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# An Inverted-U Relationship between Product Market Competition and Growth in an Extended Romerian Model : A Comment\*

July 13, 2007

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**Abstract :** *This paper shows that the results of Bucci (2005) depend critically on the assumption that there are no difference between the intermediate goods share in final output, the returns of specialization and the degree of market power of monopolistic competitors. In this paper, we disentangle the market power parameter from the intermediate goods share in final output and the returns to specialization. The main result of this paper is the death of the inverted-U shape relationship between competition and growth. Indeed, we find a decreasing relationship between competition and growth which is due to the composition of two negative effects on growth : resource allocation and Schumpeterian effects.*

**Keywords :** Endogenous growth, Horizontal differentiation, Technological change, Imperfect competition

**JEL Classification :** 031, 041

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# 1 Introduction

Bucci (2005) studies the impact of competition in the intermediate goods sector on growth. He uses the Gancia and Zilibotti (2005) model in which he introduces a different assumption concerning the production of intermediate goods. Indeed, unlike Gancia and Zilibotti (2005) which assumes that one need one unit of final good to produce one unit of intermediate good, Bucci (2005) does the hypothesis that the firm has to use one unit of labor. This assumption which is called "resource allocation effect" implies that labor can be allocated between three sectors : final good, intermediate goods and research. The interplay between this effect and the traditional Schumpeterian effect allows to obtain an interesting result. Indeed, Bucci (2005) finds an inverted-U relationship between competition and growth. For low value of competition, more competition is beneficial to growth since it allows a better allocation of resource without hampering that much innovations incentives. In this case, the resource allocation effect is bigger than the profit incentive effect. On the other hand, for high value of competition, more competition reduces strongly growth because of the reduction of profit. In this case, the profit incentive effect is bigger than the resource allocation effect.

Among the assumptions used by Bucci (2005) to derive this result is that there are no difference between the intermediate goods share in final output, the returns to specialization and the degree of market power of monopolistic competitors. This leads to the natural question whether making such a difference to the model changes its predictions. In this note, we show that including this difference into the model developed by Bucci (2005) eliminates the result mentioned above.

## 2 The model

The model developed is based on Bucci (2005).<sup>2</sup> The economy is structured by three sectors : final good sector, intermediate goods sector and R&D sector. The final output sector produces output that can be used for consumption using labor and intermediate goods. These are available in  $A$  varieties and are produced by employing only labor. The R&D sector creates the blueprints for new varieties of intermediate goods which are produced by employing labor and knowledge. These blueprints are sold to the intermediate goods sector.

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<sup>2</sup>We use the notations of Bucci (2005) in order to have a direct comparison with his model.

## 2.1 The final goods sector

In this sector atomistic producers engage in perfect competition. The final goods sector produces a composite good  $Y$  by using all the  $j$ th type of intermediate goods  $x_j$  and labor  $L_Y$ .<sup>3</sup> Production is given by :

$$Y = N^{\gamma-\lambda(1/\alpha-1)} \left[ \int_0^N x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}} L_Y^{1-\lambda}, \quad (1)$$

where  $\alpha \in [0, 1]$ ,  $\lambda \in [0, 1]$ ,  $\gamma \in ]0, 1[$  are three parameters. This production function allow us to disentangle the degree of market power of monopolistic competitors in the intermediate sector ( $\frac{1}{\alpha} - 1$ ), the intermediate goods share in final output ( $\lambda$ ) and the degree of returns from specialization ( $\gamma$ ).<sup>4</sup> In this sense, this model is a generalization of Bucci (2005) and Benassy (1998) models.<sup>5</sup> If we normalize to one the price of the final goods, the profit of the representative firm is given by :

$$\pi_Y = N^{\gamma-\lambda(1/\alpha-1)} \left[ \int_0^N x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}} L_Y^{1-\lambda} - \int_0^N p_j x_j dj - w_Y L_Y, \quad (2)$$

where  $w_Y$  is the wage rate in the final good sector and  $p_j$  is the price of the  $j$ th intermediate good. Under perfect competition in the final output market and the factor inputs markets, the representative firm chooses intermediate goods and labor in order to maximize its profit taking prices as given and subject to its technological constraint. The first order conditions are the followings :

$$\frac{\partial \pi_Y}{\partial x_j} = \lambda N^{\gamma-\frac{\lambda}{\alpha}+\lambda} \left[ \int_0^N x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}-1} L_Y^{1-\lambda} x_j^{\alpha-1} - p_j = 0, \quad (3)$$

$$\frac{\partial \pi_Y}{\partial L_Y} = (1 - \lambda) N^{\gamma-\frac{\lambda}{\alpha}+\lambda} \left[ \int_0^N x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}} L_Y^{-\lambda} - w_Y = 0. \quad (4)$$

Equation (3) is the inverse demand function for the firm that produces the  $j$ th intermediate good whereas equation (4) characterizes the demand function of labor.

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<sup>3</sup>Time subscripts are omitted whenever there is no risk of ambiguity.

<sup>4</sup>Benassy (1996) made a simple modification to the Dixit and Stiglitz (1977) model which clearly disentangles taste for variety and market power. At the same time, Benassy (1998) and de Groot and Nahuis (1998) show that the introduction of this modification in an endogenous growth model with expanding product variety *à la* Grossman and Helpman (1991) affects the welfare analysis.

<sup>5</sup>Indeed, we obtain the Bucci (2005) model by introducing the following constraints  $\lambda = \alpha$ ,  $\gamma = 1 - \alpha$  in our model. In the same way, by introducing the constraint  $\lambda = 1$ , we obtain the Benassy (1998) model.

## 2.2 The intermediate goods sector

In the intermediate goods sector, producers engage in monopolistic competition. Each firm produces one horizontally differentiated intermediate good and have to buy a patented design before producing it. Following Grossman and Helpman (1991) and Bucci (2005), we assume that each local intermediate monopolist has access to the same technology employing only labor  $l_j$  :

$$x_j = l_j. \quad (5)$$

We suppose that firms behavior which produce intermediate goods is governed by the principle of profit maximization at given factor prices under a technological constraint. The profit function of firms is the following :

$$\pi_j = p_j x_j - w_j l_j, \quad (6)$$

where  $w_j$  is wage rate in the intermediate goods. Using the first order condition, we obtain the price of the  $j$ th intermediate good :

$$p_j = \frac{w_j}{\alpha}, \quad (7)$$

At the symmetric equilibrium, all the firms produce the same quantity of the intermediate good  $x$ , face the same wage rate  $w$  and by consequence fix the same price for their production  $p$ . The price is equal to a constant mark up  $\frac{1}{\alpha}$  over the marginal cost  $w$ . Defining by  $L_j = \int_0^N l_j dj$ , the total amount of labor employed in the intermediate goods sector and under the assumption of symmetry among intermediate goods producers, we can rewrite the equation (5) as follows :

$$x_j = \frac{L_j}{N}, \quad (8)$$

Finally, the profit function of the firm which produces the  $j$ th intermediate good is

$$\pi_j = \lambda(1 - \alpha)N^{\gamma-1}L_j^\lambda L_y^{1-\lambda}. \quad (9)$$

## 2.3 The R&D sector

There are competitive research firms undertaking R&D. Following Romer (1990) and Grossman and Helpman (1991), we assume that new blueprints are produced using old blueprints  $N$  and an amount of R&D labor  $L_N$  :

$$\frac{\partial N}{\partial t} = \frac{1}{\eta}NL_N, \quad (10)$$

where  $\frac{1}{\eta} > 0$  represents the productivity of the R&D process. Because of the perfect competition in the R&D sector, we can obtain the real wage in this sector as a function of the profit flows associated to the latest intermediate in using the zero profit condition :

$$w_N L_N = \frac{\partial N}{\partial t} P_N, \quad (11)$$

where  $w_N$  represents the real wage earned by R&D labor.  $P_N$  is the real value of such a blueprint which is equal to :

$$P_N = \int_t^\infty \pi_j e^{-r(\tau-t)} d\tau, \tau > t, \quad (12)$$

where  $r$  is the real interest rate. Given  $P_N$ , the free entry conditions leads to :

$$w_N = \frac{N P_N}{\eta}. \quad (13)$$

## 2.4 The consumer behavior

The demand side is characterized by the representative household who consumes and supplies labor. Following Grossman and Helpman (1991), we assume that the utility function of this consumer is logarithmic<sup>6</sup> :

$$U = \int_0^\infty e^{-\rho t} \log(C) dt, \quad (14)$$

where  $C$  is private consumption,  $\rho > 0$  is the rate of pure time preference. The representative household is endowed with fixed quantities of labor  $L$  that are supplied inelastically. The flow budget constraint for the household is :

$$\frac{\partial W}{\partial t} = wL + rW - C, \quad (15)$$

where  $W$  is the total wealth of the agent (measured in units of final good),  $w$  is the wage rate per unit of labor services. From the maximization program of the consumer, the necessary and sufficient conditions for a solution are given by the Keynes-Ramsey rule :

$$g_C = r - \rho, \quad (16)$$

and the transversality condition :

$$\lim_{t \rightarrow \infty} \mu_t W_t = 0, \quad (17)$$

where  $\mu_t$  is the co-state variable.

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<sup>6</sup>This specification of the utility function does not alter the results.

### 3 The equilibrium and the steady state

In this section, we characterize the equilibrium and give some analytical characterization of a balanced growth path.

#### 3.1 The equilibrium

It is now possible to characterize the labor market equilibrium in the economy considered. On this market, because of the homogeneity and the perfect mobility across sectors, the arbitrage ensures that the wage rate that is earned by salaries which work in the final good sector, intermediate goods sector or R&D sector is equal. As a result, the following three conditions must simultaneously be satisfied<sup>7</sup> :

$$L = L_Y + L_j + L_N, \quad (18)$$

$$w_j = w_y, \quad (19)$$

$$w_j = w_N. \quad (20)$$

Equation (18) is a resource constraint, saying that at any point in the time the sum of the labor demands coming from each activity must be equal to the total available fixed supply. Equation (19) and equation (20) state that the wage earned by one unit of labor is to be the same irrespective of the sector where that unit of labor is actually employed. We can characterize the product market equilibrium in the economy considered. Indeed, on this market, the firms produce a final good which can be consumed. Consequently, the following condition must be satisfied :

$$Y = C. \quad (21)$$

Equation (21) is a resource constraint on the final goods sector.

#### 3.2 The steady state

At the steady state, all variables as  $Y$ ,  $C$ ,  $N$  grow at a positive constant rate. Obviously, it is easy to show from equations (21, 1 and 8) the following relationship between the economic growth rate, consumption growth rate and knowledge growth rate :

$$g_Y = g_C = \gamma g_N. \quad (22)$$

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<sup>7</sup>We assume without loss of commonalty that the total labor force is constant.

Using the previous equations, we can demonstrate the following steady state equilibrium values for the relevant variables of the model<sup>89</sup> :

$$r = \frac{L(1 - \alpha)\gamma\lambda - \eta((\alpha - 1)\lambda\gamma + \gamma - 1)\rho}{\eta}, \quad (23)$$

$$L_j = \alpha\lambda(L + \eta\rho), \quad (24)$$

$$L_Y = (1 - \lambda)(L + \eta\rho), \quad (25)$$

$$L_N = (1 - \alpha)\lambda(L + \eta\rho) - \eta\rho, \quad (26)$$

$$g_Y = \frac{\gamma((1 - \alpha)\lambda(L + \eta\rho) - \eta\rho)}{\eta}. \quad (27)$$

## 4 The relationship between product market competition and growth

In this section, we study the long run relationship between competition and growth in the model presented above. Following most authors, we use the so-called Lerner Index to gauge the intensity of market power within a market. Such an index is defined by the ratio of price ( $P$ ) minus marginal cost ( $CM$ ) over price. Using the definition of a mark up ( $Markup = \frac{P}{CM}$ ) and Lerner Index ( $LernerIndex = \frac{P - CM}{P}$ ), we can use (7) to define a proxy of competition as follows<sup>10</sup> :

$$(1 - LernerIndex) = \alpha, \quad (28)$$

We show that our simple generalization of Bucci (2005)'s model that consists in having the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization treated separately, the inverted U relationship between competition and growth no longer exists.

**Proposition 1** *The relationship between competition and growth is negative for all positive values of  $\rho$ ,  $\eta$ ,  $L$ ,  $\gamma \in ]0, 1[$  and  $\lambda \in [0, 1]$ .*

**Proof.** *The proof is obtained by differentiated (27) with respect to  $\alpha$  :*

$$\frac{\partial g_Y}{\partial \alpha} = -\frac{\gamma\lambda(L + \eta\rho)}{\eta} < 0. \quad (29)$$

<sup>8</sup>Results (23) through (27) are demonstrated in the appendix.

<sup>9</sup>In order to have a positive growth rate, we assume that  $0 < \eta < \frac{L\lambda - L\alpha\lambda}{\alpha\lambda\rho - \lambda\rho + \rho}$ .

<sup>10</sup>This is the same measure of product market competition used by Aghion, Bloom, Blundell, Griffith, and Howitt (2005), Aghion and Griffith (2005) and Aghion and Howitt (2005), contrary to Bucci and Parello (2006) which links the competition to two components : the input shares in income and the parameter of substitution between intermediates.



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In order to illustrate this result, we plot equation (27) for different values of competition ( $\alpha$ ), and returns to specialization ( $\gamma$ )<sup>11</sup> :

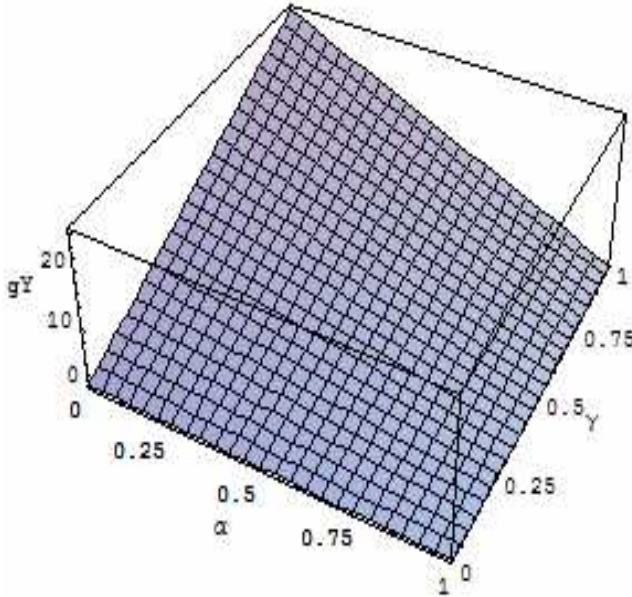


Figure 1: Relationship between competition ( $\alpha$ ), returns to specialization ( $\gamma$ ) and growth ( $g_Y$ )

According to the profit incentive effect, an increase of competition ( $\alpha$ ) reduces the price of the intermediate good and profits, what determines the incentives to innovation. Therefore, the profit incentive effect seems to predict an unambiguously negative relationship between product market competition and growth along the entire range of competition intensity. Contrary to Bucci (2005), an increase of competition reduces the amount of labor devoted to the research sector ( $L_N$ ) along the entire range of competition intensity. Moreover an increase of competition has no effect on the amount of labor allocated to the final goods sector ( $L_Y$ ) and increases the amount of labor in the intermediate goods sector ( $L_j$ ). This means that the resource allocation effect seems also to predict an unambiguously negative relationship between

<sup>11</sup>In drawing Figure 1, we take the same values of parameters as Bucci (2005) in order to be as close as possible to his model :  $\rho = 0.03$ ,  $\eta = 1$ ,  $L = 35$  and  $\lambda = 0.75$ .

product market competition and growth. Finally, we always have as we can see on the above picture a decreasing relationship between competition and growth.

## 5 Conclusion

In this paper, we presented a generalization of production function of Bucci (2005) which disentangles the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization. Our main finding is that the result of his model that close in an inverted U relationship between competition and growth depends critically on the assumptions that there are no differences between these three parameters. Indeed, for all values of parameters except to  $\lambda = \alpha$ , we could remove the inverted-U relationship between competition and growth. This result is due to the interplay of two effects : Schumpeterian and resource allocation effects. In our model, we find that the resource allocation effect is always negative which reinforces the Schumpeterian effect on growth. Consequently, we find a decreasing relationship between competition and growth.

## Appendix

In these appendix, we describe the way followed in order to obtain the main results of this paper (23 through 27). Using the equations (3, 4, 7, 8 and 19), we obtain :

$$L_Y = \frac{(1 - \lambda)L_j}{\alpha\lambda}. \quad (30)$$

Plugging this equation into equation (18) yields :

$$L_j = \frac{\alpha\lambda(L - L_N)}{1 + (\alpha - 1)\lambda}. \quad (31)$$

Consequently, the equation (30) can be re-written as :

$$L_Y = \frac{(1 - \lambda)(L - L_N)}{1 + (\alpha - 1)\lambda}. \quad (32)$$

In order to compute the wage in the research sector, we need to have the value of the blueprint in the steady state (equations 9 and 12) which is :

$$P_N = \frac{(1 - \alpha)\lambda L_Y^{1-\lambda} L_j^\lambda N^{\gamma-1}}{\frac{(1-\gamma)L_N}{\eta} + r}. \quad (33)$$

Given  $P_N$  and using equation (13), we obtain :

$$w_N = \frac{N^\gamma L_j^\lambda L_Y^{1-\lambda} (1-\alpha)\lambda}{L_N(1-\gamma) + r\eta}. \quad (34)$$

In equating  $w_j$  to  $w_N$ , we find :

$$L_j = \frac{\alpha(L_N(1-\gamma) + r\eta)}{1-\alpha}. \quad (35)$$

Using the equations (31 and 35), we obtain :

$$L_N = \frac{L(1-\alpha)\lambda - r\eta(1 + (\alpha-1)\lambda)}{1-\gamma(1 + (\alpha-1)\lambda)}. \quad (36)$$

Consequently :

$$L_j = \frac{\alpha(L(1-\gamma) + r\eta)\lambda}{1-\gamma + (1-\alpha)\lambda\gamma}. \quad (37)$$

In order to determine the equilibrium interest rate, we use these two equations :

$$g_Y = g_C = r - \rho, \quad (38)$$

$$g_Y = \gamma g_N, \quad (39)$$

After some computations, we obtain :

$$r = \frac{L(1-\alpha)\gamma\lambda - \eta((\alpha-1)\lambda\gamma + \gamma - 1)\rho}{\eta}. \quad (40)$$

Finally, from the previous equations, we obtain all the other variables of the model :

$$L_j = \alpha\lambda(L + \eta\rho), \quad (41)$$

$$L_Y = (1-\lambda)(L + \eta\rho), \quad (42)$$

$$L_N = \eta\rho + (1-\alpha)\lambda(L + \eta\rho), \quad (43)$$

$$g_Y = \frac{\gamma((1-\alpha)\lambda(L + \eta\rho) - \eta\rho)}{\eta}. \quad (44)$$

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