Endogenous fertility, endogenous lifetime and economic growth: the role of child policies

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Abstract We examine the effects of child policies on both transitional dynamics and long-term demo-economic outcomes in an overlapping-generations neoclassical growth model à la Chakraborty (2004) extended with endogenous fertility under the assumption of weak altruism towards children. The government invests in public health, and an individual's survival probability at the end of youth depends on health expenditure. We show that multiple development regimes can exist. However, poverty or prosperity do not necessarily depend on the initial conditions, since they are the result of how child policy is designed. A child tax for example can be used effectively to enable those economies that were entrapped in poverty to prosper. There is also a long-term welfare-maximising level of the child tax. We show that, a child tax can be used to increase capital accumulation, escape from poverty and maximise long-term welfare also when (*i*) a public pay-as-you-go pension system is in place, (*ii*) the government issues an amount of public debt. Interestingly, there also exists a couple child tax-health tax that can be used to find the second-best optimum optimorum. In addition, we show that results are robust to the inclusion of decisions regarding the child quantity-quality trade off under the assumption of impure altruism. In particular, there exists a threshold value of the child tax below (resp. above) which child quality spending is unaffordable (resp. affordable) and different scenarios are in existence.

Keywords Child policy; Endogenous fertility; Health; Life expectancy; OLG model

JEL Classification I1; J13; O4

1. Introduction

The past century has witnessed a dramatic increases in life expectancy (Livi-Bacci, 2006). The importance of demography (longevity and fertility) in determining the macroeconomic performances of an economy in the very long term is the focus of a growing body of economic literature (Becker and Barro, 1988; Barro and Becker, 1989;

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Ehrlich and Lui, 1991; Blackburn and Cipriani, 1998; Galor and Weil, 1999, 2000; Kalemli-Ozcan et al., 2000; Kalemli-Ozcan, 2002, 2008; Fogel, 2004; Cervellati and Sunde, 2005, 2011; Galor, 2005; Soares, 2005; Acemoglu and Johnson, 2007; Lorentzen et al., 2008).[1]

A burgeoning theoretical literature based on models with overlapping generations (OLG) has dealt with the effects of infant mortality (Cigno, 1998, Fioroni, 2010) and adult mortality (de la Croix and Licandro, 1999; Blackburn and Cipriani, 2002; Chakraborty, 2004; Chakraborty and Das, 2005; Bhattacharya and Qiao, 2007, Fanti and Gori, 2012a; Varvarigos and Zakaria, 2013) on economic growth and development. Adult mortality can be endogenised by considering how public health expenditure (Chakraborty, 2004), private health expenditure (Chakraborty and Das, 2005) or both (Bhattacharya and Qiao, 2007) affect an individual's health status.

Within the class of OLG models with *endogenous* adult mortality, for our purposes the two most relevant papers are Blackburn and Cipriani (2002) and Chakraborty (2004). The former considers a general equilibrium OLG economy with *endogenous fertility and endogenous longevity* and three-period lived individuals who accumulate human capital through education – which is the main determinant of the probability of adult survival. Individuals produce and consume output, invest in education and spend a fraction of their lifetime taking care of their descendants. An increase in the individual life span creates a virtuous cycle of events for development. It increases labour productivity by increasing the returns of capital accumulation. This creates a reduction in child bearing time as well as an increase in the time devoted to education. This chain of events promotes (human) capital accumulation which then leads to a reduction in both adult mortality and population growth. In this context, Blackburn and Cipriani found that both low and high development regimes exist, the former characterised by low income, high birth rate and a relatively short life-span, the latter by high income, low fertility and a relatively long life-span. Depending on the initial conditions, therefore, an economy may be either entrapped in poverty or prosper. Their model is in agreement with the empirical evidence of the demographic transition.[2]

Chakraborty's model introduces endogenous lifetime into Diamond's (1965) model with exogenous fertility. The probability of surviving from work to retirement is a non-decreasing concave function of an individual's health status, which is determined by public investments. A rise in health taxes to finance health expenditure may lead to individuals living longer, which in turn provides an impetus to capital accumulation together with a higher life span. Chakraborty's main finding is that, when the output elasticity of capital in the Cobb-Douglas production function is relatively high, endogenous mortality may cause development traps (represented by the stable zero equilibrium). This is turn means that low-income high mortality and high-income low-mortality societies can exist. Chakraborty (2004) considered public health investment as a prerequisite for sustained economic growth and found that improving the health

[1] See also de la Croix et al. (2012) who revisit the serendipity theorem of Samuelson (1975) with fertility and longevity. In addition, another strand of literature deals with the effects of infectious diseases on life expectancy, fertility and economic and development (Young, 2005; Chakraborty et al., 2010; Kalemli-Ozcan and Turan, 2011; Kalemli-Ozcan, 2012).

[2] In a model with educational investments and endogenous fertility, Chen (2010) developed an OLG model showing that: (*i*) with exogenous lifetime, multiple development regimes with club convergence exist when mortality rates are large, and (*ii*) with endogenous lifetime, a unique stable steady state exist when mortality rates are small.

status of people can be beneficial for growth and development. This is because it directly reduces the risk of adult mortality, which in turn causes an impetus to higher capital accumulation together with lower adult mortality.

With regard to population, it is well established that in recent decades several developed countries have experienced a dramatic drop in fertility, which has reduced the number of children even well below the replacement rate (e.g., Germany, Italy, Spain and Japan), while also causing a declining ratio of economically active to retired people. The response of many governments has been to implement child support programmes mainly based on the provision of child subsidies (e.g., a direct monetary transfer to families with children) in order to provide incentives for child care and to facilitate fertility recovery. Particularly in northern Europe, other policies have also been adopted such as child care facilities (e.g., investments in infrastructure for day-care centres and schools) and child tax credits.

However, even the opposite problem of excessive population growth could represent a serious concern for economic growth and sustainable development in some countries. In these countries, therefore, a child-tax[3] policy may be implemented in order to reduce the birth rate and thus alleviate environmental problems and social conflicts. A well known example is the one-child policy (or, alternatively, family planning policy, see Coale, 1981) which was introduced by the Chinese government in 1979, which probably represents the only case of tax penalties for couples with more than one child in the world. With regard to the technical rules of implementation, the one-child policy restricts the number of children that couples decide to have to one, although several exceptions exist, for instance, for couples living in certain rural areas of China or for ethnic minorities. Chinese families subject to the restrictions of the family planning policy have to pay fines based on their income if they choose to have more than one child. The monetary penalties, however, increase more than proportionally for any additional newborn. Of course, the enforcement of this policy is controversial because it had several unpleasant effects (especially as regards the moral feasibility of restricting the freedom of people as well as some of the methods adopted). However, such issues are beyond the scope of this paper.[4]

Since the seminal paper by Becker (1960), the economic literature has argued that the choice of the number of children should be the result of a rational choice of individuals, especially in developed economies.[5] The theoretical literature on endogenous fertility is of greater importance in the theory of economic growth (e.g., Becker and Barro, 1988; Barro and Becker, 1989), and also serves as an explanation of

[3] While public child support programmes have been extensively examined in economic literature (Peters, 1995; Momota, 2000; van Groezen et al., 2003; Apps and Rees, 2004; van Groezen and Meijdam, 2008), the theoretical analysis of the effects of child taxes on long-term demo-economic outcomes is, to the best of our knowledge, relatively scarce. For instance, in the literature with endogenous fertility, Bental (1989) represents one of the first attempt to discuss the effects of child taxes in a model where children are considered as a capital good (old-age security hypothesis). He finds that a tax on children can achieve the optimal capital-labour ratio but fails to realise the optimal population growth. Recently, Fanti and Gori (2009) have shown that a child tax can be used to actually raise population growth in the long run, while also raising per worker GDP.

[4] For empirical evidence of population policies in China see McElroy and Yang (2000), Rosenzweig and Zhang (2009) and Wu and Li (2012).

[5] For instance, van Groezen et al. (2003, p. 237) argued that "The rate of fertility should therefore be treated as an endogenous variable, that is, as the result of a rational choice which is influenced by economic constraints and incentives. Economic theory can thus help in explaining why the observed decline in the (desired) number of children would occur."

multiple regimes of development when human capital is considered (e.g., Becker et al., 1990; Blackburn and Cipriani, 2002).

This paper is an attempt to introduce endogenous fertility into a Chakraborty-like model (2004). The focus is on the crucial role that family policies, consisting of either a tax or subsidy on children, can play on both the transitional dynamics and demo-economic outcomes in the very long term.

Our main finding is that poverty or prosperity does not necessarily depend on the initial conditions, since they are the result of how child policy is designed. A fairly large increase in child tax reduces both fertility and adult mortality. This stimulates capital accumulation and eliminates the low equilibrium, so that an economy entrapped in poverty due to unfavourable initial conditions will converge towards a high development regime, where income per worker is high, life expectancy is high, and fertility is low. More importantly, in a Chakraborty-type economy a second-best optimal child tax policy exists that can be used to maximise steady-state welfare. This is because the increase in longevity and consumption by young people, which contributes to raise utility, more than compensate for the negative welfare effects of the reduction in fertility and consumption amongst older individuals.

In addition, in contrast to van Groezen et al. (2003) and van Groezen and Meijdam (2008), who showed that the optimal policy in an economy with public pay-as-you-go (PAYG) pensions under fixed contributions and endogenous fertility is the use of child allowances as this helps to eliminate the external effects of children, we found that when longevity and fertility are endogenous: a child tax can be used to maximise steady-state welfare also when a public pension system is in place. Irrespective of whether PAYG pensions are in place, there exists a couple child tax-health tax that can be used as a second-best optimum optimorum policy.

This paper differs from the above mentioned studies in terms of its specific objectives, analyses and results. From a broader perspective, our paper belongs to the demo-economic literature that treats the key demographic variables – i.e. fertility and longevity – as endogenously determined in the model rather than exogenously given. It also links them to the process of economic growth in the simple and intuitive context of the standard OLG model. This paper can also be viewed as a contribution to the wider literature on multiple equilibria, poverty traps and demographic changes over the very long term (e.g. Azariadis and Drazen, 1990).

The paper is organised as follows. Section 2 outlines the model. Section 3 studies the dynamic path of capital accumulation and provides necessary conditions for the existence of multiple (four) steady states. Section 4 analyses the effects of child taxes on economic growth, the stages of development and welfare. Section 5 introduces pay-as-you-go public pensions and Section 6 considers problems of child quantity-quality trade-off under impure altruism. Conclusions are drawn in Section 7. Appendices A and B provide proofs of the main propositions. Appendix C shows that the results of the paper hold also when the government issues an amount of public debt in every period.

2. The model

Consider a general equilibrium OLG closed economy populated by rational and identical individuals of measure N_t per generation. Each generation overlaps for one period with the previous generation and then overlaps for one period with the next generation. Time is discrete and indexed by $t = 0,1,2,...$. The life of the typical agent is divided into childhood and adulthood. As a child, an individual does not make economic decisions. He/she consumes resources directly from his/her parents and survives to the end of youth with certainty (no child mortality). As an adult, an individual works and takes care of children when young, and retires when old. The young members of a generation t are endowed with one unit of labour inelastically supplied to firms, while receiving a competitive wage w_t per unit of labour. When an adult, an individual of a generation t draws utility from consumption when young, $c_{1,t}$, consumption when old, $c_{2,t+1}$, and the number of children, n_t (Eckstein and Wolpin, 1985; Eckstein et al., 1988; Galor and Weil, 1996).[6]

We assume that the probability of surviving from youth to old age is endogenous and determined by an individual's health status, which is, in turn, improved by the public provision of health investments when young h_t (Chakraborty, 2004; Bhattacharya and Qiao, 2007; Fanti and Gori, 2012). The survival probability at the end of youth of an individual that belongs to generation t, π_t, depends on h_t and is given by a strictly increasing (though bounded) function $\pi_t = \pi(h_t)$. Following Blackburn and Cipriani (2002), we specify this relationship as follows:

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}, \tag{1}$$

where $\delta, \Delta > 0$, $0 < \pi_1 \leq 1$, $0 < \pi_0 < \pi_1$, $\pi(0) = \pi_0 > 0$, $\pi'(h) = \dfrac{\delta \Delta h^{\delta-1}(\pi_1 - \pi_0)}{(1+\Delta h^{\delta})^2} > 0$, $\lim_{h\to\infty} \pi(h) = \pi_1 \leq 1$, $\pi''(h) < 0$ if $\delta \leq 1$ and $\pi''(h) \underset{<}{\overset{>}{-}} 0$ for any $h \underset{>}{\overset{<}{-}} h_T := \left[\dfrac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Equation (1) is sufficiently general to capture the cases of both the monotonic (concave) function ($\delta = 1$) used in the numerical examples by Chakraborty (2004), and the S-shaped function ($\delta > 1$) used in the numerical examples by Blackburn and Cipriani (2002). Some clarifications regarding Eq. (1) are now useful.

Firstly, we define π_0 as an exogenous measure of the natural rate of longevity of people (e.g., Ehrlich, 2000; Leung and Wang, 2010) irrespective of health investments. This measure of adult mortality may be different depending on the country because it is affected by both economic and non-economic factors – lifestyle, education, economic growth and standard of living, the degree of culture and civilisation, weather and climate changes, ethnic and civil wars, endemic diseases, and so on. Moreover, some underdeveloped and developing countries are trapped in poverty because of weak institutions, or due to climates that foster disease, or geographies that limit access to global markets, or simply by the fact that poverty is overwhelmingly self-perpetuating. For these and other reasons, we can realistically expect the value of π_0 to be higher in developed rather than developing or under-developed countries.

Secondly, the parameter π_1 captures the intensity of the efficiency of health investments on longevity. A rise in π_1 can be interpreted as an exogenous medical advance due to scientific research, vaccination programmes and so on.

[6] The way of modelling children as a desirable good that directly enters the parents' utility is called weak form of altruism towards children (Zhang and Zhang, 1998).

Thirdly, it is reasonable to expect that health investments have a more intense effect on reducing adult mortality when a certain threshold value of health investment is achieved, while they are barely effective when longevity is close to its saturation value – for example the functional relationship between public health expenditure and longevity may be S-shaped. The parameters δ and Δ capture this idea and determine both the turning point of $\pi'(h)$ and speed of convergence from the natural length of life π_0 to the saturation value π_1. Given the value of Δ, the parameter δ represents the degree of effectiveness of public health investments as an inducement to higher longevity. In other words, it measures how an additional unit of public investment in health is transformed into higher longevity through health technologies.

If $\delta \leq 1$, $\pi(h)$ is concave for any h and, hence, longevity increases less than proportionally from the starting point π_0 to the saturation value π_1 as h rises. Figure 1 illustrates the shape of the graph of $\pi(h)$ when $\delta \leq 1$: the solid (dashed) [dotted] line refers to the case $\delta = 1$ ($\delta < 1$) [$\delta \to 0$]. As can easily be seen, the lower (higher) δ is, the more (less) efficiently an additional unit of health capital is transformed into higher longevity until a certain level of h is reached.

If $\delta > 1$, $\pi(h)$ is S-shaped and threshold effects exist. Longevity increases more (less) than proportionally until the turning point h_T is achieved. However, an increase in δ shifts the longevity function to the right, while also increasing the speed of convergence from π_0 to π_1, as clearly shown in Figure 2, where the solid (dashed) [dotted] line refers to δ_{low} (δ_{high}) [$\delta \to +\infty$]. This means that the more intense the threshold effects (high values of δ), the slower an additional unit of health investment is transformed into a higher life span when h is small, while reaching π_1 more efficiently and rapidly as h becomes larger. In other words, a rise in public expenditure on health is not effective until a specific value of health capital is achieved (and this value is higher, the higher δ is). This is because, for instance, an adequate level of knowledge to enable such investments to be effectively transformed into higher longevity has not yet been obtained (see, e.g., Egger, 2009). As an example, think of the existence of the threshold effects in the accumulation of the knowledge required for new medical advances and discoveries in the treatment of diseases (e.g. vaccines) to be effective: the public health expenditure required to finance new research projects may be high and apparently of no tangible value until a certain degree of knowledge has been achieved. Beyond such a threshold, however, there is a "sudden" effect that triggers and highlights the beneficial effects of the new discoveries, to make them efficient, usable and operative across populations and eventually transformed into higher longevity.

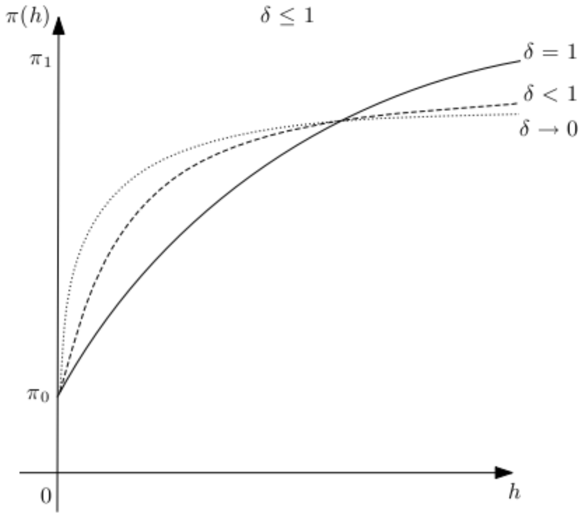


Figure 1. Longevity and public health expenditure when $\delta \leq 1$.

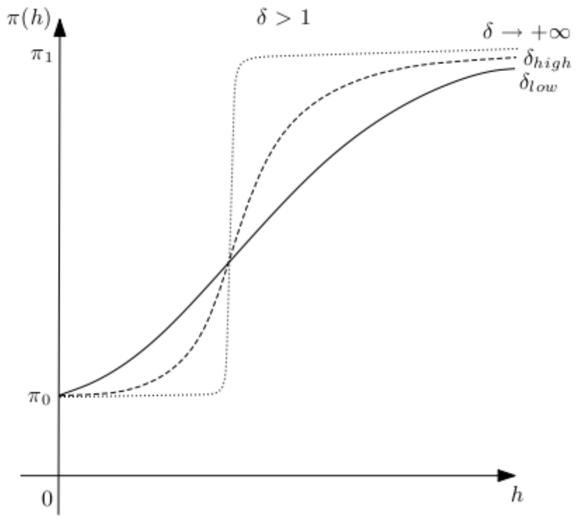


Figure 2. Longevity and public health expenditure when $\delta > 1$.

As in Chakraborty (2004), we assume that the public health expenditure per worker at time t (h_t) is financed with a balanced budget by levying a (constant) wage income tax at the rate $0 < \tau < 1$. The government health accounting rule, therefore, is given by the following formula:

$$h_t = \tau w_t . \tag{2}$$

With regard to child care activities, we assume that raising children is costly for young parents. The amount of resources needed to take care of a child is given by $q w_t$, where $0 < q < 1$ is the percentage of the cost of children of the parents' working income.[7] This element captures all needs required for the upbringing of children, included food, schooling and so on. In addition, we assume that the government finances (with a balanced budget) a wage subsidy by levying a constant per child tax.[8] Therefore, the child policy budget in per worker terms at time t reads as follows:

[7] See Wigger (1999) and Boldrin and Jones (2002).

[8] For instance, the tax penalties imposed by the Chinese birth planning programme on parents with more than one child are currently computed as a fraction of either the disposable income of people living in urban areas or cash income (estimated by the local authorities) of people living in rural area. In general, they are proportional to the number of children that exceeds the quota planned by the government.

$$\theta_t w_t = b\, w_t n_t \Rightarrow \theta_t = b\, n_t \,. \tag{3}$$

The left-hand side shows the wage subsidy expenditure, and the right-hand side, the child tax receipt, where $b > 0$ is the fixed percentage of wage income collected by the government as a tax for every additional child, $\theta_t > 0$ is the wage subsidy rate adjusted over time to balance the budget, and n_t is the individual number of children at time t.

Therefore, the budget constraint of a young individual of generation t is the following:

$$c_{1,t} + s_t + (q+b) w_t n_t = w_t (1 - \tau + \theta_t)\,, \tag{4.1}$$

i.e. the disposable (working) income is divided between material consumption when young, savings (s_t), and the cost of raising n_t children.

When old, individuals retire and live with the amount of resources saved when young plus the expected interest accrued from time t to time $t+1$ at the rate r^e_{t+1}. Following Chakraborty (2004), we assume a (perfect) market for annuities exists, so that the budget constraint when old (time $t+1$) of an individual of generation t can be expressed as follows:

$$c_{2,t+1} = \frac{R^e_{t+1}}{\pi_t} s_t \,, \tag{4.2}$$

where $R^e_{t+1} := 1 + r^e_{t+1}$ is the expected interest factor.

By taking the wage, the expected interest rate, the longevity rate and the government budget constraints Eqs. (2) and (3) as given, the individual representative of generation t chooses how many children to have and how much to save from of his/her disposable income, in order to maximise the expected lifetime utility function (see Abel, 1985). This can be expressed as:

$$\max_{\{n_t, s_t\}} U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \gamma \ln(n_t)\,, \tag{5}$$

subject to Eqs. (4.1) and (4.2), where $\gamma > 0$ captures the parents' relative desire to have children. The constrained maximisation of Eq. (5) gives the demand for children and savings, respectively:

$$n_t = \frac{\gamma(1 - \tau + \theta_t)}{(1 + \pi_t + \gamma)(q + b)}\,, \tag{6.1}$$

$$s_t = \frac{\pi_t w_t (1 - \tau + \theta_t)}{1 + \pi_t + \gamma}\,. \tag{6.2}$$

Now, using Eq. (3) to eliminate θ_t and rearrange terms, Eqs. (6.1) and (6.2) can be written as follows:

$$n_t = \frac{\gamma(1 - \tau)}{(1 + \pi_t)(q + b) + \gamma q}\,, \tag{7.1}$$

$$s_t = \frac{\pi_t w_t (1 - \tau)(q + b)}{(1 + \pi_t)(q + b) + \gamma q}\,, \tag{7.2}$$

Since one of the objectives of this paper is to study the effects of child tax on both transitional dynamics and long-term demo-economic outcomes, the role played by b in a partial equilibrium context is interesting. A rise in child tax increases the marginal cost of raising an extra child and thus makes it more convenient to opt for consumption rather than having children. As a direct partial equilibrium effect, Eqs. (7.1) and (7.2) show that a rise in child tax reduces the demand for children and increases the need to save in the short term.

In the following section we introduce production of goods and services and characterise the general equilibrium features of the model.

2.1. Production and equilibrium

We assume that firms are identical and act competitively on the market. The production function of the representative firm is the standard neoclassical Cobb-Douglas technology with constant returns to scale, that is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and labour input at time t respectively, $A > 0$ is a scale parameter and $0 < \alpha < 1$ is the output elasticity of capital. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per worker, respectively, the intensive form production function is $y_t = Ak_t^{\alpha}$. By assuming that output is sold at the unit price and capital totally depreciates at the end of every period, profit maximisation implies that factor inputs are paid their marginal products, that is:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{8}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{9}$$

On the basis that populations evolve according to the equation $N_{t+1} = n_t N_t$, the market-clearing condition in the capital market is given by $n_t k_{t+1} = s_t$, which is combined with Eqs. (7.1) and (7.2) to obtain:

$$k_{t+1} = \frac{\pi_t}{\gamma} w_t (q+b). \tag{10.1}$$

By using Eqs. (1), (2), (9) and (10.1), the dynamic path of capital accumulation is described by the following first order non-linear difference equation:

$$k_{t+1} = \frac{Dk_t^{\alpha}(\pi_0 + \pi_1 \mathrm{B} k_t^{\alpha\delta})}{\gamma(1+\mathrm{B}k_t^{\alpha\delta})} := G(k_t), \tag{10.2}$$

where $\mathrm{B} := \Delta[\tau(1-\alpha)A]^{\delta} > 0$ and $D := (q+b)(1-\alpha)A > 0$. Although Eq. (10.2) is a simple first order non-linear difference equation, the dynamic features that it gives rise to are interesting, especially from a policy perspective.

3. Dynamics and steady states

From Eq. (10.2) it is clear that when $\tau = 0$ (i.e., no health investments) a unique locally asymptotically stable steady state exists (as in Diamond, 1965), because $\mathrm{B} = 0$. In contrast, when $0 < \tau < 1$, the following proposition shows that development traps are possible.

Proposition 1. *The dynamic system described by Eq. (10.2) admits either two steady states* $\{0, \bar{k}\}$, *with* $\bar{k} > 0$ *(only the positive state being asymptotically stable) or four steady states* $\{0, \bar{k}_1, \bar{k}_2, \bar{k}_3\}$, *with* $\bar{k}_3 > \bar{k}_2 > \bar{k}_1 > 0$ *(only the second and the forth being asymptotically stable). Furthermore, (1) a sufficient condition to avoid development traps is* $\Lambda_2 > 0$ *and* $\Lambda_3 > 0$, *and (2) a necessary condition for the existence of multiple steady states is that at least either* $\Lambda_2 < 0$ *or* $\Lambda_3 < 0$ *holds, where*

$\Lambda_2 := \mathrm{B}E[1-\alpha(1-\delta)] + F[1-\alpha(1+2\delta)]$, $\Lambda_3 := \pi_0 \mathrm{B}[1-\alpha(1-2\delta)] + E[1-\alpha(1+\delta)]$, $E := [\pi_0 + \pi_1 + \delta(\pi_1 - \pi_0)]\mathrm{B} > 0$ *and* $F := \pi_1 B^2 > 0$.

Proof. See Appendix A.

Proposition 1 says that multiple development regimes are possible when longevity is endogenous and determined by an individual health measure augmented by public investments through the longevity function Eq. (1).

Note that the scenario $\pi_0 = 0$ and $\delta = 1$ resembles the case studied by Chakraborty (2004) in a model with exogenous fertility. The assumption of a positive natural rate of longevity, however, exposes the economy to a dramatic change. The zero equilibrium – which is an attractive equilibrium point when $\pi_0 = 0$ and α is sufficiently high – is always unstable when $\pi_0 > 0$, and the number of steady states passes from three to four. This thus makes the comparison of demo-economic performances between low and high income countries more plausible. In fact, although the existence of a stable zero equilibrium may be a useful abstraction to represent poorer economies, it certainly suffers from a lack of realism, especially with regard to the empirical significance of the results.

Proposition 1 provides sufficient conditions to avoid development traps and necessary but not sufficient conditions to have multiple steady states. As can easily be ascertained from Eq. (12.2), this depends on the key parameters of the problem and the policy variable τ. However, as extensive numerical simulations revealed, the existence of multiple regimes of development crucially depends on the mutual relationship between the output elasticity of capital (α) and on how effective public health investments are on longevity (δ). For any given value of δ, development traps are more likely to occur when production is relatively capital-oriented (high values of α). When $\delta = 1$ multiple steady states appear when α exceeds $1/2$, and this threshold monotonically reduces as δ increases. This is in line with Chakraborty (2004, Proposition 1, (i), p. 126).

We also note that Bunzel and Qiao (2005) have shown that the second part of Proposition 1 (i) by Chakraborty "is incorrectly stated" (Bunzel and Qiao, 2005, p. 4), because $\alpha > 1/2$ represents a necessary but not sufficient condition for the existence of multiple steady states (a high value of the scale parameter A of course is required to avoid the existence of a unique, degenerate equilibrium). Nevertheless, it still remains true that when $\alpha > 1/2$ three steady states *can* exist, so that the central result by Chakraborty (2004) is unaltered even after the critique by Bunzel and Qiao (2005). However, since $k = 0$ is an attractive equilibrium point in this context, high values A can drastically reduce the basin of attraction towards the low stable steady state. However the possibility of eliminating the poverty trap does not exist whatever the level of technological development. Therefore, when there are threshold effects of health capital on longevity (i.e. $\delta > 1$), a wider range of economies are likely to be characterised by development traps, since the output elasticity of capital that discriminates between a single regime and multiple regimes of development is empirically more plausible and smaller than $1/2$ (as shown in the numerical example below).

Now, let us assume that economies differ exclusively with regard to the initial condition k_0. Figure 3 depicts all the possible outcomes of an economy with endogenous longevity and endogenous fertility. The figure clearly shows that an economy that starts below the unstable equilibrium $\bar{k}_2$ is entrapped in the low regime ($\bar{k}_1$), where income per worker is low, fertility is high, and mortality is high. In contrast, an economy that starts beyond the threshold $\bar{k}_2$ converges towards the high regime ($\bar{k}_3$), where income per worker is high, fertility is low, and mortality is low. Therefore, an exogenous shift in the initial conditions may cause a change in the development regime. This is in agreement with Blackburn and Cipriani (2002) and Chakraborty (2004). Unlike the former paper, where the main determinant of the reduction in adult mortality is the private education expenditure that increases human capital, our findings hold in a model where an adult mortality reduction is due to increases in public health expenditure.

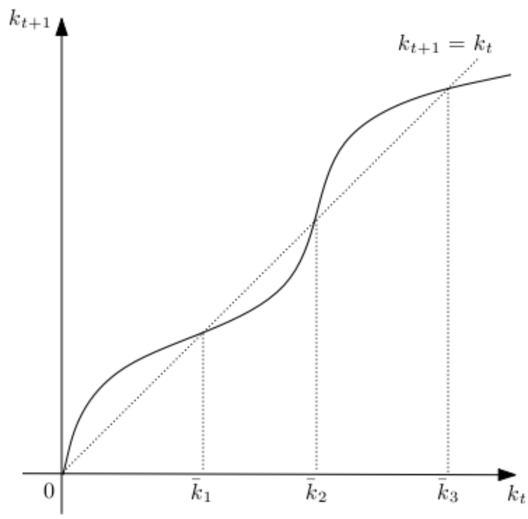


Figure 3. Multiple steady states.

A numerical experiment to illustrate Proposition 1 now follows. We take the parameter values: $A = 12.2$, $\alpha = 0.33$, $\gamma = 1$, $\pi_0 = 0.3$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.3$, $\tau = 0.1$ and $b = 0$.[9] Therefore, the low regime is characterised by the equilibrium values $\bar{k}_1 = 0.72$, $\pi(\bar{k}_1) = 0.33$ and $n(\bar{k}_1) = 1.29$. In contrast, the high regime is characterised by the equilibrium values $\bar{k}_3 = 2.88$, $\pi(\bar{k}_3) = 0.83$ and $n(\bar{k}_3) = 1.06$. The unstable equilibrium stock of capital that discriminates between poor and rich countries is $\bar{k}_2 = 1.7$. Therefore, an economy that for some exogenous reasons starts with a stock of capital below (resp. above) such a threshold level of development will end up in the low (resp. high) regime, where income per worker is small (resp. large) and mortality and

[9] A value of the output elasticity of capital (α) of one third is usual to represent developed economies (Gollin, 2002). According to Zhang et al. (2001), a value of the taste for the number of children included in the range $0.8 \le \gamma \le 1.5$ is reasonable to capture the parents' taste of children relative to material consumption in the utility function (according to the specification of preferences given by Eq. 5). Moreover, the values of both the scale parameter A and percentage of child cost on working income q, are chosen to get a value of long-run fertility close enough to unity to be as much as realistic as possible in representing actual developed economies. The value $\delta = 10$ follows de la Croix and Ponthière (2010). Finally, $\tau = 0.1$ implies a ratio of health expenditure to per worker GDP of almost 7 per cent (which is an average value for developed countries, see World Health Statistics, 2010) when $b = 0$. We note that similar results (not reported in the paper for economy of space) can be found with different parameter values.

fertility rates are high (resp. low), confirming some of the most striking aspects of the so-called demographic transition.

However poverty or prosperity may not necessarily depend on initial conditions. An added value of this paper, in fact, lies in the importance of determining the role of the child policy variable b on transitional dynamics, long-run demo-economic outcomes and welfare. The next section deals with these topics and the main findings are: (*i*) a sufficiently large increase in child tax should be able to eliminate the vicious cycle and ill-health of poverty, thus allowing those economies that were entrapped in poverty due to unfavourable initial conditions to end up in the high regime of development; (*ii*) in an economy with public health investment, a value of child tax exists that maximises long-term welfare.

4. Child policy and welfare

Child allowances in the form of direct monetary transfers entitled to families with children have often been proposed by both politicians and economists as a remedy against low fertility, and have been used extensively in several European countries, which are amongst those most plagued by sharp reductions in population growth rates over the last few decades.[10] In contrast, the Chinese one-child per family programme was enforced, among other things, as a stimulus to economic growth because over-population. As such it possibly represents the most interesting case of tax penalties on children in the world.[11]

It may be interesting, therefore, to study the effects of child taxes on demo-economic outcomes in the context of multiple steady states. An analysis of b from Eq. (10.2) gives the following proposition:

Proposition 2. *When the government invests in public health and development traps exist, a sufficiently large increase in child tax leads to the loss of the lowest stable steady state,* $\bar{k}_1$*, thus allowing poorer economies to permanently escape from poverty and converge towards the highest equilibrium,* $\bar{k}_3$.

Proof. See Appendix B.

Proposition 2 shows that in a context of multiple steady states, a rise in child tax increases extreme stable steady states, reduces intermediate unstable steady states, while reducing the size of the basin of attraction of the poverty trap. A sufficiently large increase in child tax also shifts the graph of map $G(k)$ upwards and leads to the loss of the lowest stable equilibrium. This thus allows those economies that were previously stuck in poverty due to unfavourable initial conditions, to end up in the

[10] Policies consisting in cash subsidies for children are largely adopted in several countries. As an example, in Italy a 1,000 euro child grant for each new born was introduced in the year 2005, while in Poland every woman will benefit from a one-off 258 euro payment for every child, and women from poorer families will receive double the previous amount. Evidence of a positive impact of family policy programmes (national expenditure for child allowances, maternity and so on) on fertility and women labour participation in western European countries can be found in Kalwij (2010).

[11] For instance, the one-child policy had the effect of reducing the Total Fertility Rate in China from more than five births per woman in the 1970s to slightly less than two births per woman in recent years.

high regime of development, where fertility is smaller and income per worker and longevity larger than with the lowest steady state. This because a rise in child tax, by increasing the total cost of children, reduces fertility and causes a rise in savings because of the increased needs for child-bearing purposes. This thus increases capital accumulation and, hence, the steady state stock of capital in both regimes of development.

Given Proposition 2 the following result holds.

Result 1. *For any given value of the health tax rate* τ, *a rise in child tax reduces adult mortality.*

Result 2. *For any given value of the health tax rate* τ, *poverty or prosperity do not necessarily depend on the initial conditions, since they are the result of how child policy is designed.*

As discussed in Section 2, a rise in child tax reduces fertility and increases savings due to a partial equilibrium effect. This causes a rise in capital accumulation that shifts the graph of map $G(k)$ upwards for any $k>0$, while also increasing stable steady states and wages in both the low and high income countries. Higher wages, however, translate into a larger health expenditure per worker – *ceteris paribus* with regard to the value of the health tax τ – which reduces adult mortality in the long run. The increased survival probability causes an indirect general equilibrium feedback effect which acts negatively on fertility and positively on savings, and thus works as a stimulus to accumulate capital further on. As a consequence, the equilibrium output per worker increases, while the steady-state adult mortality and fertility rates shrink in both regimes. A sufficiently large increase in child tax, however, will stimulate capital accumulation and eliminate the low equilibrium, thus allowing those economies that were previously entrapped in poverty to prosper, irrespective of the initial conditions.

We will now illustrate Proposition 2 and Results 1 and 2 with the following numerical examples. We will take the same parameter values as in Section 3 and look at the effects of changes in b on both macroeconomic and demographic performances over the very long term. Starting from the case $b=0$, where the world is divided into poor and rich countries, Table 1 shows how the main steady-state variables react following a rise in child tax. As can be seen, a slight increase in child tax (from $b=0$ to $b=0.01$) drives capital accumulation, and also reduces adult mortality and fertility in both regimes of development. If we assume that childhood is 20 years of life and adulthood is divided, in turn, into a 30-year working period and a 30-year retirement period, a rise in child tax from 0 to 0.01 increases an individual's probability of survival in poor countries from 32.8 per cent of the whole time after the end of youth (i.e. individuals live about 9.7 years beyond their working life) to 33.7 per cent of the whole time after the end of youth (i.e. individuals live about 10 years beyond their working life), with an increase of almost 0.9 years of life. Adult mortality goes down in rich countries too, but to an even greater extent because the percentage increase in capital accumulation (and, hence, in wages) is higher than in poor countries. Raising the child tax from 0 to 0.01 causes a sharp increase in the lifespan of people in rich countries, which moves from 83 per cent of the whole time after the end of youth (i.e. individuals live about 24.3 years beyond their working period) to 85 per cent of the whole time after the end of youth (i.e. individuals live about 25 years beyond their working life), with a gain of almost 1.5 years of life.

Increasing child tax further stimulates capital accumulation and produces a loss of the lowest stable steady states when b goes beyond 0.025. Therefore, the low regime of development vanishes and the economies that were entrapped in poverty due to unfavourable initial conditions converge towards the unique high stable steady state (i.e. the phase map $G(k)$ lies everywhere above the 45° line and falls below it only once the high equilibrium $\bar{k}_3$ has been achieved), with dramatic consequences for both macroeconomic and demographic outcomes. In fact, capital accumulation increases monotonically with the child tax and this increases the equilibrium wage rate. As a consequence, public health expenditure increases together with longevity, while fertility goes down considerably.

Table 1. Child tax and multiple regimes of development.

Low regime ($\bar{k}_1$)

b	0	0.01	0.02	0.025
$\bar{k}_1$	0.724	0.793	0.895	0.998
$w(\bar{k}_1)$	7.348	7.573	7.88	8.17
$h(\bar{k}_1)$	0.734	0.757	0.788	0.817
$\pi(\bar{k}_1)$	0.328	0.337	0.354	0.37
$n(\bar{k}_1)$	1.29	1.26	1.22	1.2

High regime ($\bar{k}_3$)

b	0	0.01	0.02	0.025	0.05	0.07	0.08	0.1	0.2	0.3
$\bar{k}_3$	2.88	3.19	3.45	3.58	4.16	4.6	4.82	5.26	7.5	9.9
$w(\bar{k}_3)$	11.59	12	12.3	12.45	13.08	13.53	13.73	14.13	15.9	17.42
$h(\bar{k}_3)$	1.16	1.19	1.23	1.24	1.3	1.35	1.37	1.41	1.5	1.74
$\pi(\bar{k}_3)$	0.83	0.85	0.87	0.88	0.908	0.92	0.923	0.93	0.943	0.947
$n(\bar{k}_3)$	1.06	1.02	1	0.98	0.93	0.89	0.87	0.84	0.7	0.6

So what about child allowance policy in a context of multiple steady states? We define child allowance (financed by a wage income tax $0<|\theta_t|<1$) as $0<|b|<q$, i.e. the net cost of children should remain positive to guarantee the existence of a finite positive solution for n_t. As a consequence of introducing a child allowance scheme there is a partial equilibrium effect, which increases fertility and reduces savings. Capital accumulation, therefore, will be lower. This causes a reduction in the steady-state stock of capital and, hence, in the wage rate. Therefore, for any given value of the health tax τ, the health expenditure per worker shrinks and thus adult mortality increases in equilibrium. The reduced life span induces agents to have more children and this, in turn, decreases capital accumulation further on. Moreover, a large enough increase in child subsidy can produce a loss in the high equilibrium (i.e. the phase map $G(k)$ lies everywhere below the 45° line once the low equilibrium $\bar{k}_1$ is achieved). In turn this implies that, irrespective of the initial conditions, all economies will end up

in the low regime of development, where income per worker is low, adult mortality is high, and fertility is high.

Table 2 shows the effects of child allowances on the main steady state variables in both low and high income countries. As regards the high development regime , Table 2 highlights that a slight increase in child allowance (from 0 to 0.003) increases fertility and sharply reduces both capital accumulation and adult mortality (with a loss of almost two years of life). The high equilibrium vanishes as a consequence of a further increase in child allowance and, thus, the economies converge towards the low regime where capital accumulation is low, fertility is high and people's lifespan tends to the natural rate π_0.

Table 2. Child allowance and multiple regimes of development.

High regime ($\bar{k}_3$)

$\lvert b \rvert < q$	0	-0.01
$\bar{k}_3$	2.88	2.40
$w(\bar{k}_3)$	11.59	10.91
$h(\bar{k}_3)$	1.16	1.09
$\pi(\bar{k}_3)$	0.83	0.75
$n(\bar{k}_3)$	1.06	1.11

Low regime ($\bar{k}_1$)

$\lvert b \rvert < q$	0	-0.01	-0.05	-0.1	-0.2
$\bar{k}_1$	0.724	0.668	0.503	0.35	0.122
$w(\bar{k}_1)$	7.348	7.15	6.51	5.78	4.09
$h(\bar{k}_1)$	0.734	0.715	0.651	0.578	0.4
$\pi(\bar{k}_1)$	0.328	0.322	0.308	0.302	0.3
$n(\bar{k}_1)$	1.29	1.31	1.43	1.6	2.09

4.1. Welfare

Let us now look at the welfare effects of child tax in an economy with public health investments in order to draw some conclusions regarding the desirability of the child policy. Since Golosov et al. (2007), either the P–efficiency criterion or the A–efficiency criterion can be used to compare alternatives when a population is endogenous. The former criterion implies that the preference profiles of both born and potential agents are evaluated in every state of the world (generation). The latter implies that only the welfare of those who are alive in every state of the world is evaluated to compare alternatives. However, we are now interested in maximising the steady-state expected lifetime welfare with respect to b ($b > 0$) in an economy where public health investments are in place. The use of the notion of A–efficiency will be used later in this section.

Knowing that consumption when young and consumption when old are given by $c_{1,t} = \dfrac{w_t(1-\tau)(q+b)}{(1+\pi_t)(q+b)+\gamma q}$ and $c_{2,t+1} = R^e_{t+1} c_{1,t}$, the government wants to maximise the steady-state lifetime indirect utility index $V(b)=\ln(c_1(b)^{1+\pi(b)}n(b)^{\gamma}R(b)^{\pi(b)})$ with respect to b for any $0<\tau<1$, where $x(b)$ is the steady-state value of the generic variable x.

The effects of a rise in child tax b on fertility and consumption are summarised by the following total derivatives:

$$\frac{dn}{db}=\overbrace{\frac{\partial n}{\partial b}}^{-}+\overbrace{\frac{\partial n}{\partial \pi}}^{-}\cdot\overbrace{\frac{\partial \pi}{\partial k}}^{+}\cdot\overbrace{\frac{\partial k}{\partial b}}^{+}<0, \tag{11}$$

$$\frac{dc_1}{db}=\overbrace{\frac{\partial c_1}{\partial b}}^{+}+\overbrace{\frac{\partial c_1}{\partial \pi}}^{-}\cdot\overbrace{\frac{\partial \pi}{\partial k}}^{+}\cdot\overbrace{\frac{\partial k}{\partial b}}^{+}+\overbrace{\frac{\partial c_1}{\partial w}}^{+}\cdot\overbrace{\frac{\partial w}{\partial k}}^{+}\cdot\overbrace{\frac{\partial k}{\partial b}}^{+}\gtrless 0. \tag{12}$$

Equations (11) and (12) highlight that a rise in b negatively affects fertility, while it is ambiguous in terms of consumption. This is due to the fact that when the child tax is raised, young-age consumption increases: (*i*) due to a direct impact effect of the child tax, and (*ii*) because capital accumulation and wages become larger. However, the corresponding increase in longevity (see Result 1) tends to reduce consumption. Since interest rates also go down due to the increase in capital accumulation (thus leading towards a reduction in old-age consumption), *and given the positive direct effect of increased longevity on expected lifetime utility* (i.e., individuals enjoy a longer life span), the final effect on the steady-state welfare of an increase in child tax is ambiguous. Putting it in a more analytical form, the government's (second-best) objective can be described by:[12]

$$\max_{\{b\}}V(b)=\ln\left(\gamma^{\gamma}[(q+b)w(b)]^{1+\pi(b)}R(b)^{\pi(b)}\left(\frac{1-\tau}{(1+\pi(b))(q+b)+\gamma q}\right)^{1+\pi(b)+\gamma}\right), \tag{13}$$

for any $0<\tau<1$, and the following result holds:

Result 3. *[Second-best child policy]. In an economy with public health investments, a value of child tax exists that maximises the steady-state lifetime indirect utility index.*

Using the same parameter values as in Section 3 and Table 1, Result 3 is illustrated in Table 3.

Table 3. Child tax and long-term welfare.

Low regime ($\bar{k}_1$)

b	0	0.01	0.02	0.025
$\bar{k}_1(b)$	0.724	0.793	0.895	0.998
$\pi(b)$	0.33	0.338	0.355	0.376
$n(b)$	1.29	1.26	1.22	1.2
$c_1(b)$	2.84	2.95	3.09	3.19
$c_2(b)$	14.19	13.9	13.41	12.89
$V(b)$	2.168	2.203	2.255	2.31

High regime ($\bar{k}_3$)

b	0	0.01	0.02	0.025	0.05	0.07	0.077	0.1	0.2	0.3
$\bar{k}_3(b)$	2.88	3.19	3.45	3.58	4.16	4.6	4.76	5.26	7.5	9.9

[12] In Section 5.1 we discuss the use of both health and child taxes as a second-best optimal policy.

$\pi(b)$	0.83	0.86	0.877	0.884	0.908	0.92	0.922	0.93	0.943	0.947
$n(b)$	1.06	1.02	1	0.98	0.93	0.89	0.88	0.84	0.7	0.61
$c_1(b)$	3.68	3.81	3.93	3.99	4.25	4.46	4.52	4.74	5.62	6.4
$c_2(b)$	7.3	7.06	6.9	6.83	6.59	6.45	6.41	6.28	5.87	5.5
$V(b)$	3.012	3.045	3.064	3.07	3.089	3.0945	3.0948	3.092	3.051	2.991

The economic reason for Result 3 is simple. When there is an annuities market , individuals do not take into account the benefits of an increase in public investments on their health and longevity. By positively affecting capital accumulation, the child tax leads to an increase in consumption and longevity, which directly contributes to increasing long-term welfare (because individuals live longer), despite the reduction in fertility and old-age consumption (because of the reduction in interest rates). When longevity tends towards π_1 (i.e., the child tax goes beyond the welfare-maximising value 0.077), the rise in child tax does not actually work towards an increase in an individual's life span. Thus the negative effects on welfare of the reduction in fertility and old-age consumption prevail and long-term welfare then tends to go down. Although the child tax can be welfare-maximising in the long run, the following result shows that:

Result 4. *In an economy à la Chakraborty (2004) with endogenous fertility, a child tax policy cannot represent an* A*–Pareto improvement.*

Since in this economy only the agents who are actually born at any time have a utility function that represents their lifetime preference profiles (with respect to material consumption and the number of children), the notion of A–efficiency can be used to compare alternatives. The economic reason for Result 4 is simple. Let the health tax policy already exist at time t and assume that the child tax policy is introduced at time t. Since the child policy only involves agents of generation t, the lifetime welfare of agents of generation $t-1$ is negatively affected by the child tax exclusively through a reduction in old-age consumption ($c_{2,t}$) because the interest rate at time t becomes lower, with young-age consumption ($c_{1,t-1}$) and fertility (n_t) being unaffected. Therefore, through the child policy, some people end up better off and others are worse off (see Section 4.2 for details). This implies that a rise in child tax cannot be an A–Pareto improvement.

4.2. Transitional dynamics

In order to evaluate the political feasibility of the child policy, it may be useful to study the (short-term) transitional effects of introducing a child tax in an economy with public health investments, as well as how much time is required to approach the steady-state welfare level. We recall that the parameter values are the following: $A=12.2$, $\alpha=0.33$, $\gamma=1$, $\pi_0=0.3$, $\pi_1=0.95$, $\delta=10$, $\Delta=1$, $q=0.3$, $\tau=0.1$. The low (resp. high) regime of development is thus characterised by the steady-state stock of capital $\bar{k}_1=0.72$ (resp. $\bar{k}_3=2.88$), and the stock of capital that discriminates between poor and rich countries is $\bar{k}_2=1.7$. As usual, we assume that a generation consists of almost thirty years (see de la Croix and Michel, 2002). Then, by contrasting the lifetime indirect utility index at different dates (V_t, where $t = 0,1,2,...$), we compare a Chakraborty-type economy without a child policy ($b = 0$) with a Chakraborty-type economy where the government also adopts a second-best optimal child tax policy ($b = 0.077$) introduced at time $t = 0$.

Table 4. Transitional dynamics. Low regime (initial condition: $k_{-1} = 0.65$).

	$b = 0$ (no child tax)	$b = 0.077$ (second-best optimal policy)
V_{-1}	2.12	2.07
V_0	2.14	2.18
V_1	2.152	2.27
V_2	2.159	2.35
V_3	2.163	2.44
V_4	2.165	2.55
V_5	2.166	2.69
V_6	2.167	2.82
V_7	2.168	2.94
V_{20}	2.168	3.0948

Table 5. Transitional dynamics. High regime (initial condition: $k_{-1} = 2.7$).

	$b = 0$ (no child tax)	$b = 0.077$ (second-best optimal policy)
V_{-1}	2.977	2.85
V_0	2.986	2.95
V_1	2.993	3.02
V_2	2.998	3.06
V_3	3.002	3.081
V_4	3.005	3.089
V_{13}	3.012	3.0948

As can be seen from Tables 4 and 5, the child tax policy does not represent an A – Pareto improvement (see Result 4) because the agents that belong to generation $t-1$, with regard to the low regime of development, and the agents that belong to generations $t-1$ and t, with regard to the high regime of development, are worse off when the second-best optimal policy is implemented. However, from time $t+1$ onwards, individuals of both the low and high regimes are better off. The poor economy escapes from the poverty trap and both the poor and rich countries approach the high-regime steady-state welfare level in almost twenty generations with regard to poor countries, and thirteen generations with regard to rich countries.

4.3. A *–Pareto improving health policy*

In previous sections we have studied the effects on macroeconomic and demographic variables of the use of child taxes in an economy with health investments as in

Chakraborty (2004). We now go further and study the desirability of the introduction of health tax τ (for the generations alive at the moment of the introduction of the policy), by considering for analytical simplicity the absence of child taxes ($b=0$) in a Chakraborty-type environment with endogenous fertility and $\delta=1$. Results are summarised in the following proposition (which hold also in the case of exogenous fertility).

Proposition 3. *Financing a public health programme can represent an* A*–Pareto improvement for the generations involved at the time of the introduction of it.*

Proof. We assume that a public programme is introduced at the beginning of time t. The generations involved are $t-1$ and t. If none of them suffer, implementing it represents an A–Pareto improvement. Putting it in an analytical form, the indirect expected lifetime utility of generation $t-1$ is $V_{t-1}=\ln(c_{1,t-1}^{1+\pi_{t-1}} n_{t-1}^{\gamma} R_t^{\pi_{t-1}})$, where $c_{1,t-1}=\dfrac{w_{t-1}}{1+\pi_{t-1}+\gamma}$, $n_{t-1}=\dfrac{\gamma}{(1+\pi_{t-1}+\gamma)q}$, $\pi_{t-1}=\pi_0$, $w_{t-1}=(1-\alpha)Ak_{t-1}^{\alpha}$, $R_t=\alpha Ak_t^{\alpha-1}$ and $k_t=\dfrac{\pi_0}{\gamma}w_{t-1}q$, while expected utility of generation t is $V_t=\ln(c_{1,t}^{1+\pi_t} n_t^{\gamma}(R_{t+1}^{e})^{\pi_t})$, where $c_{1,t}=\dfrac{w_t(1-\tau)}{1+\pi_t+\gamma}$, $n_t=\dfrac{\gamma(1-\tau)}{(1+\pi_t+\gamma)q}$, π_t is governed by Eq. (1), $w_t=(1-\alpha)Ak_t^{\alpha}$, $R_{t+1}^{e}=\alpha Ak_{t+1}^{\alpha-1}$ and $k_{t+1}=\dfrac{\pi_t}{\gamma}w_t q$. Let $t=0$. As can easily be ascertained, for any given initial condition k_{-1}, generation -1 is A–Pareto neutral because all arguments of V_{-1} are independent of τ. With regard to generation 0, there exist counterbalancing effects on welfare at work: on the one hand, health tax reduces disposable income, and increase longevity and capital accumulation in period 1. Then, consumption, fertility and the interest factor tend to be reduced by the policy. Overall, this causes a negative welfare effect. However, the increase in longevity causes a positive welfare effects because individuals directly enjoy by living longer. If the latter effect is large enough, the introduction of a public health programme is A–Pareto improving for the generations involved. This of course depends on key parameters of the problem and initial condition. Analytically, we get:

$$\frac{\partial V_t}{\partial \tau}\Big|_{\tau=0}=w_t(\pi_1-\pi_0)\ln\left(\frac{\alpha A w_t}{1+\pi_0+\gamma}\left(\frac{\pi_0 w_t q}{\gamma}\right)^{\alpha-1}\right)-w_t(\pi_1-\pi_0)(2-\alpha)-(1+\pi_0+\gamma).$$

When $\dfrac{\partial V_t}{\partial \tau}\Big|_{\tau=0}>0$ we get the result. **Q.E.D.**

By using the parameter set $A=12.2$, $\alpha=0.33$, $\gamma=1$, $\pi_0=0.3$, $\pi_1=0.95$, $\delta=1$, $\Delta=1$ and $q=0.3$, it can easily be verified that a health policy is A–Pareto improving depending on initial condition. In fact, by assuming for instance $k_{-1}=1$, we get $\dfrac{\partial V_0}{\partial \tau}\Big|_{\tau=0}=4.057$.

5. Public pensions with defined contribution

In this section we extend the model developed in Section 2 to allow for the existence of a public PAYG pension plan with defined contribution, i.e. the contribution is specified period by period. There is a burgeoning theoretical literature based on models with endogenous fertility that have studied the interaction of changes in fertility behaviour and pensions with different objectives (e.g., Cigno, 1992, 1995, 2010; Nishimura and Zhang, 1992; Cigno and Rosati, 1992; Wigger, 1999; Corneo and Marquardt, 2000; van Groezen et al., 2003; Abio et al., 2004; Cigno and Werding, 2007; van Groezen and Meijdam, 2008; Hirazawa and Yakita, 2009; Fenge and von Weizsäcker 2010; Fanti and Gori, 2012b, 2012c, 2013).[13]

As clearly pointed out by Cigno (1993), when fertility is endogenous, two positive externalities of children exist. One is due to the positive effect that an extra child implies on future output. The other is due to the reduction in the capital-labour ratio. When there are pensions, there is also an additional external effect of children because the benefit of having a child is too small to be internalised by parents (with the contribution to the PAYG system being shared among all the members of the working population). This makes the child allowance/tax policy important as an instrument to correct offspring externalities in economies with PAYG pensions (van Groezen et al., 2003; Fenge and Meier, 2005, 2009; van Groezen and Meijdam, 2008) in order to achieve the social optimum.

The aim of this section is to show the effects of child taxes on economic growth and development, in an economy à la Chakraborty (2004) augmented with unfunded public pensions. In fact, it is useful to relate our analysis to the above mentioned literature justifying subsidising fertility in view of the positive externalities of having children when a PAYG scheme is in place. Therefore, in addition to the health expenditure financed by labour income taxes and the wage subsidy expenditure financed by child taxes (see Eqs. 2 and 3, respectively), we now assume that PAYG pensions $P_t = p_t N_{t-1}$ exist to be redistributed across generations, where p_t represents pension expenditure per older people in period t. This expenditure is constrained by the amount of tax receipts $\eta w_t N_t$, where η is the contribution rate to the PAYG system. Since $N_t = n_{t-1} N_{t-1}$, the per pensioner budget constraint of the government in period t is:

$$\pi_{t-1} p_t = \eta w_t n_{t-1} . \tag{14}$$

The budget constraints when young (Eq. 15) and when old (Eq. 16) of an individual of generation t become :

$$c_{1,t} + s_t + (q+b) w_t n_t = w_t (1 - \tau - \eta + \theta_t) , \tag{15}$$

and

$$c_{2,t+1} = \frac{R^e_{t+1}}{\pi_t} s_t + p^e_{t+1} . \tag{16}$$

By taking as given the government pension budget, the other child policy variables and factor prices, the maximisation of the expected lifetime utility function Eq. (5) by the representative individual subject to Eqs. (15) and (16) gives the demand for children and savings as follows:

[13] Other interesting papers that deal with this topic in models with exogenous (resp. endogenous) uncertain lifetime are Yakita (2001) and Hirazawa et al. (2010) (resp. Pestieau et al., 2008).

$$n_t = \frac{\gamma w_t(1-\tau-\eta)}{w_t[(1+\pi_t)(q+b)+\gamma q]-\gamma\eta\dfrac{w_{t+1}^e}{R_{t+1}^e}}, \tag{17}$$

$$s_t = \frac{w_t(1-\tau-\eta)[\pi_t w_t(q+b)-\gamma\eta\dfrac{w_{t+1}^e}{R_{t+1}^e}]}{w_t[(1+\pi_t)(q+b)+\gamma q]-\gamma\eta\dfrac{w_{t+1}^e}{R_{t+1}^e}}, \tag{18}$$

where $0<\tau+\eta<1$ must hold. Since market clearing in the capital market is still given by $n_t k_{t+1} = s_t$, then together with Eqs. (17) and (18) we get the equilibrium condition:

$$k_{t+1} = \frac{\pi_t}{\gamma} w_t(q+b) - \eta\frac{w_{t+1}^e}{R_{t+1}^e}. \tag{19.1}$$

By assuming that individuals have perfect foresight, i.e. $R_{t+1}^e = \alpha A k_{t+1}^{\alpha-1}$ and $w_{t+1}^e = (1-\alpha)Ak_{t+1}^{\alpha}$, Eq. (19.1) can be rewritten as follows:

$$k_{t+1} = \frac{\alpha}{\alpha+\eta(1-\alpha)}\cdot\frac{\pi_t}{\gamma}w_t(q+b), \tag{19.2}$$

where $\alpha/[\alpha+\eta(1-\alpha)]<1$. Now, by using Eqs. (1), (2), (9) and (19.2), the dynamic path of capital accumulation is described by the following first order non-linear difference equation:

$$k_{t+1} = \frac{\alpha}{\alpha+\eta(1-\alpha)}G(k_t). \tag{20}$$

As expected, Eq. (20) reveals that when a public pension expenditure is in place (positive values of η), capital accumulation is lower than when it is absent, and a rise in η, which only affects the constant that multiplies $G(k_t)$, monotonically reduces the accumulation of capital and the steady-state equilibria. From a graphical point of view, this means that when η rises, everything else being equal the graph of $G(k)$ shifts downwards but the shape is the same as in the model in Section 2. Therefore, Propositions 1 and 2 also hold when PAYG pensions exist. The following results can thus be stated.

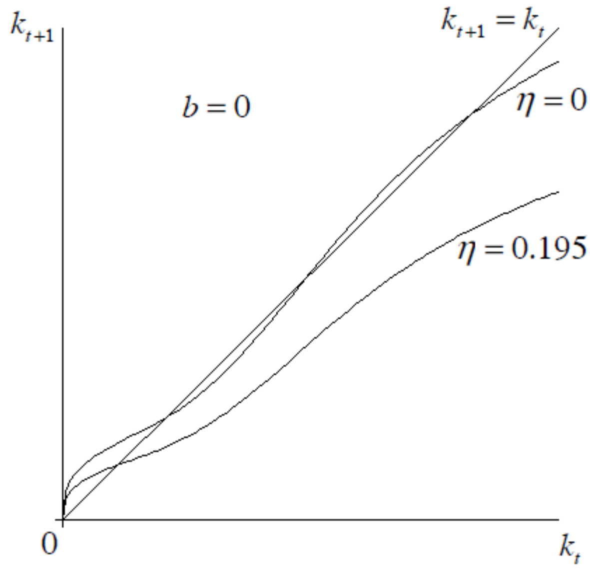
Result 5. *Unlike van Groezen et al. (2003) and van Groezen and Meijdam (2008), who show that when a PAYG pension system is in place, a subsidy on children can be used to maximise long-term lifetime welfare in an economy with endogenous fertility, we find that in a Chakraborty-type economy with endogenous fertility and pensions, a child tax can be used to both escape from poverty and to maximise long-term lifetime welfare.*

Result 6. *In addition, in a Chakraborty-type economy with PAYG pensions, the values of child taxes that can be used to escape from poverty and maximise long-term lifetime welfare are higher than the corresponding values in the absence of PAYG pensions, everything else being equal.*

The increase in the long-term lifetime welfare of individuals due to a rise in child tax is due to the fact that in addition to the externalities of children, another externality exists due to the increased life span, which is not taken into account by individuals in the market (the existence of a market for annuities in fact implies that the factor of interest is divided by *average* longevity). A rise in child tax, therefore, contributes to reduce fertility and increase longevity in such a way as to maximise lifetime welfare in the long term.

In addition, Result 6 is due to the fact that when a PAYG system is in place savings are lower and fertility is higher than in the absence of it (crowding out effect). Since capital accumulation is lower with PAYG pensions (see Eqs. 10.2 and 20) then, everything else being equal, higher values of the child tax (which causes an increase in savings and a reduction in fertility) must be used to both escape from poverty and maximise the lifetime welfare of individuals in the long term as compared with an economy without pensions.

Results 5 and 6 are illustrated in Figures 4 and 5, and in Table 6. In particular, Figure 4 shows that the introduction of PAYG pensions reduces capital accumulation with respect to an economy without pensions. Then, higher values of the child tax should be used (ceteris paribus) to trigger the beneficial effects on macroeconomic variables than when no PAYG pensions exist. The figure exemplifies the case without child policy for the parameter set used in Sections 3 and 4. When pensions are absent, two regimes of development exist. The introduction of public pensions (by using a value of the contribution rate close to that of the German economy, i.e. $\eta = 0.195$ see Cigno and Werding, 2007) causes the disappearance of the high regime and thus the economies converge to the low regime irrespective of initial conditions.



(a)

Figure 4. Capital accumulation in an economy with PAYG pensions ($\eta = 0.195$, see Feldstein, 2005 and Cigno and Werding, 2007 for realistic values of the contribution rate to the PAYG system in several countries. In particular, we have followed Cigno and Werding, 2007 and we have chosen a value of η close to the value of Germany), and without PAYG pensions ($\eta = 0$) when no child tax exists ($b = 0$). Parameter set: $A = 12.2$, $\alpha = 0.33$, $\gamma = 1$, $\pi_0 = 0.3$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.3$ and $\tau = 0.1$ (see Sections 3 and 4).

Figure 5 and Table 6 exemplify the effects of child taxes in an economy with pensions for the following parameter set (used also in Table 6): $A = 13.7$, $\alpha = 0.33$, $\gamma = 1$, $\pi_0 = 0.3$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.3$, $\tau = 0.1$ and $\eta = 0.195$. Starting from a situation where two regimes of development are in existence when child tax is absent, the use of tax on children can allow to both escape from poverty (for a value of b close to 2 per cent) and maximise long-term welfare (for a value of b slightly higher than 11 per cent). In fact, the positive effects on welfare of the increase in longevity and consumption when young more than compensate the negative effects of the reduction in both fertility and consumption when old.

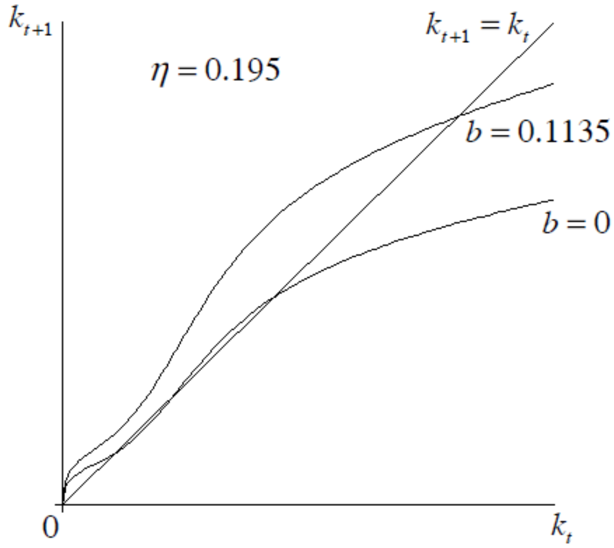


Figure 5. Capital accumulation in an economy with PAYG pensions, with and without a child tax policy. The child tax is fixed at the steady-state welfare-maximising level ($b = 0.1135$). Parameter set (used also in Table 6): $A = 13.7$, $\alpha = 0.33$, $\gamma = 1$, $\pi_0 = 0.3$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.3$, $\tau = 0.1$ and $\eta = 0.195$.

Table 6. Child tax and long-term welfare in an economy with PAYG pensions.

Low regime ($\bar{k}_1$)

b	0	0.01	0.015	0.02
$\bar{k}_1(b)$	0.53	0.59	0.63	0.73
$\pi(b)$	0.33	0.34	0.35	0.38
$n(b)$	1.5	1.02	1.01	0.99
$c_1(b)$	2.35	2.45	2.52	2.64
$c_2(b)$	16.17	15.76	15.44	14.7
$p(b)$	4.58	4.47	4.38	4.16
$V(b)$	1.83	1.88	1.91	2

High regime ($\bar{k}_3$)

b	0	0.01	0.015	0.02	0.05	0.07	0.1	0.1135	0.2	0.3
$\bar{k}_3(b)$	2.15	2.35	2.44	2.53	3.02	3.34	3.81	4.02	5.42	7.16
$\pi(b)$	0.84	0.87	0.878	0.88	0.91	0.92	0.93	0.935	0.944	0.947
$n(b)$	0.9	0.87	0.86	0.85	0.8	0.77	0.73	0.71	0.62	0.54
$c_1(b)$	3.19	3.31	3.36	3.42	3.71	3.89	4.16	4.27	4.96	5.68
$c_2(b)$	8.64	8.44	8.36	8.29	8	7.85	7.67	7.6	7.23	6.87
$p(b)$	2.45	2.39	2.37	2.35	2.26	2.22	2.17	2.15	2.05	1.95
$V(b)$	2.88	2.92	2.93	2.94	2.99	3	3.009	3.01	2.99	2.94

5.1. Second-best health and child policies

Up to now we have considered a situation for which, for any given value of the health tax rate (τ), the government had the purpose of using child tax b to maximise expected lifetime utility at the stationary state (second-best policy), given that Result 4 prevents A –Pareto improvements of the child policy. Nevertheless, has Chakraborty (2004) shown that a welfare-maximising value of the health tax rate does exist, while Proposition 3 in this paper has shown that the introduction of health policy can be A – Pareto improving for the generations involved.

Then, government's objective would be that of maximising expected steady-state welfare by choosing both the health tax rate and child tax. Unfortunately, although preferences are logarithmic, analytic expressions for welfare-maximising tax rates cannot explicitly be derived. Then, we proceed through numerical simulations to show that a couple (τ,b) exists as a second-best optimal policy.

We assume the government has the objective of maximising

$$V(\tau,b)=\ln(c_1(\tau,b)^{1+\pi(\tau,b)}n(\tau,b)^{\gamma}R(\tau,b)^{\pi(\tau,b)}), \tag{21}$$

with respect to τ and b, for any $0\le\eta<1-\tau$, where $x(\tau,b)$ is the steady-state value of the generic variable x. The maximisation of (21) implies

$$\begin{cases}\dfrac{\partial V(\tau,b)}{\partial\tau}=0\Leftrightarrow\tau=\tau(b)\\[2mm]\dfrac{\partial V(\tau,b)}{\partial b}=0\Leftrightarrow b=b(\tau)\end{cases}, \tag{22}$$

From Eq. (22) it can be shown through numerical simulation that $\tau(b)$ (resp. $b(\tau)$) is a monotonic decreasing function of b (resp. τ) for several parameter values. This implies that b and τ can be used as substitute policy instruments in this context. In fact, both policies act in the same direction on demo-economic variables by increasing longevity and consumption when young, and reducing fertility at the steady-state. Although closed-form expressions for $\tau(b)$ and $b(\tau)$ are prevented, it is possible to find the coordinates of a point where the graph of $\tau(b)$ and the graph of $b(\tau)$ intersect each other and expected steady-state welfare is at the highest possible level. Therefore,

Result 7. *There exists a couple of tax rates (τ_V,b_V) corresponding to which the second-best optimum optimorum is obtained for every $0\le\eta<1-\tau$.*

In order to illustrate Result 7, we adopt the usual parameter values: $A=13.7$, $\alpha=0.33$, $\gamma=1$, $\pi_0=0.3$, $\pi_1=0.95$, $\delta=10$, $\Delta=1$, $q=0.3$ and $\eta=0.195$.[14] In this context, $(\tau_V,b_V)\cong(0.0877,0.18)$, corresponding to which only the high regime exists and $\bar{k}_3=4.92$, $\pi(\tau_V,b_V)=0.921$, $n(\tau_V,b_V)=0.653$ and $V(\tau_V,b_V)=3.025818$. Any deviations from the couple $(0.0877,0.18)$ reduces steady-state welfare.

6. The child quantity-quality trade-off

[14] Result 7 holds for several parameter constellations (with and without pensions) that are reported to save space.

In this section we introduce an additional element into the analysis: the child quality expenditure. For doing this, in order to preserve the structure of the OLG economy with endogenous fertility presented in previous sections under weak altruism towards children, we follow Strulik (2004a, 2004b) and consider an economy where parents derive utility from both the family size and quality of children they have. This line of reasoning have been initiated by the theory of child demand of Becker (1960), for which perfectly rational parents enjoy having children as well as putting effort in raising them (child quality), that is parental utility positively depends on having well educated children. Following Andreoni (1989), this individual behaviour is called impure altruism towards children. This approach markedly differs from the approach called pure altruism towards children (e.g., Becker et al., 1990), for which perfectly rational parents derive utility from the utility of their descendants.

To capture elements of impure altruism, we consider the following expected lifetime utility function where child quality is measured by the expenditure on children:

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \gamma_1 \ln(n_t) + \gamma_2 \ln((q+Q_t)w_t) , \tag{23}$$

where $(q+Q_t)w_t$ is total expenditure to take care of children, Q_t (control variable) is the portion of wage income devoted to schooling expenditure and q can now be interpreted as the constant fraction of wage income spent on nourishing activities. The parameters γ_1 and γ_2 capture the parents' relative desire to have children and parents' relative desire to have well educated children, respectively. We assume that $\gamma_1 > \gamma_2$ to ensure the existence of some positive expenditure on child quality as an interior solution (maximum).[15]

This simple utility function leads to two different scenarios with regard to long-term demo-economic outcomes depending of the relative size of the child tax. In the former scenario, characterised by either a sufficiently low value of the child tax or child subsidy, parents do not voluntarily spend on child quality (corner solution) and choose to several children. In the latter one, characterised by a sufficiently high value of the child tax, parents voluntarily spend to have less and well-educated children (interior solution). This will be clear later in this section.

By also assuming the existence of PAYG pensions, the lifetime budget constraint of an individual representative of generation t can be written as follows:

$$c_{1,t} + \frac{\pi_t c_{2,t+1}}{R^e_{t+1}} + (q + Q_t + b)w_t n_t = w_t(1-\tau-\eta+\theta_t) + \frac{p^e_{t+1}}{R^e_{t+1}} . \tag{24}$$

The problem of the representative individual at time t is to maximise utility function (23) with respect to consumption when young ($c_{1,t}$), consumption when old ($c_{2,t+1}$), the number of children (n_t) and the quality of children (Q_t) subject to Eq. (24) and $Q_t \geq 0$. Then, the first order condition are the following:

$$\frac{c_{2,t+1}}{c_{1,t}} = R^e_{t+1} , \tag{25}$$

$$\frac{c_{1,t}}{n_t}\gamma_1 = (q + Q_t + b)w_t , \tag{26}$$

[15] To clarify this assumption, we note that utility function (23) can be rewritten as $U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + (\gamma_1 - \gamma_2)\ln(n_t) + \gamma_2 \ln((q+Q_t)w_t n_t)$. For $\gamma_1 < \gamma_2$ a sequence of feasible allocation may exist such that utility diverges to positive values. Then, $\gamma_1 > \gamma_2$ is required.

$$\frac{c_{1,t}}{q+Q_t}\gamma_2 = w_t n_t \,. \tag{27}$$

Eq. (25) is the standard condition that equates marginal rates of substitution of consumption when young and consumption when old to prices. Eq. (26) drives the substitution between consumption when young and the family size. As far as marginal costs of raising an extra child increase (e.g., because child taxes increase), it becomes convenient to substitute children with consumption. Eq. (27) implies that at an interior solution child quality increase when consumption increase. Conversely, a rise in the family size implies a reduction in the expenditure for child quality because spending to have well educated children becomes more expensive.

By combining Eqs. (24)-(27) two distinct scenarios can be obtained depending on the value of the child tax. First of all, let

$$\bar{b} := \frac{q(\gamma_1 - \gamma_2)}{\gamma_2}, \tag{28}$$

be a threshold value of the child tax.

Scenario 1 (no child quality expenditure). If $\bar{b} < b$ (i.e., the government levies a low value of the child tax or provides child allowances) then child quality expenditure is not affordable ($Q = 0$) and demand for children and savings are given by Eqs. (17) and (18), respectively. This can be explained by the fact that a low value of child tax (alternatively, the provision of a child allowance) implies a sufficiently low marginal cost of children, which makes it convenient to increase the family size, reduce consumption and do not spend for schooling (corner solution). Therefore, the findings of previous sections hold.

Scenario 2 (positive child quality expenditure). If $\bar{b} > b$ (the child tax is sufficiently high) then child quality expenditure per child is affordable and the optimal solution is given by the following equations:

$$Q_t = Q = \frac{\gamma_2 b}{\gamma_1 - \gamma_2} - q\,, \tag{29}$$

$$n_t = \frac{(\gamma_1 - \gamma_2) w_t (1-\tau-\eta)}{(1+\pi_t+\gamma_2) b\, w_t - (\gamma_1-\gamma_2)\eta \dfrac{w^e_{t+1}}{R^e_{t+1}}}, \tag{30}$$

$$s_t = \frac{w_t(1-\tau-\eta)\left[\pi_t b\, w_t - (\gamma_1-\gamma_2)\eta \dfrac{w^e_{t+1}}{R^e_{t+1}}\right]}{(1+\pi_t+\gamma_2) b\, w_t - (\gamma_1-\gamma_2)\eta \dfrac{w^e_{t+1}}{R^e_{t+1}}}, \tag{31}$$

When child tax becomes higher, individuals tend to have less children and spend for schooling because of the increase in marginal cost of raising a child and then marginal rates of substitution change accordingly at the optimum. In addition, the following proposition holds.

Proposition 4. *Given the parents' relative desire to have children, the parents' relative desire to have children of high quality and the fraction of wage income spent on nourishing activities, there exists a threshold value of the child tax above which* *existence of child quality spending is guaranteed. An increase in the child tax increases child quality spending per child.*

Proof. The proof is straightforward from (28) and (29). **Q.E.D.**

From Proposition 4 it is interesting to note that child tax has the potential to modify marginal conditions for the substitution of child quantity and child quality by individuals, thus allowing to end up in two different macroeconomic environments, where difference in longevity can also be observed. Everything else being equal, Scenario 1 in fact implies no child quality expenditure, lower capital accumulation and longevity and higher fertility than Scenario 2, where child quality spending is positive. These results are similar in spirit to main findings of Strulik (2004a, p. 438), where "parental behavior at the corner may generate economic stagnation while behavior at the interior may generate perpetual growth". In his model, however, it is child mortality that determines either the corner solution or interior solution of child quality spending.

We now turn to the analysis of the macro-economy given firm's behaviour. Market clearing in the capital market is $n_t k_{t+1} = s_t$. Then, by using Eqs. (1), (2) and (9), and assuming individuals have perfect foresight, capital accumulation is described by:

$$\begin{cases} k_{t+1} = \dfrac{\alpha}{\alpha + \eta(1-\alpha)} G(k_t) \quad if \quad b < \bar{b} \\ k_{t+1} = \dfrac{\alpha}{\alpha + \eta(1-\alpha)} \bar{G}(k_t) \quad if \quad b > \bar{b} \end{cases}, \tag{32}$$

where $G(k_t)$ is defined in (10.2), $\bar{G}(k_t) := \dfrac{\bar{D} k_t^{\alpha}(\pi_0 + \pi_1 \mathrm{B} k_t^{\alpha\delta})}{(\gamma_1 - \gamma_2)(1 + \mathrm{B} k_t^{\alpha\delta})}$ and $\bar{D} := b(1-\alpha)A$. Of course $\gamma_1 = \gamma$ when child quality expenditure is binding. From map (32) the following proposition holds.

Proposition 5. *Capital accumulation under Scenario 2 is higher than capital accumulation under Scenario 1 for every* $b > \bar{b}$.

Proof. This statement can easily be proved by comparing the functions G and $\bar{G}$. **Q.E.D.**

What is interesting to note is that Scenario 1 becomes the unique environment when governments provide child allowances to families. Differently, when child tax is applied the marginal cost or raising an extra child increases so that individuals want substitute child quantity for child quality and the shift from Scenario 1 to Scenario 2 is made possible depending on the relative size of child tax.

To illustrate this result, let us assume the following parameters values: $A = 13.7$, $\alpha = 0.33$, $\gamma_1 = 1$, $\gamma_2 = 0.5$, $\pi_0 = 0.3$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.15$, $\tau = 0.1$ and $\eta = 0.195$. These are the same parameters than those used to exemplify the model with endogenous fertility and pensions under weak altruism, with the only difference that now parents also have preference for child quality (γ_2), and q is lower than 0.3 because it now only captures the nourishing cost of children, rather than summarising the whole cost to take care of them (nourishing plus schooling). To avoid to lengthen the paper further, however, we do not present figures or tables to show this result, while restricting attention to a discussion of the main findings. This parameters generate $\bar{b} = 0.15$. A child tax per child smaller (resp. larger) than 15 per cent implies no child quality spending and a sufficiently large family size (resp. child quality spending and sufficiently small family size).

Therefore, for any $b < 0.15$ Scenario 1 applies: there only exists the low regime of development when $b < 0.134$ and individuals do not spend on child quality. For instance, for $b = 0.1$ we get $Q = 0$, $\bar{k}_1 = 0.36$, $\pi(b) = 0.31$, $n(b) = 1.55$ and $V(b) = 2.33$. An increases in the child tax at $b = 0.14$ causes the appearance of two regimes of development. In the low regime, $Q = 0$, $\bar{k}_1 = 0.49$, $\pi(b) = 0.325$, $n(b) = 1.39$ and $V(b) = 2.39$. In the high regime, $Q = 0$, $\bar{k}_3 = 1.9$, $\pi(b) = 0.8$, $n(b) = 1.16$ and $V(b) = 3.44$. A government which has the objective of maximising lifetime welfare in the long-term wants increase further the child tax. A rise in b over 0.15 makes it convenient for individuals to spend for child quality and Scenario 2 then applies in a Chakraborty-type economy. In particular, a child tax close to 0.16 allows to escape from poverty and long-term welfare is maximised in the high regime at $b = 0.1855$, corresponding to which $Q = 0.035$, $\bar{k}_3 = 3.35$, $\pi(b) = 0.92$, $n(b) = 0.88$ and $V(b) = 3.858$. As can seen, child tax favours the shift from nourishing expenditure to child quality expenditure which, in turn, implies different scenarios with regard to demographic and macroeconomic outcomes.

With regard to the second-best optimum optimorum in Scenario 1 (resp. Scenario 2), it can be reached at $(\tau_V, b_V) \cong (0.1155, 0.12)$ (resp. $(\tau_V, b_V) \cong (0.110851, 0.151187)$), corresponding to which only the high regime exists and $V(\tau_V, b_V) = 3.557863$ (resp. $V(\tau_V, b_V) = 3.86124021$).

7. Conclusions

We have studied the effects of child policies in an overlapping generations model with endogenous fertility (under the assumptions of both weak altruism and impure altruism towards children) and endogenous longevity determined by public health investments (Chakraborty, 2004). We have assumed a sufficiently general form of the relationship between longevity and public health spending to include realistic features of the evolution of the lifespan of agents. When there are threshold effects of health capital on longevity, development traps appear under less stringent (and more empirically significant) conditions in the output elasticity of capital with respect to Chakraborty (2004). Thus, depending on the initial conditions, an economy may be either entrapped in a low development regime, where income and life expectancy are low and fertility is high, or converge towards a high development regime, where income and life expectancy are high and fertility is low.

However, the main message of this paper is that the limiting outcomes of the economy do not necessarily depend on the initial conditions. Regardless of whether an economy starts out with a low or high stock of capital, a child tax programme can be adopted to permanently escape the vicious cycle of poverty and ill-health, since it works as a stimulus to accumulate capital, increase income per worker and life expectancy, reduce population growth and maximise lifetime welfare in the long term. In addition, we have also shown that in a Chakraborty-type economy with endogenous fertility, a child tax can also be used as a second-best optimal policy when public pensions are in place in order to escape from poverty and maximise steady-state welfare. Interestingly, in the case of impure altruism towards children we find that the child tax plays a preeminent role in determining both the substitution between the quantity and quality of children and the long-term demo-economic outcomes of an economy. In particular, when the government provides child allowances or levies a sufficiently low child tax, the marginal cost of raising a child is low and individuals do not want to spend on child quality and decide to have larger families. This causes lower capital accumulation and income per young person than when the child tax is high, because individuals tend to substitute quantity with quality of children in that case given the rise in marginal costs of having an additional child. Therefore, the child tax favours child quality spending which in turn causes higher income per young person and welfare of individuals.

This paper suggests that the child tax policy may have favoured the economic growth in recent years of economies such as China. Conversely, our results may also constitute a warning with regard to the effects of the more traditional child-subsidy policies: the price to be paid for stimulating a recovery in fertility in developed countries may be not only the expected reduction in GDP, but also the formation of development traps which may also be attractive for developing countries in the case of negative economic shocks.

Of course we are aware of the limitations of our analysis, especially with regard to policy applications. In fact we have presented a highly stylised theoretical contribution to the topic of endogenous longevity and endogenous fertility in a growth model with overlapping generations. However, it does represent a useful theoretical basis for future empirical contributions. Finally, this model could be extended along other lines including the following: (*i*) a private provision of health capital could be introduced and compared with the public health system; (*ii*) both young and old people could be entitled to public healthcare services; (*iii*) improvements in health care can have important economic effects in addition to a higher life expectancy: for instance, lower morbidity can increase both the productivity and wages of individuals (Strauss and Thomas, 1998) and thus may affect economic growth (Egger, 2009). With regard to a model with disease and infection dynamics, endogenous fertility could be introduced in an economy à la Chakraborty et al. (2010).

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Appendix A. Proof of Proposition 1

Lemma 1. *Define the right-hand side of Eq. (10.2) as* $G(k)$. *Then, we have: (1.i)* $G(0)=0$, *(1.ii)* $G'(k)>0$ *for any* $k>0$, *(1.iii)* $\lim_{k\to 0^+} G'(k)=+\infty$, *(1.iv)* $\lim_{k\to +\infty} G'(k)=0$, *(1.v)* $G''(k)$ *admits at most three roots and* $G''(0)\neq 0$.

From Eq. (10.2), property (1.*i*) is straightforward. Differentiating the right-hand side of Eq. (10.2) with respect to k gives

$$G'(k)=\frac{\alpha D(Fk^{2\alpha\delta}+Ek^{\alpha\delta}+\pi_0)}{\gamma k^{1-\alpha}(1+\mathrm{B}k^{\alpha\delta})^2}. \tag{A.1}$$

By defining $k^{\alpha\delta}:=x$ as a new supporting variable, (A.1) can be transformed to:

$$g(k,x)=\frac{\alpha D(Fx^2+Ex+\pi_0)}{\gamma k^{1-\alpha}(1+\mathrm{B}x)^2}. \tag{A.2}$$

Since no positive real roots of (A.2) exist, then (A.1) implies that $G'(k)>0$ for any $k>0$. This proves (1.*ii*).

Moreover,

$$\lim_{k\to 0^+} G'(k)=\frac{\alpha D}{\gamma}\lim_{k\to 0^+}\frac{Fk^{2\alpha\delta}+Ek^{\alpha\delta}+\pi_0}{k^{1-\alpha}(1+\mathrm{B}k^{\alpha\delta})^2}=+\infty .$$

and

$$\lim_{k\to+\infty} G'(k)=\lim_{k\to+\infty}\frac{\alpha D(Fk^{2\alpha\delta}+Ek^{\alpha\delta}+\pi_0)}{\gamma k^{1-\alpha}(1+\mathrm{B}k^{\alpha\delta})^2}=\frac{\alpha D}{\gamma}\lim_{k\to+\infty}\frac{\frac{\pi_0}{k^{2\alpha\delta}}+\frac{E}{k^{\alpha\delta}}+F}{k^{1-\alpha}\left(\frac{1}{k^{2\alpha\delta}}+\frac{2\mathrm{B}}{k^{\alpha\delta}}+\mathrm{B}^2\right)}=0,$$

which prove (1.*iii*) and (1.*iv*), respectively. Now, differentiating (A.1) with respect to k gives:

$$G''(k)=\frac{-\alpha D(\Lambda_1 k^{3\alpha\delta}+\Lambda_2 k^{2\alpha\delta}+\Lambda_3 k^{\alpha\delta}+\Lambda_4)}{\gamma k^{2-\alpha}(1+\mathrm{B}k^{\alpha\delta})^3}, \tag{A.3}$$

where $\Lambda_1:=(1-\alpha)\mathrm{B}F>0$ and $\Lambda_4:=\pi_0(1-\alpha)>0$. Knowing that $k^{\alpha\delta}:=x$, Eq. (A.3) can be rewritten as follows:

$$f(k,x)=\frac{-\alpha D(\Lambda_1 x^3+\Lambda_2 x^2+\Lambda_3 x+\Lambda_4)}{\gamma k^{2-\alpha}(1+\mathrm{B}x)^3}. \tag{A.4}$$

From (A.4), it is clear that $f(k,x)$ admits at most three roots for x and $f(k,0)\neq 0$. Hence, from (A.3), $G''(k)$ admits at most three roots for k and $G''(0)\neq 0$ for any $k>0$. This proves (1.*v*).

Proposition 1 therefore follows. In fact, by properties (1.*i*) and (1.*iii*), zero is always an unstable steady state of Eq. (10.2). By (1.*ii*)-(1.*iv*), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line, so that at least one positive stable steady state exists for any $k>0$.

Now, assume *ad absurdum* the existence of an odd number of equilibria. By (1.*ii*)-(1.*iv*), there cannot be an odd number of inflection points for any $k>0$. By property (1.*v*), therefore, the number of inflection points of $G(k)$ is either zero or two for any $k>0$. Since at least one positive stable steady state exists, then for any $k>0$ the phase map $G(k)$ may intersect the 45° line from below *at most* once before falling below it. Hence, an even number of equilibria must exist. There are either two steady states, with the positive one being the unique asymptotically stable equilibrium, or *at most* one positive steady state separates the lowest asymptotically stable steady state from the highest asymptotically stable one, and, thus, the number of equilibria is four.

In addition, from Eqs. (A.3) and (A.4) we observe that if $\Lambda_2>0$ and $\Lambda_3>0$ then no inflection points of $G(k)$ exist for any $k>0$, $G''(k)<0$ and two steady states exist in that case. In contrast, $G(k)$ has two inflection points for any $k > 0$ if at least either $\Lambda_2 < 0$ or $\Lambda_3 < 0$ is fulfilled and, hence in this case four steady states *can* exist.

Appendix B. Proof of Proposition 2

Differentiating Eq. (10.2) with respect to b gives:

$$G_b'(k,b) = \frac{k^{\alpha}(\pi_0 + \pi_1 \mathrm{B} k^{\alpha\delta})}{\gamma(1+\mathrm{B}k^{\alpha\delta})} \cdot \frac{\partial D}{\partial b}, \tag{B.1}$$

where $\partial D / \partial b = (1-\alpha)A > 0$ for any $b \in [0,+\infty)$. Since $G_b'(k,b) > 0$ for any $k > 0$ and $b \in [0,+\infty)$, then for any $k^* > 0$ such that $G(k^*, b_1) < k^*$ with $b_1 \in [0,+\infty)$, a threshold value $b^* > b_1$ exists such that $G(k^*, b^*) = k^*$. Therefore, $G(k^*, b) > k^*$ holds for any $b > b^*$.

Appendix C. Public debt

In this appendix we show that the main results of this paper also hold when the government at each date t issues an amount Z_t of public debt and levies lump sum taxes (τ_t^z) on the young workers (Diamond, 1965; Jaeger and Kuhle, 2009; Spataro and Fanti, 2011).

The level of national debt evolves according to the following equation $Z_{t+1} = Z_t R_t - \tau_t^z N_t$, which can be transformed in per worker terms as follows:

$$n_t z_{t+1} = z_t R_t - \tau_t^z, \tag{C.1}$$

where $z_t := Z_t / N_t$. Following Diamond (1965), we assume that the (non-negative) level of debt is constant over time, i.e. $z_{t+1} = z_t = z$. Thus, (C.1) becomes the following:

$$\tau_t^z = z(R_t - n_t). \tag{C.2}$$

The maximisation of expected utility function (5) subject to the lifetime budget constraint

$$c_{1,t} + \frac{\pi_t c_{2,t+1}}{R_{t+1}^e} + (q+b) w_t n_t = w_t(1-\tau+\theta_t) - \tau_t^z, \tag{C.3}$$

gives the following demand for children and savings (upon substitution of (C.2) for τ_t^z), respectively:

$$n_t = \frac{\gamma[w_t(1-\tau) - zR_t]}{w_t[(1+\pi_t)(q+b)+\gamma q] - \gamma z}, \tag{C.4}$$

$$s_t = \frac{\pi_t w_t (q+b)[w_t(1-\tau) - zR_t]}{w_t[(1+\pi_t)(q+b)+\gamma q] - \gamma z}. \tag{C.5}$$

Market clearing can now be expressed as $n_t(k_{t+1} + z) = s_t$. Then, by using (1), (2), (9), (C.4) and (C.5) we get:

$$k_{t+1} = G(k_t) - z. \tag{C.6}$$

Since the capital accumulation equation is the same as in the basic model of Section 2 minus a constant, the main findings of the paper also hold qualitatively when the government issues a constant (non-negative) amount of public debt.

We have not pursued this analysis further to save space. However, numerical simulations (available on request) show that a child tax can be used to permanently escape from poverty and maximise steady-state welfare, and their values are higher than when $z = 0$. Unlike the basic model, a child tax now increases the debt per child (z/n_t) due to a lower population growth in the short run. However, the increase in capital accumulation implies that the ratio of debt per young person over GDP per young person goes down in the long term.

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