The capital asset pricing model and the investment horizon: Comment

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TABLE 1.—INVENTORY AND RETALIATORY STRATEGIC STRENGTHS, 1960–73

	Total Inventory Warheads		Reliable and Penetrating Equivalent Megatons Surviving An All Out Attack	
	U.S.	Soviet	U.S.	Soviet
1960	2720	1415	2680	1194
1961	2839	1460	2283	714
1962	3268	1535	1956	387
1963	3228	1620	1329	362
1964	3700	1710	1413	330
1965	4255	1700	1376	309
1966	4176	1725	1369	283
1967	4260	1900	1451	315
1968	3970	2220	1369	659
1969	4070	2560	1350	739
1970	4896	3480	1352	995
1971	5510	3770	1290	1144
1972	6240	3910	1260	1154
1973	7862	3976	1330	1134

ber of warheads) is not a crucial test of success; the most desirable dependent variable, I should think, is *cost* under the assumption that combinations of W, Y, and other weapons characteristics which minimize cost for a given deterrence-defense combination are observed. And lastly, if a set of estimates shows no positive solution, we may lose interest in the stability of the solution without rejecting the estimates, provided they appear meaningful. No one has said the arms race *must* be stable.[1]

REFERENCES

Desai, Meghnad, and David Blake, "Modelling the Ultimate Absurdity: A Comment on 'A Quantitative Study of the Strategic Arms Race in the Missile Age'," this REVIEW 63 (Nov. 1981).

McGuire, Martin C., "A Quantitative Study of the Strategic Arms Race in the Missile Age," this REVIEW 59 (Aug. 1977), 328–329.

[1] Checking my back-up notes, I found the following regression result for the period 1960–73:

$$\dot{A} = 167 + .49A - .09R$$

$$\dot{R} = 779 + .21R - .75A$$

where R indicates number of Soviet megatons, while A indicates total number of U.S. warheads. Both equations are "perverse," indicating individually "unstable" behavior and a continuing "unstable" perpetuation of the arms race. This is just an illustration of "perverse" results which can fall out of the estimating process.

THE CAPITAL ASSET PRICING MODEL AND THE INVESTMENT HORIZON: COMMENT

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I. Introduction

In a recent paper in this REVIEW, Levhari and Levy (1977, p. 104) show that "the systematic risk of defensive stocks tends to decline while that for aggressive stocks tends to increase with increases in the investment horizon." They "illustrate the relationship between the systematic risk and the investment horizon with ten stocks that were undoubtedly defensive ($\beta_j < 1$) and ten stocks that were clearly aggressive ($\beta_j > 1$)" (p. 101). These 20 stocks are selected from a sample of 101 stocks traded on the New York Stock Exchange (NYSE) for the period 1948–1968.

The purpose of this comment is to investigate the validity of L-L's findings for a more comprehensive sample of 1,115 stocks. We find that L-L's conclusions do not hold when the larger sample is used. Specifically, we show that the changes in the value of a stock's systematic risk induced by a lengthening of the investment horizon follow a random walk. The changes in a stock's systematic risk seem to conform to a binomial distribution rather than exhibit a tendency to either rise or fall in response to an increase or a decrease in the investment horizon. In the conclusion we suggest an explanation for the fact that our empirical results do not support L-L's findings. The next section describes the sample. Section III discusses the methodology. The empirical findings are presented in section IV. The last section contains concluding remarks.

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II. The Sample

The data are from the Center for Research in Security Prices (CRSP). The sample consists of 828 common stocks traded on the NYSE and 287 common stocks traded on the American Stock Exchange (AMEX). A security is included in the sample only if it has 180 observations of monthly returns beginning in July 1962 and ending in June 1976. Over this interval of

15 years there are 3,950 securities listed on these two exchanges. The proxy for the general market movement is the return on a value-weighted index of all the securities listed during a particular month. The weight is equal to the total market value of an individual security divided by the total market value of all the securities generating that index.

III. Methodology

Following L-L, the single-period single-index market model of Sharpe is used to investigate the effect of increasing investment horizon (n) on systematic risk (β_j^n). From L-L (p. 101) we have

$$\hat{\beta}_j^n = \text{cov}[R_i^n, R_m^n]/\text{var}\,[R_m^n] \quad (1)$$

where R_i^n and R_m^n are, respectively, the returns of security i and the market portfolio in a given period for a given investment horizon. The estimate of systematic risk derived in (1) is labeled "arithmetic" beta. In order to investigate the effect of the definition of a security's returns we also use the following alternative formulation,

$$\hat{\beta}_j^n = \text{cov}[\ln(1 + R_i^n), \ln(1 + R_m^n)]/\text{var}[\ln(1 + R_m^n)] \quad (2)$$

and label it "logarithmic" beta.

In this comment we consider 10 investment horizons. The lengths of the investment horizons (in months), and their associated number of observations (in parentheses) are 1(180), 2(90), 3(60), 4(45), 5(36), 6(30), 9(20), 10(18), 12(15), and 15(12). In their paper L-L use 240 monthly returns from 1948 to 1968 to generate the estimates of systematic risk. Their longest horizon is of 30 months, i.e., 8 observations.

IV. The Empirical Findings

The securities in the sample are divided into two groups: those with one-month beta less than 1 (397 for the arithmetic and 405 for the logarithmic specifications), and those with one-month beta greater than 1 (718 and 710, respectively). In tables 1a and 1b we present the average of betas for these two groups as well as the full sample, for all the ten horizons.

For the defensive stocks, the average beta changes from 0.732 to 0.908 (0.735 to 0.879 for the logarithmic beta) as the investment horizon is increased from 1 to 15 months. However, a closer look at tables 1a and 1b indicates that this increase has not been uniform. For both specifications, of the 9 changes, 4 are negative and 5 are positive. For the aggressive stocks, the average beta changes from 1.418 to 1.542 (1.416 to 1.542 for the logarithmic beta) for the same change in the investment horizon. Again, this increase is not uniform. Of the 9 changes, 3 are negative and 6 positive. For the full sample of 1,115 stocks, the average beta increases from 1.174 to 1.315 (1.168 to 1.300 for the logarithmic beta), again, in a non-uniform fashion. Note that the average beta of the full sample does not equal to one for two main reasons. First, while 3,950 securities over the period 1962–1976 generate the market index, only 1,115 are analyzed here. Second, while the market index used is value-weighted, the beta averages reported in tables 1a and 1b are equally-weighted.

Because the averages in tables 1a and 1b present the aggregate behavior of the sample they may fail to convey the behavior of individual securities. For each security, we have 10 estimates of beta, 1 for each investment horizon. From these 10 successive betas, we can compute 9 changes in beta. According to L-L, for aggressive stocks we should observe a statistically significant number of positive changes and for defensive stocks a statistically significant number of negative changes.

Statistically, each security's beta can have 1 of the 10 possible combinations of positive (+) and negative (−) changes. These are (0+, 9−), (1+, 8−), ... , (8+, 1−), and (9+, 0−). If the changes in beta are randomly distributed, i.e., the probability of positive

TABLE 1A.—AVERAGE OF BETAS FOR DEFENSIVE AND AGGRESSIVE SECURITIES FOR VARIOUS INVESTMENT HORIZONS $\hat{\beta}_j^n = \text{cov}(R_i^n, R_m^n)/\text{var}(R_m^n)$

	$\hat{\beta}_i^1 \leqq 1$ 397 Securities		$\hat{\beta}_i^1 > 1$ 718 Securities		All $\hat{\beta}_i^1$ 1,115 Securities	
Horizon	Average	Change	Average	Change	Average	Change
1	.732	—	1.418	—	1.174	—
2	.824	.092	1.513	.095	1.268	.094
3	.813	−.011	1.613	.100	1.328	.060
4	.822	.009	1.645	.032	1.352	.024
5	.776	−.046	1.511	−.134	1.249	−.103
6	.859	.083	1.669	.158	1.380	.131
9	.849	−.010	1.427	−.242	1.220	−.160
10	.874	.025	1.440	.013	1.237	.017
12	.858	−.016	1.383	−.057	1.195	−.042
15	.908	.050	1.542	.159	1.315	.120

TABLE 1B.—AVERAGE OF BETAS FOR DEFENSIVE AND AGGRESSIVE SECURITIES FOR VARIOUS INVESTMENT HORIZONS
$\hat{\beta}_j^n = \text{cov}(\ln(1 + R_i^n), \ln(1 + R_m^n))/\text{var}(\ln(1 + R_m^n))$

	$\hat{\beta}_i^1 \leqq 1$ 405 Securities		$\hat{\beta}_i^1 > 1$ 710 Securities		All $\hat{\beta}_i^1$ 1,115 Securities	
Horizon	Average	Change	Average	Change	Average	Change
1	.735	—	1.416	—	1.168	—
2	.818	.083	1.498	.082	1.251	.083
3	.791	−.027	1.503	.005	1.244	−.007
4	.820	.029	1.573	.070	1.299	.055
5	.785	−.035	1.502	−.071	1.241	−.058
6	.847	.062	1.580	.078	1.313	.072
9	.842	−.005	1.370	−.210	1.177	−.136
10	.872	.030	1.482	.112	1.259	.082
12	.838	−.034	1.407	−.075	1.199	−.060
15	.879	.041	1.542	.135	1.300	.101

changes and the probability of negative changes each equals ½, then the proportion of securities having these 10 possible combinations will conform to a particular binomial distribution. These theoretical proportions are shown in the last columns of tables 2a, 2b, and 2c.

In table 2a we present the numbers and proportions of securities' betas having each of the 10 possible combinations of changes based on the arithmetic betas. Of the 397 defensive securities, 29.22% are in the group (4+, 5−), and 31.99% are in the group (5+, 4−). Contrary to L-L's findings only 12.09% of the betas have 6 or more negative changes. Overall, these values indicate a very strong concentration in the middle groups implying a random-walk in betas with respect to an increasing investment horizon. Of the 718 aggressive securities, contrary to L-L's findings, only 22.71% have 6 or more positive changes in betas. For the aggressive securities, the proportions in different groups are not significantly different from those of the defensive securities.

While in table 2a the results are based on the arithmetic betas (i.e., multiplicative returns), in table 2b we present the results based on the logarithmic betas (i.e., additive returns). There is no significant difference between the two sets of results and hence the definition of a security's return has no significant effect on the temporal behavior of its estimated beta coefficient.[1] In both tables, as can be predicted from the upward drift in average betas shown in tables 1a and 1b, there is a slight bias in favor of positive changes for the defensive as well as the aggressive securities.

As we pointed out earlier, if we had been able to use the complete sample of 3,950 securities and derive a value-weighted average of the betas, *or* if we had used an equally-weighted index of only 1,115 securities, then the grand-average of betas would have been equal to 1 for all investment horizons. In this case the upward drift from 1.174 to 1.315 (table 1a) and from 1.168 to 1.300 (table 1b) would not exist.[2]

In table 2c we present the behavior of the changes in betas after the effect of the upward drift has been neutralized by subtracting from each security's beta

[1] Note that for small returns relative to one ($R < .10$), log $(1 + R)$ is approximately equal to R.

[2] This drift must also be present in the L-L data since they used the Fisher Arithmetic Index as a proxy for the market portfolio.

TABLE 2A.—CONFIGURATION OF POSITIVE AND NEGATIVE CHANGES IN BETAS OF DEFENSIVE AND AGGRESSIVE SECURITIES ("arithmetic-beta")

Changes		$\hat{\beta}_i^1 \leqq 1$ 397 Securities		$\hat{\beta}_i^1 > 1$ 718 Securities		All $\hat{\beta}_i^1$ 1,115 Securities		Random-Walk
+	−	Number	Proportion	Number	Proportion	Number	Proportion	Proportion
0	9	0	—	0	—	0	—	.0020
1	8	2	.0050	2	.0028	4	.0036	.0176
2	7	5	.0126	21	.0293	26	.0233	.0703
3	6	41	.1033	92	.1281	133	.1193	.1640
4	5	116	.2922	211	.2939	327	.2933	.2461
5	4	127	.3199	229	.3189	356	.3193	.2461
6	3	92	.2317	118	.1644	210	.1883	.1640
7	2	14	.0353	43	.0599	57	.0511	.0703
8	1	0	—	2	.0028	2	.0018	.0176
9	0	0	—	0	—	0	—	.0020

TABLE 2B.—CONFIGURATION OF POSITIVE AND NEGATIVE CHANGES IN BETAS OF DEFENSIVE AND AGGRESSIVE SECURITIES ("logarithmic-beta")

Changes		$\hat{\beta}_i^1 \leqq 1$ 405 Securities		$\hat{\beta}_i^1 > 1$ 710 Securities		All $\hat{\beta}_i^1$ 1,115 Securities		Random-Walk
+	−	Number	Proportion	Number	Proportion	Number	Proportion	Proportion
0	9	0	—	1	.0014	1	.0009	.0020
1	8	0	—	3	.0042	3	.0027	.0176
2	7	10	.0247	15	.0211	25	.0224	.0703
3	6	42	.1037	95	.1338	137	.1229	.1640
4	5	103	.2543	183	.2578	286	.2565	.2461
5	4	141	.3482	232	.3268	373	.3345	.2461
6	3	75	.1852	137	.1930	212	.1901	.1640
7	2	27	.0667	40	.0563	67	.0601	.0703
8	1	7	.0173	4	.0056	11	.0099	.0176
9	0	0	—	0	—	0	—	.0020

TABLE 2C.—CONFIGURATION OF POSITIVE AND NEGATIVE CHANGES IN BETAS OF DEFENSIVE AND AGGRESSIVE SECURITIES AFTER ADJUSTING FOR THE SAMPLE DRIFT ("arithmetic-beta")

Changes		$\hat{\beta}_i^1 \leqq 1$ 397 Securities		$\hat{\beta}_i^1 > 1$ 718 Securities		All $\hat{\beta}_i^1$ 1,115 Securities		Random-Walk
+		Number	Proportion	Number	Proportion	Number	Proportion	Proportion
0	9	0	—	0	—	0	—	.0020
1	8	3	.0076	5	.0070	8	.0072	.0176
2	7	13	.0328	38	.0529	51	.0457	.0703
3	6	73	.1839	122	.1699	195	.1749	.1640
4	5	109	.2746	224	.3120	333	.2987	.2461
5	4	110	.2771	210	.2925	320	.2870	.2461
6	3	69	.1738	95	.1323	164	.1471	.1640
7	2	18	.0453	23	.0320	41	.0368	.0703
8	1	2	.0050	1	.0014	3	.0027	.0176
9	0	0	—	0	—	0	—	.0020

the drift of the grand-average. This adjustment, common for both the defensive and aggressive securities, makes the distributions of changes in betas symmetric for both groups. Yet, the proportions for the two middle groups, (4+, 5−) and (5+, 4−), are much higher than what would be indicated by pure randomness. This implies that, not only is there *no* systematic upward or downward drift for a security's betas with respect to changes in the investment horizon, but these betas are less likely to wander away from their central value than what even pure randomness would suggest.

V. Conclusion

In this comment we have re-examined L-L's proposition regarding the effect of changes in the investment horizon on the value of a stock's systematic risk. Our empirical findings based on a sample of 1,115 securities do not support L-L's conclusions. The changes in the beta coefficient of a stock induced by changes in the investment horizon seem to follow a random walk rather than display the systematic increase or decrease predicted by L-L. This phenomenon may be explained by the fact that L-L have assumed in their model that securities' returns are independently distributed. There is evidence, however, that stocks' returns are intertemporally cross-correlated and that market indexes exhibit significant positive autocorrelation.[3] The presence of these correlations may affect the temporal behavior of systematic risk differently than predicted by L-L and possibly produce the reported random-walk behavior.[4]

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[3] In this respect see the recent contribution of Cohen et al. (1980).

[4] See propositions 5 and 6 in Cohen et al. (1980).