journal of Financial and Quantitative Analysis* (June 1980), Forthcoming.
14. G. Hawawini and A. Vora, "Intertemporal Risk in Common Stocks: An Examination of Risk Classes and Portfolio Effect," Working Paper, (1980).
15. C.F. Lee and K. Morimune, "Time Aggregation, Coefficient of Determination, and Systematic Risk of the Market Model," *The Financial Review* (Spring 1978), 36-47.
16. D. Levhari and H. Levy, "The Capital Asset Pricing Model and the Investment Horizon," *The Review of Economics and Statistics,* (February 1977), 92-104.
17. J. Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *The Review of Economics and Statistics* (February 1965), 13-37.
18. R. Schwartz and D. Whitcomb, "The Time-Variance Relationship: Evidence on Autocorrelation in Common Stock Returns," *Journal of Finance* (March 1977), 41-55.
19. R. Schwartz and D. Whitcomb, "Evidence on the Presence and Causes of Serial Correlation in Market Model Residuals," *Journal of Financial and Quantitative Analysis* (June 1977), 291-311.
20. W. Sharpe, "Capital Aset Prices: A Theory of Market Equilibrium Under Conditions of Risk." *Journal of Finance* (September 1964), 425-442.
21. K.V. Smith, "The Effect of Intervaling on Estimating Parameters of the Capital Asset Pricing Model," *Journal of Financial and Quantitative Analysis* (June 1978), 313-332.